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Digital Signal Processing 2

Les 5: Detectieproblemen

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Digital Signal Processing 2: Vakinhoud

- Les 1: Eindige woordlengte
- Les 2: Lineaire predictie
- Les 3: Optimale filtering
- Les 4: Adaptieve filtering
- Les 5: Detectieproblemen
- Les 6: Spectrale signaalanalyse
- Les 7: Schattingsproblemen 1
- Les 8: Schattingsproblemen 2
- Les 9: Sigma-Deltamodulatie
- Les 10: Transformatiecodering

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Les 5: Detectieproblemen

- **Signal detection**
  introduction, binary hypothesis testing, …
- **Example: detection of sinusoid in noise**
  energy detection, periodogram-based detection, …
- **Detection performance analysis**
  confusion matrix, receiver operating characteristic (ROC), …

Les 5: Detectieproblemen

- **Signal detection**
- **Example: detection of sinusoid in noise**

  Research paper:

  H. C. So *et al.*, “Comparison of various periodograms for sinusoid detection and frequency estimation,” *IEEE Trans. Aerospace Electron. Syst.*, vol. 35, no. 3, July 1999, pp. 945-952.

  (Section I–II)

  Matlab code:

  DSP2_sinusoid_detection.m (available on Toledo)

- **Detection performance analysis**

  Research paper:

  T. Fawcett, “An introduction to ROC analysis,” *Patt. Recognition Lett.*, vol. 27, 2006, pp. 861-874.

  (Section 1–3)

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Les 5: Detectieproblemen

- **Signal detection**
  introduction, binary hypothesis testing, …
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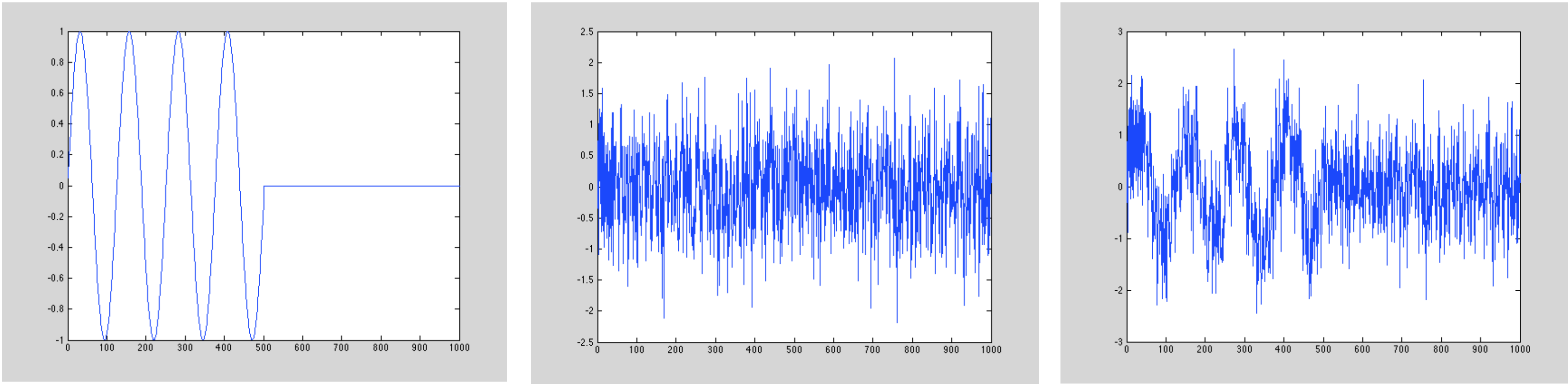
Signal detection

- Introduction
- Binary hypothesis testing

Introduction

- **Signal detection**
  - problem of determining whether or not signal is active
  - trivial problem in noiseless case (amplitude ≠ 0 ?)
  - challenging problem in noisy case (e.g. sinusoid in noise)



- **Detection ≠ Estimation !!!**
  - estimation: real-valued result
  - detection: binary result

Signal detection

- Introduction
- Binary hypothesis testing

Binary hypothesis testing (1)

- **Two hypotheses**
  - null hypothesis $\mathcal{H}_0$: signal is not active
  - alternative hypothesis $\mathcal{H}_1$: signal is active

- **Two detection outcomes**
  - null hypothesis is rejected (signal is detected)
  - null hypothesis is not rejected (no signal is detected)

Binary hypothesis testing (2)

- **Observed signal**
  - observation: data vector of length *N*

$$\mathbf{y} = \begin{bmatrix} y(0) & y(1) & \dots & y(N-1) \end{bmatrix}^T$$

- **Signal features**
  - signal properties that are most characteristic of signal (and can hence be used to detect the signal)

$$\mathbf{f} = \begin{bmatrix} f_0 & f_1 & \dots & f_{P-1} \end{bmatrix}^T$$

- **Detection function**
  - function that "summarizes" feature values to single value

$$\delta(\mathbf{f}) \in \mathbb{R}$$

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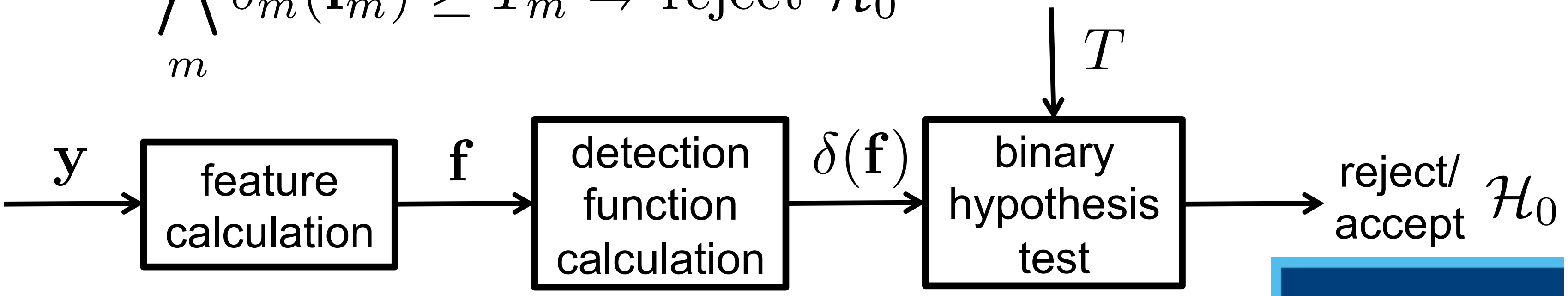
Binary hypothesis testing (3)

- **Binary hypothesis test**
  - single-feature test: comparison of detection function to threshold

$$\delta(\mathbf{f}) \geq T \Rightarrow \text{reject } \mathcal{H}_0$$

  - multiple-feature test: logical conjunction of single-feature tests, using multiple feature vectors, detection functions and thresholds

$$\bigwedge_m \delta_m(\mathbf{f}_m) \geq T_m \Rightarrow \text{reject } \mathcal{H}_0$$



Binary hypothesis testing (3)

- **Discriminative power**
  - how easy is it to discriminate $\mathcal{H}_1$ from $\mathcal{H}_0$ based on detection function value $\delta(\mathbf{f})$?
  - denote detection function value
    - by $\delta(\mathbf{f}, \mathcal{H}_1)$ for observation with active signal
    - by $\delta(\mathbf{f}, \mathcal{H}_0)$ for observation without active signal
  - optimal threshold value *T* should be in between both detection function values:

$$\delta(\mathbf{f}, \mathcal{H}_1) \geq T > \delta(\mathbf{f}, \mathcal{H}_0)$$

  - discriminative power of detection algorithm
    - is determined by choice of signal features and detection function
    - can be quantified by means of ratio $\dfrac{\delta(\mathbf{f}, \mathcal{H}_1)}{\delta(\mathbf{f}, \mathcal{H}_0)}$

Les 5: Detectieproblemen

- **Signal detection**
  introduction, binary hypothesis testing, …

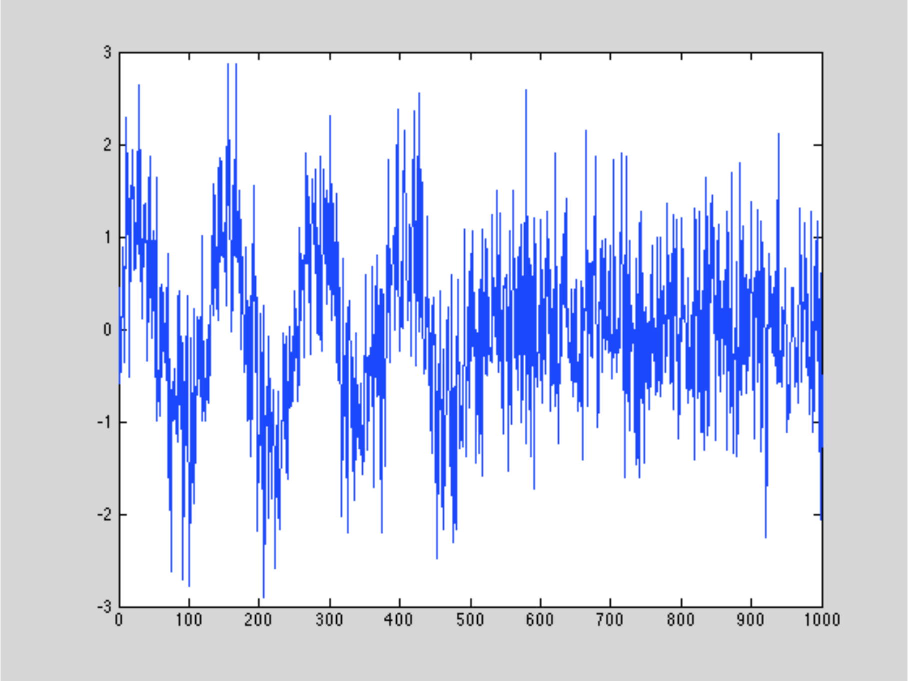
- **Example: detection of sinusoid in noise**
  energy detection, periodogram-based detection, …

- **Detection performance analysis**
  confusion matrix, receiver operating characteristic (ROC), …

Example: detection of sinusoid in noise (1)

- **Problem statement**
  - observation consisting of two fragments (active/inactive):

$$\begin{cases} y(n) = \sin(\omega n) + e(n), & n = 0, \ldots, N/2 - 1 \\ y(n) = e(n), & n = N/2, \ldots, N-1 \end{cases}$$



  - Gaussian white noise

$$e(n) \sim \mathcal{N}(0, \sigma_e^2)$$

  - SNR = 0 dB
  - N = 1000
  - ω = 0.05 rad/s

Example: detection of sinusoid in noise (2)

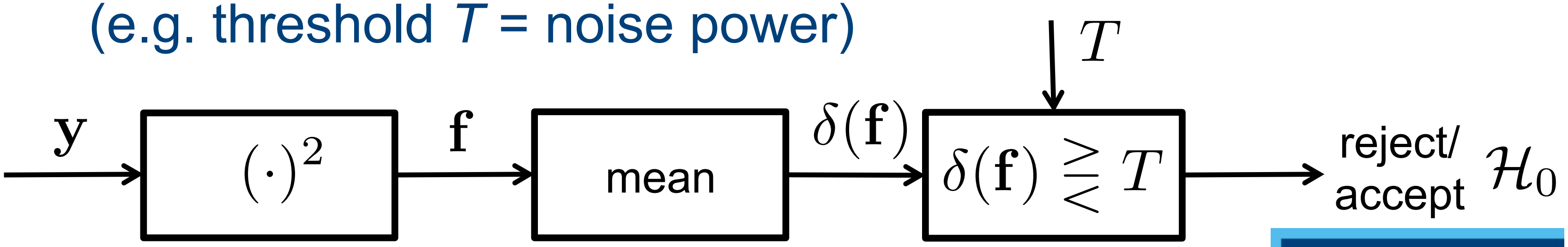
- **Method 1: energy detection**
  - features: squared signal values

$$\mathbf{f} = \left[y^2(0) \quad y^2(1) \quad \ldots \quad y^2(N-1)\right]^T$$

  - detection function: mean of feature values

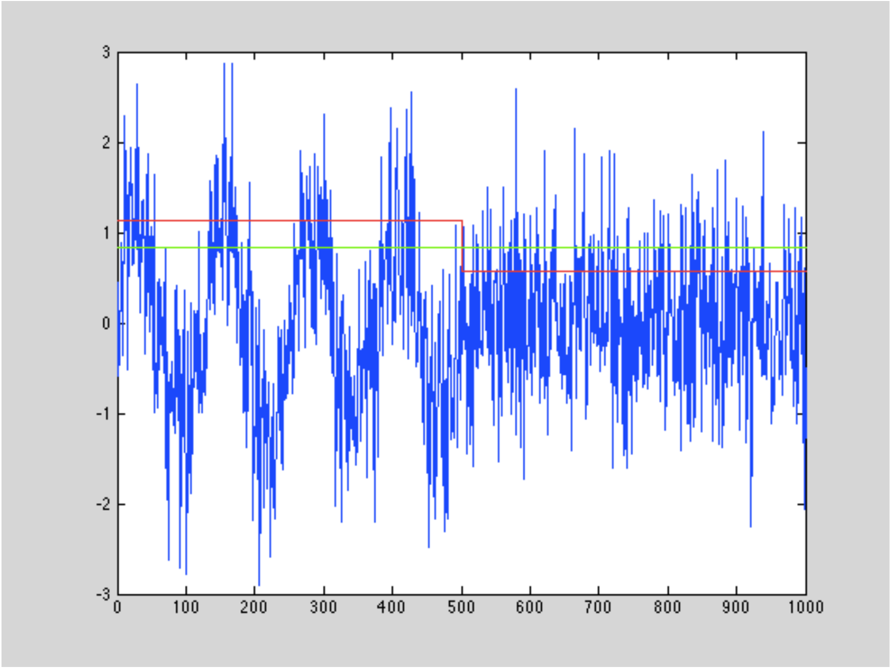
$$\delta(\mathbf{f}) = \frac{1}{N}\sum_{p=0}^{N-1} f_p = \frac{1}{N}\sum_{n=0}^{N-1} y^2(n) = \frac{1}{N}\mathbf{y}^T\mathbf{y}$$

  - hypothesis test: $\delta(\mathbf{f}) \geq T \Rightarrow \text{reject } \mathcal{H}_0$
    (e.g. threshold T = noise power)



Example: detection of sinusoid in noise (3)

- **Method 1: energy detection**



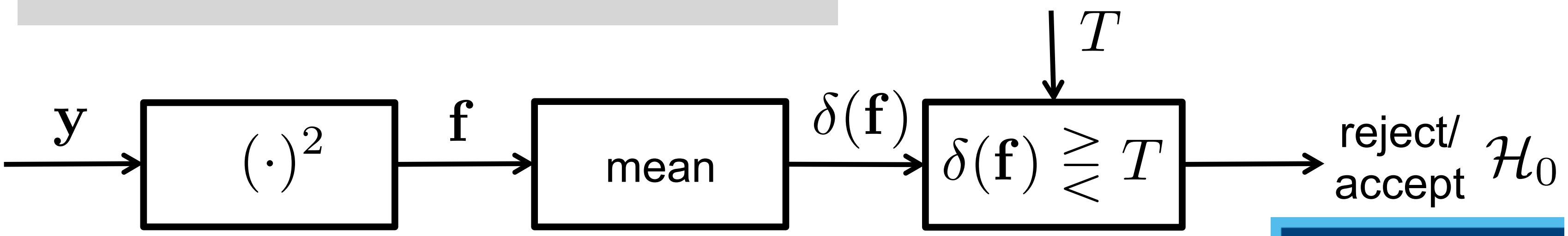
– detection function

– threshold

$$T = 1.5\sigma_e^2$$

– discriminative power

$$\frac{\delta(\mathbf{f}, \mathcal{H}_1)}{\delta(\mathbf{f}, \mathcal{H}_0)} = 2$$



Example: detection of sinusoid in noise (4)

- **Method 2: periodogram-based energy detection**
  - features: squared DFT magnitude values (= periodogram)

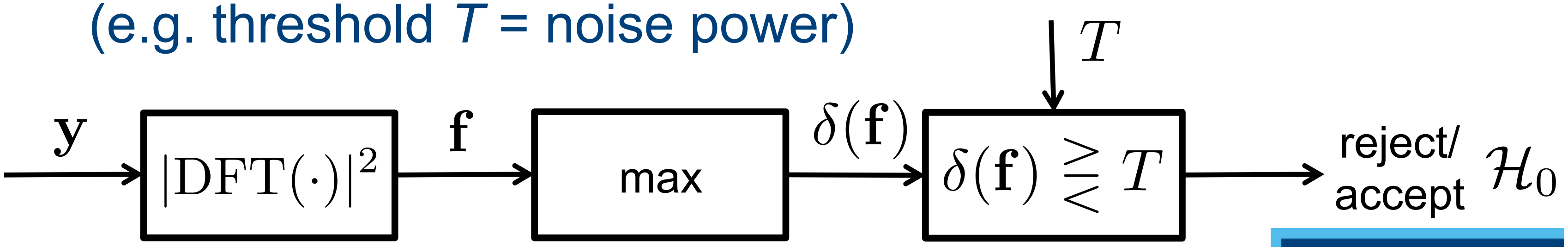
$$\mathbf{f} = \left[|Y(0)|^2 \quad |Y(1)|^2 \quad \ldots \quad |Y(N-1)|^2\right]^T$$

with Y = DFT(y)

  - detection function: maximum of feature values (why?)

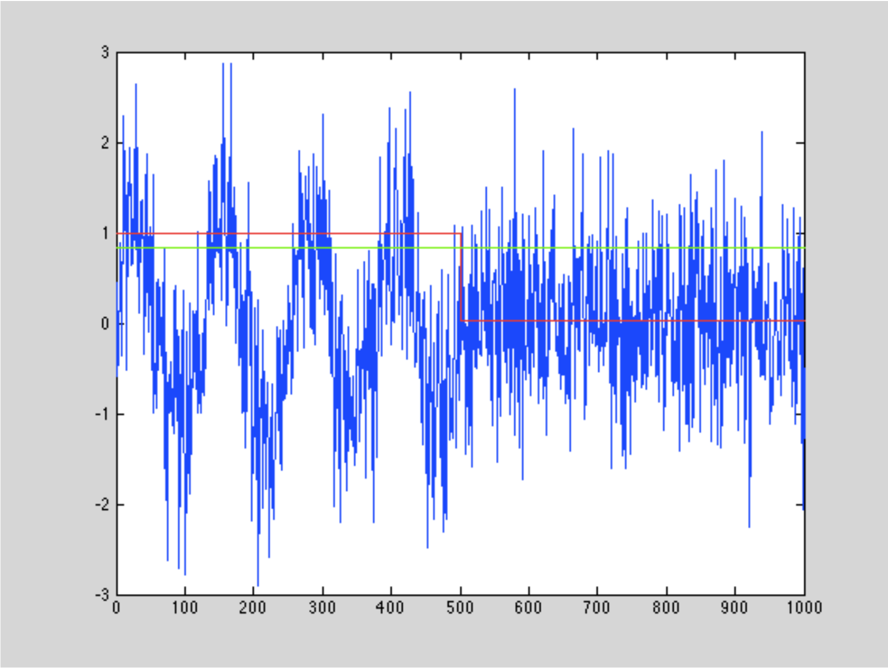
$$\delta(\mathbf{f}) = \max(\mathbf{f}) = \max_k |Y(k)|^2$$

  - hypothesis test: $\delta(\mathbf{f}) \geq T \Rightarrow \text{reject } \mathcal{H}_0$
    (e.g. threshold T = noise power)



Example: detection of sinusoid in noise (5)

- **Method 2: periodogram-based energy detection**

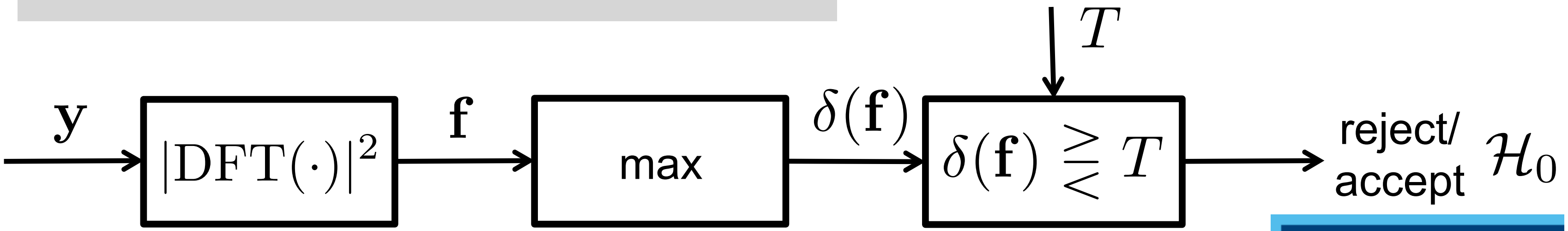


- detection function
- threshold

$$T = 1.5\sigma_e^2$$

- discriminative power

$$\frac{\delta(\mathbf{f}, \mathcal{H}_1)}{\delta(\mathbf{f}, \mathcal{H}_0)} \approx 50$$



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Example: detection of sinusoid in noise (6)

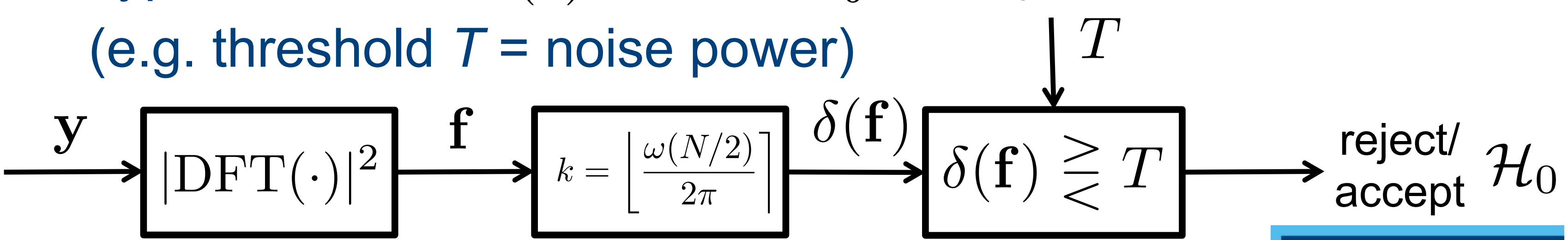
- **Method 3: periodogram-based energy detection with known sinusoid frequency**
  - features: squared DFT magnitude values (= periodogram)

$$\mathbf{f} = \left[|Y(0)|^2 \quad |Y(1)|^2 \quad \ldots \quad |Y(N-1)|^2 \right]^T$$

    with Y = DFT(y)
  - detection function: feature value at DFT bin $k = \left\lfloor \frac{\omega(N/2)}{2\pi} \right\rceil$

$$\delta(\mathbf{f}) = |Y(k)|^2$$

  - hypothesis test: $\delta(\mathbf{f}) \geq T \Rightarrow \text{reject } \mathcal{H}_0$
    (e.g. threshold T = noise power)



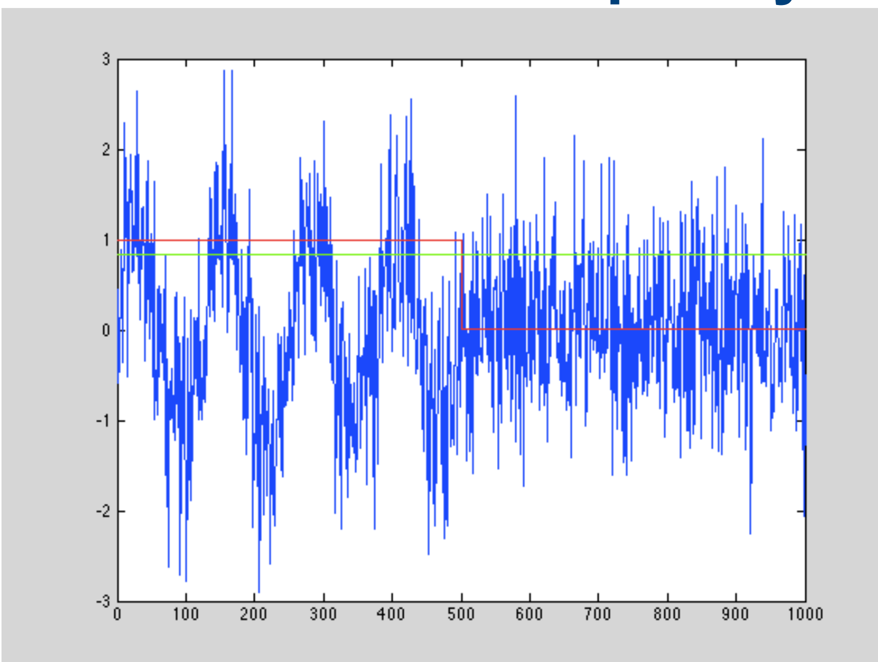
Example: detection of sinusoid in noise (7)

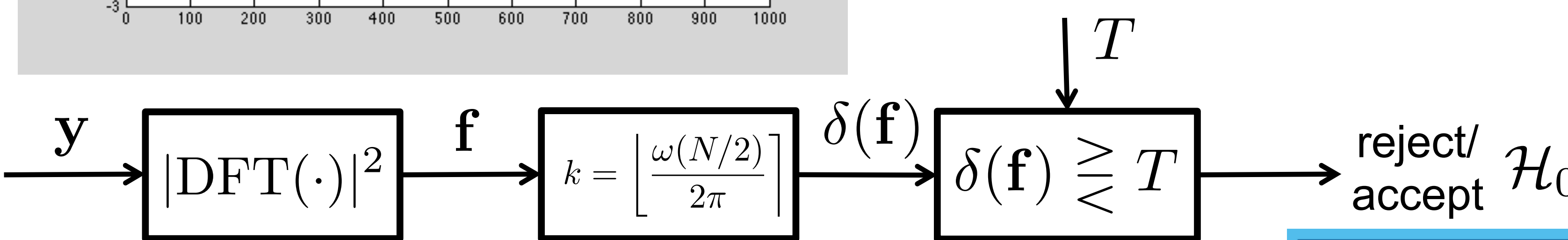
- **Method 3: periodogram-based energy detection with known sinusoid frequency**
  - detection function
  - threshold

$$T = 1.5\sigma_e^2$$

  - discriminative power

$$\frac{\delta(\mathbf{f}, \mathcal{H}_1)}{\delta(\mathbf{f}, \mathcal{H}_0)} \approx 500$$





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- **Detection performance analysis**
  confusion matrix, receiver operating characteristic (ROC), …

Detection performance analysis

- Performance measures
- Confusion matrix
- Receiver operating characteristic

Performance measures (1)

- **Data realizations:**
  - suppose we have R realizations of the observation vector $\mathbf{y}$

$$\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(R)}$$

  - each realization either corresponds to:
    - active signal (positive realization), or
    - no active signal (negative realization)
  - number of positive and negative realizations:

$$R_P + R_N = R$$

  - performance analysis should be based on “balanced” set of data realizations, i.e. $R_P \approx R_N$

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Performance measures (2)

- **Detection outcomes**
  - true positive: $\mathcal{H}_0$ rejected for positive realization
  - false positive: $\mathcal{H}_0$ rejected for negative realization
  - true negative: $\mathcal{H}_0$ not rejected for negative realization
  - false negative: $\mathcal{H}_0$ not rejected for positive realization

- **Counting detection outcomes**
  - R_{TP}: number of true positives
  - R_{FP}: number of false positives
  - R_{TN}: number of true negatives
  - R_{FN}: number of false negatives

Performance measures (3)

- **Performance measures:**
  - probability of detection = true positives rate = recall

$$P_D = P(\mathcal{H}_1; \mathcal{H}_1) = \frac{R_{TP}}{R_P}$$

  - probability of false alarm = false positives rate

$$P_{FA} = P(\mathcal{H}_1; \mathcal{H}_0) = \frac{R_{FP}}{R_N}$$

  - precision $= \dfrac{R_{TP}}{R_{TP} + R_{FP}}$
  - accuracy $= \dfrac{R_{TP} + R_{TN}}{R}$
  - F-measure $= \dfrac{2}{1/\text{precision} + 1/\text{recall}}$

Detection performance analysis

- Performance measures
- Confusion matrix
- Receiver operating characteristic

Confusion matrix

- **Confusion matrix**
  - 2 x 2 representation of counted detection outcomes

True class

Hypothesized class	p	n
Y	True Positives	False Positives
N	False Negatives	True Negatives
Column totals:	P	N

$$\text{fp rate} = \frac{FP}{N} \qquad \text{tp rate} = \frac{TP}{P}$$

$$\text{precision} = \frac{TP}{TP+FP} \qquad \text{recall} = \frac{TP}{P}$$

$$\text{accuracy} = \frac{TP+TN}{P+N}$$

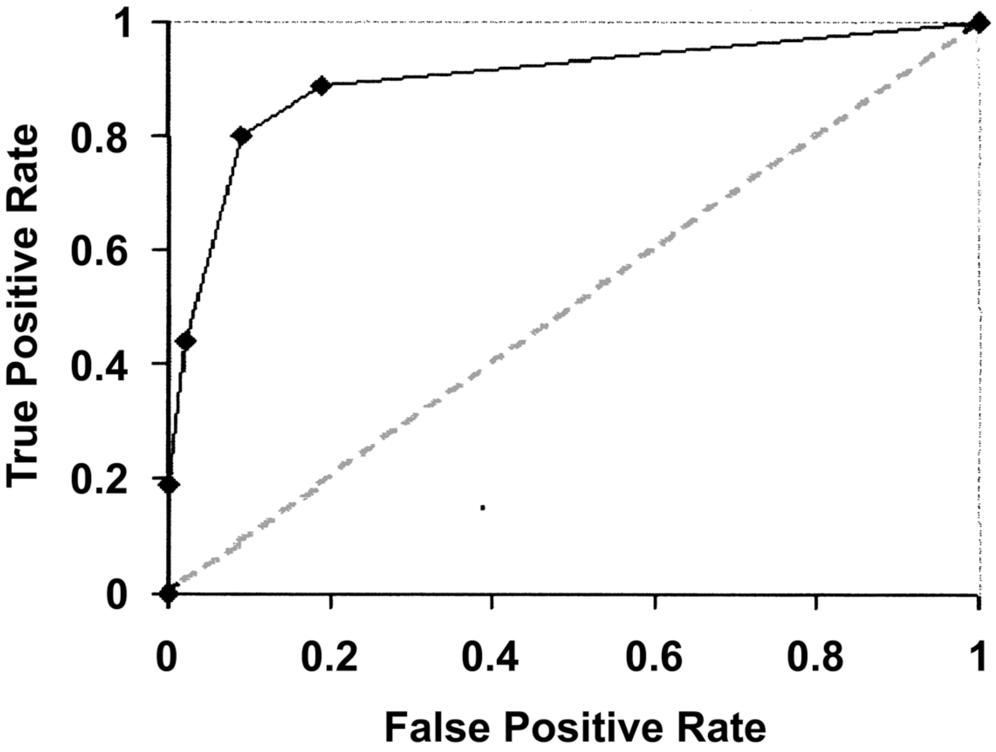
$$\text{F-measure} = \frac{2}{1/\text{precision}+1/\text{recall}}$$

Detection performance analysis

- Performance measures
- Confusion matrix
- Receiver operating characteristic

Receiver operating characteristic (1)

- **Receiver operating characteristic (ROC)**
  - 2-dimensional representation of (R_{FP}, R_{TP})-curve obtained for different threshold values in range $T \in [0, \infty]$



Receiver operating characteristic (2)

- **ROC interpretation:**
  - ROC curve = on or below diagonal line from (0,0) to (1,1): useless detector (not better than random decision)
  - ROC curve = line segments from (0,0) and (1,1) to (0,1): ideal detector
  - area under ROC curve = number to compare different ROC curves