Optimization

Lecture 1: Introduction & Fundamental Concepts of Optimization

Prof. dr. ir. Toon van Waterschoot

Faculty of Engineering Science
ESAT – Department of Electrical Engineering
KU Leuven, Belgium

Research Division

- **STADIUS** Center for Dynamical Systems, Signal Processing and Data Analytics:
  - Dynamical Systems:
    - identification
    - optimization
    - systems & control
  - Signal Processing:
    - speech & audio processing
    - digital communications
    - biomedical signal processing
  - Data Analysis:
    - machine learning
    - bio-informatics

STADIUS
Center for Dynamical Systems, Signal Processing and Data Analytics

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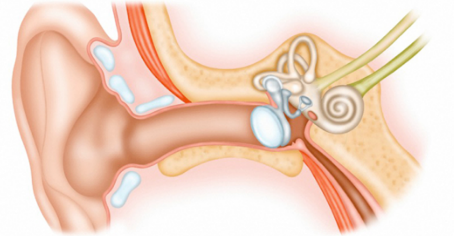
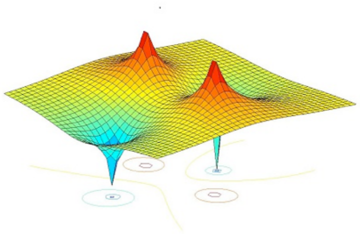
Research Disciplines

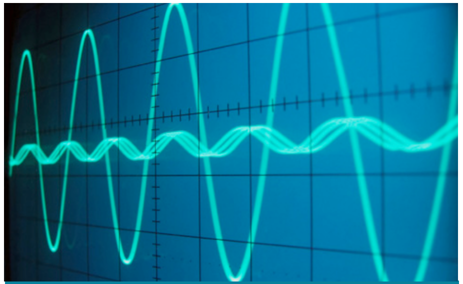
Room Acoustics



Psycho-acoustics

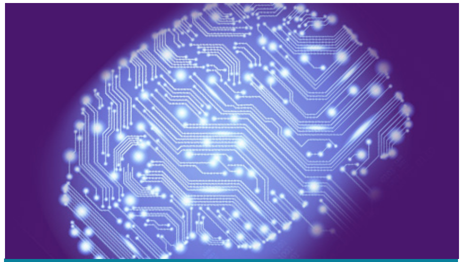


Numerical Optimization



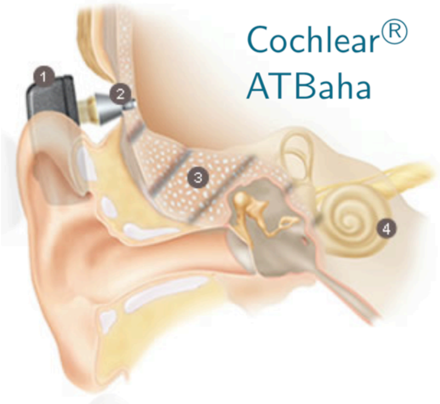
Signal processing

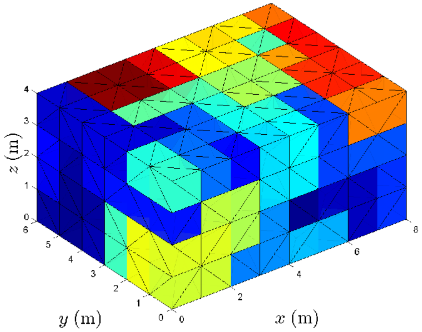
Audio Research



Machine Learning

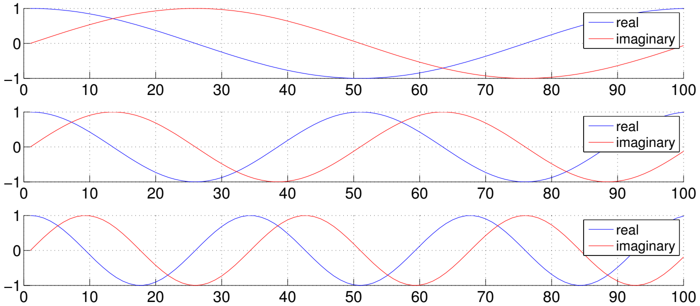
Research Topics





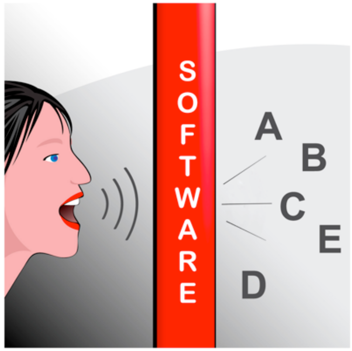
Acoustic modeling

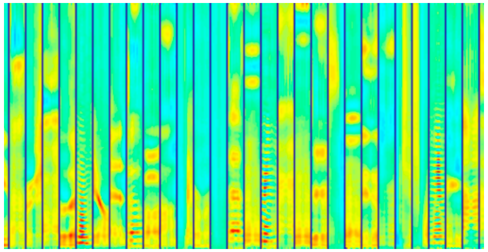
- ear modeling
- room modeling
- loudspeaker modeling
- signal modeling

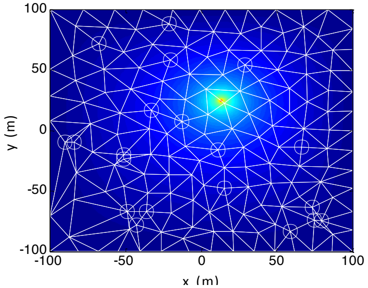


Audio signal analysis

- speech recognition
- event detection
- source localization
- audio classification







Acoustic signal enhancement

- noise reduction
- echo/feedback control
- room equalization

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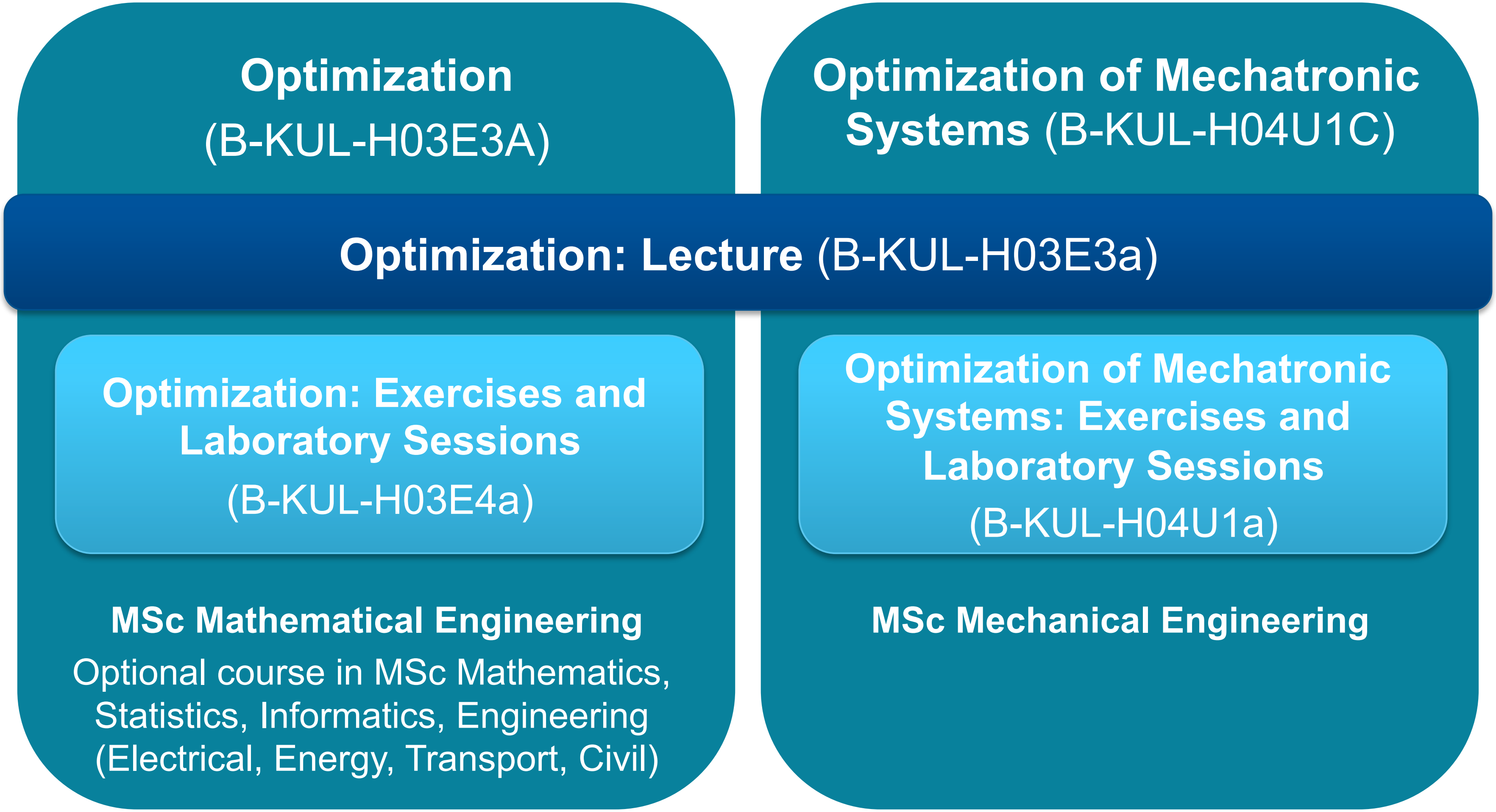
Contact

Toon van Waterschoot

- Mail: toon.vanwaterschoot@esat.kuleuven.be
- Office: Department of Electrical Engineering (ESAT), room 01.91
- Phone: +32 16 321788

Course Structure



Optimization
(B-KUL-H03E3A)

Optimization of Mechatronic Systems (B-KUL-H04U1C)

Optimization: Lecture (B-KUL-H03E3a)

Optimization: Exercises and Laboratory Sessions
(B-KUL-H03E4a)

Optimization of Mechatronic Systems: Exercises and Laboratory Sessions
(B-KUL-H04U1a)

MSc Mathematical Engineering

Optional course in MSc Mathematics, Statistics, Informatics, Engineering (Electrical, Energy, Transport, Civil)

MSc Mechanical Engineering

Optimization: Lecture

- **Study load: 4 ECTS**
- **Schedule**
  - 15 lectures of 2 hours each
  - Tuesdays, 14:00 – 16:00
    - weekly until 02/12
    - MTM 00.13 (23/9) or MOLE 00.07 (Aud. De Molen)
  - Wednesdays, 10:30 – 12:30
    - weekly until 29/10
    - ESAT 00.54 (Aud. A)
  - *note:* online schedule has 17 lectures
    - no lecture on 24/9! (will be announced on Toledo)
    - no lecture on x/x! (will be announced on Toledo)

Optimization: Lecture

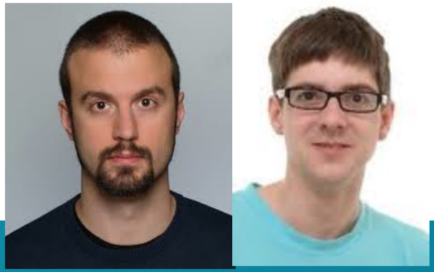
- **Lecture 1:** Introduction & Fundamental Concepts of Optimization
- **Lecture 2:** Types of Optimization Problems
- **Lecture 3:** Convex Optimization
- **Lecture 4:** The Lagrangian Function and Duality
- **Lecture 5:** Optimality Conditions for Unconstrained Optimization
- **Lecture 6:** Estimation and Fitting Problems
- **Lecture 7:** Newton Type Optimization
- **Lecture 8:** Globalisation Strategies
- **Lecture 9:** Calculating Derivatives
- **Lecture 10:** Optimality Conditions for Constrained Optimization
- **Lecture 11:** Equality Constrained Optimization Algorithms
- **Lecture 12:** Inequality Constrained Optimization Algorithms
- **Lecture 13:** Optimal Control Problems
- **Lecture 14:** Summary

Optimization: Lecture

- **Course materials**
  - syllabus “Numerical Optimization” by Moritz Diehl
    - printed version available at VTK
    - PDF version available on Toledo
  - textbook “Numerical Optimization” by Nocedal & Wright
    - printed version available at VTK
    - PDF version available at SpringerLink (http://link.springer.com)
    - important chapters & sections: see App. B.1 of syllabus
  - textbook “Convex Optimization” by Boyd & Vandenberghe
    - PDF version available at http://stanford.edu/~boyd/cvxbook/
    - important chapters & sections: see App. B.1 of syllabus
  - slides for Lectures 1 & 2
    - PPT & PDF version available on Toledo
  - optional: lecture videos & research papers (Toledo)

Exercises and Laboratory Sessions

- **Master of Mathematical Engineering**
  - study load: 2 ECTS
  - 8 sessions of 2.5 hours each, starting at 14/10
  - schedule: http://people.cs.kuleuven.be/~btw/roosters.html
  - session 1 – 6: guided exercises
  - session 7 – 8: support for individual project work
  - individual project:
    - individually or in groups of two
    - topic from list or own topic
    - 5-page report + Matlab software to be delivered by 19/12
    - oral part of exam = project discussion
  - lecturers: Milan Vukov & Joris Gillis
  - location: ESAT PC rooms (see schedule)



Exercises and Laboratory Sessions

- **Master of Mathematical Engineering**
  - **Session 1:** Fitting problems
  - **Session 2:** Hanging chain
  - **Session 3:** Steepest descent & Newton
  - **Session 4:** Gauss-Newton
  - **Session 5:** SQP
  - **Session 6:** Optimal control

  - exercise assignments available on Toledo page for "Optimization: Exercises and Laboratory Sessions"
  - solutions available on Toledo by mid December

Exercises and Laboratory Sessions

- **Master of Mechanical Engineering**
  - study load: 2 ECTS
  - 13 sessions of 2.5 hours each, starting at 7/10
  - schedule: KU Leuven programmes website
  - session 1 – 6: guided exercises
  - session 7 – 13: case studies with individual assignment
  - coordination: Joris De Schutter & Goele Pipeleers
  - location: MECH PC rooms (see schedule)

Exercises and Laboratory Sessions

- **Master of Mechanical Engineering**
  - **Session 1:** Fitting problems
  - **Session 2:** Hanging chain
  - **Session 3:** Steepest descent & Newton
  - **Session 4:** Gauss-Newton
  - **Session 5:** SQP
  - **Session 6:** Optimal control
  - **Session 7 – 10:** Case studies "Optimal motion trajectories"
  - **Session 11 – 13:** Case studies "Optimal balancing of linkages"
  - exercise assignments available on Toledo page for "Optimization of Mechatronic Systems: Exercises and Laboratory Sessions"

Evaluation

- **Master of Mathematical Engineering**
  - written part: theory + exercises
  - oral part: individual project discussion (T. van Waterschoot)
  - weights: written part (2/3), oral part (1/3)
- **Master of Mechanical Engineering**
  - written part: theory + exercises
  - oral part: case studies (J. De Schutter, G. Pipeleers)
  - weights: written part (2/3), oral part (1/3)

  - list of rehearsal questions (+ answers) included in syllabus (App. B.2)

Optimization: Lecture

- **Lecture 1:** Introduction & Fundamental Concepts of Optimization
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Lecture 1: Introduction & Fundamental Concepts of Optimization

- **Introduction**
  - motivation
  - research examples

- **Fundamental Concepts of Optimization**
  - fundamental concepts: variables, objective function, …
  - research examples revisited

Motivation: Engineering = Optimization

- Computer simulation nowadays ubiquitous in design of cars, aircraft, reactors, ships, …
- Engineer usually “plays” with input parameters (sizes, lengths, …) until satisfied by results
- Question: Can’t computer directly OPTIMIZE?
- New paradigm: engineer plays NOT with input parameters, but with objective, constraints, …
- Need reliable optimization methods (course topic)

OPTEC - Optimization in Engineering Center

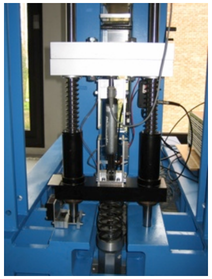
***Center of Excellence of KU Leuven**, since 2005*

*70 people, working jointly on **methods and applications of optimization,***

in 5 departments:

- Electrical Engineering
- Mechanical Engineering
- Chemical Engineering
- Computer Science
- Civil Engineering

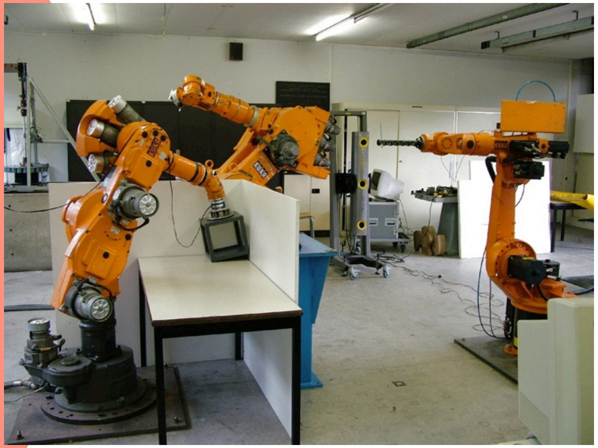
Many real world applications at OPTEC...



OPTEC Aim: Connect Optimization Methods & Applications

Applications: Smart problem formulations allow efficient solution (e.g. convexity)

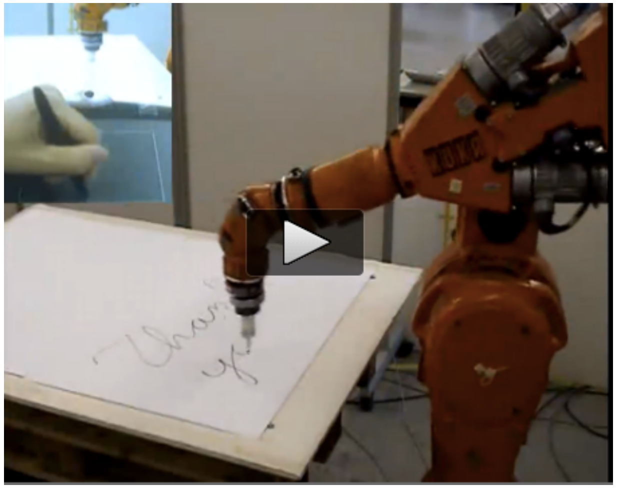
$$
\begin{aligned}
\min_{\substack{u_0,\ldots,u_{N-1}\\ x_1,\ldots,x_N}} \quad & x_N^T P x_N + \sum_{i=0}^{N-1} \left(x_i^T Q x_i + u_i^T R u_i \right) \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k \,, \\
& (x_0 \text{ given}) \,, \\
& \underline{c} \leq C x_k \leq \overline{c} \,, \\
& \underline{d} \leq D u_k \leq \overline{d} \,, \\
& c_T \leq C_T x_N \,,
\end{aligned}
$$



Methods: New developments are inspired and driven by application needs

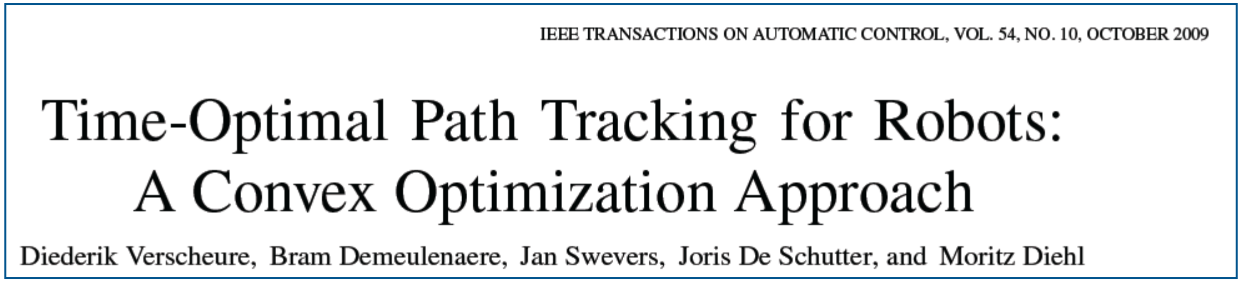
OPTEC Research Example: Time Optimal Robot Motion

- **Objective:**
  - follow given writing trajectory as close as possible
  - while maximizing the speed of writing
- **Solution:**
  - convex reformulation
  - global solution found in 2 ms



PhD work of Diederik Verscheure

https://www.mech.kuleuven.be/en/pma/research/robotics/media#section-1



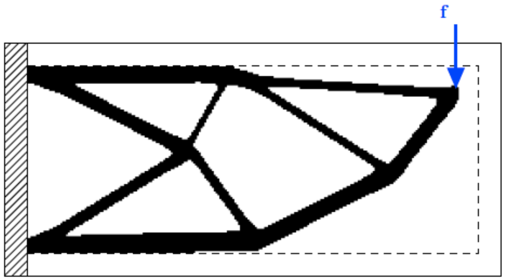
IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 54, NO. 10, OCTOBER 2009

Time-Optimal Path Tracking for Robots:
A Convex Optimization Approach

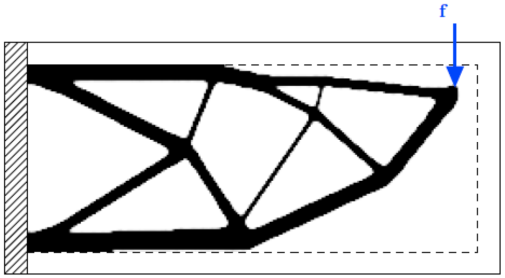
Diederik Verscheure, Bram Demeulenaere, Jan Swevers, Joris De Schutter, and Moritz Diehl

OPTEC Research Example: Topology Optimization

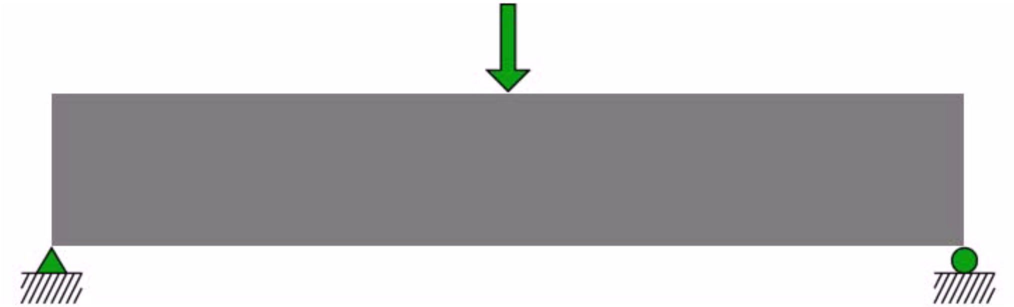
- **Objective:**
  - minimize amount of material used
  - while keeping sufficiently high stiffness
  - and reducing sensitivity to geometric imperfection



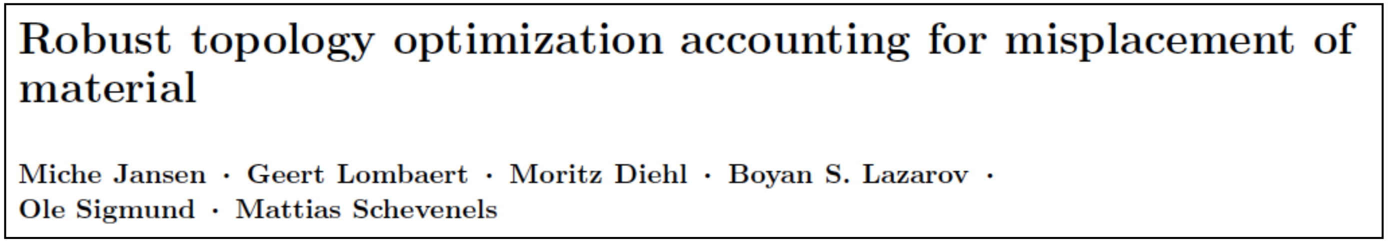
Deterministic optimized design



Robust optimized design



PhD work of Miche Jansen



Robust topology optimization accounting for misplacement of material

Miche Jansen · Geert Lombaert · Moritz Diehl · Boyan S. Lazarov · Ole Sigmund · Mattias Schevenels

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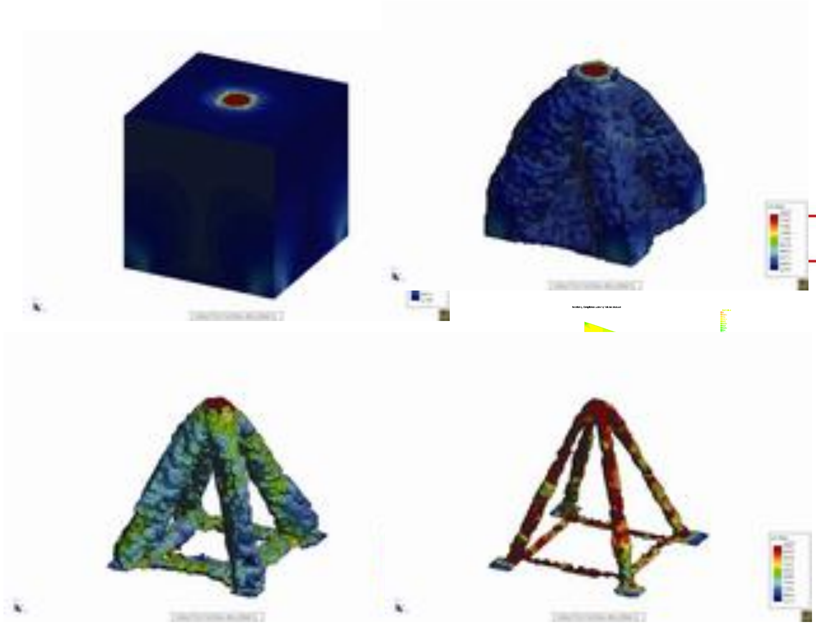
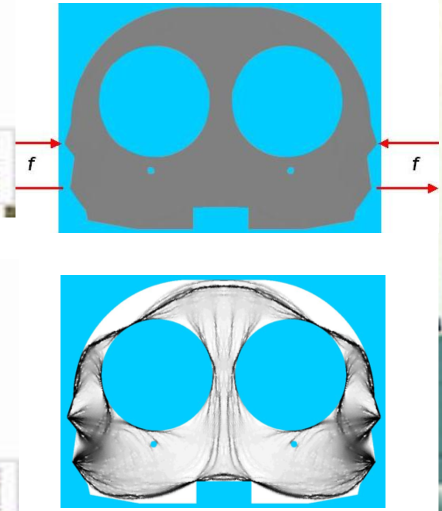
OPTEC Research Example: Topology Optimization

- **Other applications:**

  Bridges, roofs, aircraft wings/ fuselages…



Camera support



aircraft section

roof structures

OPTEC Research Example: Time Optimal Control of Crane

- **Objective:**
  - fast crane movement
  - minimal residual payload vibration
- **Solution:**
  - Time Optimal MPC (TOMPC) impementation using xPC target
  - time sampling at 60 Hz
  - at each time, solve series of medium scale QPs in <10 ms

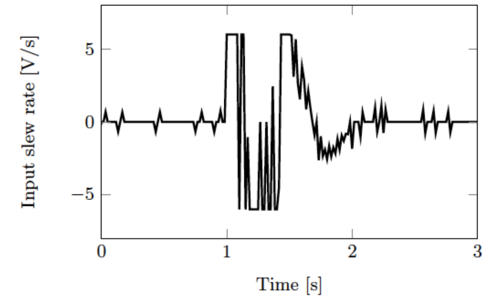
PhD work of Lieboud Van den Broeck

https://www.mech.kuleuven.be/en/pma/research/robotics/media#section-2

A model predictive control approach for time optimal point-to-point motion control

Lieboud Van den Broeck [a,*], M. Diehl [b], J. Swevers [a]

[a] *Department of Mechanical Engineering, Celestijnenlaan 300 B, B-3001 Leuven, Belgium*
[b] *Department of Electrical Engineering, Kasteelpark Arenberg 10, B-3001 Leuven, Belgium*



OPTEC Research Example: Real-Time Perception-Based Clipping of Audio Signals

- **Objective:**
  - constrain amplitude level of audio signal
  - while minimizing perceived signal distortion
- **Solution:**
  - convex optimization formulation
  - FPGA implementation

IEEE TRANSACTIONS ON AUDIO, SPEECH, AND LANGUAGE PROCESSING, VOL. 20, NO. 10, DECEMBER 2012

Real-Time Perception-Based Clipping of Audio Signals Using Convex Optimization

Bruno Defraene, *Student Member, IEEE*, Toon van Waterschoot, *Member, IEEE*, Hans Joachim Ferreau,
Moritz Diehl, *Member, IEEE*, and Marc Moonen, *Fellow, IEEE*

demo videos:

see website

PhD work of Bruno Defraene

ftp://ftp.esat.kuleuven.be/pub/SISTA/vanwaterschoot/abstracts/11-127.html

OPTEC Research Example: Acoustic Room Modeling using Sparse Approximation

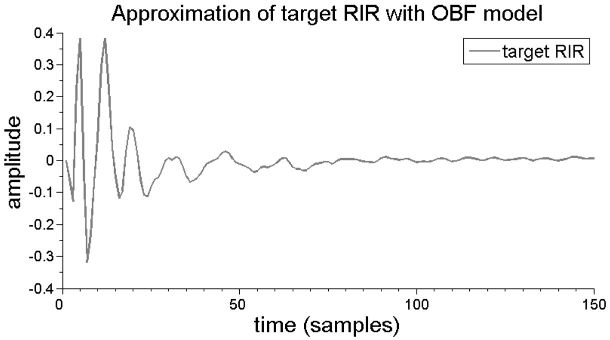
- **Objective:**
  - estimate efficient and scalable acoustic room model
  - with minimal and user-specified model complexity…
  - … but maximal accuracy in approximating room response

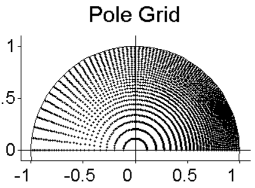
AN AUTOMATIC MODEL-BUILDING ALGORITHM FOR SPARSE APPROXIMATION OF ROOM IMPULSE RESPONSES WITH ORTHONORMAL BASIS FUNCTIONS

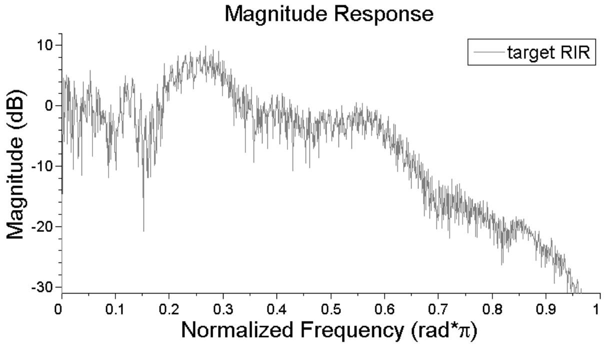
Giacomo Vairetti[1], *Toon van Waterschoot*[1,2], *Marc Moonen*[1], *Michael Catrysse*[3], *and Søren Holdt Jensen*[4]

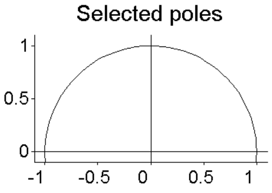
- **Solution:**
  - sparse & iterative approximation of room response using linear combination of orthogonal basis functions

PhD work of Giacomo Vairetti









OPTEC Research Example: Kite Power

- **Objective:**
  - optimize kite trajectory (stable orbit)
  - maximize output power

ERC Project “Highwind” of Moritz Diehl

http://homes.esat.kuleuven.be/~highwind/

https://www.youtube.com/playlist?list=UUxMrjYTMI_qny20p0jnBbhQ

demo videos:
see website

Lecture 1: Introduction & Fundamental Concepts of Optimization

- **Introduction**
  - motivation
  - research examples

- **Fundamental Concepts of Optimization**
  - fundamental concepts: variables, objective function, …
  - research examples revisited

Fundamental Concepts of Optimization

- Optimization problem consists of **three ingredients**:
  - objective function, to be maximized or minimized

    $$f(x)$$

  - decision variables, to be calculated

    $$x$$

  - constraints, to be respected

    $$g(x) = 0 \text{ (equality constraints)}$$

    $$h(x) \geq 0 \text{ (inequality constraints)}$$

Fundamental Concepts of Optimization

- Optimization problem in **standard form**:

$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g(x) = 0, \\ & h(x) \geq 0. \end{aligned}$$

  - assumptions: differentiability of objective and constraint functions $f : \mathbb{R}^n \rightarrow \mathbb{R}$, $g : \mathbb{R}^n \rightarrow \mathbb{R}^p$, $h : \mathbb{R}^n \rightarrow \mathbb{R}^q$
  - inequalities should hold for all components:

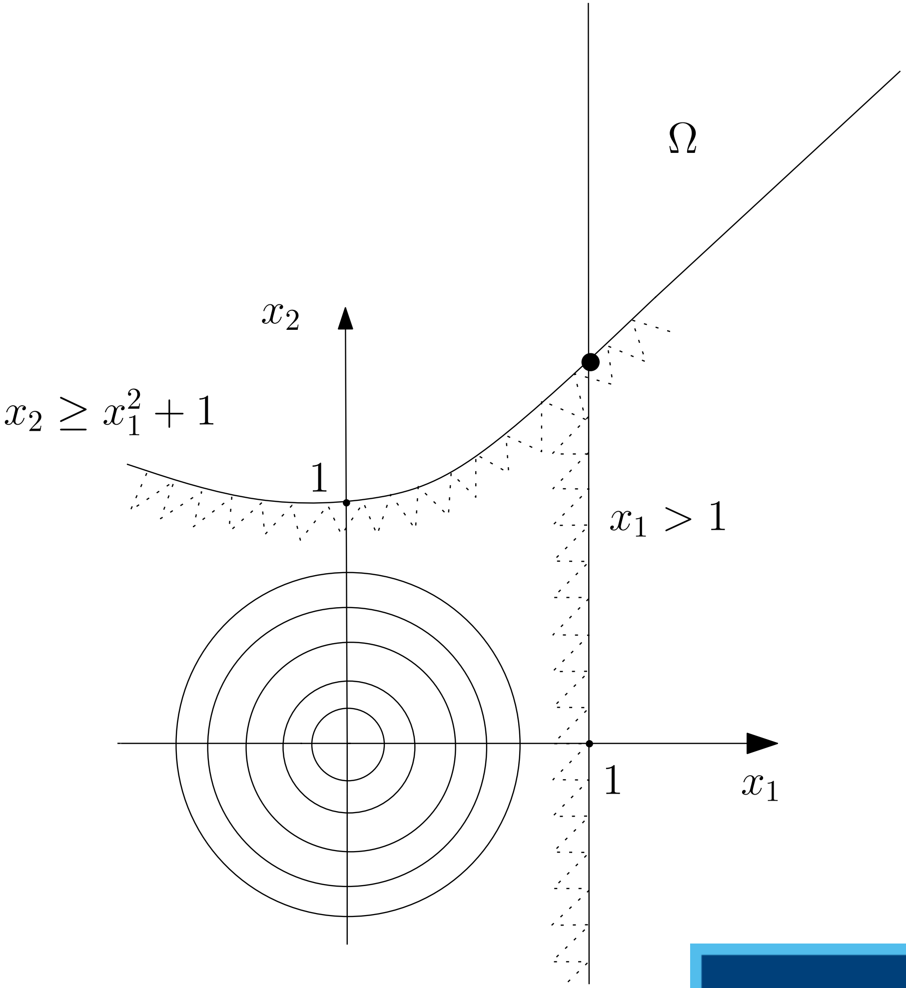
$$h(x) \geq 0 \quad \Leftrightarrow \quad h_i(x) \geq 0, \quad i = 1, \ldots, q.$$

Fundamental Concepts of Optimization

- Example:

$$\begin{aligned} \underset{x \in \mathbb{R}^2}{\text{minimize}} \quad & x_1^2 + x_2^2 \\ \text{subject to} \quad & x_2 - 1 - x_1^2 \geq 0, \\ & x_1 - 1 \geq 0. \end{aligned}$$

  - Ω = “feasible set”
  - solution at intersection of constraint functions



OPTEC Research Example Revisited: Time Optimal Robot Motion

- **Objective:**
  - follow given writing trajectory as close as possible
  - while maximizing the speed of writing

minimize time

$$\boxed{\min_{T,s(\cdot),\boldsymbol{\tau}(\cdot)} \quad T}$$

obey laws of motion

$$\boxed{\begin{aligned}\text{subject to} \quad & \boldsymbol{\tau}(t) = \mathbf{m}\left(s(t)\right)\ddot{s}(t) + \mathbf{c}\left(s(t)\right)\dot{s}(t)^2 \\ & + \mathbf{g}\left(s(t)\right)\end{aligned}}$$

$$s(0) = 0$$

$$s(T) = 1$$

$$\dot{s}(0) = \dot{s}_0$$

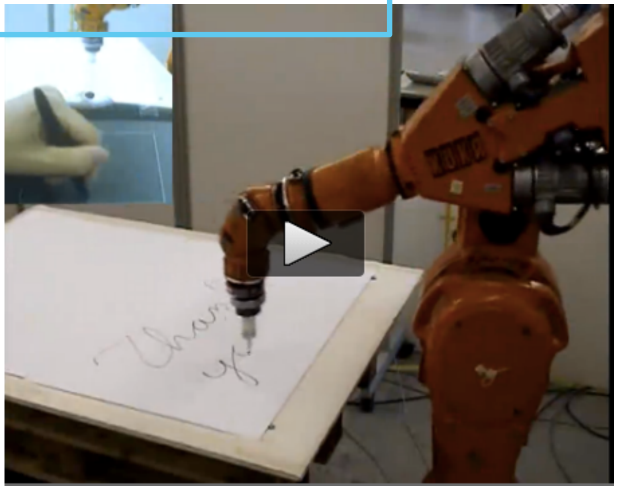
$$\dot{s}(T) = \dot{s}_T$$

$$\dot{s}(t) \geq 0$$

within trajectory error bounds

$$\boxed{\underline{\boldsymbol{\tau}}\left(s(t)\right) \leq \boldsymbol{\tau}(t) \leq \overline{\boldsymbol{\tau}}\left(s(t)\right)}$$

$$\text{for } t \in [0, T]$$



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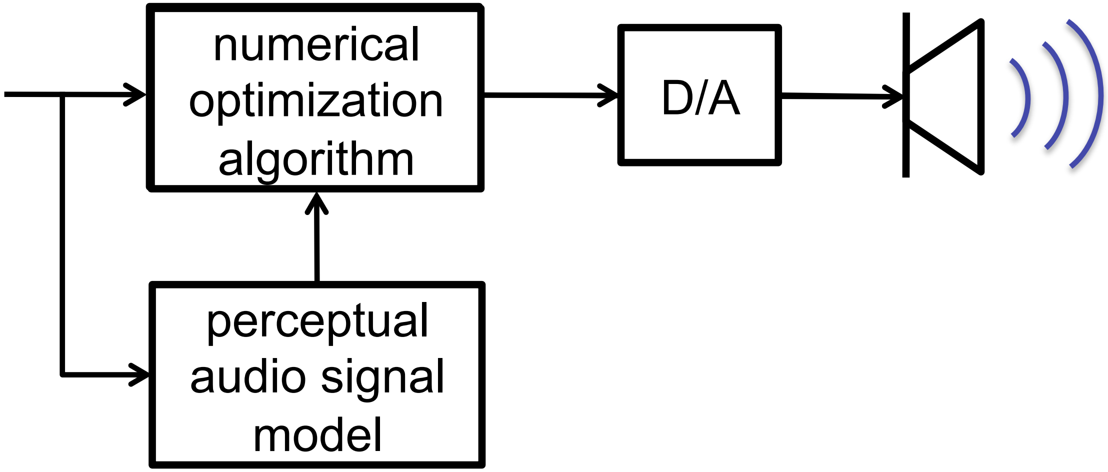
OPTEC Research Example: Real-Time Perception-Based Clipping of Audio Signals

- **Objective:**
  - constrain amplitude level of audio signal
  - while minimizing perceived signal distortion

$$\min_{\mathbf{y}} \sum_{i=0}^{N-1} \boxed{w_i |Y(e^{j\omega_i}) - X(e^{j\omega_i})|^2} \quad \text{s.t.} \quad \boxed{-l \leq \mathbf{y} \leq l}$$

minimize perceived distortion

constrain amplitude level



numerical optimization algorithm

D/A

perceptual audio signal model

OPTEC Research Example: Acoustic Room Modeling using Sparse Approximation

- **Objective:**
  - estimate efficient and scalable acoustic room model
  - with minimal and user-specified model complexity…
  - … but maximal accuracy in approximating room response

$$\hat{\bar{\boldsymbol{\theta}}}_N = \arg\min_{\boldsymbol{\theta}} \left\{ \boxed{\frac{1}{2}\sum_{t=1}^{N}\left(y(t) - \bar{\boldsymbol{\varphi}}(t,\boldsymbol{\xi})^T\bar{\boldsymbol{\theta}}\right)^2} + \boxed{\lambda\|\bar{\boldsymbol{\theta}}\|_1} \right\}$$

minimize response approximation error

penalize model complexity