**European Embedded Control Institute**

**Graduate School on Control — Spring 2010**

# The Behavioral Approach to Modeling and Control

## Lecture VIII

# SYSTEM INTERCONNECTION

# Theme

**How are systems interconnected?**

**How are interconnected systems modeled?**

**How does control fit in?**

# Theme

**How are systems interconnected?**

**How are interconnected systems modeled?**

**How does control fit in?**

**We deal with very simple examples,**
**mainly electrical circuits and**
**1-dimensional mechanical systems.**

**Other applications: hydraulic systems**
**chemical systems**
**thermal systems, ...**

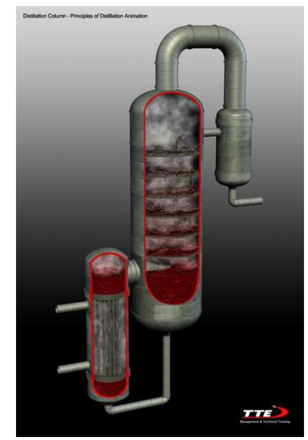
# Outline

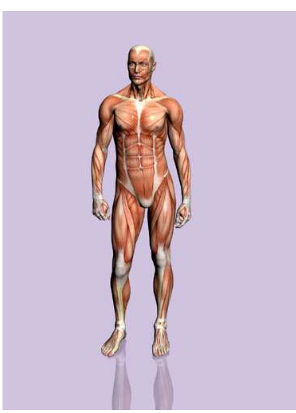
- Motivation
- Modeling by tearing, zooming, and linking
- An example
- Control as interconnection
- Pole assignment and stabilization

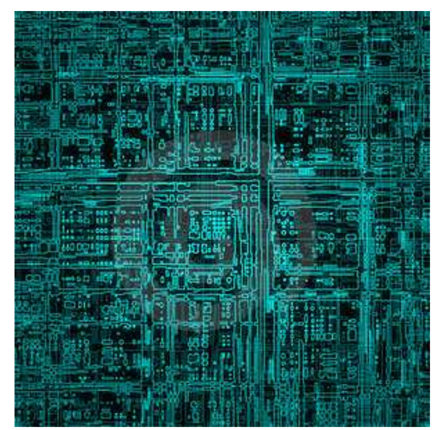
# Systems



OIL REFINERY (GVG / PD)





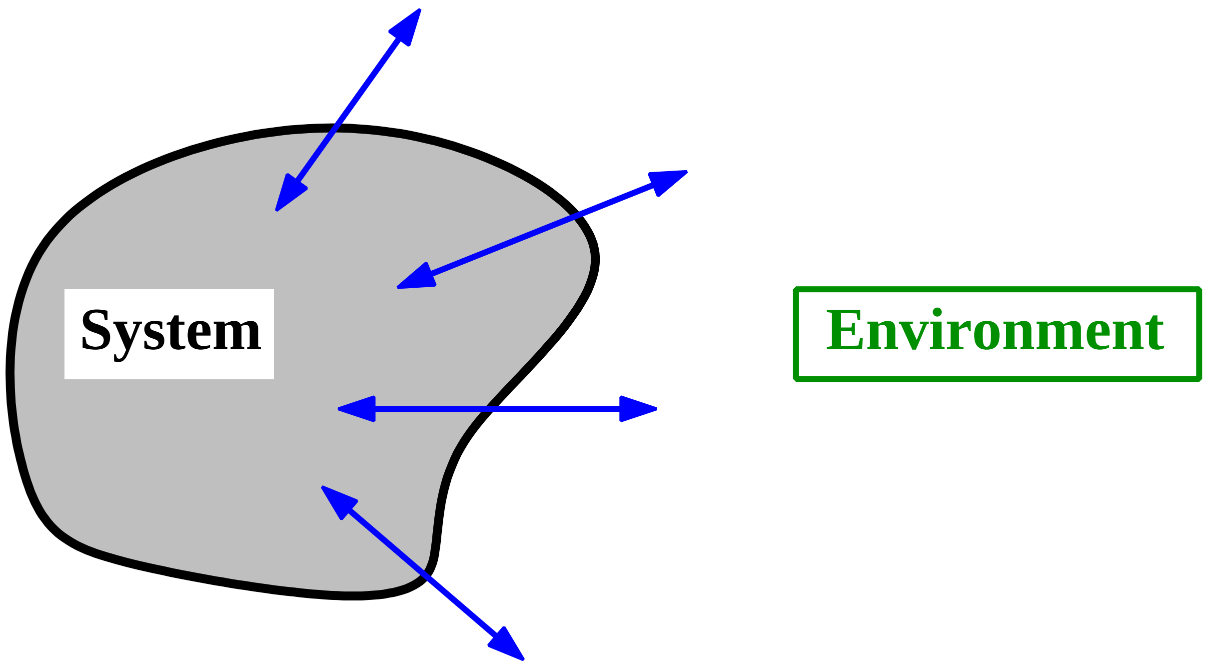
# Features

- **Open**
- **Interconnected**
- **Modular**
- **Dynamic**

**The ever-increasing computing power allows to model such complex interconnected systems accurately by tearing, zooming, and linking.**

$\leadsto$ **Simulation, model based design, ...**

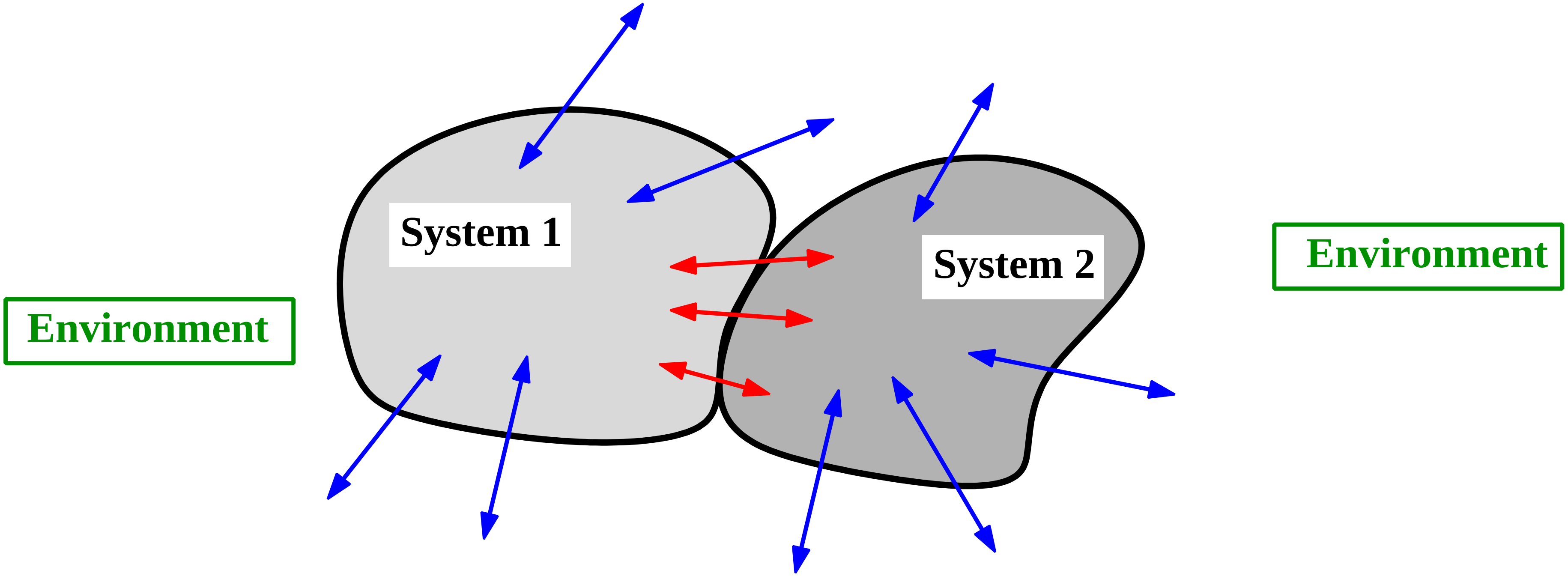
# Open systems




**Systems are 'open', they interact with their environment.**

**In the previous lectures, we have seen that thinking of systems in terms of their behavior captures the 'open' nature of systems very well.**

# Interacting systems




**Interconnected systems interact.**

**How is interaction formalized?**

# Motivation

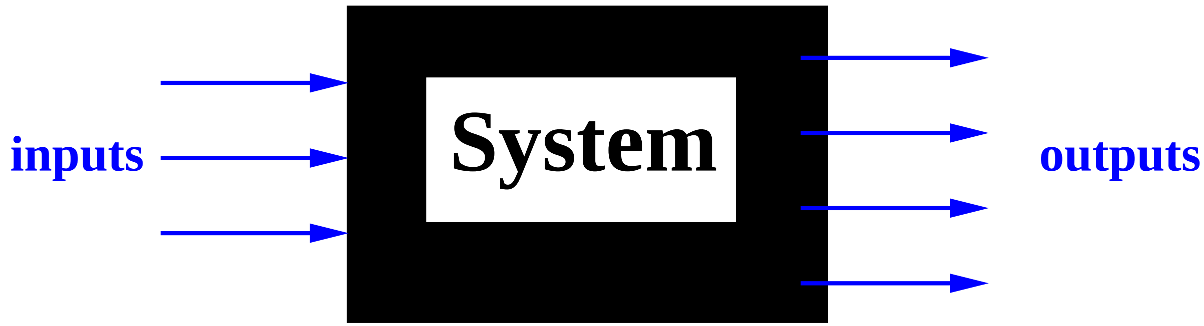
**The ever-increasing computing power allows to model complex interconnected systems accurately.**

**Requires the right mathematical concepts**

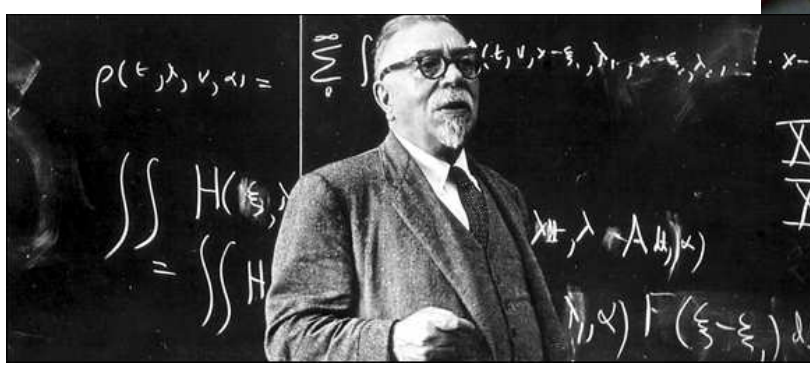
- **for dynamical system (the behavior),**
- **for interconnection (this lecture),**
- **for interconnection architecture (this lecture).**

# Classical view

# Input/output systems



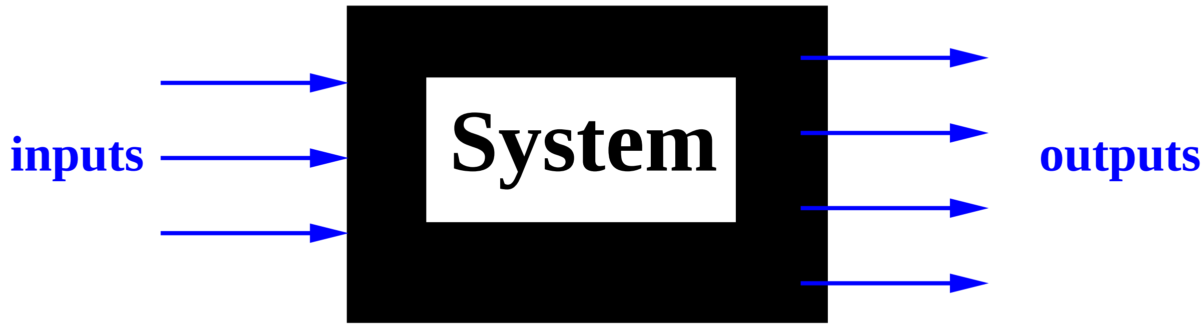

**Oliver Heaviside**
**(1850-1925)**



**Norbert Wiener**
**(1894-1964)**

**Rudy Kalman**
**(1930-     )**

# Input/output systems

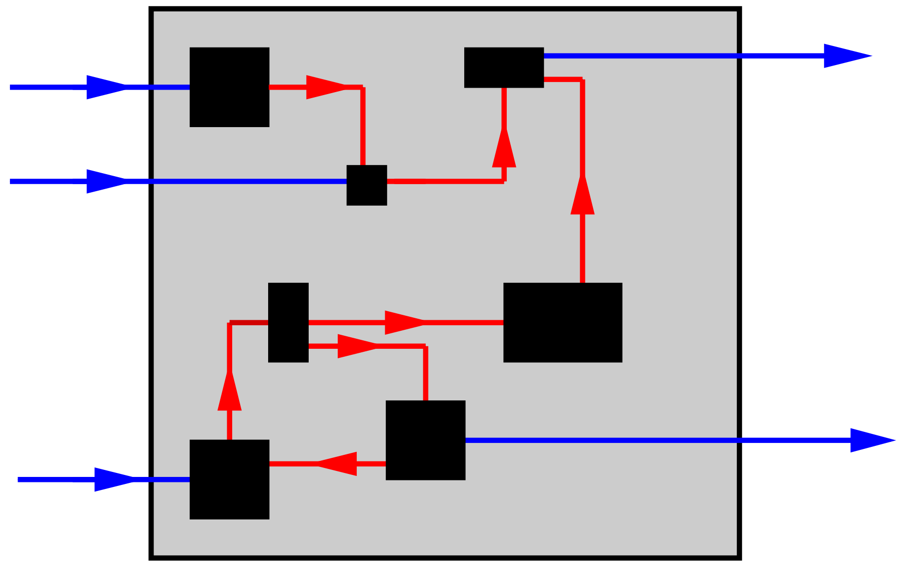



**Input/output thinking is *inappropriate* for describing the functioning of physical systems.**

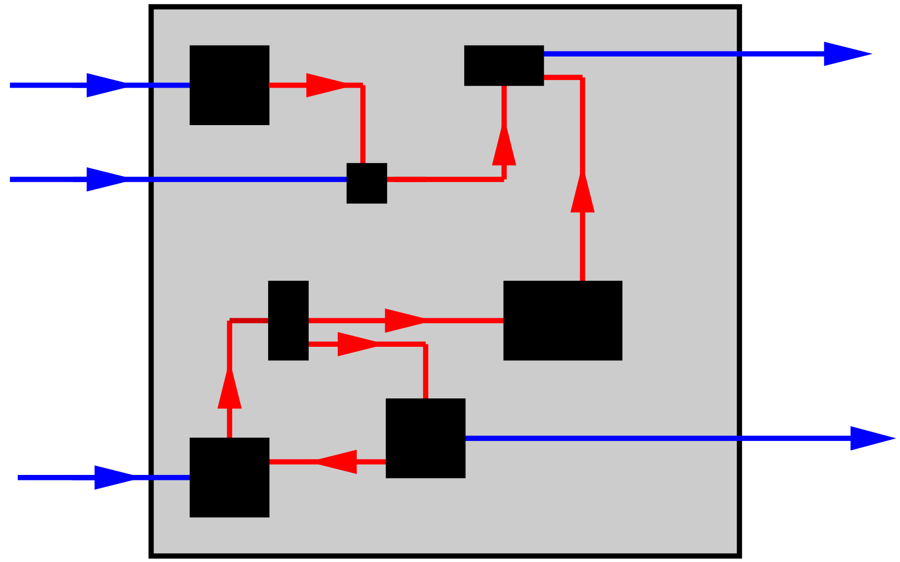
**A physical system is not a signal processor.**

**Better concept: the behavior.**

# Signal flow graphs



# Signal flow graphs



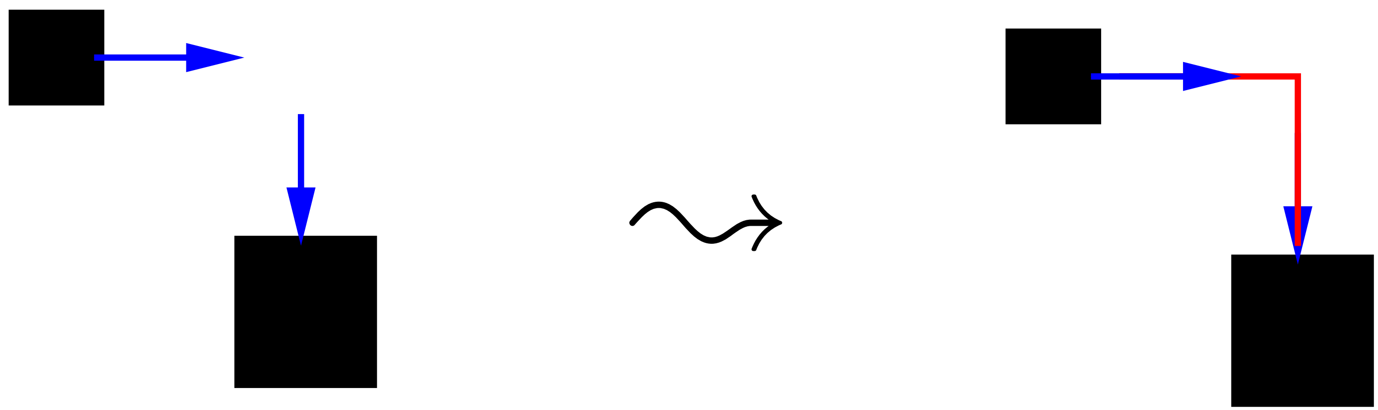
**Signal flow graphs are *inappropriate* for describing the interaction architecture of physical systems.**

**A physical system is not a signal processor.**

**Better concept: graph with leaves.**

# Interconnection

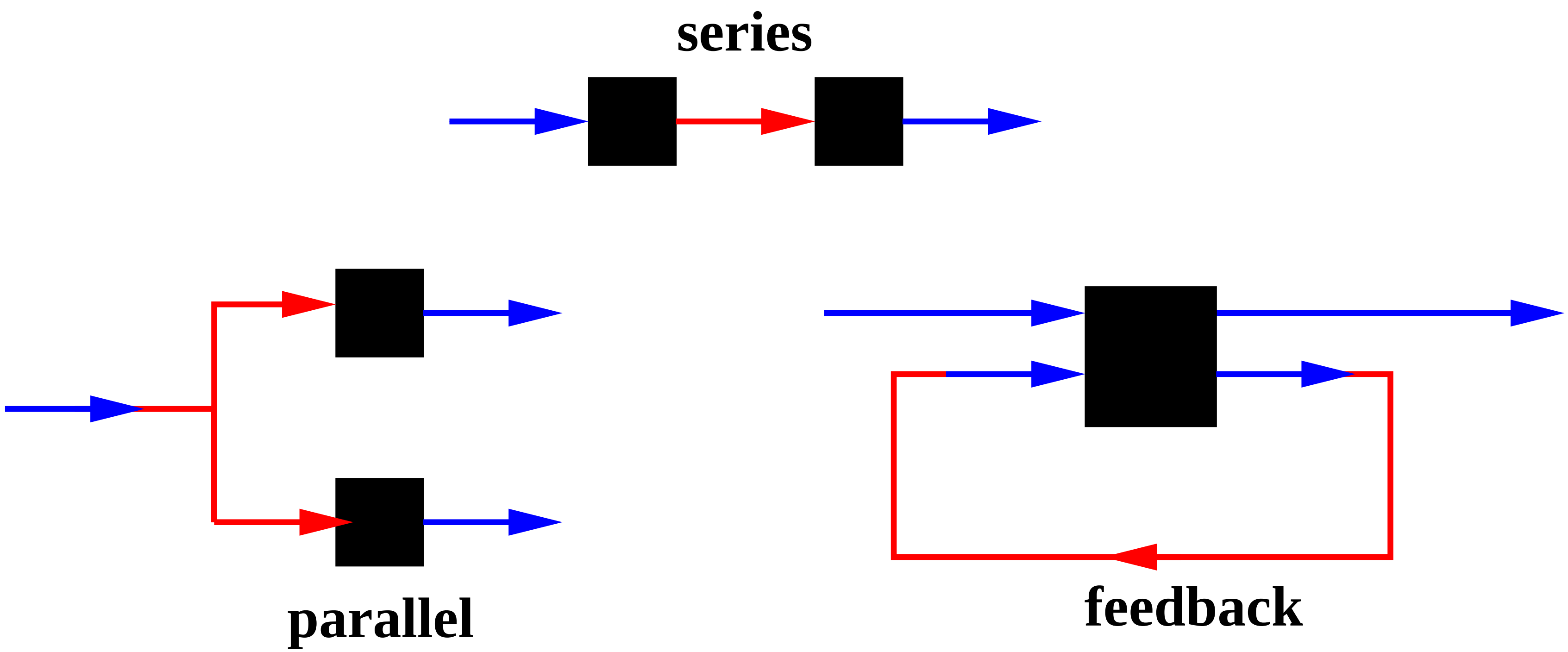
**Interconnection as output-to-input assignment.**



# Interconnection

**Interconnection as output-to-input assignment.**

**Examples:**



# Interconnection

**Interconnection as output-to-input assignment.**

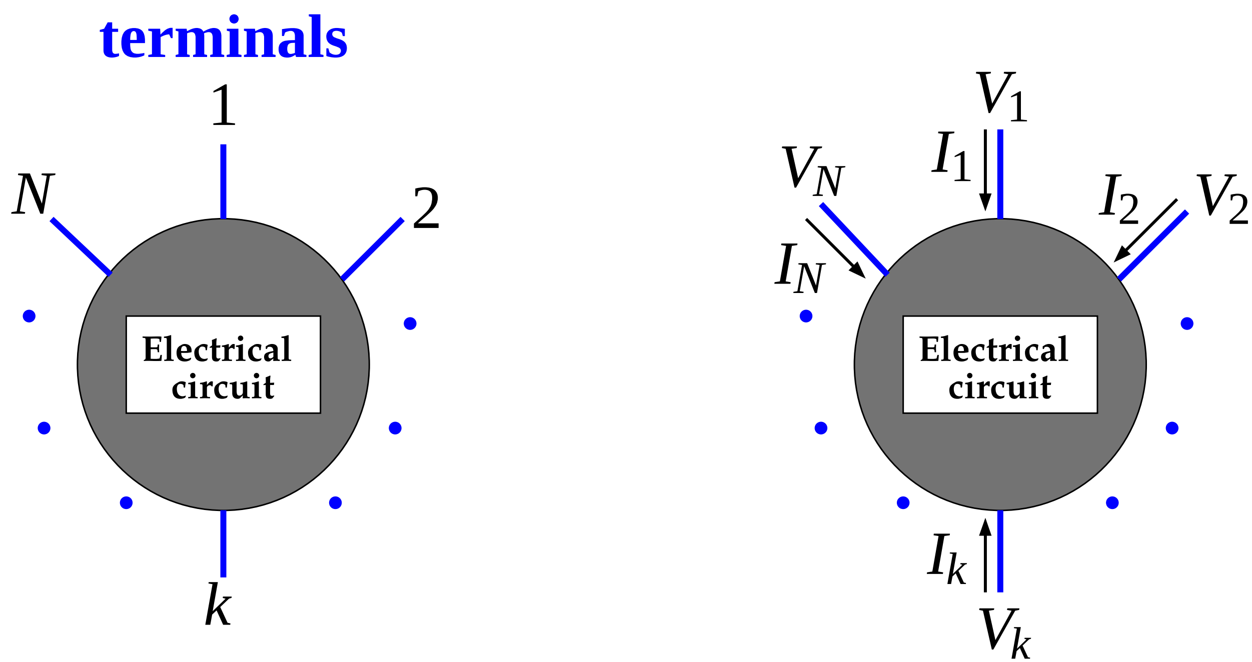
**Output-to-input assignment is *inappropriate* for describing the interconnection of physical systems.**

**A physical system is not a signal processor.**

**Better concept: variable sharing.**

# Examples

# Electrical circuit

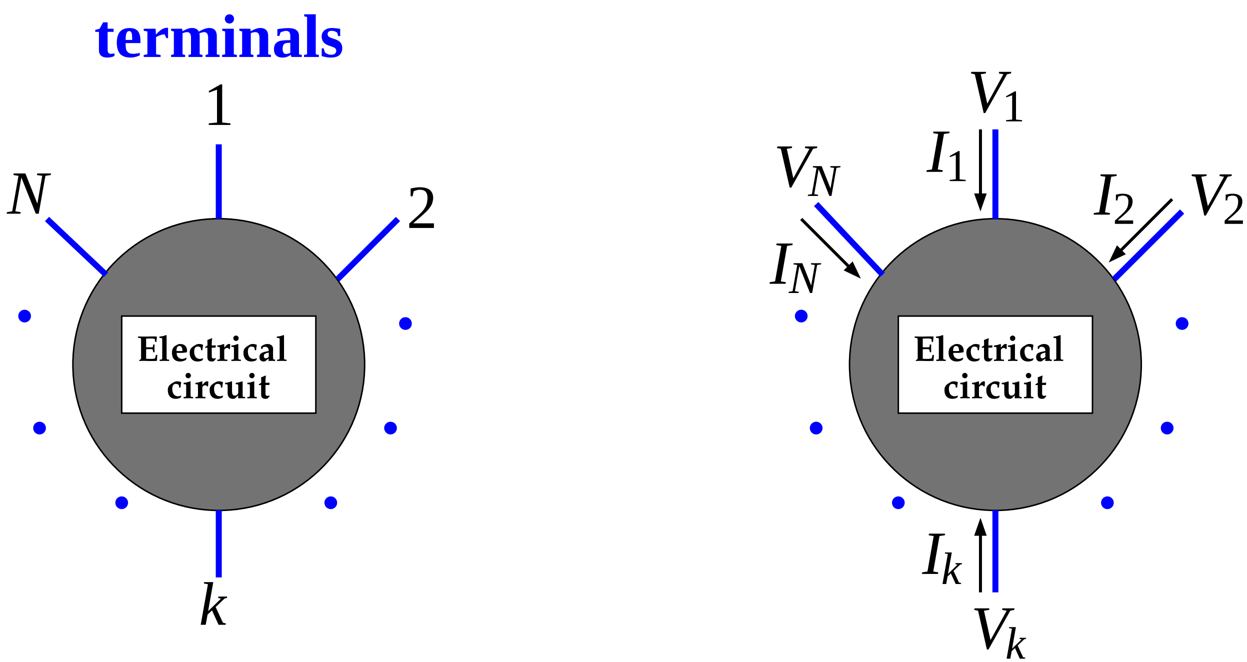



**<u>At each terminal</u>:**

**a potential (!) and a current (counted $> 0$ into the circuit),**

**The relation between potentials of the terminals and voltages across the terminals is discussed ielsewhere.**

# Electrical circuit




**At each terminal:**

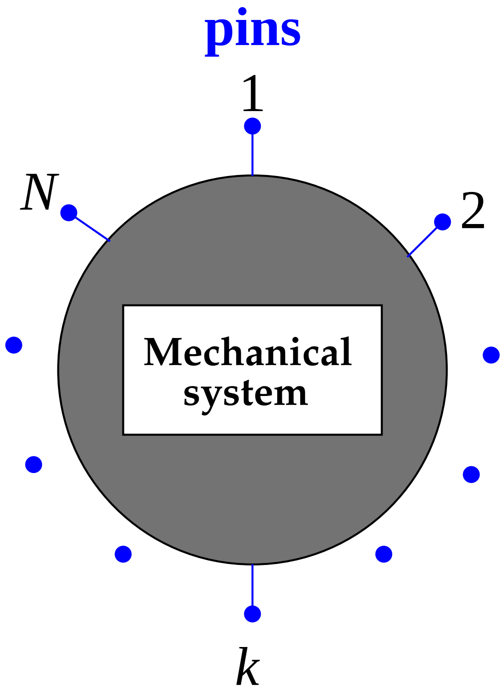
**a potential (!) and a current (counted $> 0$ into the circuit),**

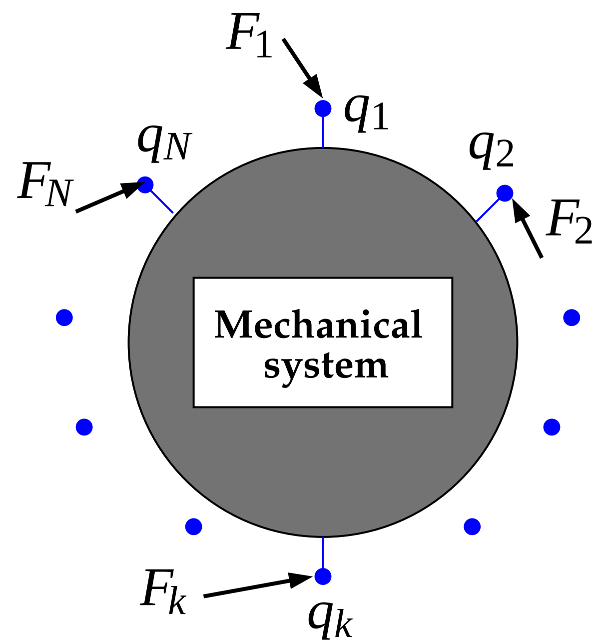
$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

**this potential/current trajectory is compatible with the circuit architecture and its element values.**

# Mechanical device




**At each terminal: a position and a force.**

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

**More generally, a position, force, angle, and torque.**

# Other domains

- **Thermal systems:**

  **At each terminal: a temperature and a heat flow.**

- **Hydraulic systems:**

  **At each terminal: a pressure and a mass flow.**

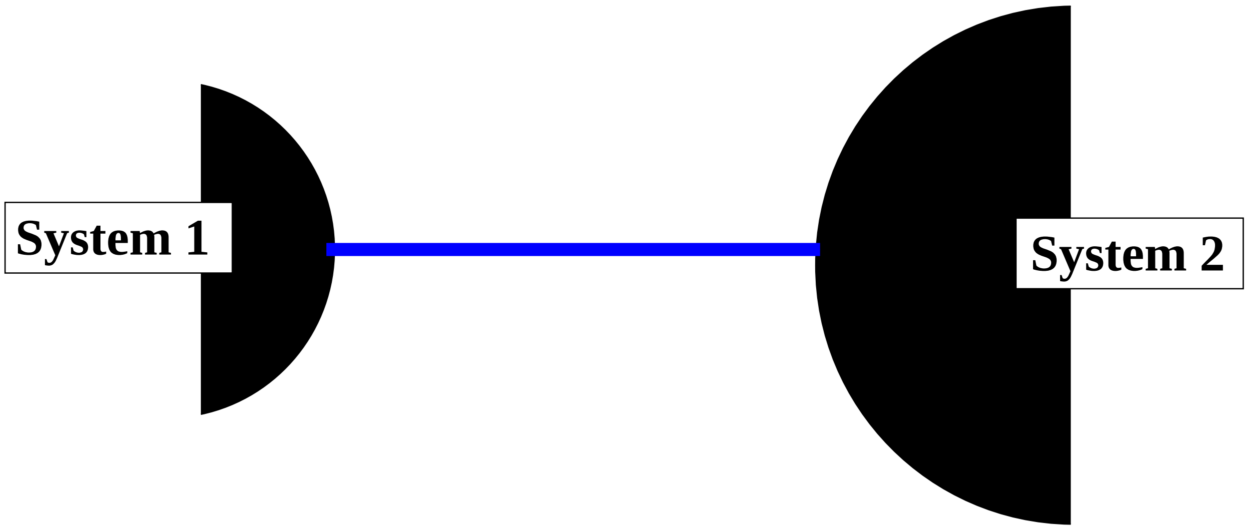
- **Multidomain systems:**

  **Systems with terminals of different types, as motors, pumps, etc.**

- **...**

# Interconnection

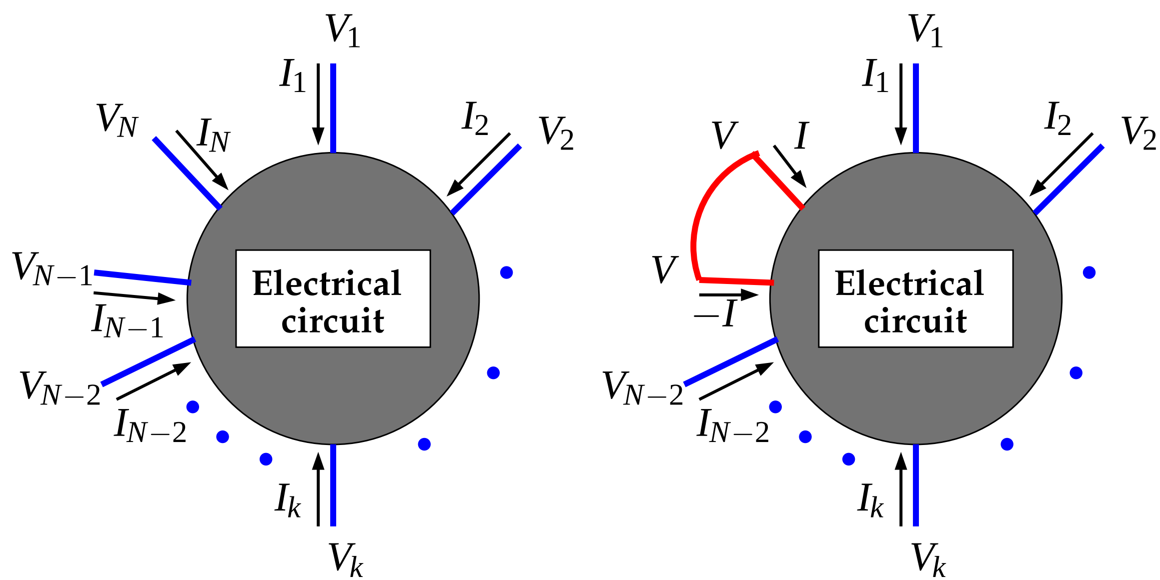
# Connection of terminals




**By interconnecting, the terminal variables are equated.**

# Connection of circuit terminals

**Interconnection = connecting terminals, like soldering wires together.**

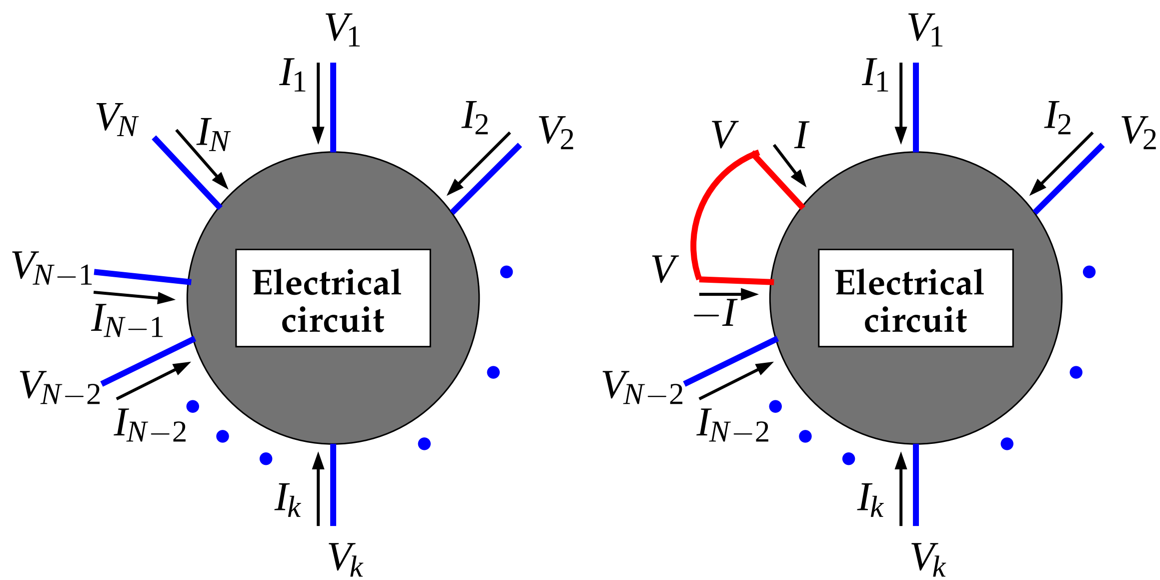



**Connecting terminals $N-1$ and $N$ leads to**

$$V_{N-1} = V_N, \quad I_{N-1} + I_N = 0.$$

**After interconnection the terminals share the variables $V_{N-1}, V_N$, and $I_{N-1}, I_N$ (up to a sign).**

# Connection of circuit terminals




**Connecting terminals $N-1$ and $N$ leads to**

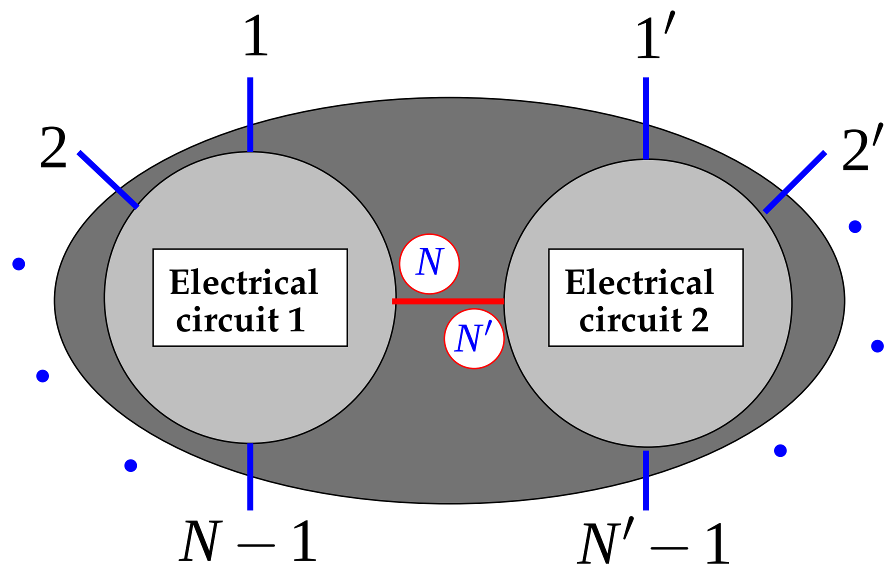
$$V_{N-1} = V_N, \quad I_{N-1} + I_N = 0.$$

**The interconnected circuit has $N-2$ terminals. Its behavior =**

$$\mathscr{B}' = \{(V_1, I_1, V_2, I_2, \ldots, V_{N-2}, I_{N-2}) : \mathbb{R} \to \mathbb{R}^{2(N-2)} \mid \exists\ V, I$$

$$\text{such that } (V_1, I_1, V_2, I_2, \ldots, V_{N-2}, I_{N-2}, V, I, V, -I) \in \mathscr{B}\}.$$

# Interconnection of circuits



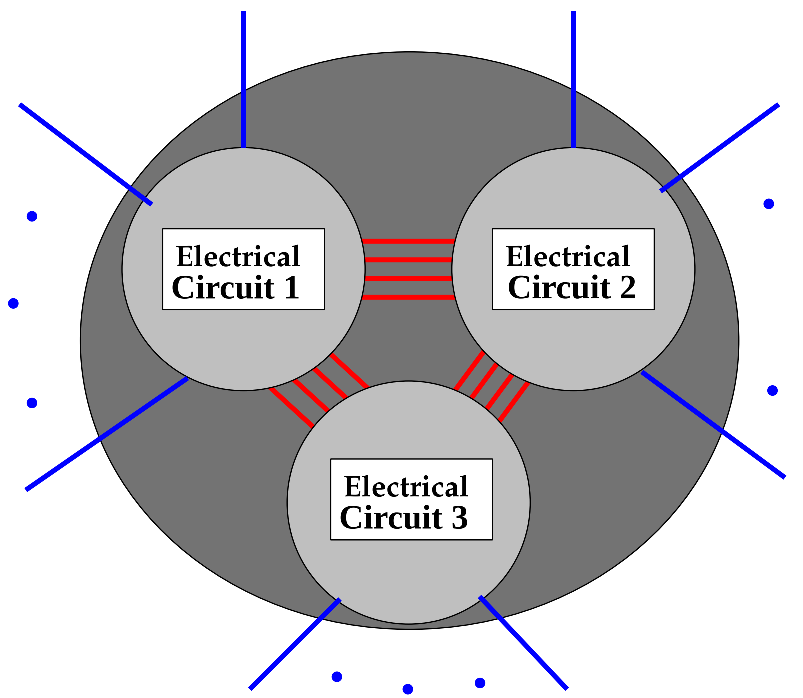

$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

**Behavior after interconnection:**

$$\mathscr{B}_1 \sqcap \mathscr{B}_2$$
$$:= \{(V_1, \ldots, V_{N-1}, V_{1'}, \ldots, V_{N'-1}, I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}) \mid$$
$$\exists\, V, I \text{ such that}$$
$$(V_1, \ldots, V_{N-1}, V, I_1, \ldots, I_{N-1}, I) \in \mathscr{B}_1 \textbf{ and}$$
$$(V_{1'}, \ldots, V_{N'-1}, V, I_{1'}, \ldots, I_{N'-1}, -I) \in \mathscr{B}_2\}.$$

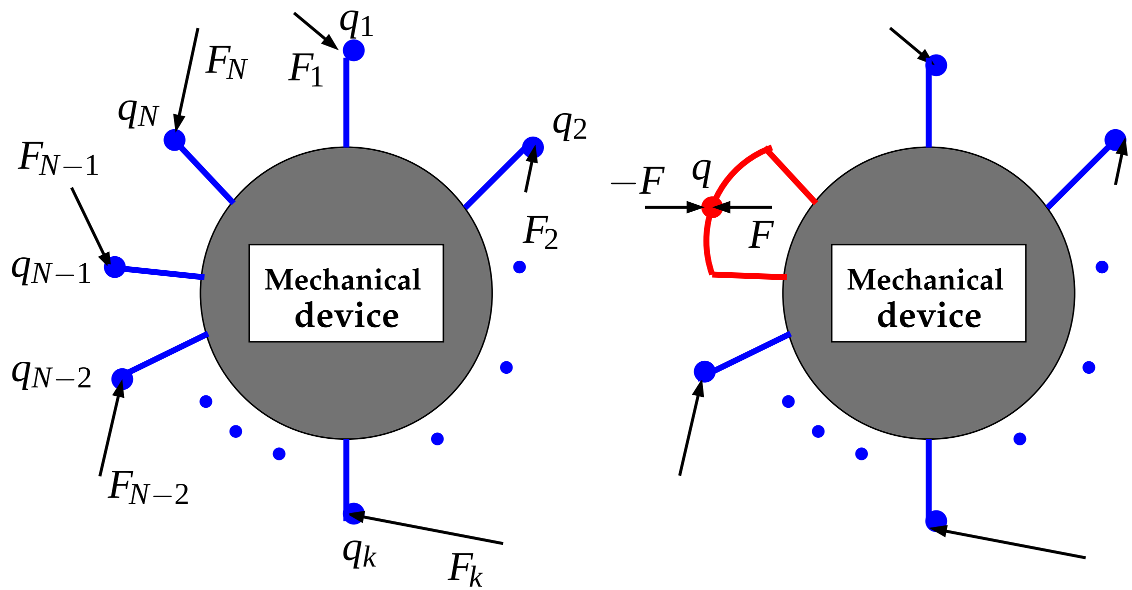
# Interconnection of circuits

⇝ **more terminals and more circuits connected**



# Connection of mechanical terminals

**Interconnection = connecting terminals, like screwing pins together.**

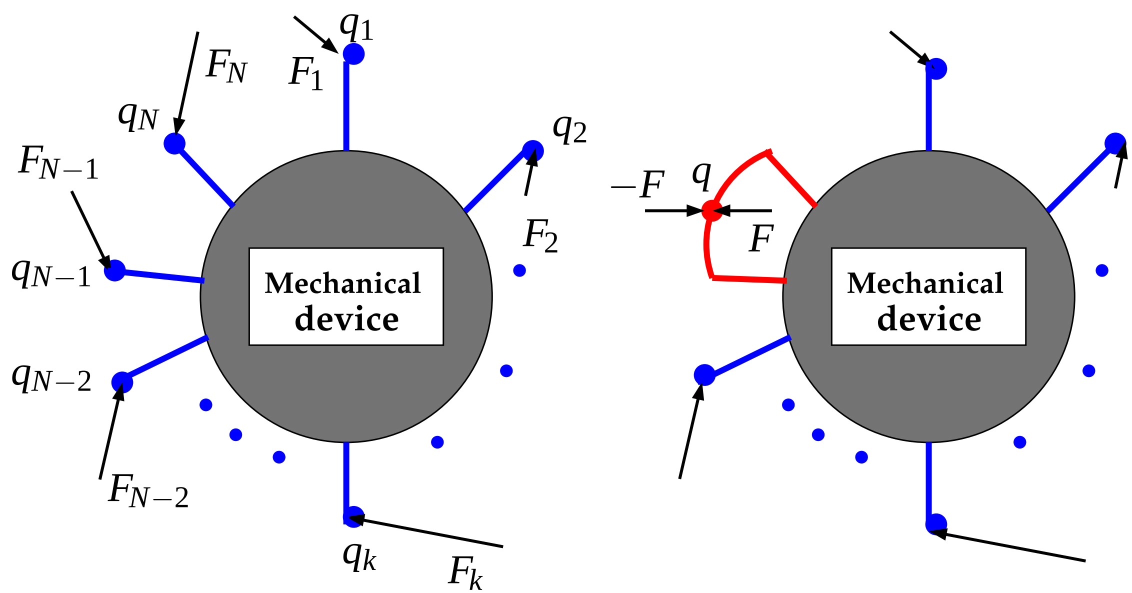



**Connecting terminals $N-1$ and $N$ leads to**

$$q_{N-1} = q_N, \quad F_{N-1} + F_N = 0.$$

**After interconnection the terminals share the variables $q_{N-1}, q_N$, and $F_{N-1}, F_N$ (up to a sign).**

# Connection of mechanical terminals




**Connecting terminals $N-1$ and $N$ leads to**

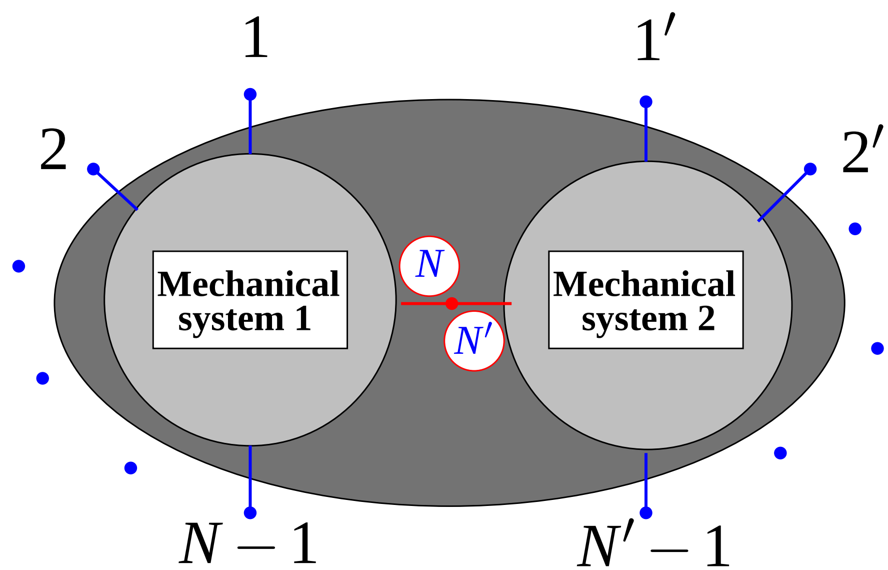
$$q_{N-1} = q_N, \quad F_{N-1} + F_N = 0.$$

**The interconnected circuit has $N-2$ terminals. Its behavior =**

$$\mathscr{B}' = \{(q_1, F_1, q_2, F_2, \ldots, q_{N-2}, F_{N-2}) : \mathbb{R} \to \mathbb{R}^{2(N-2)} \mid \exists\ q, F$$

$$\textbf{such that } (q_1, F_1, q_2, F_2, \ldots, q_{N-2}, F_{N-2}, q, F, q, -F) \in \mathscr{B}\}.$$

# Interconnection of mechanical systems




$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

# Other terminal types

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- ...

# Sharing variables

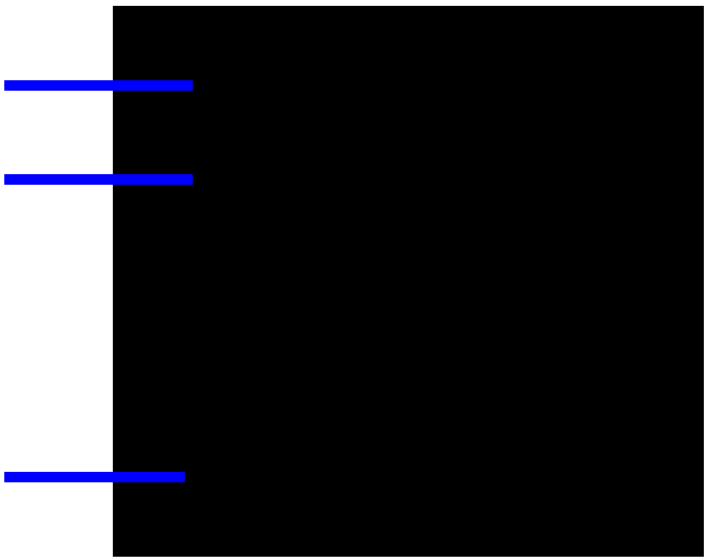
$$
\begin{aligned}
V_N = \; V_{N'} \quad &\textbf{and} \quad I_N + I_{N'} = \; 0, \\
q_N = \; q_{N'} \quad &\textbf{and} \quad F_N + F_{N'} = \; 0, \\
T_N = \; T_{N'} \quad &\textbf{and} \quad Q_N + Q_{N'} = \; 0, \\
p_N = \; p_{N'} \quad &\textbf{and} \quad f_N + f_{N'} = \; 0, \\
&\vdots
\end{aligned}
$$

**Interconnection means variable sharing.**

# Tearing, zooming, and linking

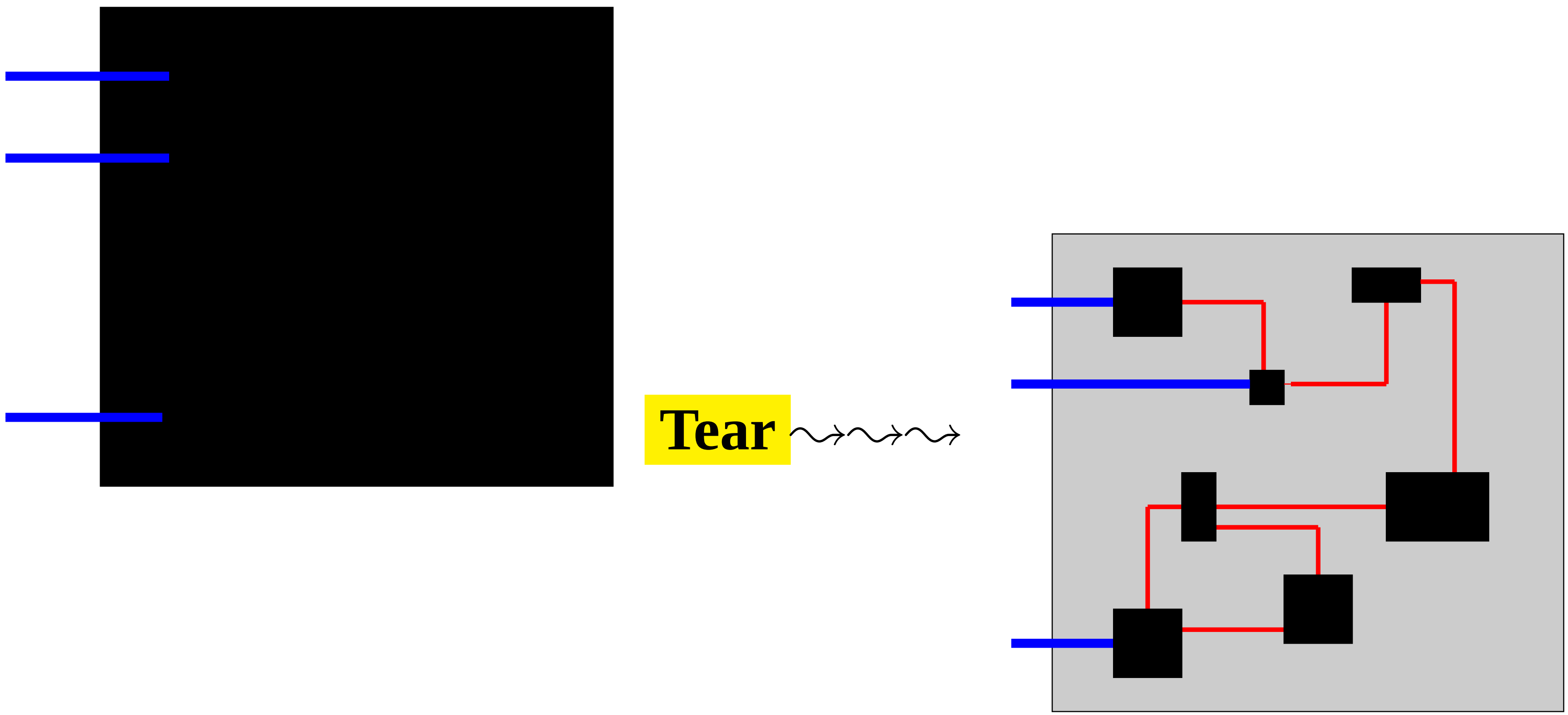
# Tearing

**¡¡ Model the behavior of selected variables !!**

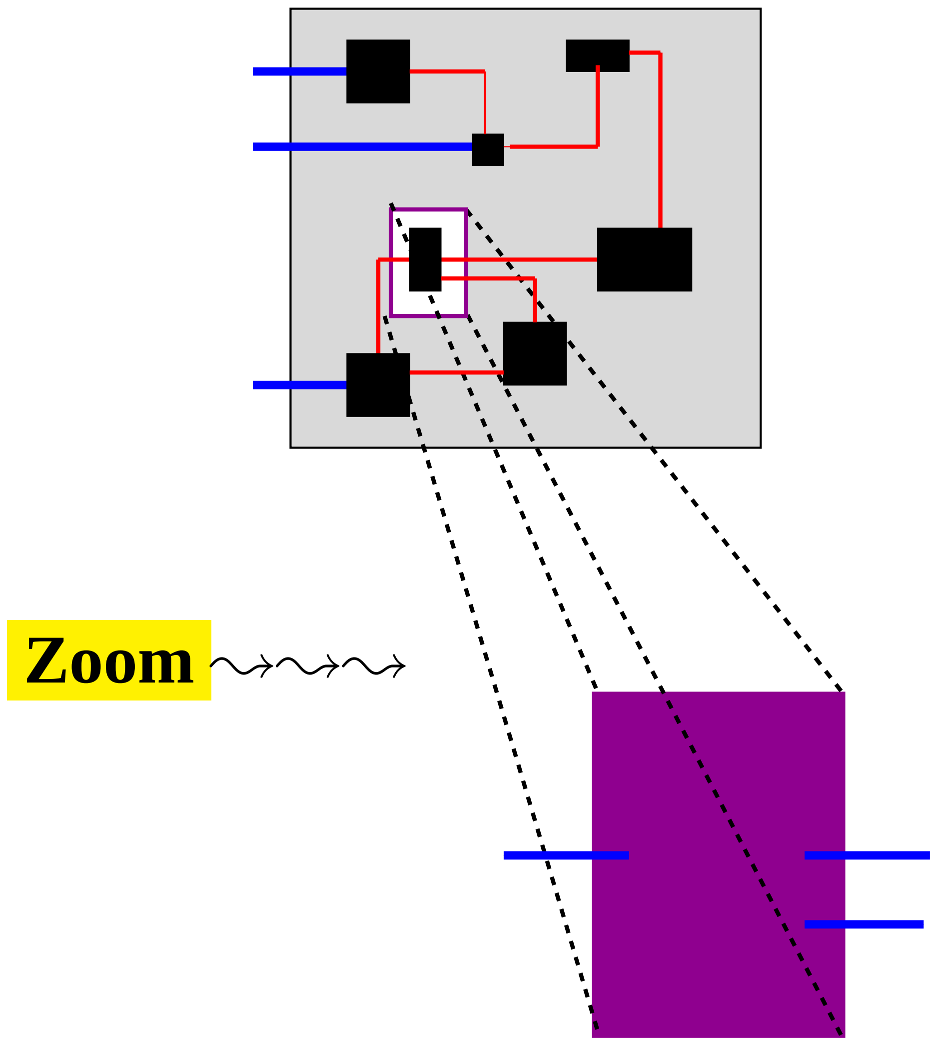


# Tearing

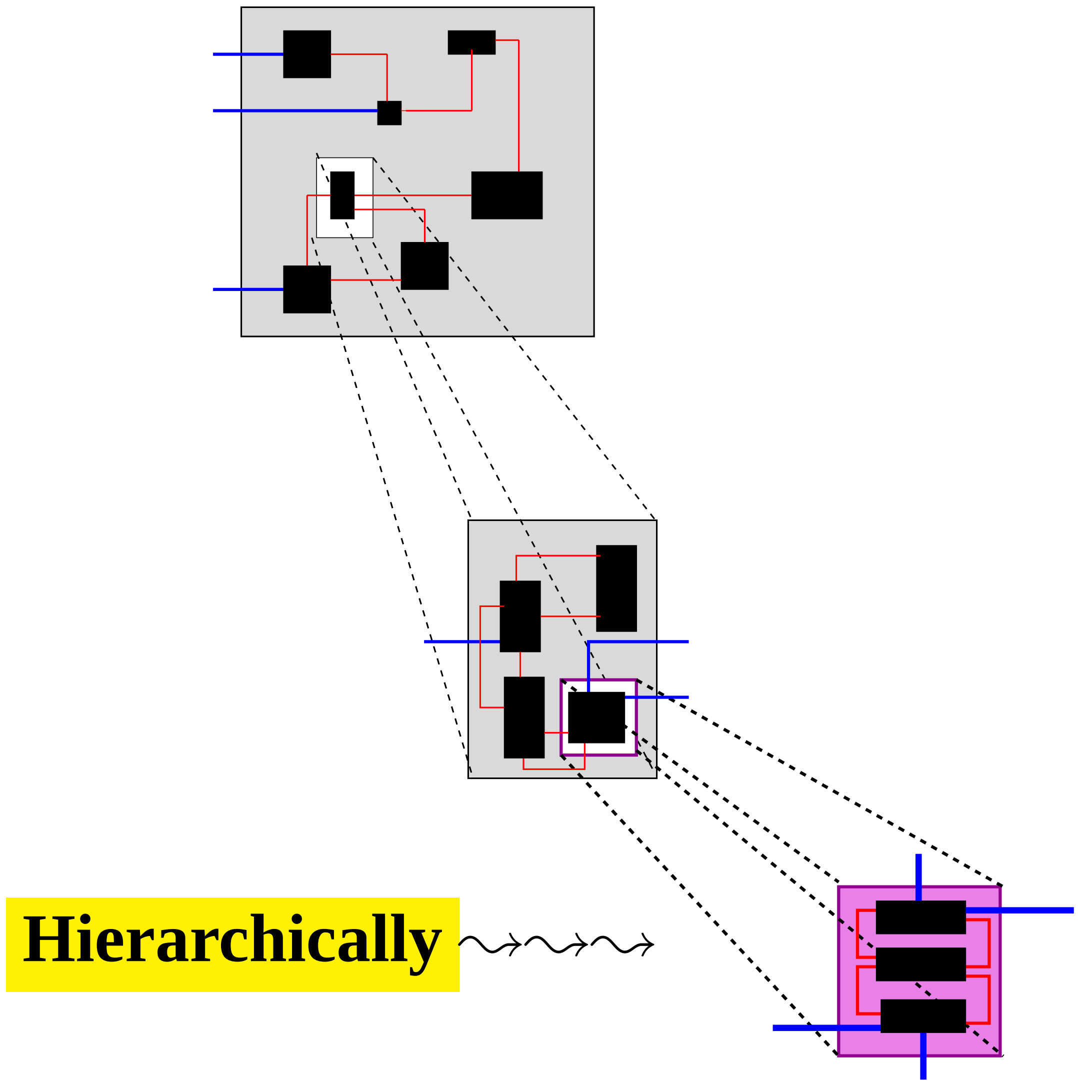
## ¡¡ Model the behavior of selected variables !!



# Zooming

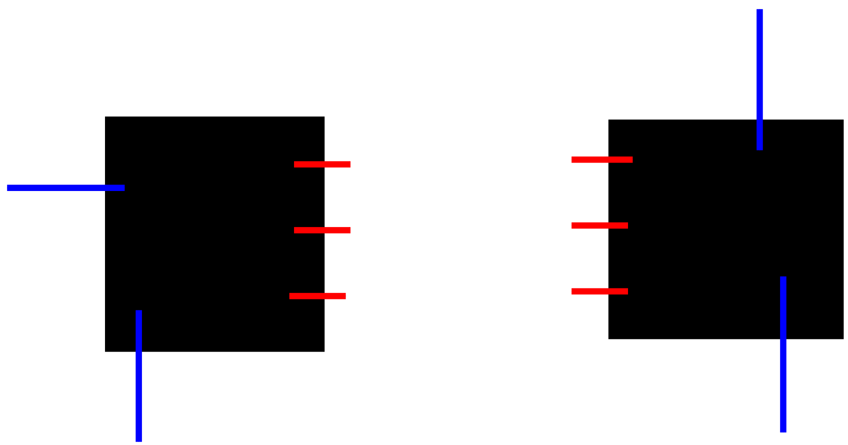


# Zooming

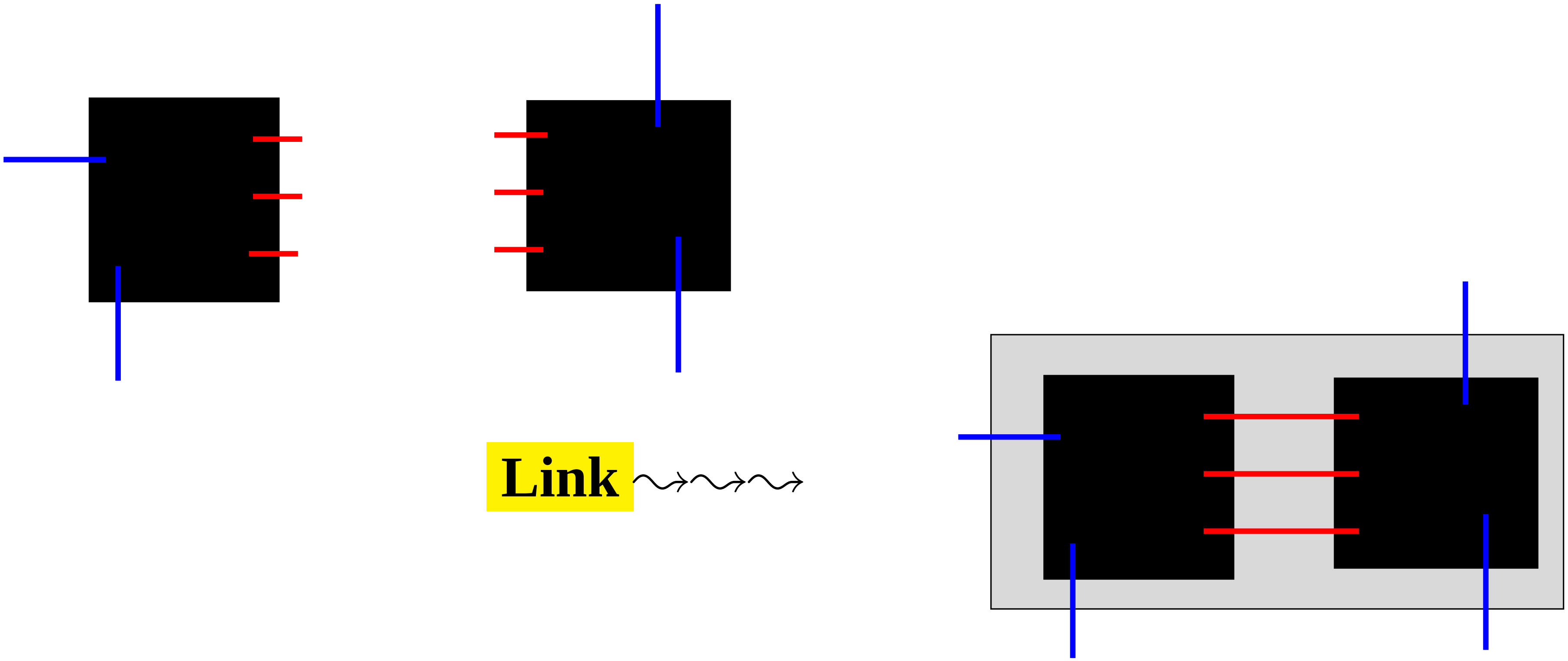



**Proceed until subsystems ('modularity') are obtained whose model is known, from first principles, or stored in a database.**

# Linking



# Linking



# Interconnection architecture

# Graph with leaves

**A graph with leaves :⇔**

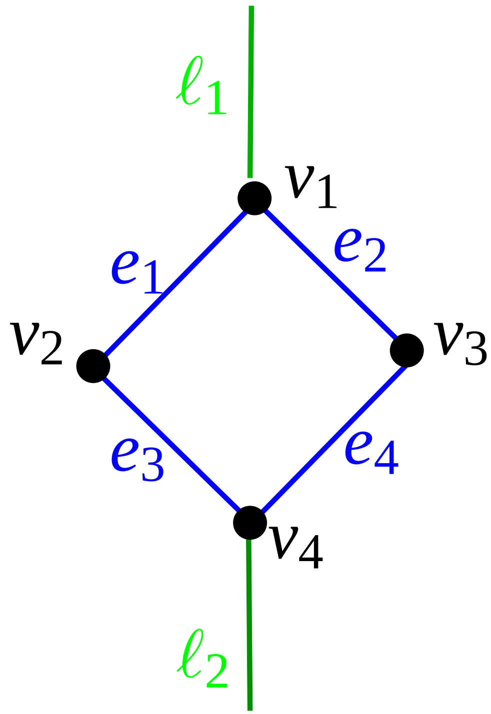
$$\mathscr{G} = (\mathbb{V}, \mathbb{E}, \mathbb{L}, f_{\mathbb{E}}, f_{\mathbb{L}})$$

$\mathbb{V}$ **a finite set of *vertices*,**
$\mathbb{E}$ **a finite set of *edges*,**
$\mathbb{L}$ **a finite set of *leaves*,**
$f_{\mathbb{E}}$ **the edge incidence map,**
$f_{\mathbb{L}}$ **the leaf incidence map.**

$f_{\mathbb{E}}$ **maps each element** $e \in \mathbb{E}$ **into an unordered pair** $[v_1, v_2]$**, with** $v_1, v_2 \in \mathbb{V}$**,**
$f_{\mathbb{L}}$ **is a map from** $\mathbb{L}$ **to** $\mathbb{V}$**, it maps each element** $\ell \in \mathbb{L}$ **into an element** $v \in \mathbb{V}$**.**

# Graph with leaves

**Example:**




$$f_{\mathbb{E}} : e_1 \mapsto [v_1, v_2], e_2 \mapsto [v_1, v_3], e_3 \mapsto [v_2, v_4], e_4 \mapsto [v_3, v_4].$$

$$f_{\mathbb{L}} : \ell_1 \mapsto v_1, \ell_2 \mapsto v_4.$$

# Formalization of interconnected system

**An interconnected system is identified with a graph with leaves**

$$\mathscr{G} = (\mathbb{V}, \mathbb{E}, \mathbb{L}, f_{\mathbb{E}}, f_{\mathbb{L}}).$$

**The vertices** $\leftrightarrow$ **subsystems**
**The edges** $\leftrightarrow$ **connections,**
**The leaves** $\leftrightarrow$ **external terminals.**

# Model specification

**A model is obtained as follows.**

- **For each subsystem, specify the behavior of the variables on its terminals, i.e. on the edges and the leaves that are incident to the vertex corresponding to the subsystem.**
- **For each connection, specify the sharing variable conditions on the connected terminals. I.e., for each edge, specify the interconnection constraints on the variables of the subsystem terminals that correspond to the edges.**
- **Specify the manifest variables.**

**Subsystems in the vertices.**

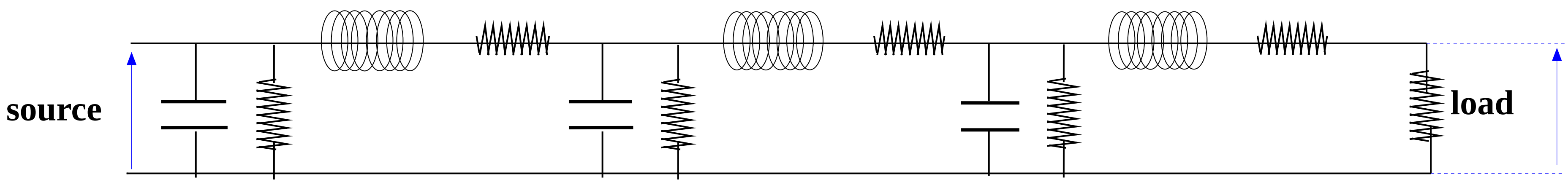
**Connections in the edges.**

**External terminals in the leaves.**

# Example

# A transmission line

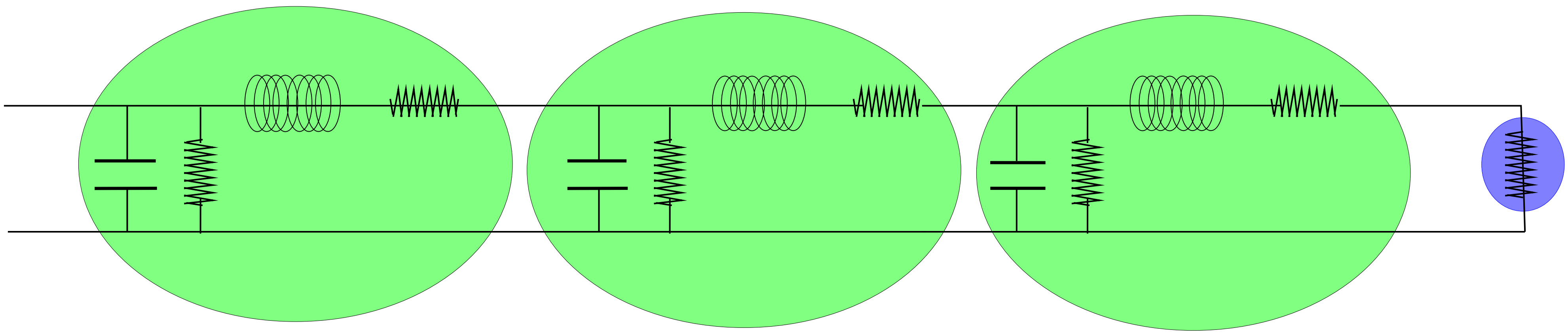
**Consider the transmission line shown below.**




**The aim is to model the relation between the voltage of the source on the left and the voltage across the load on the right.**

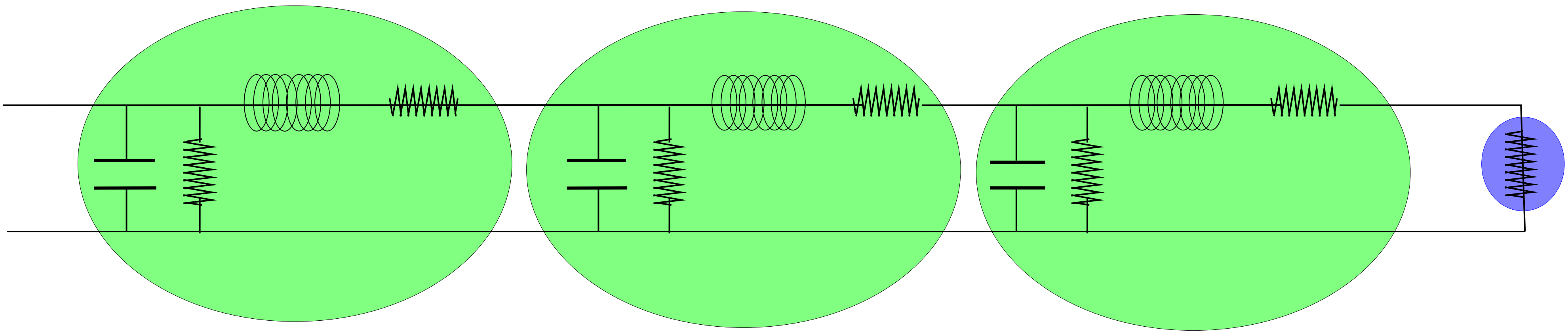
# A transmission line

**View the system as an interconnection of 4 subsystems.**

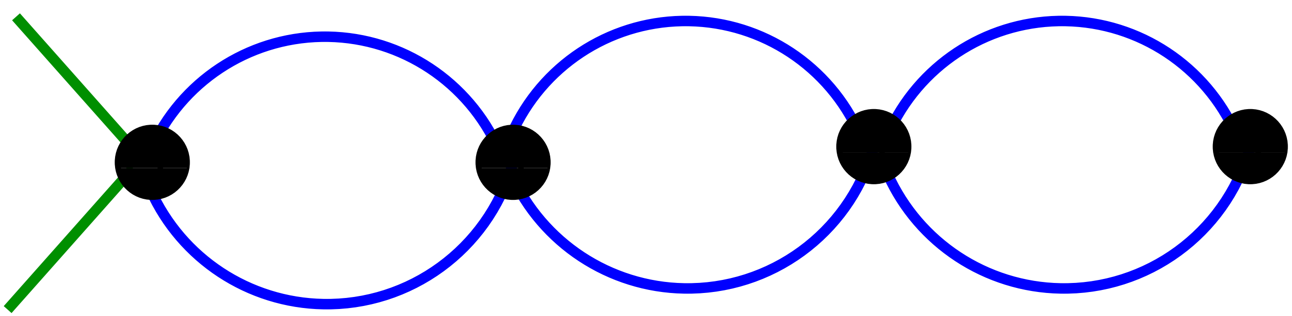


# A transmission line

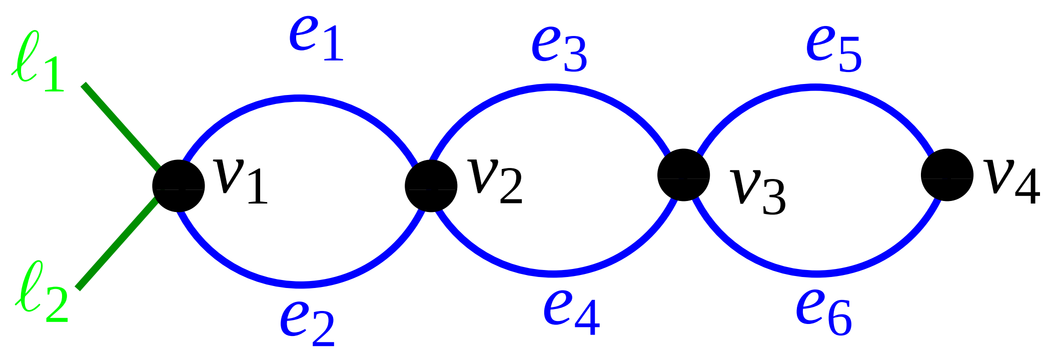
**View the system as an interconnection of 4 subsystems.**



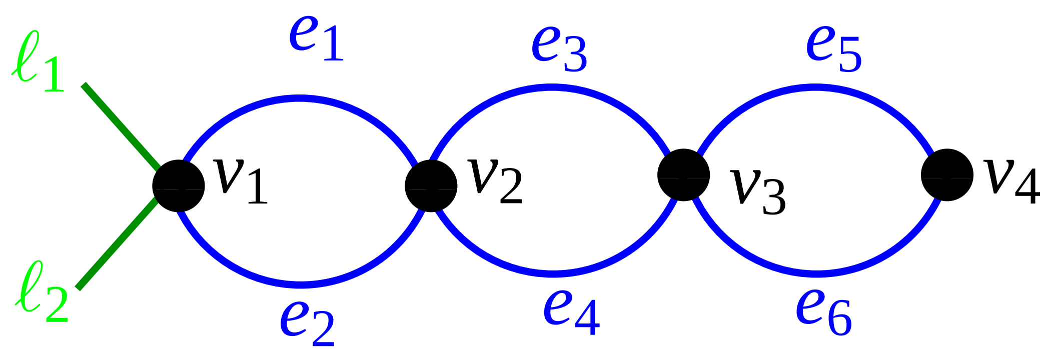
**The interconnection architecture $\rightsquigarrow$ the graph with leaves**



# A transmission line

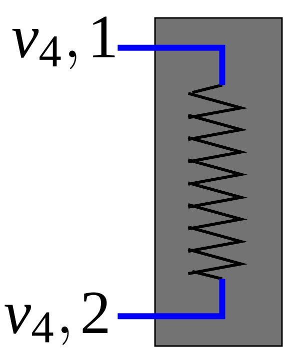


# A transmission line




**In vertices $v_1, v_2, v_3$ we have identical subsystems.**
**We deal with them later.**

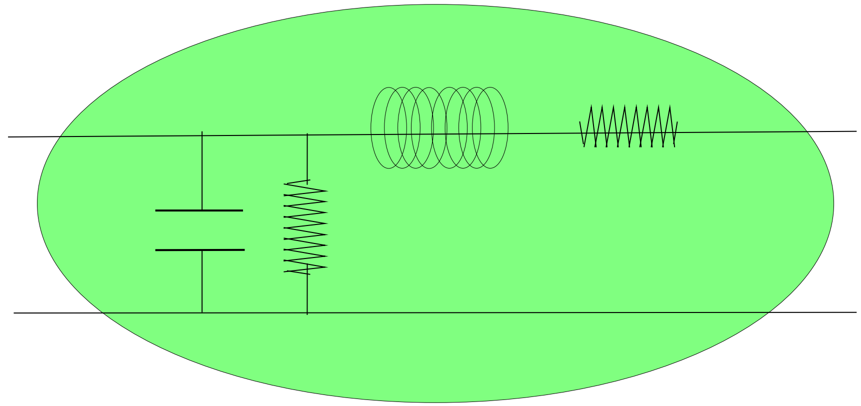
**In vertex $v_4$ there is a resistor** $\rightsquigarrow$



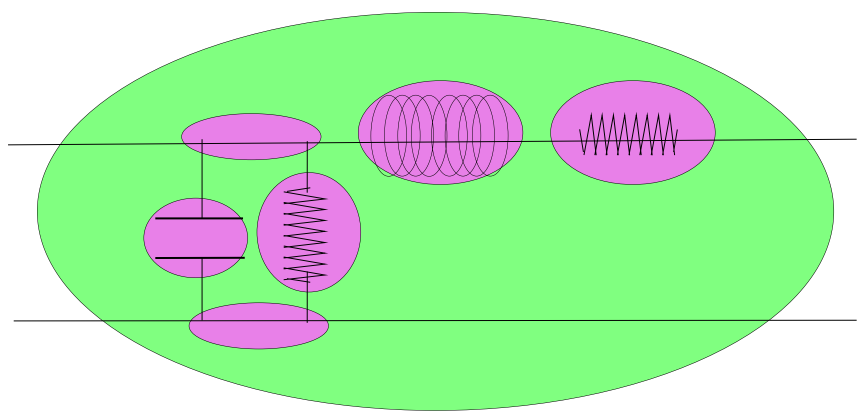

$$V_{v_4,1} - V_{v_4,2} = R\, I_{v_4,1}, \quad I_{v_4,1} + I_{v_4,2} = 0.$$

# A transmission line section

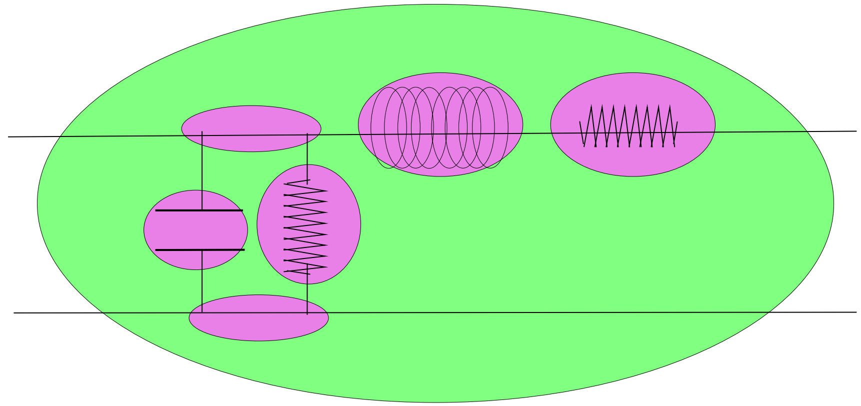
**In each of the vertices $v_1, v_2, v_3$ we have:**



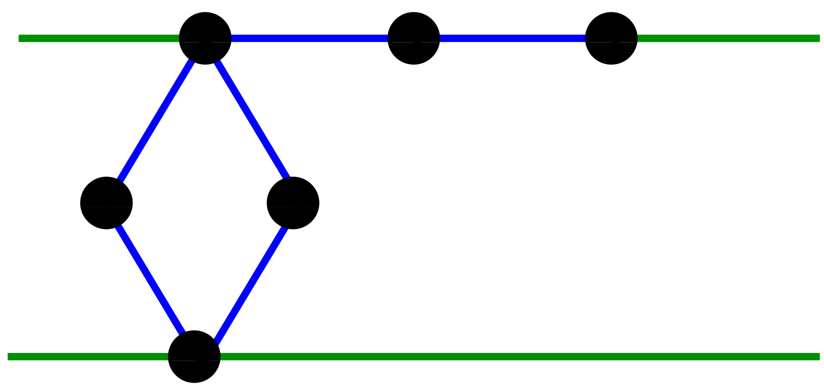
**This system can be viewed as the interconnection of 6 subsystems:**



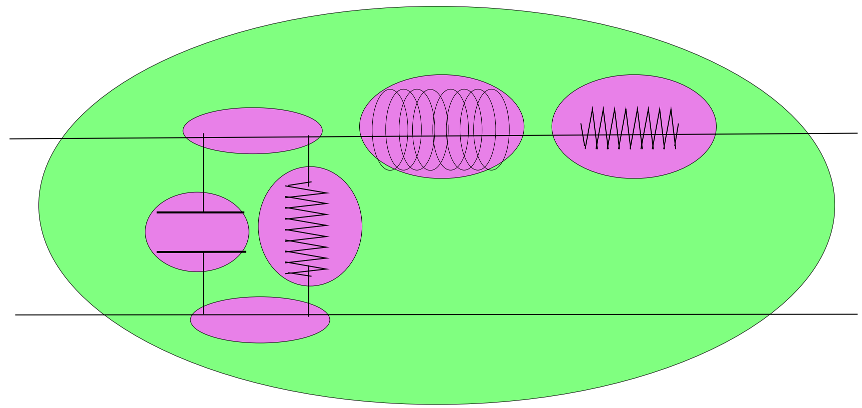
# A transmission line section



**The associated interconnection architecture is**



# A transmission line section



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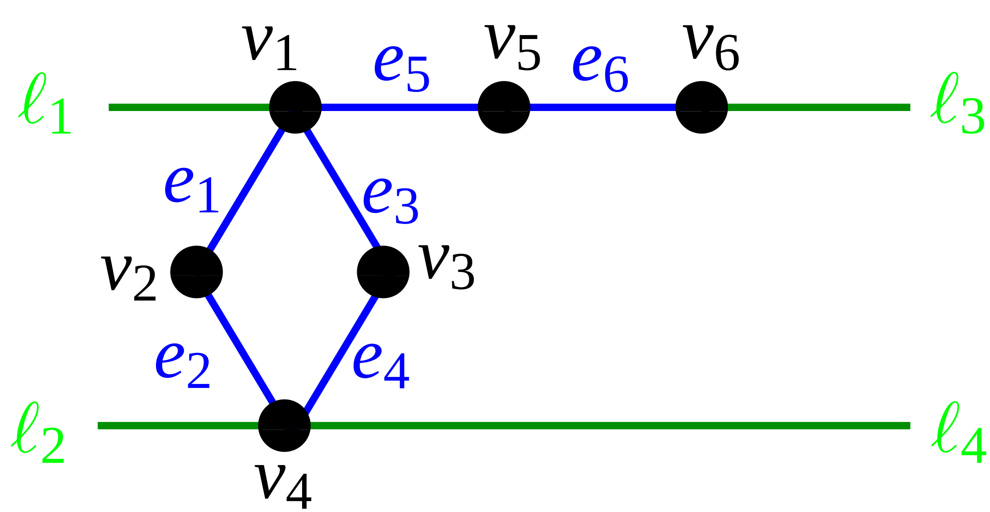
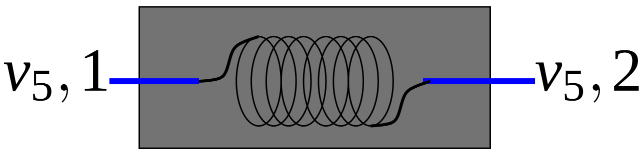


# Modeling the transmission line section

**Subsystems: 2 resistors, 1 inductor, 1 capacitor, 2 connectors.**

**The inductor in vertex $v_5$, for example, $\leadsto$**

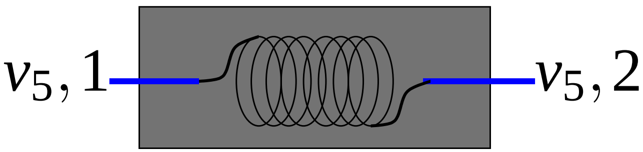



$$V_{v_5,1} - V_{v_5,2} = L\,\frac{d}{dt} I_{v_5,1}, \qquad I_{v_5,1} + I_{v_5,2} = 0.$$

# Modeling the transmission line section

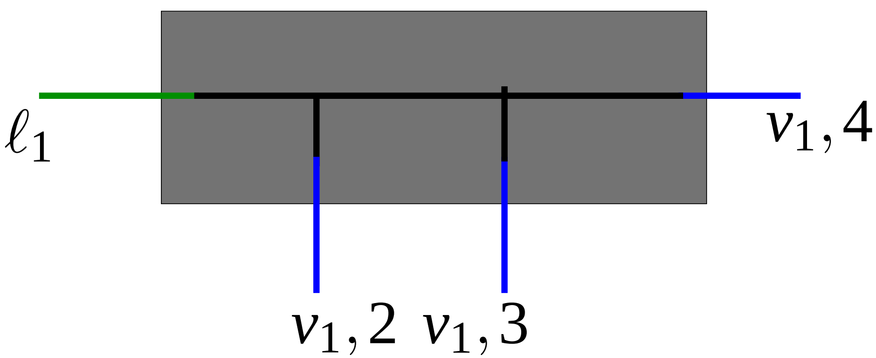
**Subsystems: 2 resistors, 1 inductor, 1 capacitor, 2 connectors.**

**The inductor in vertex $v_5$, for example, $\leadsto$**




$$V_{v_5,1} - V_{v_5,2} = L\,\frac{d}{dt} I_{v_5,1}, \quad I_{v_5,1} + I_{v_5,2} = 0.$$

**The connector in vertex $v_1$, for example, $\leadsto$**




$$V_{\ell_1} = V_{v_1,2} = V_{v_1,3} = V_{v_1,4}, \quad I_{\ell_1} + I_{v_1,2} + I_{v_1,3} + I_{v_1,4} = 0.$$

**For each vertex we obtain a set of equations.**

# Modeling the transmission line section

**The connection equations for edge $e_5$, for example, $\rightsquigarrow$**

$$V_{v_1,4} = V_{v_5,1}, \quad I_{v_1,4} + I_{v_5,1} = 0.$$

**For each edge we obtain two such equations.**

# Modeling the transmission line section

**For each vertex and for each edge we obtain a set of equations. The manifest variables for this subsystem are**

$$V_{\ell_1}, I_{\ell_1}, V_{\ell_2}, I_{\ell_2}, V_{\ell_3}, I_{\ell_3}, V_{\ell_4}, I_{\ell_4}.$$

**The latent variables are**

$$V_{v_1,1}, I_{v_1,1}, \ldots, V_{v_6,1}, I_{v_6,1}.$$

# Modeling the transmission line section

**For each vertex and for each edge we obtain a set of equations. The manifest variables for this subsystem are**

$$V_{\ell_1}, I_{\ell_1}, V_{\ell_2}, I_{\ell_2}, V_{\ell_3}, I_{\ell_3}, V_{\ell_4}, I_{\ell_4}.$$

**The latent variables are**

$$V_{v_1,1}, I_{v_1,1}, \ldots, V_{v_6,1}, I_{v_6,1}.$$

**Eliminating the latent variables**

$\leadsto$ **a LTIDS with (4) ODEs in the variables**

$$V_{\ell_1}, I_{\ell_1}, V_{\ell_2}, I_{\ell_2}, V_{\ell_3}, I_{\ell_3}, V_{\ell_4}, I_{\ell_4}.$$

# Modeling the transmission line section

**Denote these equations as**

$$R\left(\frac{d}{dt}\right)\begin{bmatrix} V_{\ell_1} \\ I_{\ell_1} \\ V_{\ell_2} \\ I_{\ell_2} \\ V_{\ell_3} \\ I_{\ell_3} \\ V_{\ell_4} \\ I_{\ell_4} \end{bmatrix} = 0.$$

**Note: For RLC-circuits, as is the case here, there are more efficient ways to arrive at these equations (see Lecture IV).**

# A transmission line

**Going back to the transmission line yields the subsystem equations**

$$R\left(\frac{d}{dt}\right)\begin{bmatrix}\begin{bmatrix}V_{\ell_1}\\ I_{\ell_1}\\ V_{v_1,2}\\ I_{v_1,2}\\ V_{v_1,3}\\ I_{v_1,3}\\ V_{v_1,4}\\ I_{v_1,4}\end{bmatrix} & \begin{bmatrix}V_{v_2,1}\\ I_{v_2,2}\\ V_{v_2,2}\\ I_{v_2,2}\\ V_{v_2,3}\\ I_{v_2,3}\\ V_{v_2,4}\\ I_{v_2,4}\end{bmatrix} & \begin{bmatrix}V_{v_3,1}\\ I_{v_3,2}\\ V_{v_3,2}\\ I_{v_3,2}\\ V_{v_3,3}\\ I_{v_3,3}\\ V_{v_3,4}\\ I_{v_3,4}\end{bmatrix}\end{bmatrix} = 0,$$

$$V_{v_4,1} - V_{v_4,2} = R\, I_{v_4,1}, \qquad I_{v_4,1} + I_{v_4,2} = 0,$$

# A transmission line

## and the interconnection equations

$$
\begin{aligned}
V_{v_1,3} = V_{v_2,1}, &\quad I_{v_1,3} + I_{v_2,1} = 0,\\
V_{v_1,4} = V_{v_2,2}, &\quad I_{v_1,4} + I_{v_2,2} = 0,\\
V_{v_2,3} = V_{v_3,1}, &\quad I_{v_2,3} + I_{v_3,1} = 0,\\
V_{v_2,4} = V_{v_3,2}, &\quad I_{v_2,4} + I_{v_3,2} = 0,\\
V_{v_3,3} = V_{v_4,1}, &\quad I_{v_4,3} + I_{v_4,1} = 0,\\
V_{v_3,4} = V_{v_4,2}, &\quad I_{v_3,4} + I_{v_4,2} = 0.
\end{aligned}
$$

# A transmission line

## and the interconnection equations

$$
\begin{aligned}
V_{v_1,3} &= V_{v_2,1}, \quad I_{v_1,3} + I_{v_2,1} = 0, \\
V_{v_1,4} &= V_{v_2,2}, \quad I_{v_1,4} + I_{v_2,2} = 0, \\
V_{v_2,3} &= V_{v_3,1}, \quad I_{v_2,3} + I_{v_3,1} = 0, \\
V_{v_2,4} &= V_{v_3,2}, \quad I_{v_2,4} + I_{v_3,2} = 0, \\
V_{v_3,3} &= V_{v_4,1}, \quad I_{v_4,3} + I_{v_4,1} = 0, \\
V_{v_3,4} &= V_{v_4,2}, \quad I_{v_3,4} + I_{v_4,2} = 0.
\end{aligned}
$$

## Finally, there is the manifest variable assignment

$$
w_1 = V_{\ell_1} - V_{\ell_2}, \quad w_2 = V_{v_4,1} - V_{v_4,2}.
$$

# A transmission line

**After elimination of the latent variables, we obtain the desired differential equation that describes the behavior of $(w_1, w_2)$**

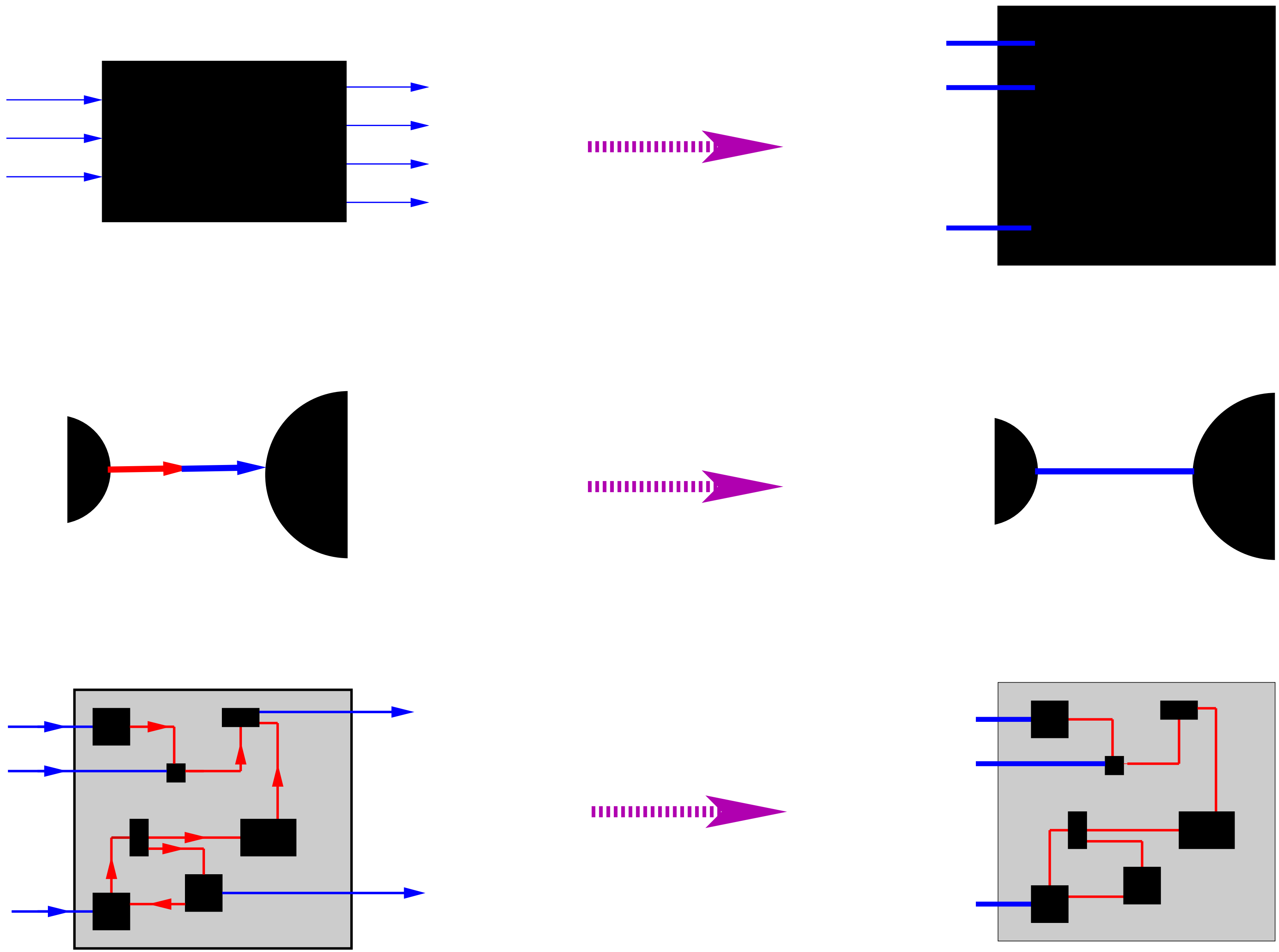
$$r_1\left(\frac{d}{dt}\right)w_1 = r_2\left(\frac{d}{dt}\right)w_2.$$

**In practice, all these steps need to be carried out more explicitly, faster, better, and more reliably with the help of software and a computer toolbox.**

# Input/output

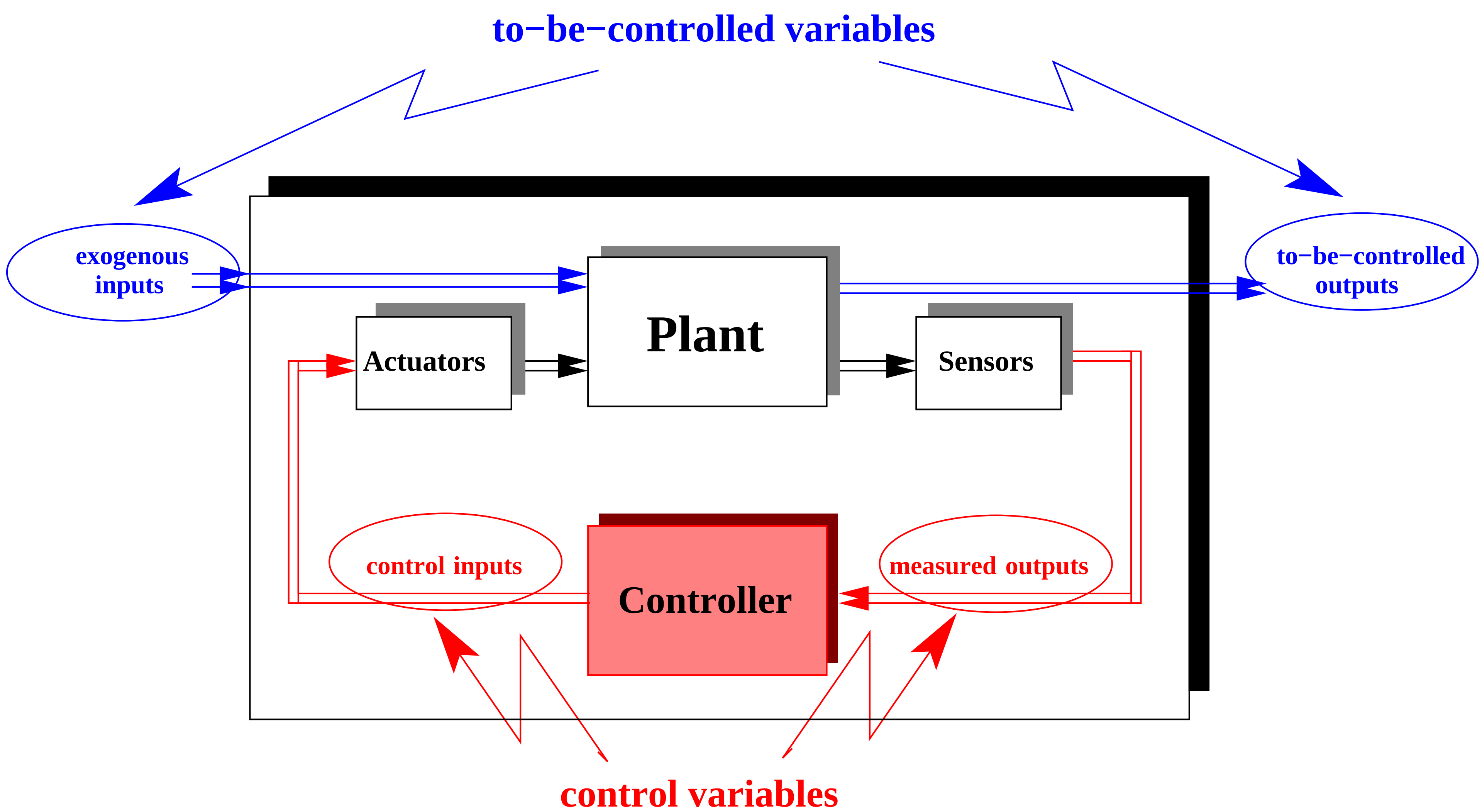
# versus

# tearing, zooming, and linking

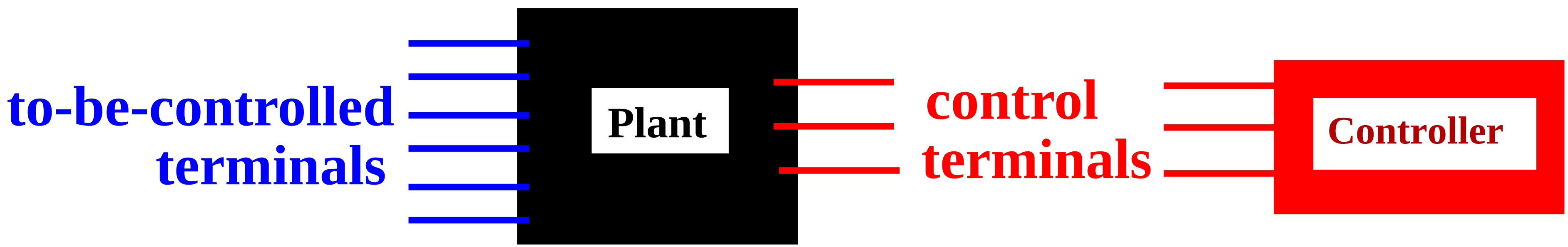


# Control as interconnection

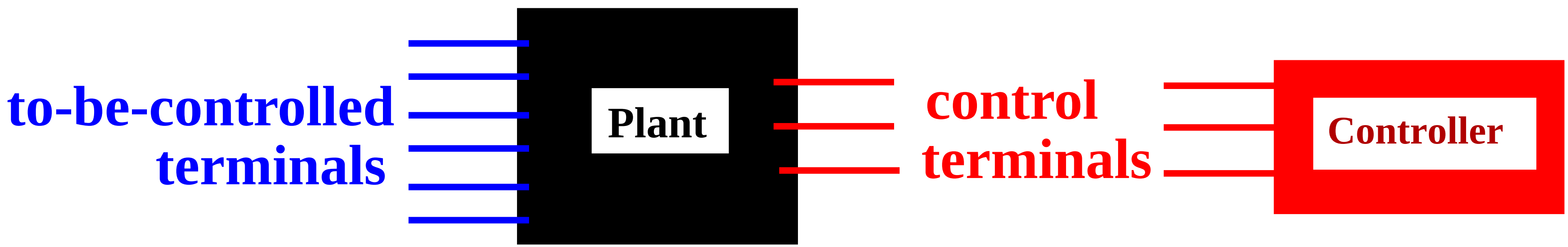
# Feedback control



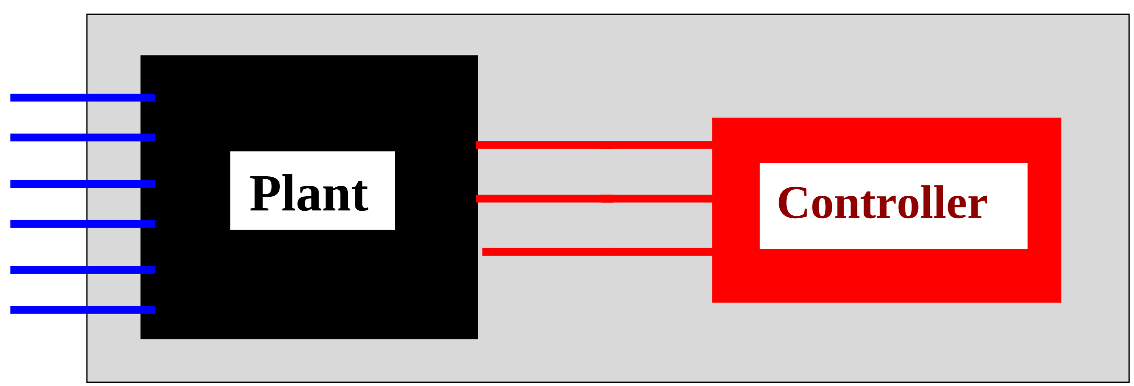
# Behavioral control



# Behavioral control

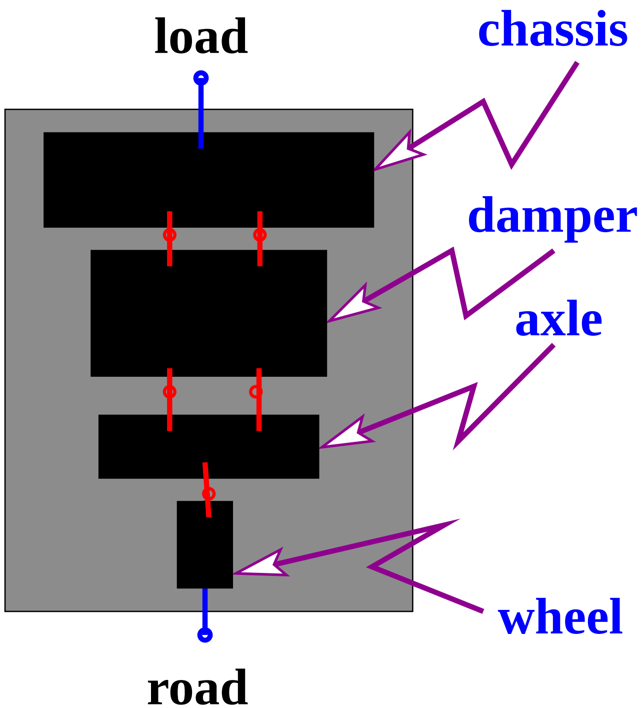



**control = interconnection.**

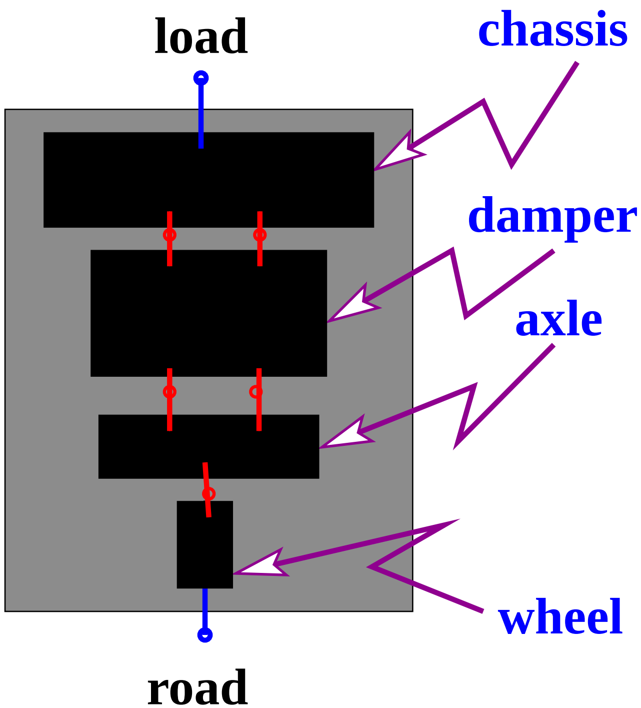


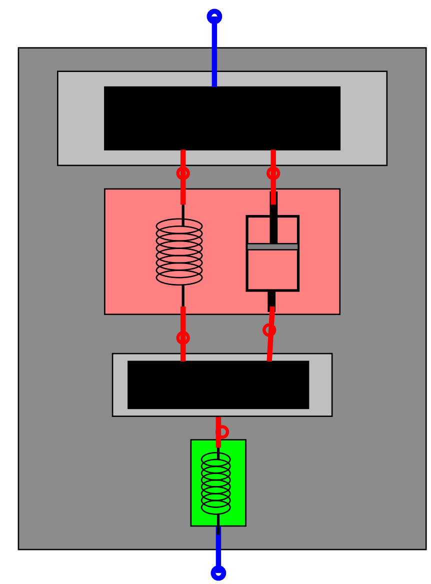

controlled system

# A ‘quarter car’

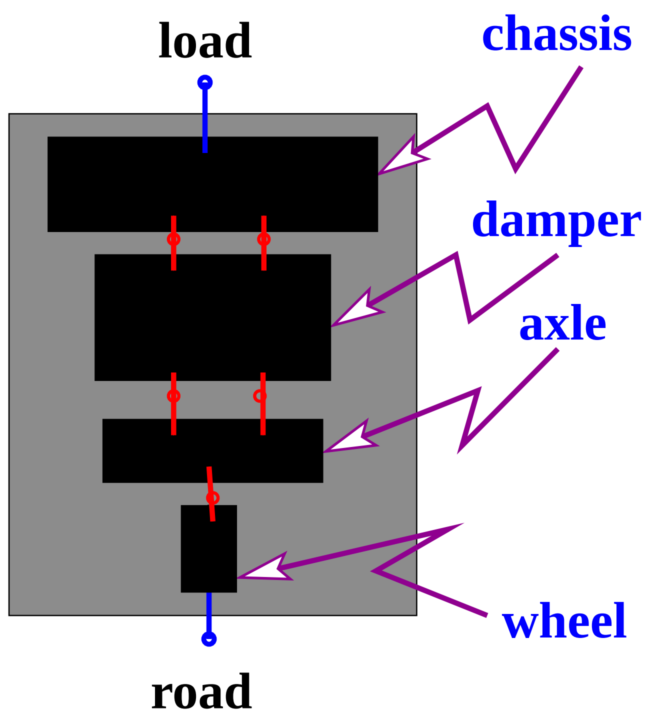


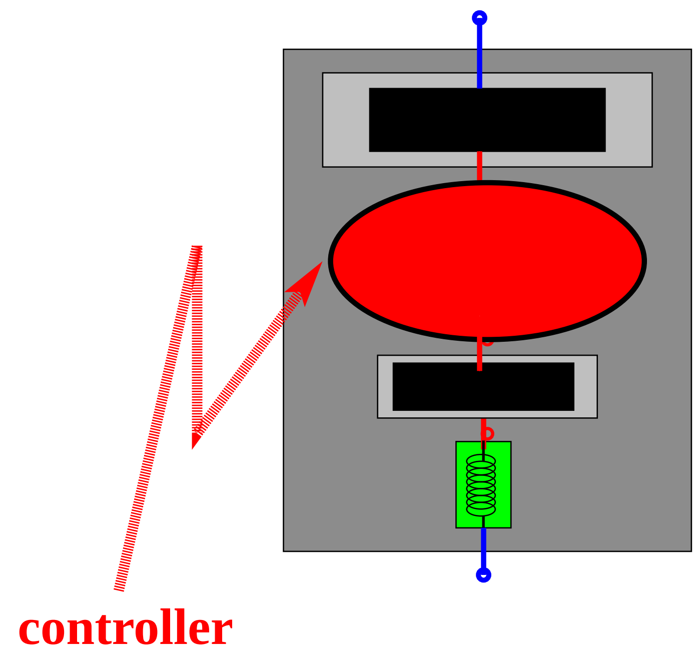
# A ‘quarter car’



# A ‘quarter car’



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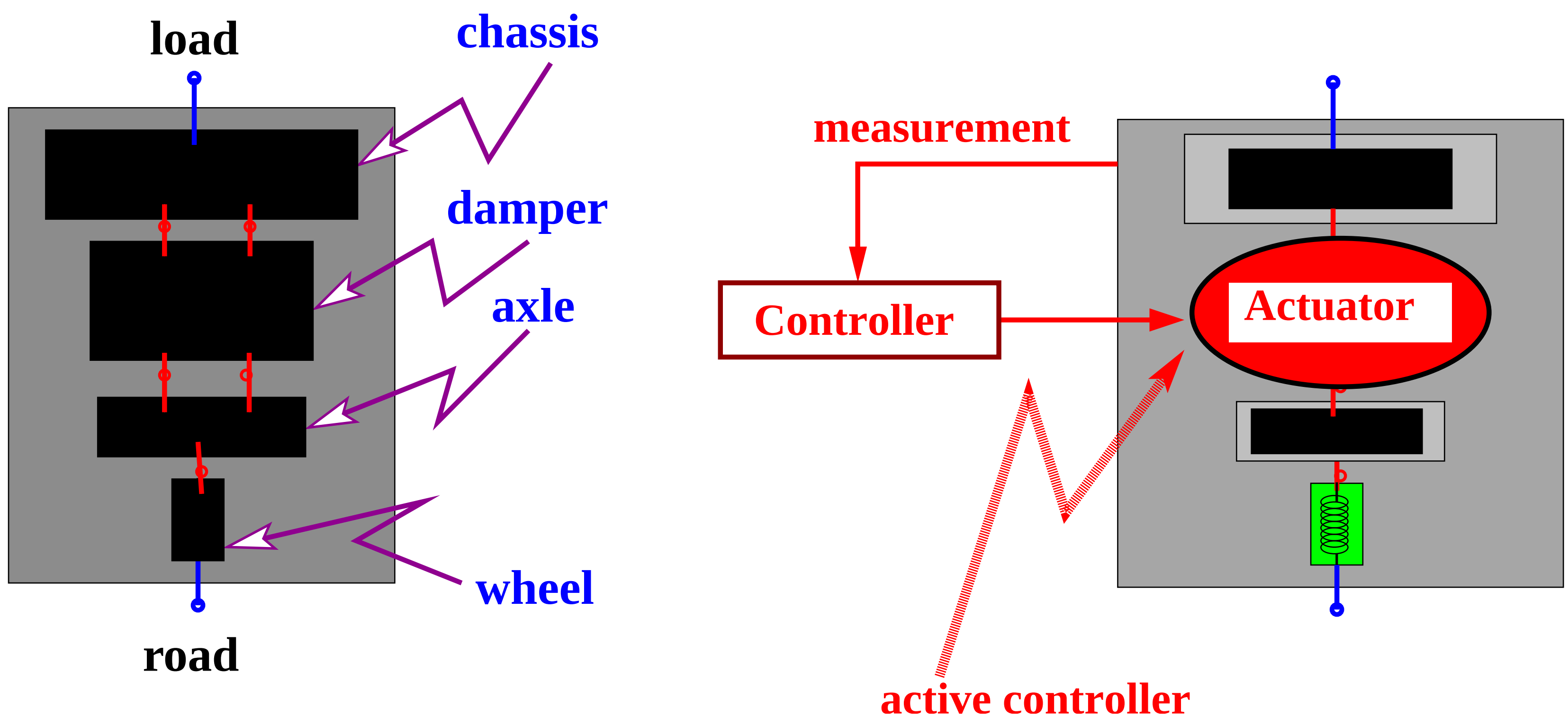
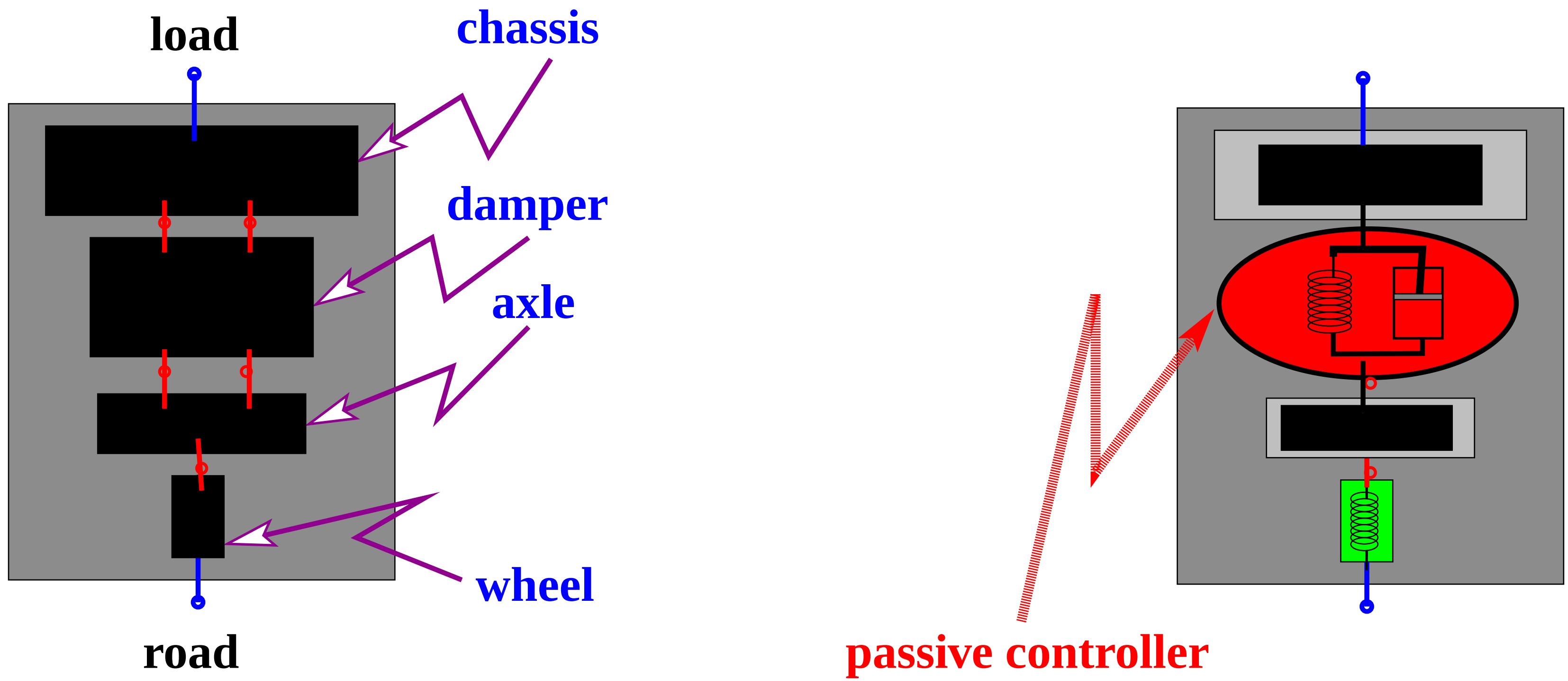


# A 'quarter car'

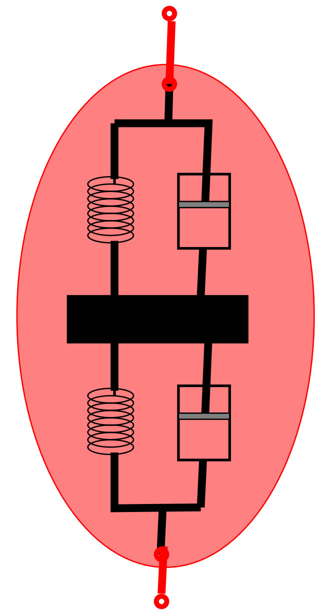


# Suspension controllers in Formula 1

**Nigel Mansell victorious in 1992 with an active damper suspension.**

**Active dampers were banned in 1994 to break the dominance of the Williams team.**

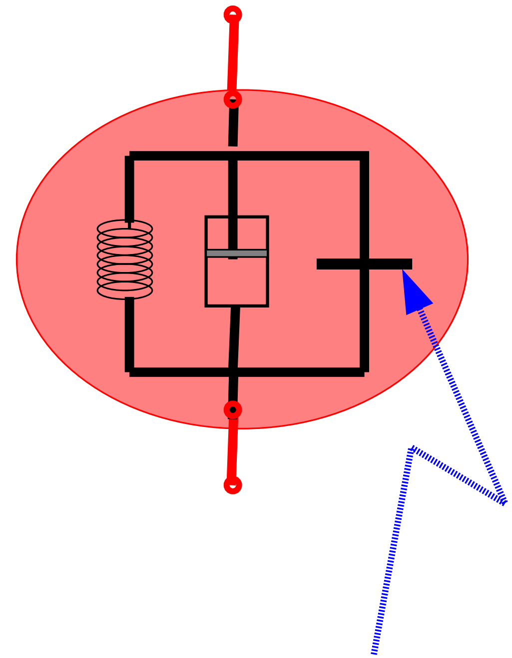
# Suspension controllers in Formula 1



**Renault successfully uses a passive 'tuned mass damper' in 2005/2006.**

**Banned in 2006, under the 'movable aerodynamic devices' clause.**

# Suspension controllers in Formula 1

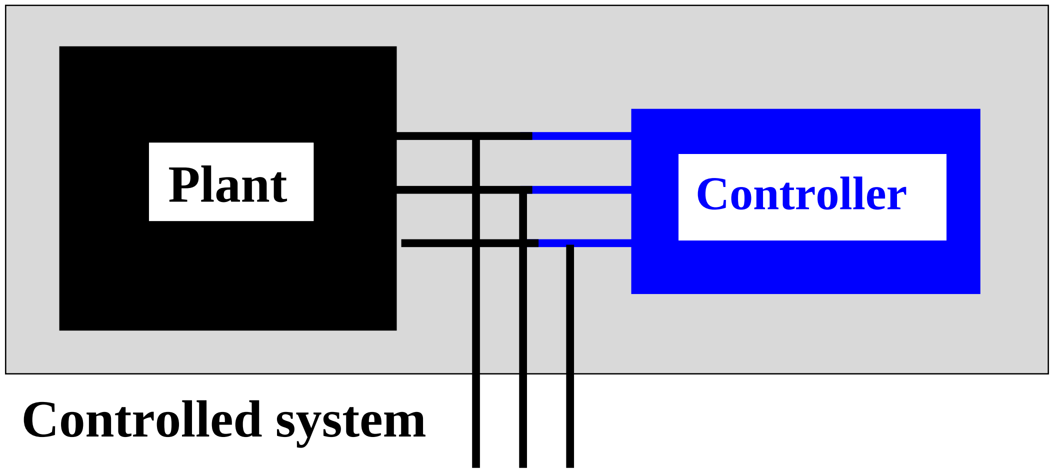



**Kimi Räikkönen wins the 2005 Grand Prix in Spain with McLaren's 'J-damper' (see Lecture V).**

# Pole placement and stabilization

# LTIDSs as controllers

**We consider only 'full control'.**




**and controllers such that the controlled system is autonomous.**

# LTIDSs as controllers

**Consider a LTIDS $\mathscr{P} \in \mathscr{L}^{\mathtt{w}}$, the *plant*,**
**and a LTIDS $\mathscr{C} \in \mathscr{L}^{\mathtt{w}}$ the *controller*.**

$[\![\mathscr{C}$ **is called a *regular controller* for** $\mathscr{P}]\!] :\Leftrightarrow$

$[\![$**(i)** $\mathtt{p}(\mathscr{C}) = \mathtt{m}(\mathscr{P})$ **and** **(ii)** $\mathscr{P} \cap \mathscr{C}$ **is autonomous** $]\!]$.

# LTIDSs as controllers

**Consider a LTIDS $\mathscr{P} \in \mathscr{L}^{\mathtt{w}}$, the *plant*,**
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$[\![$ $\mathscr{C}$ **is called a *regular controller* for** $\mathscr{P}$ $]\!]$ $:\Leftrightarrow$

$[\![$**(i)** $\mathtt{p}(\mathscr{C}) = \mathtt{m}(\mathscr{P})$ **and (ii)** $\mathscr{P} \cap \mathscr{C}$ **is autonomous** $]\!]$.

**In terms of minimal kernel representations**

$$P\left(\frac{d}{dt}\right)w = 0$$
$$C\left(\frac{d}{dt}\right)w = 0$$

**for $\mathscr{P}$ and $\mathscr{C}$, regularity** $\Leftrightarrow \begin{bmatrix} P \\ C \end{bmatrix}$ **square and nonsingular.**

# The characteristic polynomial

**Let** $\mathscr{B} \in \mathscr{L}^{\mathtt{w}}$ **be autonomous, and** $R\left(\frac{d}{dt}\right)w = 0$ **be a minimal kernel representation of** $\mathscr{B}$**.**
**Assume that** $\mathtt{determinant}(R)$ **is monic (otherwise change** $R \longrightarrow \alpha R$ **with** $0 \neq \alpha \in \mathbb{R}$ **suitably chosen).**

**The characteristic polynomial of** $\mathscr{B}$**,** $\chi_{\mathscr{B}} \in \mathbb{R}[\xi]$**, is defined as**

$$\chi_{\mathscr{B}} := \mathtt{determinant}(R).$$

**Note:** $\chi_{\mathscr{B}}$ **is independent of the** $R$ **chosen to represent** $\mathscr{B}$**.**

# The characteristic polynomial

**Let** $\mathscr{B} \in \mathscr{L}^{\mathtt{w}}$ **be autonomous, and** $R\left(\frac{d}{dt}\right) w = 0$ **be a minimal kernel representation of** $\mathscr{B}$**.**
**The minimal polynomial of** $\mathscr{B}$**,** $\mu_{\mathscr{B}} \in \mathbb{R}[\xi]$**, is defined as the monic polynomial of least degree that annihilates** $\mathscr{B}$**, i.e.**

$$\mu_{\mathscr{B}}\left(\frac{d}{dt}\right)\mathscr{B} = 0,$$

$$[\![p_{\mathscr{B}}\left(\frac{d}{dt}\right)\mathscr{B} = 0, 0 \neq p \in \mathbb{R}[\xi]]\!] \Rightarrow [\![\mathtt{degree}(p) \geq \mathtt{degree}(\mu_{\mathscr{B}})]\!].$$

**Note:** $\mu_{\mathscr{B}}$ **is a factor of** $\chi_{\mathscr{B}}$**.**

> **For** $\mathscr{B}$ **described by** $\frac{d}{dt}x = Ax, w = Cx$ **with** $(A, C)$ **observable,** $\chi_{\mathscr{B}}$ **and** $\mu_{\mathscr{B}}$ **are equal to the characteristic and minimal polynomial of** $A$**.**

# Pole placement

## Theorem

**Let $\mathscr{P} \in \mathscr{L}^{\mathtt{w}}$ be controllable. Then for all monic polynomials $\pi \in \mathbb{R}[\xi]$, there exists a regular controller $\mathscr{C} \in \mathscr{L}^{\mathtt{w}}$ such that**

$$\chi_{\mathscr{P} \cap \mathscr{C}} = \pi.$$

# Pole placement

**Theorem**

**Let $\mathscr{P} \in \mathscr{L}^{\mathtt{w}}$ be controllable. Then for all monic polynomials $\pi \in \mathbb{R}[\xi]$, there exists a regular controller $\mathscr{C} \in \mathscr{L}^{\mathtt{w}}$ such that**

$$\chi_{\mathscr{P}\cap\mathscr{C}} = \pi.$$

**Proof: Let $P\left(\frac{d}{dt}\right)w = 0$ be a minimal kernel representation of $\mathscr{P}$. The Smith form yields $P = U\begin{bmatrix}0_{\mathtt{p}(\mathscr{P})\times\mathtt{m}(\mathscr{P})} & I_{\mathtt{p}(\mathscr{P})\times\mathtt{p}(\mathscr{P})}\end{bmatrix}V$, with $U$ and $V$ unimodular. Take for $\mathscr{C}$ the system $C\left(\frac{d}{dt}\right)w = 0$, with $C = \begin{bmatrix}\mathtt{diag}(\pi, 1, \ldots, 1) & 0_{\mathtt{m}(\mathscr{P})\times\mathtt{p}(\mathscr{P})}\end{bmatrix}V$.**

# Pole placement

The above proof can be refined in numerous directions (see Exercise IV.5).

In particular, we obtain

**Theorem**

Let $\mathscr{P} \in \mathscr{L}^{\mathtt{w}}$ be controllable. Then for all monic polynomials $\nu \in \mathbb{R}[\xi]$, there exists a regular controller $\mathscr{C} \in \mathscr{L}^{\mathtt{w}}$ such that

$$\mu_{\mathscr{P} \cap \mathscr{C}} = \nu.$$

# Stabilization

**and**

**Theorem**

**Let $\mathscr{P} \in \mathscr{L}^{\mathtt{w}}$ be stabilizable. Then there exists a regular controller $\mathscr{C} \in \mathscr{L}^{\mathtt{w}}$ such that the autonomous system**

$\mathscr{P} \cap \mathscr{C}$ **is stable.**

# Recapitulation

# Summary

- **Complex systems consist of interconnections of subsystems.**

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- Input/output thinking and signal flow graphs provide a very limited view of interconnected physical systems.

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# Summary

- Complex systems consist of interconnections of subsystems.
- Input/output thinking and signal flow graphs provide a very limited view of interconnected physical systems.
- Modeling of interconnected systems proceeds by tearing, zooming, and linking.
- A graph with leaves is a useful formalization of the interconnection architecture of an interconnected system.
- Control as interconnection is an effective way of thinking about control, with feedback control as a very useful special case.

# Summary

- **Complex systems consist of interconnections of subsystems.**
- **Input/output thinking and signal flow graphs provide a very limited view of interconnected physical systems.**
- **Modeling of interconnected systems proceeds by tearing, zooming, and linking.**
- **A graph with leaves is a useful formalization of the interconnection architecture of an interconnected system.**
- **Control as interconnection is an effective way of thinking about control, with feedback control as a very useful special case.**
- **Controllable LTIDSs allow regular controllers that achieve an arbitrary characteristic polynomial for the controlled system. Stabilizable LTIDSs can be stabilzed by regular controllers.**

# End of Lecture VIII