MECHANICAL PORTS

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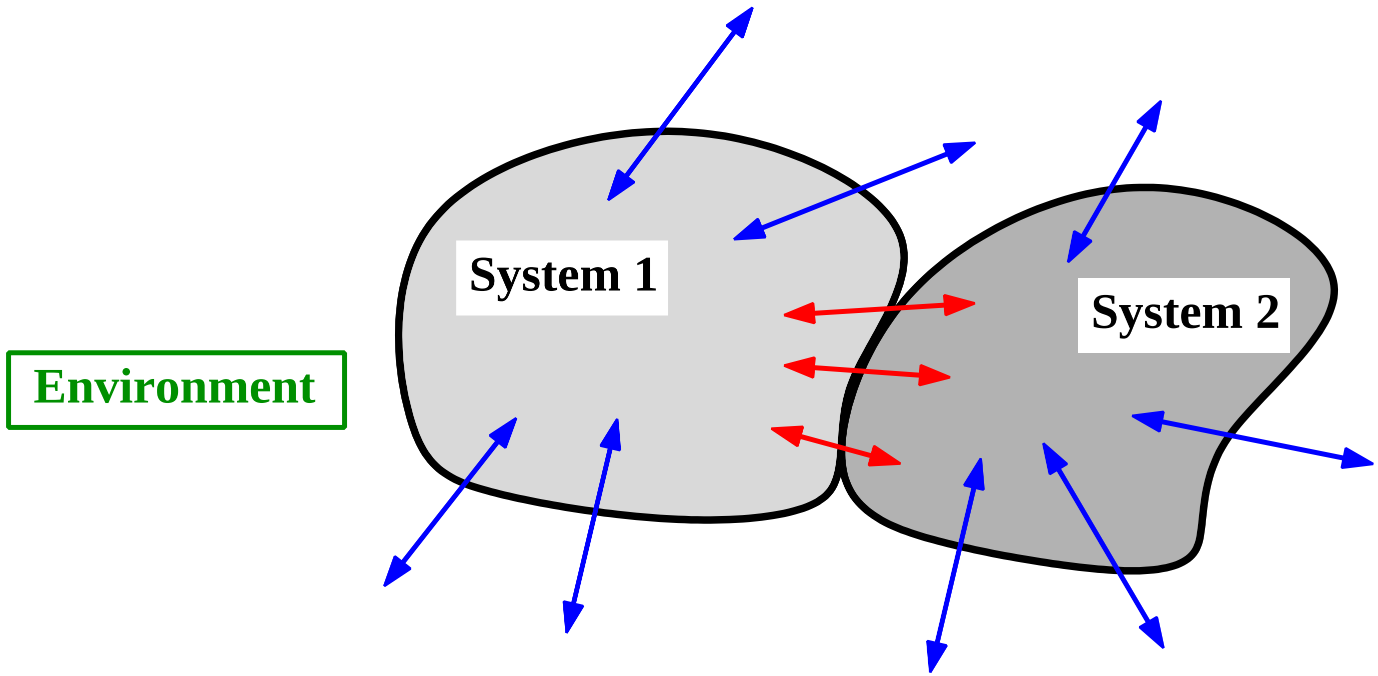
YYFest **March 29, 2010**

In honor of Yutaka Yamamoto on the occasion of his sixtieth birthday.

Theme: energy transfer

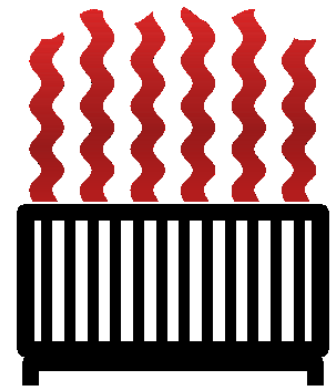


How is energy transferred from the environment to a system?

How is energy transferred between systems?

Energy

Energy := a physical quantity transformable into heat.



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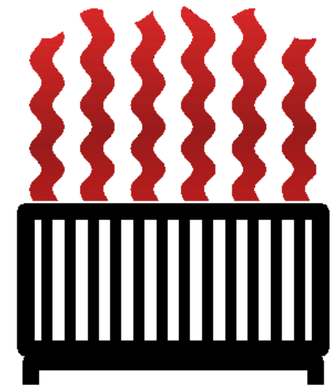
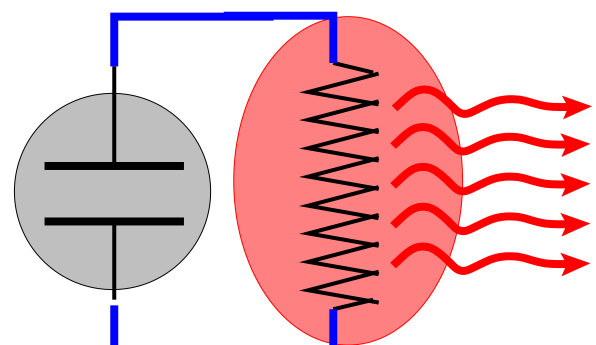


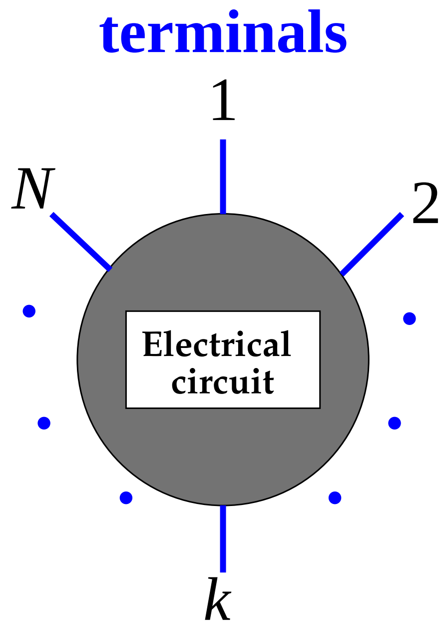
For example capacitor → resistor → heat.

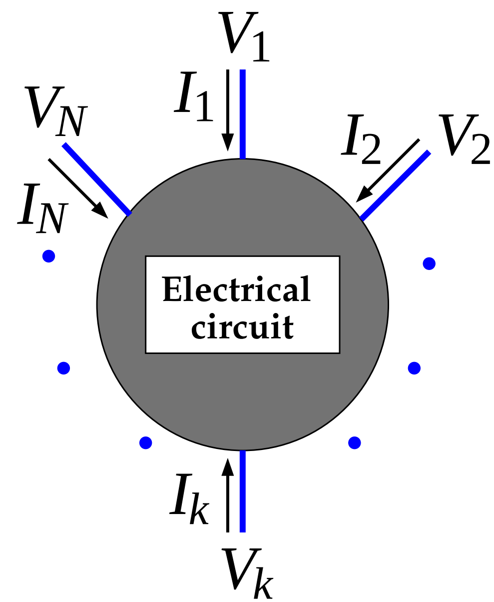
Energy on capacitor = $\frac{1}{2}CV^2$



Electrical ports

Electrical circuit





<u>At each terminal</u>:

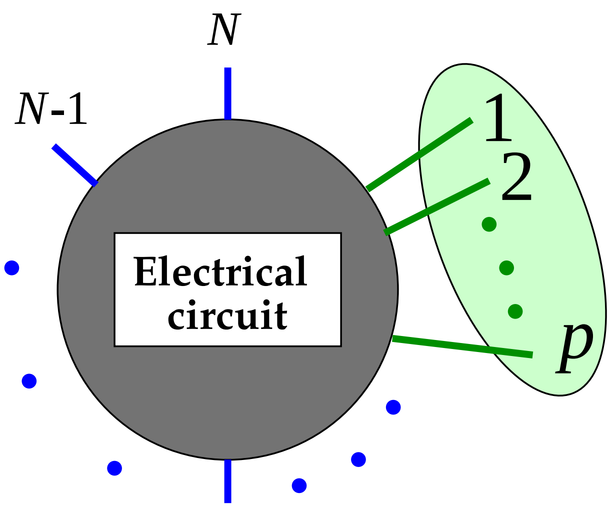
a potential (!) and a current (counted > 0 into the circuit),

$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

this potential/current trajectory is compatible with the circuit architecture and its element values.

Ports

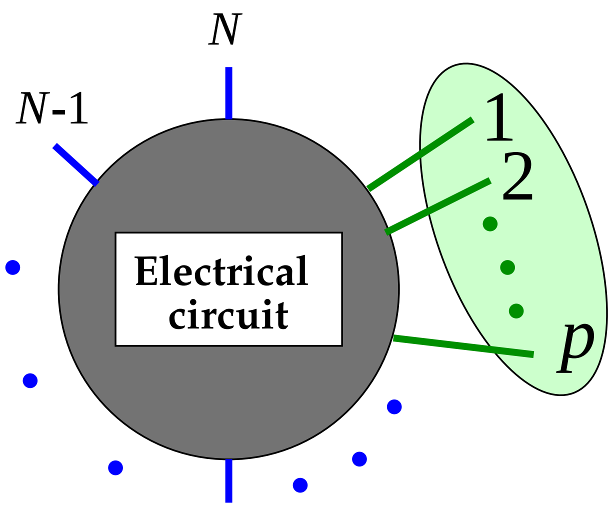


Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

$$(V_1, \ldots, V_p, V_{p+1}, \ldots, V_N, I_1, \ldots, I_p, I_{p+1}, \ldots, I_N) \in \mathscr{B}$$

$$\Rightarrow \quad I_1 + \cdots + I_p = 0.$$ ***'port KCL'***.

Ports



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

power in along these terminals = $V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)$,

energy in =

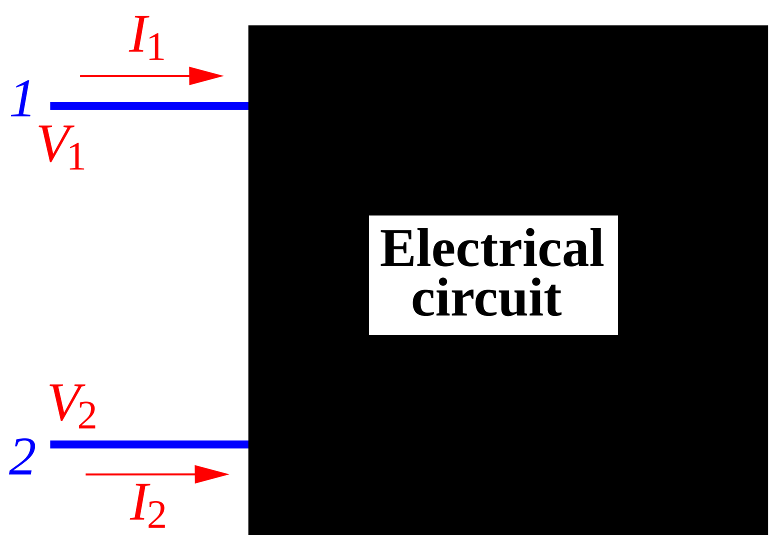
$$\int_{t_1}^{t_2} (V_1(t)I_1(t) + \cdots + V_p(t)I_p(t))\, dt.$$

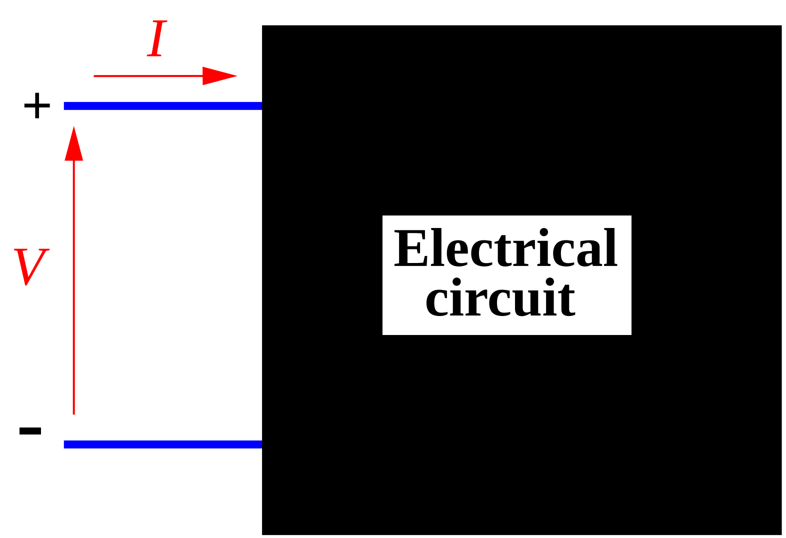
This interpretation in terms of power and energy is not valid unless these terminals form a port !

Examples

2-terminal 1-port devices:

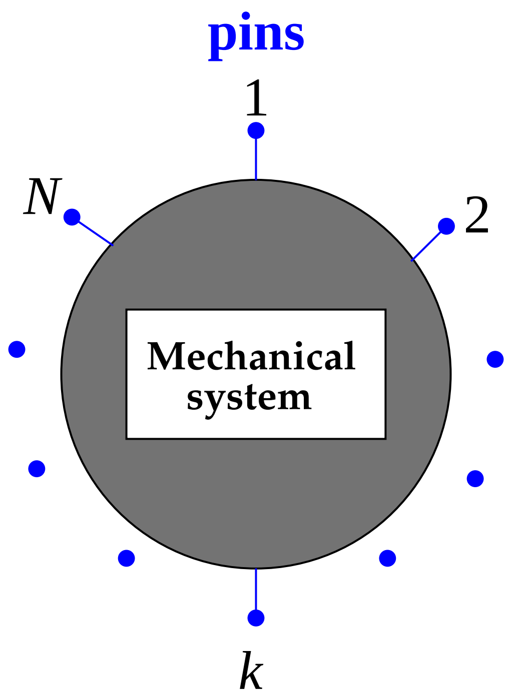
resistors, inductors, capacitors, memristors, etc.
any 2-terminal circuit composed of these.

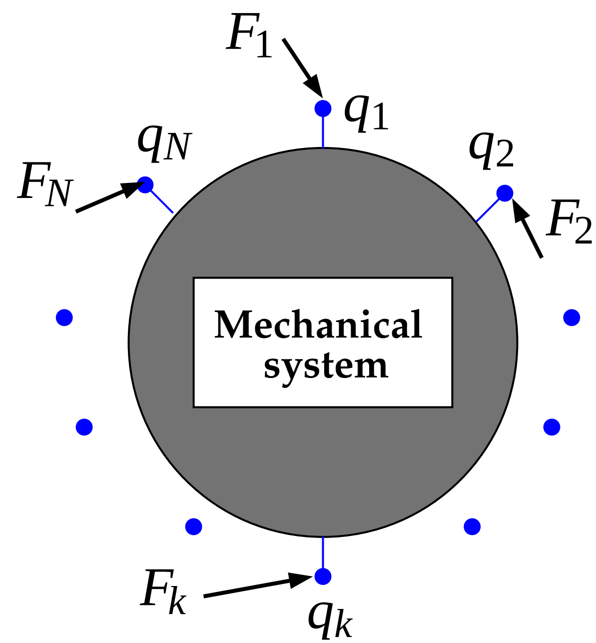




Mechanical ports

Mechanical systems



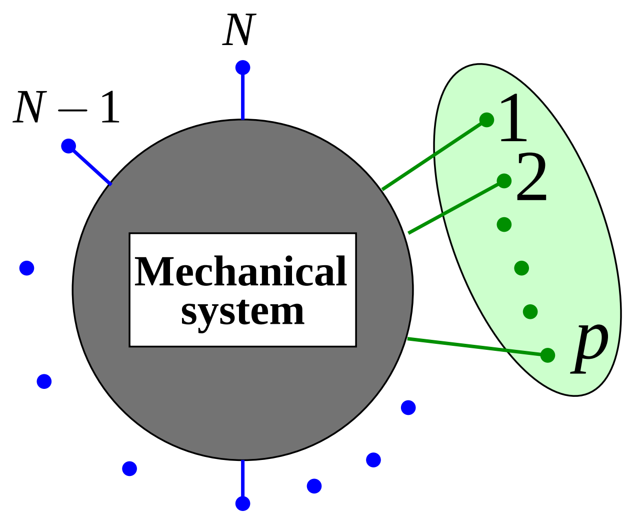


At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

More generally, a position, force, angle, and torque.

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \quad F_1 + F_2 + \cdots + F_p = 0.$$

'port KFL'

Power and energy

If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

$$\textbf{power in} \;=\; F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

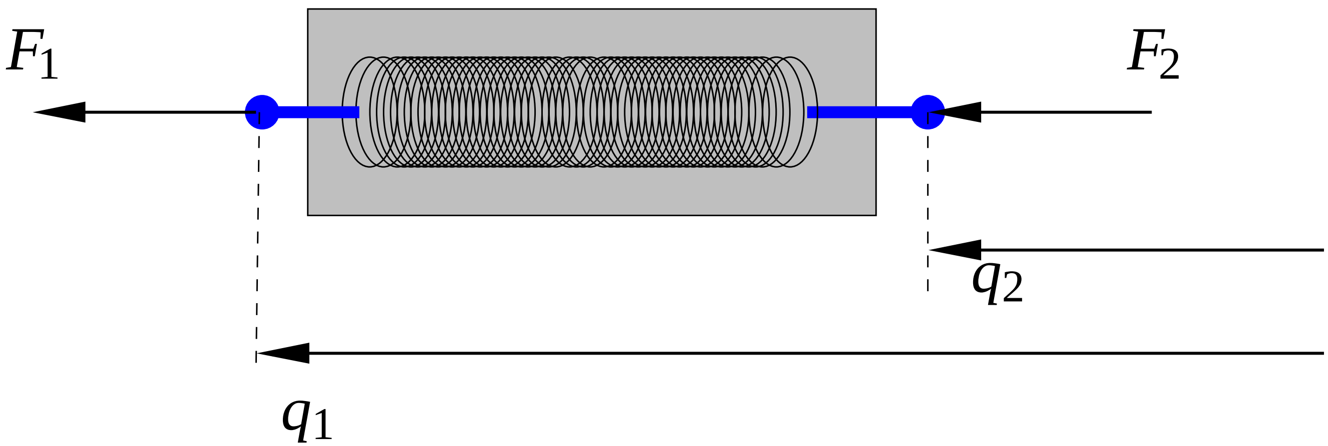
and

$$\textbf{energy in} \;=\; \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

Example

Spring

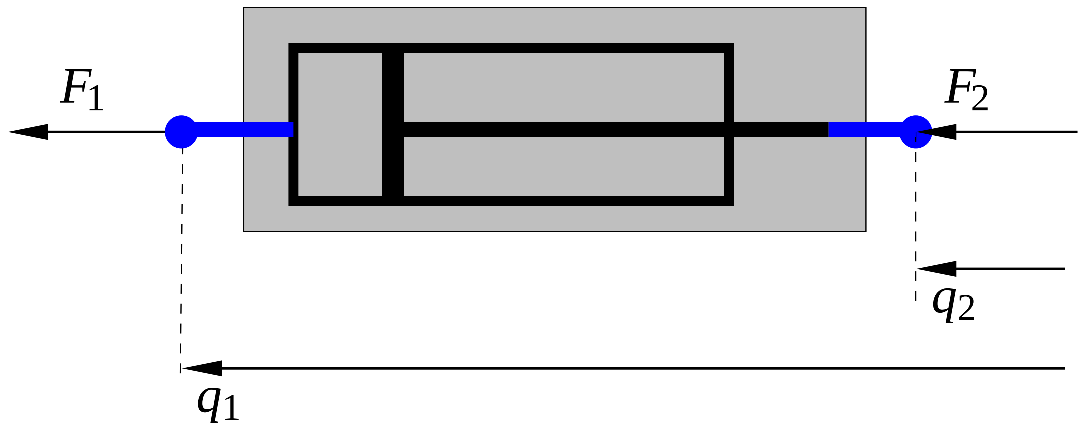


$$F_1 + F_2 = 0, \quad K(q_1 - q_2) = F_1$$

satisfies KFL

Examples

Damper

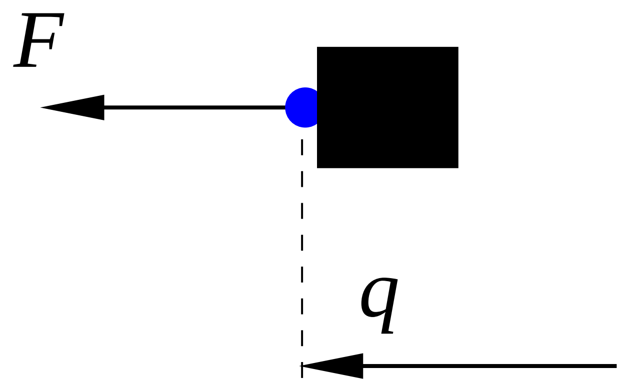


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1.$$

satisfies KFL

Springs and dampers, and the interconnection of springs and dampers form ports.

A mass



$$M \frac{d^2}{dt^2} q = F.$$

does not satisfy KFL

Not a port!!!

Consequences

We discuss three consequences of the fact that a mass is not a port.

- **The inerter**
- **Motion energy**
- **Energy as an extensive quantity**

The inerter

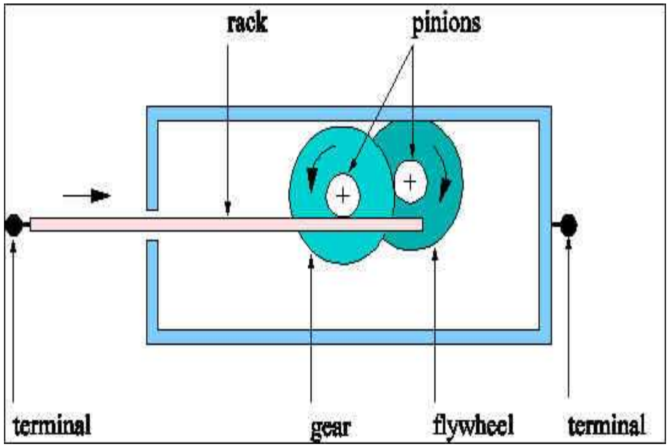
Mechanical synthesis

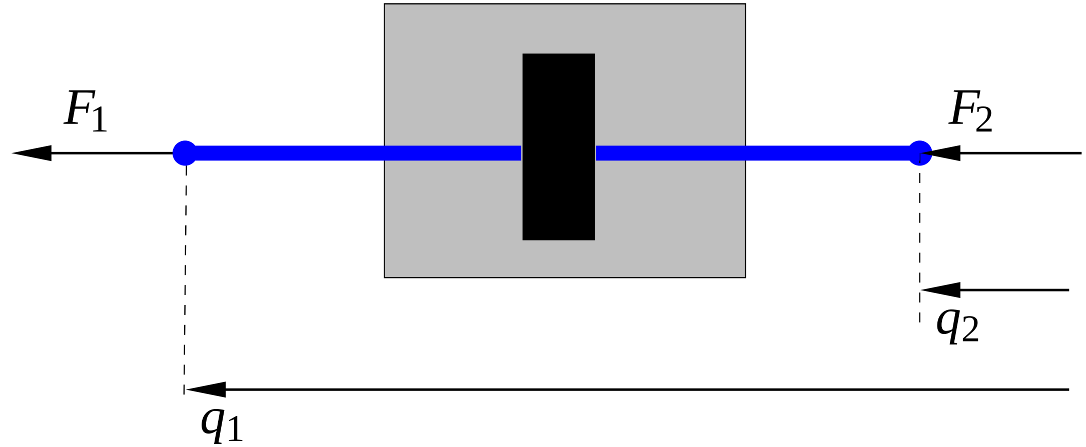
The mass is NOT the mechanical analogue of a capacitor.

RLC synthesis $\not\Leftrightarrow$ Damper-Spring-Mass synthesis

Is there a mechanical analogue of a capacitor ?

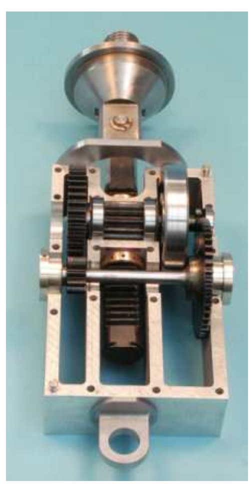
The inerter





$$B\frac{d^2}{dt^2}(q_1 - q_2) = F_1, \quad F_1 + F_2 = 0$$

satisfies KFL $\rightsquigarrow$ a port



Malcolm Smith

Electrical-mechanical analogies

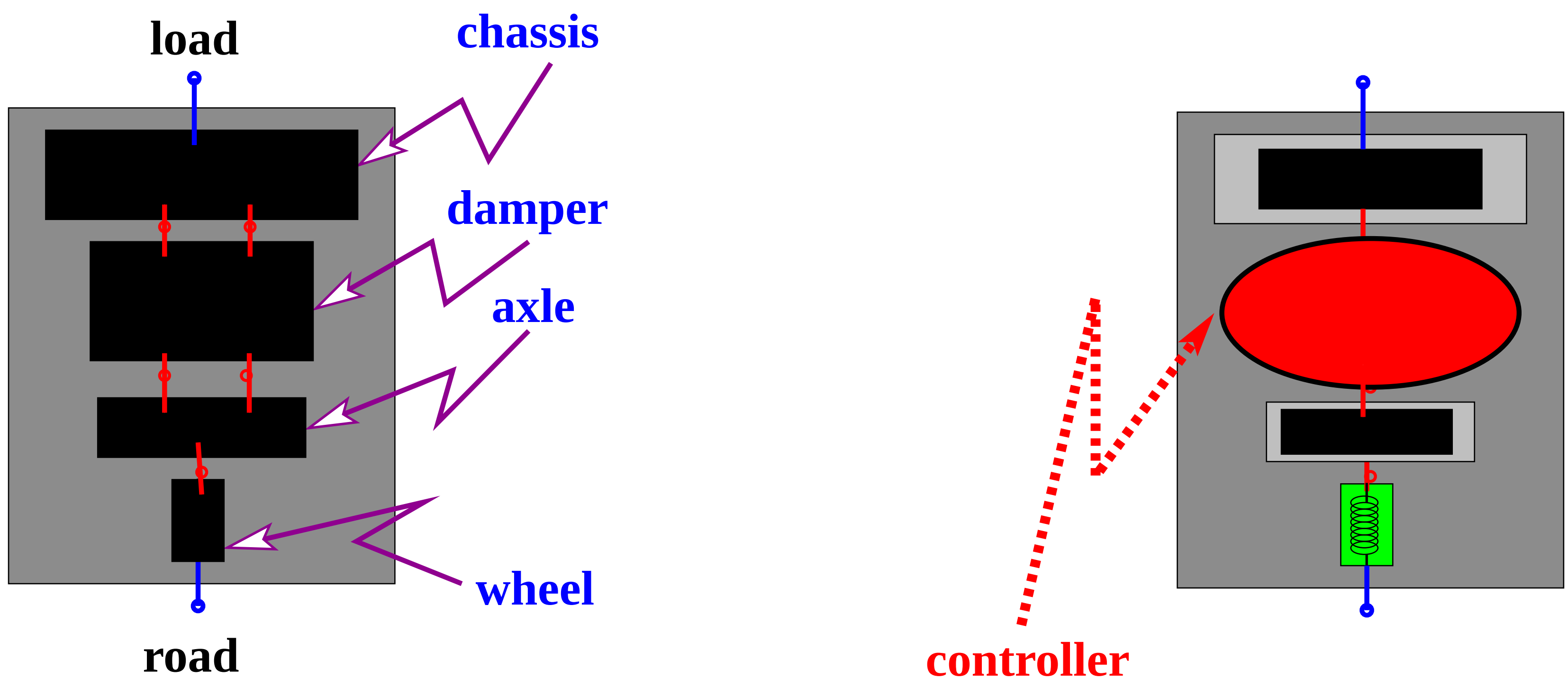
voltage $V \leftrightarrow v$ **velocity** **current** $I \leftrightarrow F$ **force**

Resistor	Damper
$\frac{1}{R}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	$D(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$
Inductor	**Spring**
$\frac{1}{L}(V_1 - V_2) = \frac{d}{dt} I_1, \;\; I_1 + I_2 = 0$	$K(v_1 - v_2) = \frac{d}{dt} F_1, \;\; F_1 + F_2 = 0$
Capacitor	**Inerter**
$C\frac{d}{dt}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	$B\frac{d}{dt}(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$

electrical RLC synthesis $\Leftrightarrow$ **mechanical SDI synthesis**

Springs, dampers, inerters, and their interconnections form ports!

Example of behavioral control: A ‘quarter car’



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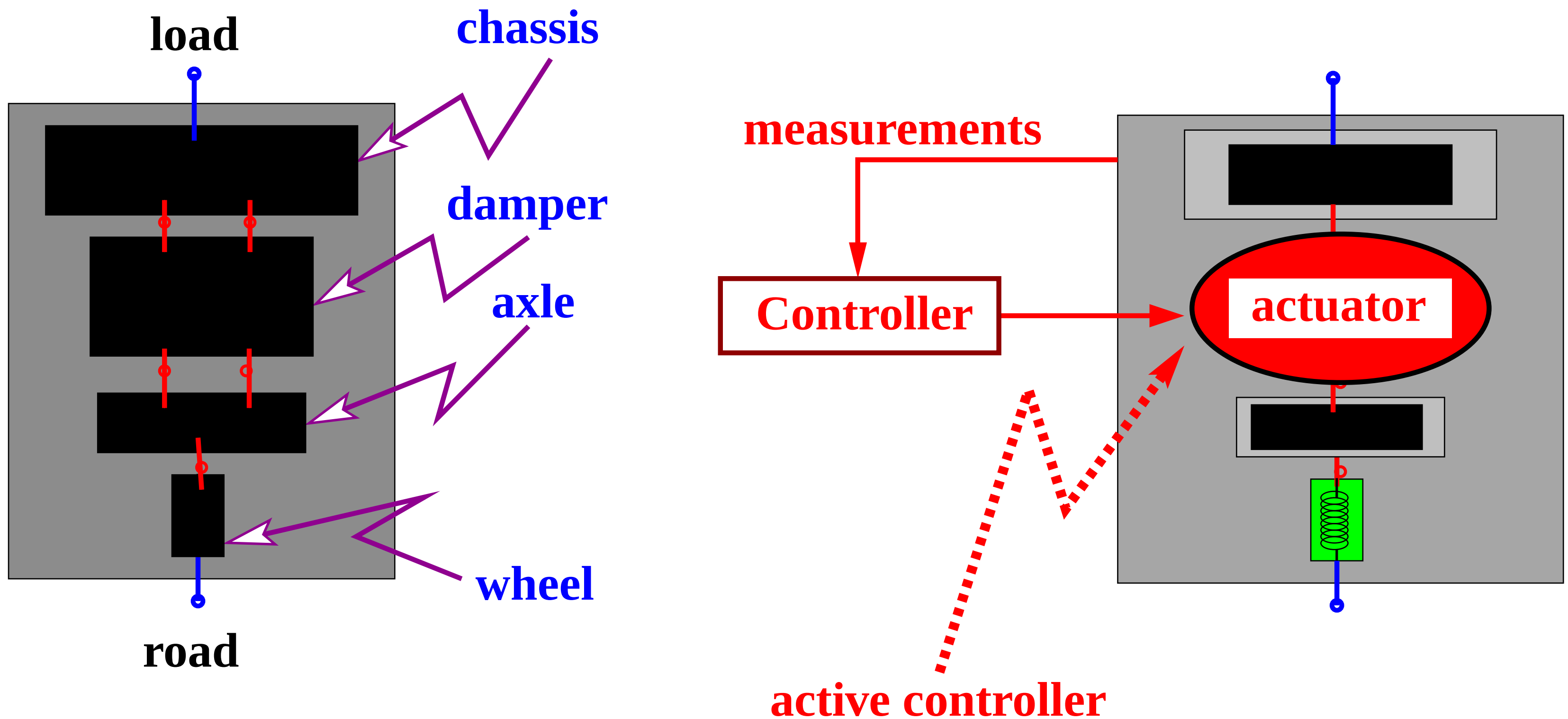

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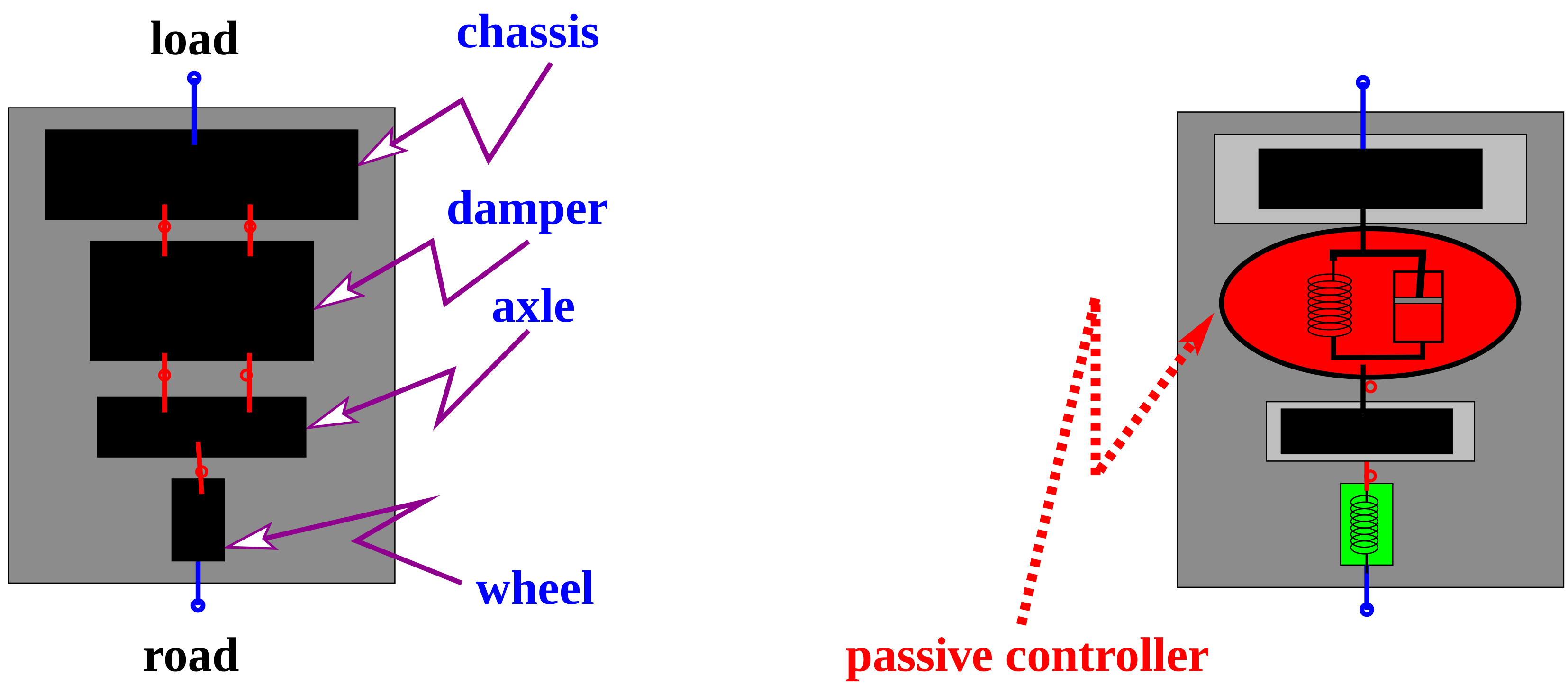
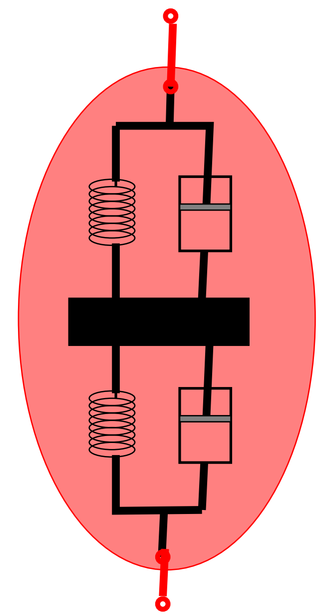


Suspension controller in Formula 1

Nigel Mansell victorious in 1992 with an active damper suspension.

Active dampers were banned in 1994 to break the dominance of the Williams team.

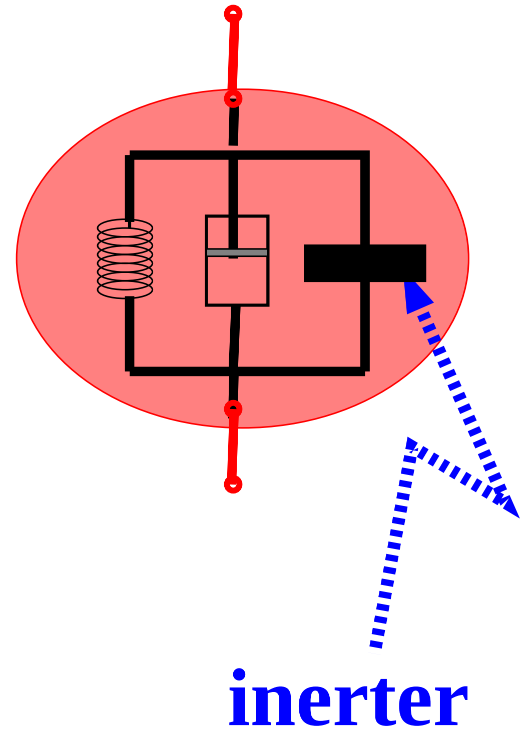
Suspension controller in Formula 1



Renault successfully use a passive ‘tuned mass damper’ in 2005/2006.

Banned in 2006, under the ‘movable aerodynamic devices’ clause.

Suspension controller in Formula 1



Kimi Räikkönen wins the 2005 Grand Prix in Spain with McLaren's 'J-damper', i.e., Smith's inerter.

![grandprix.com]

AUGUST 21, 2008

Ingenuity still brings success in Formula 1

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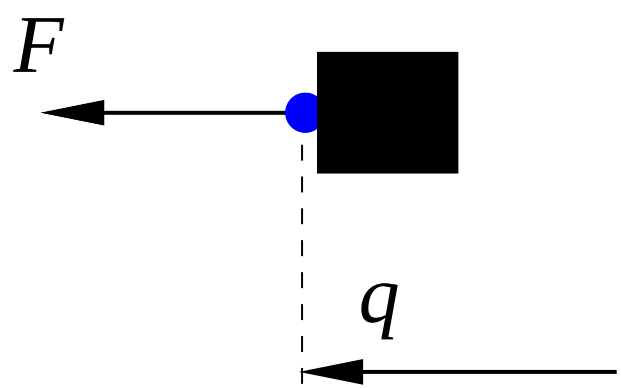
For years engineers have complained that the rules of Formula 1 mean that there is little room left for innovation but Cambridge University's engineering department has just revealed that this is not the case at all.

Professor Malcolm Smith, a fellow of Gonville and Caius College, created an innovative suspension system in the late 1990s and this was patented by the university. The first details were published in 2002 in the obscure Institute of Electrical and Electronics Engineers's publication called Transactions on Automatic Control. This was spotted by the boffins at McLaren and an exclusive deal was negotiated to allow the team to use the technology in F1. The new system was first used at the Spanish GP in 2005 and Kimi Raikkonen won.

The team used the name "J Damper" to describe the unit - in an effort to confuse the opposition - but it has now been revealed that it is actually called "an inerter". This is

Motion energy

Back to the mass

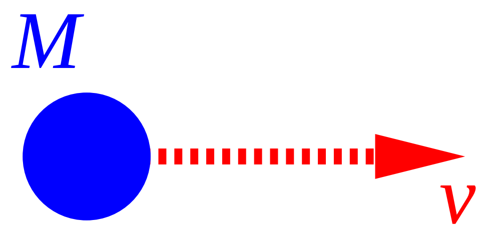


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q$$

If $F^\top v$ is not power,

is $\frac{1}{2}M||\frac{d}{dt}q||^2$ not stored (kinetic, motion) energy ???

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

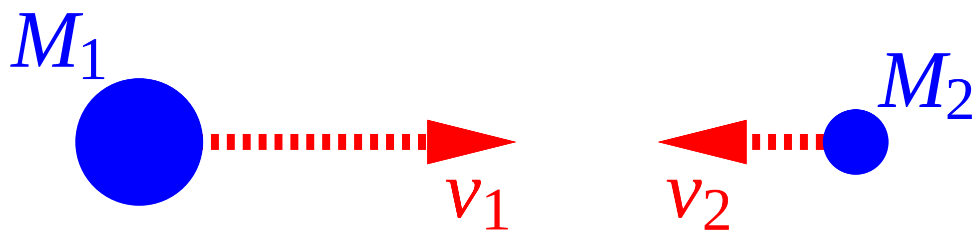
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

This expression is not invariant under uniform motion.

Motion energy



What is the motion energy?

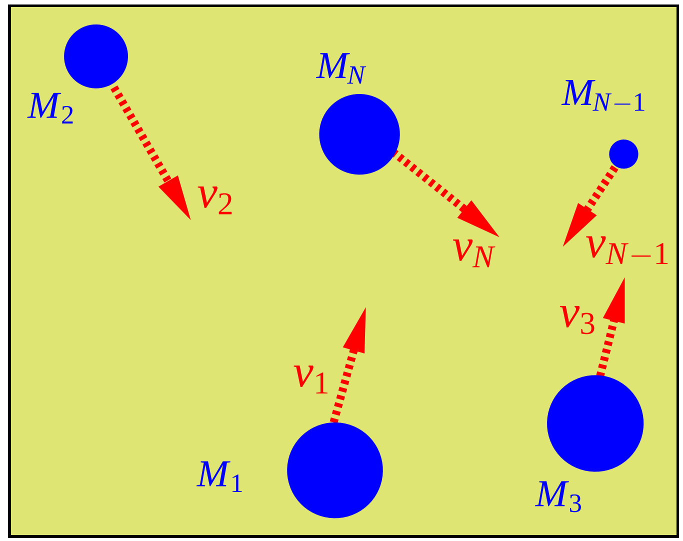
What quantity is transformable into heat?

$$\mathscr{E}_{\textbf{motion}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1-v_2||^2$$

Invariant under uniform motion.

Motion energy

Generalization to N masses.



$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

$$\textbf{KFL} \quad \Rightarrow \qquad \frac{d}{dt}\mathscr{E}_{\textbf{motion}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^{\top} v_i.$$

Motion energy

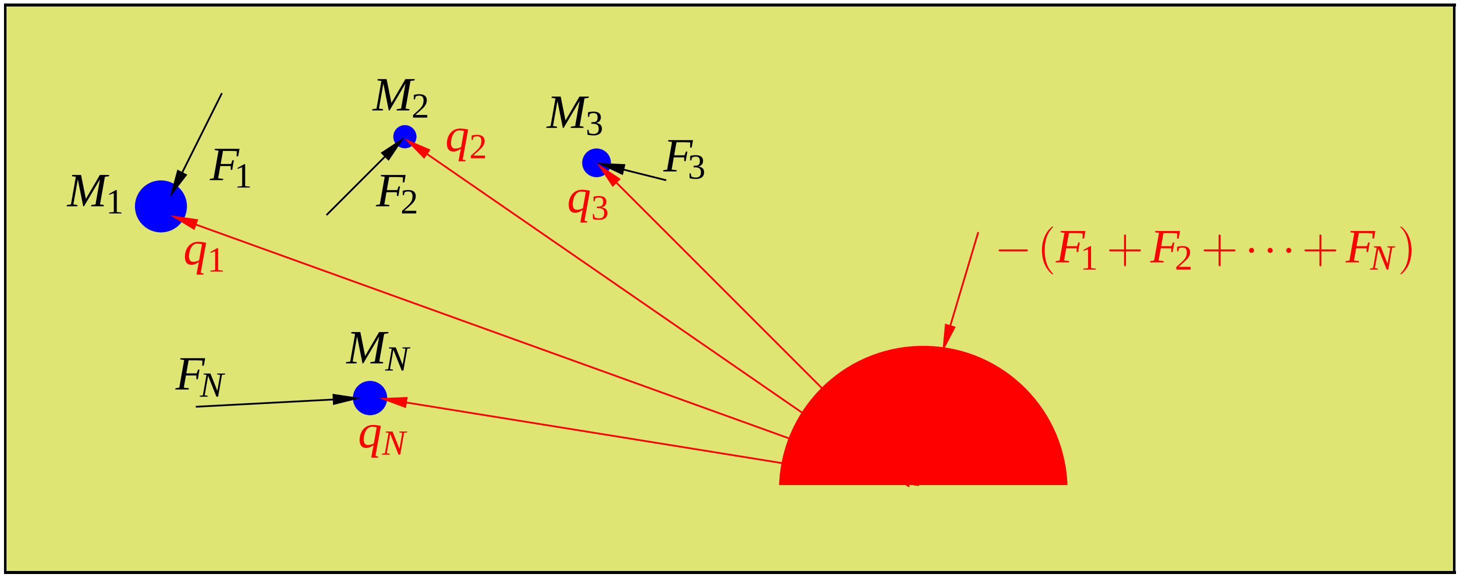
$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2 .$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2 .$$

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



measure velocities w.r.t. this infinite mass ('ground'), then

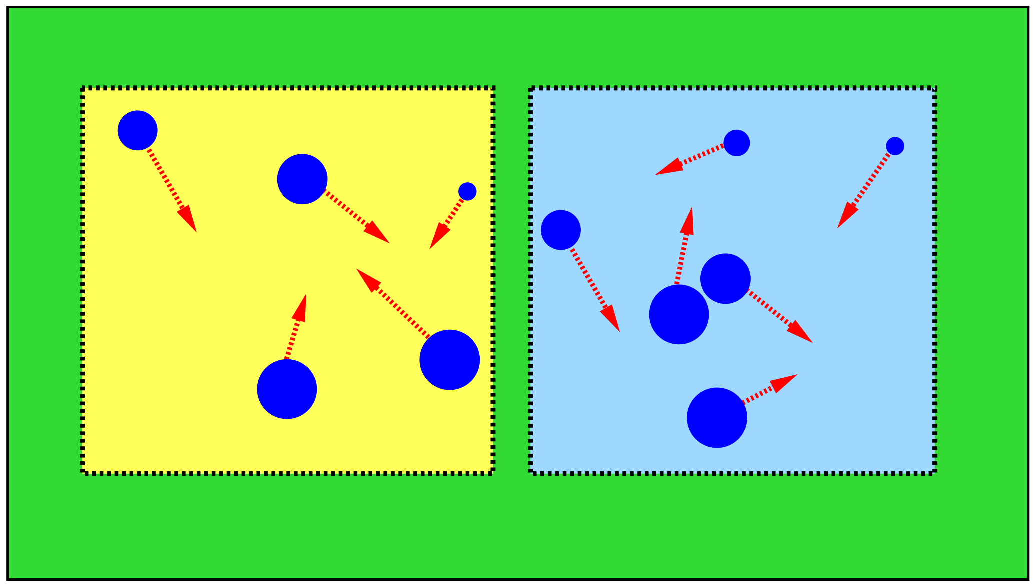
$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2.$$

Energy as an extensive quantity

Motion energy

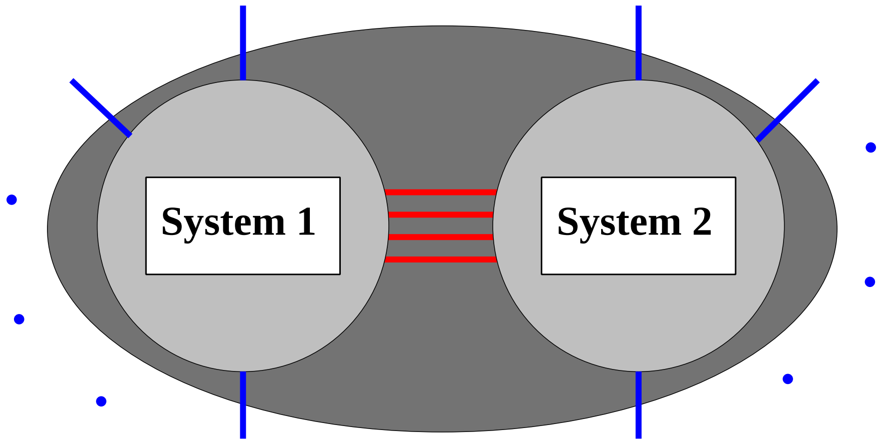
Motion energy is not an extensive quantity, it is not additive.

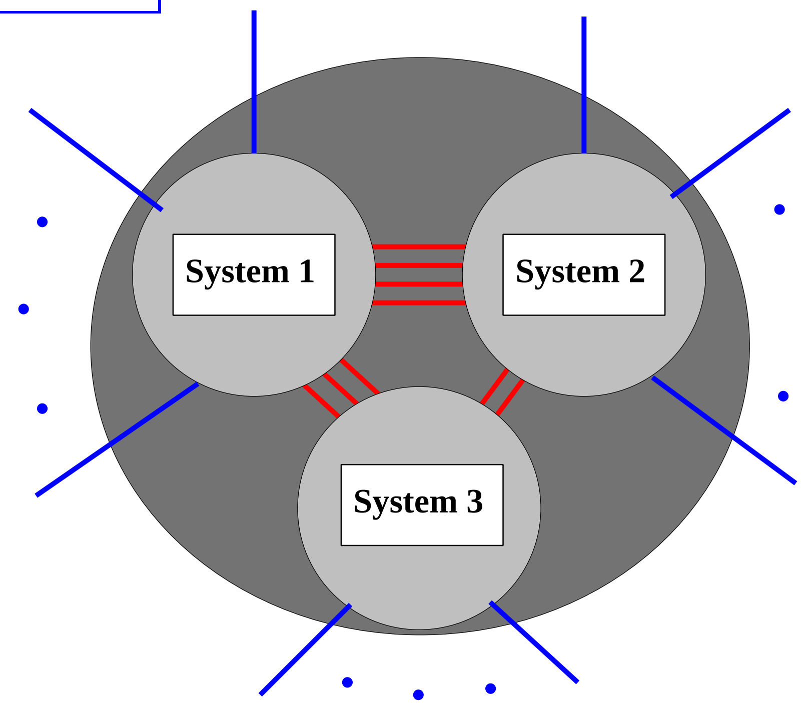


Total motion energy $\neq$ sum of the parts.

Ports and terminals

Energy transfer





One cannot speak about

"the energy transferred from system 1 to system 2"
or *"from the environment to system 1"*,

unless the relevant terminals form a port.

Happy birthday, Yutaka

Het ga je goed, en nog veel gelukkige jaren!