MODELING, INTERCONNECTION

and ENERGY FLOW

for DYNAMICAL SYSTEMS

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Theme

How are open systems formalized?

How are systems interconnected?

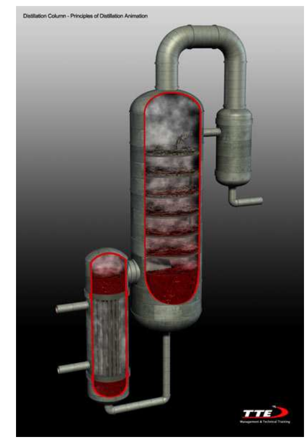
How is energy transferred between systems?

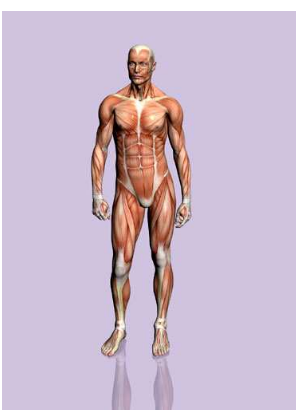
**We deal with very simple examples,
mainly electrical circuits.**

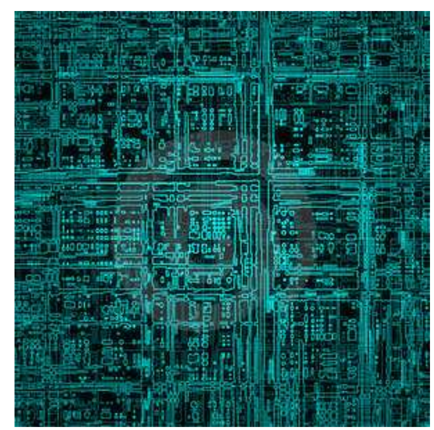
Expect a kind of ‘Back to Basics’ lecture.

SYSTEMS

OIL REFINERY (GVG / PD)



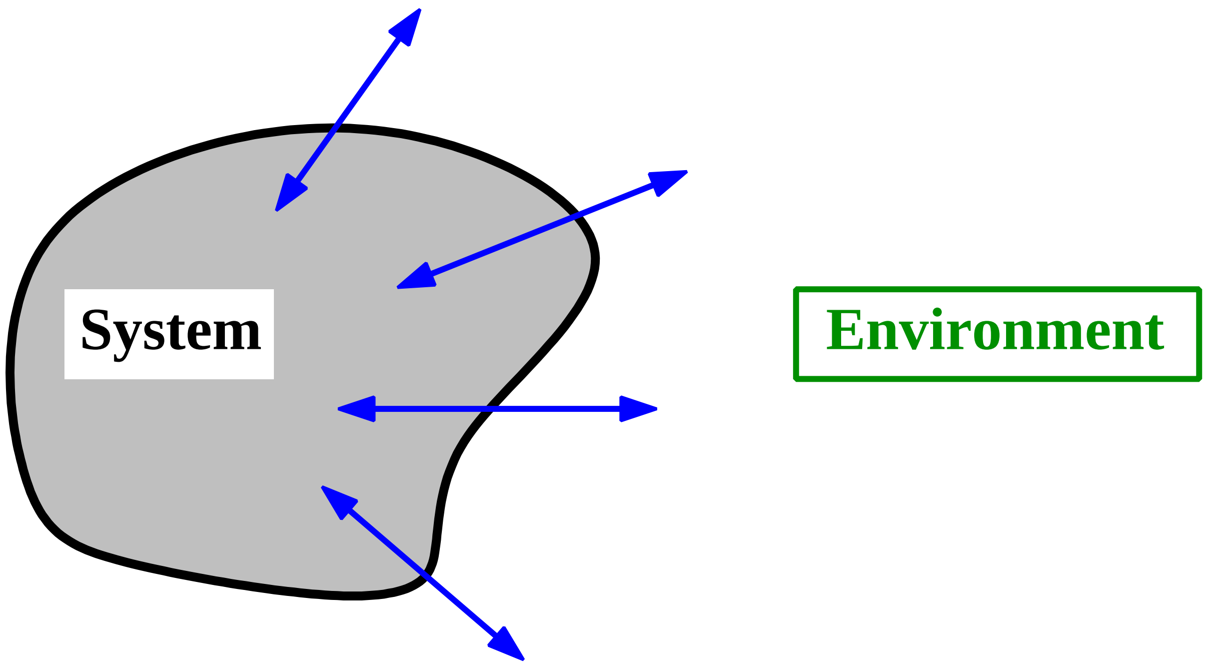




Features

- **Open**
- **Interconnected**
- **Modular**

Open systems

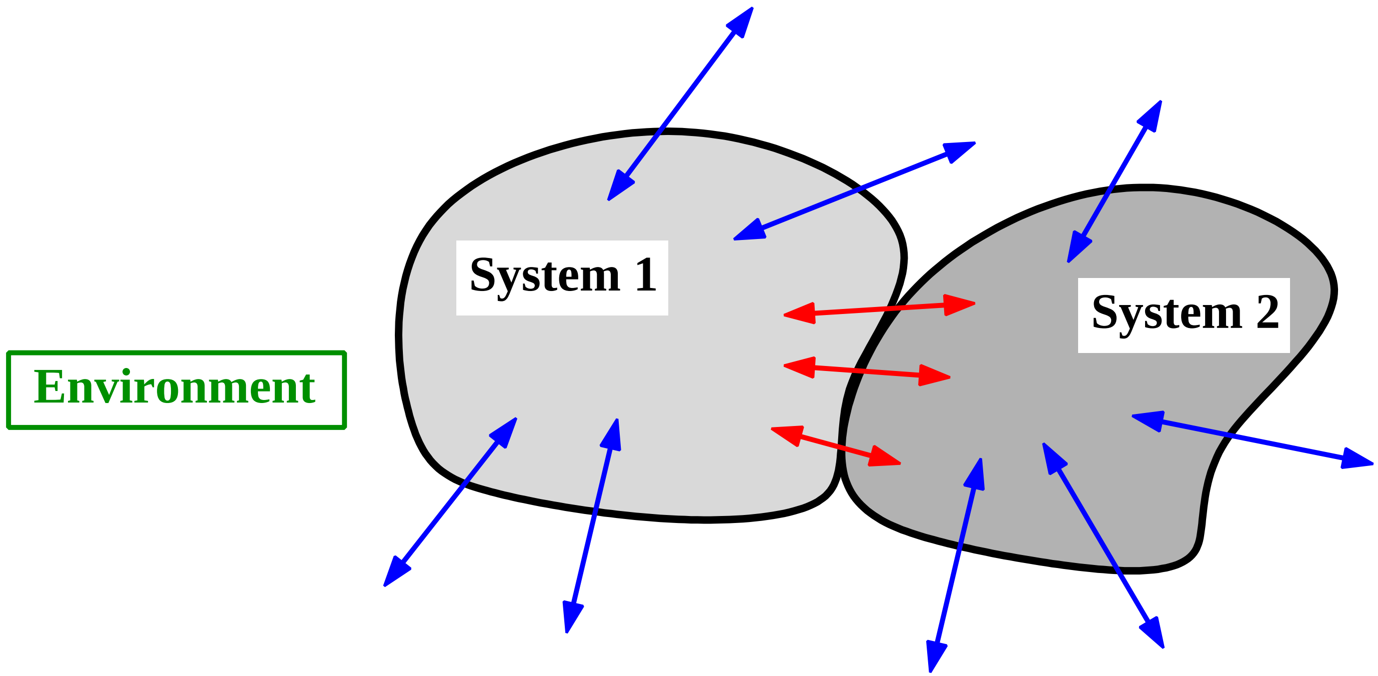


Systems are ‘open’, they interact with their environment.

How are such systems formalized?

How is energy transferred from the environment to a system?

Interacting systems



Interconnected systems interact.

How is this interaction formalized?

How is energy transferred between systems?

Premise

The ever-increasing computing power allows to model complex interconnected systems accurately by

tearing, zooming, and linking.

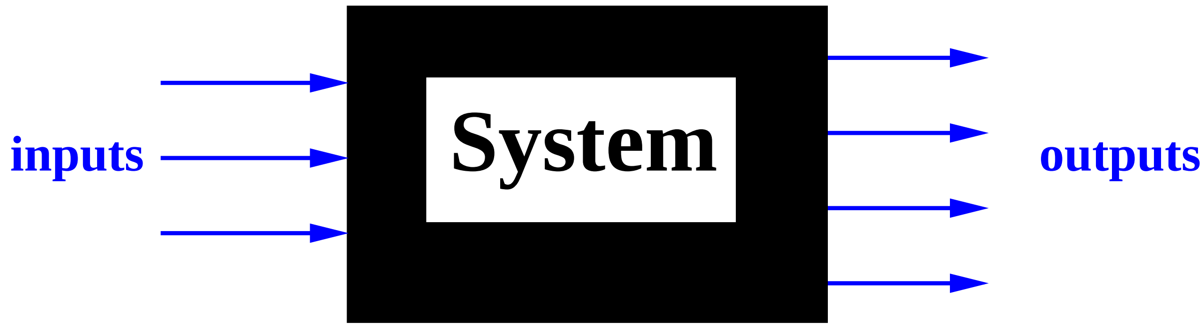
$\rightsquigarrow$ **Simulation, model based design, ...**

Requires the right mathematical concepts

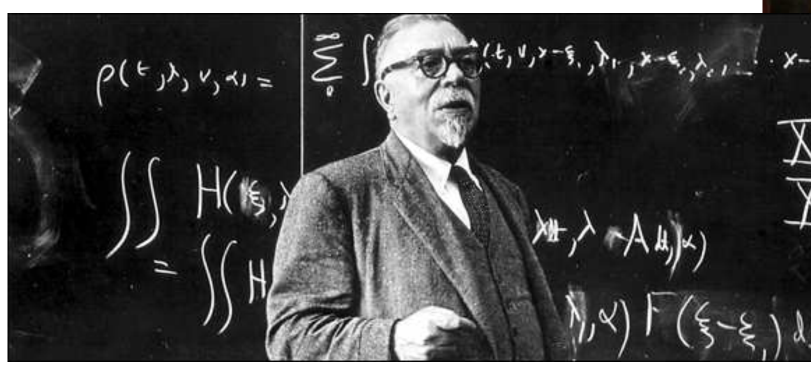
- **for *'dynamical system'***
- **for *'interconnection'***
- **for *'interconnection architecture'***

CLASSICAL VIEW

Input/output systems



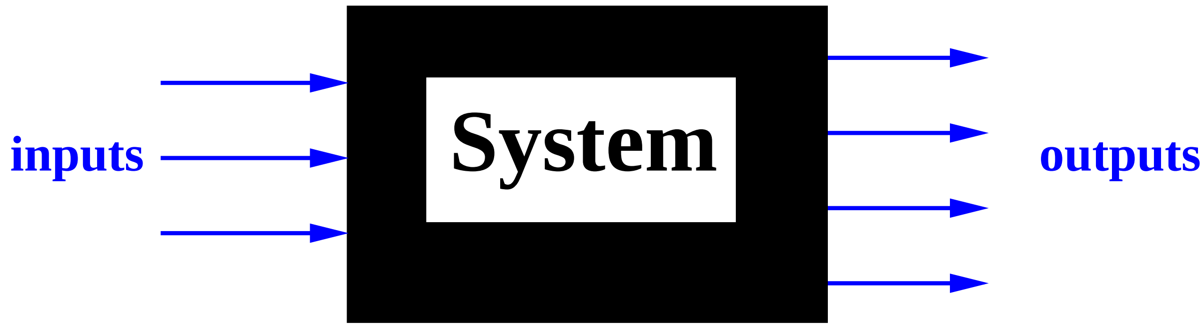
Oliver Heaviside



Norbert Wiener

Rudy Kalman

Input/output systems



Input/output thinking is ***inappropriate*** **for describing the functioning of open physical systems.**

A physical system is not a signal processor.

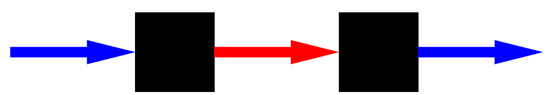
Better concept: a behavior.

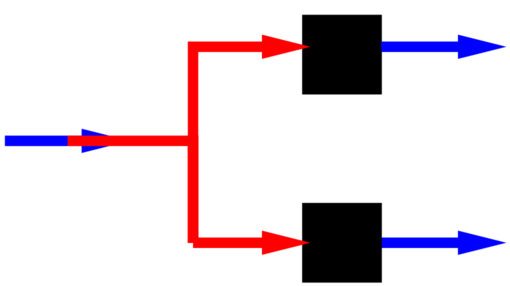
Interconnection

Interconnection as **output-to-input assignment.**

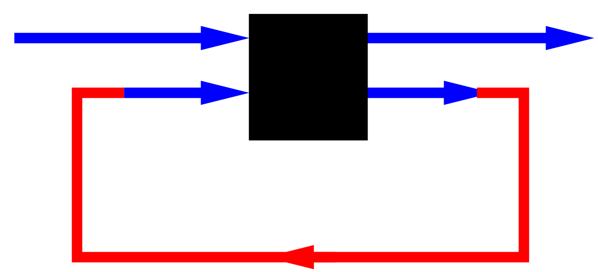
Examples:

series



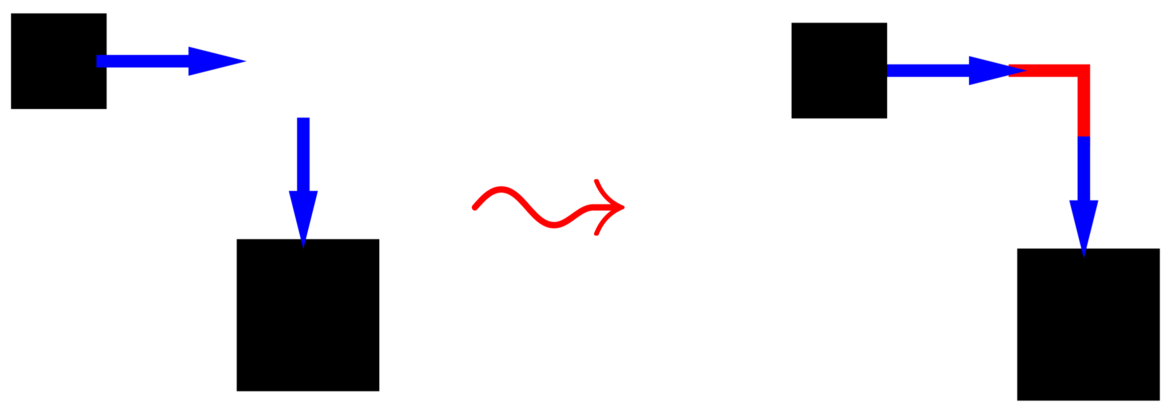


parallel



feedback

Interconnection

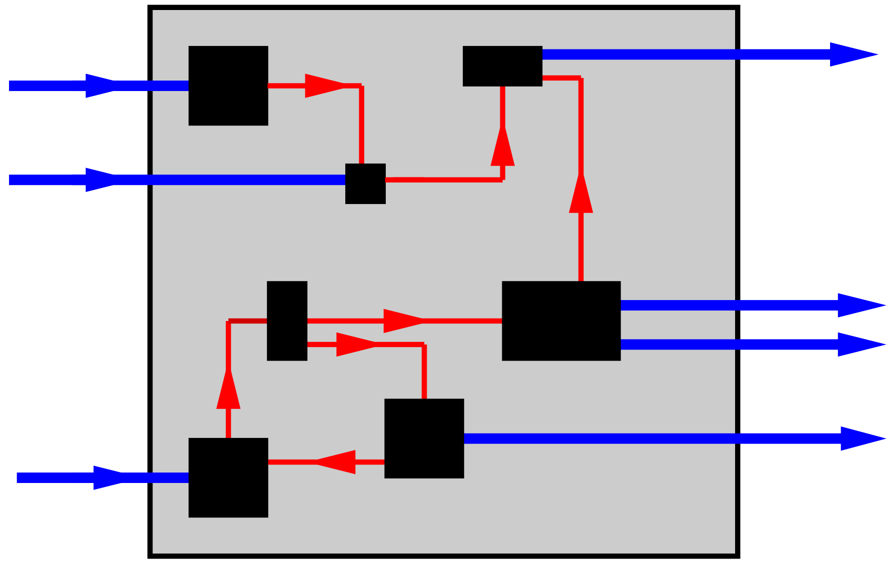


Output-to-input assignment is *inappropriate* for describing the interconnection of physical systems.

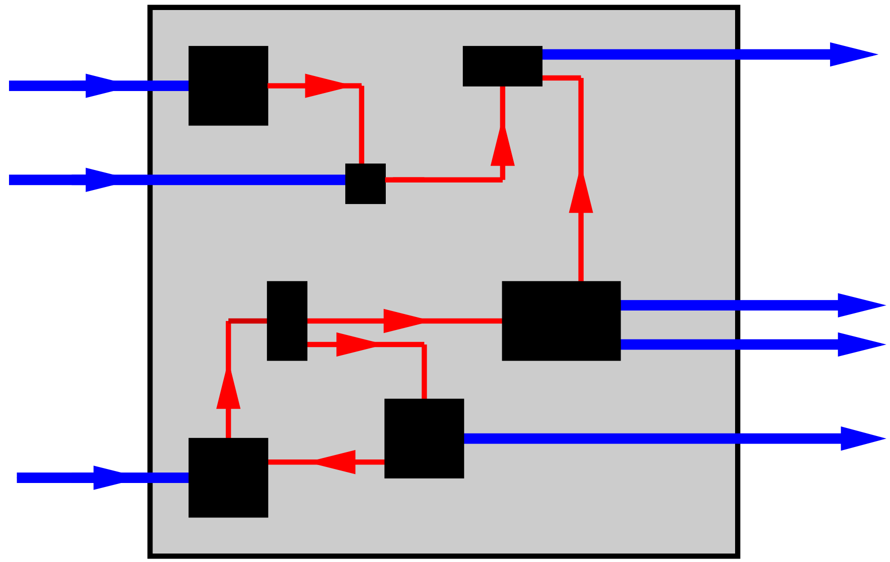
A physical system is not a signal processor.

Better concept: **variable sharing**

Signal flow graphs



Signal flow graphs



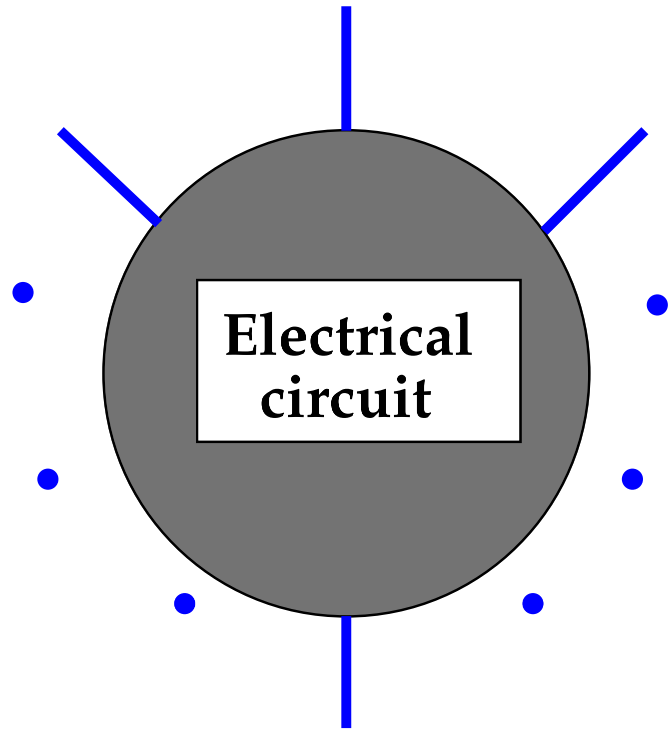
Signal flow graphs are ***inappropriate*** **for describing the interaction architecture of physical systems.**

A physical system is not a signal processor.

Better concept: a graph with leaves.

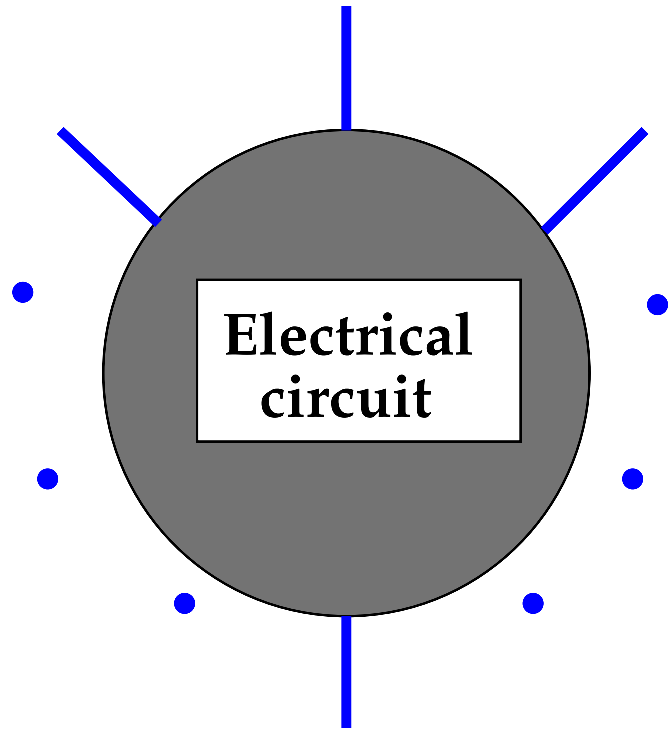
Terminal behavior

A circuit with external terminals



Describe the dynamic terminal behavior!

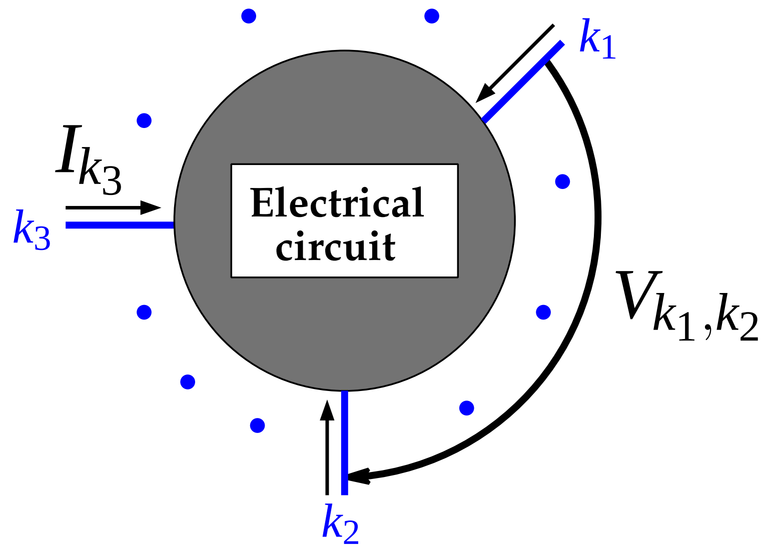
A circuit with external terminals



Describe the dynamic terminal behavior!

What are the interaction variables?

Currents and voltages



Interaction variables: **currents in & voltages across.**

$$\rightsquigarrow \qquad I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \quad V = \begin{bmatrix} V_{1,1} & V_{1,2} & \cdots & V_{1,N} \\ V_{2,1} & V_{2,2} & \cdots & V_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N,1} & V_{N,2} & \cdots & V_{N,N} \end{bmatrix}.$$

Currents and voltages

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$(I,V) \in \mathscr{B}_{IV}$ **means**

$$\left(I_1, I_2, \ldots, I_k, \ldots, I_N, V_{1,1}, V_{1,2}, \ldots, V_{k_1,k_2}, \ldots, V_{N,N}\right) : \mathbb{R} \longrightarrow \mathbb{R}^N \times \mathbb{R}^{N \times N}$$

is compatible with circuit architecture and element values.

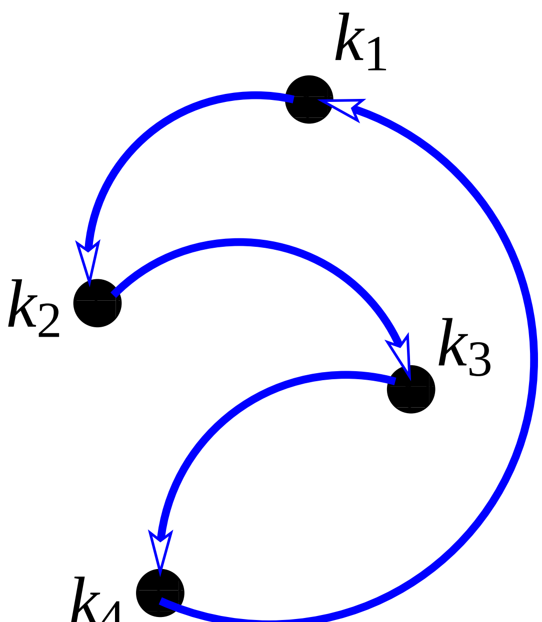
The trajectories $(I,V) \in \mathscr{B}_{IV}$ **are those that can conceivably occur.**

KVL

Kirchhoff voltage law:

$$[\![(I,V) \in \mathscr{B}_{IV}]\!] \Rightarrow [\![V_{k_1,k_2} + V_{k_2,k_3} + \cdots + V_{k_{n-1},k_n} + V_{k_n,k_1} = 0$$

for all $k_1, k_2, \ldots, k_n \in \{1, 2, \ldots, N\}]\!].$



Physically, KVL is evident
(No EM fields outside the wires)
We henceforth assume it

Potentials

Thm: $V : \mathbb{R} \longrightarrow \mathbb{R}^{N \times N}$ **satisfies KVL** $\quad \Leftrightarrow$

$$\exists\, P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} : \mathbb{R} \longrightarrow \mathbb{R}^N \text{ such that } V_{k_1,k_2} = P_{k_1} - P_{k_2}.$$

Potentials

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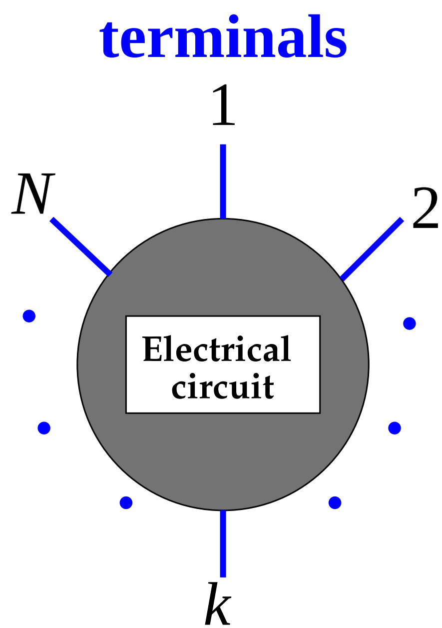
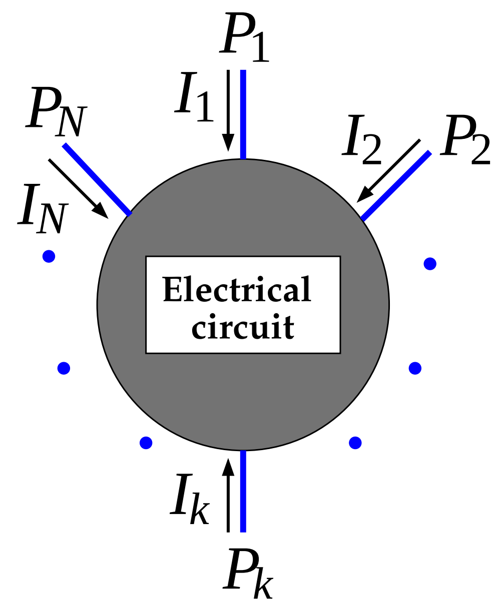
$$P \text{ 'potential'} \Rightarrow \begin{bmatrix} P_1 + \alpha \\ P_2 + \alpha \\ \vdots \\ P_N + \alpha \end{bmatrix} \text{ potential } \forall\, \alpha : \mathbb{R} \rightarrow \mathbb{R}.$$

Electrical circuit





At each terminal:

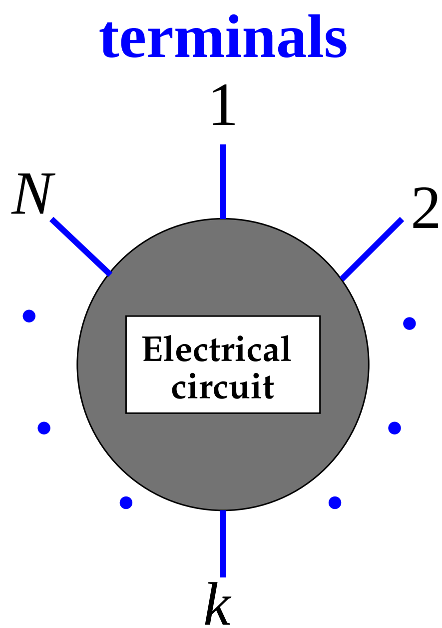
a current (counted > 0 into the circuit) and a potential

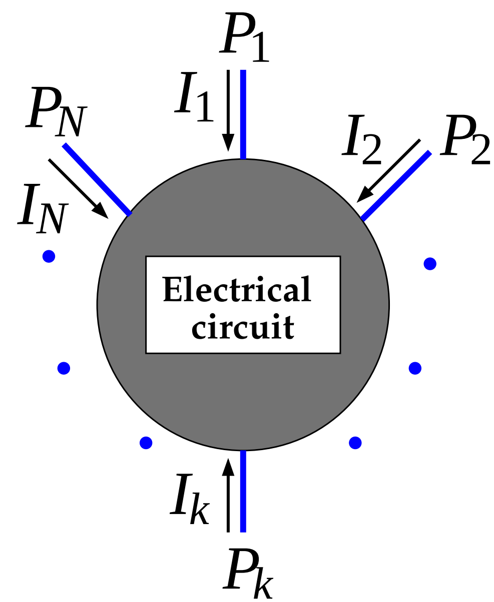
$$\rightsquigarrow \textbf{behavior } \mathscr{B}_{IP} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$\left(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N\right) \in \mathscr{B}_{IP}$ **means:**

this current/potential trajectory is compatible with the circuit architecture and its element values.

Electrical circuit

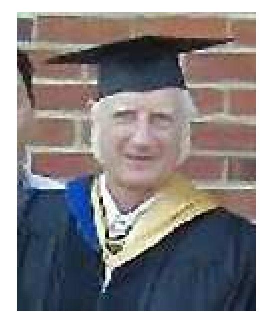




$\rightsquigarrow$ **behavior** $\mathscr{B}_{IP} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}$.

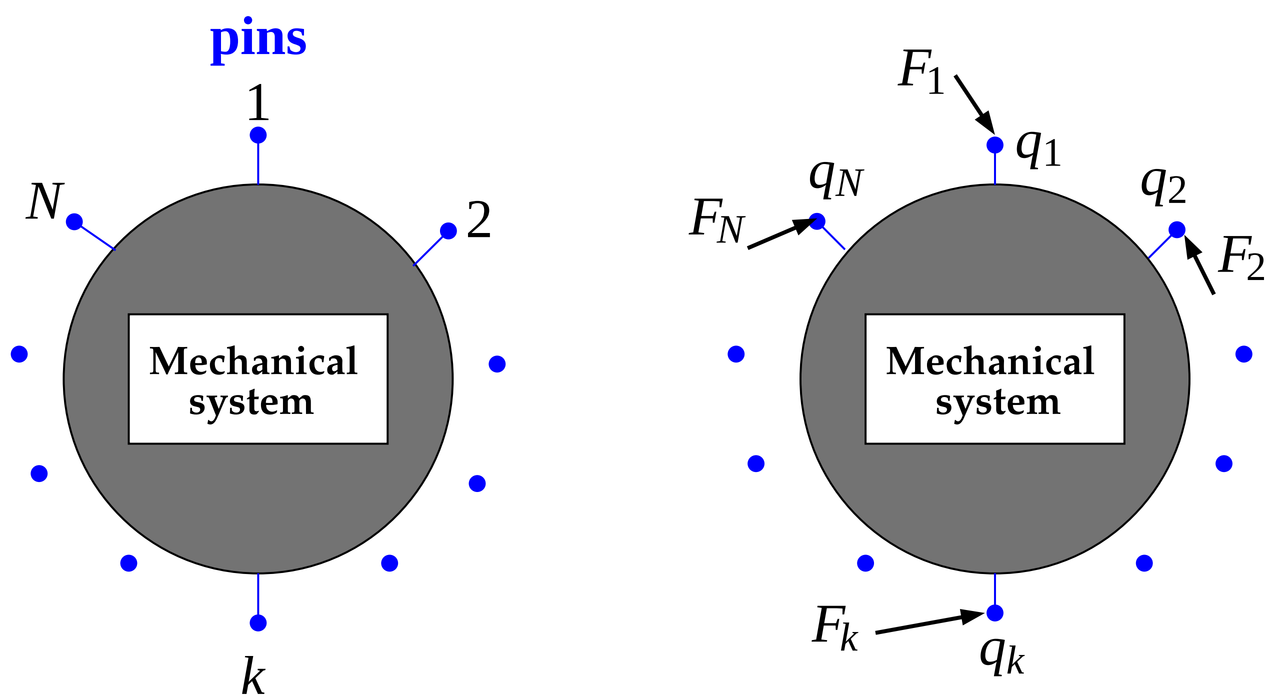
Early sources:

Brockway McMillan



Robert Newcomb

Mechanical device



At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

More generally, a position, force, angle, and torque.

Other domains

- **Thermal systems:**

  At each terminal: a temperature and a heat flow.

- **Hydraulic systems:**

  At each terminal: a pressure and a mass flow.

- **Multidomain systems:**

  Systems with terminals of different types, as motors, pumps, loudspeakers, etc.

- **...**

The BEHAVIORAL APPROACH

The dynamic behavior

Definition: A *dynamical system* $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathscr{B})$, with

- $\mathbb{T} \subseteq \mathbb{R}$ **the time set,**
- $\mathbb{W}$ **the signal space,**
- $\mathscr{B} \subseteq (\mathbb{W})^{\mathbb{T}}$ **the behavior,**
  that is, $\mathscr{B}$ is a family of maps from $\mathbb{T}$ to $\mathbb{W}$.

$w : \mathbb{T} \to \mathbb{W} \in \mathscr{B}$ **means: the model allows the trajectory w,**

$w : \mathbb{T} \to \mathbb{W} \notin \mathscr{B}$ **means: the model forbids the trajectory w.**

Behavioral models

The behavior captures the essence of what a model is.

The behavior is all there is.
Equivalence of models, properties of models,
symmetries, system identification, etc.
must all refer to the behavior.

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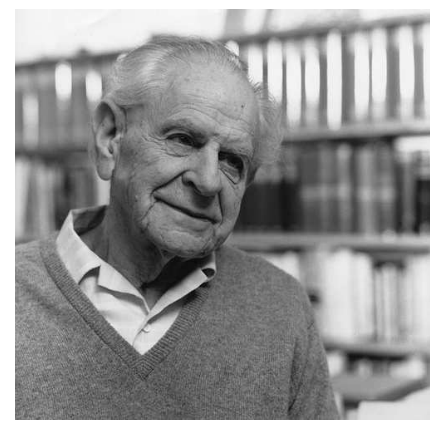
Every 'good' scientific theory is prohibition:
it forbids certain things to happen.
The more it forbids, the better it is.



Karl Popper (1902-1994)

The behavior

There has been an extensive development that deals with

system theory, control, system identification, etc.

from this point of view.

System representations

While $\mathscr{B}$ is the basic object of study, it allows many representations. For LTIDSs, we have

- **kernels of differential operators, images,**
- **transfer fs, i/s/o (A, B, C, D), coprime fact.,**
- **DAEs,**
- **general ODEs with general latent variables,**
- **etc., etc.**

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While $\mathscr{B}$ is the basic object of study, it allows many representations. For LTIDSs, we have

- **kernels of differential operators, images,**
- **transfer fs, i/s/o (A, B, C, D), coprime fact.,**
- **DAEs,**
- **general ODEs with general latent variables,**
- **etc., etc.**

Some representations more convenient than others.

- **Concepts: $\mathscr{B}$ itself.**
- **Math & intuition : kernels, images, tf f's, (A, B, C, D),**
- **first principles models: general ODEs with general latent variables, DEAs,**
- **numerical algorithms: DAEs, (A, B, C, D).**

WHAT NEW DOES THIS BRING?

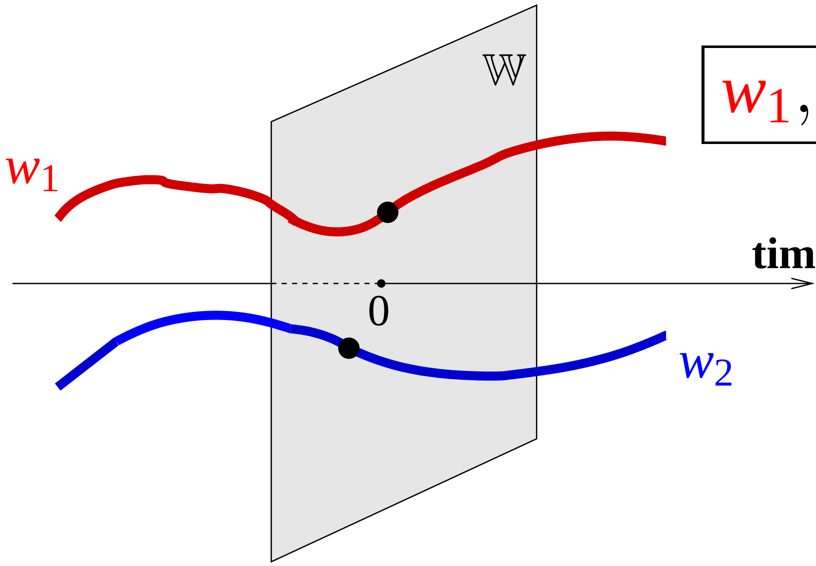
Controllability

The dynamical system $\Sigma = (\mathbb{T}, \mathbb{W}, \mathscr{B})$**, with** $\mathbb{T} = \mathbb{R}$ **or** $\mathbb{Z}$**, is said to be controllable** $:\Leftrightarrow$

for all $w_1, w_2 \in \mathscr{B}$**, there exist** $T \in \mathbb{T}, T \geq 0$**, and** $w \in \mathscr{B}$**, such that**

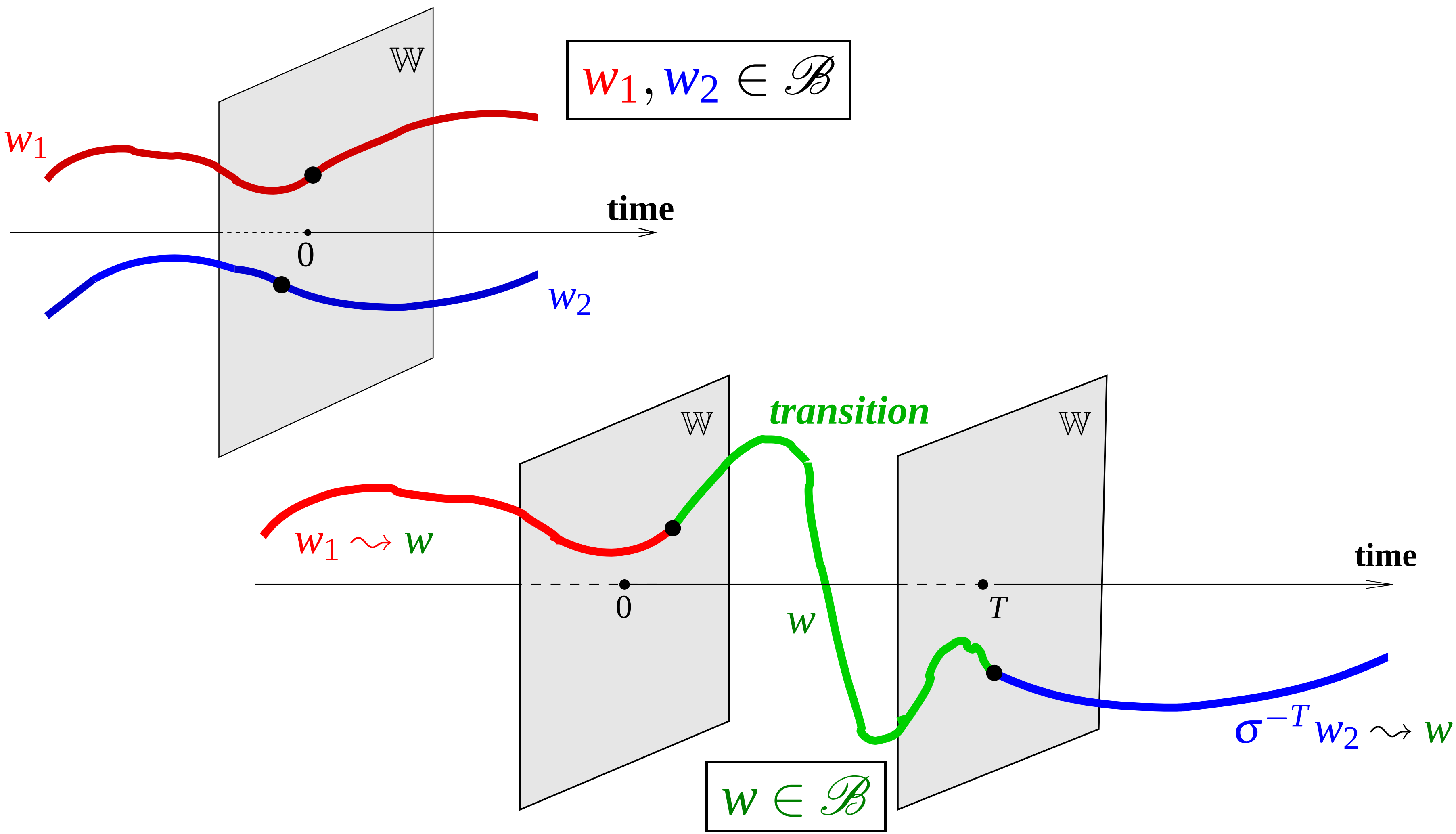
$$w(t) = \begin{cases} w_1(t) & \textbf{for } t < 0, \\ w_2(t-T) & \textbf{for } t \geq T. \end{cases}$$

Controllability in a picture



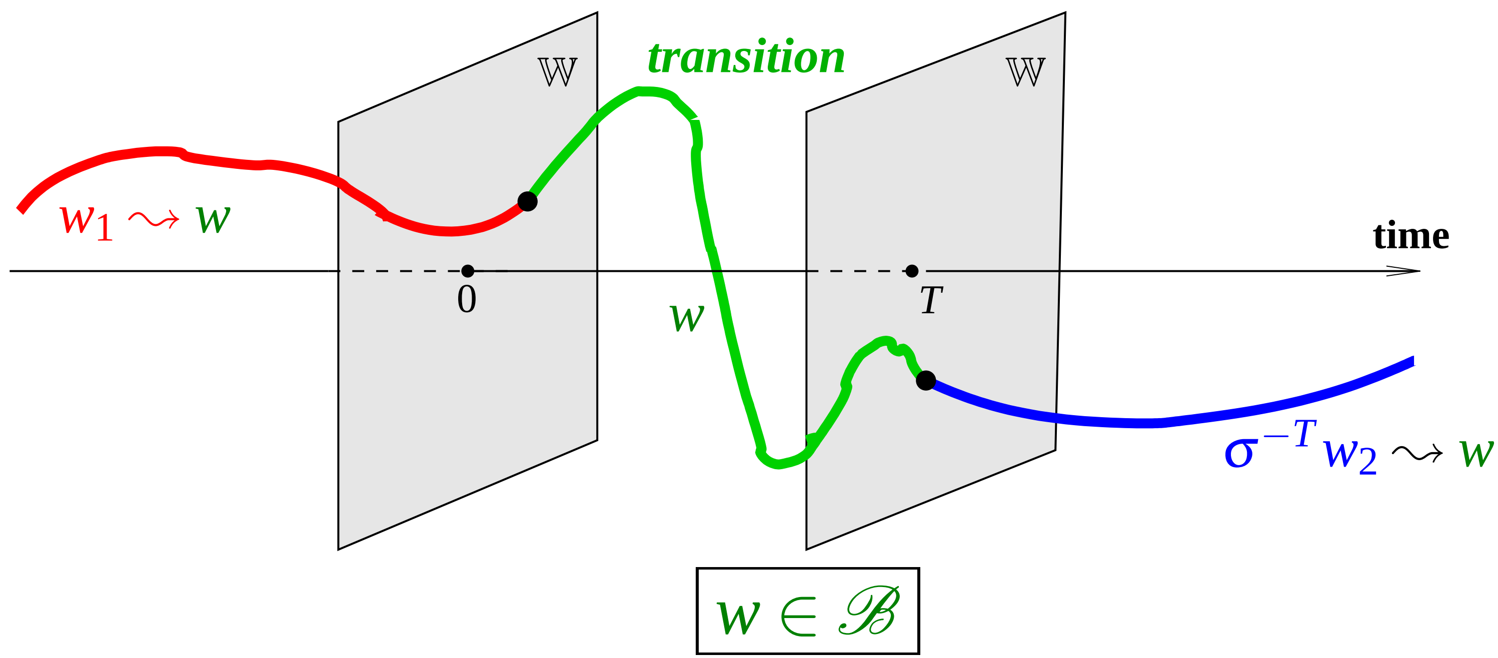
$w_1, w_2 \in \mathscr{B}$

Controllability in a picture



controllability :$\Leftrightarrow$ concatenability of trajectories after a delay

Controllability in a picture

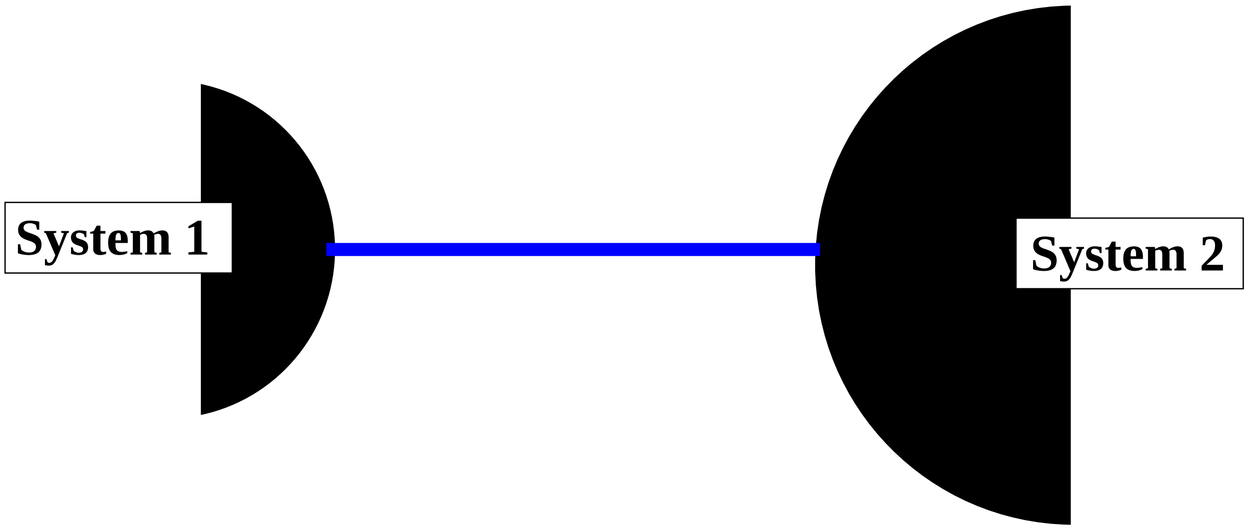


controllability :⇔ concatenability of trajectories after a delay

Makes controllability into a genuine, an intrinsic property of a system, rather than merely of a state representation.

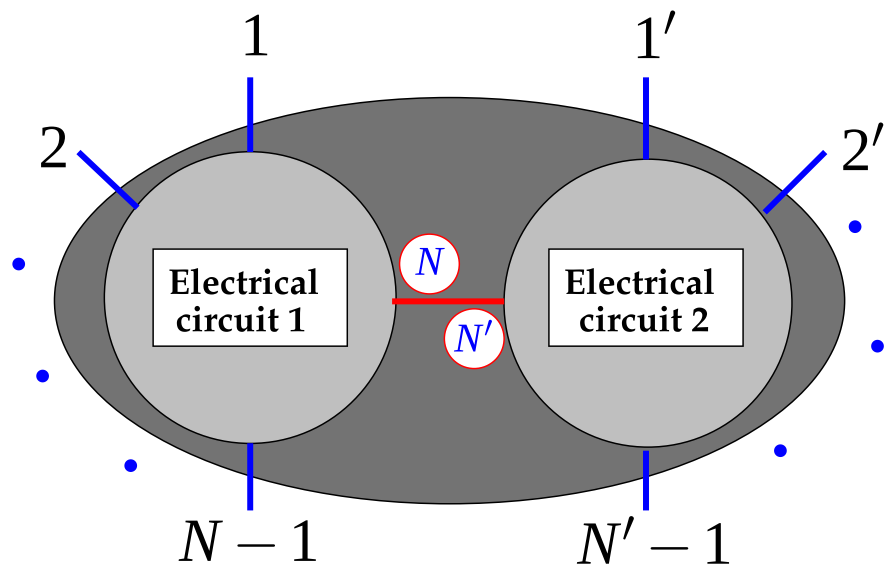
INTERCONNECTION

Connection of terminals



By interconnecting, the terminal variables are equated.

Electrical interconnection



$$I_N + I_{N'} = 0 \quad \textbf{and} \quad P_N = P_{N'}.$$

Behavior after interconnection:

$$\mathscr{B}_1 \sqcap \mathscr{B}_2$$

$$:= \{(I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}, P_1, \ldots, P_{N-1}, P_{1'}, \ldots, P_{N'-1}) \mid$$

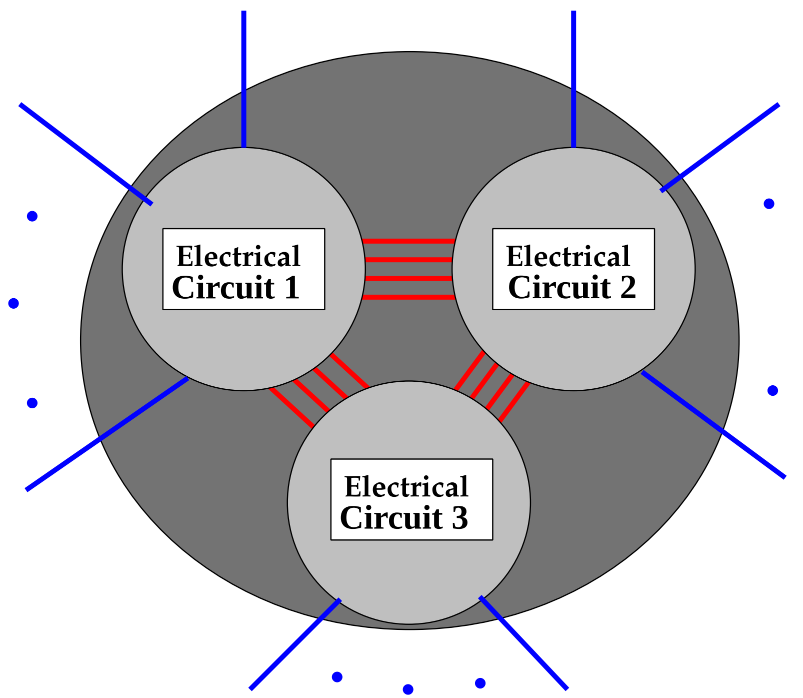
$$\exists\, I,\ P \ \textbf{such that}$$

$$(I_1, \ldots, I_{N-1}, I, P_1, \ldots, P_{N-1}, P) \in \mathscr{B}_1 \ \textbf{and}$$

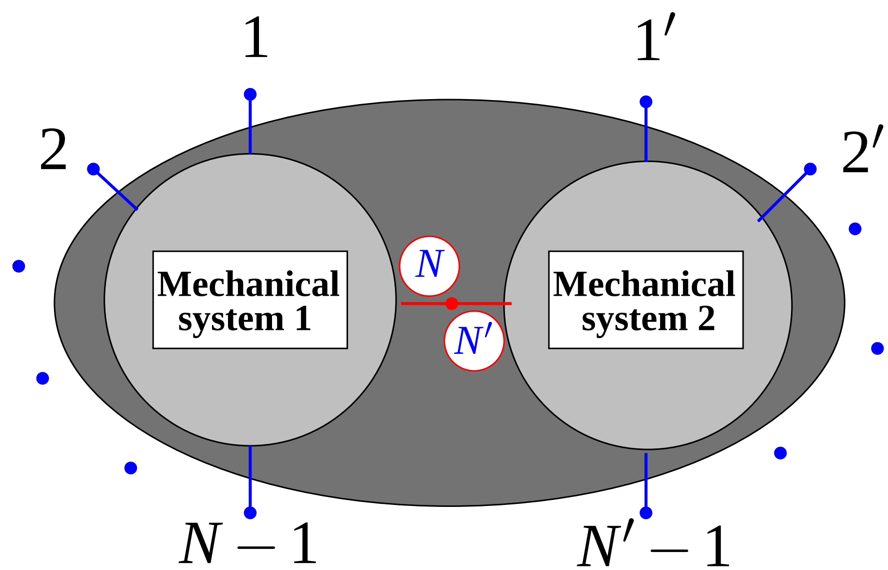
$$(I_{1'}, \ldots, I_{N'-1}, -I, P_{1'}, \ldots, P_{N'-1}, P) \in \mathscr{B}_2\}.$$

Electrical interconnection

⇝ **more terminals and more circuits connected**



Interconnection of 1-D mechanical systems



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Other terminal types

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **...**

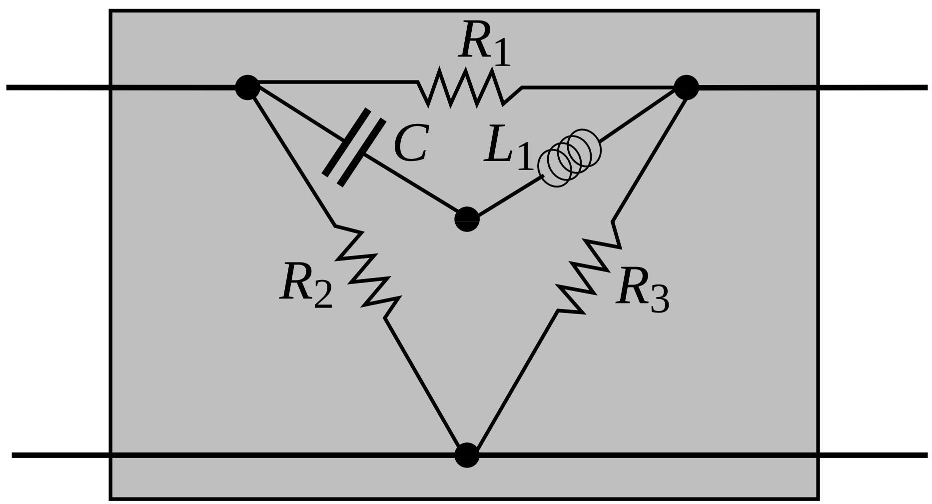
Sharing variables

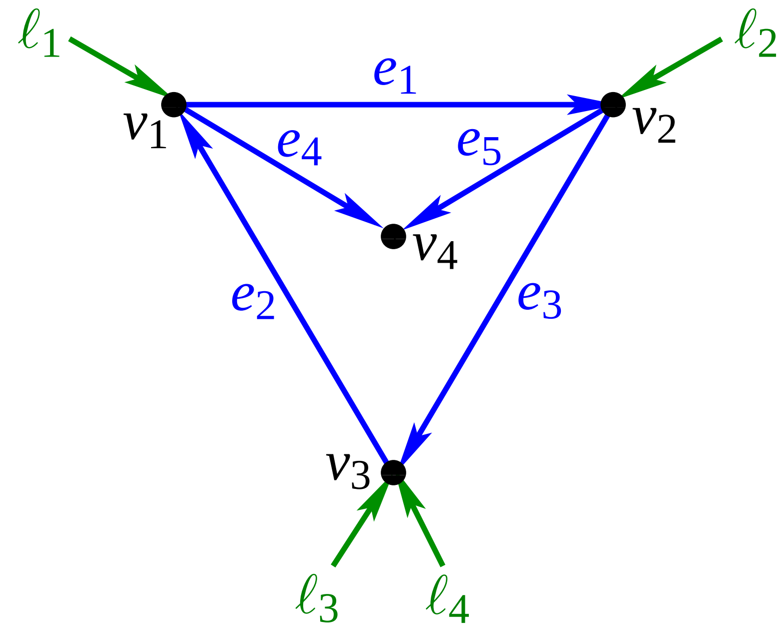
$$
\begin{aligned}
V_N = V_{N'} \quad &\textbf{and} \quad I_N + I_{N'} = 0, \\
q_N = q_{N'} \quad &\textbf{and} \quad F_N + F_{N'} = 0, \\
T_N = T_{N'} \quad &\textbf{and} \quad Q_N + Q_{N'} = 0, \\
p_N = p_{N'} \quad &\textbf{and} \quad f_N + f_{N'} = 0, \\
&\vdots
\end{aligned}
$$

Interconnection means variable sharing.

RLC circuits

Circuit architecture





<u>Circuit architecture</u> :=

digraph with leaves $\cong (\mathbb{A}_{\mathbb{E}}, \mathbb{A}_{\mathbb{L}})$

Element specification

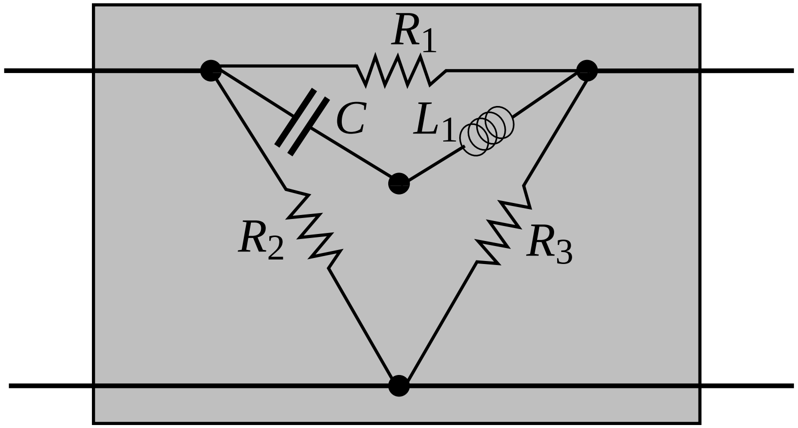
The elements of the circuit (the R's, L's, and C's) correspond to the edges.

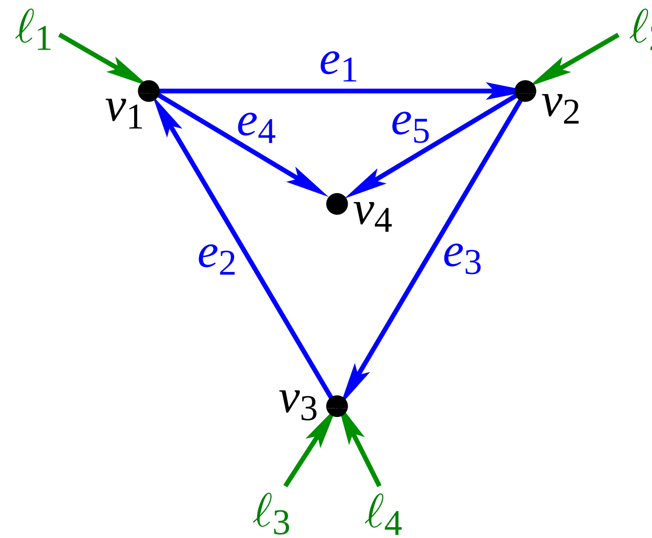
$\rightsquigarrow$ **a map that associates with each edge a resistance, an inductance, or a capacitance of a given value.**

$\rightsquigarrow$

3 $|\mathbb{E}| \times |\mathbb{E}|$ **diagonal polynomial matrices** $R, L\xi, C\xi$

Element specification





$$R + L\xi = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & L_1\xi \end{bmatrix}, \quad C\xi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & C\xi & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Circuit equations

Manifest variables:

the leaf currents I and the leaf potentials P.

Latent variables:

the edge currents $I_{\mathbb{E}}$ and the vertex potentials $P_{\mathbb{V}}$.

$$I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{|\mathbb{L}|} \end{bmatrix}, \quad P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{|\mathbb{L}|} \end{bmatrix}, \quad I_{\mathbb{E}} = \begin{bmatrix} I_{e_1} \\ I_{e_2} \\ \vdots \\ I_{e_{|\mathbb{E}|}} \end{bmatrix}, \quad P_{\mathbb{V}} = \begin{bmatrix} P_{v_1} \\ P_{v_2} \\ \vdots \\ P_{v_{|\mathbb{V}|}} \end{bmatrix}.$$

Circuit equations

Edges $\rightsquigarrow$ **constitutive equations:**

$$(R + L\frac{d}{dt})I_{\mathbb{E}} = C\frac{d}{dt}A_{\mathbb{E}}^{\top}P_{\mathbb{V}}$$

Vertices $\rightsquigarrow$ **KCL:**

$$A_{\mathbb{E}}I_{\mathbb{E}} + A_{\mathbb{L}}I = 0$$

Leaves $\rightsquigarrow$ **potential assignment:**

$$P + A_{\mathbb{L}}^{\top}P_{\mathbb{V}} = 0$$

Circuit properties

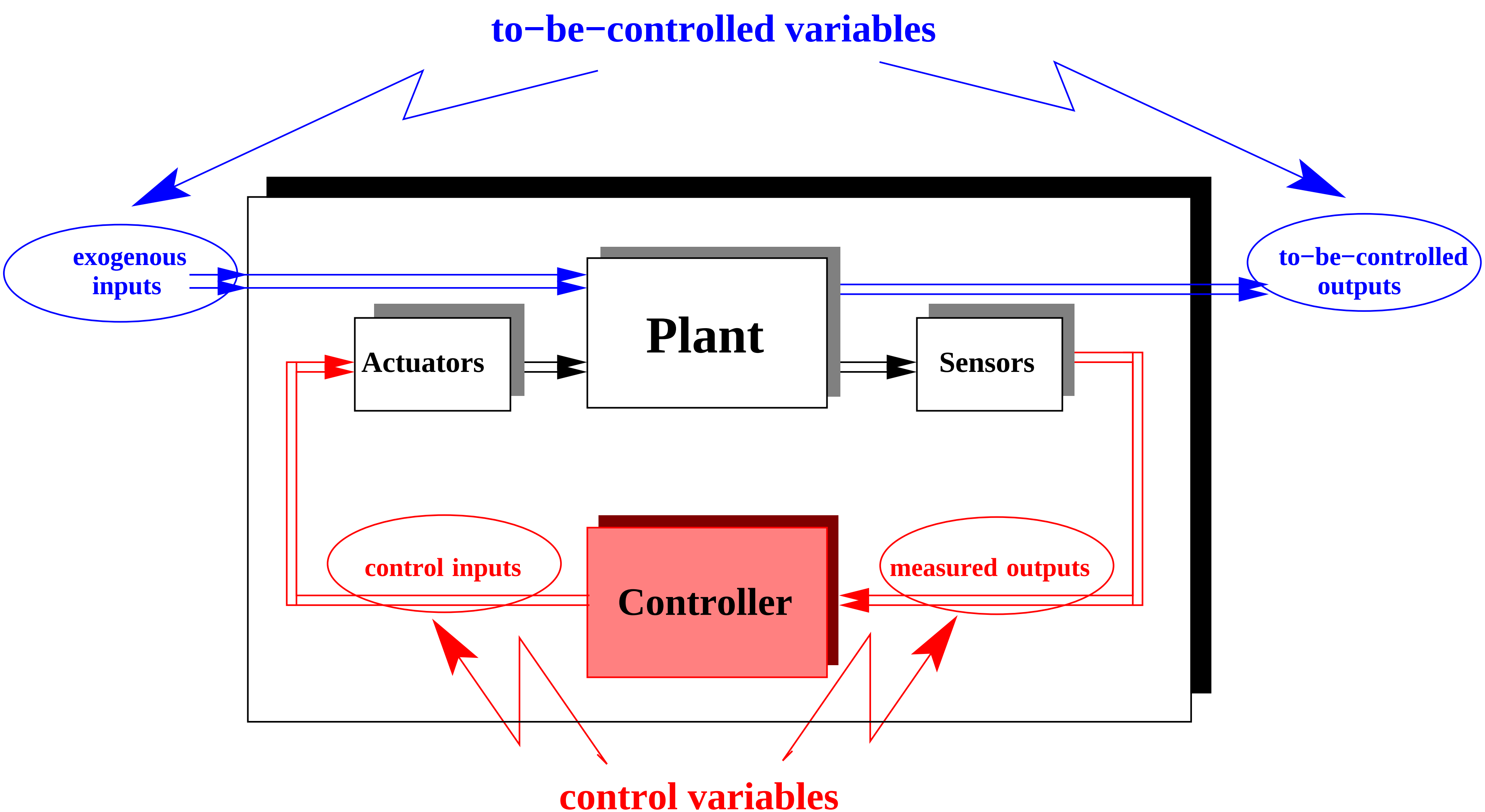
- **Elimination of $I_{\mathbb{E}}$ and $P_{\mathbb{V}}$** $\Rightarrow$

$$F\left(\frac{d}{dt}\right)\begin{bmatrix} I \\ P \end{bmatrix} = 0, \quad F \in \mathbb{R}[\xi]^{N \times 2N}.$$

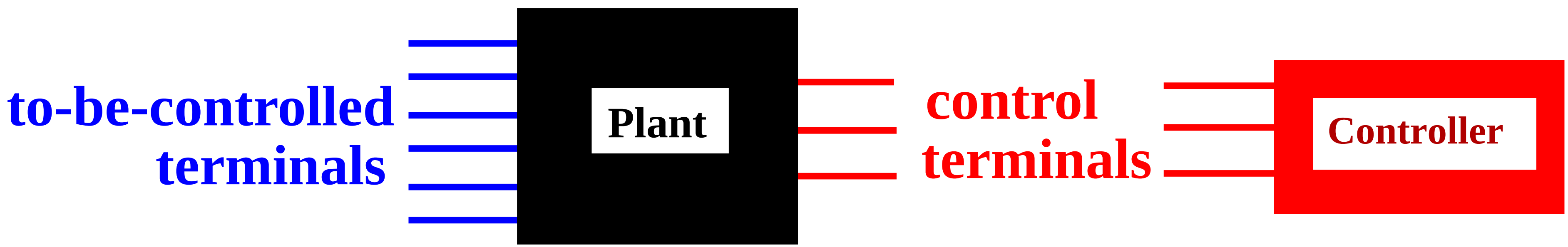
- **KCL and KVL**
- **Passivity**
- **Hybridicity**
- **Reciprocity**
- **etc.**

CONTROL as INTERCONNECTION

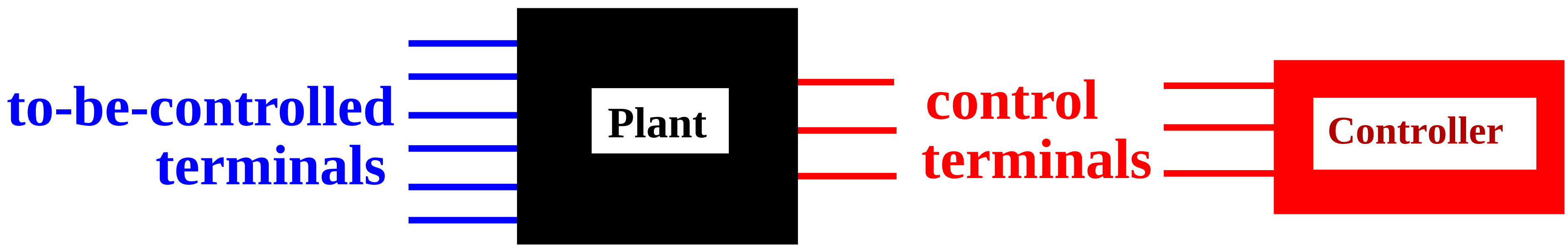
Feedback control



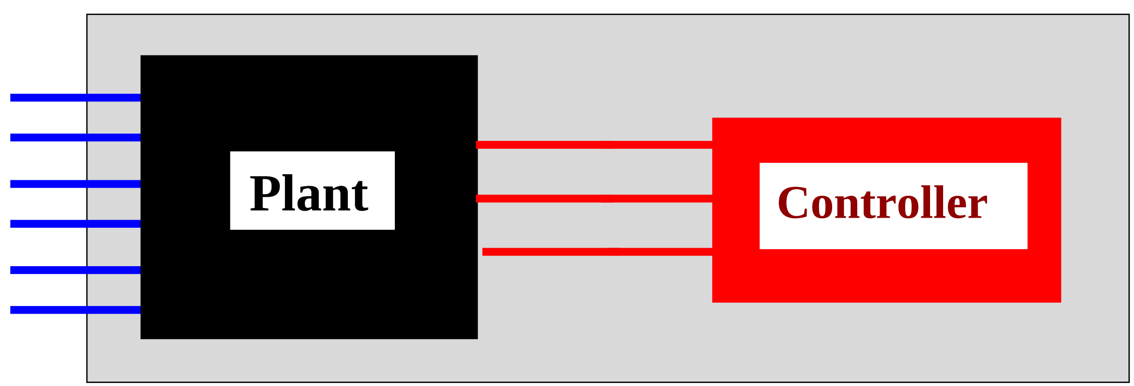
Behavioral control



Behavioral control

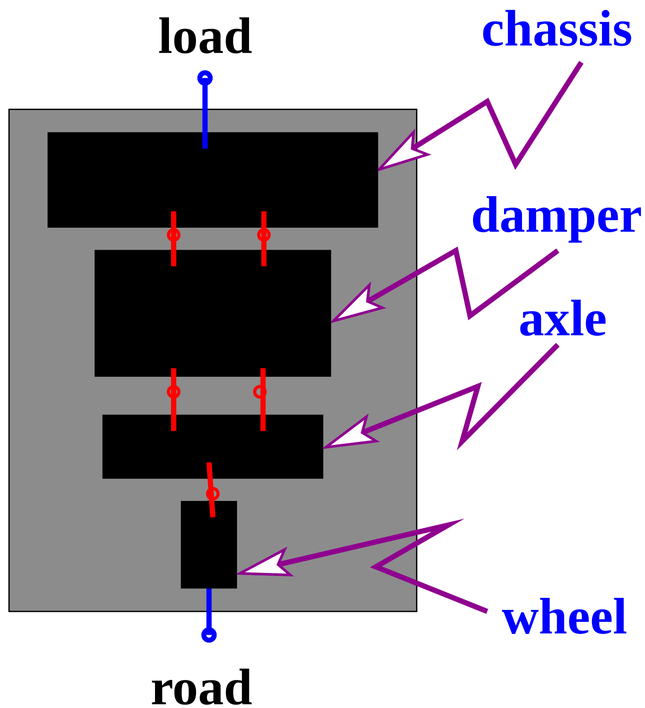


control = interconnection.

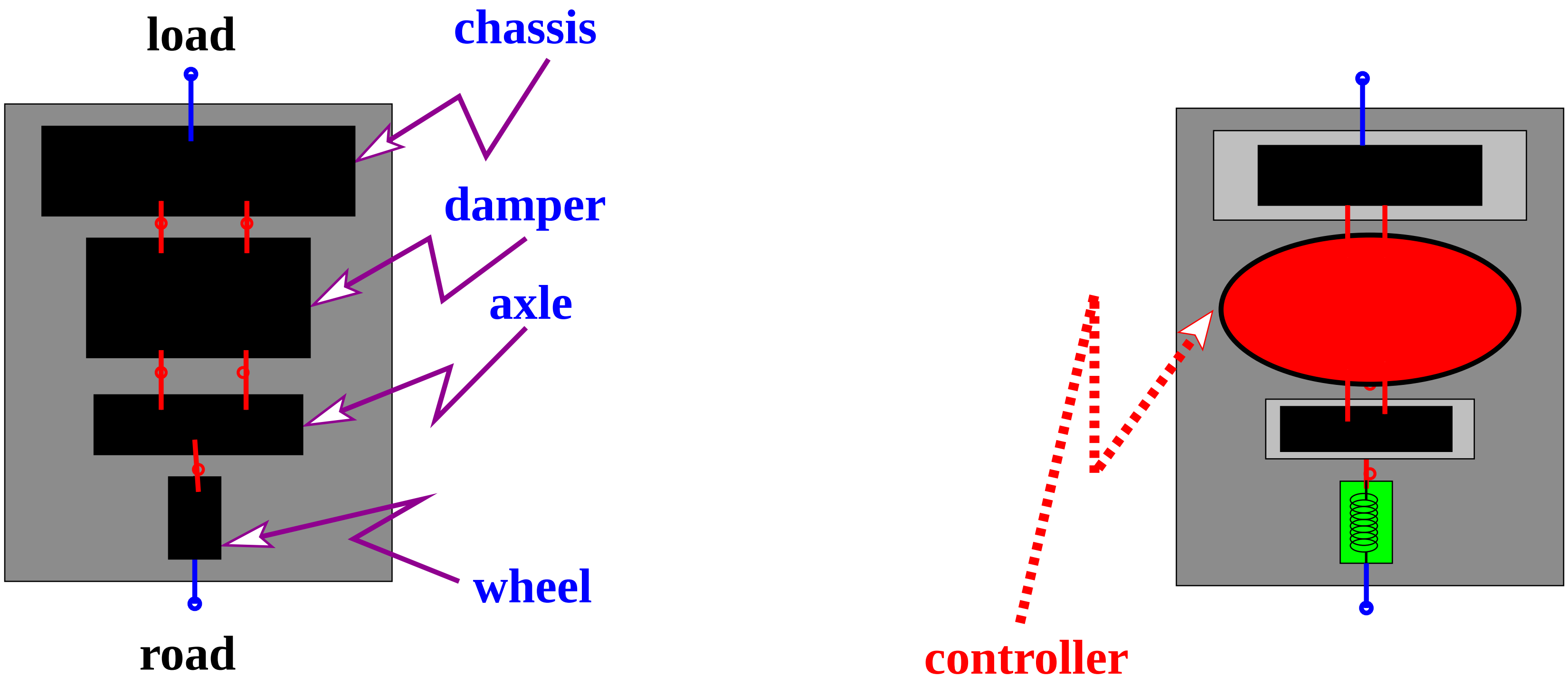


controlled system

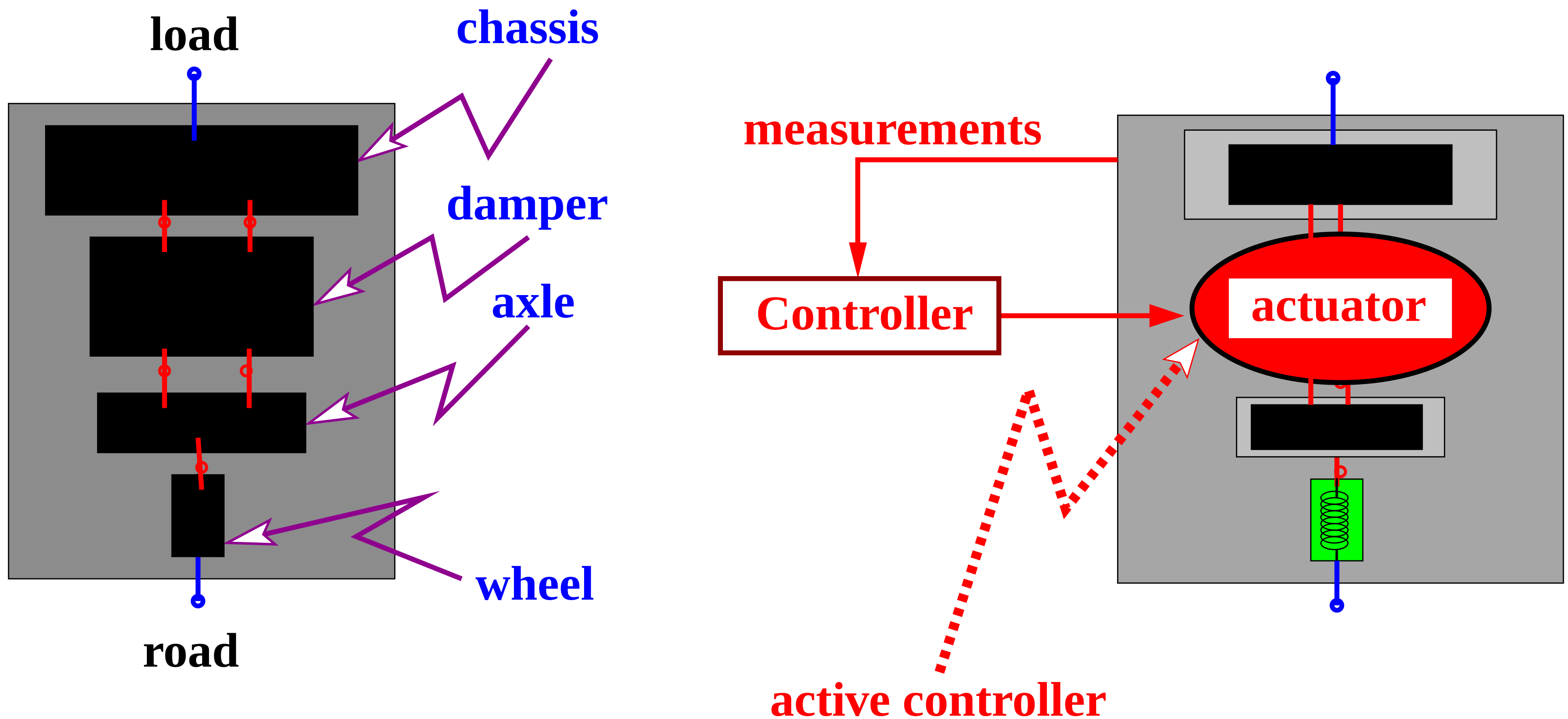
Example of behavioral control: A ‘quarter car’



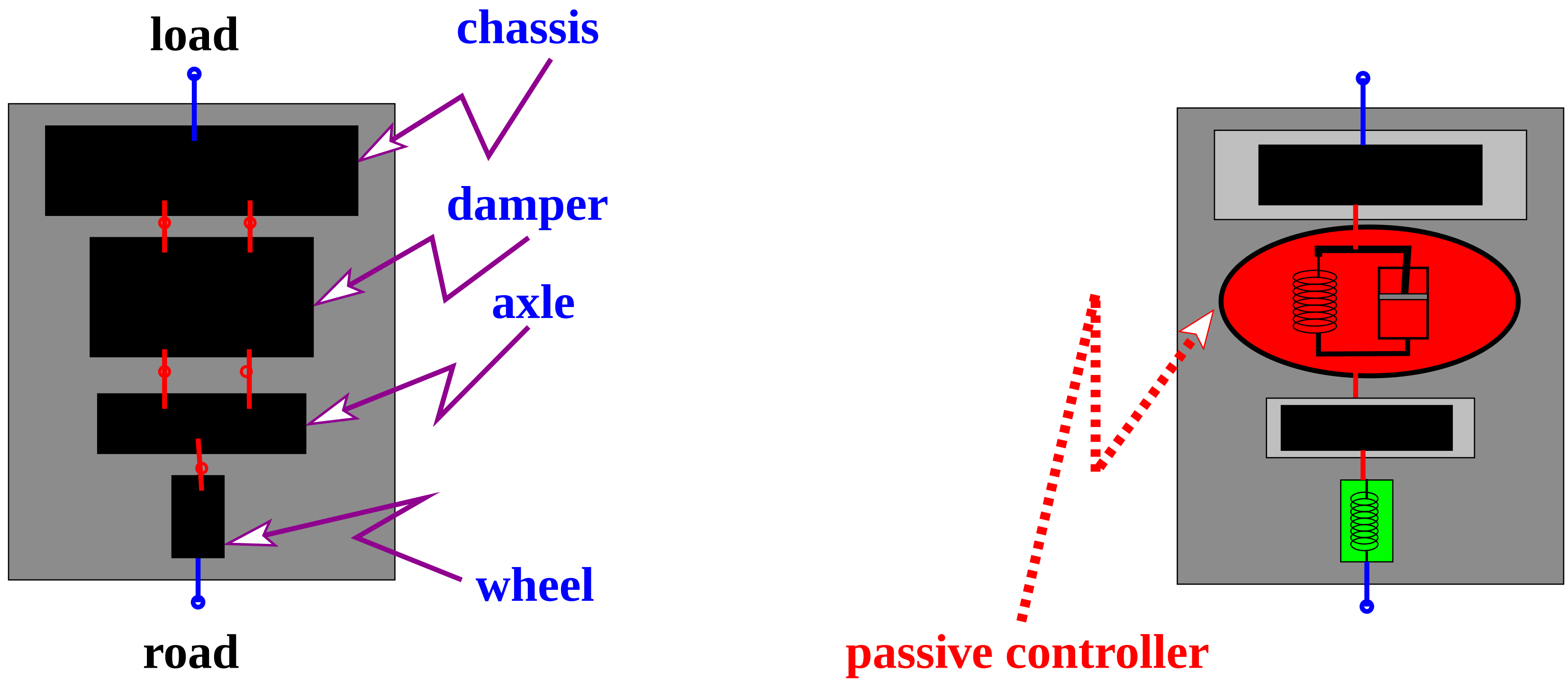
Example of behavioral control: A ‘quarter car’



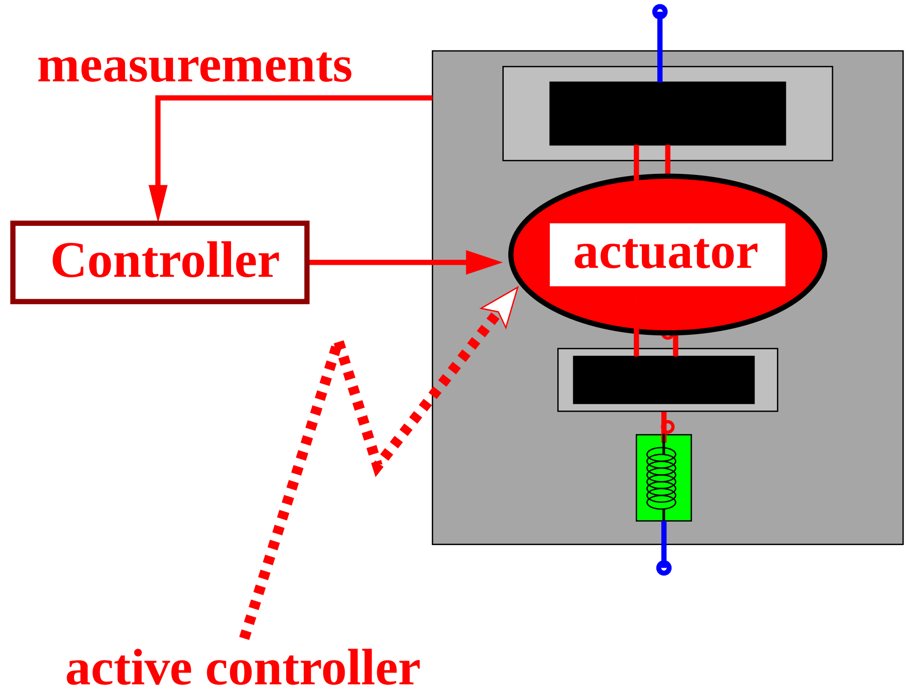
Example of behavioral control: A 'quarter car'



Example of behavioral control: A ‘quarter car’



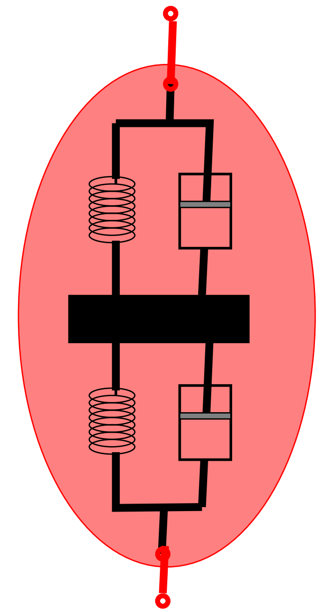
Suspension control in Formula 1



Nigel Mansell victorious in 1992
with an active damper suspension.

Active dampers were banned in 1994
to break the dominance of the Williams team.

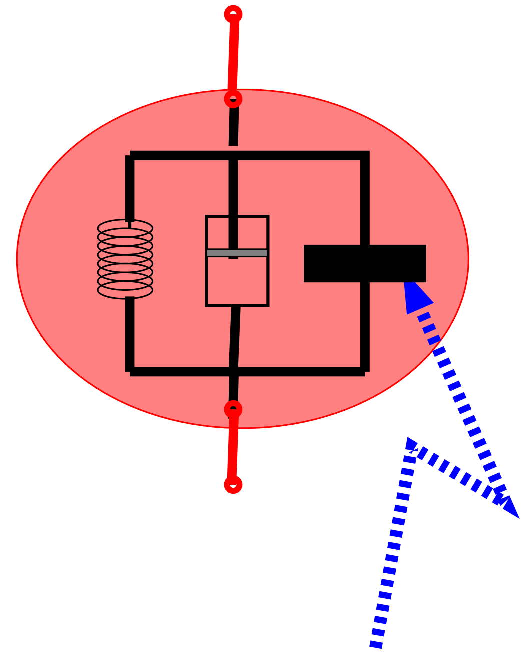
Suspension control in Formula 1



Renault successfully used a passive 'tuned mass damper' suspension in 2005/2006.

Tuned mass dampers were banned in 2006, under the 'movable aerodynamic devices' clause.

Suspension control in Formula 1

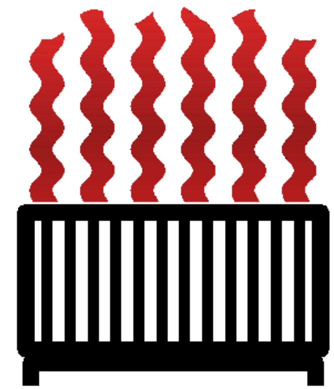


Kimi Räikkönen wins the 2005 Grand Prix in Spain with McLaren's 'J-damper', i.e., an inerter.

ENERGY

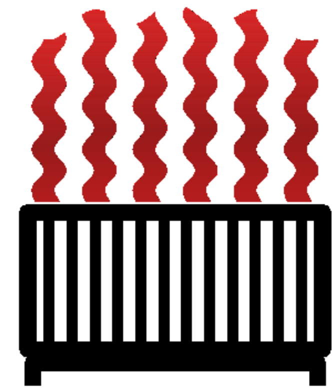
Energy

Energy **:= a physical quantity transformable into heat.**



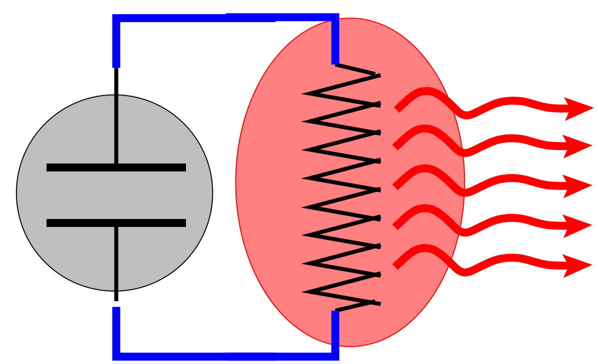
Energy

Energy **:= a physical quantity transformable into heat.**



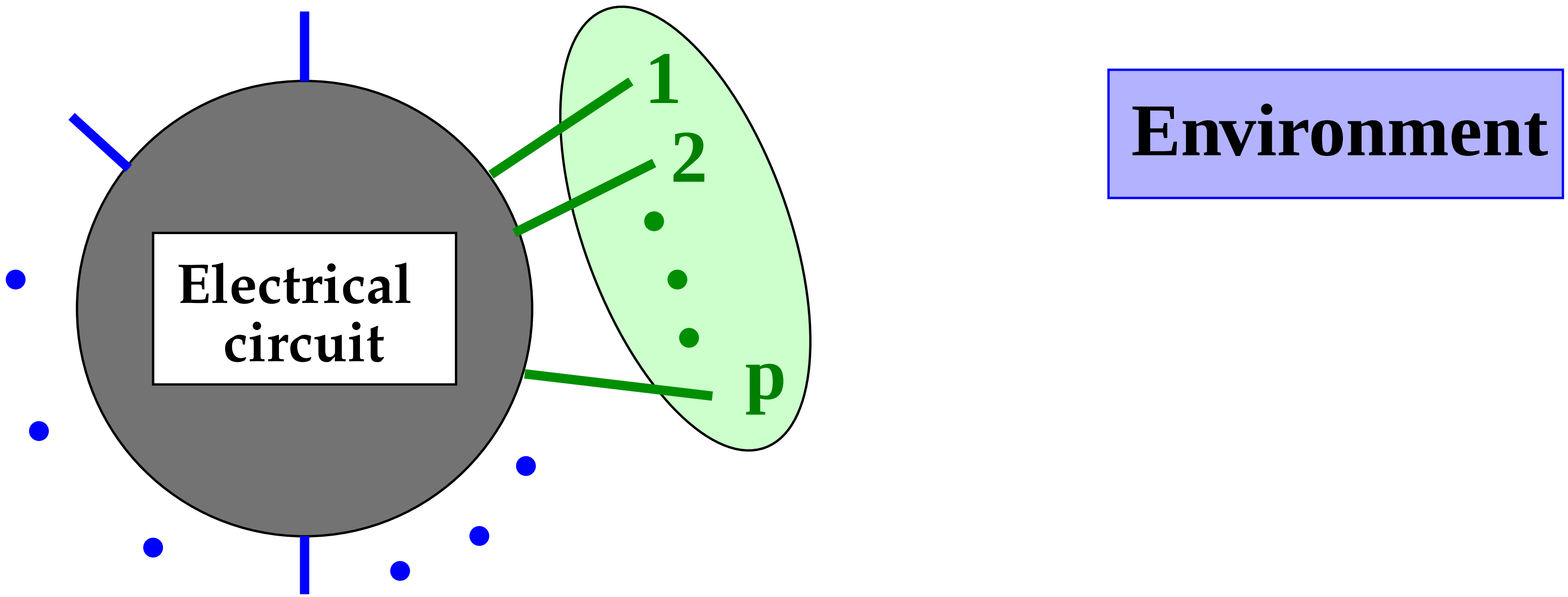
For example capacitor → resistor → heat.

Energy on capacitor = $\frac{1}{2}CV^2$



PORTS

Energy transfer

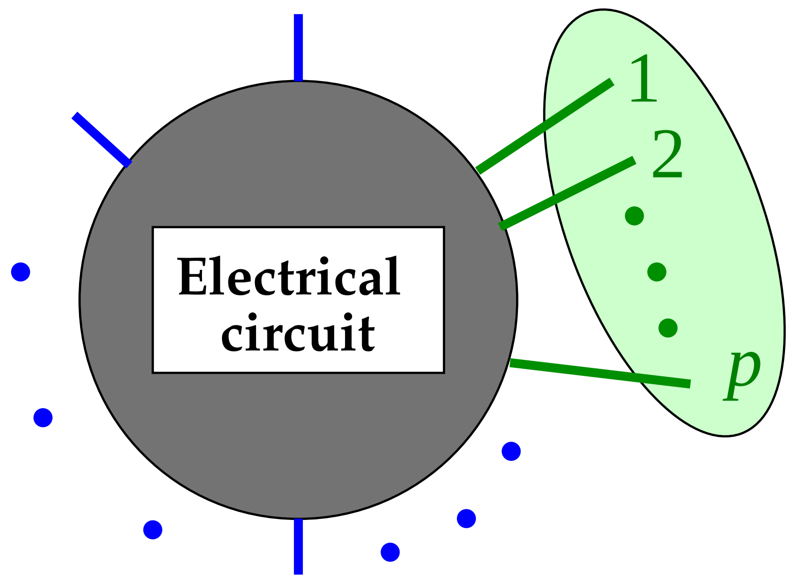


Can we speak about

the energy transferred from the environment to the circuit along these terminals?

Electrical ports

Assume KVL.



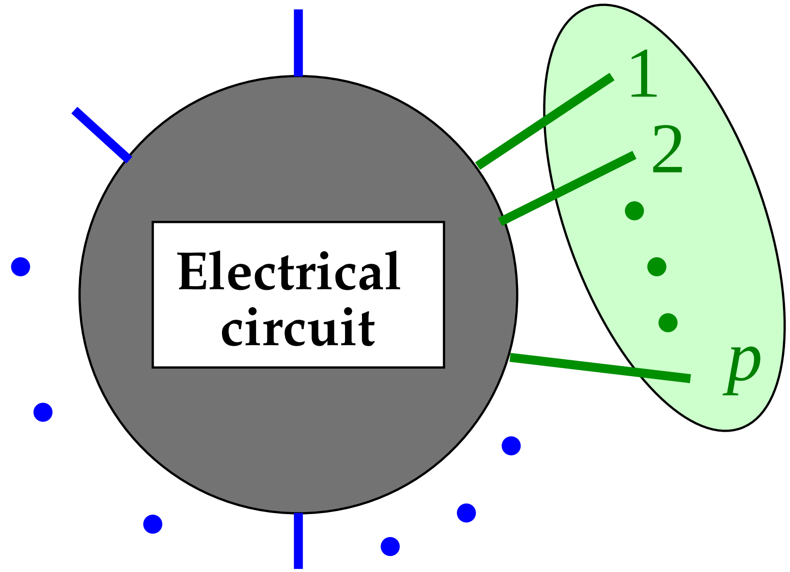
Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

$$[\![\left(I_1, \ldots, I_p, I_{p+1}, \ldots, I_N, V_{1,1}, \ldots, V_{k_1,k_2}, \ldots, V_{N,N}\right) \in \mathscr{B}_{IV}]\!]$$

$$\Rightarrow \quad [\![I_1 + I_2 + \cdots + I_p = 0]\!].$$

'port KCL'

Energy



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

power in = $I_1(t)P_1(t) + \cdots + I_p(t)P_p(t)$

energy in = $\int_{t_1}^{t_2} [I_1(t)P_1(t) + \cdots + I_p(t)P_p(t)]\, dt$

This interpretation in terms of power and energy is not valid
unless these terminals form a port !

Internal ports

Analogous definition for internal terminals

$\rightsquigarrow$ **internal ports,**

combinations of external and internal terminals

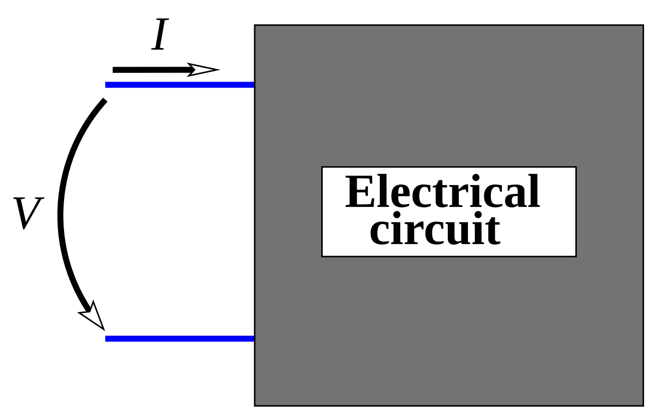
$\rightsquigarrow$ **mixed ports.**

Examples

2-terminal 1-port devices:

resistors, inductors, capacitors, memristors, etc.,

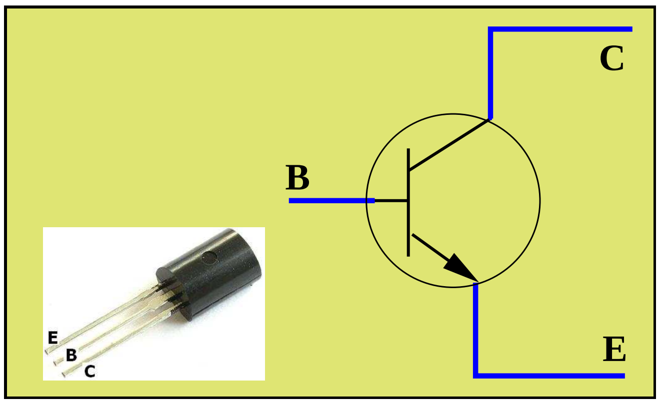
any 2-terminal circuit composed of these.

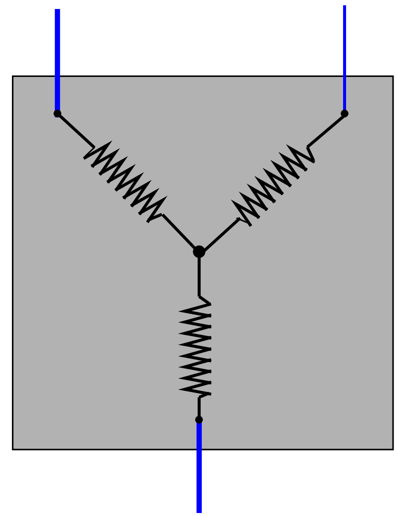


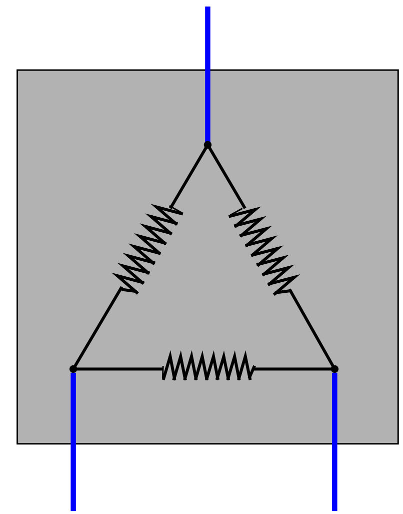
KVL $\Rightarrow$ **only** $V_{1,2} := V$ **matters,**

KCL $\Rightarrow I_1 = -I_2 =: I$**.**

3-terminal circuits



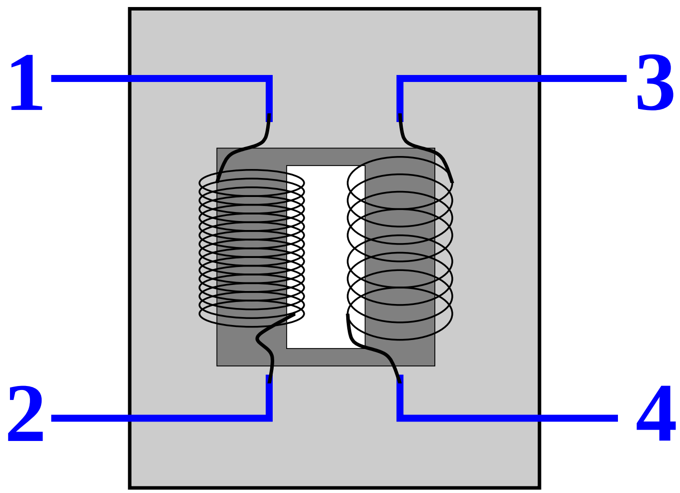




3-terminal 1-ports.

Transformer

A transformer:



$$P_3 - P_4 = n\,(P_1 - P_2),$$
$$I_1 = -n I_3,$$
$$I_1 + I_2 = 0,\ I_3 + I_4 = 0.$$

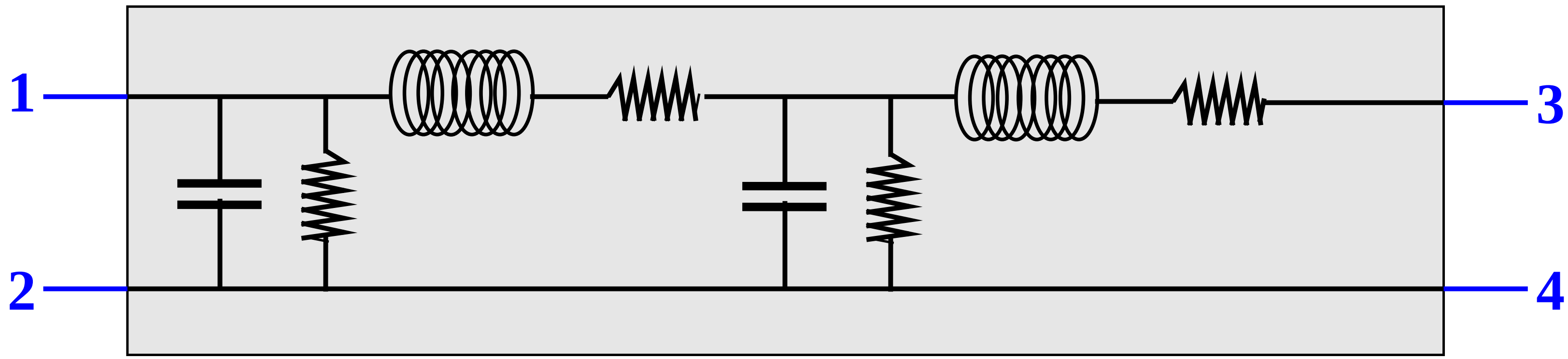
$\{1,2\}$ **and** $\{3,4\}$ **form ports.**

A transformer = a 2-port with two 2-terminal ports.

Interconnected circuits

The set of external terminals of a circuit composed of elements that individually satisfy KCL satisfies KCL and is therefore a port.

Transmission line



Terminals $\{1, 2, 3, 4\}$ **form a port;**

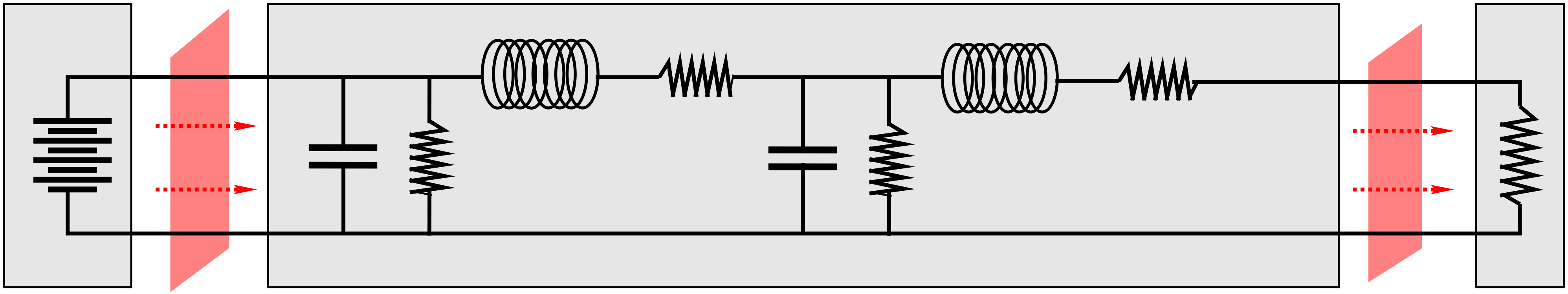
$\{1, 2\}$ **and** $\{3, 4\}$ **do not.**

We cannot speak about

"the energy transferred from terminals $\{1, 2\}$ ***to*** $\{3, 4\}$***"***,

or ***"from the environment to the circuit through*** $\{1, 2\}$***"***.

Transmission line

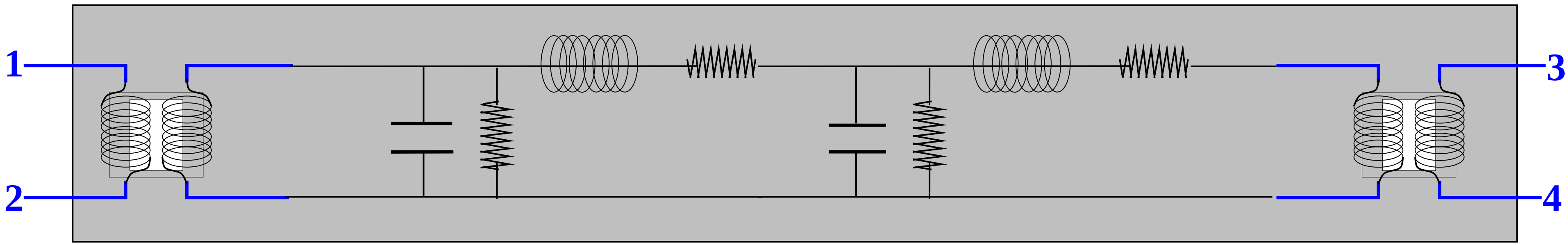


The energy flows from the source and to the load are well-defined, since the terminals form internal ports.

Therefore we can speak about

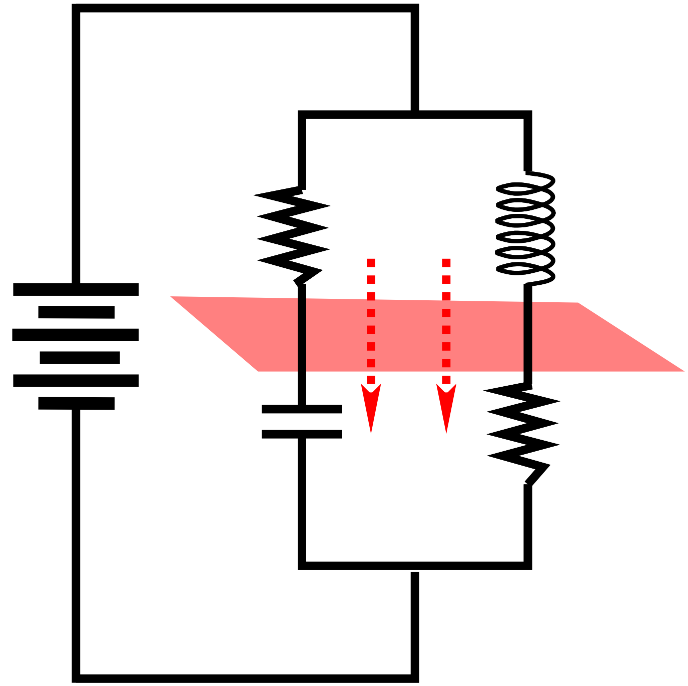
"the energy transferred from the source to the load".

Transmission line



Terminals $\{1,2\}$ and $\{3,4\}$ now form a port.

RLC circuit



Not an internal port: energy flow not well-defined.

Are ports common?

<u>Theorem</u>: Consider an electrical circuit consisting of an interconnection of (linear passive) R's, L's, C's.

Assume that every pair of terminals is connected by the circuit graph. Then

the only port is the one that consists of all the terminals.

Are ports common?

Theorem: Consider an electrical circuit consisting of an interconnection of (linear passive) R's, L's, C's.

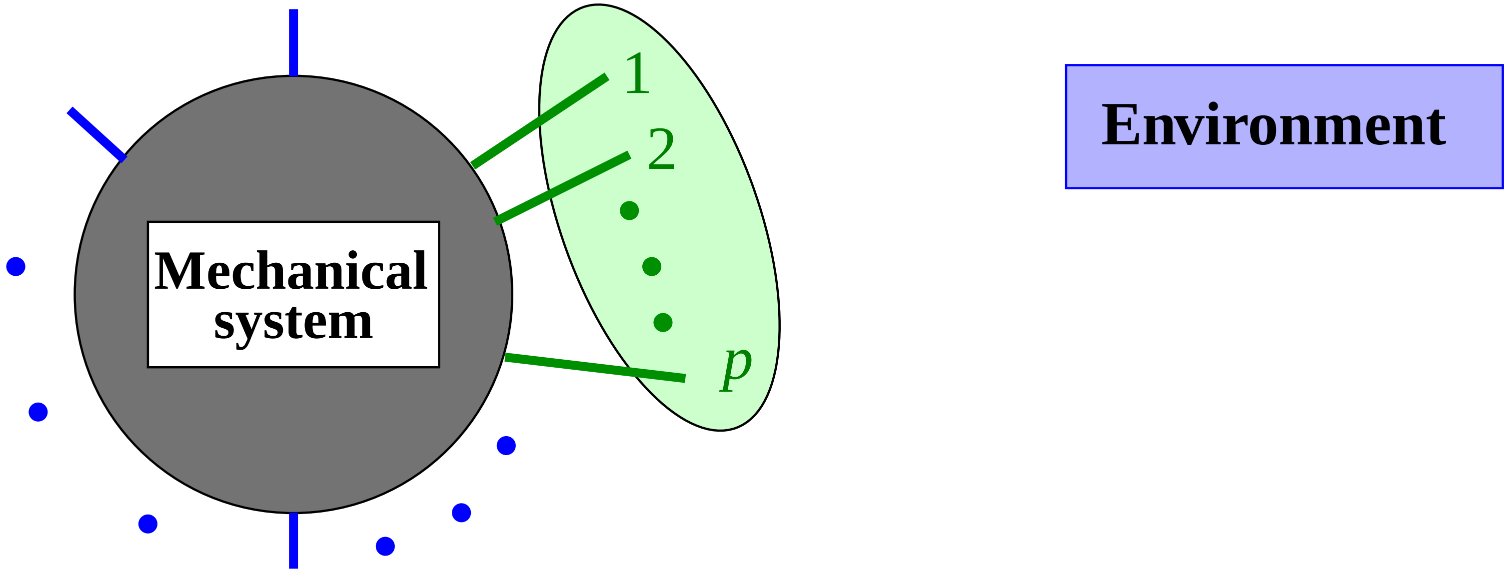
Assume that every pair of terminals is connected by the circuit graph. Then

the only port is the one that consists of all the terminals.

For non-trivial ports, we need multi-port elements, as transformers or gyrators.

MECHANICAL PORTS

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \quad F_1 + F_2 + \cdots + F_p = 0.$$ ***'port KFL'***

Power and energy

If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

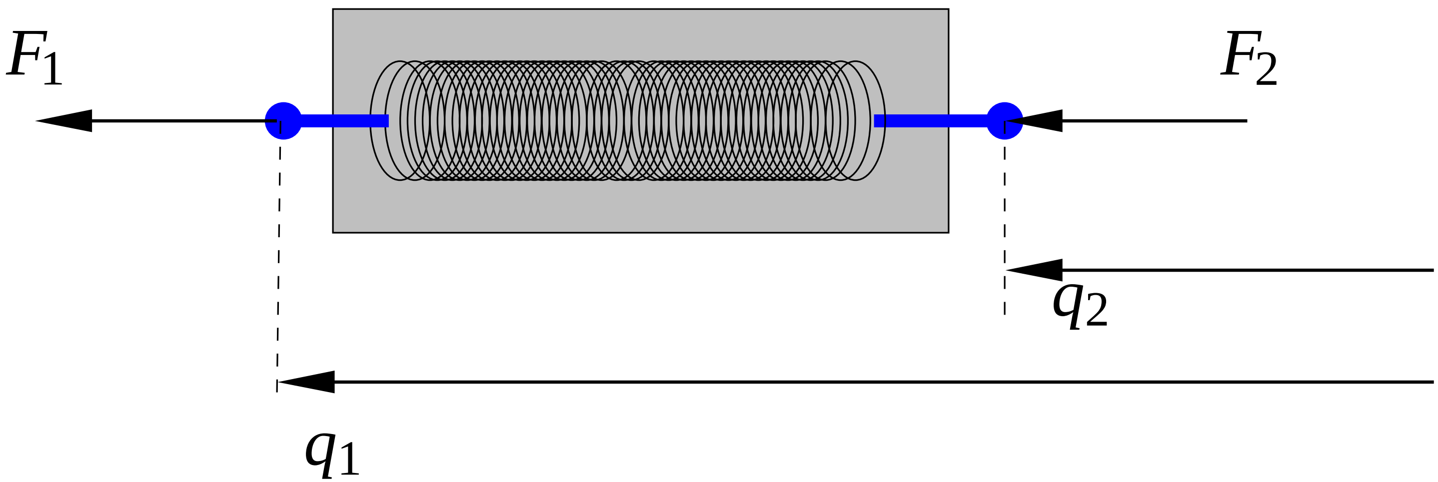
$$\textbf{power in} \quad = F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

$$\textbf{energy in} \quad = \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

Example

Spring

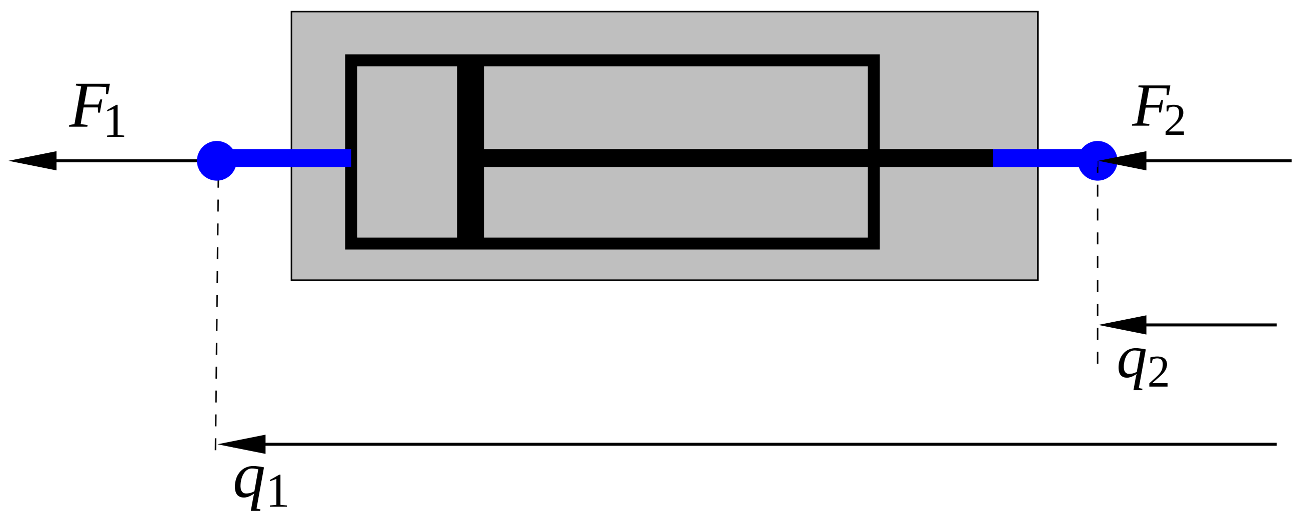


$$F_1 + F_2 = 0, \quad K(q_1 - q_2) = F_1$$

satisfies KFL

Example

Damper

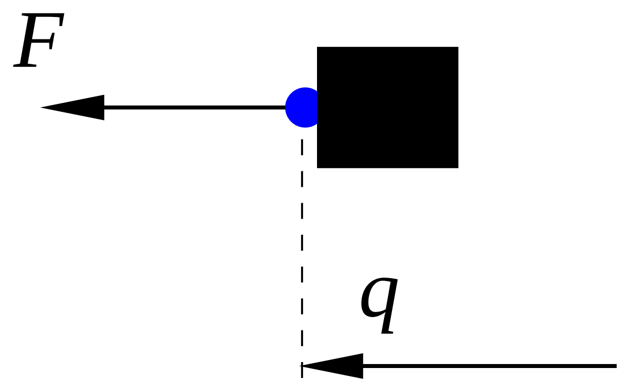


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1$$

satisfies KFL

Springs and dampers, and their interconnection form ports.

Example



$$M\frac{d^2}{dt^2}q = F$$

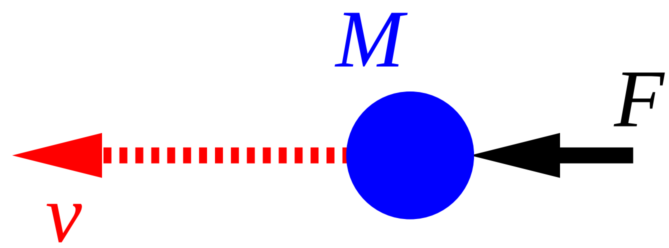
does <u>not</u> satisfy KFL

Not a port!!!

Interconnections of springs, dampers, and masses do not necessarily form ports.

MOTION ENERGY

Conservation law

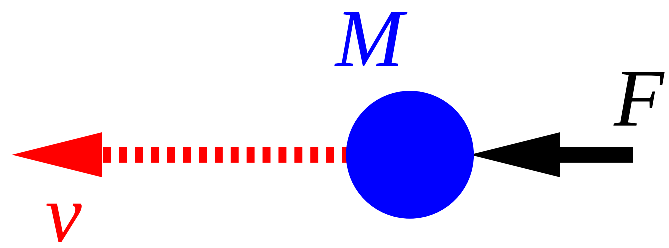


$$M \frac{d^2}{dt^2} q = F \quad \Rightarrow \quad \frac{d}{dt} \frac{1}{2} M \left|\left| \frac{d}{dt} q \right|\right|^2 = F^\top \frac{d}{dt} q$$

If $F^\top v$ is not power,

is $\frac{1}{2} M ||v||^2$ not stored (kinetic, motion) energy ???

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

$$\mathcal{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem 's Gravesande
1688–1742

Émilie du Châtelet
1706–1749

This expression is not invariant under uniform motion.

Motion energy

What is the motion energy?

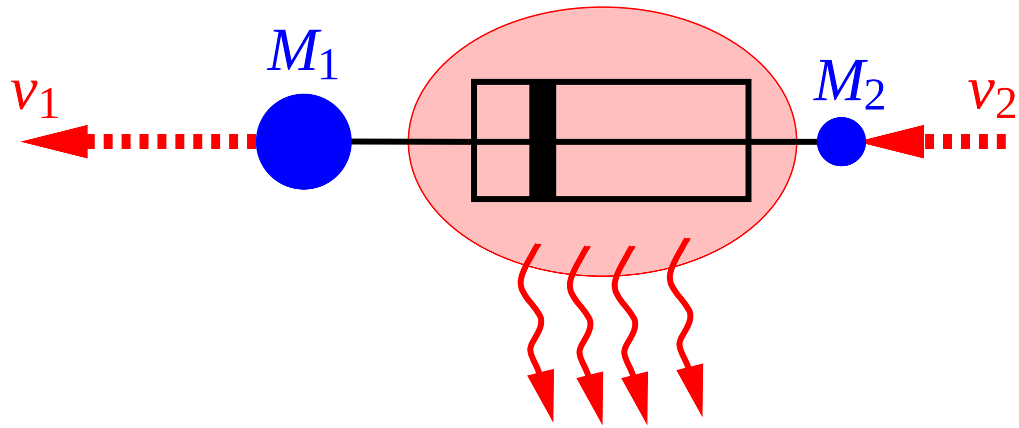
What quantity is transformable into heat?

$$\mathscr{E}_{\textbf{motion}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1 - v_2||^2$$

Invariant under uniform motion.

Dissipation into heat

Can be justified by mounting a damper between the masses.

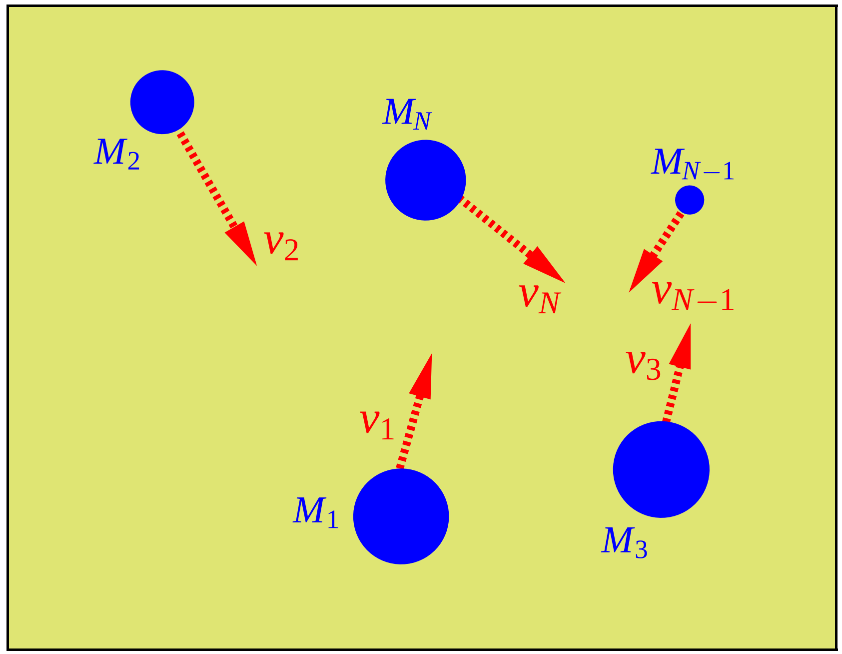


$$\frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1-v_2||^2$$

is the heat dissipated in the damper.

Motion energy

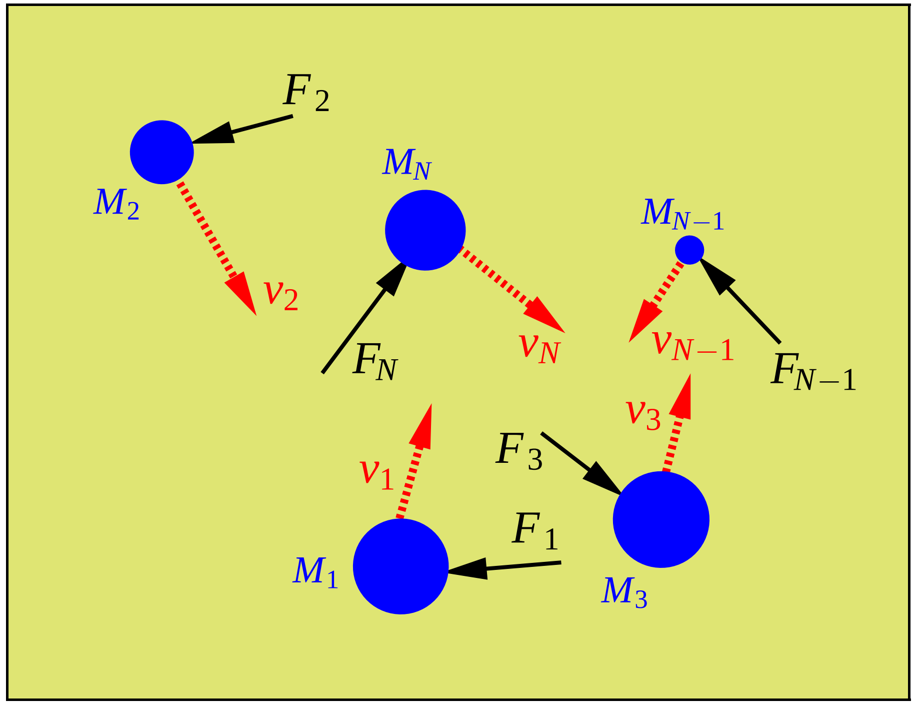
Generalization to N masses.



$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} \|v_i - v_j\|^2.$$

Motion energy

With external forces.



$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

(KFL) $\sum_{i \in \{1,2,\ldots,N\}} F_i = 0 \;\Rightarrow\; \boxed{\frac{d}{dt} \mathscr{E}_{\textbf{motion}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^{\top} v_i.}$

Motion energy

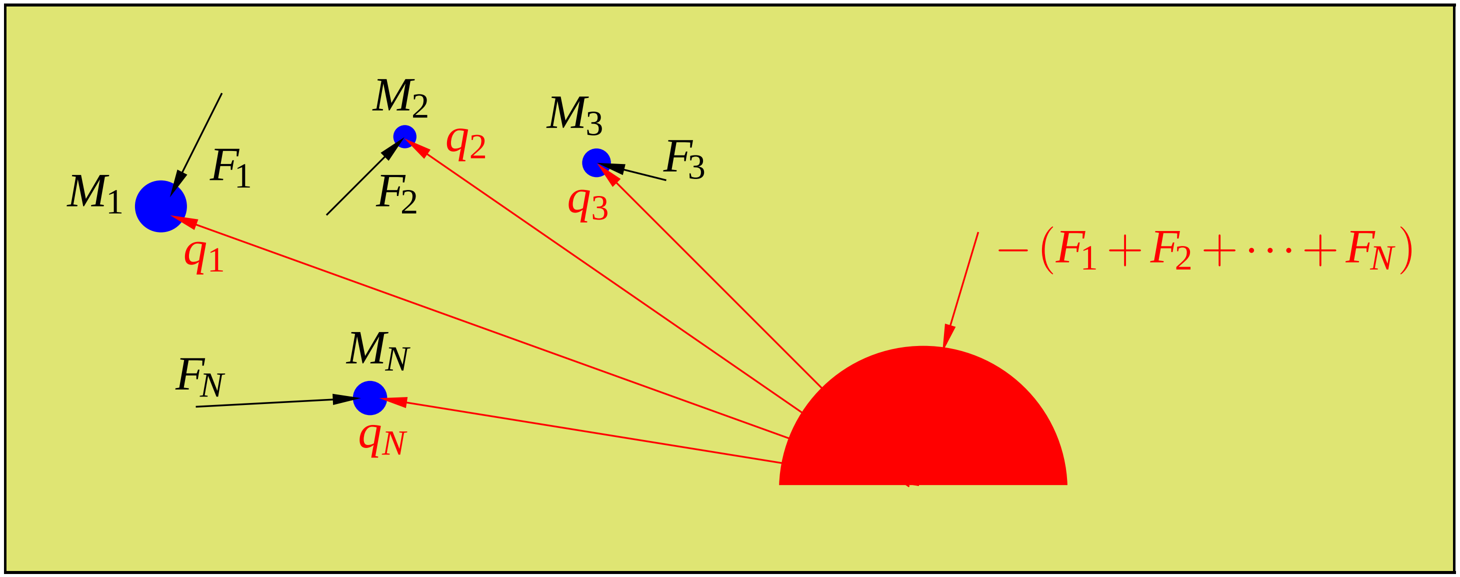
$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2 .$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2 .$$

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



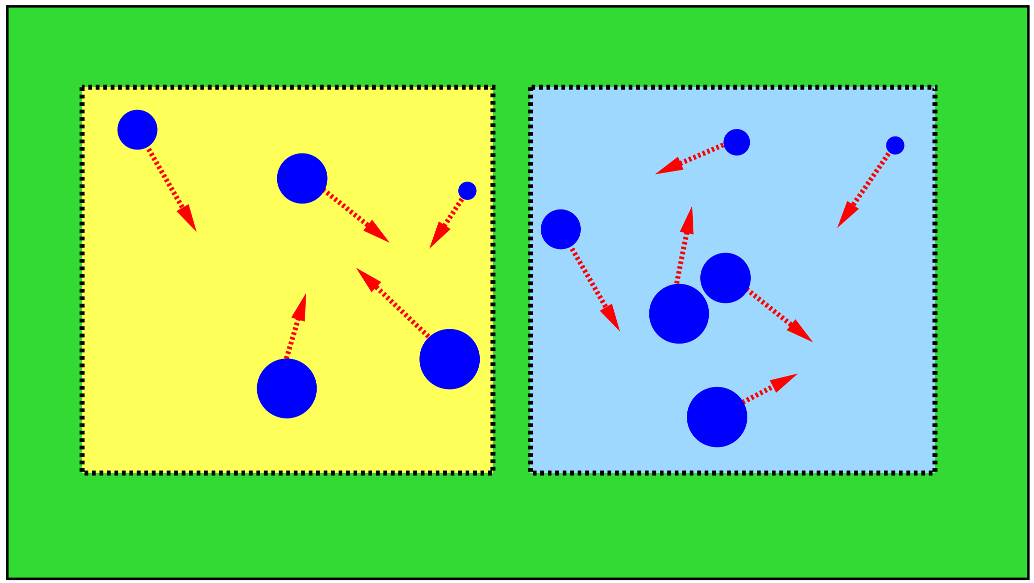
measure velocities w.r.t. this infinite mass ('ground'), then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\xrightarrow[M_N \to \infty]{} \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

Motion energy

Motion energy is not an extensive quantity, it is not additive.

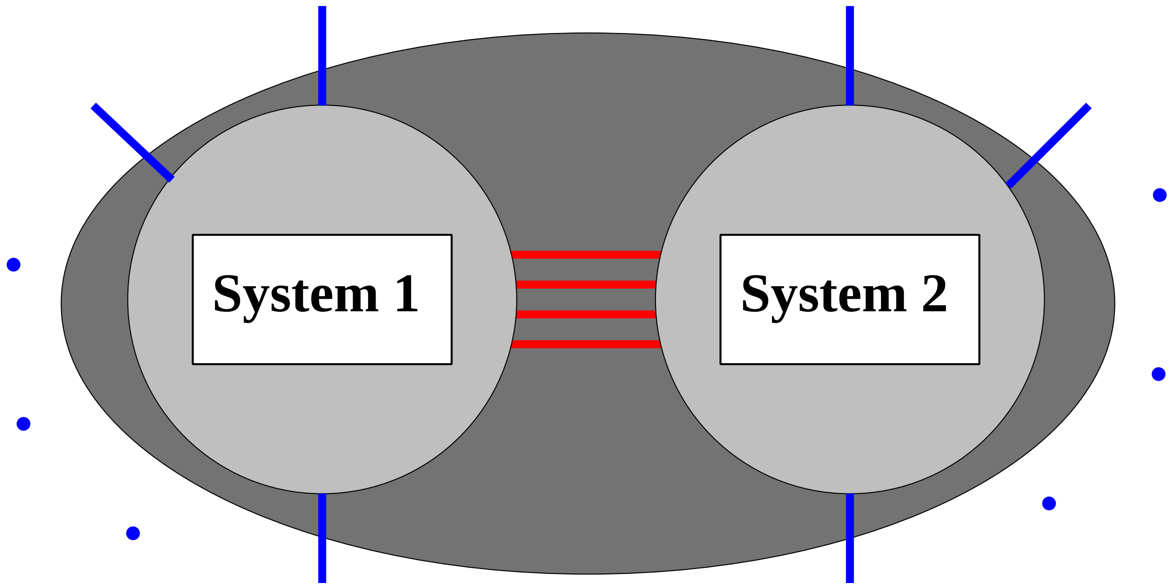


Total motion energy $\neq$ sum of the parts.

Power and energy involve ‘action at a distance’.

ENERGY TRANSFER

Energy transfer



One cannot speak about

“ the energy transferred from system 1 to system 2 ”
or ***“ from the environment to system 1 ”*****,**

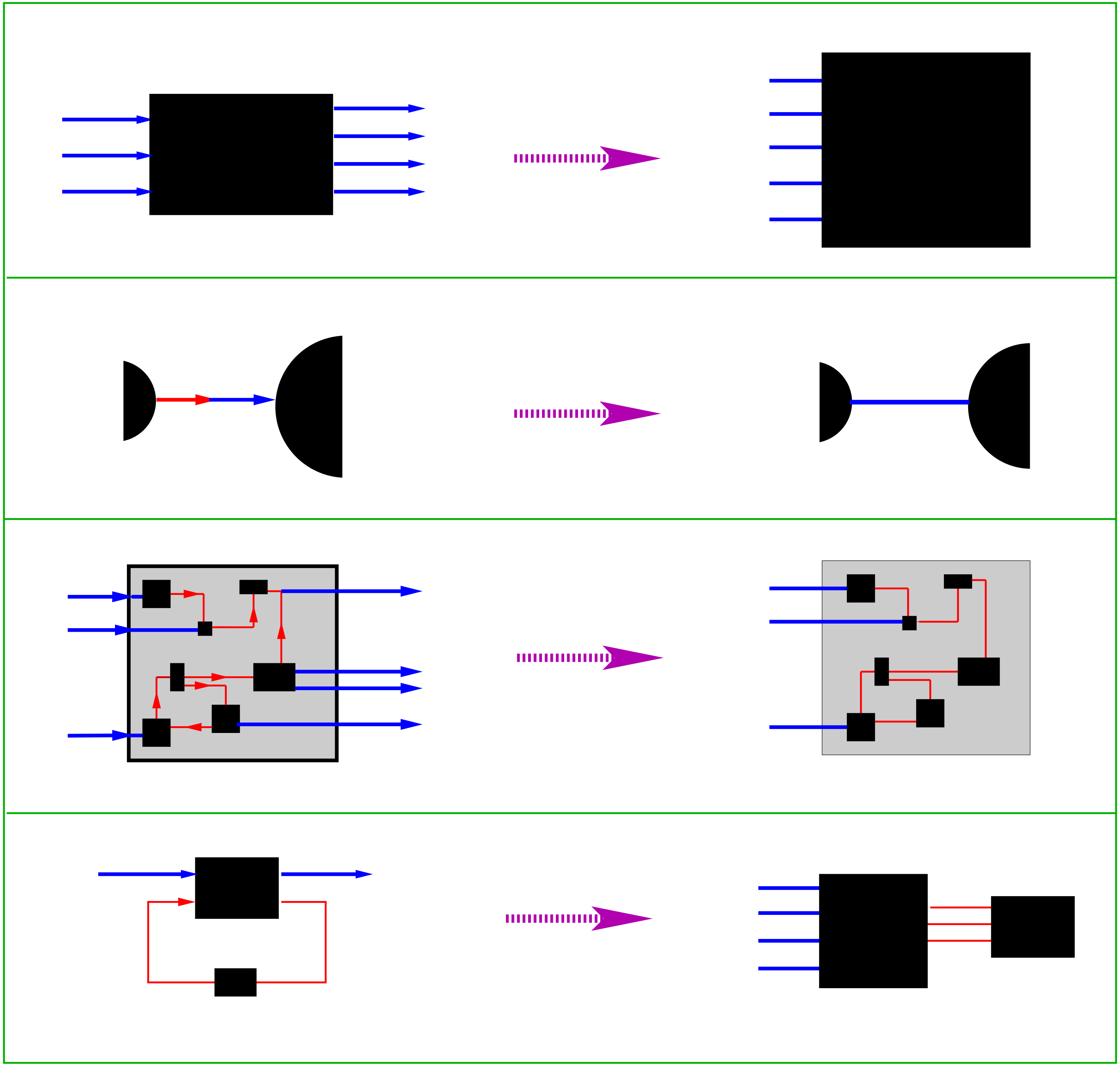
unless the relevant terminals form a port.

Ports and terminals

Terminals are for interconnection,

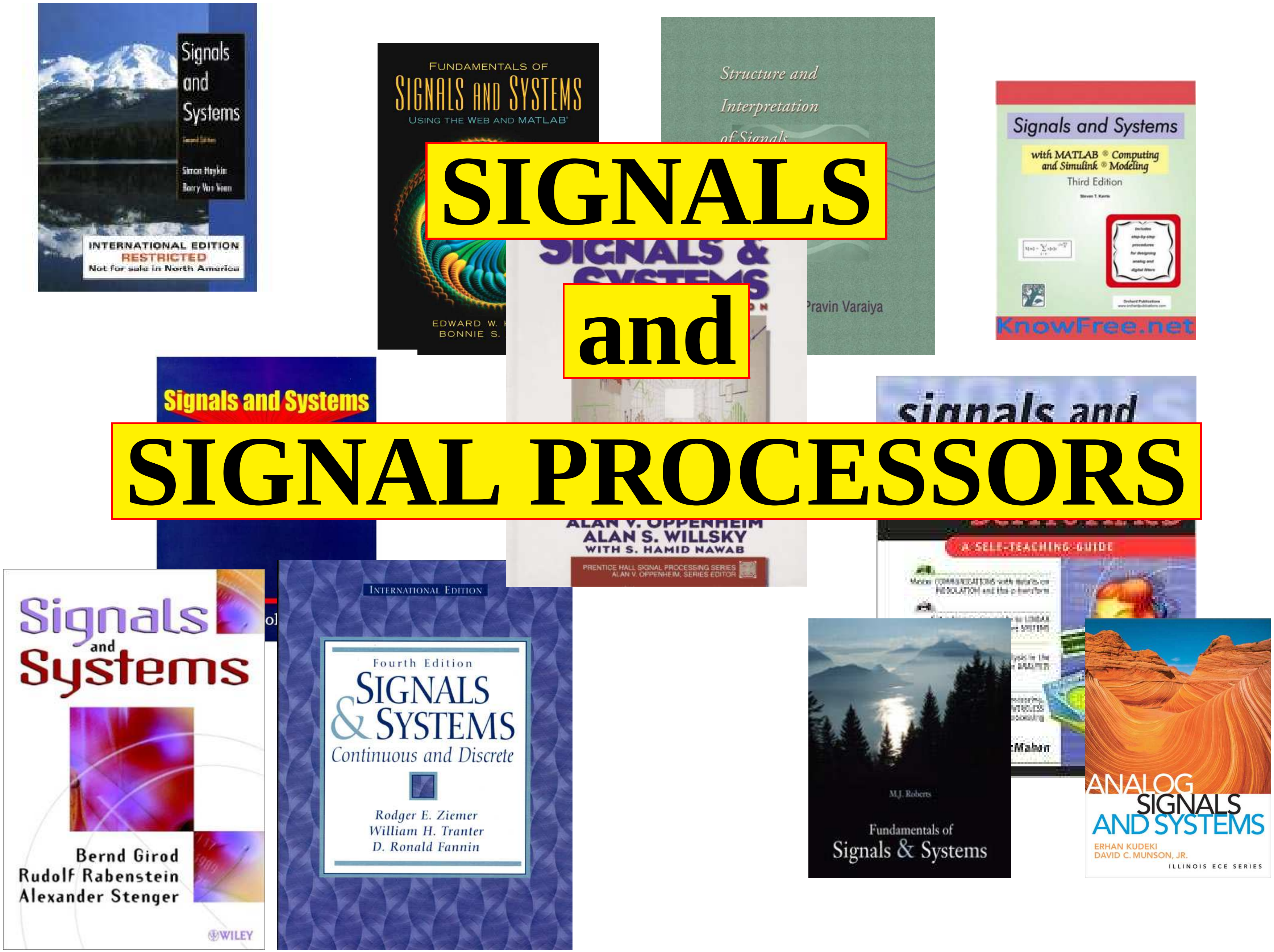
ports are for energy transfer.

CONCLUSION



Favorite textbooks

Favorite textbooks



References: 1. The behavioral approach to open and interconnected systems, ***Control Systems Magazine*****, volume 27, pages 46-99, 2007.**

2. Terminals and Ports, ***Circuits and Systems Magazine*****,**
volume 10, issue 4, pages 8-16, December 2010.

Copies of the lecture frames available from/at
http://www.esat.kuleuven.be/∼jwillems

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you