An internal model principle for observers

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Outline

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The observation problem

Given a set of *variables* (signals) whose interaction is described by a known *dynamical system* and given *measurements* of (some values of) some of the variables, can you provide *good estimates* of (some values of) other variables in the system? How?

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Can you do it with an *observer*?

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Can you do it with an *observer*?

Observer = system interconnected with the observed system
Estimate = value of variable in the observer

Ingredients for a theory of observers

- Model class for the observed system (incl. measurement model)
- What makes an estimate a *good* estimate?
- Is the problem solvable (*observability*)?
- Model class for candidate observers
- Is the problem still solvable (*existence*)?
- How do you recognize a solution (*characterization*)?
- How do you build an observer (*construction*)?
- Describe the set of all solutions (*parametrization*).
- Find a “perfect” estimator (*optimization, ...*)

Our main motivation

A *good* characterization result solves several of these problems in one go.

Given a characterization in terms of equations (for example), then

- existence = solvability of the equations
- construction = find a solution
- parametrization = describe the *solution set*

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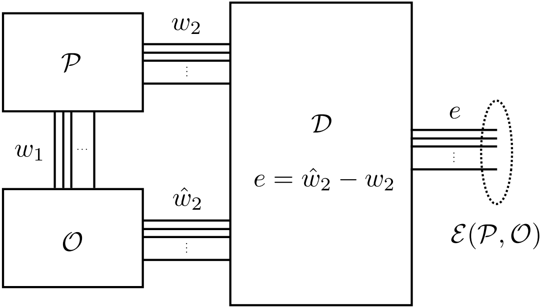
Model class

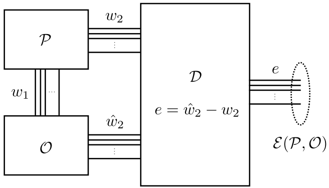


$$\mathcal{P} = \ker \left(R_1(\tfrac{\mathrm{d}}{\mathrm{d}t}) \quad R_2(\tfrac{\mathrm{d}}{\mathrm{d}t}) \right)$$
$$\mathcal{O} = \ker \left(\hat{R}_1(\tfrac{\mathrm{d}}{\mathrm{d}t}) \quad \hat{R}_2(\tfrac{\mathrm{d}}{\mathrm{d}t}) \right)$$

$w_i, \hat{w}_i$ are C^∞

cf. Bisiacco/Valcher/Willems
"irrelevant" variables have been eliminated

Estimation goal



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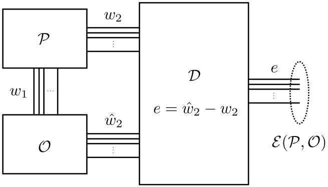
$w_i, \hat{w}_i$ are C^∞

$\mathcal{E}(\mathcal{P}, \mathcal{O})$ autonomous $\longrightarrow$ tracking observer (Fuhrmann/T.)
$\mathcal{E}(\mathcal{P}, \mathcal{O})$ autonomous stable $\longrightarrow$ asymptotic observer
$\mathcal{E}(\mathcal{P}, \mathcal{O}) = \{0\}$ $\longrightarrow$ exact observer

Existence



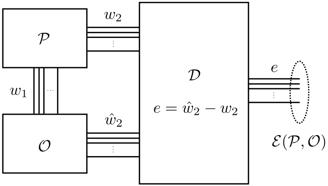
$$\begin{aligned}\mathcal{E}(\mathcal{P},\mathcal{O}) &= \mathcal{N}_{w_2}(\mathcal{P}+\mathcal{O})\\ &\supset \mathcal{N}_{w_2}(\mathcal{P})\\ &= \{w_2 \mid (0, w_2) \in \mathcal{P}\}\end{aligned}$$

Given a plant $\mathcal{P}$, a specific error behavior is *achievable* if and only if it contains the *hidden behavior* of w_2 in $\mathcal{P}$.

If such an error behavior is autonomous/stable/... then the hidden behavior has this property: this is an *observability* property for the plant (and analogously for the observer).

existence $\Leftrightarrow$ observability

Reasonable observers



w_2 observable from w_1 in $\mathcal{P}$

$\mathcal{O} = \{0\}$???

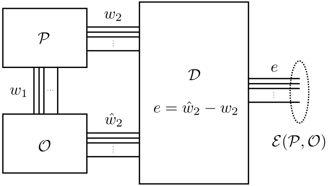
A *reasonable* observer should not disturb the plant.

$$\mathcal{O} \text{ nonintrusive} \Leftrightarrow (\mathcal{P} \wedge_{w_1} \mathcal{O})_{(w_1,w_2)} = \mathcal{P}$$
$$\Leftrightarrow \mathcal{P}_{w_1} \subset \mathcal{O}_{w_1} \quad \text{“acceptor” (Valcher/Willems)}$$

i/o-observers are nonintrusive and exist w.l.o.g.

Outline

An internal model principle



$$\mathcal{P} = \ker \begin{pmatrix} R_1(\tfrac{\mathrm{d}}{\mathrm{d}t}) & R_2(\tfrac{\mathrm{d}}{\mathrm{d}t}) \end{pmatrix} \text{ (obs.)}$$

$$\mathcal{O} = \ker \begin{pmatrix} \hat{R}_1(\tfrac{\mathrm{d}}{\mathrm{d}t}) & \hat{R}_2(\tfrac{\mathrm{d}}{\mathrm{d}t}) \end{pmatrix}$$

$w_i, \hat{w}_i$ are C^∞

A nonintrusive observer, i.e. $(\mathcal{P} \wedge_{w_1} \mathcal{O})_{(w_1,w_2)} = \mathcal{P}$, is

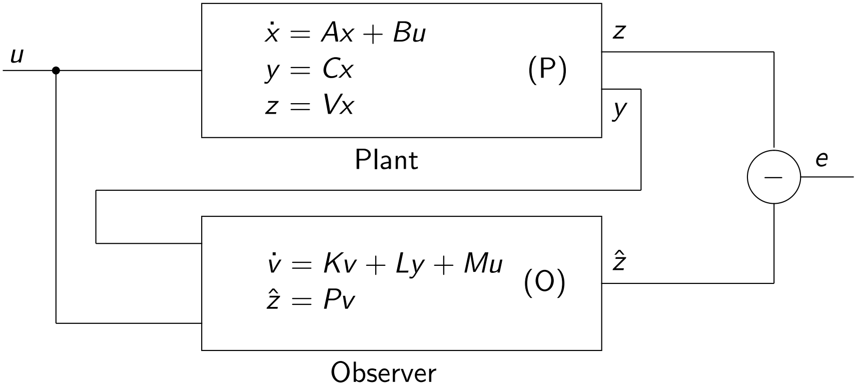
tracking	$(\mathcal{E}(\mathcal{P},\mathcal{O})$ autonomous)	$\Leftrightarrow$	$\mathcal{P}_{\mathrm{cont}} \subset \mathcal{O}$
asymptotic	$(\mathcal{E}(\mathcal{P},\mathcal{O})$ stable)	$\Leftrightarrow$	$\mathcal{P}_{\mathrm{cont}} \oplus \mathcal{P}_{\mathrm{antistab}} \subset \mathcal{O}$
exact	$(\mathcal{E}(\mathcal{P},\mathcal{O})$ trivial)	$\Leftrightarrow$	$\mathcal{P} \subset \mathcal{O}$

(& resp. necessary observability condition on $\mathcal{O}$)

Outline

1. Motivation
2. Problem setup
3. Main result
4. A special case
5. Outlook

The state space setting



Internal model equations

$$\dot{x} = Ax + Bu \qquad \dot{v} = Kv + Ly + Mu$$
$$y = Cx \qquad \hat{z} = Pv$$
$$z = Vx$$

Fuhrmann/T.: $\mathcal{P}_{(u,y,z)} \subset \mathcal{O}_{(u,y,\hat{z})}$ if and only if there exists a (constant) matrix U such that

$$UA - KU = LC$$
$$M = UB$$
$$V = PU.$$

Then $d := v - Ux$ fulfills

$$\dot{d} = Kd$$
$$e = Pd.$$

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Future work

- All this can be done for injective cogenerator signal modules over the polynomial ring $F[s]$ (F a field)
- Using Oberst's quotient signal modules (Gabriel localization) one can derive a kernel representation for the internal model part of $\mathcal{P}$
- This leads to very nice parametrization results (joint work with Ingrid Blumthaler)
- General duality between nonintrusive observer design and open-loop reference tracking control

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Thanks.