MATHEMATICAL MODELS of SYSTEMS

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IUAP Graduate Course **Fall 2002**

Lecture 8

DISSIPATIVE SYSTEMS

THEME

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How do we formalize this?

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How is it constructed? Is it unique?

$\leadsto$ KYP, LMI's, ARE's, QDF's, polynomial matrix factorization.

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$\rightsquigarrow$ KYP, LMI's, ARE's, QDF's, polynomial matrix factorization.

Where is this notion applied in systems and control?

OUTLINE

1. **Lyapunov theory**
2. **Dissipative dynamical systems**
3. **The construction of storage functions**
4. **LQ theory; the KYP lemma**

LYAPUNOV THEORY

Consider the classical 'dynamical system', the *flow*

$$\Sigma : \quad \frac{d}{dt}x = f(x)$$

with $x \in \mathbb{X} = \mathbb{R}^{n}$, the *state space*, $f : \mathbb{X} \to \mathbb{X}$.

Denote the set of solutions $x : \mathbb{R} \to \mathbb{X}$ by $\mathfrak{B}$, the 'behavior'.

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Denote the set of solutions $x : \mathbb{R} \to \mathbb{X}$ by $\mathfrak{B}$, the '**behavior**'.

The function

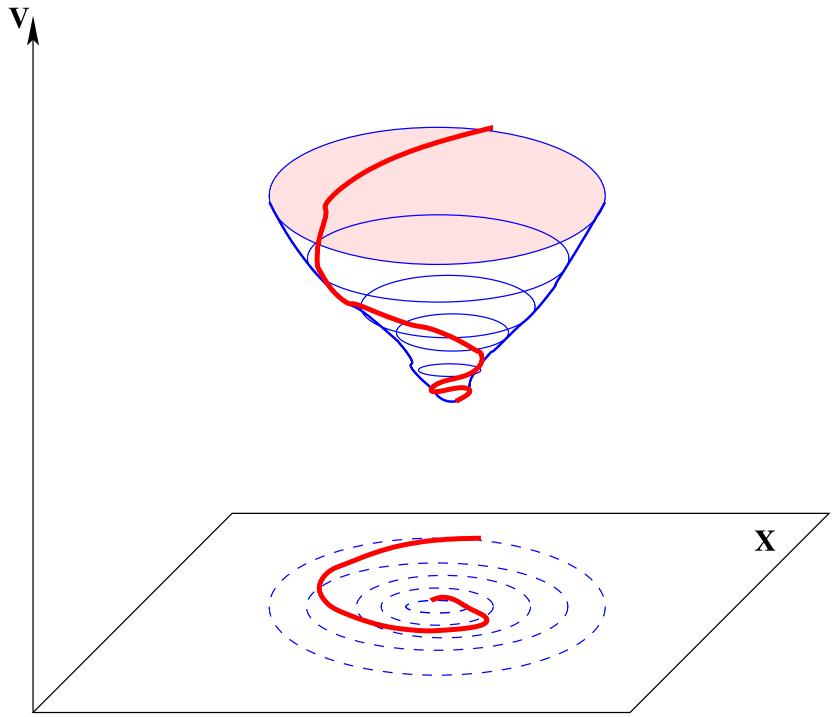
$$V : \mathbb{X} \to \mathbb{R}$$

is said to be a **Lyapunov function** for Σ if along $x \in \mathfrak{B}$

$$\frac{d}{dt} V(x(\cdot)) \leq 0$$

Equivalent to $\quad \overset{\bullet}{V}^{\Sigma} := \nabla V \cdot f \leq 0$

Typical Lyapunov 'theorem':



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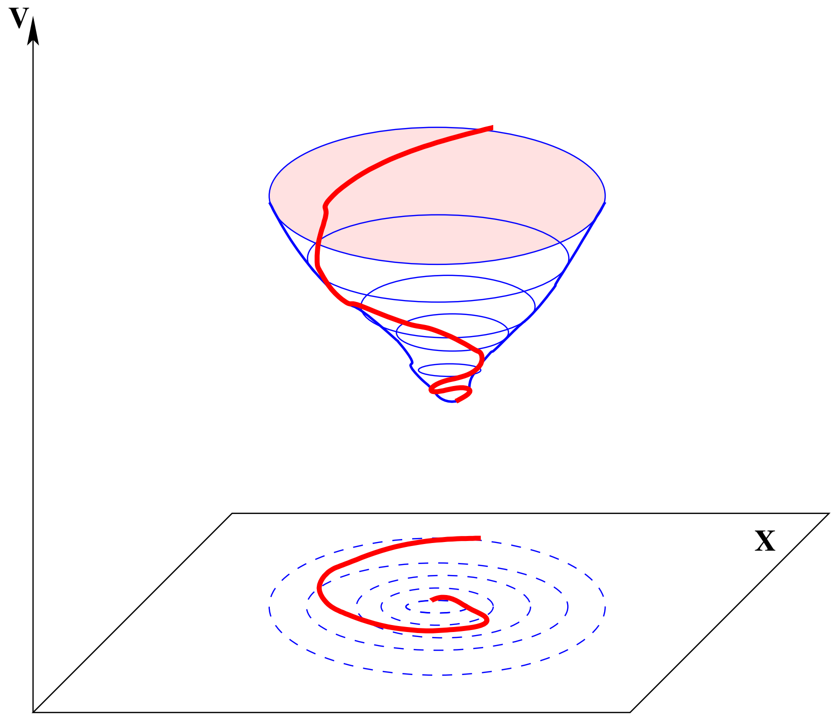


$$V(x) > 0 \text{ and } \overset{\bullet}{V}^{\Sigma}(x) < 0 \text{ for } 0 \neq x \in \mathbb{X}$$

$$\Longrightarrow$$

$\forall\ x \in \mathfrak{B}$, there holds $x(t) \to 0$ for $t \to \infty$ **‘global stability’**

Refinements: LaSalle's invariance principle.

Converse: Kurzweil's thm.

LQ theory $\rightsquigarrow A^\top X + XA = Y$ 'Lyapunov (matrix) equation'.

A linear system is stable iff it has a quadratic pos. def. Lyapunov function.

Basis for most stability results in diff. eq'ns, physics, (adaptive) control, even numerical analysis, system identification.

Plays a remarkably central role in the field.

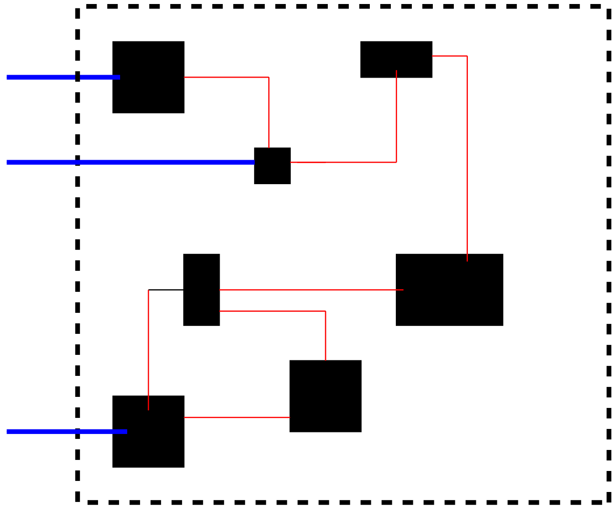
Aleksandr Mikhailovich Lyapunov (1857-1918)

Studied mechanics, differential equations.

Introduced Lyapunov's 'second method' in his Ph.D. thesis (1899).

DISSIPATIVE DYNAMICAL SYSTEMS

A much more appropriate starting point for the study of dynamics are 'open' systems. $\leadsto$



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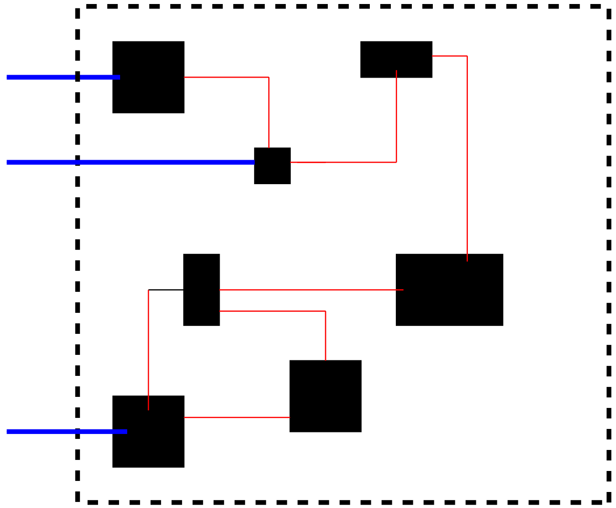

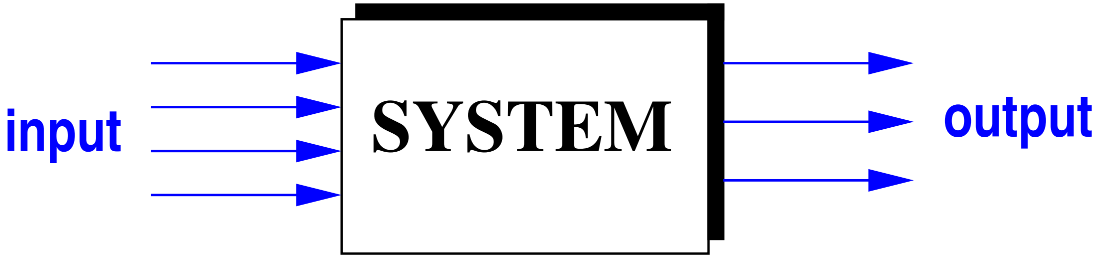

INPUT/STATE/OUTPUT SYSTEMS

Consider the 'dynamical system'

$$\Sigma : \quad \frac{d}{dt}x = f(x,u), \quad y = h(x,u).$$

$u \in \mathbb{U} = \mathbb{R}^{\mathtt{m}}, y \in \mathbb{Y} = \mathbb{R}^{\mathtt{p}}, x \in \mathbb{X} = \mathbb{R}^{\mathtt{n}}$**: the input, output, state.**

Behavior $\mathfrak{B} =$ **all sol'ns** $(u,y,x) : \mathbb{R} \to \mathbb{U} \times \mathbb{Y} \times \mathbb{X}$.

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Let

$$s : \mathbb{U} \times \mathbb{Y} \to \mathbb{R}$$

be a function, called the *supply rate*.

DISSIPATIVITY

Σ is said to be *dissipative* w.r.t. the supply rate s if $\exists$

$$V : \mathbb{X} \rightarrow \mathbb{R},$$

called the *storage function,* such that

$$\frac{d}{dt} V(x(\cdot)) \leq s(u(\cdot), y(\cdot))$$

along input/output/state trajectories ($\forall$ $(u(\cdot), y(\cdot), x(\cdot)) \in \mathfrak{B}$).

This inequality is called the *dissipation inequality.*

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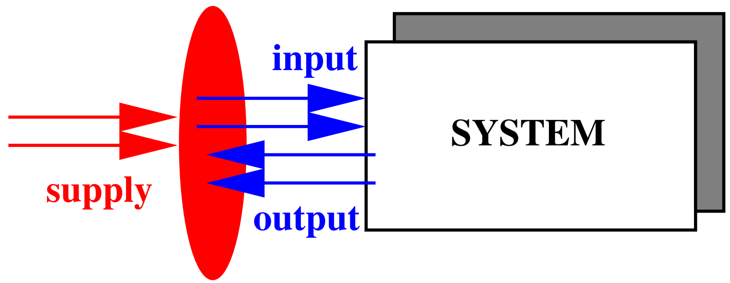
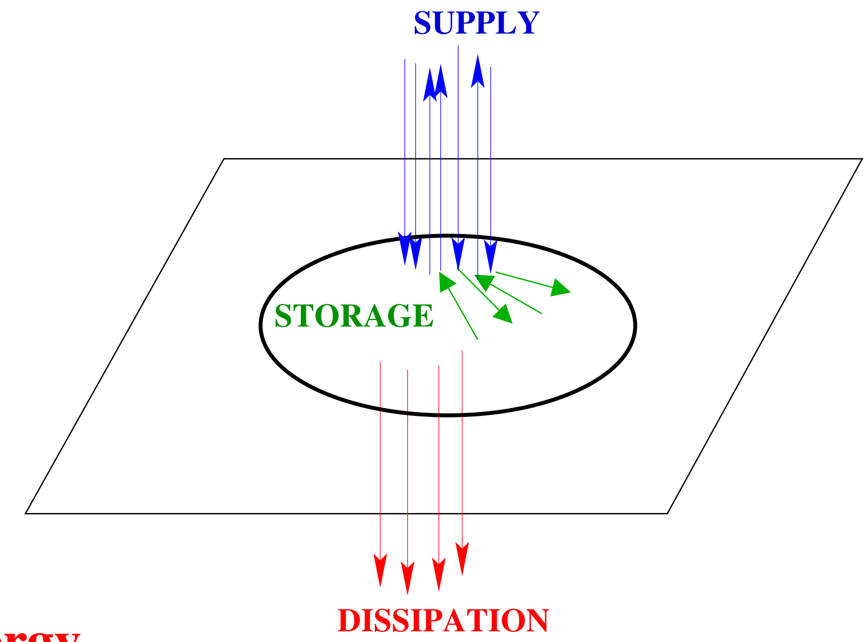
If equality holds: 'conservative' system.

$s(u, y)$ models something like the power delivered to the system when the input value is u and output value is y.





$V(x)$ then models the internally stored energy.

Dissipativity $:\Leftrightarrow$

rate of increase of internal energy $\leq$ supply rate.

Special case: 'closed system': $\boxed{s = 0}$

then dissipativeness $\leftrightarrow$ V is a Lyapunov function.

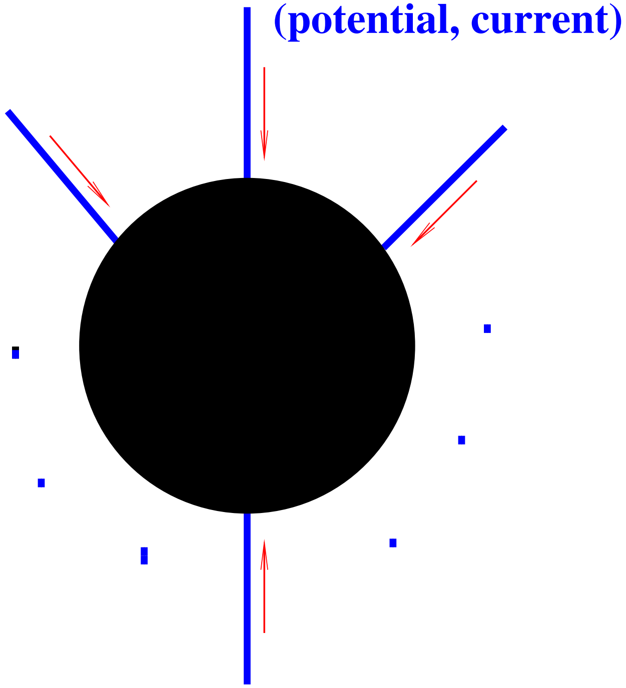
Dissipativity is a natural generalization of LF to open systems.

Stability for closed systems $\simeq$ Dissipativity for open systems.

PHYSICAL EXAMPLES

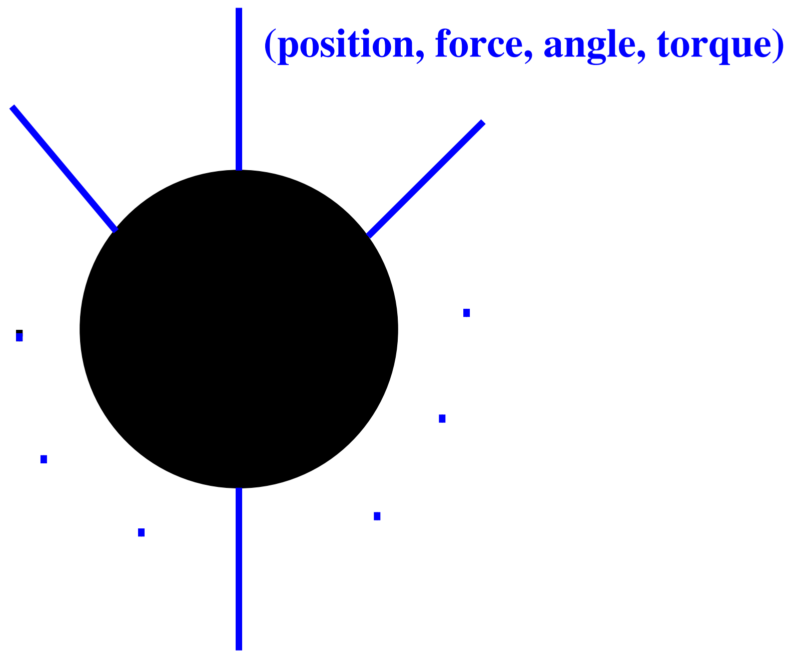
System	Supply	Storage
Electrical circuit	$V^\top I$ V : voltage I : current	energy in capacitors and inductors
Mechanical system	$F^\top v + (\frac{d}{dt}\theta)^\top T$ F : force, v : velocity θ: angle, T : torque	potential + kinetic energy
Thermodynamic system	$Q + W$ Q : heat, W : work	internal energy
Thermodynamic system	$-Q/T$ Q : heat, T : temp.	entropy
etc.	etc.	etc.

Electrical circuit:



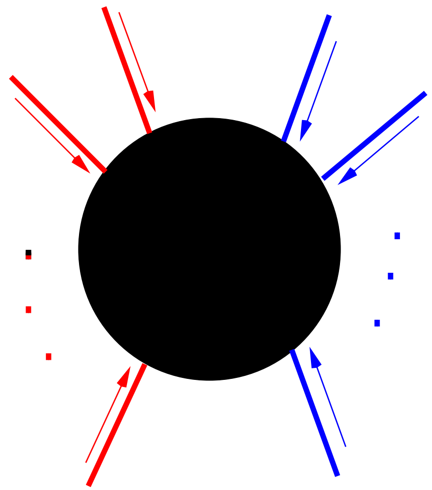
Dissipative w.r.t. $\Sigma_{\ell=1}^{N} V_\ell I_\ell$ **(electrical power).**

Mechanical device:



Dissipative w.r.t. $\Sigma_{\ell=1}^{\mathrm{N}}((\frac{d}{dt}q_\ell)^\top F_\ell + (\frac{d}{dt}\theta_\ell)^\top T_\ell)$ **(mech. power).**

Thermodynamic system:



Conservative w.r.t. $\Sigma_{\ell=1}^{N} Q_\ell + \Sigma_{\ell=1}^{N'} W_\ell$; **First law.**

Dissipative w.r.t. $-\Sigma_{\ell=1}^{N} \frac{Q_\ell}{T\ell}$; **Second law.**

THE CONSTRUCTION OF STORAGE FUNCTIONS

Central question:

Given (a representation of) Σ, the dynamics, and

given s, the supply rate,

is the system dissipative w.r.t. s, i.e.,

does there exist a storage function V such that the

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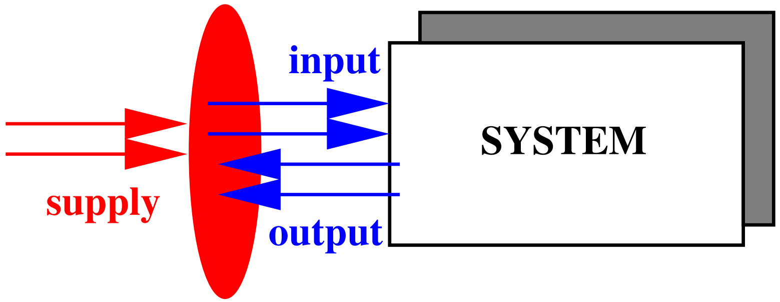

Assume s 'power', known dynamics, what is the internal energy?

Assume that a number of (reasonable) conditions hold:

$f(0,0) = 0, h(0,0) = 0, s(0,0) = 0;$

Maps and functions (including V) smooth;

State space $\mathbb{X}$ of Σ 'connected':

every state reachable from every other state;

Observability.

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<u>'Thm'</u>: Let Σ and s be given.

Then Σ is dissipative w.r.t. s iff

$$\boxed{\oint s(u(\cdot), y(\cdot)) \; dt \geq 0}$$

for all periodic $(u(\cdot), y(\cdot), x(\cdot)) \in \mathfrak{B}$.

Two universal storage functions:

1. <u>The available storage</u>

$V_{\mathrm{available}}(x_0) :=$

$$\sup_{(u(\cdot),y(\cdot),x(\cdot))\in\mathfrak{B},x(0)=x_0,x(\infty)=0} \{-\int_0^{+\infty} s(u(\cdot),y(\cdot))\ dt\}$$

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V is in general far from unique. Storage f'ns form convex set, every storage function satisfies

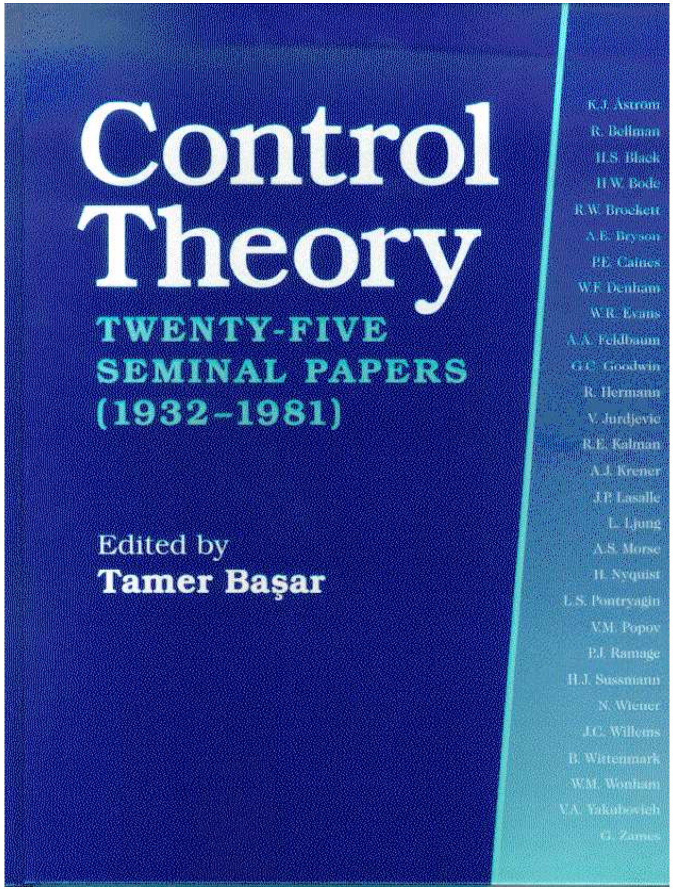
$$\boxed{V_{\text{available}} \leq V \leq V_{\text{required}}.}$$

For conservative systems, V is unique.

The construction of storage f'ns is very well understood, particularly for linear input/state/output systems and quadratic supply rates.

Leads to the KYP-lemma, LMI's, ARIneq, ARE, semi-definite programming, spectral factorization, Lyapunov functions, robust control, electrical circuit synthesis, stochastic realization theory.

Dissipative systems play a remarkably central role in the field.



LINEAR SYSTEMS with QUADRATIC SUPPLY RATES

Assume Σ linear, time-invariant, finite-dimensional:

$$\boxed{\frac{d}{dt}x = Ax + Bu, \quad y = Cx,}$$

and s quadratic: for example,

$$\boxed{s : (u, y) \mapsto ||u||^2 - ||y||^2.}$$

E.g., for circuits $u = \frac{V+I}{2}$, $y = \frac{V-I}{2}$, etc.

Assume (A, B) controllable, (A, C) observable.

$G(s) := C(Is - A)^{-1}B$, the transfer function of Σ.

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Solution set (of LMI, ARineq) is convex, compact, and attains its infimum and its supremum:

$$\boxed{K^- \leq K \leq K^+.}$$

These extreme sol'ns K^- and K^+ themselves satisfy the ARE, associated with the available storage and the required supply.

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Extensive theory, relation with other system representations, many applications, well-understood (also algorithmically). Connection with optimal LQ control, semi-definite programming.

Important refinement:

Existence of a $V \geq 0$ (i.e., bounded from below) (energy?)

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In LQ case $\Leftrightarrow$

- $\int_{-\infty}^{0} ||u(\cdot)||^2 \, dt \geq \int_{-\infty}^{0} ||y(\cdot)||^2 \, dt,$
- $\sup_{\{s \in \mathbb{C} | \mathrm{Re}(s) > 0\}} ||G(s)|| =: ||G||_{\mathcal{H}_\infty} \leq 1,$

  Note def. of $\mathcal{H}_\infty$-norm !
- **$\exists$ sol'n $K = K^\top \geq 0$ to LMI, ARineq, ARE.**

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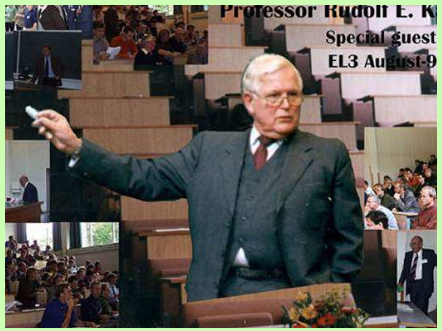

$\rightsquigarrow$ **KYP-lemma.**

Another situation

$$\boxed{\tfrac{d}{dt}x = Ax + Bu, \quad y = Cx + Du, \;\; \mathtt{m} = \mathtt{p},}$$

$$\boxed{s : (u, y) \mapsto u^\top y.}$$

E.g., for circuits $u = V, y = I$, etc.

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3. **Electrical circuit synthesis.**

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- LQ theory $\rightsquigarrow$ LMI, ARIneq, ARE, KYP

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- ▶ **Central question: construction of storage functions**
- ▶ **LQ theory $\rightsquigarrow$ LMI, ARIneq, ARE, KYP**
  - ▶ **Applications: robust stability, stabilization, circuit synthesis**

RECAP

- Dissipative system: involves supply rate, storage f'n.
- Dissipative system: net flow of supply into the system
- Natural generalization of Lyapunov f'n to open systems
- Central question: construction of storage functions
- LQ theory $\rightsquigarrow$ LMI, ARIneq, ARE, KYP
  - Applications: robust stability, stabilization, circuit synthesis
  - For differential systems $\rightsquigarrow$ QDF's $\longrightarrow$ next lecture

End of Lecture 8