Summer Course

Linear System Theory

Control

&

Matrix Computations

Monopoli, Italy **September 8-12, 2008**

Lecture 13

Modeling

Interconnected Systems

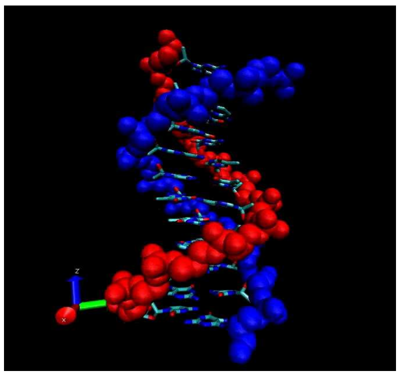
Lecturer: Jan C. Willems

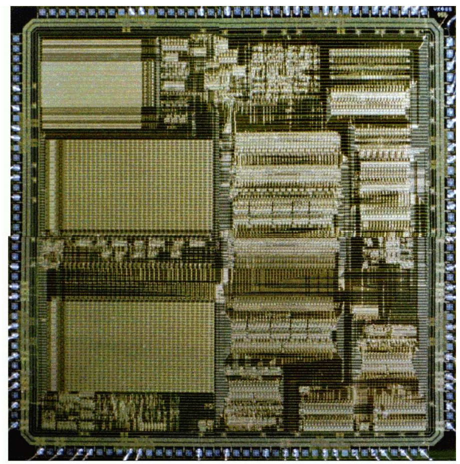
Outline

- Open, connected, and modular
- How do paradigms cope with this?
- Classical dynamical systems
- Input/output systems
- Modeling by tearing, zooming, and linking
- Bond graphs
- Control as interconnection

Systems


ZYGOTE





Features

- **open**
- **interconnected**
- **modular**
- **dynamic**

Features

- open
- interconnected
- modular
- dynamic

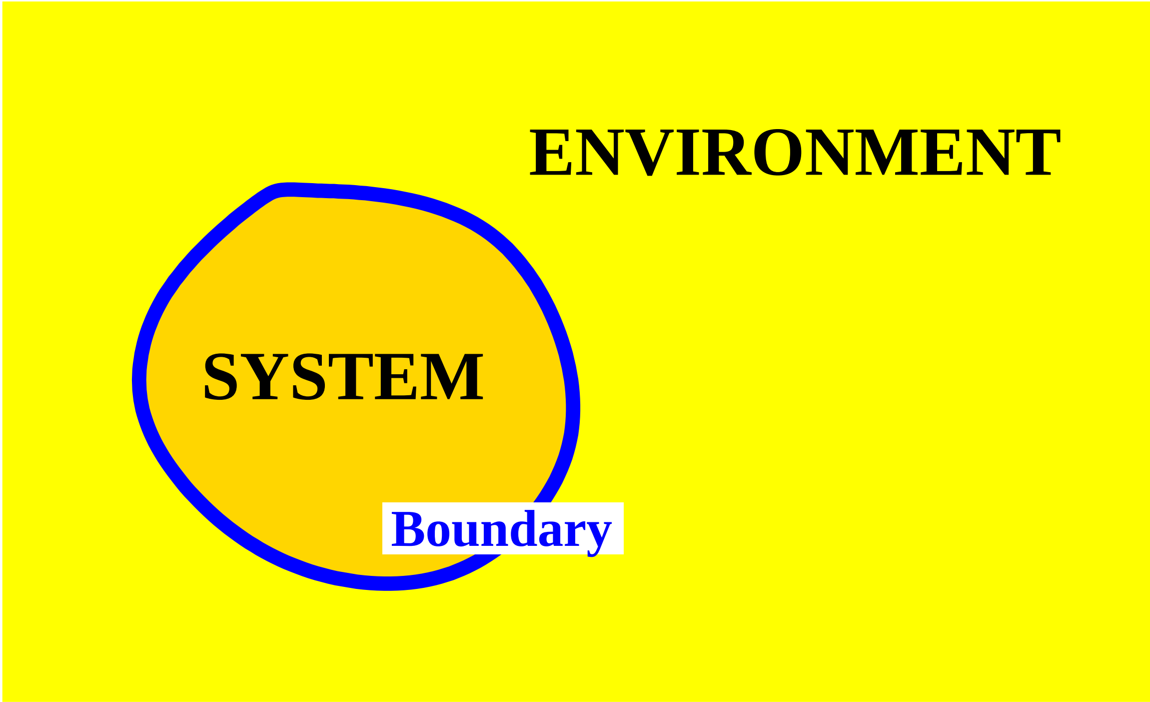
Theme of this lecture:

develop a suitable mathematical language

aimed at computer-assisted modeling.

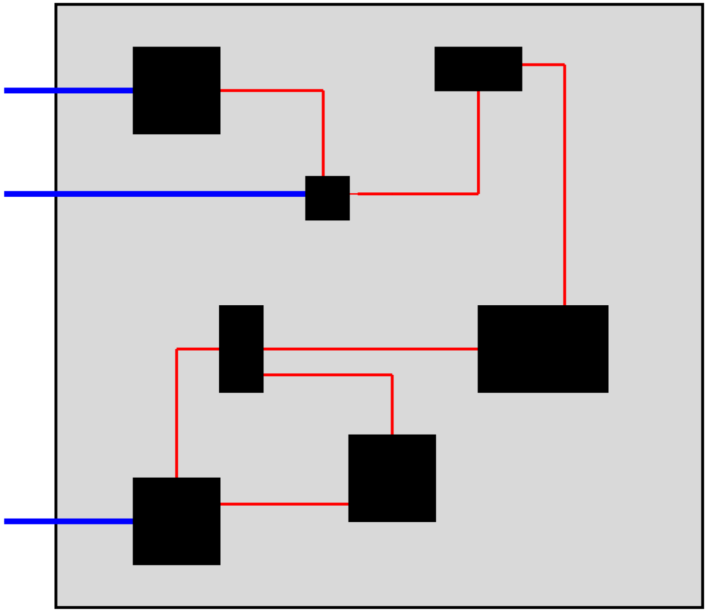
Open, connected, and modular

Open



Systems interact with their environment

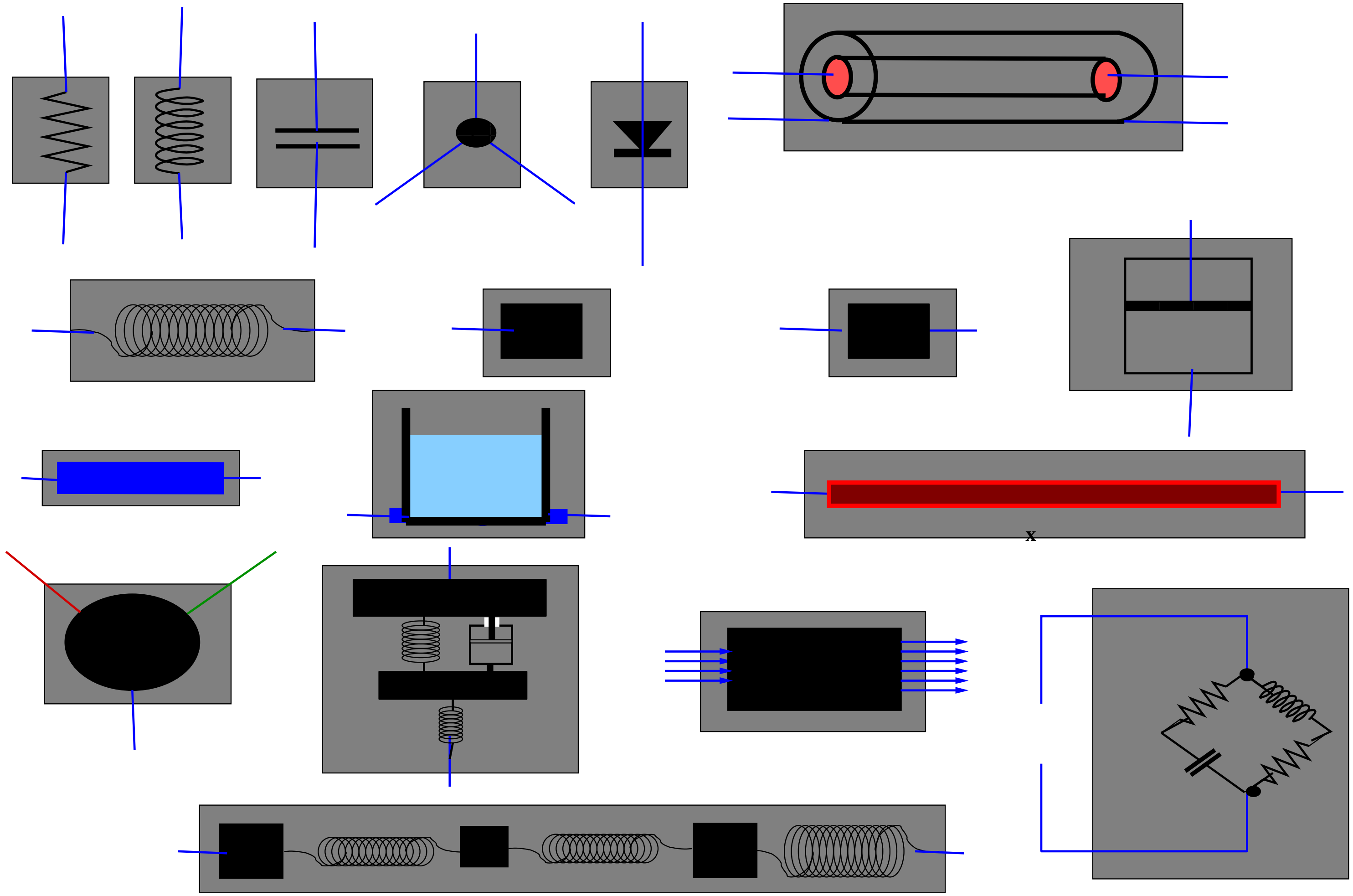
Architecture



Systems consist of subsystems, interconnected

Modular

Systems consist of an interconnection of ‘building blocks’



The development of the notion of a dynamical system

a brief causerie

Mathematization

1. **Get the physics right**
2. **The rest is mathematics**

R.E. Kalman
Opening lecture
IFAC World Congress
Prague, July 4, 2005

Mathematization

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Prima la fisica, poi la matematica

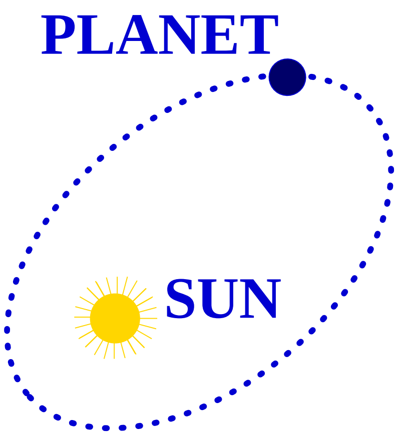
How it all began ...

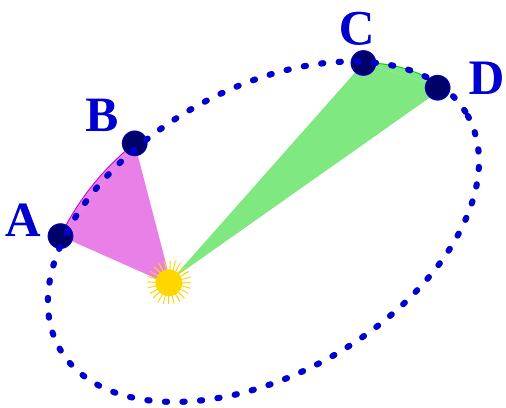
The celestial question

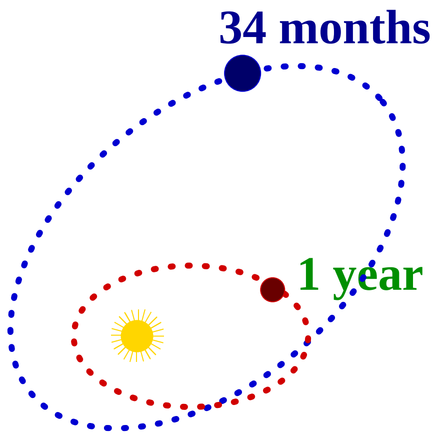
How, for heaven's sake, does it move?

Kepler's laws

Johannes Kepler
1571-1630







Kepler's laws:

Ellipse, sun in focus;
= areas in = times;
(period)$^2 \cong$ (diameter)^3

The equation of the planet

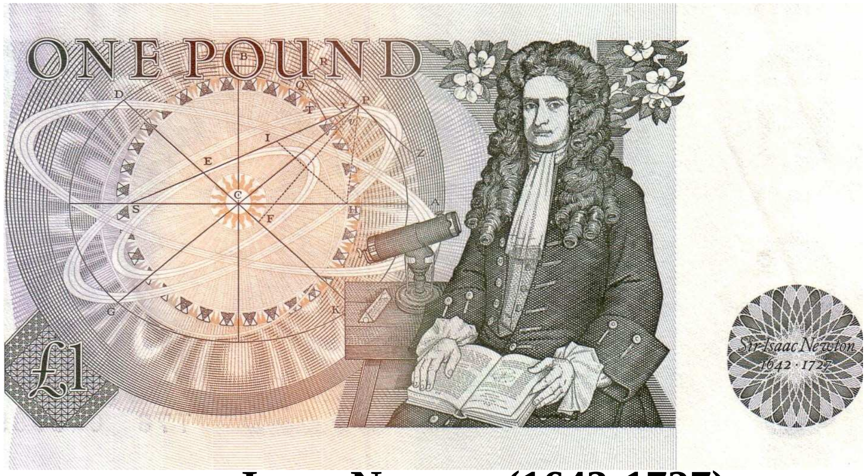
Consequence:

acceleration = function of position and velocity

$$\frac{d^2}{dt^2}w(t) = A(w(t), \frac{d}{dt}w(t))$$

$\rightsquigarrow$ **via calculus and calculation**

$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0$$



Isaac Newton (1643-1727)

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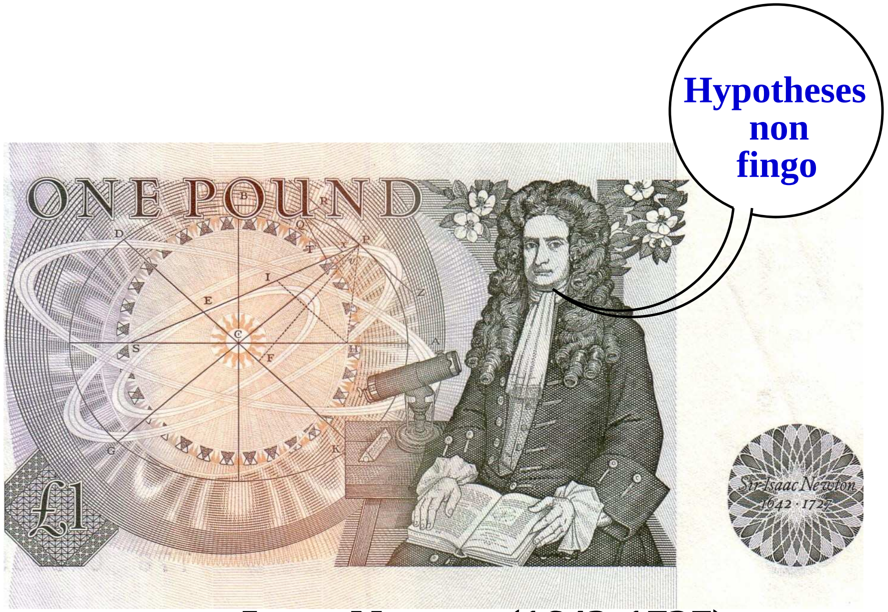
$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0$$

$\cong$ **another representation of K.1, K.2, K.3**



Isaac Newton (1643-1727)

Newton's laws

2-nd law $F'(t) = m\frac{d^2}{dt^2}w(t)$

gravity $F''(t) = m\frac{1_{w(t)}}{|w(t)|^2}$

3-rd law $F'(t) + F''(t) = 0$

$$\Downarrow$$

$$\boxed{\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0}$$

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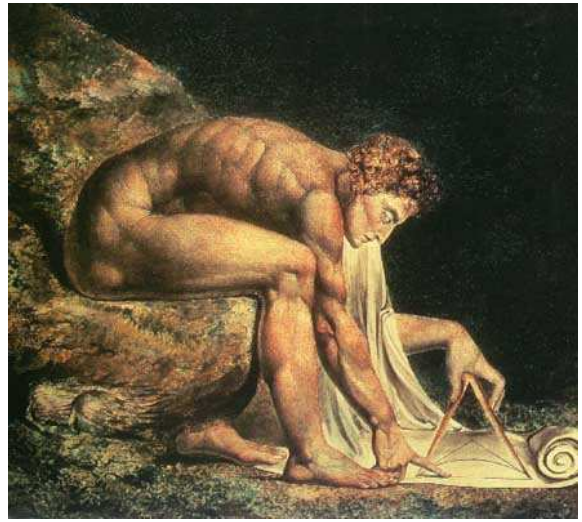

Isaac Newton by William Blake

$$\Downarrow$$

$$\boxed{\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0}$$

Viewing as interconnection is the key to generalization

The paradigm of *closed* systems

‘Axiomatization’

K.1, K.2, & K.3

$$\rightsquigarrow \quad \frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|\frac{d}{dt}w(t)|^2} = 0$$

$\rightsquigarrow$ **with** $x = (w, \frac{d}{dt}w)$ $\frac{d}{dt}x = f(x)$

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$\rightsquigarrow$ **with** $x = (w, \frac{d}{dt}w)$ $\frac{d}{dt}x = f(x)$

$\rightsquigarrow$ **generalization** $\frac{d}{dt}x = f(x)$

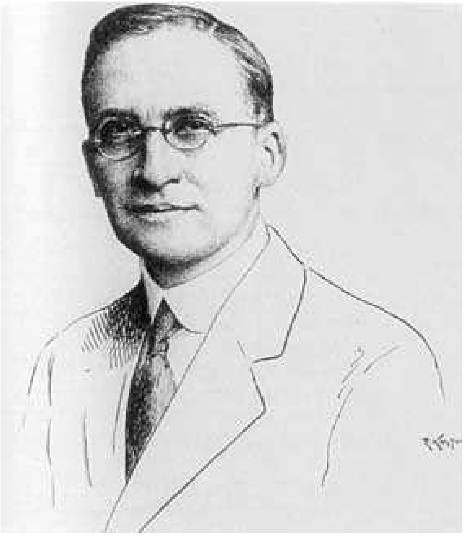
$\rightsquigarrow$ **‘dynamical systems’, flows**

$\rightsquigarrow$ **flows as paradigm of dynamics** $\rightsquigarrow$ **closed systems**

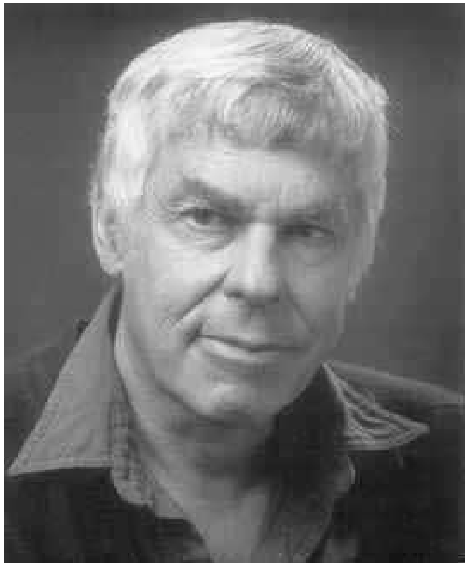
Motion determined by internal initial conditions.

‘Axiomatization’

Henri Poincaré (1854-1912)



George Birkhoff (1884-1944)



Stephen Smale (1930-   )

‘Axiomatization’

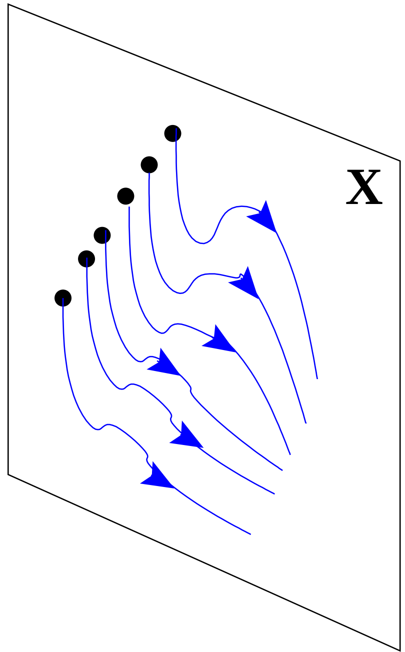
A *dynamical system* is defined by

a state space X and

a state transition function

ϕ : $\cdots$ **such that** $\cdots$

$\phi(t, \mathrm{x})$ **= state at time** t **starting from state** x



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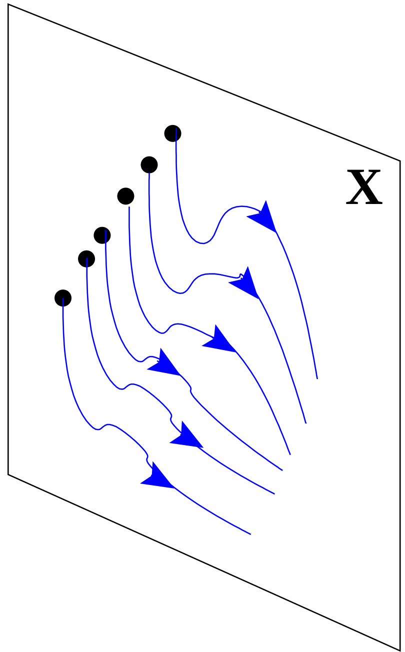

$\phi(t, x)$ **= state at time t starting from state x**

This framework of closed systems is universally used for dynamics in mathematics and physics

‘Axiomatization’

How could they forget Newton’s 2^{nd} **law,**
about Maxwell’s eq’ns,
about thermodynamics,
about tearing & zooming & linking, ...?

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Reply: assume ‘fixed boundary conditions’

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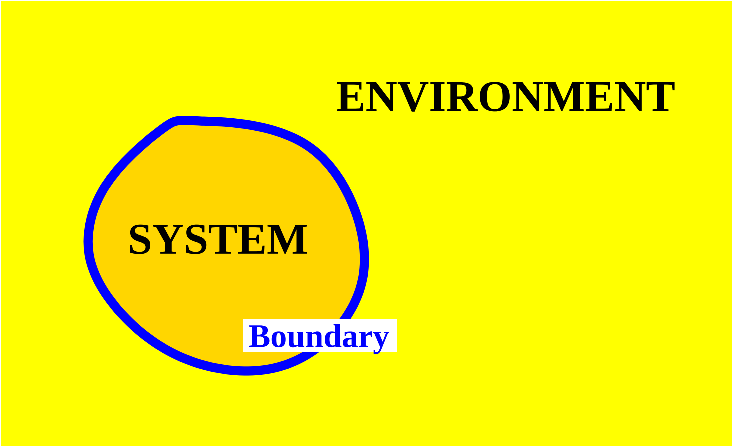

$\leadsto$ **to model a system, we have to model also the environment!**

‘Axiomatization’

How could they forget Newton’s 2^{nd} law,
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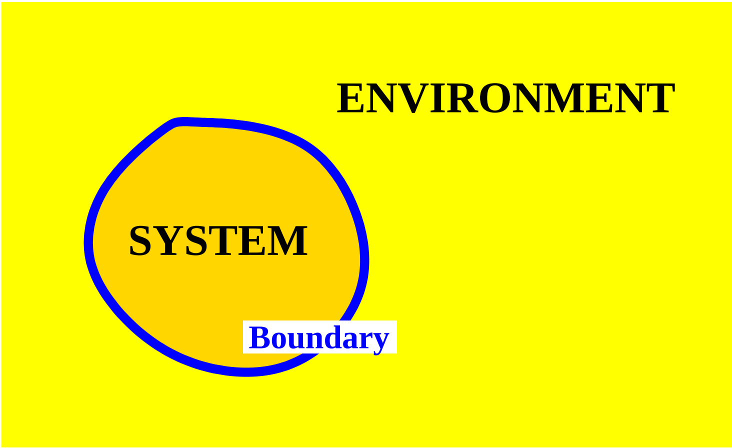
Reply: assume ‘fixed boundary conditions’



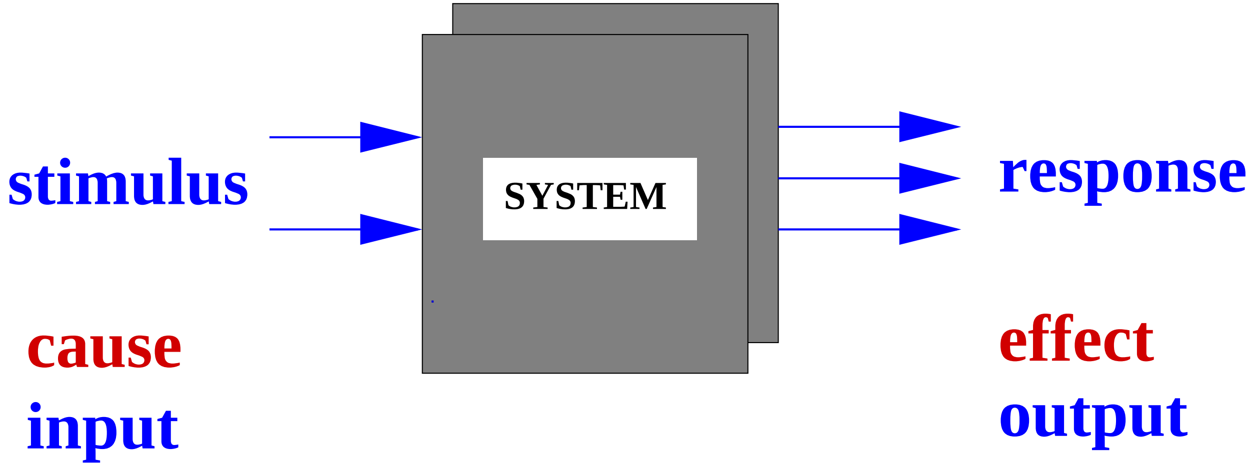
$\leadsto$ to model a system, we have to model also the environment!

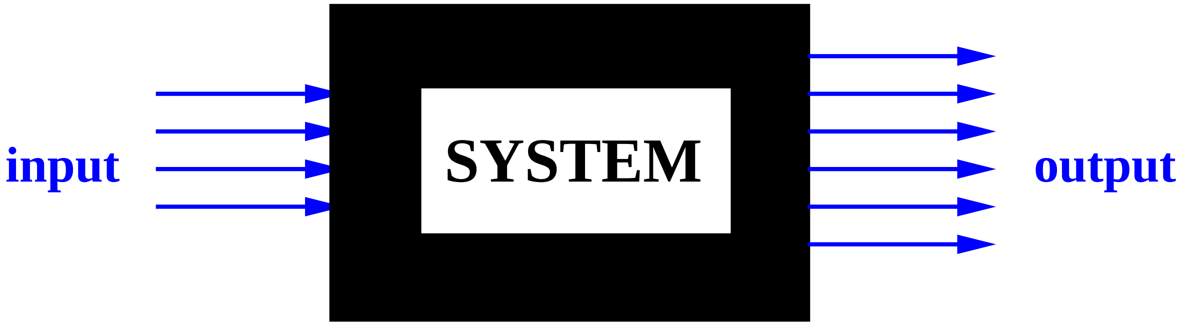
Chaos theory, cellular automata, sync, etc., function in this framework ...

Inputs and outputs

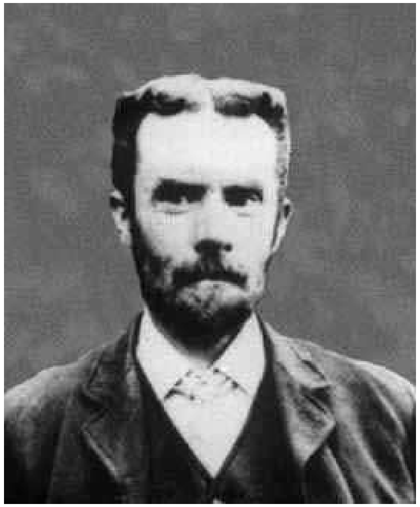
meanwhile, in engineering...

Input/output systems

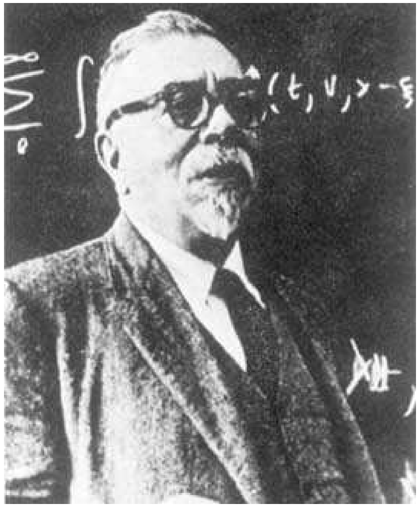




The originators



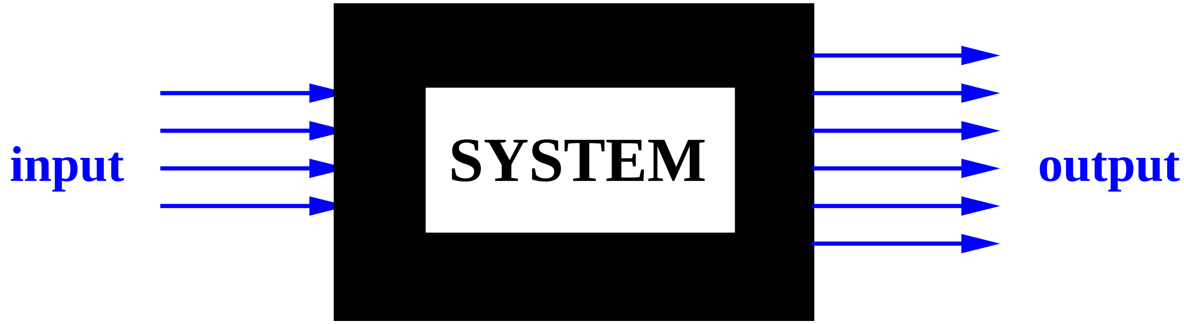
Oliver Heaviside (1850-1925)



Norbert Wiener (1894-1964)

and the many electrical circuit theorists ...

Mathematical description

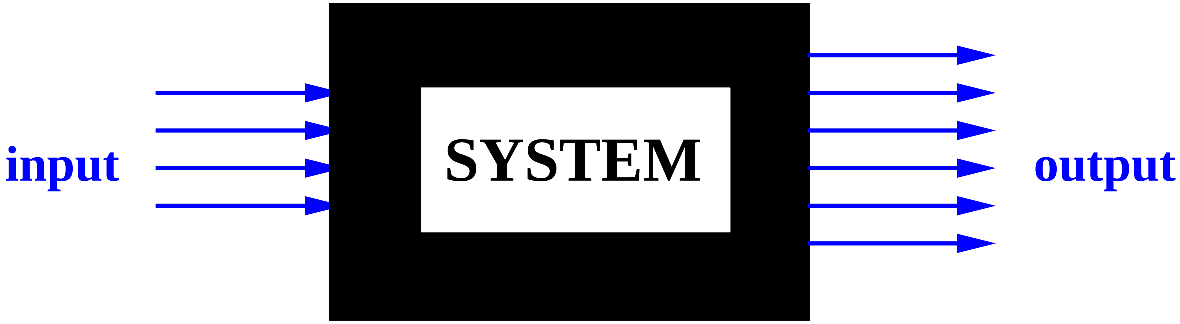


u: **input**, y: **output**,

SISO, LTI case $\rightsquigarrow$ $G(s) = \frac{q(s)}{p(s)}$ **transfer functions, impedances, admittances.**

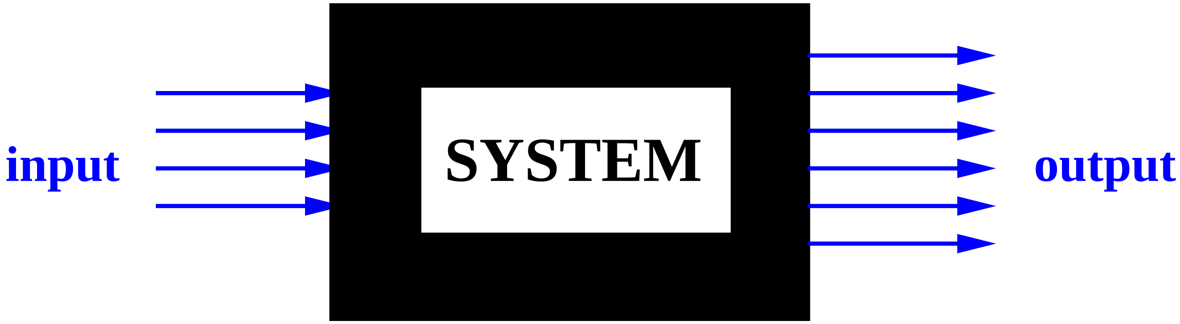
Circuit analysis and synthesis
Classical control
Bode, Nyquist, root-locus.

Mathematical description



$$y(t) = \int_{0 \textbf{ or } -\infty}^{t} H(t-t')u(t')\, dt'$$

Mathematical description



$$y(t) = \int_{0 \textbf{ or } -\infty}^{t} H(t-t')u(t')\,dt'$$

$$y(t) = H_0(t) + \int_{-\infty}^{t} H_1(t-t')u(t')\,dt' + \int_{-\infty}^{t}\int_{-\infty}^{t'} H_2(t-t', t'-t'')u(t')u(t'')\,dt'dt'' + \cdots$$

Awkward nonlinear — far from the physics

Fail to deal with ‘initial conditions’.

Input/state/output systems

Around 1960: a paradigm shift to

$$\frac{d}{dt}x = f(x,u),\ y = g(x,u)$$

Rudolf Kalman (1930-)

Input/state/output systems

Around 1960: a paradigm shift to

$$\frac{d}{dt}x = f(x,u),\ y = g(x,u)$$

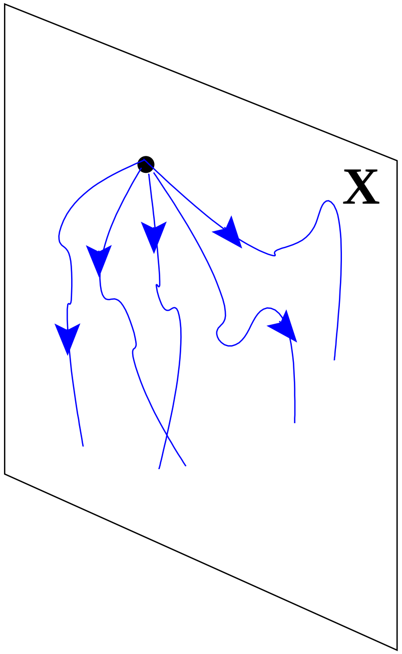
Rudolf Kalman (1930-)

- **open**
- **ready to be interconnected**
  outputs of one system $\mapsto$ inputs of another
- **deals with initial conditions**
- **incorporates nonlinearities, time-variation**
- **models many physical phenomena**
- $\cdots$

‘Axiomatization’

State transition function:

$\phi(t, \mathrm{x}, u)$: **state reached at time t from x using input u.**



$\frac{d}{dt}x = f(x, u),\ y = g(x, u)$

Read-out function:

$g(\mathrm{x}, \mathrm{u})$: **output value with state x and input value u.**

The input/state/output paradigm

The input/state/output view turned out to be very effective and fruitful

- **for modeling**
- **for control (stabilization, robustness, ...)**

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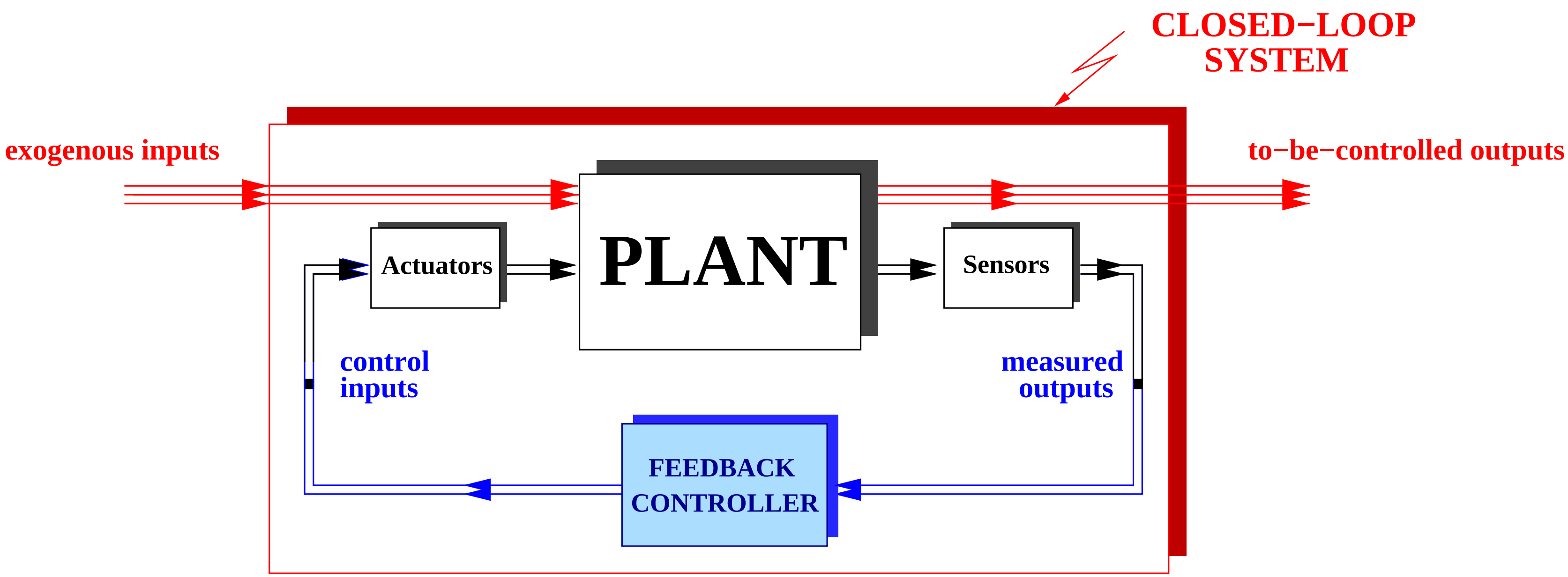


The input/state/output paradigm

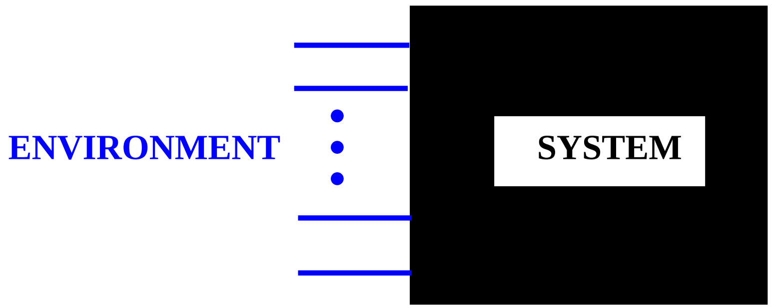
The input/state/output view turned out to be very effective and fruitful

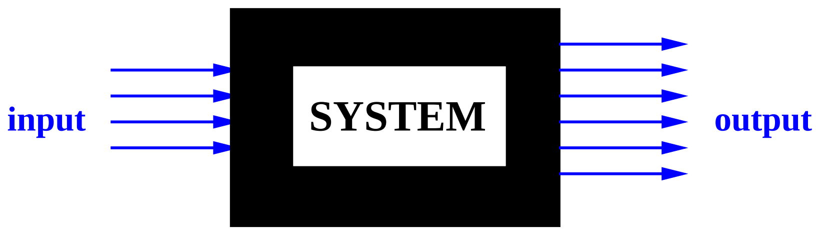
- **for modeling**
- **for control (stabilization, robustness, ...)**
- **prediction of one signal from another, filtering**
- **understanding system representations (transfer f'n, input/state/output repr., etc.)**
- **model simplification, reduction**
- **system ID: models from data**
- **etc., etc., etc.**

Theme

Theme of this lecture

We are accustomed to view an open dynamical system as an input/output structure

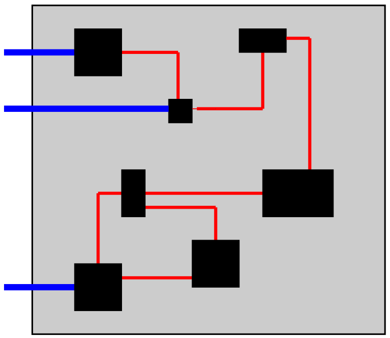


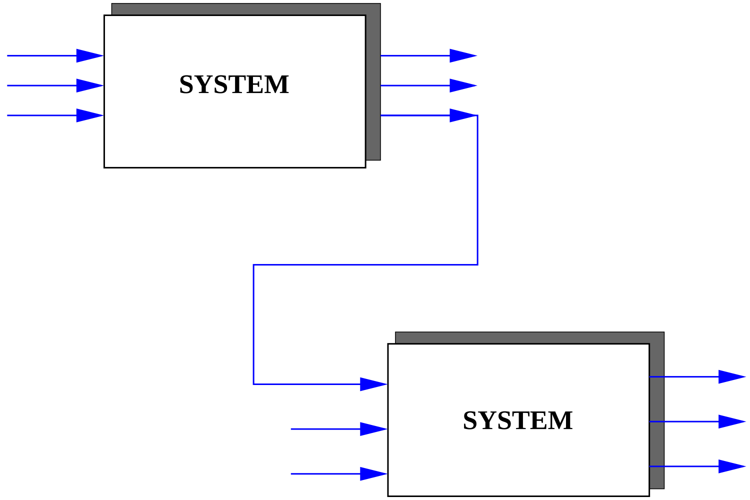


Is this appropriate for modeling physical systems?

Theme of this lecture

and interconnection as output-to-input assignment





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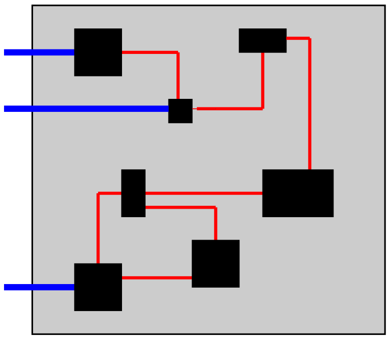
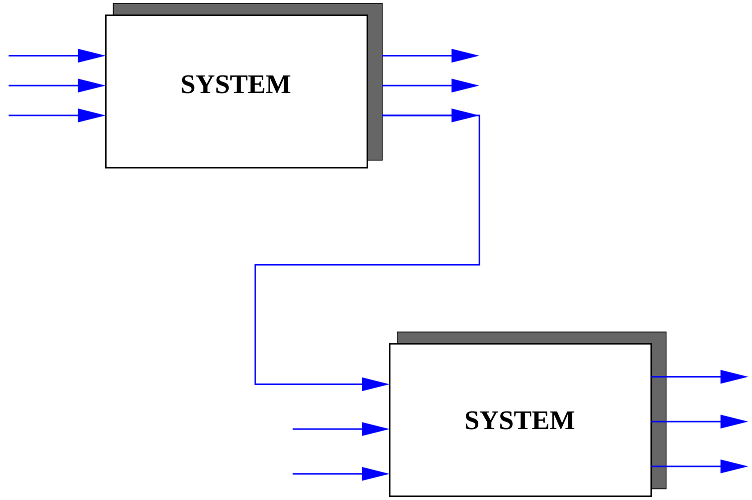
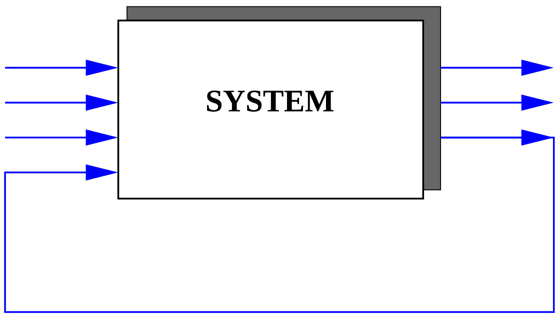
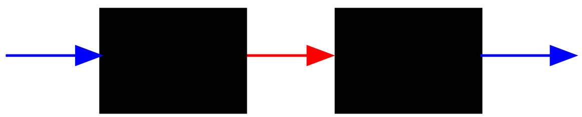


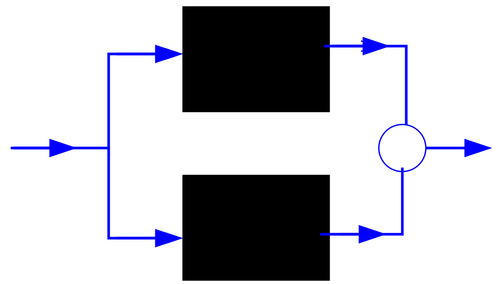




Feedback



Series



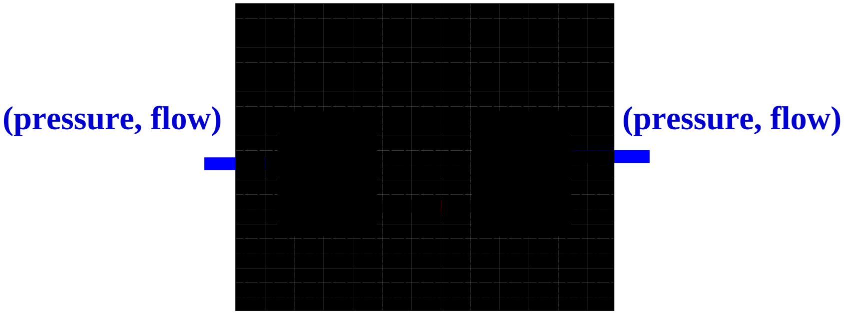
Parallel

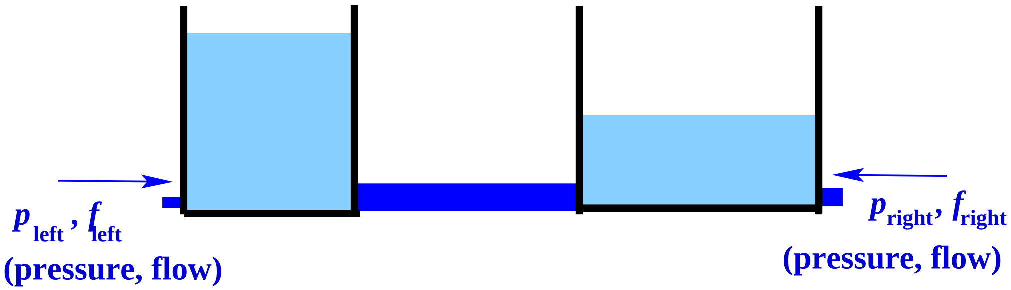
Is this appropriate for modeling physical systems?

Interconnection in physical systems

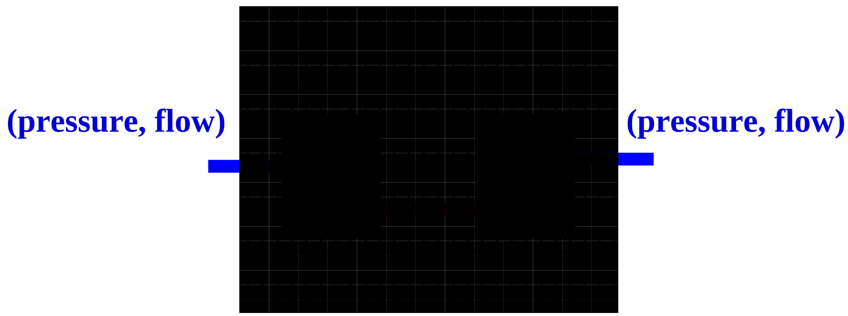
We have seen an extensive example in lecture 1.
We now give a simple example from hydraulics.

Tearing


(pressure, flow)
(pressure, flow)


p_{left}, f_{left}
(pressure, flow)
p_{right}, f_{right}
(pressure, flow)

Tearing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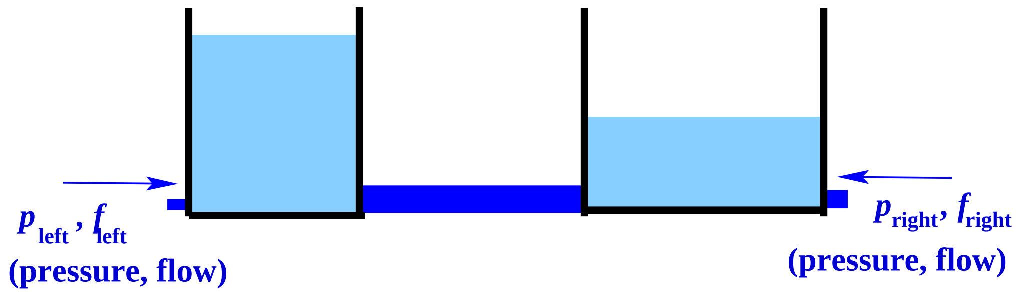
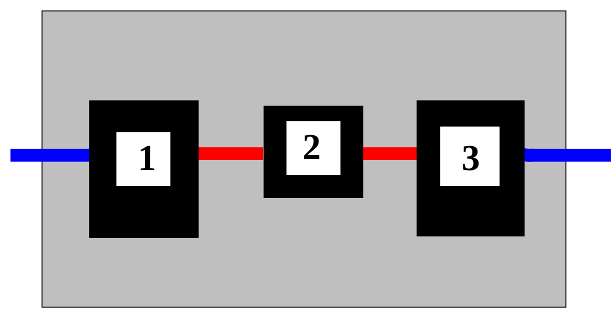
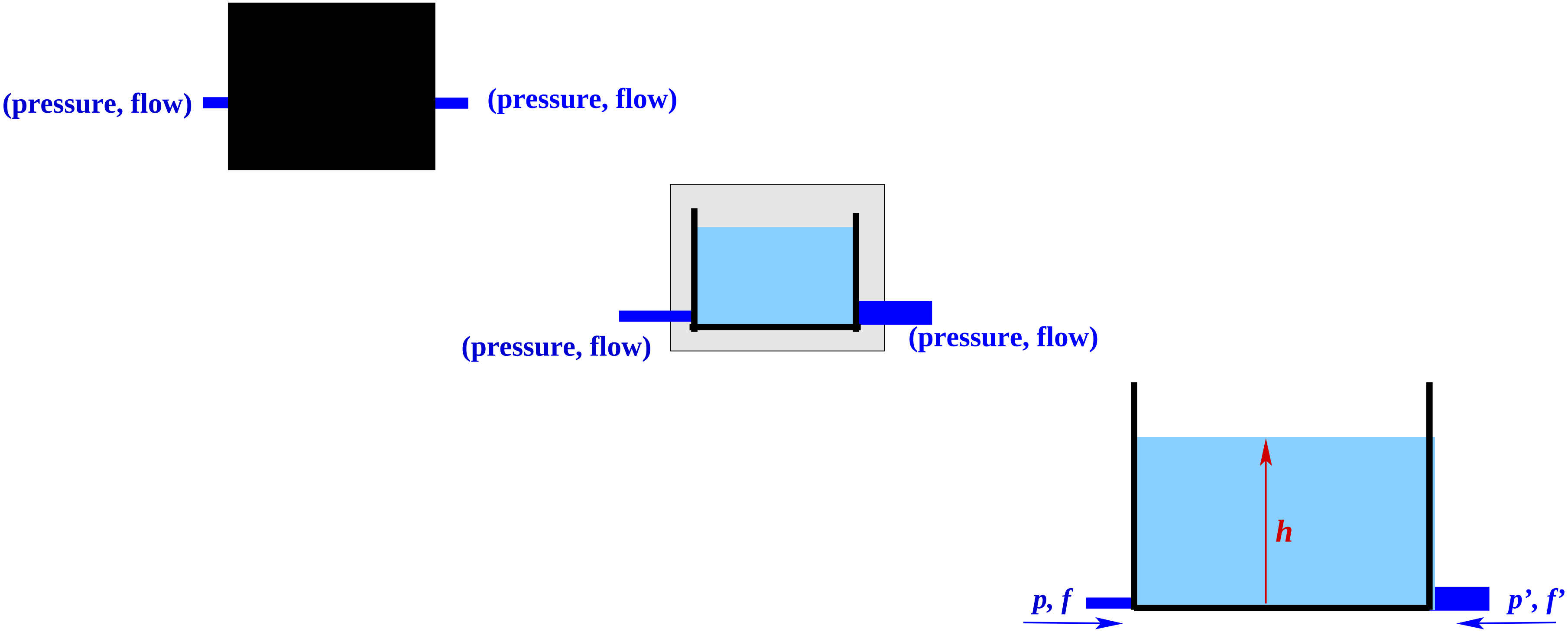

p_{left} , f_{left}
(pressure, flow)
p_{right}, f_{right}
(pressure, flow)


1
2
3

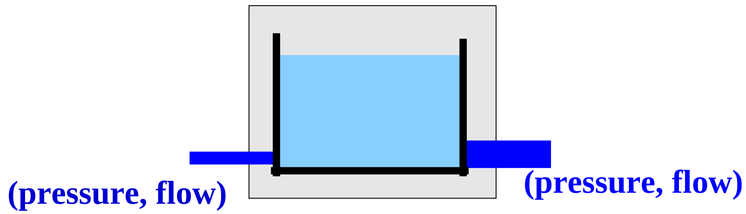
Zooming

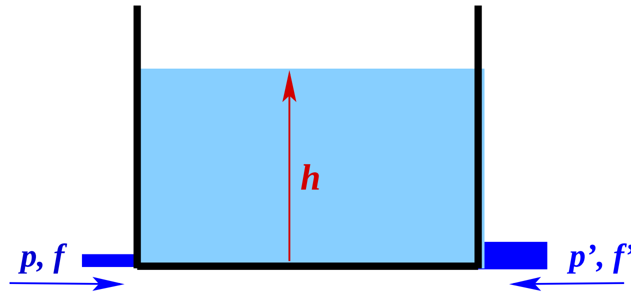
Subsystems 1 and 3:



Zooming

Subsystems 1 and 3:





$$A\frac{d}{dt}h = f + f',$$

$$Bf = \begin{cases} \sqrt{|p - p_0 - \rho h|} & \textbf{if } p - p_0 \geq \rho h, \\ -\sqrt{|p - p_0 - \rho h|} & \textbf{if } p - p_0 \leq \rho h, \end{cases}$$

$$Cf' = \begin{cases} \sqrt{|p' - p_0 - \rho h|} & \textbf{if } p' - p_0 \geq \rho h, \\ -\sqrt{|p' - p_0 - \rho h|} & \textbf{if } p' - p_0 \leq \rho h, \end{cases}$$

Zooming

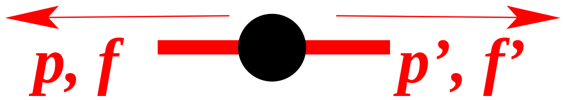
Subsystem 2:

Zooming

Subsystem 2:

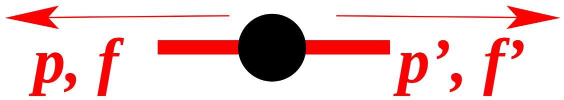
$$f = -f', \quad p - p' = \alpha f$$

Interconnection laws:



Linking

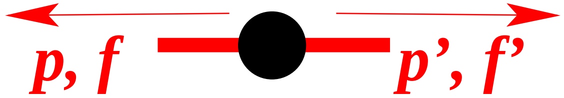
Interconnection laws:



$$p = p', \qquad f + f' = 0.$$

Linking

Interconnection laws:



$$p = p', \qquad f + f' = 0.$$

Leads to the complete model:

$$A_1 \frac{d}{dt} h_1 = f_1 + f_1',$$

$$B_1 f_1 = \begin{cases} \sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \geq \rho h_1, \\ -\sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \leq \rho h_1, \end{cases} \qquad \textbf{(blackbox 1)}$$

$$C_1 f_1' = \begin{cases} \sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \geq \rho h_1, \\ -\sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \leq \rho h_1, \end{cases}$$

$$f_2 = -f_2', \quad p_2 - p_2' = \alpha f_2, \qquad \textbf{(blackbox 2)}$$

$$A_3 \frac{d}{dt} h_3 = f_3 + f_3',$$

$$C f_3 = \begin{cases} \sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \geq \rho h_3, \\ -\sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \leq \rho h_3, \end{cases} \qquad \textbf{(blackbox 3)}$$

$$C_3 f_3' = \begin{cases} \sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \geq \rho h_3, \\ -\sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \leq \rho h_3, \end{cases}$$

$$p_1' = p_2, \quad f_1' + f_2 = 0, \; p_2' = p_3, \quad f_2' + f_3 = 0. \qquad \textbf{(interconnection)}$$

$$p_{\text{left}} = p_1, \; f_{\text{left}} = f_1, \; p_{\text{right}} = p_3', \; f_{\text{right}} = f_3'. \qquad \textbf{(manifest variable assignment)}$$

- **Unclear input/output structure for terminal variables**
- **Many variables, indivisibly, at the same terminal**
- **Interconnection = variable sharing**
- **No signal flows, no output-to-input assignment**

**These remarks pertain to every physical interconnection.
And, ultimately, every interconnection is physical**

- Unclear input/output structure for terminal variables
- Many variables, indivisibly, at the same terminal
- Interconnection = variable sharing
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These remarks pertain to every physical interconnection. And, ultimately, every interconnection is physical

"Block diagrams unsuitable for serious physical modeling

- the control/physics barrier"

"Behavior based (declarative) modeling is a good alternative"

Karl Åström (born 1934)

from K.J. Åström, *Present Developments in Control Applications*

IFAC 50

IFAC 50-th Anniversary Celebration
Heidelberg, September 12, 2006.

Behavioral systems

Behavioral approach

A dynamical system

:⇔ **a family of time functions, *'the behavior'***

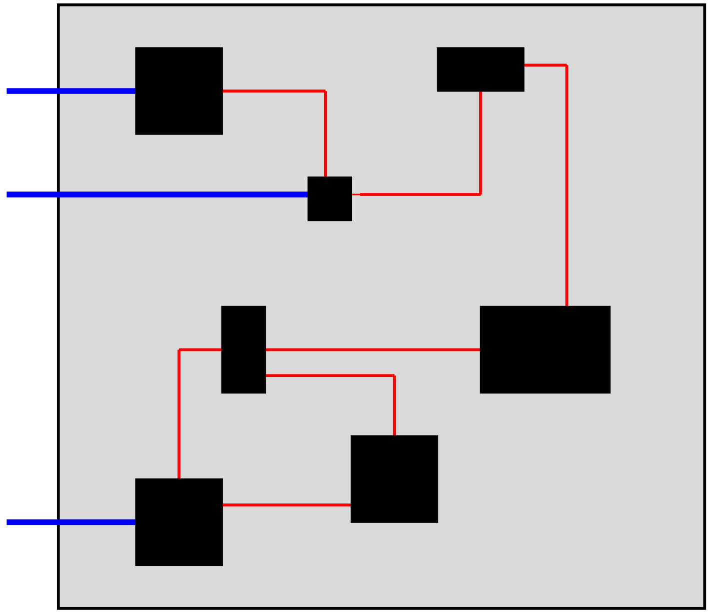
Interconnection :⇔ ***'variable sharing'*.**

Control :⇔ ***interconnection*.**

Modeling of interconnected physical systems is the strongest case for 'behaviors'.

Interconnection architecture

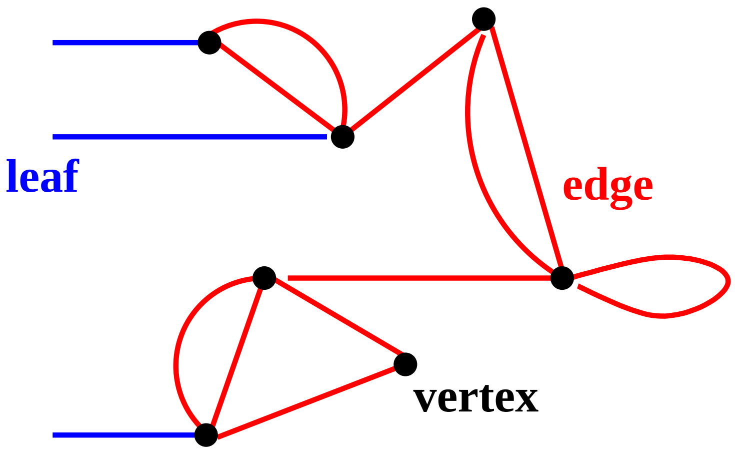
Objective



Formalize mathematically interconnection of systems.

Architecture & module embedding

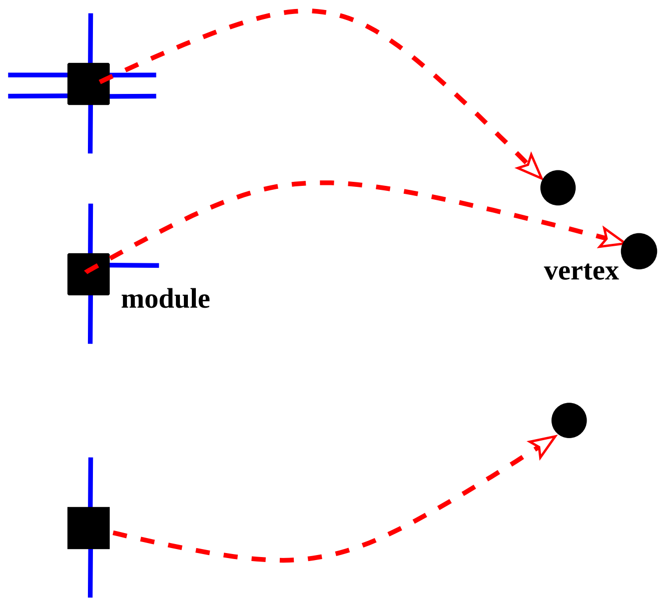
Architecture



nodes	$\rightsquigarrow$	systems with terminals
edges	$\rightsquigarrow$	connected terminals
leaves	$\rightsquigarrow$	interaction with environment

Architecture & module embedding

Modules (systems) in the vertices



nodes	$\rightsquigarrow$	systems with terminals
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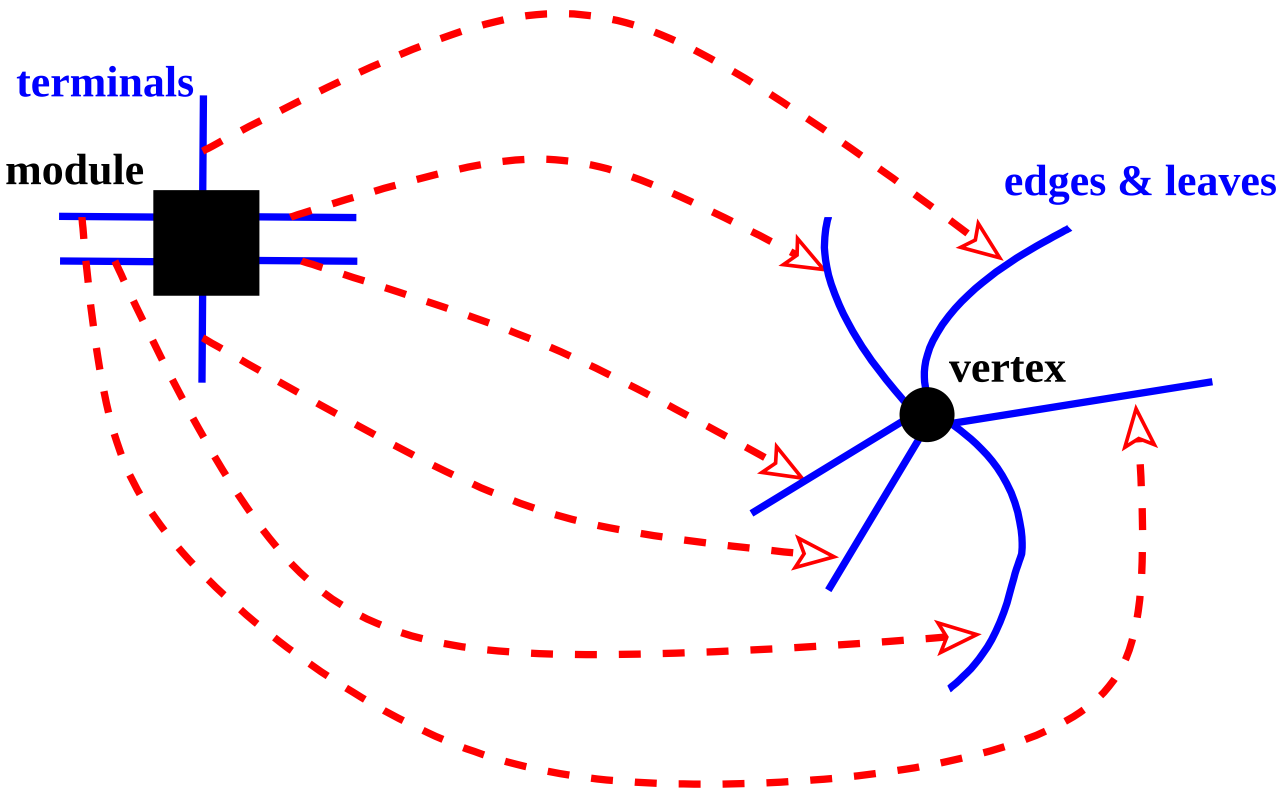
Architecture & module embedding

Terminals in the edges



nodes $\rightsquigarrow$ **systems with terminals**

edges $\rightsquigarrow$ **connected terminals**

leaves $\rightsquigarrow$ **interaction with environment**

Interconnection architecture

A *graph with leaves* is defined as $\mathscr{G} = (\mathbb{V}, \mathbb{E}, \mathbb{L}, \mathscr{A})$

- $\mathbb{V}$ **the set of *vertices*,**
- $\mathbb{E}$ **the set of *edges*,**
- $\mathbb{L}$ **the set of *leaves*,**
- $\mathscr{A}$ **the *adjacency map*.**

$\mathscr{A}$ **associates**

with each edge $e \in \mathbb{E}$ **an unordered pair**

$$\mathscr{A}(e) = [v_1, v_2] \quad v_1, v_2 \in \mathbb{V},$$

with each leaf $\ell \in \mathbb{L}$ **an element** $\mathscr{A}(\ell) = v \in \mathbb{V}$**.**

Module embedding

The ***module embedding*** **associates**

a module with each vertex,
a $1 \leftrightarrow 1$ **assignment between the**
edges and leaves adjacent to the vertex and
the terminals of the associated module.

Module embedding

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vertices specify the subsystems,

edges how terminals of subsystems are connected,

leaves how the interconnected system interacts with the environment.

Module embedding

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leaves how the interconnected system interacts with the environment.

Vertices $\rightsquigarrow$ Subsystems **Edges $\rightsquigarrow$ Interconnections**

Contrast with circuit theory:
systems in the edges, interconnections in vertices.

Manifest variables

The *manifest variable assignment* is a map that assigns the manifest variables as a function of the terminal (or, more generally, the module) variables.

The terminal variables are henceforth considered as latent (auxiliary) variables.

Behavioral equations

1. **Module equations** for each vertex.

   Relation among the variables on the terminals.

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

3. **Manifest variable assignment**

   Specifies the variables of interest.

Behavioral equations

1. **Module equations** for each vertex.

   Relation among the variables on the terminals.

   Behavioral equations stored as (parametrized) modules in a data-base

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

   Interconnection laws stored in a data-base.
   Laws depend on terminal type:
   electrical / mechanical / hydraulic / etc.

3. **Manifest variable assignment**

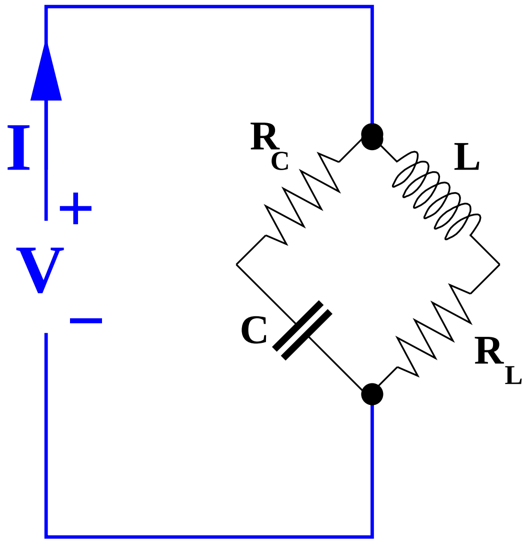
   Specifies the variables of interest.

A classical example

already discussed in lecture 1

RLC circuit

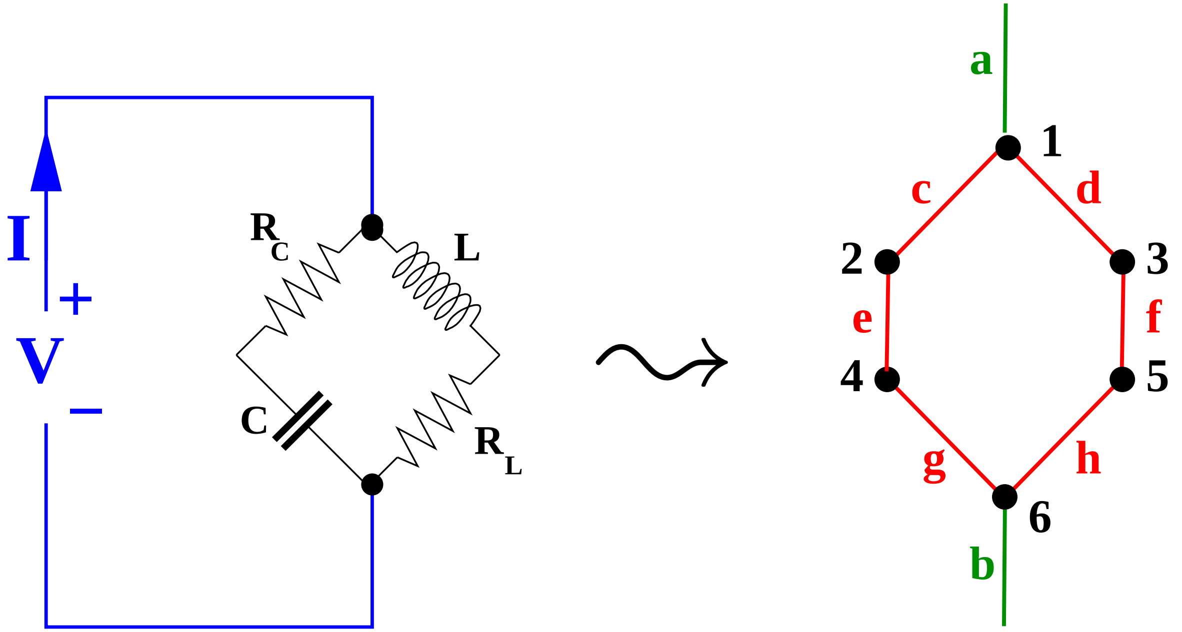
Model the port behavior of



by tearing, zooming, and linking.

RLC circuit

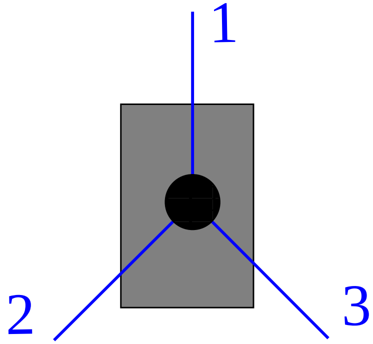
Model the port behavior of



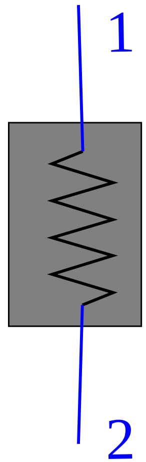
by tearing, zooming, and linking.

In each node there is a module ⇝ module equations
each terminal involves 2 variables (potential, current)
in each branch an electrical interconnection ⇝
interconnection equations

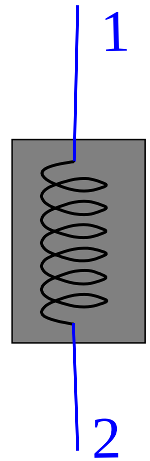
Modules



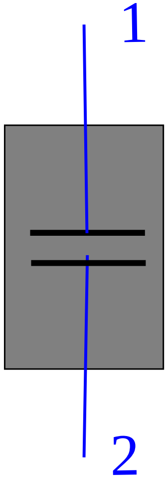
connector1 $\mathtt{n} = 3$



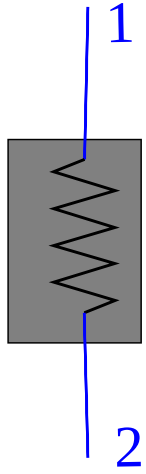
resistor1 R_C



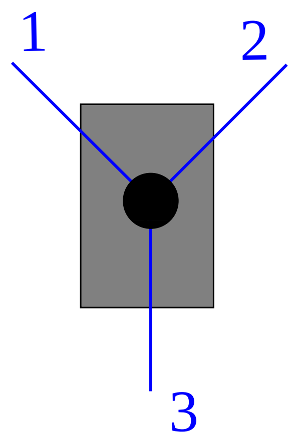
inductor L



capacitor C



resistor2 R_L



connector2 $\mathtt{n} = 3$

Module equations

vertex 1 : $V_{\text{connector}_1,1} = V_{\text{connector}_1,2} = V_{\text{connector}_1,3}$

$I_{\text{connector}_1,1} + I_{\text{connector}_1,2} + I_{\text{connector}_1,3} = 0$

vertex 2 : $V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1},\ I_{R_C,1} + I_{R_C,2} = 0$

vertex 3 : $L\frac{d}{dt} I_{L,1} = V_{L,1} - V_{L,2},\ I_{L,1} + I_{L,2} = 0$

vertex 4 : $C\frac{d}{dt}\left(V_{C,1} - V_{C,2}\right) = I_{C,1},\ I_{C,1} + I_{C,2} = 0$

vertex 5 : $V_{R_L,1} - V_{R_L,2} = R_L I_{R_L,1}$

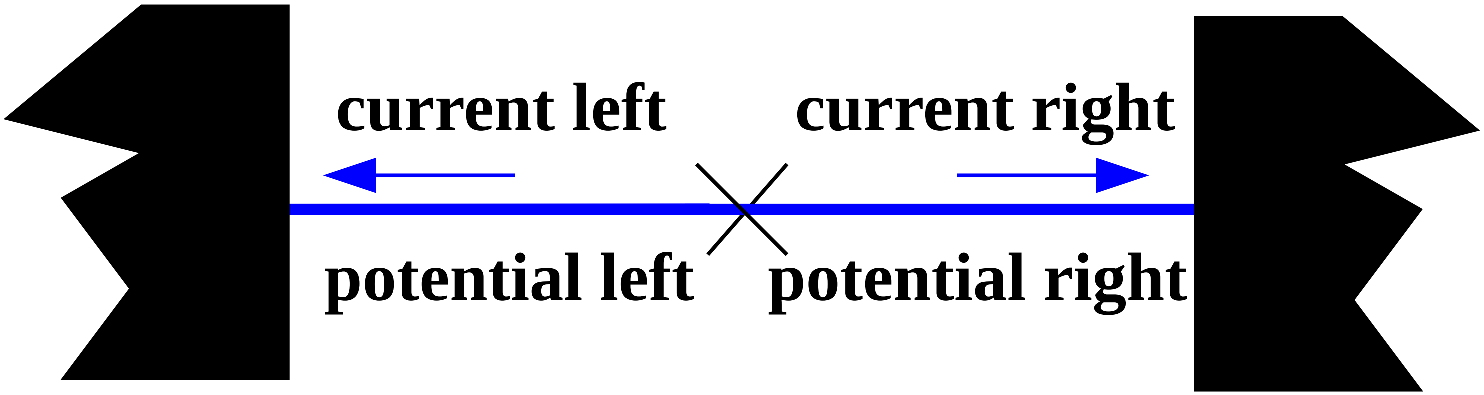
$I_{R_L,1} + I_{R_L,2} = 0$

vertex 6 : $V_{\text{connector}_2,1} = V_{\text{connector}_2,2} = V_{\text{connector}_2,3}$

$I_{\text{connector}_2,1} + I_{\text{connector}_2,2} + I_{\text{connector}_2,3} = 0$

Interconnection

All interconnection are of electrical type



Interconnection equations:

$$\textbf{potential left} = \textbf{potential right}$$

$$\textbf{current left} + \textbf{current right} = 0$$

Interconnection equations

$$\begin{array}{llll}
\textbf{edge c}: & V_{R_{C,1}} = V_{\text{connector}1_2} & I_{R_{C,1}} + I_{\text{connector}1,2} & = 0 \\
\textbf{edge d}: & V_{L_1} = V_{\text{connector}1_3} & I_{L_1} + I_{\text{connector}1_3} & = 0 \\
\textbf{edge e}: & V_{R_{C,2}} = V_{C_1} & I_{R_{C,2}} + I_{C_1} & = 0 \\
\textbf{edge f}: & V_{L_2} = V_{R_{C,1}} & I_{L_2} + I_{R_{L,1}} & = 0 \\
\textbf{edge g}: & V_{C_2} = V_{\text{connector}2_1} & I_{C_2} + I_{\text{connector}2_1} & = 0 \\
\textbf{edge h}: & V_{R_{L,2}} = V_{\text{connector}2_2} & I_{R_{L,2}} + I_{\text{connector}2_2} & = 0
\end{array}$$

Manifest variable assignment

$$V_{\text{externalport}} = V_{\text{connector}_1,1} - V_{\text{connector}_2,3}$$

$$I_{\text{externalport}} = I_{\text{connector1}_1}$$

Manifest behavior

$\rightsquigarrow$ **the dynamical system with behavior $\mathscr{B}$ specified by:**

<u>Case 1</u>: $CR_C \neq \dfrac{L}{R_L}$

$$\boxed{\left(\frac{R_C}{R_L} + \left(1 + \frac{R_C}{R_L}\right) CR_C \frac{d}{dt} + CR_C \frac{L}{R_L} \frac{d^2}{dt^2}\right) V = \left(1 + CR_C \frac{d}{dt}\right) \left(1 + \frac{L}{R_L} \frac{d}{dt}\right) R_C I}$$

<u>Case 2</u>: $CR_C = \dfrac{L}{R_L}$

$$\boxed{\left(\frac{R_C}{R_L} + CR_C \frac{d}{dt}\right) V = \left(1 + CR_C \frac{d}{dt}\right) R_C I}$$

$\rightsquigarrow$ **behavior $\mathscr{B}$ = all solutions $(V, I) : \mathbb{R} \to \mathbb{R}^2$**

Manifest behavior

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$\rightsquigarrow$ **behavior $\mathscr{B}$ = all solutions $(V, I) : \mathbb{R} \to \mathbb{R}^2$**

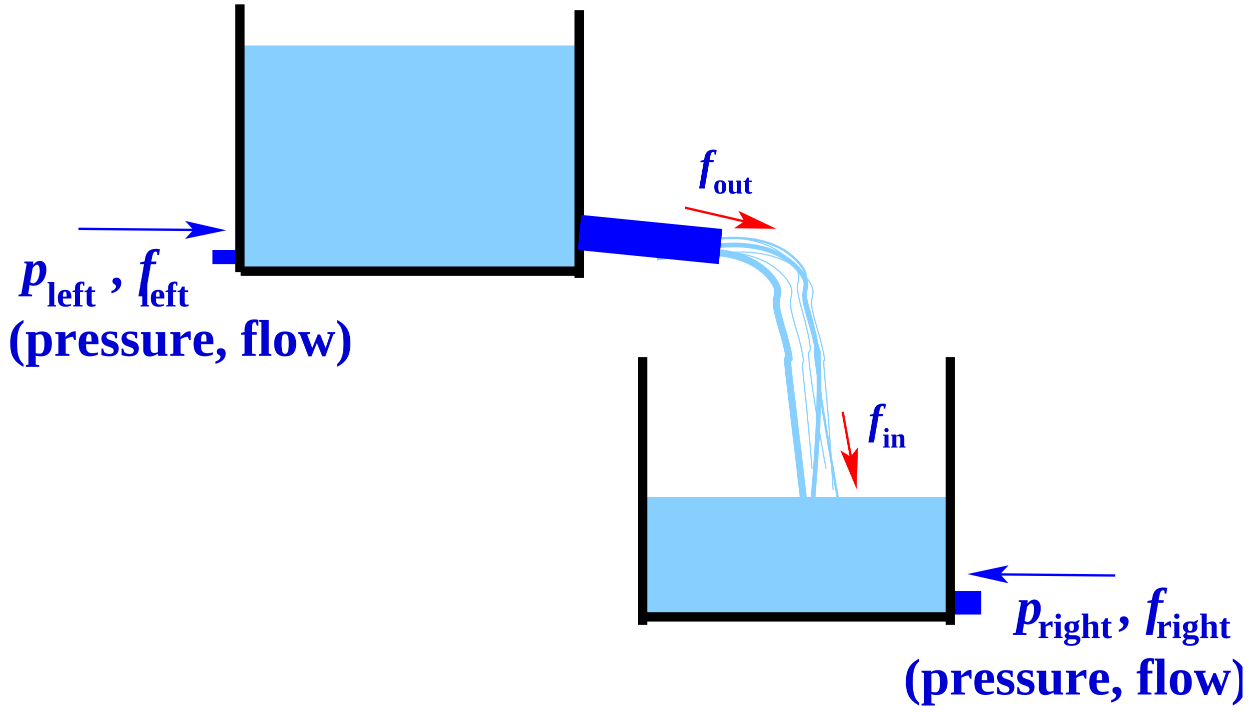
Theorem: In LTIDSs latent variables can be eliminated

Other methodologies

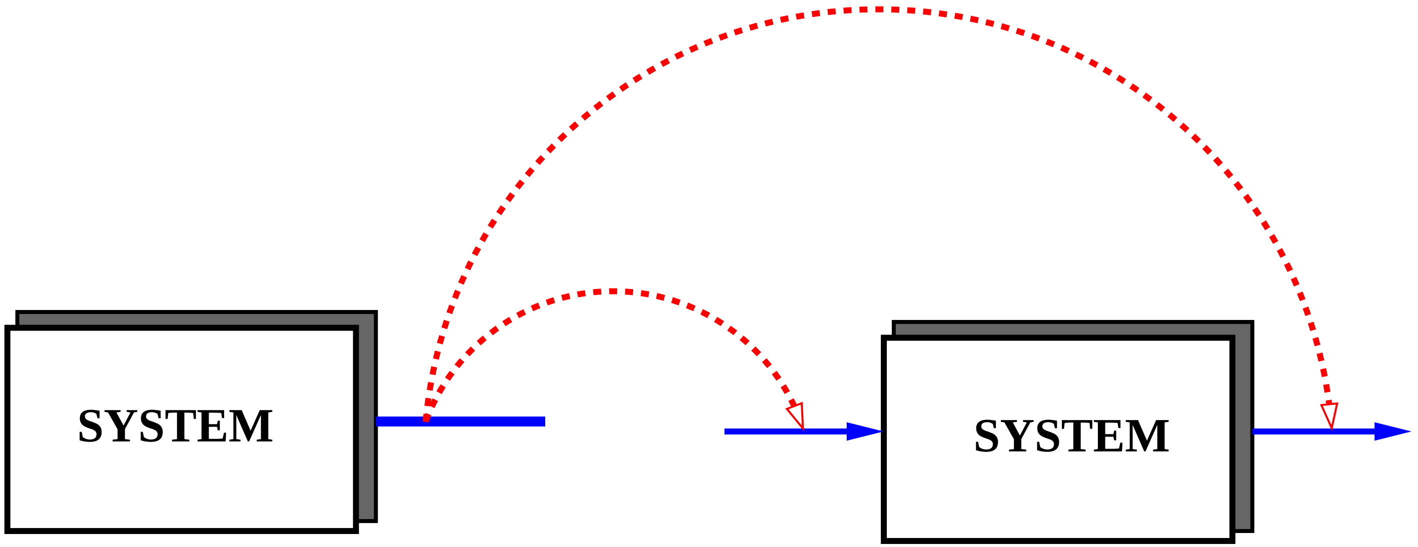
Signal flow graphs

input/output thinking

There are many many examples where output-to-input connection is eminently natural:



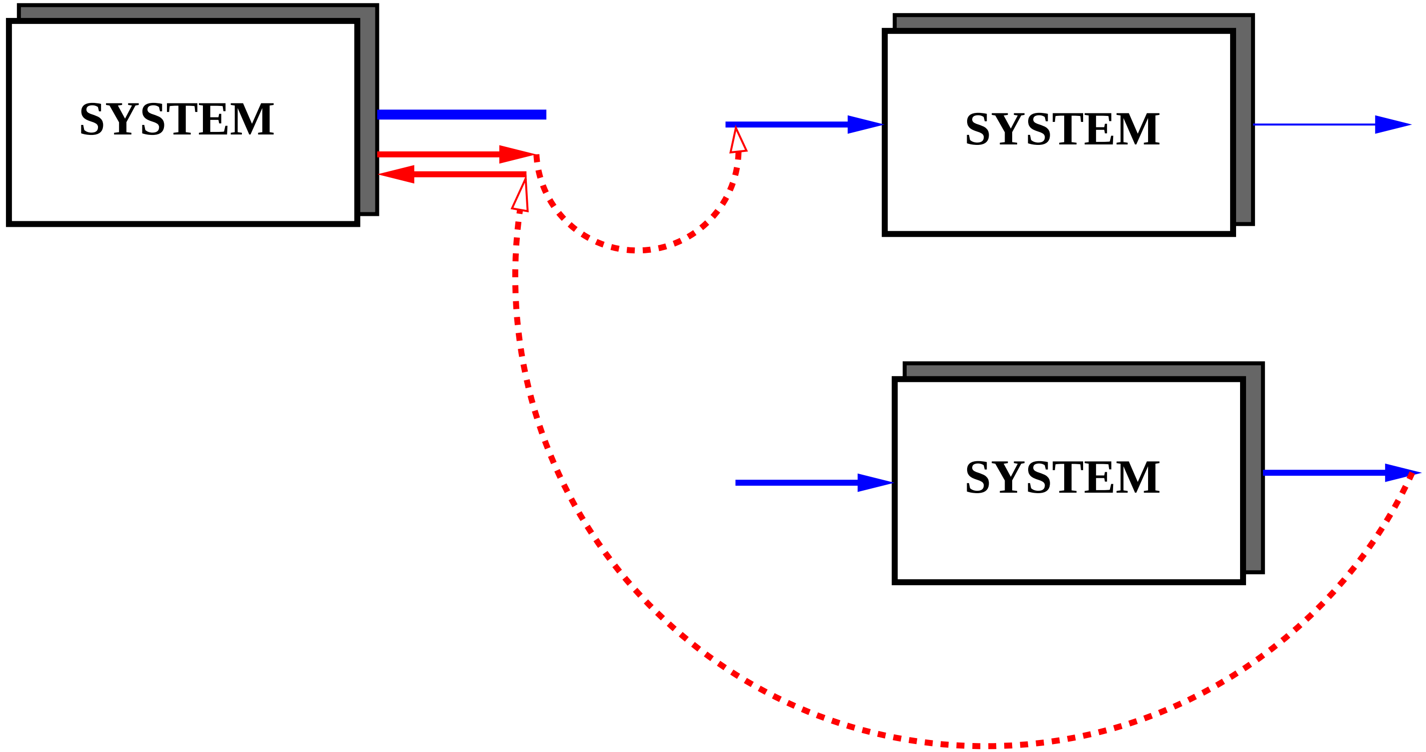
input/output thinking



- shows terminal variables separate
- suggests that inputs and outputs occur at different physical points

Does not respect the physics

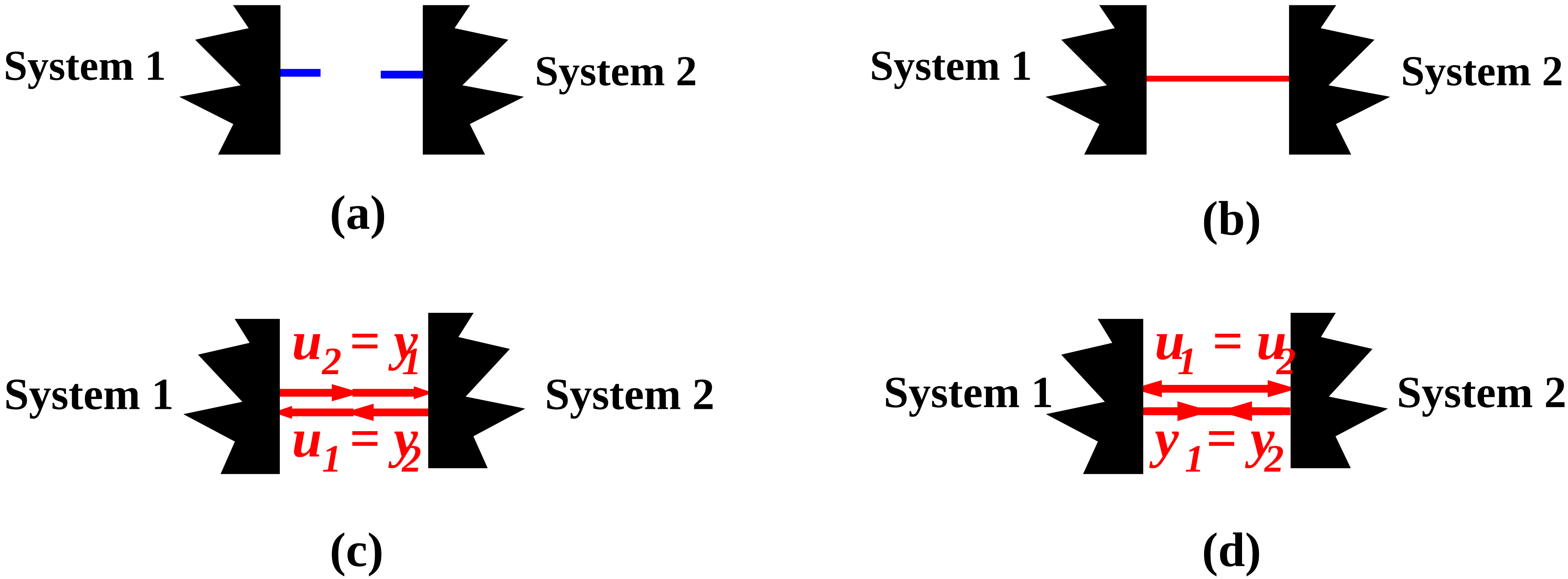
input/output thinking



- **allows impossible input-output connections**

Does not respect the physics

input/output thinking



For physical systems

input-to-input & output-to-output

assignment very prevalent:

force to force; pressure to pressure; ...

Physical systems are not signal processors.

input/output thinking

"Block diagrams unsuitable for serious physical modeling

- the control/physics barrier"

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Karl Åström
(born 1934)

from K.J. Åström, *Present Developments in Control Applications*

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Bond graphs

Bond graphs

Interconnection variables:

a flow and an effort **product = power**

- **current & voltage**
- **velocity & force**
- **mass flow & pressure**
- **heat flow& temperature**

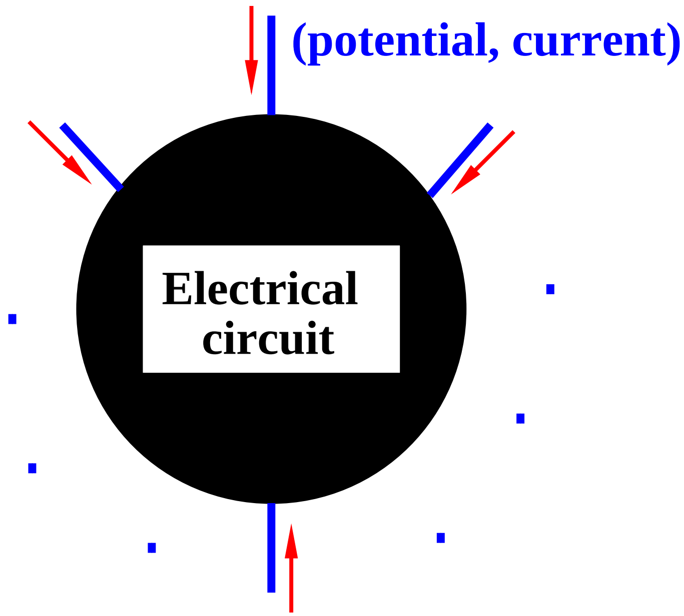
  $$\frac{\text{heat flow}}{\text{temperature}} \text{ \& temperature}$$

- **...**

Bond graphs

1. Mechanical interconnections equate positions, not velocities
2. Not all interconnections involve equating energy transfer
3. Terminals are for interconnection,
   ports are for energy transfer

Terminals versus ports

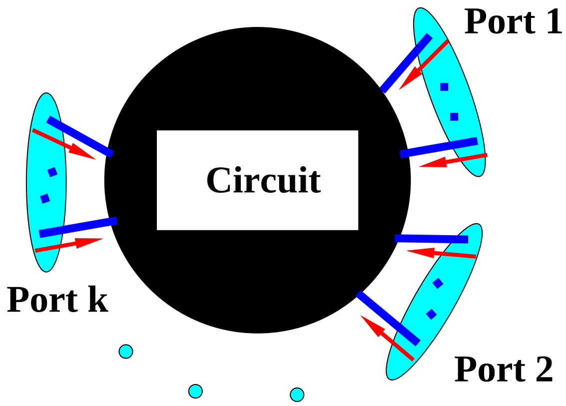


Terminal variables and behavior:

$$(V_1, I_1, V_2, I_2, \ldots, V_{\mathtt{n}}, I_{\mathtt{n}}) \rightsquigarrow \textbf{ behavior } \mathscr{B} \subseteq \left(\mathbb{R}^{2\mathtt{n}}\right)^{\mathbb{R}}$$

Energy does not flow along the terminals!

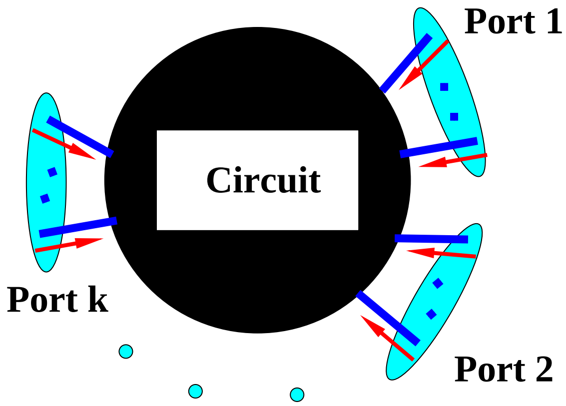
Terminals versus ports



Port :$\Leftrightarrow$ sum currents = 0

potentials + constant $\Rightarrow$ potentials

Terminals versus ports



Port $:\Leftrightarrow$ **sum currents = 0**
potentials + constant $\Rightarrow$ potentials

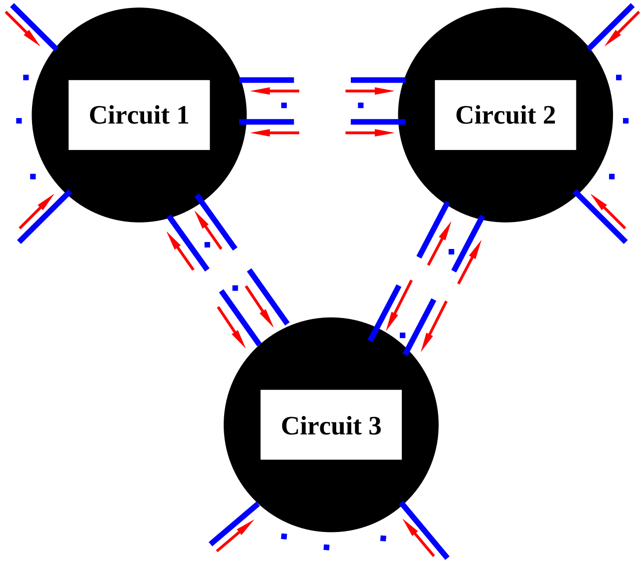
$$\left(\boxed{V_1, I_1 \ldots, V_p, I_p}, V_{p+1}, \ldots, I_n\right) \in \mathscr{B}, \alpha : \mathbb{R} \longrightarrow \mathbb{R}$$

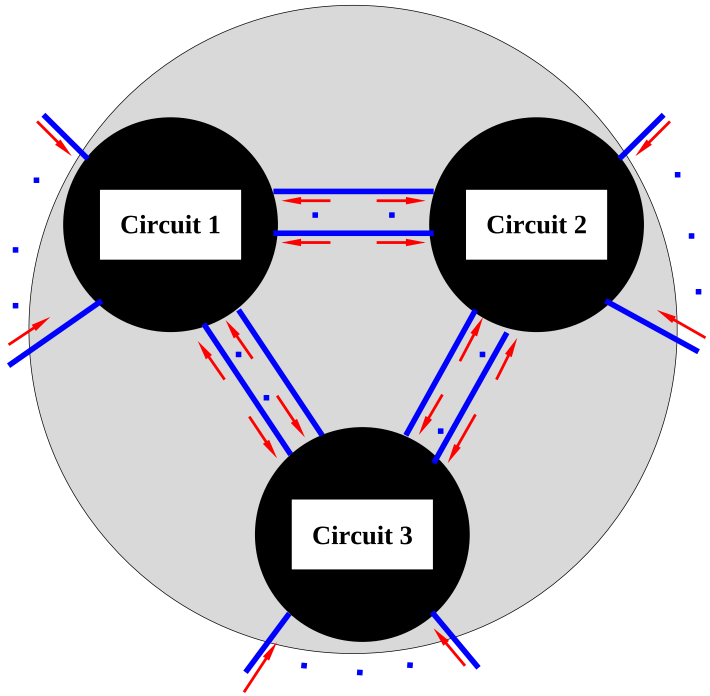
$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_p + \alpha, I_p}, V_{p+1}, \ldots, I_n\right) \in \mathscr{B}$$

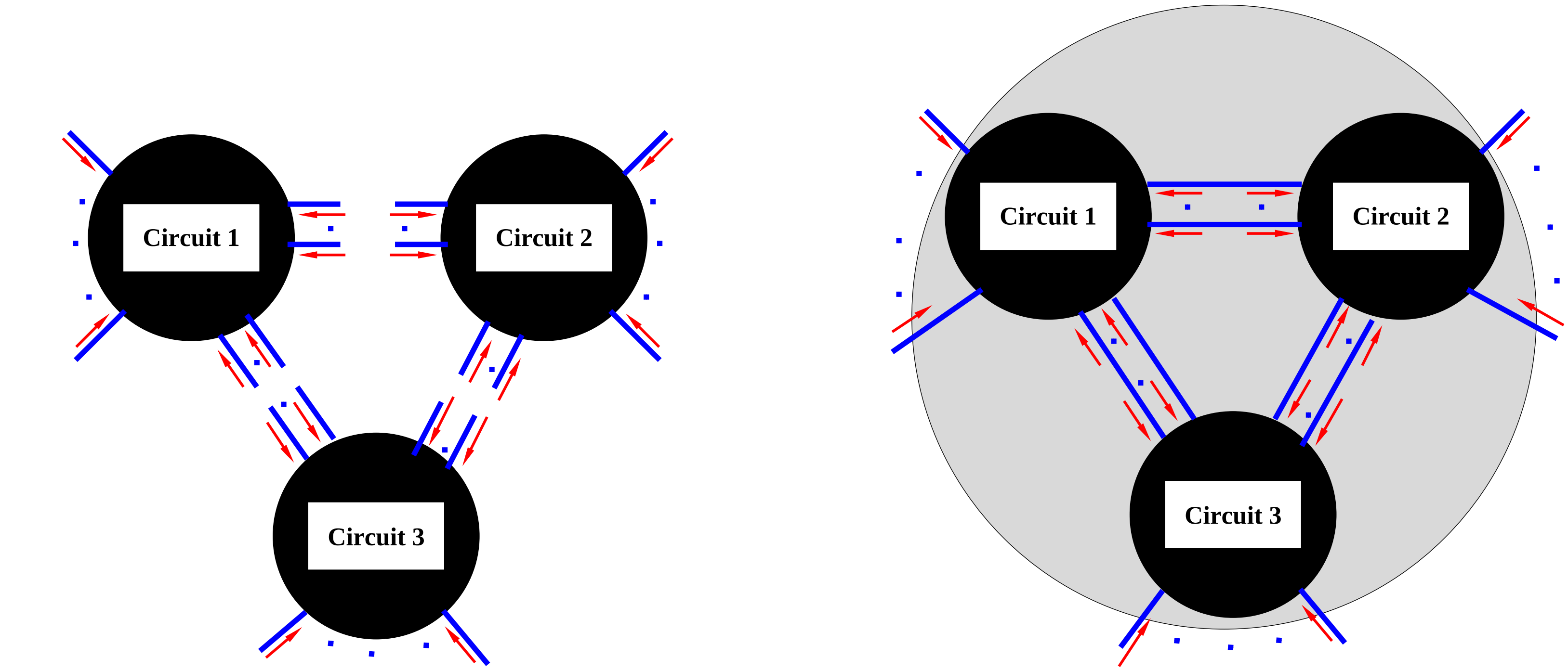
$$\boxed{I_1 + \cdots + I_p} = 0$$

Terminals versus ports





Terminals versus ports



Interconnection via terminals, energy transfer via ports; one cannot talk about

"the energy transferred from circuit 1 to circuit 2"

unless their interconnected terminals form a port.

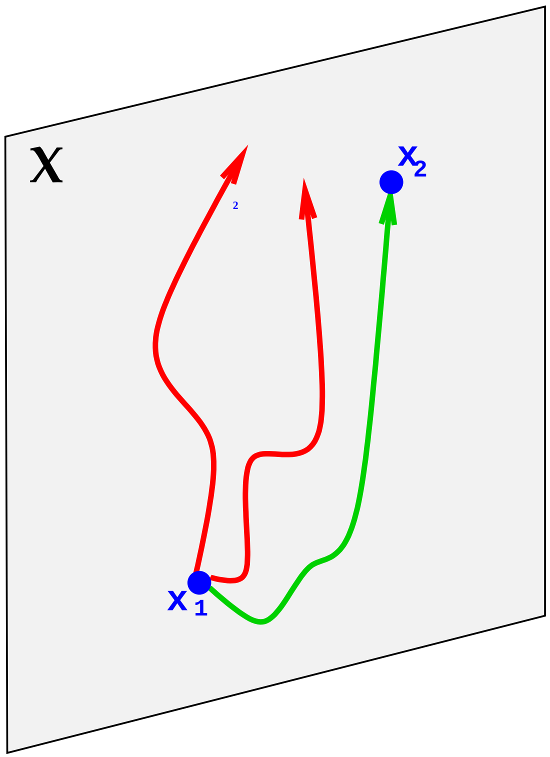
Various facets of control

Path planning

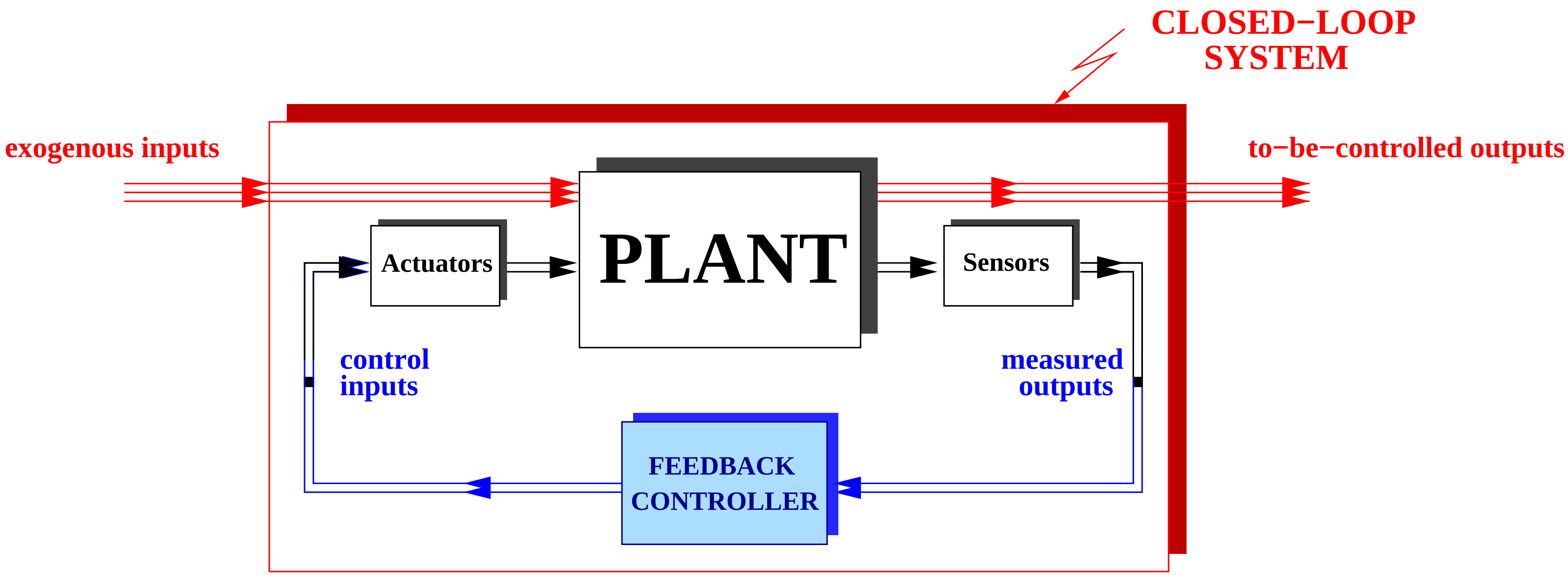
$$\frac{d}{dt}x = f(x,u)$$

Choose time-function $u(\cdot) : [0,T] \rightarrow \mathbb{U}$ **so as to achieve (optimal) state transfer.**

'open loop control'



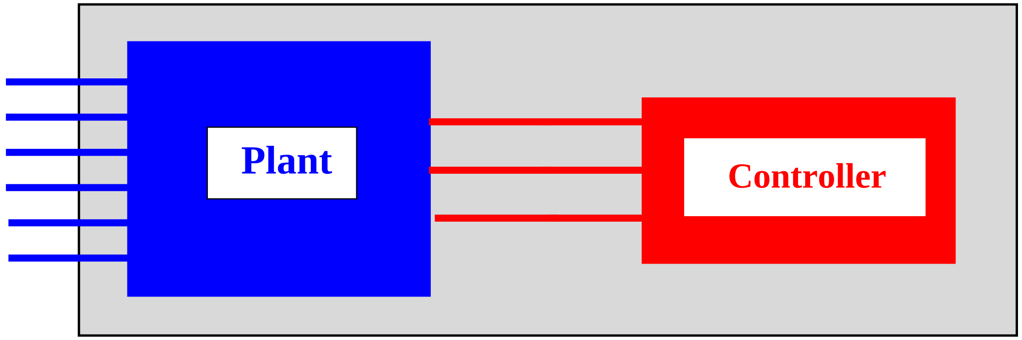
Decision making



Choose a feedback system that processes sensor outputs and generates actuator inputs so as to achieve good (optimal) performance.

'feedback control'
'closed loop control'
'intelligent control'

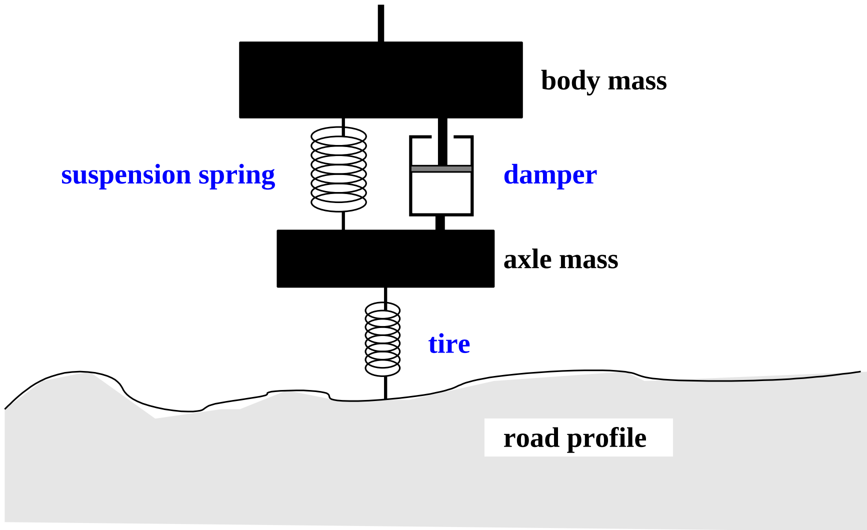
Embedded systems control

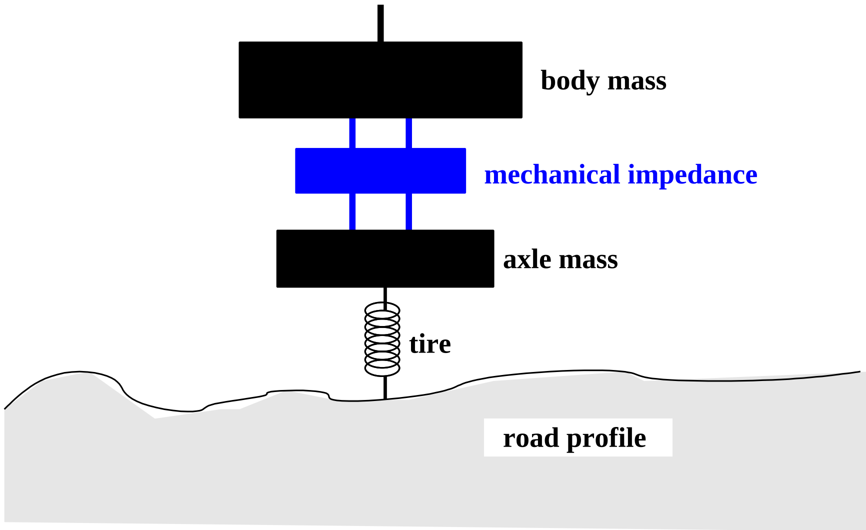


Choose controller so as to achieve good (optimal) performance of the interconnected system

'control as interconnection'
'integrated system design'

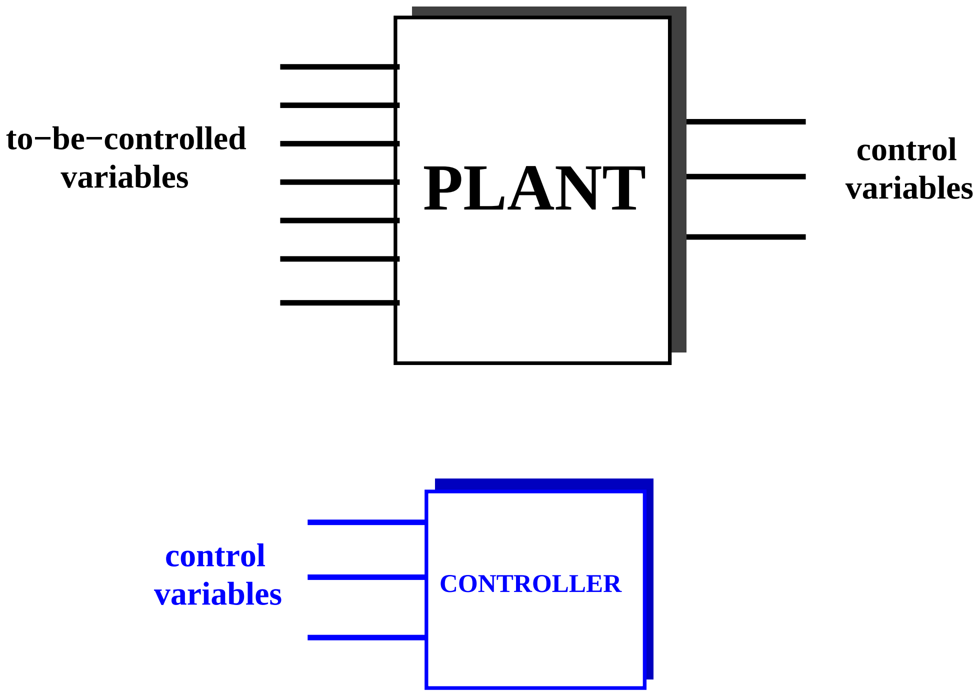
Example


body mass
suspension spring
damper
axle mass
tire
road profile

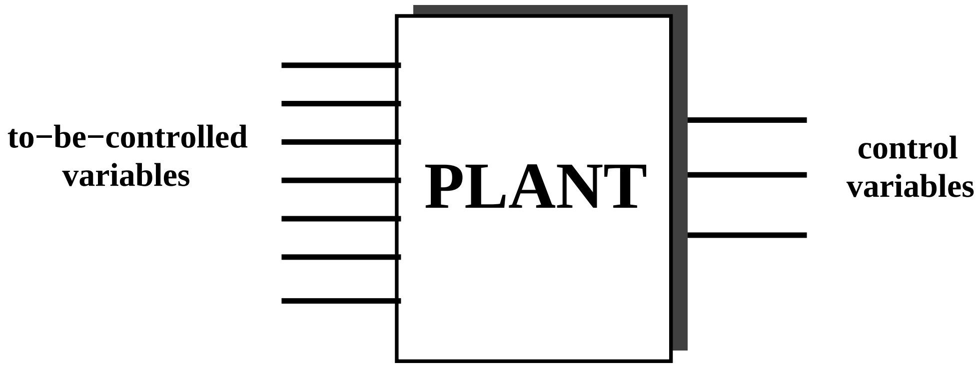

body mass
mechanical impedance
axle mass
tire
road profile

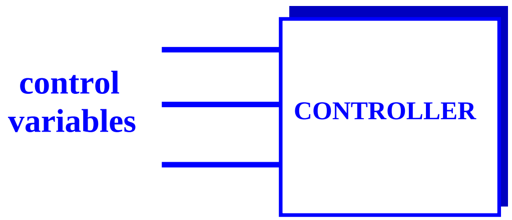
Control as interconnection

Interconnecting a controller

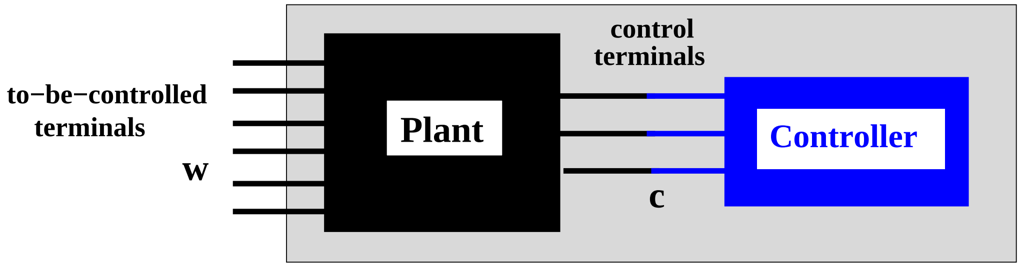


Interconnecting a controller





Interconect via control terminals:



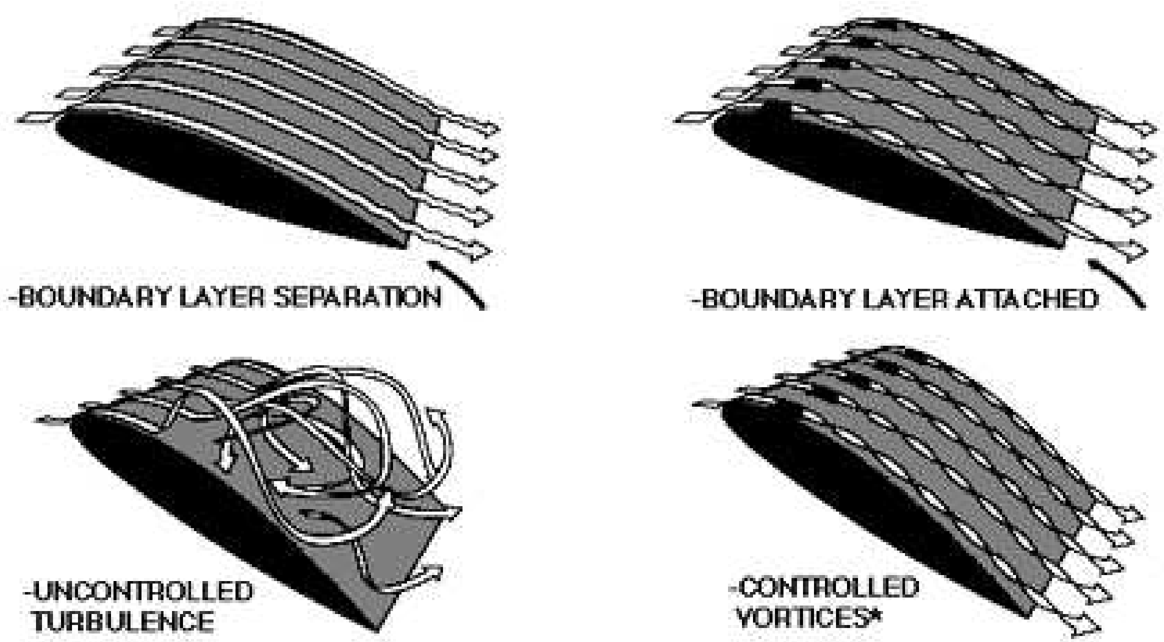
Controlled system

Many controllers are not sensor-to-actuator

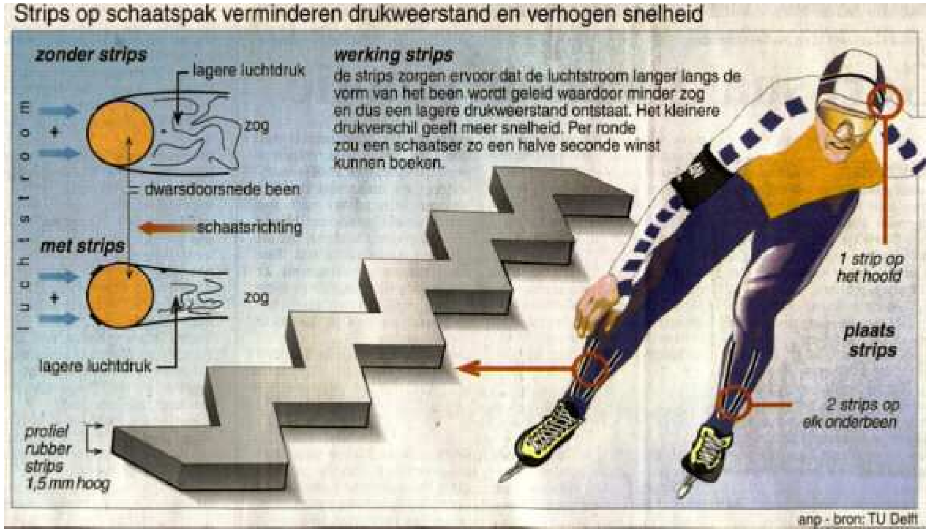
Controlling turbulence:

Many controllers are not sensor-to-actuator

Controlling turbulence:



Nagano 1998 Winter Olympics

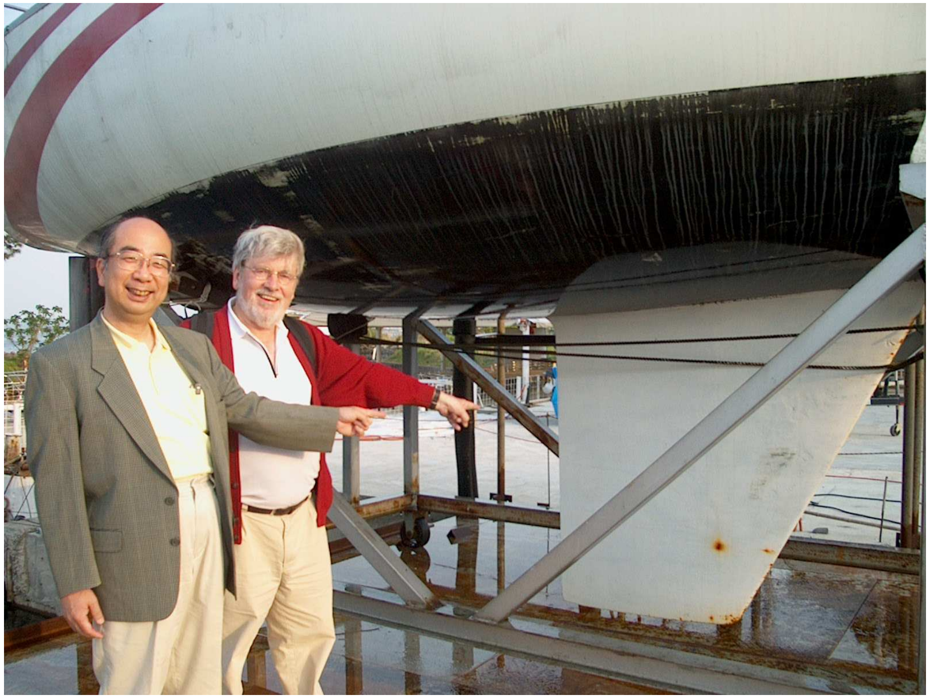


Many controllers are not sensor-to-actuator

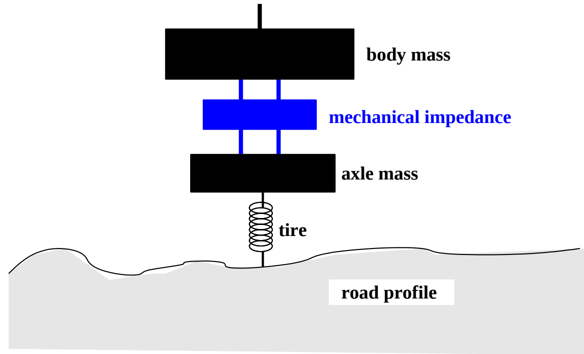
controlling drag:

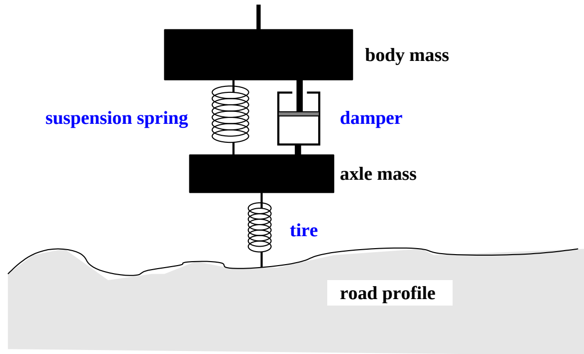
Many stabilizers are not sensor-to-actuator

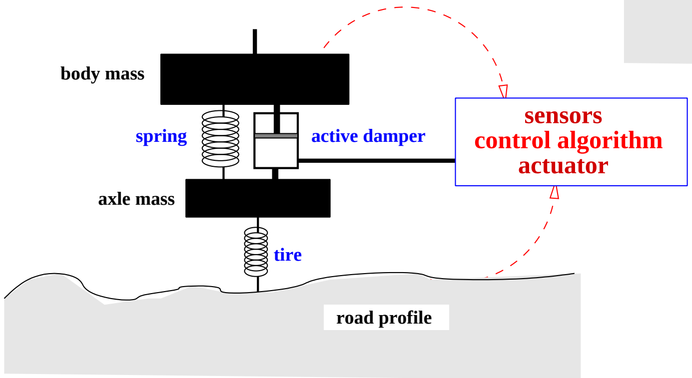
Stabilization:



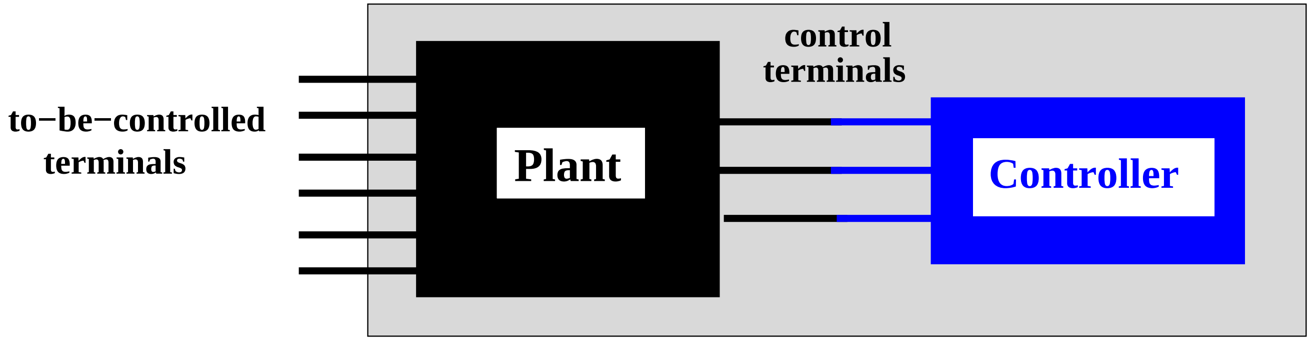
Disturbance attenuation







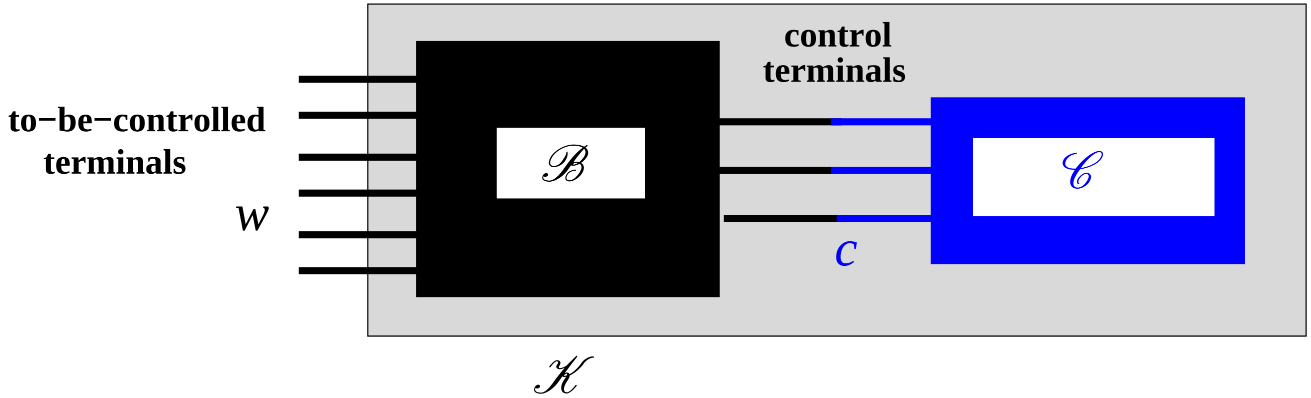
Control as Interconnection



Controlled system

- Are all interconnections ‘reasonable’?
- Which controlled behaviors can be achieved?
- Parametrize all stabilizing controllers
- ...

Implementability



For simplicity, restrict attention to LTIDSs, $\mathscr{L}^\bullet$.

Let $\mathscr{B} \in \mathscr{L}^{\mathtt{w}+\mathtt{c}}$ be the plant behavior,
$\mathscr{C} \in \mathscr{L}^{\mathtt{c}}$ be the controller behavior, and

$$\mathscr{K} = \{w : \mathbb{R} \to \mathbb{R}^{\mathtt{w}} \mid \exists\, c \in \mathscr{C} \text{ such that } (w,c) \in \mathscr{B}\}$$

be the *controlled behavior*

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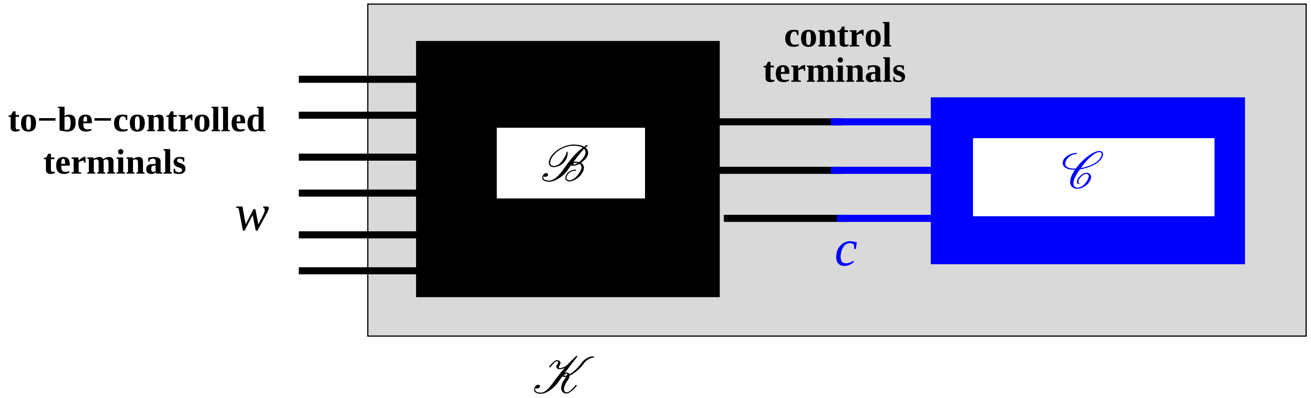

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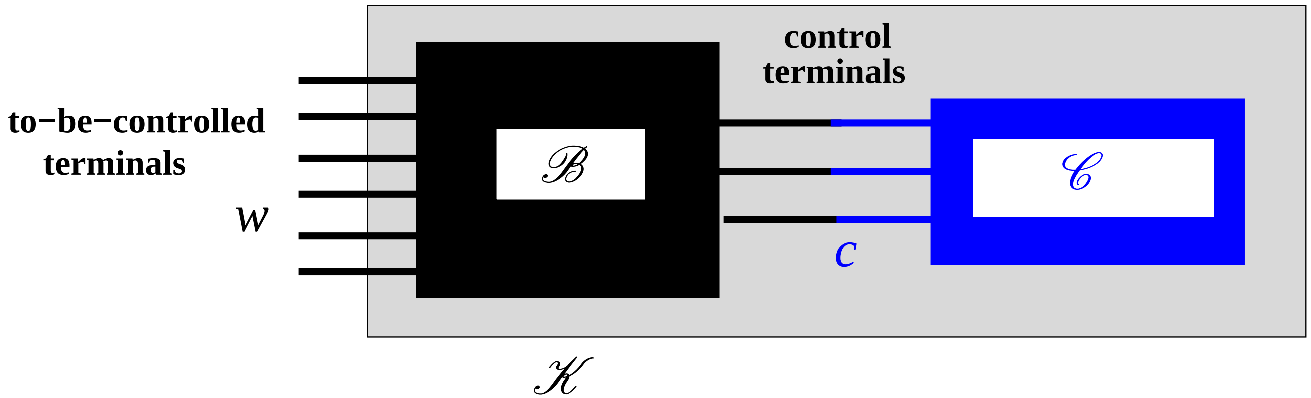
Elimination theorem $\Rightarrow$ $\mathscr{K} \in \mathscr{L}^{\mathtt{w}}$

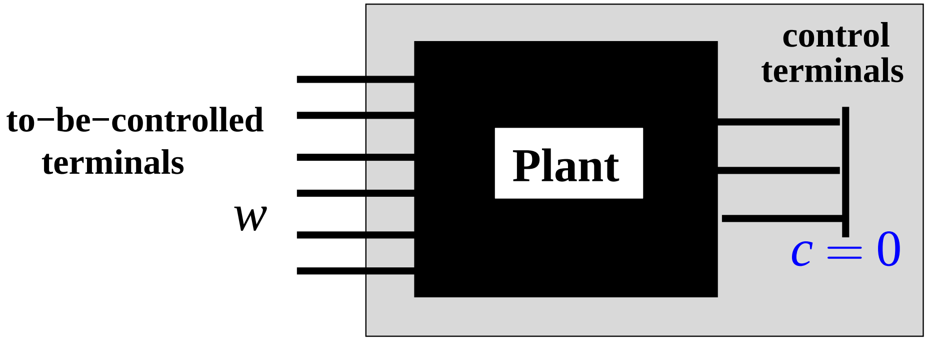
Implementability



For a given $\mathscr{B} \in \mathscr{L}^{\mathtt{w}+\mathtt{c}}$, call $\mathscr{K} \in \mathscr{L}^{\mathtt{w}}$ implementable if there exists $\mathscr{C} \in \mathscr{L}^{\mathtt{c}}$ such that $\mathscr{K}$ is the controlled behavior.

Which $\mathscr{K} \in \mathscr{L}^{\mathtt{w}}$ are implementable ?

Every lecture must have at least one theorem

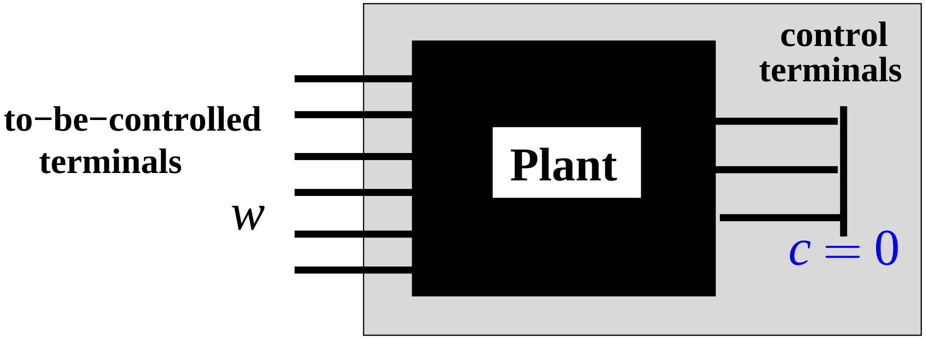


Hidden behavior

Define the *hidden behavior*

$$\mathscr{N} := \{w \mid (w, 0) \in \mathscr{B}\}$$

Every lecture must have at least one theorem



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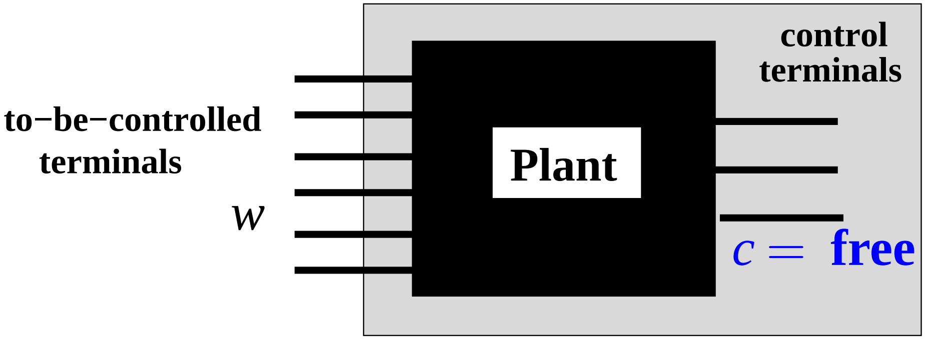

Uncontrolled plant behavior

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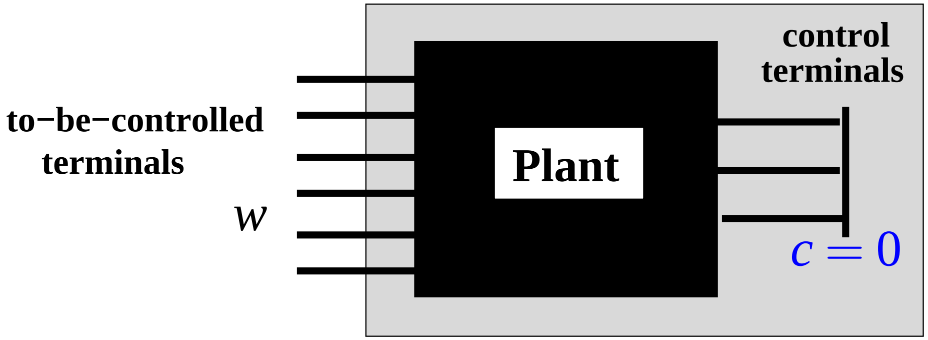

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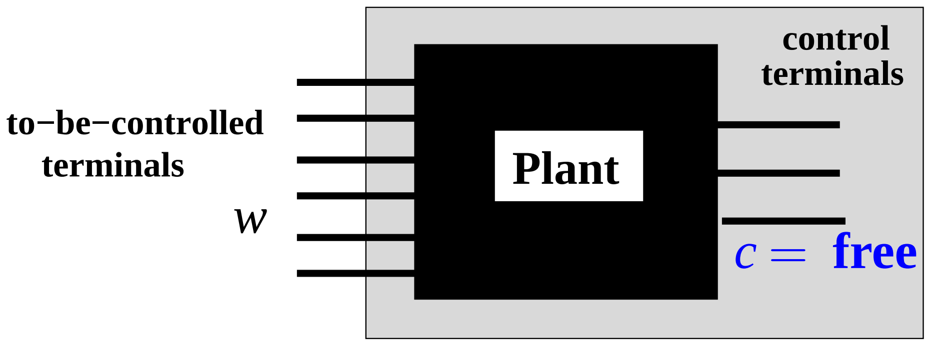

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Implementability theorem

$\mathscr{K} \in \mathscr{L}^{\mathtt{w}}$ **is implementable** $\Leftrightarrow$ $\boxed{\mathscr{N} \subseteq \mathscr{K} \subseteq \mathscr{P}}$

Proof of the implementability theorem

Summary of lecture 13

Main points

- **Interconnection = variable (terminal) sharing**

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- **Hierarchical procedure**
- **Importance of latent variables and the elimination theorem**
- **Limitations of input/output thinking**
- **Control is interconnection, sensor output to actuator input feedback important special case**

Overview

Behavioral systems

- **Gets the physics right**

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- **Starts with first principles models**

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Behavioral systems

- **Controllability becomes genuine system property**

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- **Natural approach to dissipative systems**

Behavioral systems

- Controllability becomes genuine system property
- Deals faithfully with interconnections: variable sharing
- Views control as interconnection
- Advantages in SYSID with the MPUM, etc.
- Natural approach to dissipative systems
- Far easier pedagogically

1. **A dynamical system = a family of trajectories.**
2. **Interconnection = variable sharing**
3. **Control = interconnection**

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End of lecture 13