MATHEMATICAL MODELS OF SYSTEMS

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Tentative outline

The aim of this course is to introduce fundamental notions in the theory of dynamical systems. We take the behavior of a mathematical model and a dynamical system as the central concept. The notions will be often introduced for general nonlinear systems, but the course emphasizes linear time-invariant differential systems described by differential or difference equations.

A behavioral description of a system is the natural outcome of first principles modeling. It also provides a cogent mathematical framework for dealing with interconnected systems based on modularity.

Throughout, the theoretical constructs will be illustrated by means of physical examples, in particular, electrical circuits, mechanical systems, thermodynamics, electromagnetism, etc.

- **Lecture 1: Basic concepts**
  The behavior, behavioral equations, latent variables. Dynamical systems.
- **Lecture 2: Linear differential systems**
  Linearity. Time-invariance. Polynomial matrices. Linear systems as submodules. Structure of kernel representations. The elimination theorem.
- **Lecture 3: Controllability and observability**
  Definitions. Tests for controllability and observability. Image representations. Autonomous systems. Stabilizability.
- **Lecture 4: Inputs and outputs**
  Input/outputs representations. The transfer function and the frequency response. Left and right coprime factorizations. Relation with controllability.

- **Lecture 5: Modeling by tearing and zooming**
  Terminals. Modules. The interconnection architecture. Electrical RLCT circuits. Input/output connections. Bondgraphs.

- **Lecture 6: State models**
  The notion of state. Abstract state construction. State construction for differential systems. State construction for convolution systems. The Hankel matrix.

- **Lecture 7: Balanced state representations and model reduction**
  Representations of controllable systems. Matrix norms. The SVD and low rank approximations. Balanced realizations and order reduction.

- **Lecture 8: Dissipative dynamical systems**
  Construction of the storage function. Relations with LMI's, the Riccati equation, and polynomial matrix factorization.

- **Lecture 9: Control in a behavioral setting**
  Control as interconnection. Controller implementability. Pole placement. Stabilization.

- **Lecture 10: Distributed parameter systems**
  Systems described by PDE's. Elimination. controllability, image representations, and potentials. Dissipative distributed systems.