Digital Signal Processing-2
Adaptive notch filters for acoustic feedback control

Prof. dr. ir. Toon van Waterschoot
KU Leuven, Faculty of Engineering Technology,
Department of Electrical Engineering (ESAT)

October 2014

This text provides an introduction to the problem of acoustic feedback control, and details on the use of adaptive notch filters to tackle the acoustic feedback problem. A more detailed treatment of the state of the art in acoustic feedback control can be found in [1].

1 The Acoustic Feedback Problem

A Public Address (P.A.) system typically consists of one or more microphones, an amplifier and several loudspeakers. Acoustic feedback occurs when loudspeaker sound is picked up again by a microphone. Even when direct sound transmission from loudspeaker to microphone is avoided, sound is fed back due to reflections against walls and other objects. A closed loop is thus born and gives rise to system instability.

Considering a simple 1-microphone/1-loudspeaker setup in **Figure 1**, the closed loop transfer function from source signal $V(z)$ to feedback signal $X(z)$ at the microphone is given in the z-domain by

$$\frac{X(z)}{V(z)} = \frac{G(z)F(z)}{1 - G(z)F(z)} \tag{1}$$

System stability depends on both the forward path amplification transfer function $G(z)$ and the acoustic feedback path transfer function $F(z)$. Nyquist's stability criterion says that if there exists a frequency ω for which

$$|G(e^{i\omega})F(e^{i\omega})| \geq 1 \quad and \quad \angle G(e^{i\omega})F(e^{i\omega}) = n2\pi, n \in \mathbb{Z} \tag{2}$$

the system will be unstable and start oscillating at frequency ω. From this criterion it is clear that the peaks in the room's frequency response $F(e^{i\omega})$ are particularly apt to give rise to system instability. Oscillation will be perceived as a sharp, narrow-band howling sound. Even before the onset of oscillation the sound quality may be severely degraded by distortion and ringing effects. This

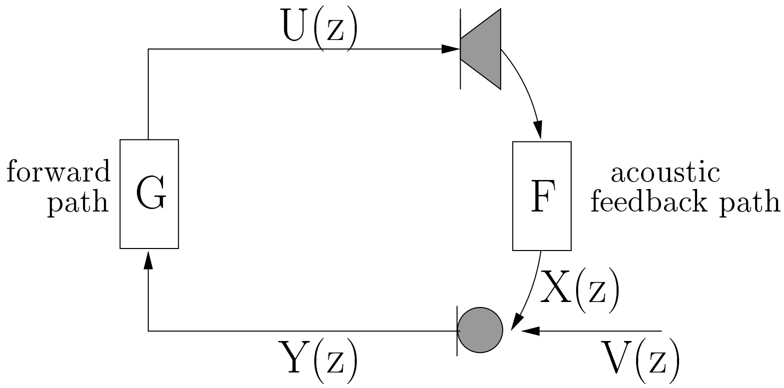

Figure 1: The acoustic feedback problem in a 1-microphone/1-loudspeaker setup

howling is often referred to as "acoustic feedback". We prefer though to point to the complete feedback signal $X(z)$ when using the term "acoustic feedback", even when no oscillation occurs.

2 A survey of existing solutions

The problem of acoustic feedback has been a challenge for many researchers over the past few decades. In the early days acoustic feedback was a typical P.A. problem but more recently, with the advent of hearing aids, research has focused especially on the latter application. The following list gives a non-exhaustive overview of proposed feedback remedies in both P.A. and hearing aid applications (see [1] for a more in-depth literature study):

1. careful microphone and/or loudspeaker selection and positioning [2]
2. manual equalization and/or gain reduction [3]
3. inclusion of frequency shifter in $G(z)$ [4]
4. automatic gain reduction [5]
5. inclusion of fixed phase shift in $G(z)$ [6]
6. inclusion of time-varying delay in $G(z)$ [7]
7. adaptive inverse filtering [7]
8. beam dithering [8]
9. inclusion of notch filters (NF) in $G(z)$
10. adaptive feedback cancellation (AFC)

The first two *manual techniques* are conceptually straightforward but in practice quite elaborate operations. Several other techniques (e.g. 4 and 9) are an attempt to automate these manual procedures. Translating sound engineering skills into an algorithm is not an easy task though. Many sound engineers prefer even today to control feedback manually.

Frequency shifting [4] causes a signal that enters the microphone at frequency ω to be shifted to $\omega + \Delta\omega$ before being played back. In this way buildup of the microphone signal at a single frequency is avoided. With a frequency shift of $\Delta\omega = 5$ Hz an amount of 10 dB in added stable gain can be obtained. This means the use of the frequency shifter allows the amplifier gain to be raised by 10 dB (relative to the maximum stable gain of the system without feedback control) before the stability limit is reached. The major drawback of this technique is the auditive distortion. Schroeder [4] suggests a 5 Hz shift will be "hardly perceptible for speech and many types of music, too". We believe this is an understatement because, among other reasons, the frequency shift alters the relationship between the harmonics of the signal thereby causing harmonic distortion. This distortion can be avoided by making the frequency shift proportional to the frequency of the component being shifted. This frequency compression/expansion is desribed in more detail in [9] as a decorrelation technique for adaptive feedback cancellation in hearing aids.

Automatic Gain Reduction as proposed in [5] is not frequency-selective, i.e. reduces the overall gain, and therefore has become obsolete. Techniques 5 and 6 try to overcome system instability by avoiding the phase of the feedback signal to approach a multiple of 2π at the microphone. Including a *fixed phase shift* [6] may work as long as the acoustic feedback path does not change, a condition which is obviously not satisfied in a P.A. system. Another way of altering the phase response is the inclusion of a *time-varying delay* resulting in a time-varying phase response. This introduces a "warbling" sound in the loudspeaker signal though, and not more than 1 or 2 dB in added stable gain can be obtained [7].

Adaptive inverse filtering implies an adaptive filter is included in the forward path $G(z)$ (i.e. in series with $G(z)$ in contrast to the AFC technique where an adaptive filter is placed in parallel). Depending on the filter order, different results are obtained. A low filter order ($N \sim 10$) gives the filter a notch filter behavior since only sharp frequency peaks in the input spectrum $Y(e^{j\omega})$ are compensated for [7]. We will come back to this in **Section 3.1**. A high filter order ($N \sim 10^3$ or more) turns the feedback problem into an equalization problem (see e.g. [10], [11], [12]), which is however difficult to solve if the "desired" signal $V(z)$ is not available.

Beam dithering is a loudspeaker array beamforming technique in which the radiation pattern (and hence the room excitation) will vary in time. An addition of 6 dB in stable gain was reported in [8]. Spatial constraints are quite hard to accomplish though: the public should be seated in the main lobe while all microphones should be positioned in side lobes.

Finally, techniques 9 and 10 are the most popular approaches to acoustic feedback control, on which more details can be found in [1] and references therein. *Notch filters (NF)* are the state of the art in P.A. feedback controllers that are commercially available. *Adaptive feedback cancellation (AFC)* is a promising research topic for P.A. systems, given its success in hearing aid feedback control.

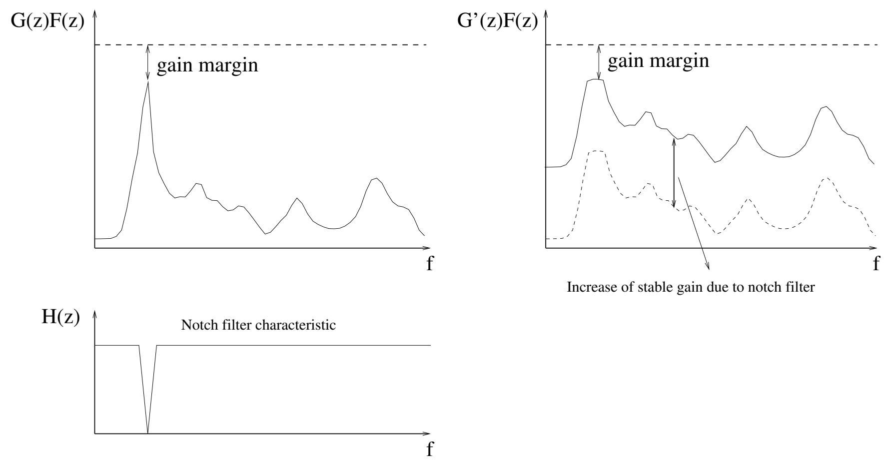

Figure 2: The concept of notch filtering for increasing the overall stable gain in a P.A. system

3 Notch Filtering (NF)

When a frequency exists for which the amplitude and phase conditions (2) are both met, a very fast signal buildup will occur at this frequency. The system instability is thus perceived as a narrow-band interference, more frequently called "howling" or "screeching". The notch filtering idea is to detect these sinusoid-like components in the microphone signal and subsequently filter them out with a very narrow bandstop filter. Even before the system becomes unstable a notch filter can be applied to filter out "candidate" feedback frequencies. In this way the P.A. system can operate at a higher overall gain without decreasing the gain margin. The gain margin is defined as the magnitude difference between the stability limit and the highest peak in the system's frequency response. This is illustrated in **Figure 2**.

In the following we distinguish between two types of notch filter implementations. The first one, the *Adaptive Notch Filter (ANF)*, was developed in the 1980's as an early application of adaptive filter theory [13]. It is a relatively autonomous algorithm with few parameters to be adjusted by the user. The second one is an attempt to automate the sound engineer's manual operations to detect and eliminate howling. Different implementations of this idea are described in a dozen patents and have led to various commercial audio products. Although in these patents the technique is often called "adaptive", we prefer to call it *Self-Adjusting Notch Filter (SANF)* to emphasize the difference with the Adaptive Notch Filter. The ANF is described in more detail in Section 3.1 below, whereas details on the SANF can be found in [1] and [14].

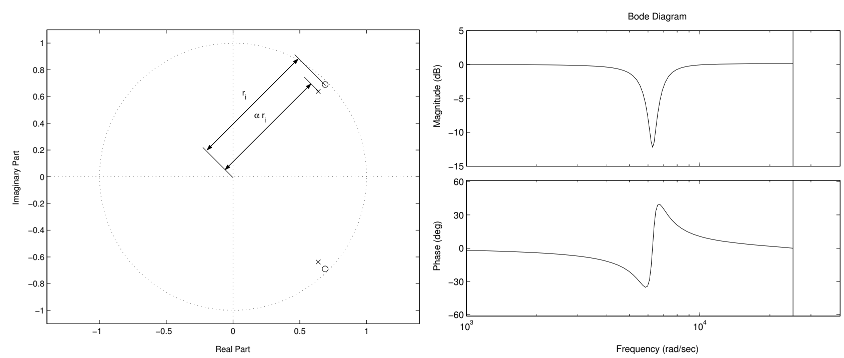

Figure 3: Pole-zero map (left) and Bode plot (right) of an 2^{nd} order ANF with zeros $z_i = r_i e^{\pm j\omega_i}$ and poles $p_i = \alpha r_i e^{\pm j\omega_i}$

3.1 Adaptive Notch Filter (ANF)

3.1.1 Overview of the literature

The ANF filter structure. The Adaptive Notch Filter (ANF) was first conceived by Rao et al. [15] as a means for retrieving sinusoids or narrow-band signals buried in broadband noise. Their idea was based on Widrow's Adaptive Line Enhancer (ALE, Widrow et al. [16]), implemented as an adaptive FIR filter preceded by a decorrelating delay. This implementation was copied by Bustamante et al. [7] for suppressing acoustic feedback in hearing aids but provided a smaller increase in stable gain than desired.

Rao et al. believed though that a constrained IIR filter would suit the problem better than an unconstrained FIR filter. Their *constraint* was that poles and zeros should lie on the same radial lines, both inside the unit circle, with the zeros lying between the poles and the unit circle, see **Figure 3** on the left. Intuitively this constraint can be understood as follows: a zero $z_i = r_i e^{j\omega_i}$ close to the unit circle ($0 \ll r_i \leq 1$) attenuates all frequencies in the neighbourhood of ω_i. A pole $p_i = \alpha r_i e^{j\omega_i}$ lying on the same radial line causes a resonance at frequency ω_i, thereby narrowing the bandwidth of the notch. This is probably the reason Bustamante et al. [7] found that the FIR adaptive notch filter (i.e. without poles) produced very broad notches.

Rao et al. called α the debiasing parameter, since for $\alpha \to 1$ the "ideal" unbiased notch filter is approached. "Ideal" here means that the frequency response magnitude equals 0 dB at all frequencies, except at the notch frequencies where it equals $-\infty$ dB. Taking into account the proposed filter structure, the

ANF transfer function in the z-domain looks like

$$H(z^{-1}) = \frac{\prod_{i=1}^{2n}(1 - z_i z^{-1})}{\prod_{i=1}^{2n}(1 - \alpha z_i z^{-1})} \qquad \text{where } 0 \le \alpha < 1 \tag{3}$$

$$= \frac{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_{2n} z^{-2n}}{1 + \alpha a_1 z^{-1} + \alpha^2 a_2 z^{-2} + ... + \alpha^{2n} a_{2n} z^{-2n}} \tag{4}$$

$$= \frac{A(z^{-1})}{A(\alpha z^{-1})} \tag{5}$$

With this structure a filter of order $2n$ has $2n$ unknown parameters and may suppress at most n narrow-band components. A Bode plot of a 2^{nd} order ANF is shown on the right in **Figure 3**.

Nehorai [17] proposed an adaptive notch filter with half as much parameters by imposing a *second constraint*: the zeros z_i should lie on the unit circle. A necessary condition to meet this constraint is that the numerator coefficients have a mirror symmetric form (i.e. when z_i is a zero, $\frac{1}{z_i}$ will also be a zero). A $2n^{th}$ order ANF with n unknown parameters thus has a transfer function

$$H(z^{-1}) = \frac{1 + a_1 z^{-1} + ... + a_n z^{-n} + ... + a_1 z^{-2n+1} + z^{-2n}}{1 + \rho a_1 z^{-1} + ... + \rho^n a_n z^{-n} + ... + \rho^{2n-1} a_1 z^{-2n+1} + \rho^{2n} z^{-2n}} \tag{6}$$

$$= \frac{A(z^{-1})}{A(\rho z^{-1})} \tag{7}$$

where the debiasing parameter α has been replaced by the "pole radius ρ".

Estimating the filter coefficients. Including an adaptive notch filter in the forward path of the 1-microphone/1-loudspeaker setup results in the scheme depicted in **Figure 4**. An estimate of parameter vector $\theta = [a_1 \ a_2 \ ... \ a_n]^T$ is obtained by minimizing the cost function $V_N(\theta)$:

$$\hat{\theta} = \arg\min_{\theta} V_N(\theta) \tag{8}$$

$$= \arg\min_{\theta} \sum_{t=1}^{N} e^2(\theta, t) \tag{9}$$

$$= \arg\min_{\theta} \sum_{t=1}^{N} \left[\frac{A(\theta, z^{-1})}{A(\theta, \rho z^{-1})} y(t) \right]^2 \tag{10}$$

When the ANF order is chosen approximately twice the number of expected narrow-band components, minimizing the square of the filter output $e(\theta, t)$ with the filter stucture as defined in (6) results in a filter with n notches at the desired frequencies. As for the bandwidth of the notches, the pole radius ρ plays a crucial role: the closer ρ is to 1, the narrower the notches will be. Choosing ρ

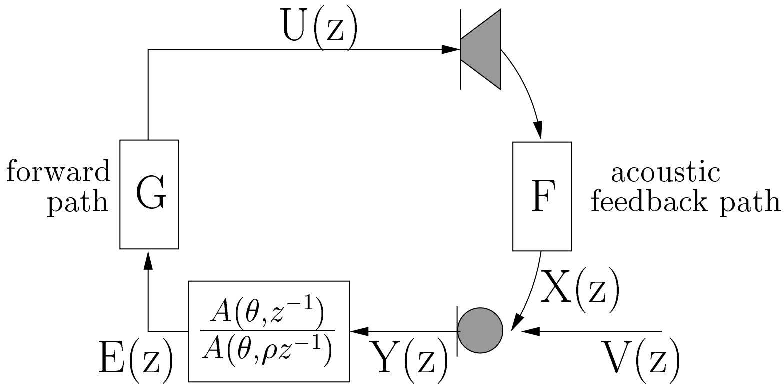

Figure 4: Including an adaptive notch filter in the 1-microphone/1-loudspeaker setup

too close to 1 can result in an unstable filter though. For optimal convergence a time-varying pole radius $\rho(t)$ could be applied, starting at a smaller value $\rho(0)$ (i.e. wider notches) and exponentially growing towards a final value $\rho(\infty)$:

$$\rho(t+1) = \lambda\rho(t) + (1-\lambda)\rho(\infty) \tag{11}$$

where λ corresponds to an exponential decay time constant [17]. For more details about the ANF's convergence and stability properties we refer to [18] and [19].

3.1.2 The ANF-LMS algorithm

A 2^{nd} order ANF of the type described above was applied to hearing aids by Kates [20], only to detect oscillations due to acoustic feedback. Later on, Maxwell et al. [21] employed Kates' algorithm to suppress feedback oscillations for comparison with adaptive feedback cancellation techniques. Their implementation in Direct Form II follows directly from the ANF tranfer function (6):

$$x(t) = y(t) + \rho(t)a(t-1)x(t-1) - \rho^2(t)x(t-2) \tag{12}$$
$$e(t) = x(t) - a(t-1)x(t-1) + x(t-2) \tag{13}$$

where $y(t)$ and $e(t)$ represent the ANF input resp. output as before, $x(t)$ is introduced as an auxiliary variable and some signs have been changed. This 2^{nd} order ANF has only one parameter $a(t)$ that appears in both transfer function numerator and denominator. Instead of solving the nonlinear minimization problem (10) the filter coefficient update is done in an approximate way as suggested by Travassos-Romano et al. [19]. Only the FIR portion of the filter is adapted to track the frequency of the narrow-band components and the coefficients are then copied to the IIR portion of the filter. The filter coefficient update can thus be calculated as follows:

$$
\begin{aligned}
a_{upd}(t) &= \arg\min_a (e^2(t)) && (14)\\
&= \arg\left(\frac{d}{da}e^2(t) = 0\right) && (15)\\
&= \arg\left(2e(t)\frac{d}{da}e(t) = 0\right) && (16)\\
&= \arg\Big(2e(t)(-x(t-1)) = 0\Big) && (17)
\end{aligned}
$$

where the last equality follows from (13). In this way we obtain an LMS filter update which completes the filter implementation given by (12)-(13):

$$
a(t) = a(t-1) + 2\mu e(t)x(t-1) \qquad (18)
$$

The 2^{nd} order ANF-LMS algorithm is summarized in **Algorithm 1**.

Algorithm 1 : 2^{nd} order ANF-LMS algorithm

Input step size μ, initial pole radius $\rho(0)$, final pole radius $\rho(\infty)$, exponential decay time constant λ, input data $\{y(t)\}_{t=1}^{N}$, initial conditions $x(0), x(-1), a(0)$

Output 2^{nd} order ANF parameter $\{a(t)\}_{t=1}^{N}$

for $t = 1, \ldots, N$ **do**

  $\rho(t) = \lambda\rho(t-1) + (1-\lambda)\rho(\infty)$

  $x(t) = y(t) + \rho(t)a(t-1)x(t-1) - \rho^2(t)x(t-2)$

  $e(t) = x(t) - a(t-1)x(t-1) + x(t-2)$

  $a(t) = a(t-1) + 2\mu e(t)x(t-1)$

end for

Higher order ANF's can be implemented in a similar way. As an example we give the difference equations describing an 8^{th} order ANF with LMS update:

$$
\begin{aligned}
x(t) = y(t) &+ \rho a_1(t-1)x(t-1) - \rho^2 a_2(t-1)x(t-2) + \rho^3 a_3(t-1)x(t-3)\\
&- \rho^4 a_4(t-1)x(t-4) + \rho^5 a_3(t-1)x(t-5) - \rho^6 a_2(t-1)x(t-6)\\
&+ \rho^7 a_1(t-1)x(t-7) - \rho^8 x(t-8)\\
e(t) = x(t) &- a_1(t-1)x(t-1) + a_2(t-1)x(t-2) - a_3(t-1)x(t-3)\\
&+ a_4(t-1)x(t-4) - a_3(t-1)x(t-5) + a_2(t-1)x(t-6)\\
&- a_1(t-1)x(t-7) + x(t-8)\\
\hat{\theta}(t) = \begin{bmatrix} a_1(t)\\ a_2(t)\\ a_3(t)\\ a_4(t) \end{bmatrix} &= \begin{bmatrix} a_1(t-1)\\ a_2(t-1)\\ a_3(t-1)\\ a_4(t-1) \end{bmatrix} + 2\mu e(t) \begin{bmatrix} x(t-1) + x(t-7)\\ -x(t-2) - x(t-6)\\ x(t-3) + x(t-5)\\ -x(t-4) \end{bmatrix}
\end{aligned}
\qquad (19)
$$

3.1.3 Experiments

Experimental setup. Computer simulations with the proposed ANF-LMS algorithm were done in MATLAB/SIMULINK, using a prerecorded speech signal for $V(z)$ (8 kHz, 16 bit). The forward path $G(z)$ was a simple amplifier model consisting of a user-adjustable gain K and a saturation. The acoustic feedback path transfer function $F(z)$ was assumed time-invariant and was calculated with the Image Source Method [22] for an empty room of dimensions 4.20 m $\times$ 3.70 m $\times$ 3.20 m with strongly reflective walls, floor and ceiling. Microphone and loudspeaker were assumed to be omnidirectional, be at fixed positions and have an ideally flat frequency response.

Simulations. For stability reasons the pole radius ρ and the adaptation step size μ were assigned fixed values. A pole radius $\rho = 0.9$ gives sufficiently narrow notches while ensuring a stable filter. The adaptation step size μ has to be chosen small enough to minimize the effect of changes in the input speech spectrum $V(e^{j\omega})$ on the adaptation. Moreover a μ chosen too large will threaten the ANF stability. We found $\mu = 10^{-6}$ to be an optimal choice in most cases.

Without feedback control the system reaches its stability limit when the amplifier gain is set to $K = 9$ dB. This means the system does not become unstable but on the other hand there is a continuous ringing sound which does not disappear. A 2^{nd} order ANF succeeds at suppressing this feedback oscillation although not immediately. When raising the gain to $K = 11$ dB several oscillating feedback frequencies arise making the need for a higher filter order obvious. An 8^{th} order ANF can do the job, but not in an efficient way. We notice that, when a new oscillating feedback frequency arises, all four filter coefficients move in its direction instead of only one coefficient. As we raise the gain to $K = 15$ dB the system has become fundamentally unstable and even a 32^{th} order ANF does not succeed in suppressing all oscillating feedback frequencies.

Conclusions. An adaptive notch filter can be applied very efficiently to remove "soft" ringing sounds in a system at the edge of instability. Once the gain is raised beyond this point the number of oscillating feedback frequencies increases rapidly, requiring a higher ANF order. There is a limit to this though: a gain increase of 5 dB (relative to the maximum stable gain of the system without feedback control) does not allow the ANF to converge, no matter how high the order is chosen. For systems that require an addition in stable gain of more than 5 dB, another feedback control solution has to be sought (e.g. AFC).

Moreover we noticed that a higher order ANF (≥ 8) suffers from the fact that several filter coefficients try to converge to the same oscillating feedback frequency. As suggested in [23] this problem could be tackled by introducing subband filtering, assigning one 2^{nd} order ANF to each subband. Simulations showed that subband filtering can improve convergence but at high gains leads to broadband attenuation of the microphone signal.

References

[1] T. van Waterschoot and M. Moonen, "Fifty years of acoustic feedback control: state of the art and future challenges," *Proc. IEEE*, vol. 99, no. 2, pp. 288–327, Feb. 2011.

[2] P. Mapp and C. Ellis, "Improvements in acoustic feedback margin in sound reinforcement systems," in *Preprints AES 105th Convention*, San Francisco, CA, USA, Sept. 1998, AES Preprint 4850.

[3] E. Straus, "The art of equalization – sound systems and narrow-band anomalies revisited," S&VC Magazine, Dec. 2000.

[4] M. R. Schroeder, "Improvement of acoustic-feedback stability by frequency shifting," *J. Acoust. Soc. Amer.*, vol. 36, no. 9, pp. 1718–1724, Sept. 1964.

[5] E. T. Patronis, Jr., "Acoustic feedback detector and automatic gain control," Mar. 1978.

[6] D. A. Preves, "The effect of phase compensation on the group delay of hearing aids," in *Proc. IEEE 7th Annual Conf. Eng. Med. Biol. Society*, 1985, pp. 1109–1113.

[7] D. K. Bustamante, T. L. Worrall, and M. J. Williamson, "Measurement and adaptive suppression of acoustic feedback in hearing aids," in *Proc. 1989 IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP '89)*, Glasgow, UK, 1989, vol. 3, pp. 2017–2020.

[8] G. W. Elko and M. M. Goodwin, "Beam dithering: Acoustic feedback control using a modulated-directivity loudspeaker array," in *Proc. 1993 IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP '93)*, Minneapolis, MN, USA, Apr. 1993, vol. 1, pp. 173–176.

[9] H. A. L. Joson, F. Asano, Y. Suzuki, and T. Sone, "Adaptive feedback cancellation with frequency compression for hearing aids," *J. Acoust. Soc. Amer.*, vol. 94, no. 6, pp. 3248–3254, Dec. 1993.

[10] M. Miyoshi and Y. Kaneda, "Inverse filtering of room acoustics," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-36, no. 2, pp. 145–152, Feb. 1988.

[11] P. A. Nelson, H. Hameda, and S. J. Elliot, "Adaptive inverse filters for stereophonic sound reproduction," *IEEE Trans. Signal Process.*, vol. 40, no. 7, pp. 1621–1632, July 1992.

[12] Y. Haneda, S. Makino, and Y. Kaneda, "Multiple-point equalization of room transfer functions by using common acoustical poles," *IEEE Trans. Speech Audio Process.*, vol. 5, no. 4, pp. 325–333, July 1997.

[13] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Prentice-Hall, Englewood Cliffs, New Jersey, 1985.

[14] T. van Waterschoot and M. Moonen, "Comparative evaluation of howling detection criteria in notch-filter-based howling suppression," *J. Audio Eng. Soc.*, vol. 58, no. 11, pp. 923–940, Nov. 2010.

[15] D. V. B. Rao and S.-Y. Kung, "Adaptive notch filter for the retrieval of sinusoids in noise," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-32, no. 4, pp. 791–802, Aug. 1984.

[16] B. Widrow et al., "Adaptive noise cancelling: Principles and applications," *Proc. IEEE*, vol. 63, pp. 1692–1716, Dec. 1975.

[17] A. Nehorai, "A minimal parameter adaptive notch filter with constrained poles and zeros," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-33, no. 4, pp. 983–996, Aug. 1985.

[18] T. S. Ng, "Some aspects of an adaptive digital notch filter with constrained poles and zeros," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-35, no. 2, pp. 158–161, Feb. 1987.

[19] J. M. Travassos-Romano and M. Bellanger, "Fast least squares adaptive notch filtering," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-36, no. 9, pp. 1536–1540, Sept. 1988.

[20] J. M. Kates, "Feedback cancellation in hearing aids: Results from a computer simulation," *IEEE Trans. Signal Process.*, vol. 39, no. 3, pp. 553–562, Mar. 1991.

[21] J. A. Maxwell and P. M. Zurek, "Reducing acoustic feedback in hearing aids," *IEEE Trans. Speech Audio Process.*, vol. 3, no. 4, pp. 304–313, July 1995.

[22] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Amer.*, vol. 65, no. 4, pp. 943–950, Apr. 1979.

[23] Y. Kinugasa, J. Okello, Y. Itoh, M. Kobayashi, and Y. Fukui, "A new algorithm for adaptive notch filter with sub-band filtering," in *Proc. 2001 IEEE Int. Symp. Circuits Syst. (ISCAS '01)*, May 2001, vol. 2, pp. 817–820.