MODELING & INTERCONNECTION

of

DYNAMICAL SYSTEMS

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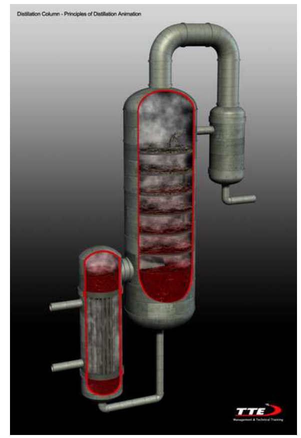
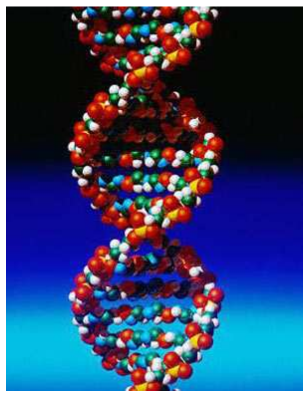
Theme

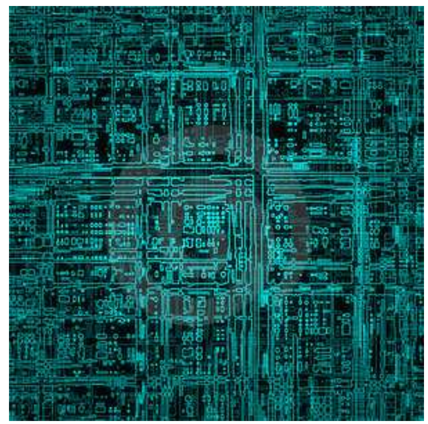
What is the 'right' pedagogical paradigm for dynamic modeling?

SYSTEMS


OIL REFINERY (GVG / PD)



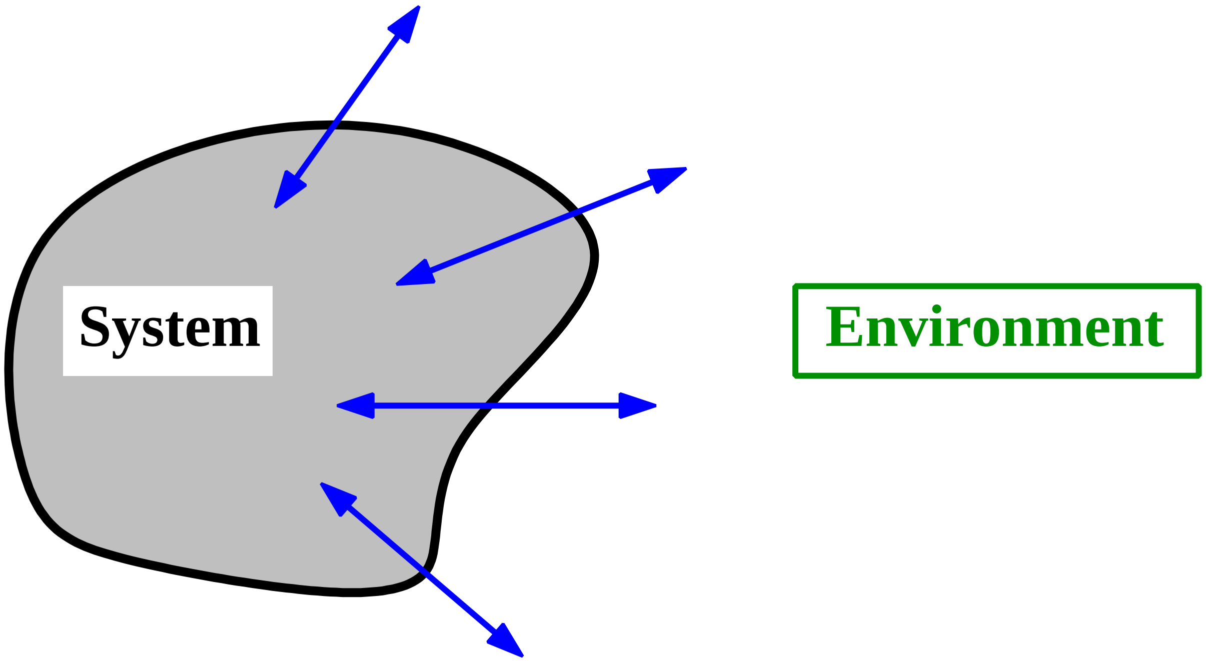




Features

- **Open**
- **Interconnected**
- **Modular**
- **Dynamical**

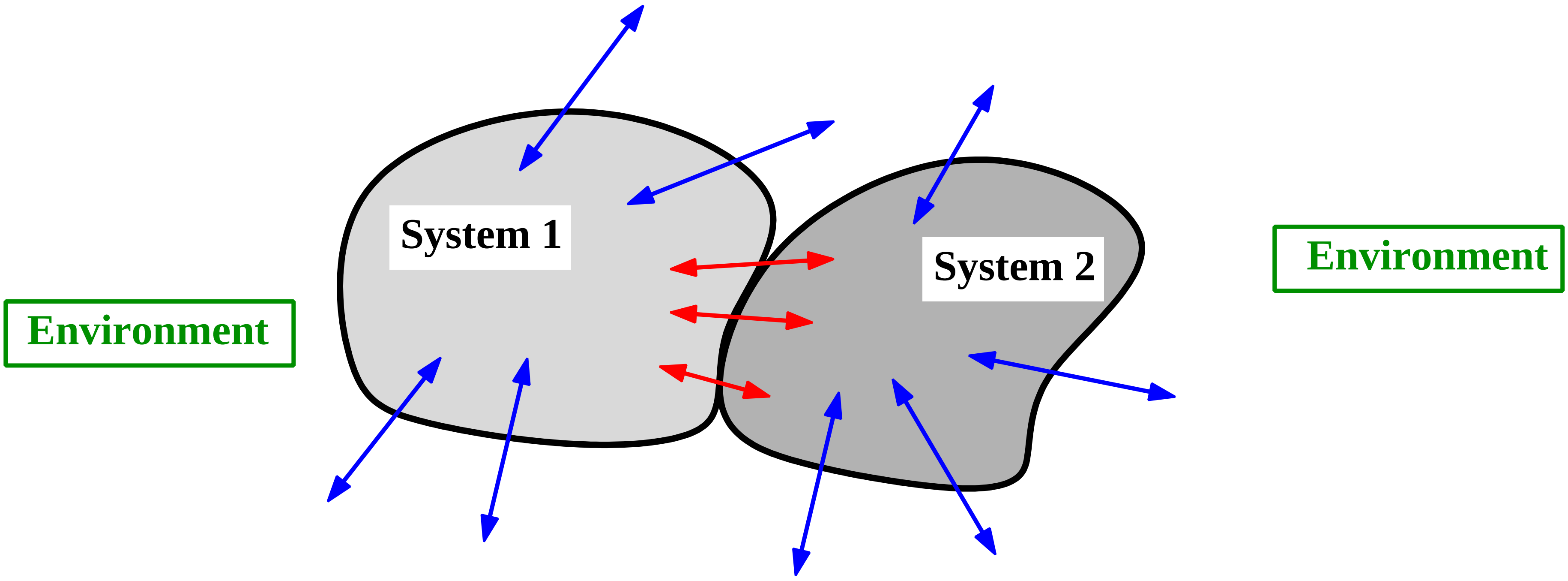
Open



Systems are ‘open’, they interact with their environment.

How are such systems formalized?

Interconnected



Interconnected systems interact.

How is this interaction formalized?

Modularity

Systems consist of the interconnection of repeated building blocks.

Essential for computer-assisted modeling.

Modularity

Systems consist of the interconnection of repeated building blocks.

Essential for computer-assisted modeling.

Examples:

electrical circuits $\rightsquigarrow$

resistors, capacitors, inductors, transistors, diodes, sources, etc.

mechanical devices $\rightsquigarrow$

masses, springs, dampers, connecting bars, joints, etc.

etc.

Dynamical

Main interest: the evolution over time.

How do the variables evolve in the long-term?

Are there excessive transients?

Do small variations drastically change the future?

etc.

TEARING, ZOOMING, & LINKING

Modeling

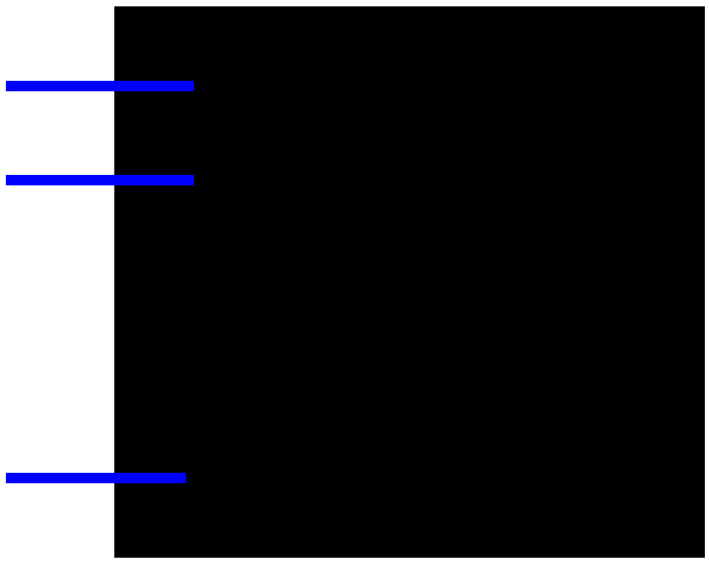
The ever-increasing computing power allows to model complex interconnected systems accurately by

tearing, zooming, and linking.

$\rightsquigarrow$ **Simulation, model based design, ...**

Tearing

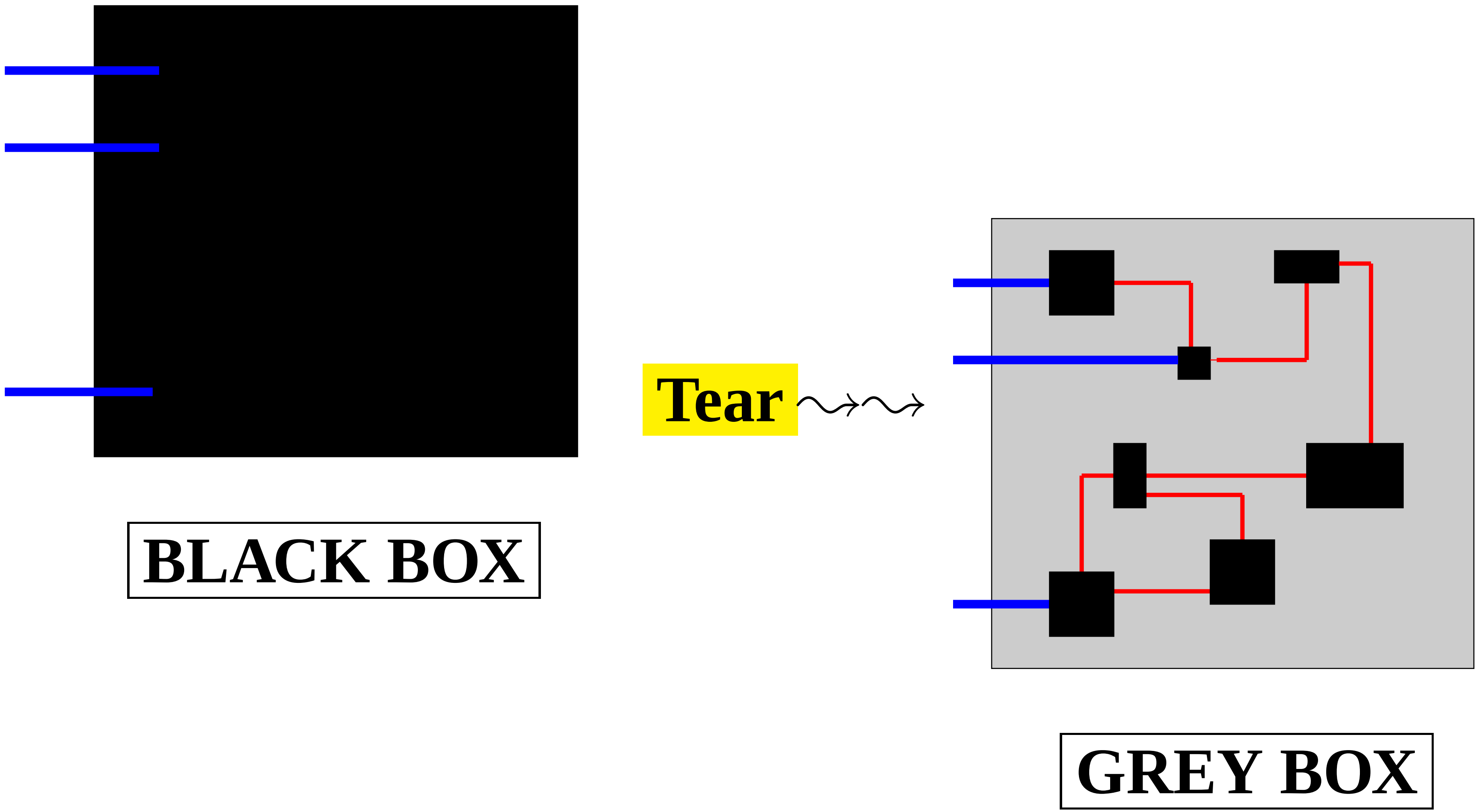
¡¡ Model the behavior of selected variables !!



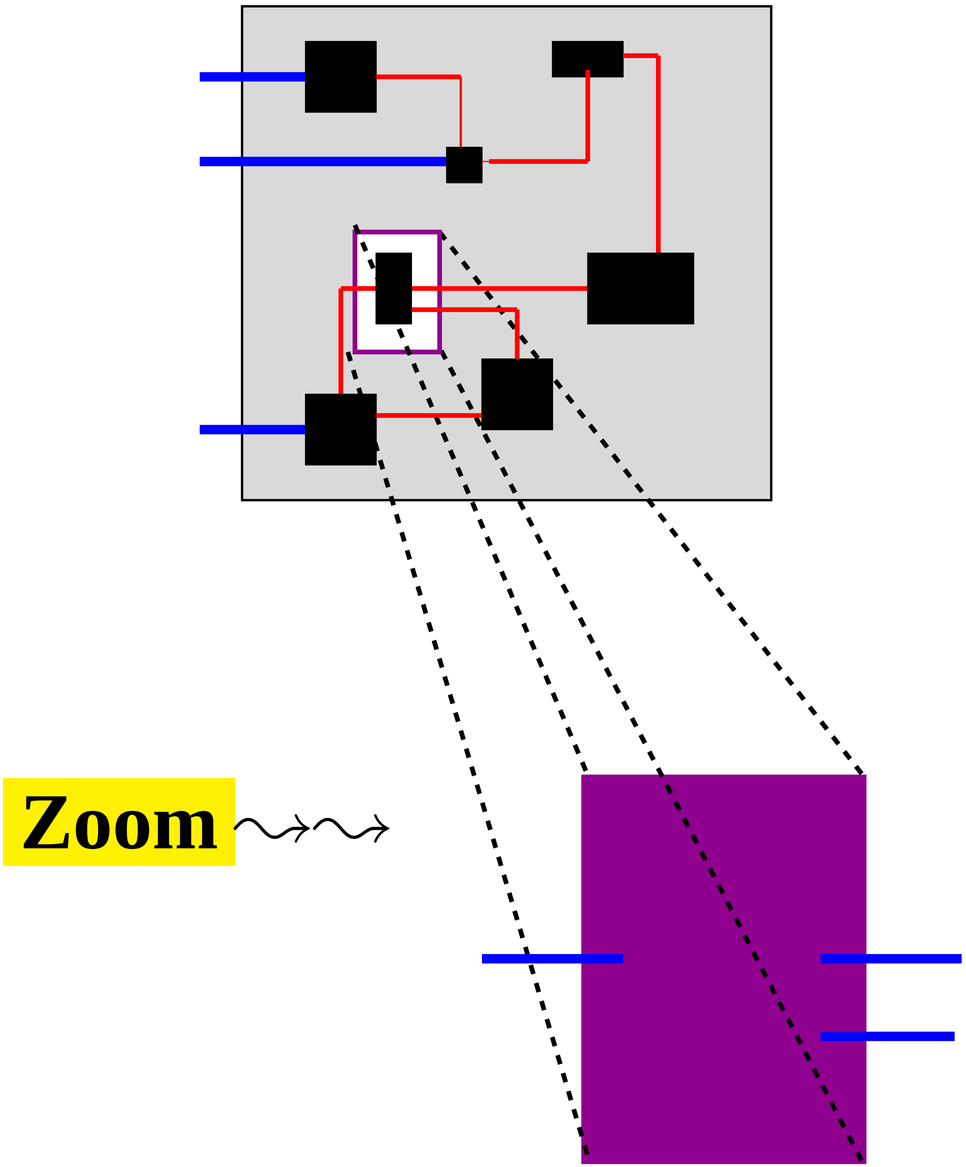
BLACK BOX

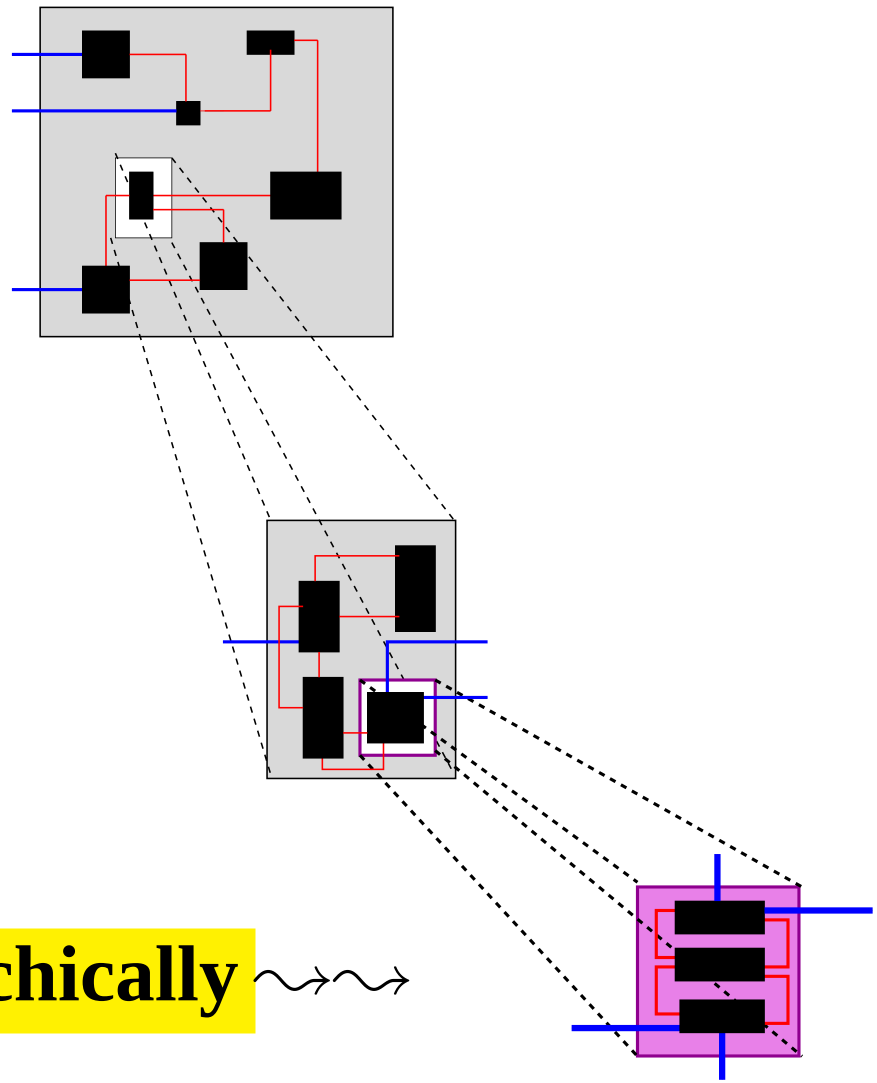
Tearing

¡¡ Model the behavior of selected variables !!



Zooming

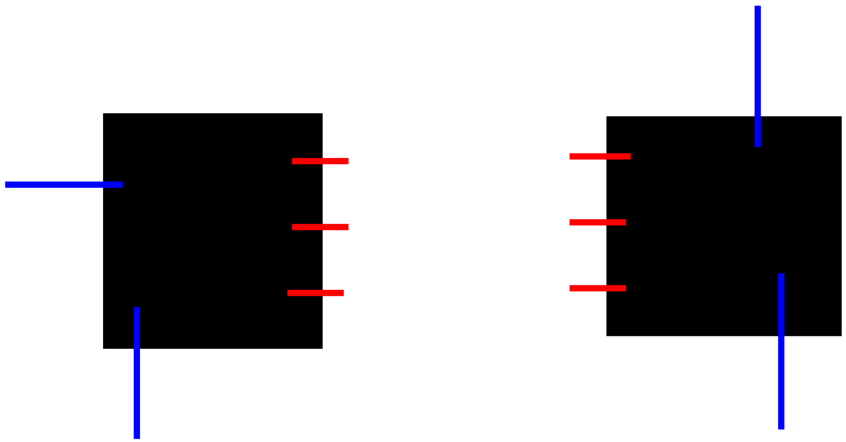




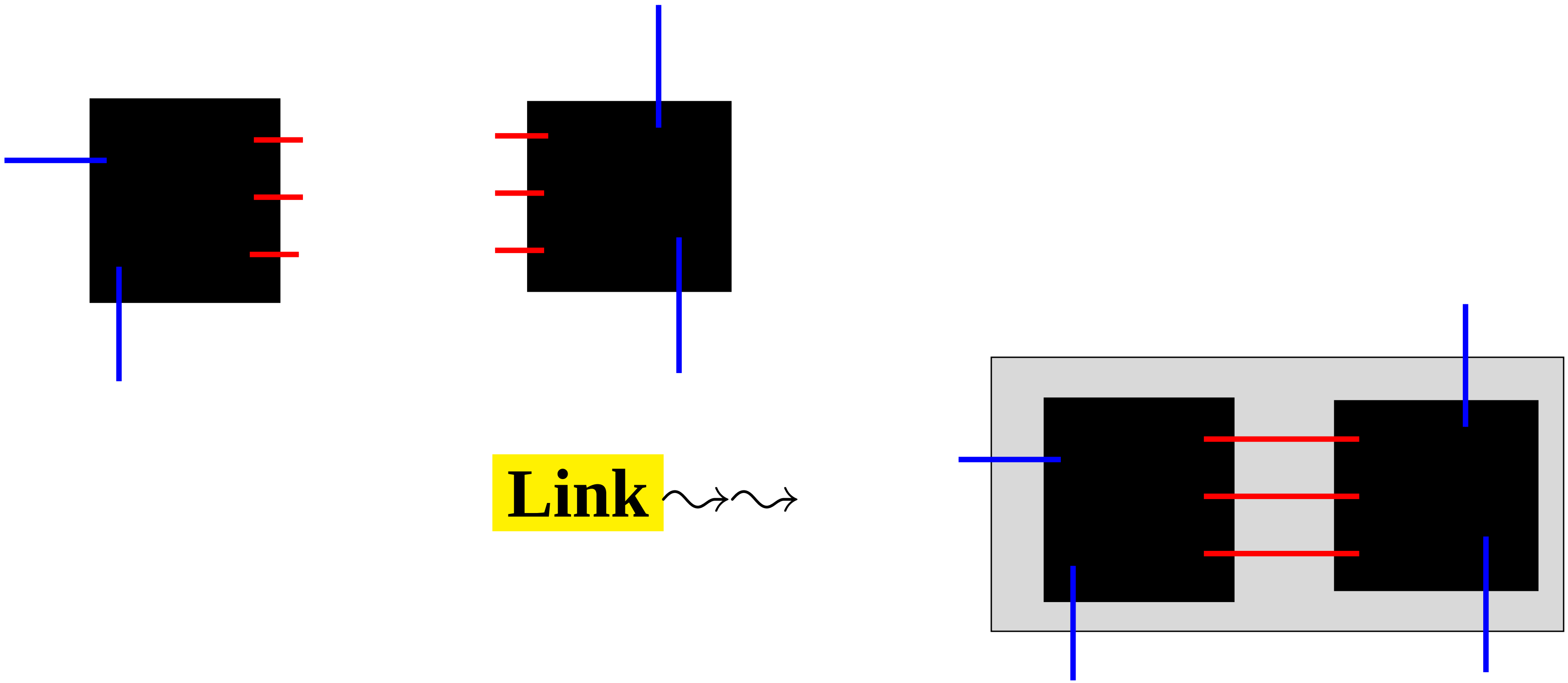
Zoom hierarchically ⇝⇝

Proceed until subsystems are obtained whose model is known from first principles, or stored in a database ('modularity').

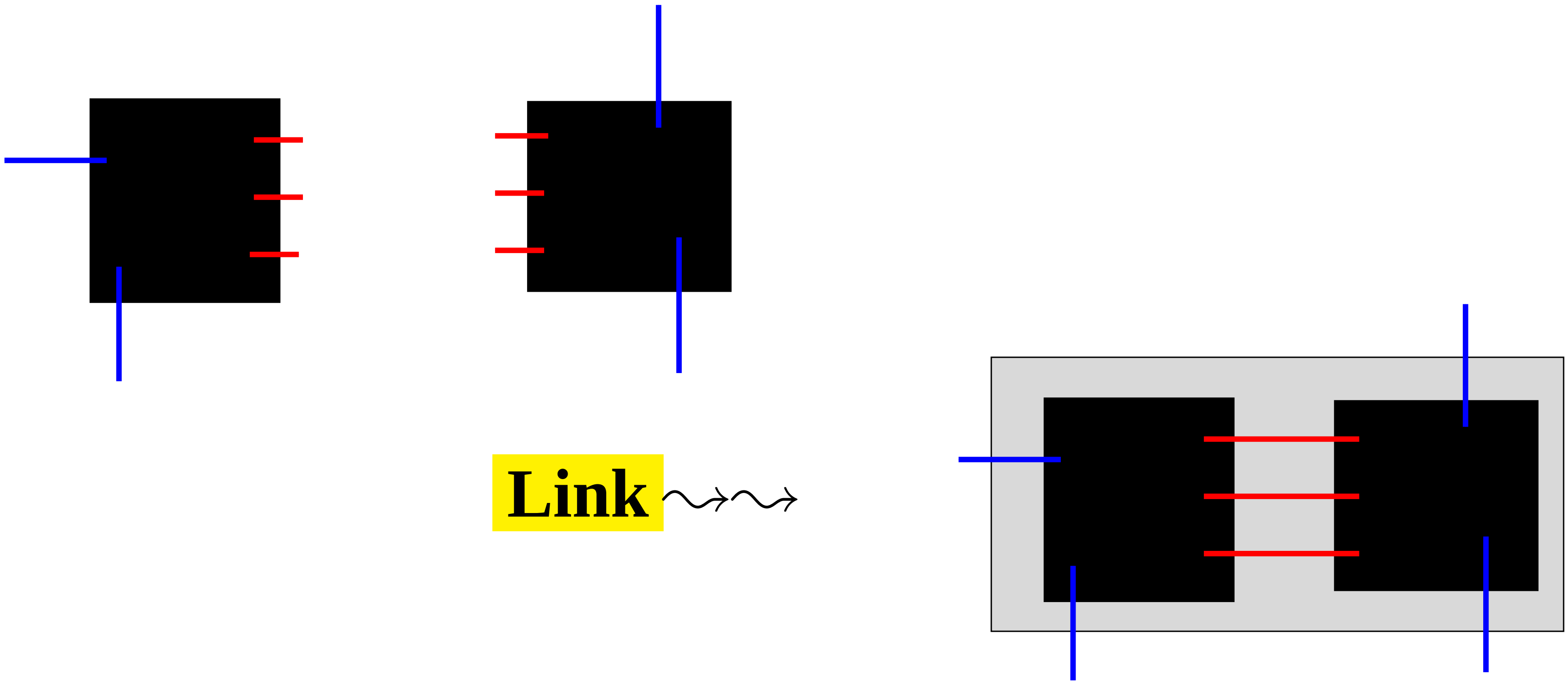
Linking



Linking



Linking



model for component variables + linking equations

$\Rightarrow$ model of behavior of the black box variables.

Tearing, zooming, & linking $\Leftrightarrow$ basis for modeling.

Theme

This modeling methodology requires the right mathematical concepts

- **for dynamical system,**
- **for interconnection,**
- **for interconnection architecture.**

What are these concepts?

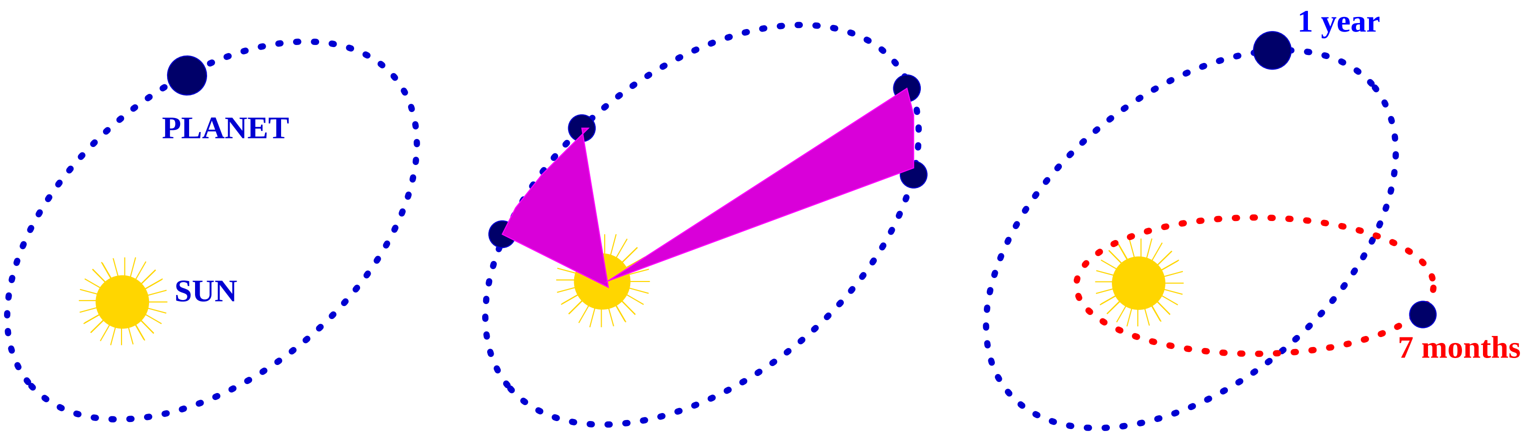
HOW IT ALL BEGAN ...

Planetary motion

How, for heaven's sake, does it move?

Kepler's laws

Variable: the position as a function of time.



- **K1: ellipse, sun in focus,**
- **K2: = areas in = times,**
- **K3: (period)^2 = (major axis)^3.**

Johannes Kepler (1571–1630)

Newton's version

Acceleration = function of position and velocity $\rightsquigarrow$

$$\frac{d^2}{dt^2}w(t) = A\left(w(t), \frac{d}{dt}w(t)\right).$$

Via calculus and calculations: K1, K2, & K3 $\Leftrightarrow$

$$\frac{d^2}{dt^2}w(t) + \frac{\vec{1}_{w(t)}}{||w(t)||^2} = 0$$

Isaac Newton (1643–1727)

From Newton to flows

$$\frac{d^2}{dt^2}w(t) + \frac{\vec{1}_{w(t)}}{||w(t)||^2} = 0 \quad \rightsquigarrow \quad \textbf{with } x = \begin{bmatrix} w \\ \frac{d}{dt}w \end{bmatrix}$$

$$\rightsquigarrow \quad \frac{d}{dt}x(t) = f(x(t)) \quad \rightsquigarrow \quad x(0) \Rightarrow x(\cdot)$$

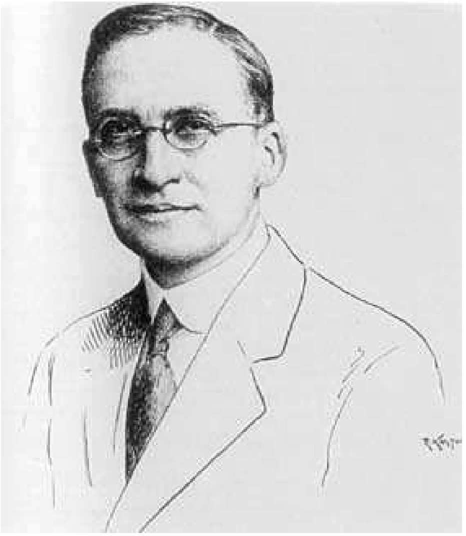
Motion determined by its initial conditions.

$\rightsquigarrow$ **Idea of a ‘flow’.**

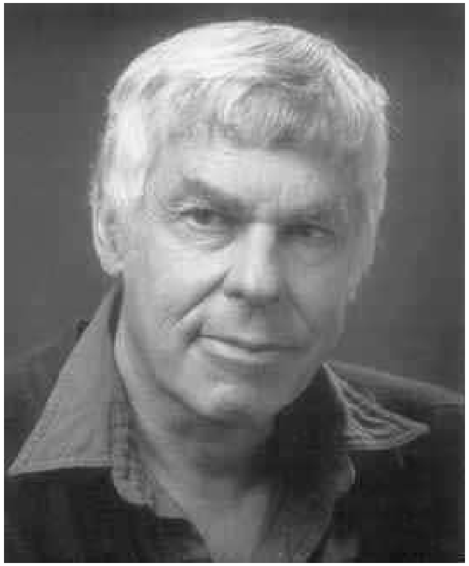
Flows, $\frac{d}{dt}x(t) = f(x(t))$

Motion completely determined by initial conditions.

Henri Poincaré (1854-1912)



George Birkhoff (1884-1944)



Stephen Smale (1930-)

$\rightsquigarrow$ **differential equations, chaos, cellular automata, etc.**

Flows, $\frac{d}{dt}x(t) = f(x(t))$

Motion completely determined by initial conditions.

Inadequate:

How could they forget

about Newton's second law,
about Maxwell's equations,
about thermodynamics,
about tearing, zooming, & linking?

Not a good paradigm for teaching dynamic modeling!

Newton's laws & interconnection

Gravitation: $F_1(t) = \frac{\vec{1}_{w(t)}}{||w(t)||^2}$

Second law: $F_2(t) = \frac{d^2}{dt^2} w(t)$

Third law: $F_1(t) + F_2(t) = 0$

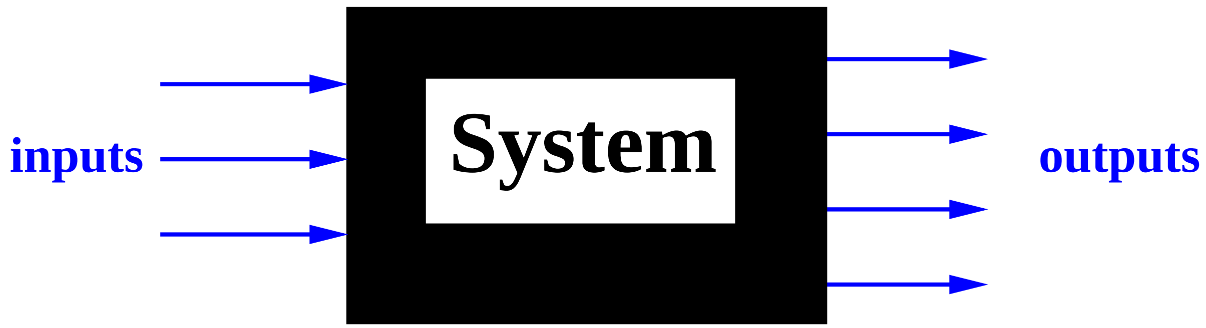
$$\Downarrow$$

$$\frac{d^2}{dt^2} w(t) + \frac{\vec{1}_{w(t)}}{||w(t)||^2} = 0$$

Newton painted by William Blake

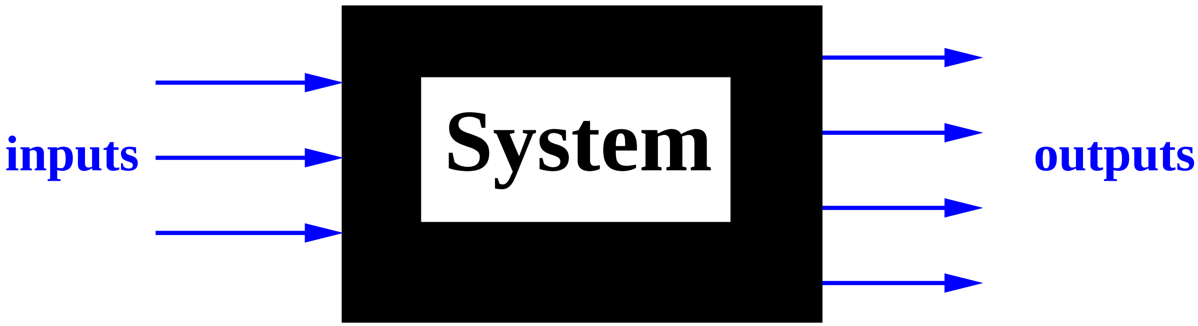
INPUT/OUTPUT VIEW

Input/output systems



Appealing: cause & effect, stimulus & response, etc.

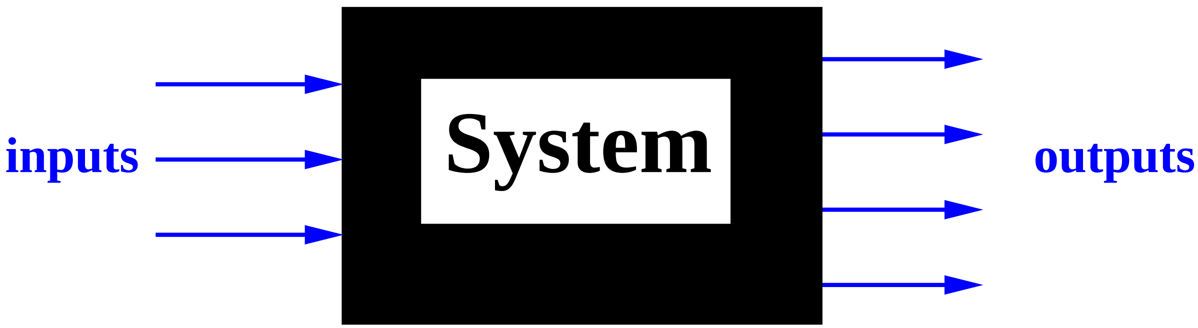
Input/output systems

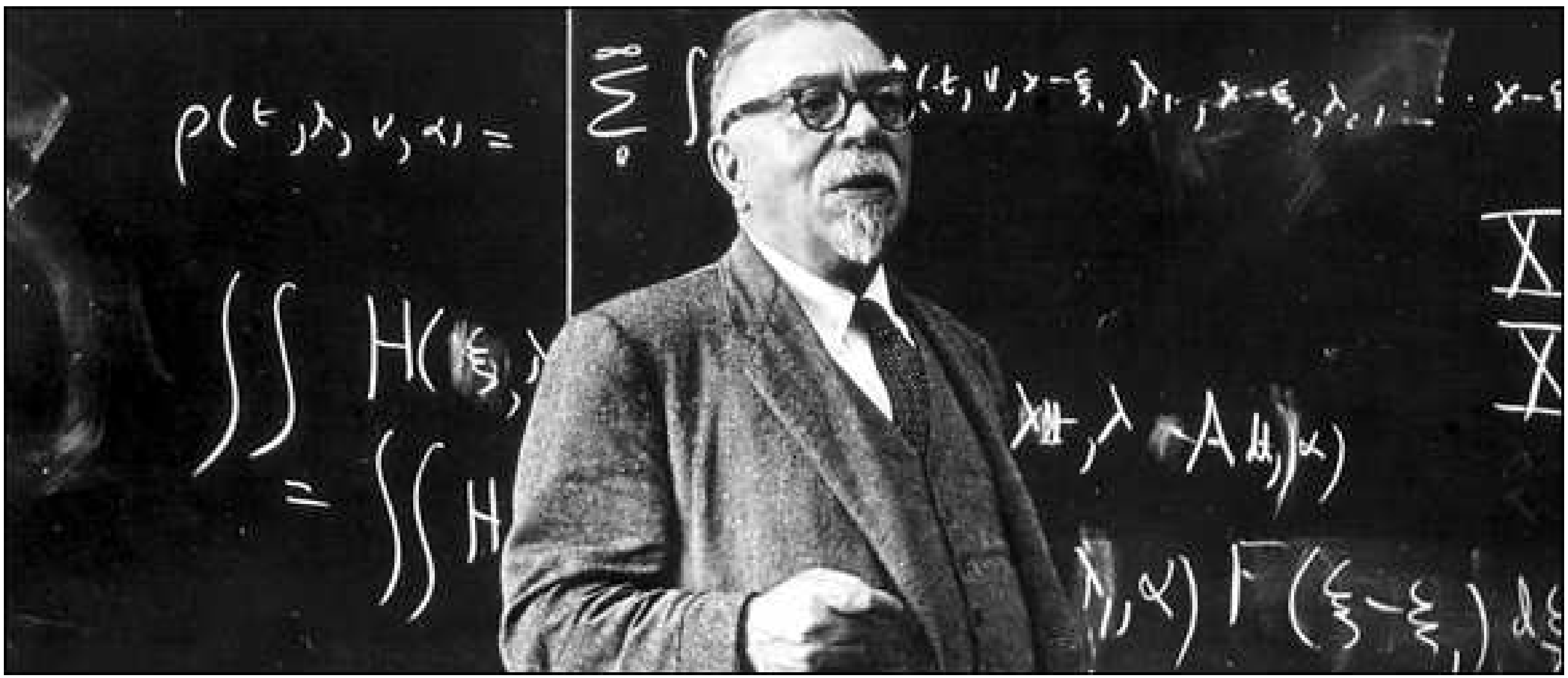


Lord Rayleigh (1842-1919)

Oliver Heaviside
(1850-1925)

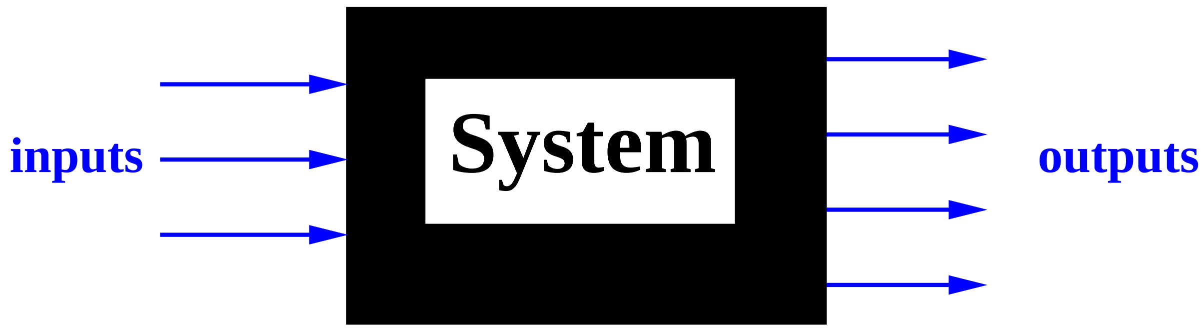
Input/output systems





Norbert Wiener (1894-1964)

Input/output systems



Appealing: cause & effect, stimulus & response, etc.

I/O maps, developed mainly in electrical engineering

since $\pm$ 1900, for circuits, signal processing, control, ...

These models do not cope well with initial conditions, very awkward framework for nonlinear models.

Input/state/output models

Around 1960, paradigm shift to

$$\frac{d}{dt}x(t) = f(x(t), u(t)), \qquad y(t) = h(x(t), u(t)).$$

The generation of outputs from inputs is viewed as follows

$x(0)$ **and** $u(\cdot)$ **lead to** $x(\cdot)$ **through** $\frac{d}{dt}x(t) = f(x(t), u(t))$

$x(\cdot)$ **and** $u(\cdot)$ **lead to** $y(\cdot)$ **through** $y(t) = h(x(t), u(t))$**.**

Input/state/output models

Around 1960, paradigm shift to

$$\frac{d}{dt}x(t) = f(x(t), u(t)), \qquad y(t) = h(x(t), u(t)).$$

The generation of outputs from inputs is viewed as follows

$$x(0) \text{ and } u(\cdot) \text{ lead to } x(\cdot) \text{ through} \quad \frac{d}{dt}x(t) = f(x(t), u(t))$$
$$x(\cdot) \text{ and } u(\cdot) \text{ lead to } y(\cdot) \text{ through} \quad y(t) = h(x(t), u(t)).$$

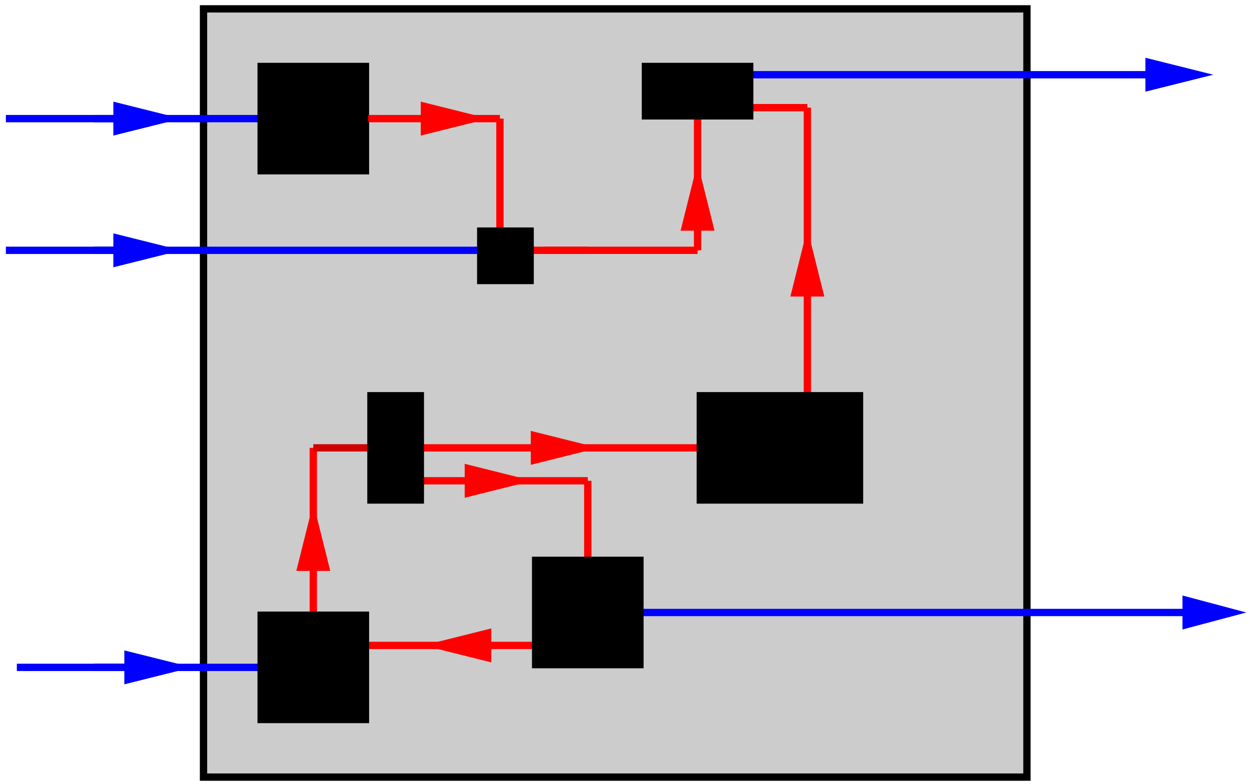
Combines $\frac{d}{dt}x(t) = f(x(t))$ **with** $u(\cdot) \mapsto y(\cdot)$

$\leadsto$ **a vigorous program, encompassing all aspects of dynamical modeling, signal processing, control, ...**

Rudolf Kalman (1930-   )

INTERCONNECTION

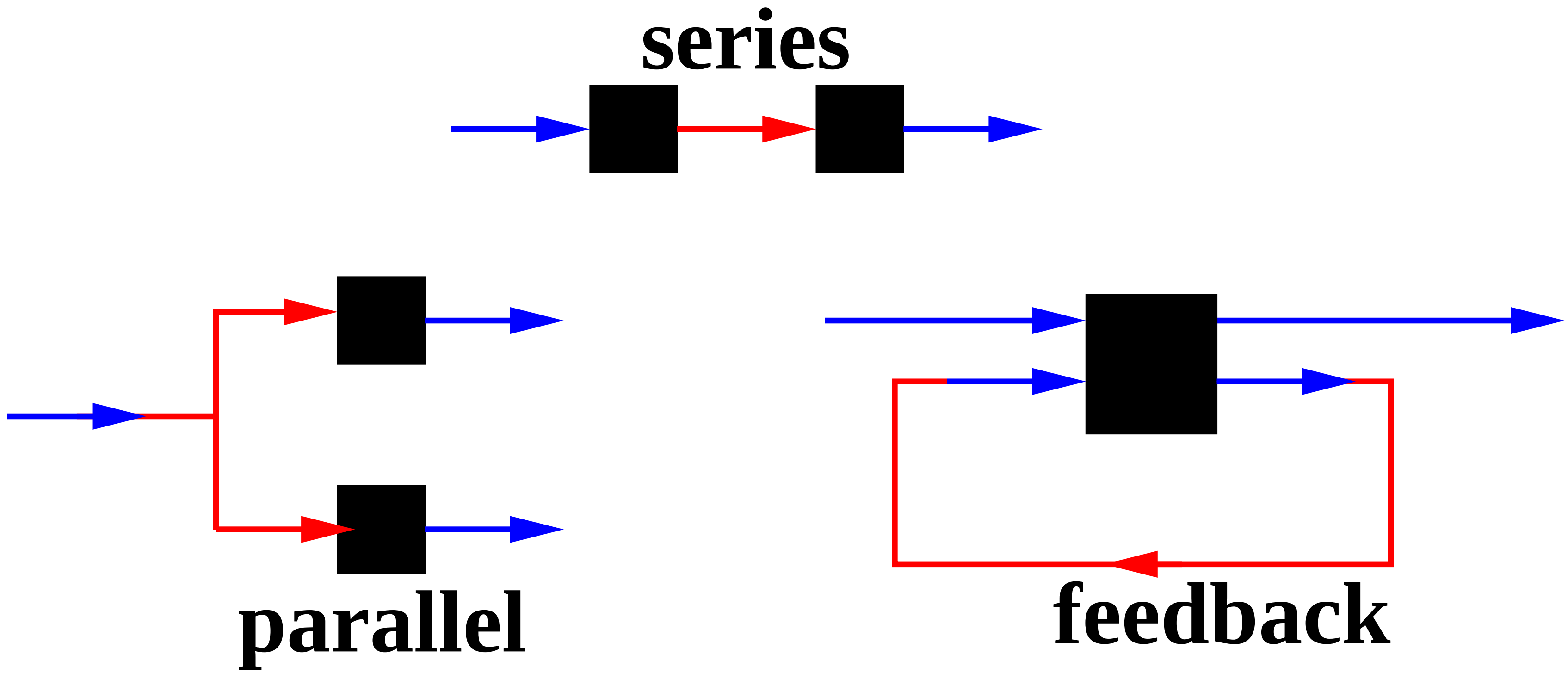
Signal flow graphs



‘Pathways’

Signal flows graphs

Examples: **combinations of**

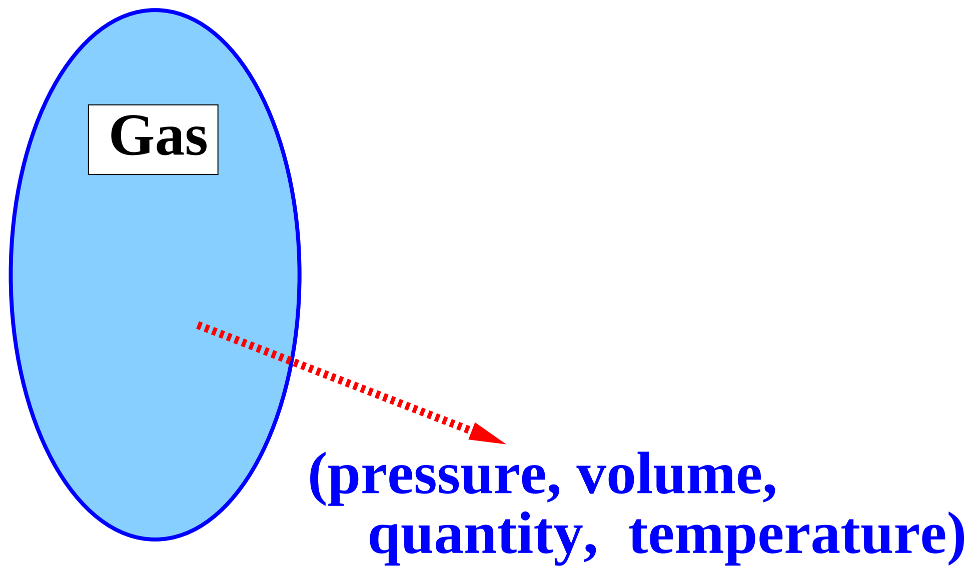


INADEQUACIES of I/O THINKING

Problems with I/O

- **Physical laws dictate the simultaneous occurrence of events.**
  No cause/effect is implied.

  E.g., the gas law



$$PV = NT$$

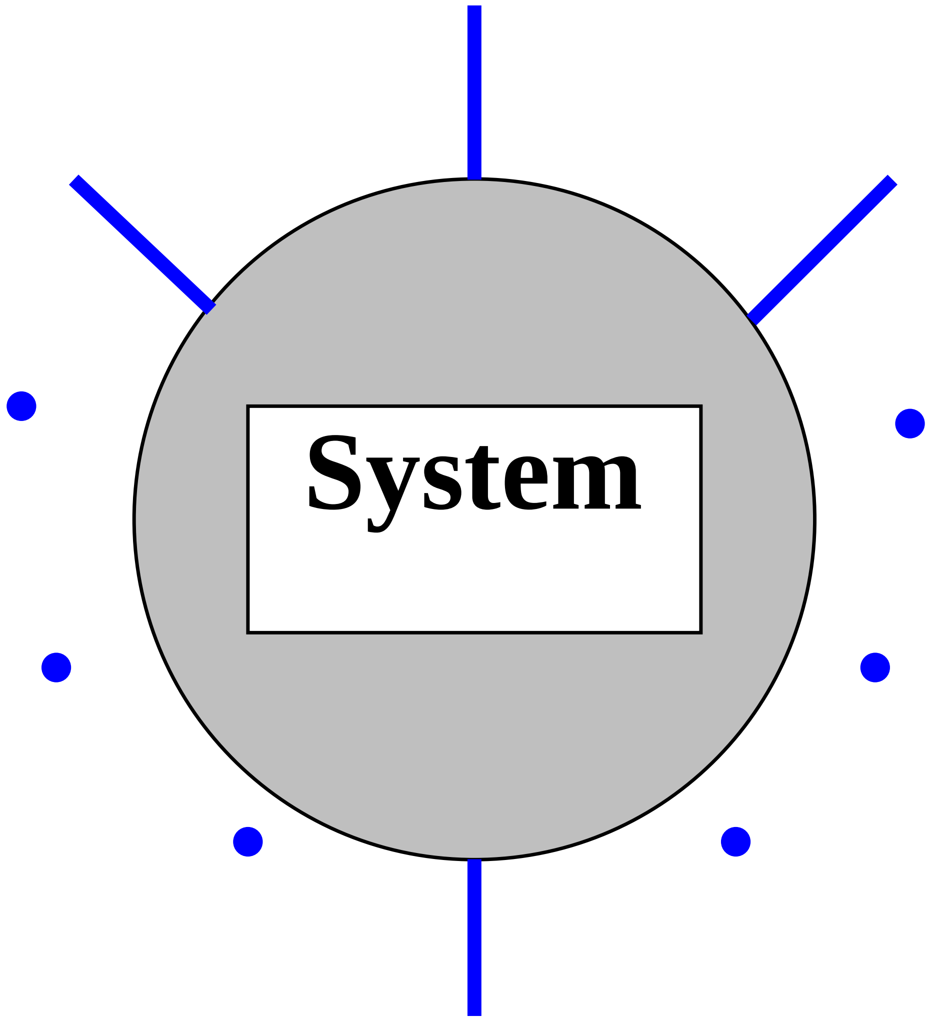
Problems with I/O

- **Physical laws dictate the simultaneous occurrence of events.**
  No cause/effect is implied.

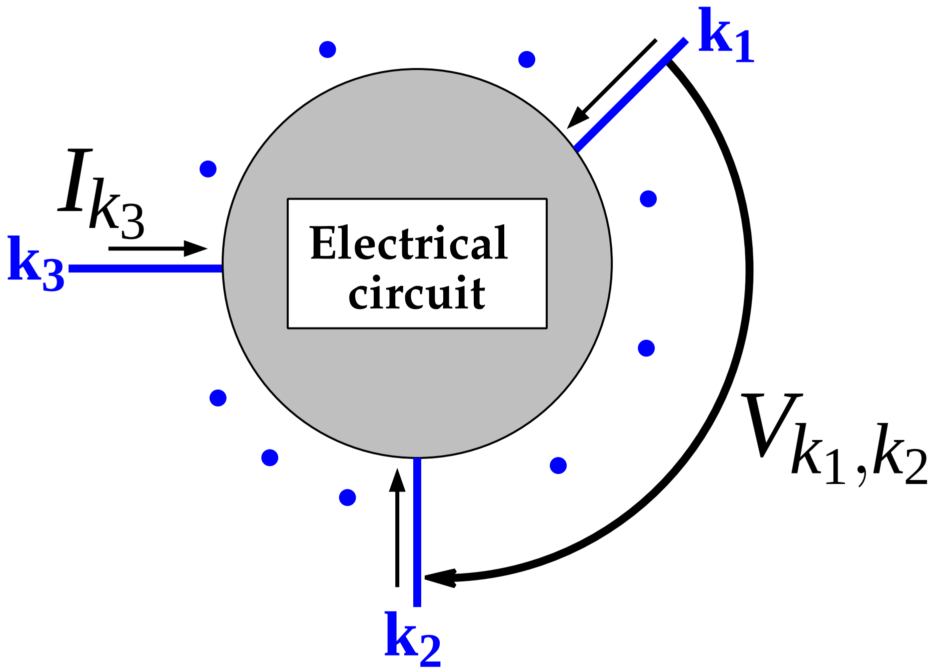
- **Interconnection of physical systems leads to variable sharing, not signal transmission.**

A physical system is not a signal processor.

Systems with terminals



Electrical terminals



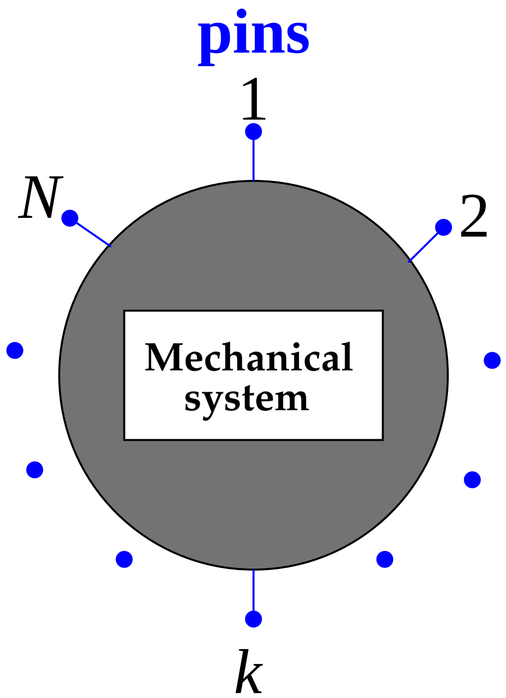
<u>interaction variables</u>: currents & voltages.

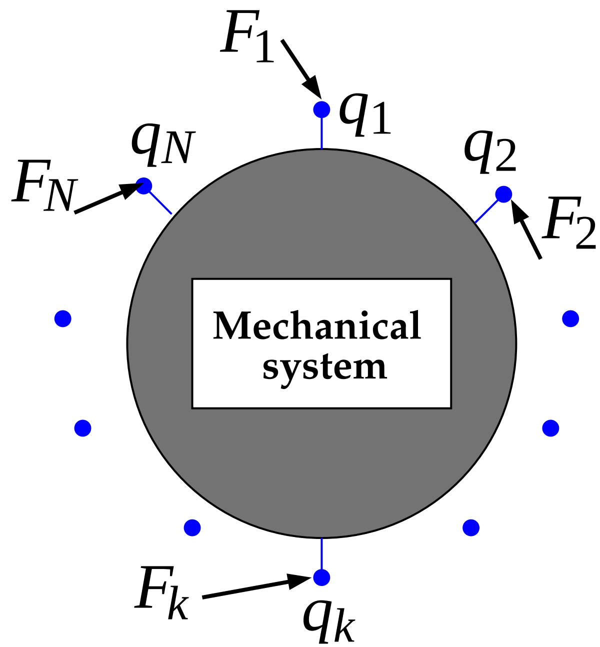
measurable by ammeters and voltmeters.

What is the cause and what is the effect?

What is the stimulus and what is the response?

Mechanical terminals





At each terminal: a position and a force.

More generally, position, force, angle, torque.

What is the cause and what is the effect?

What is the stimulus and what is the response?

Other domains

- **Thermal systems:**

  At each terminal: a temperature and a heat flow.

- **Hydraulic systems:**

  At each terminal: a pressure and a mass flow.

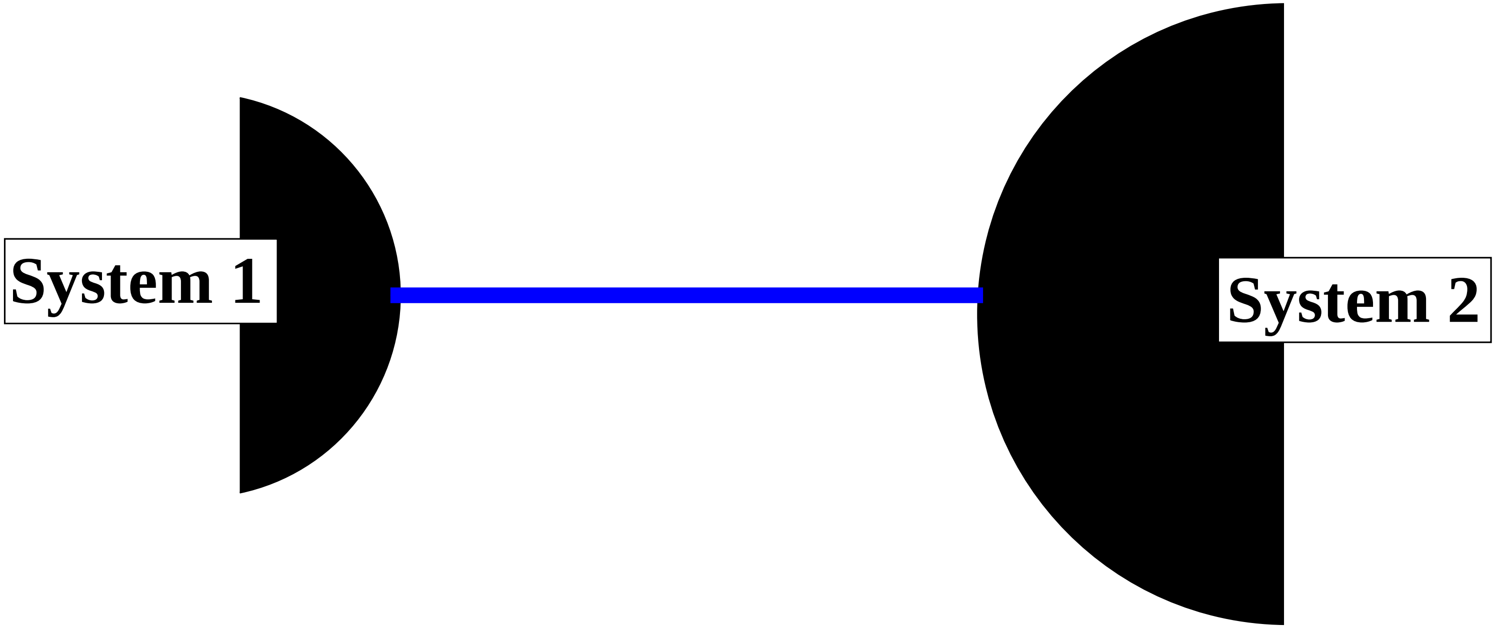
- **Multidomain systems:**

  Systems with terminals of different types, as motors, pumps, etc.

At each terminal, there are many simultaneous variables.

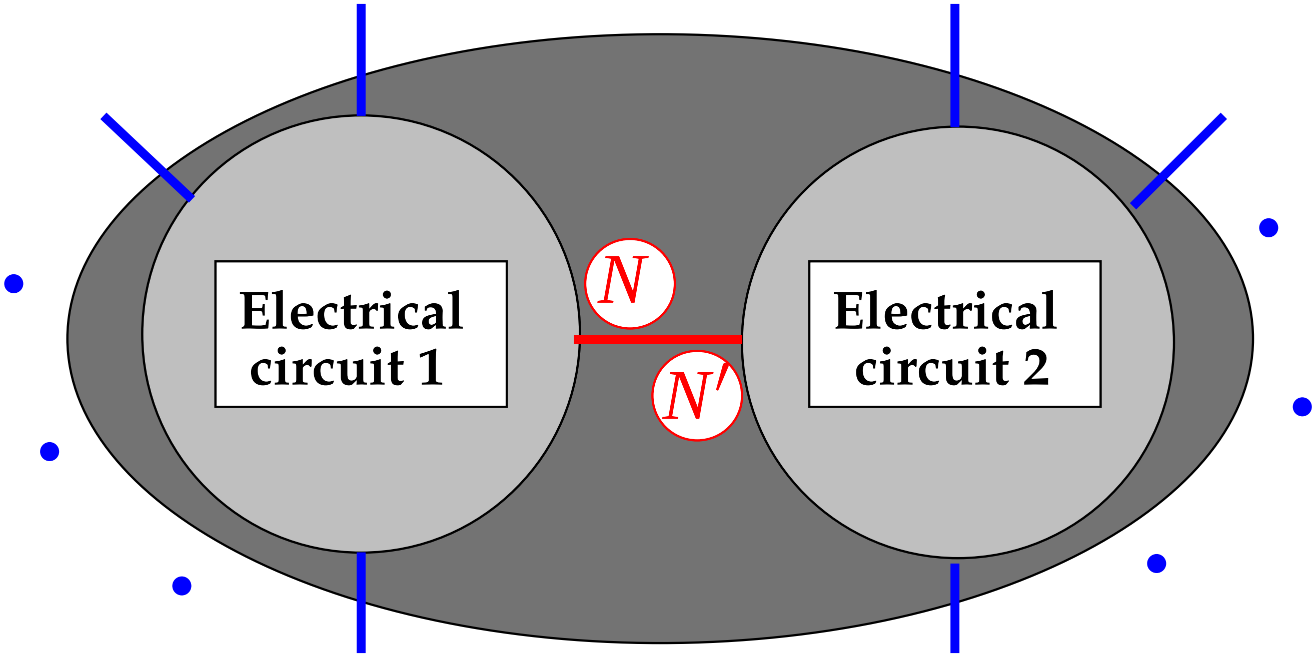
Why and how should we separate these in stimulus and response?

Connection of terminals



By interconnecting, the terminal variables are equated.

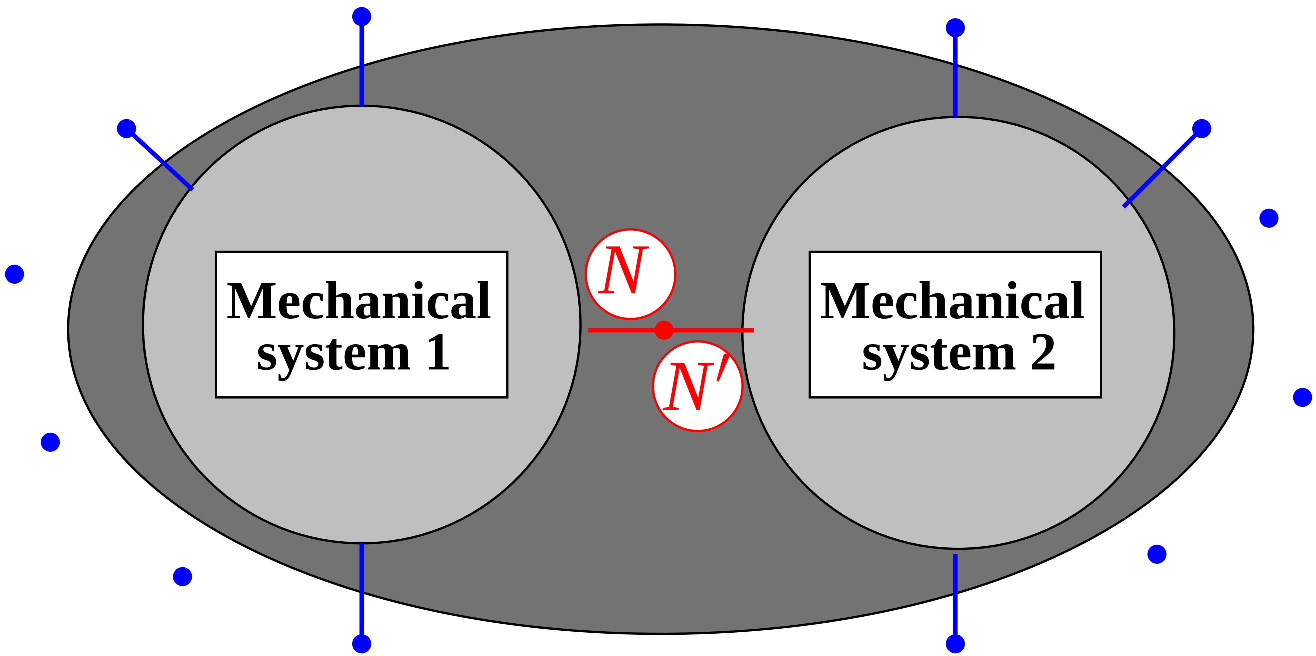
Interconnection of electrical circuits



$$P_N = P_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

The *P*'s are potentials. We used Kirchhoff's voltage law.

Interconnection of mechanical devices



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Other domains

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **etc.**

Linking

$$\begin{aligned}
V_N &= V_{N'} \quad \textbf{and} \quad & I_N + I_{N'} &= 0, \\
q_N &= q_{N'} \quad \textbf{and} \quad & F_N + F_{N'} &= 0, \\
T_N &= T_{N'} \quad \textbf{and} \quad & Q_N + Q_{N'} &= 0, \\
p_N &= p_{N'} \quad \textbf{and} \quad & f_N + f_{N'} &= 0,
\end{aligned}$$

$$\vdots$$

Interconnection ⇔ variable sharing.

In contrast to output-to-input assignment.

Linking

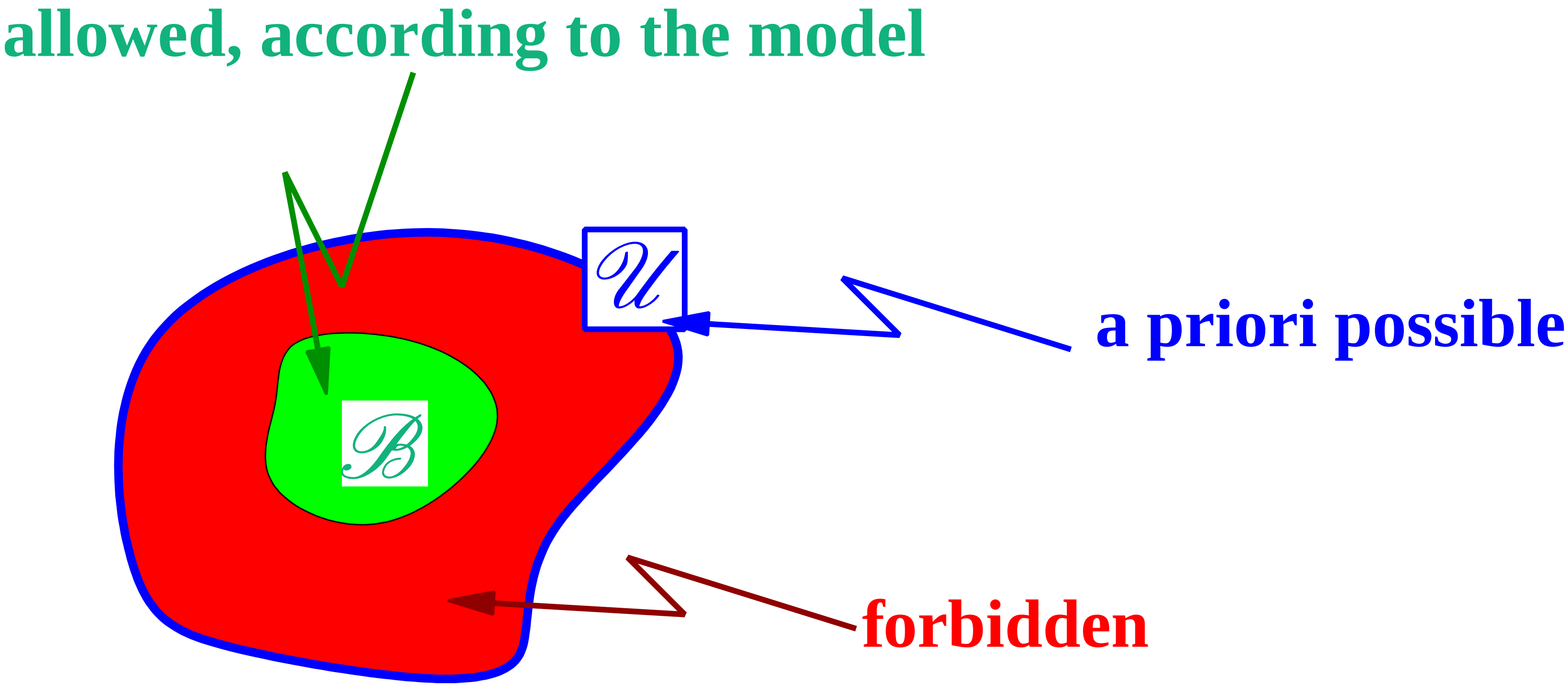
$$\begin{aligned} V_N &= V_{N'} \quad \textbf{and} \quad & I_N + I_{N'} &= 0, \\ q_N &= q_{N'} \quad \textbf{and} \quad & F_N + F_{N'} &= 0, \\ T_N &= T_{N'} \quad \textbf{and} \quad & Q_N + Q_{N'} &= 0, \\ p_N &= p_{N'} \quad \textbf{and} \quad & f_N + f_{N'} &= 0, \end{aligned}$$

$$\vdots$$

An interconnection usually involves *more than one* variable. Signal flow pathways involving *a single* variable should be scrutinized with skepticism.

The BEHAVIORAL APPROACH

The behavior



A model tells which events are allowed.

It does not articulate a cause/effect,

stimulus/response relation.

The dynamic behavior

Definition: A *dynamical system* $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathscr{B})$**, with**

- $\mathbb{T} \subseteq \mathbb{R}$ **the time set,**
- $\mathbb{W}$ **the signal space,**
- $\mathscr{B} \subseteq \mathbb{W}^{\mathbb{T}}$ **the behavior.**

$\mathscr{B}$ **= a family of time trajectories.**

$w \in \mathscr{B}$ **means: the model allows the trajectory** w**,**

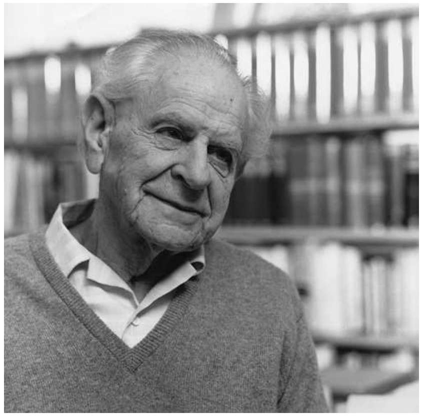
$w \notin \mathscr{B}$ **means: the model forbids the trajectory** w**.**

Behavioral models

The behavior captures the essence of what a model is.

The behavior is all there is.
Equivalence of models, properties of models,
symmetries, system identification, etc.
must all refer to the behavior.

Every 'good' scientific theory is prohibition:
it forbids certain things to happen.
The more it forbids, the better it is.



Karl Popper (1902-1994)

Technical development

There has been an extensive program that deals with system theory, control, identification, etc.

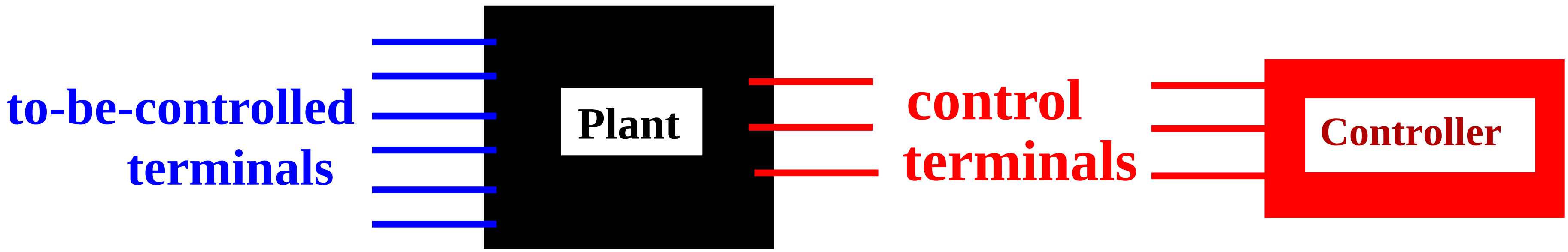
from this point of view,

with systems as behaviors and

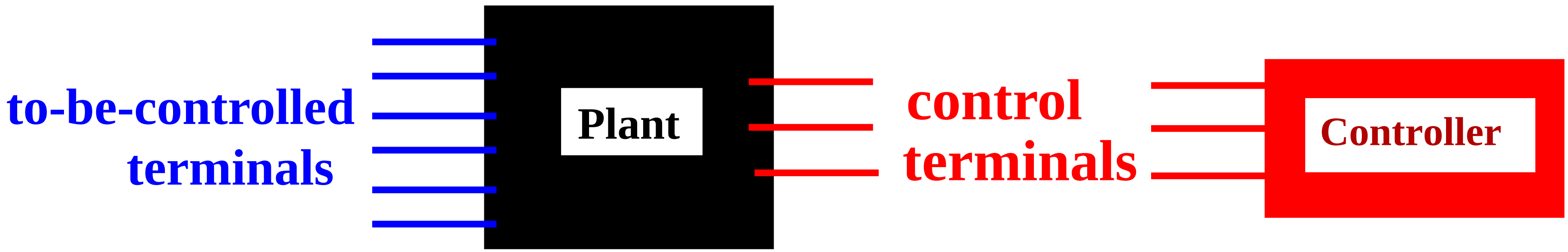
interconnection as variable sharing.

CONTROL as INTERCONNECTION

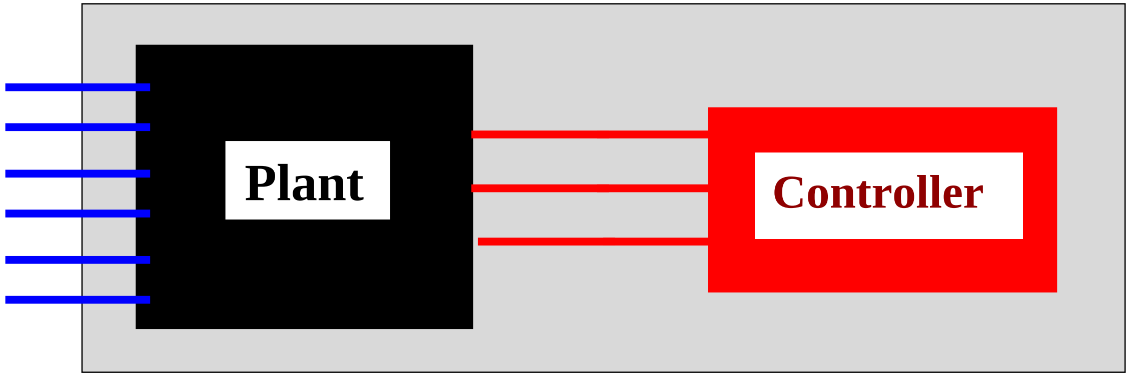
Behavioral control



Behavioral control



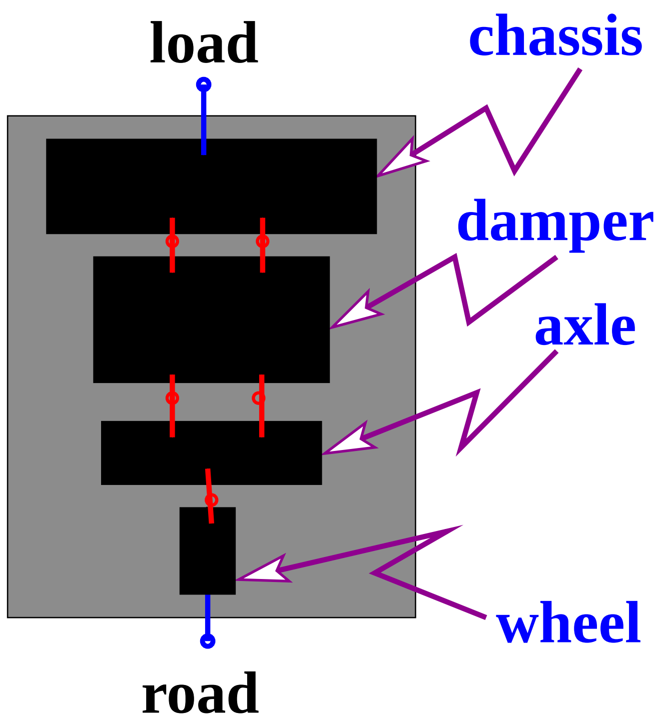
control = interconnection.



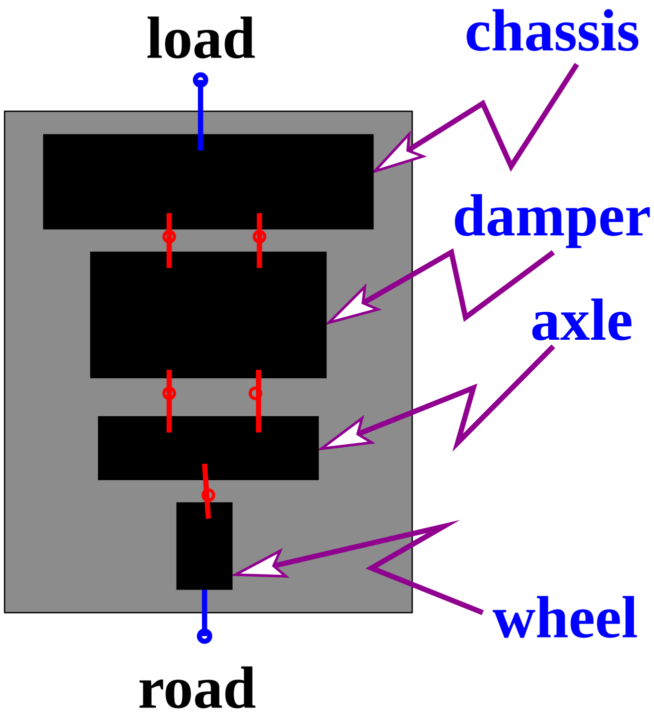
controlled system

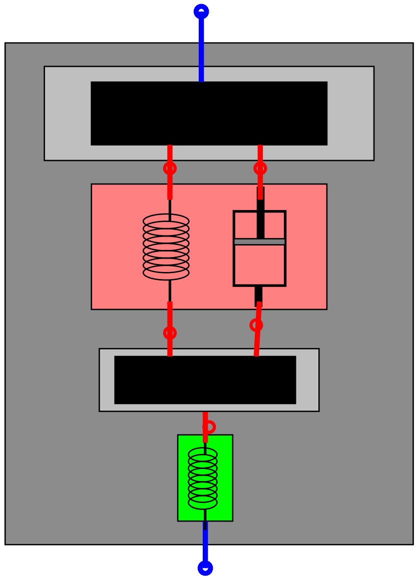
control = integrated system design.

Example: A ‘quarter car’



Example: A ‘quarter car’





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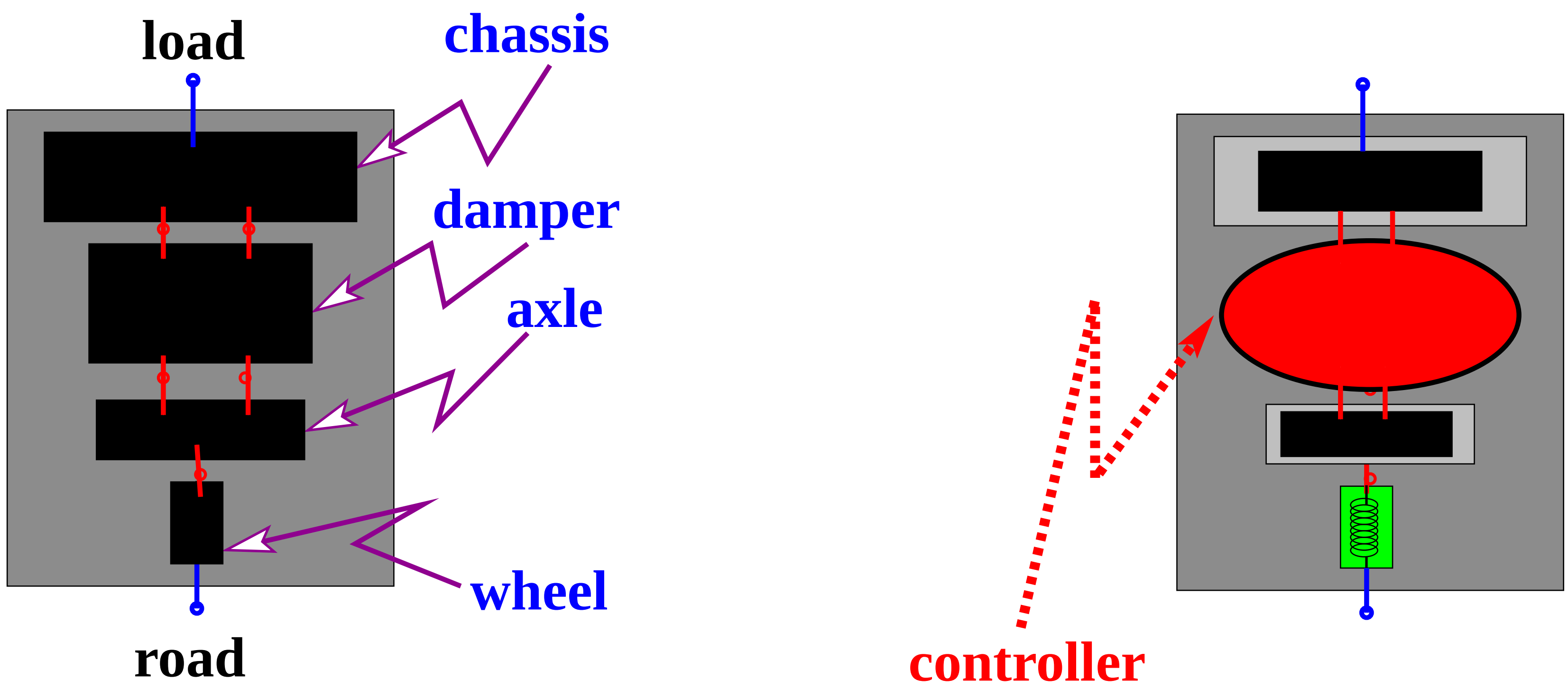

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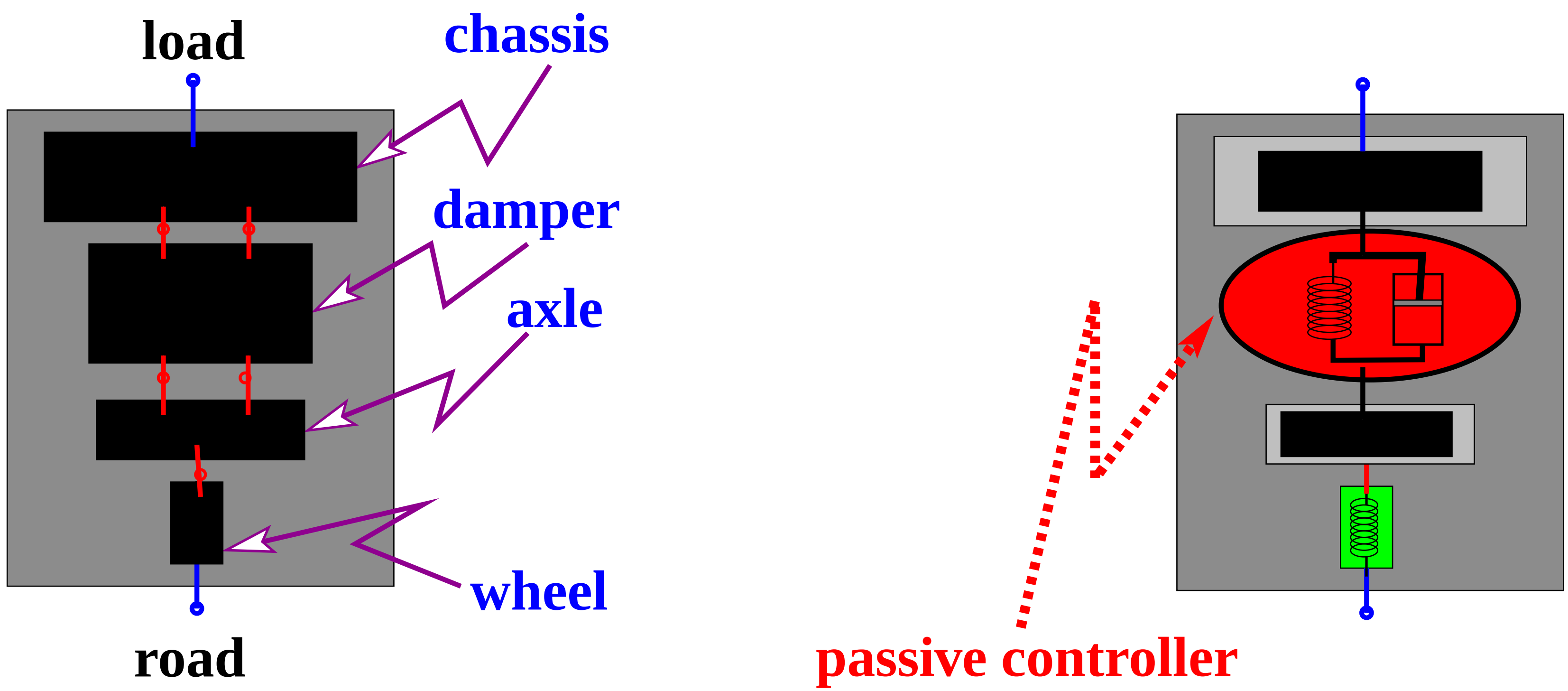

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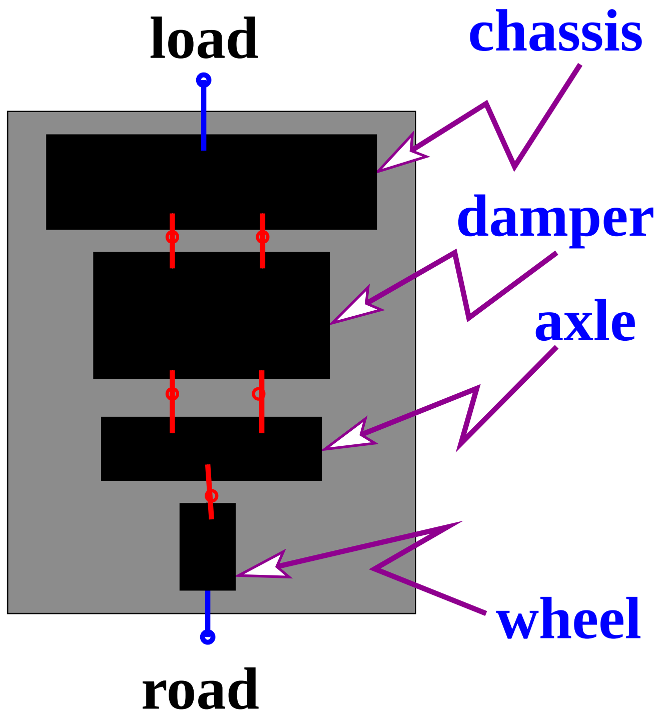
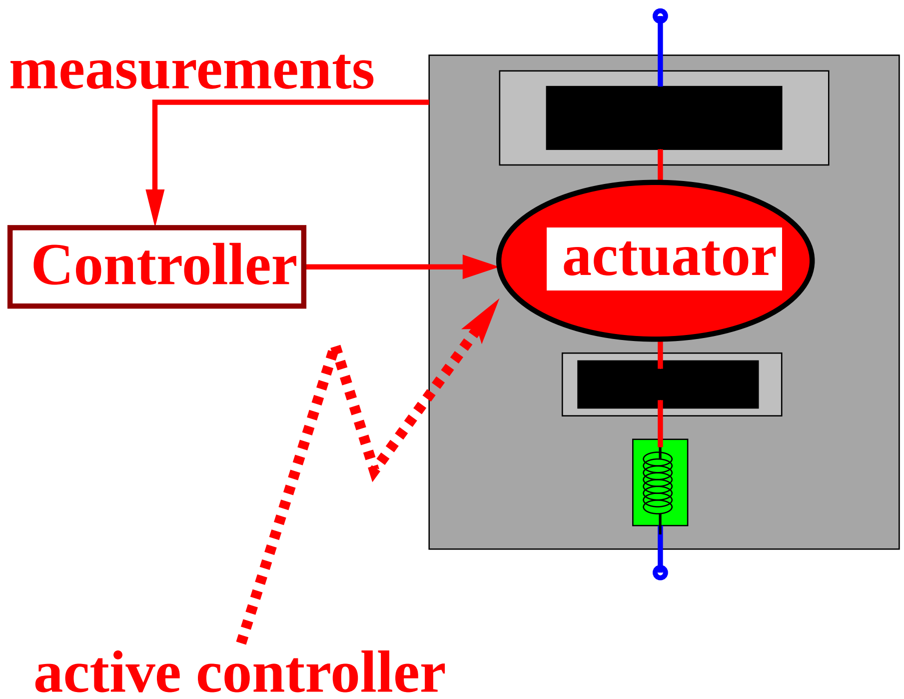




WHAT NEW DOES THIS BRING?

Controllability

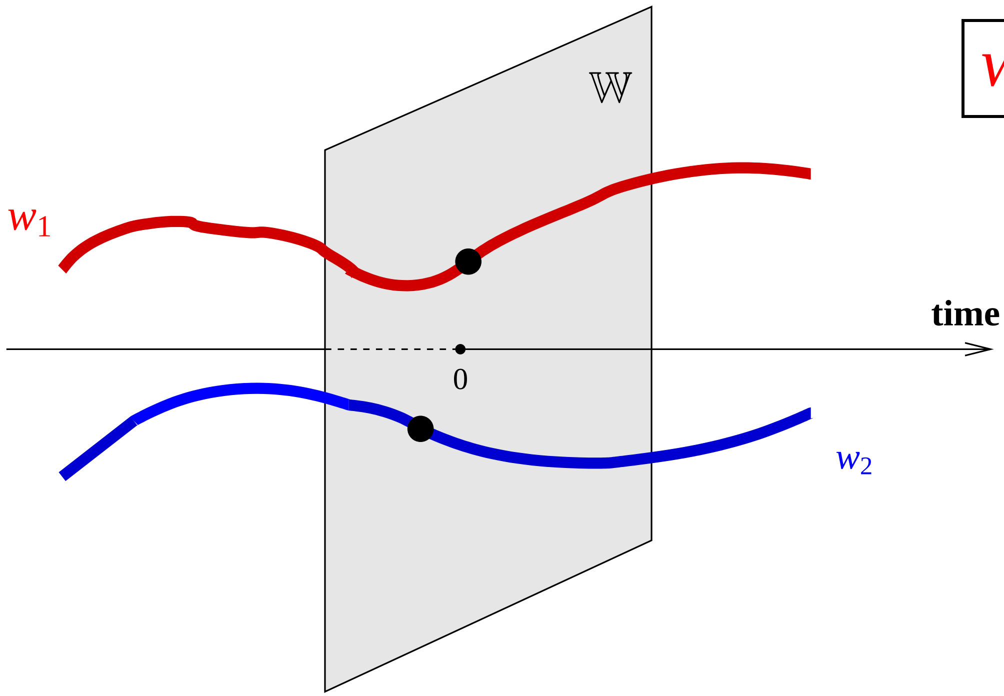
The dynamical system $\Sigma = (\mathbb{T}, \mathbb{W}, \mathscr{B})$, with $\mathbb{T} = \mathbb{R}$ or $\mathbb{Z}$, is said to be controllable $:\Leftrightarrow$

for all $w_1, w_2 \in \mathscr{B}$, there exist $T \in \mathbb{T}, T \geq 0$, and $w \in \mathscr{B}$, such that

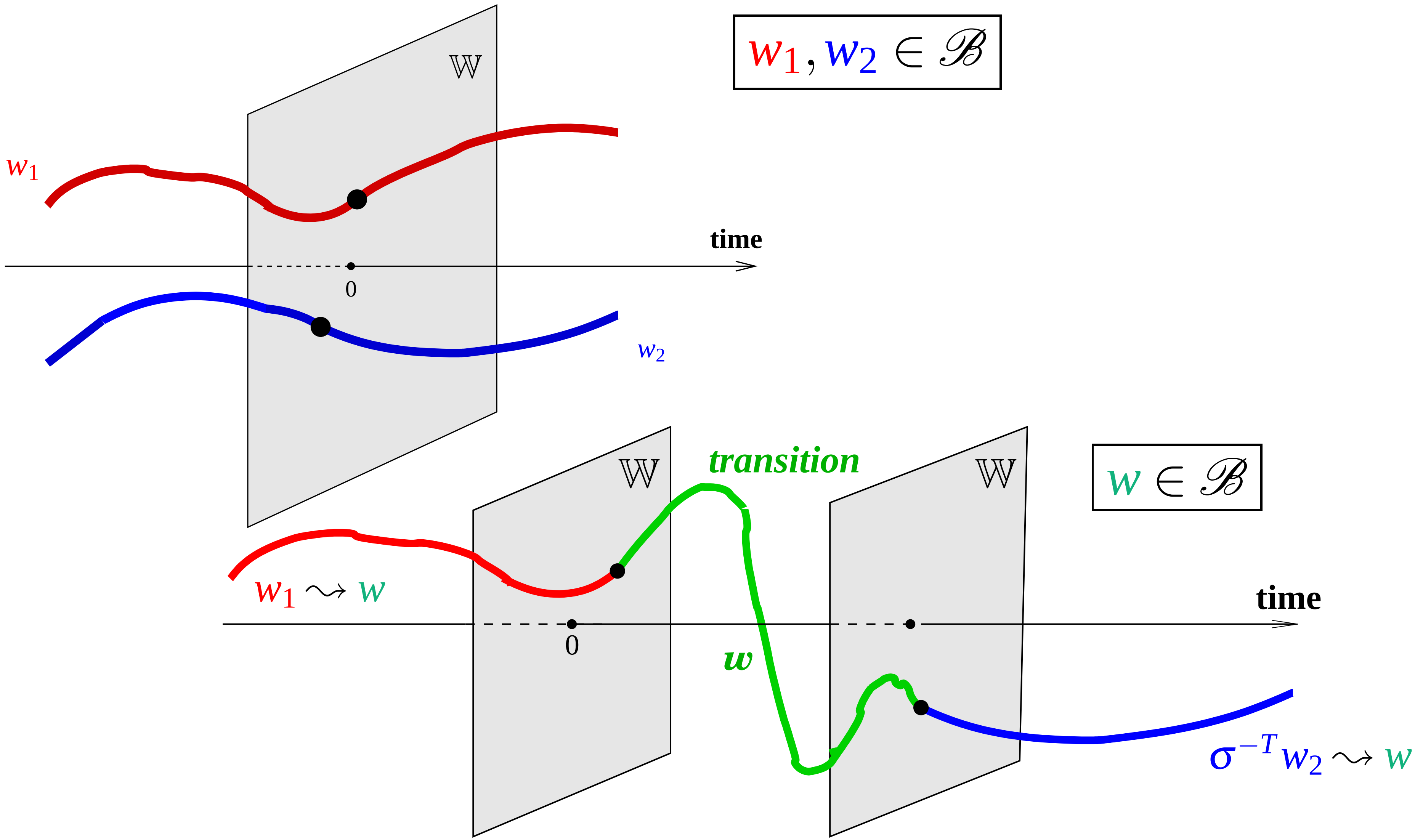
$$w(t) = \begin{cases} w_1(t) & \textbf{for } t < 0; \\ w_2(t - T) & \textbf{for } t \geq T. \end{cases}$$

Controllability in pictures

$w_1, w_2 \in \mathscr{B}$

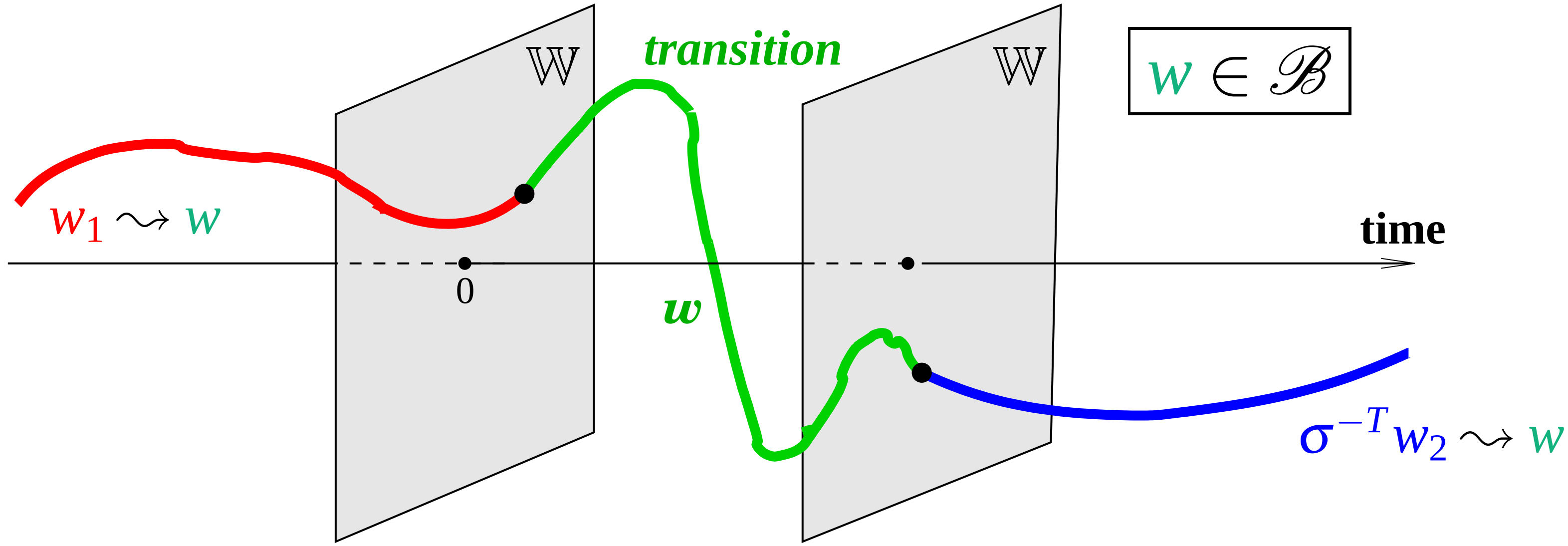


Controllability in pictures



controllability :⇔ concatenability of trajectories after a delay

Controllability in pictures



controllability :⇔ concatenability of trajectories after a delay

Makes controllability into an intrinsic property of a system, rather than a property of a state representation.

LTIDSs

A linear time-invariant differential system (LTIDS) $:\Leftrightarrow$

the behavior $\mathscr{B} \subseteq (\mathbb{R}^{\mathtt{w}})^{\mathbb{R}}$ **is the set of solutions of a system of linear constant-coefficient ODEs**

$$R_0 w + R_1 \frac{d}{dt} w + \cdots + R_{\mathtt{n}} \frac{d^{\mathtt{n}}}{dt^{\mathtt{n}}} w = 0,$$

with $R_0, R_1, \ldots, R_{\mathtt{n}} \in \mathbb{R}^{\bullet \times \mathtt{w}}$ **real matrices that parametrize the system, and** $w : \mathbb{R} \to \mathbb{R}^{\mathtt{w}}$**.**

In polynomial matrix notation

$$R\left(\frac{d}{dt}\right) w = 0$$

with $R(\xi) = R_0 + R_1 \xi + \cdots + R_{\mathtt{n}} \xi^{\mathtt{n}} \in \mathbb{R}[\xi]^{\bullet \times \mathtt{w}}$**.**

Examples of $R\left(\frac{d}{dt}\right)w = 0$

- $\frac{d}{dt}x = Ax$,
- $p(\frac{d}{dt})w = 0$,
- $\frac{d}{dt}x = Ax + Bu, y = Cx + Du$,
- $P(\frac{d}{dt})y = Q(\frac{d}{dt})u$.

R is usually 'wide'.

3 theorems for LTIDSs

1. **There exists a $1 \leftrightarrow 1$ relation between the LTIDSs and the $\mathbb{R}[\xi]$-submodules of $\mathbb{R}[\xi]^{\bullet}$.**

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2. **In LTIDSs, variables can be eliminated:**

$$R\left(\frac{d}{dt}\right)w = M\left(\frac{d}{dt}\right)\ell \qquad \Rightarrow \qquad \tilde{R}\left(\frac{d}{dt}\right)w = 0.$$

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3. **A LTIDS is controllable if and only if its behavior can be expressed as**

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A kernel is an image iff the system is controllable.

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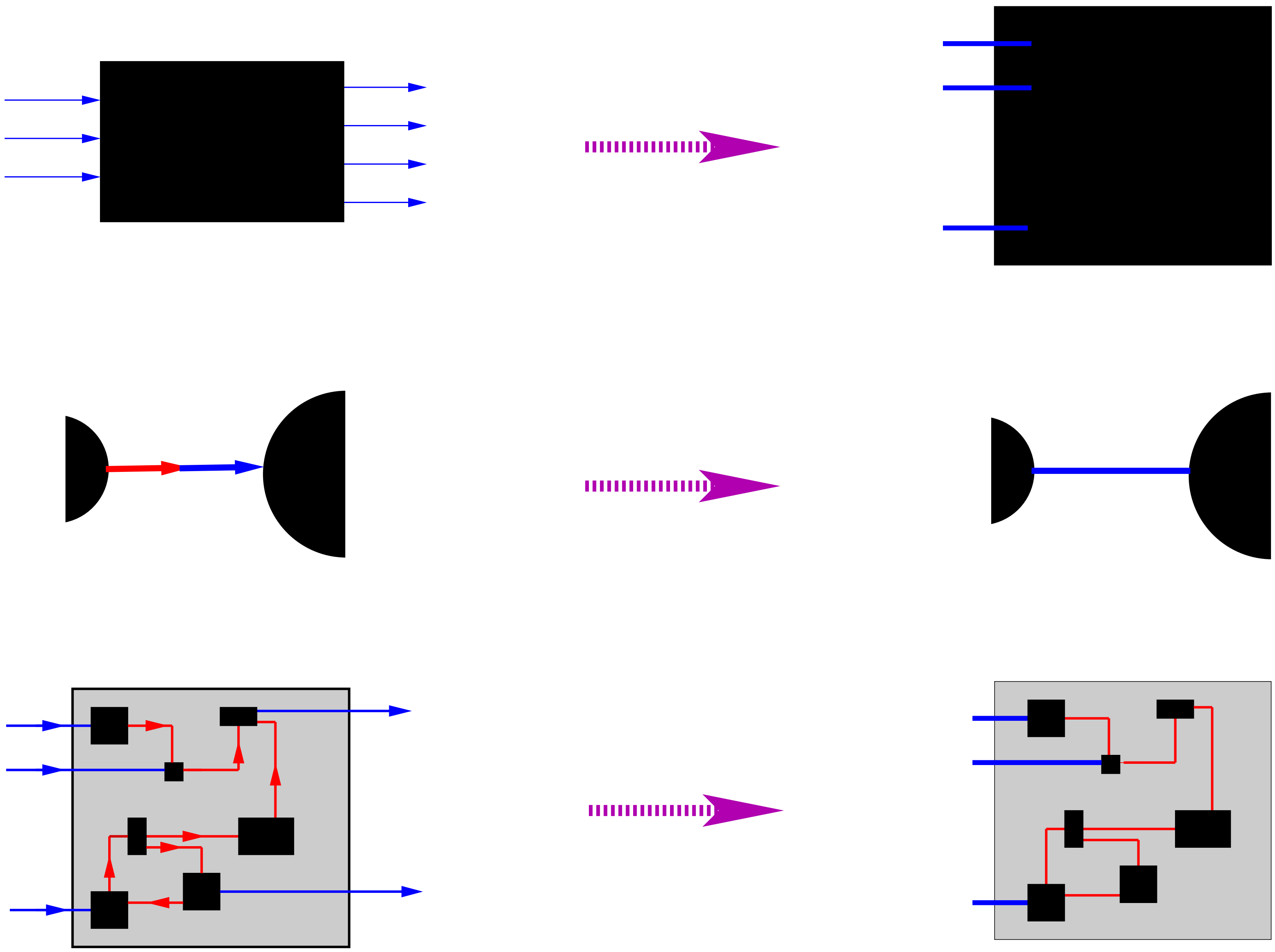
These theorems hold *mutatis mutandis* for discrete-time LTIDSs and for systems described by linear PDEs.

CONCLUSION



<u>Reference</u>: The behavioral approach to open and interconnected systems, *Control Systems Magazine*, volume 27, pages 46-99, 2007.

Copies of the lecture frames available from/at

`http://www.esat.kuleuven.be/~jwillems`

Thank you

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