MOVING and MOTION

JAN C. WILLEMS

K.U. Leuven, Flanders, Belgium

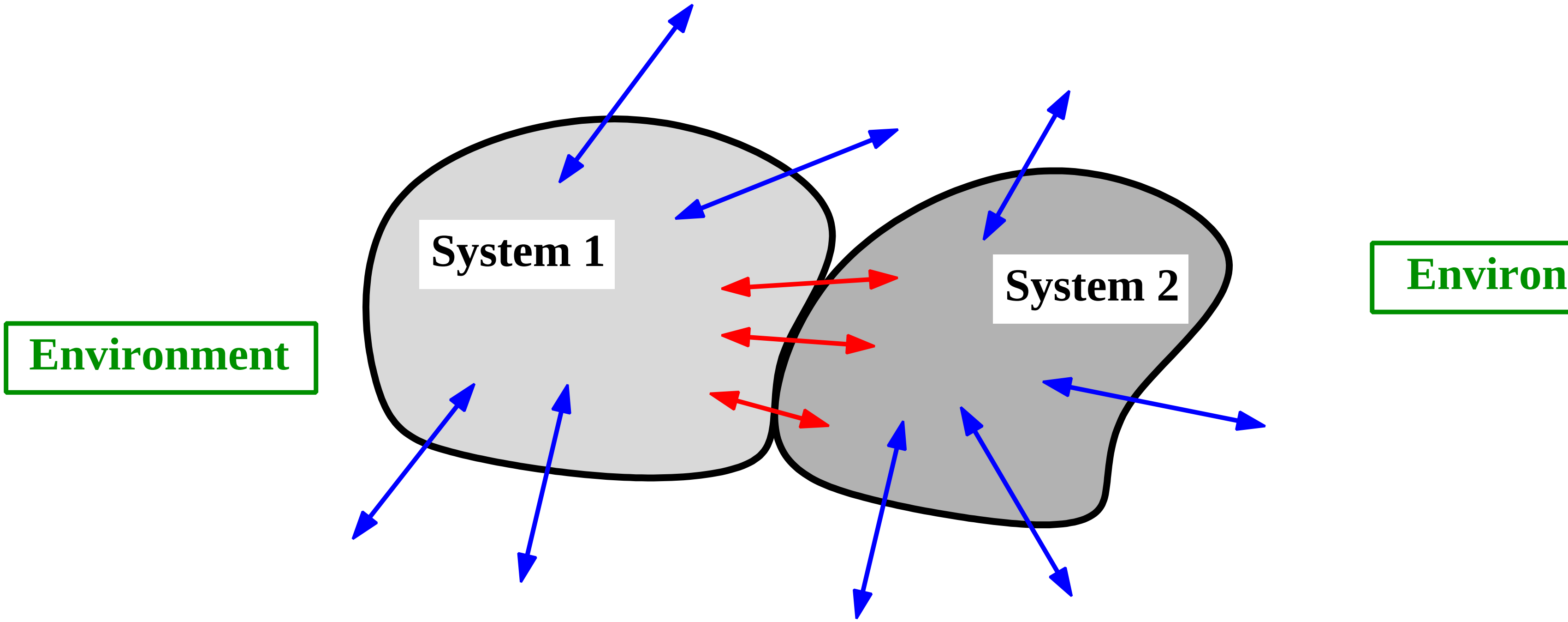
Afscheid Carsten Scherer **February 26, 2010**

In honor of Carsten on the occasion of his moving from Delft to Stuttgart.

De LMI-les van Dr. Carsten Scherer

Let's get serious!

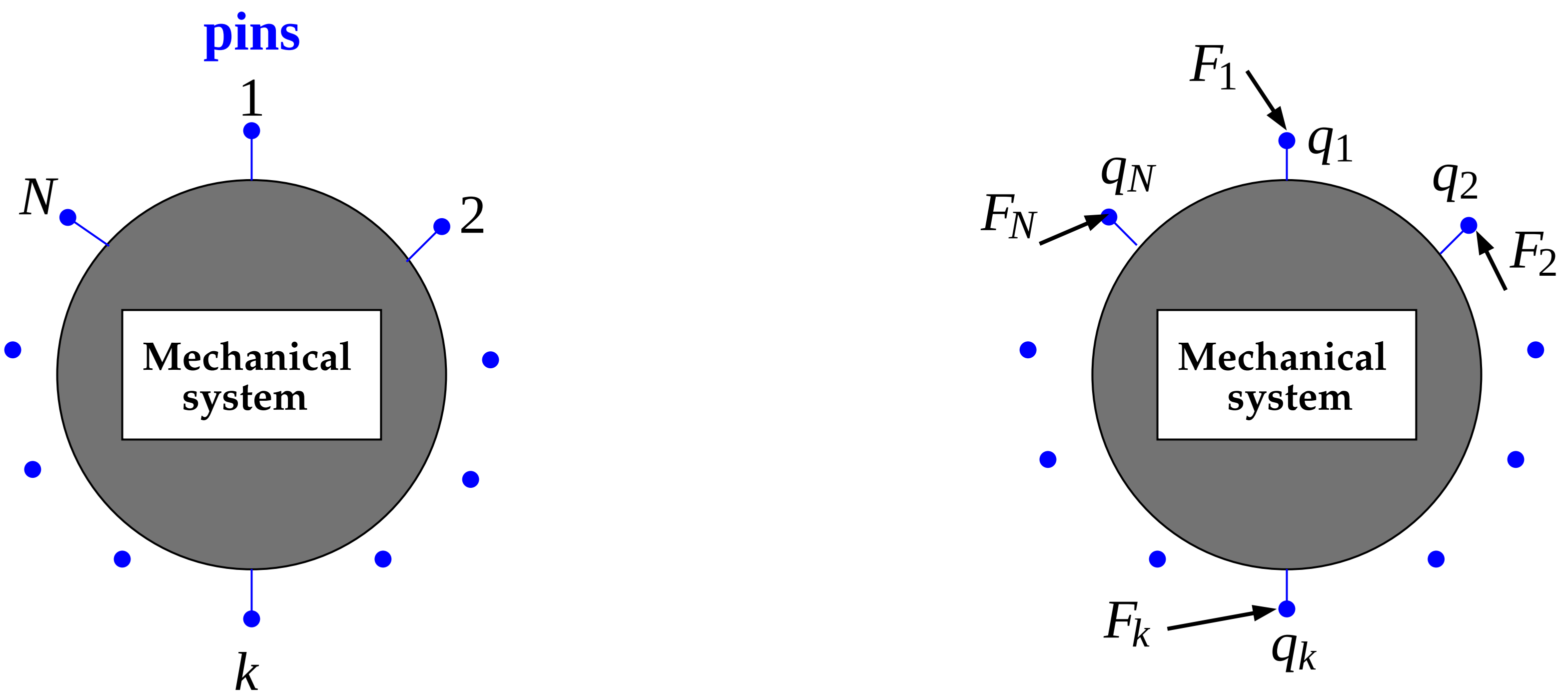
Theme: energy transfer



How is energy transferred from the environment to a system?

How is energy transferred between systems?

Mechanical systems

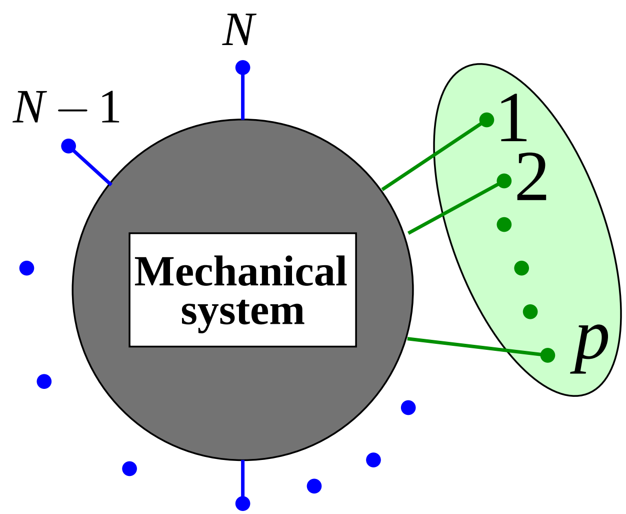


At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$**.**

More generally, a position, force, angle, and torque.

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \quad F_1 + F_2 + \cdots + F_p = 0.$$

'port KFL'

Power and energy

If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

$$\textbf{power in} \;=\; F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

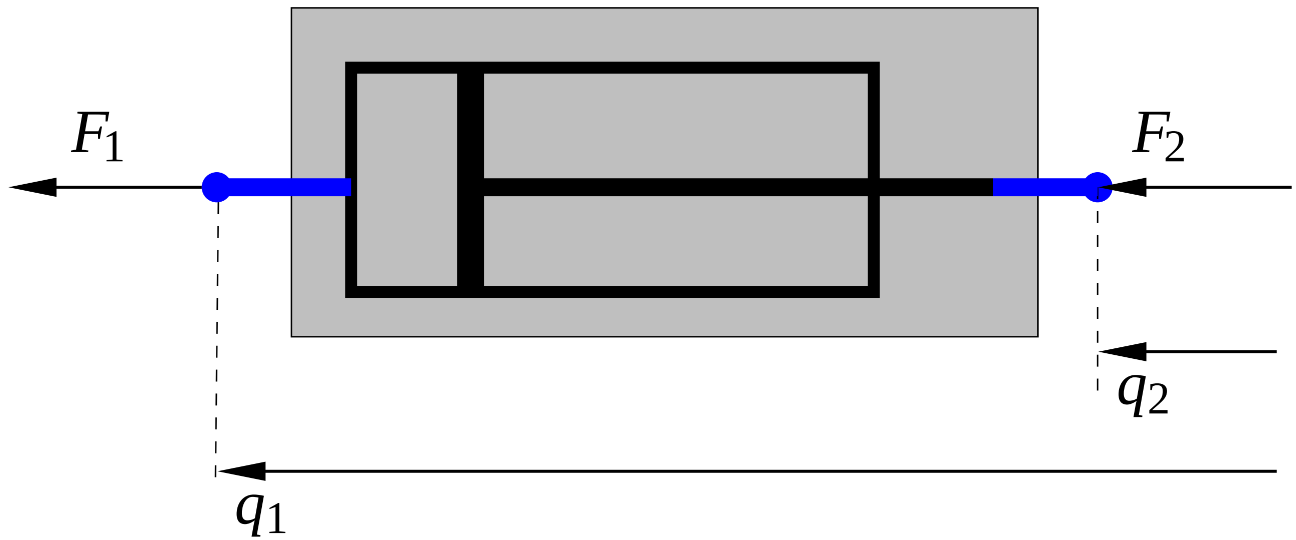
and

$$\textbf{energy in} \;=\; \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid
unless these terminals form a port !

Examples

A damper

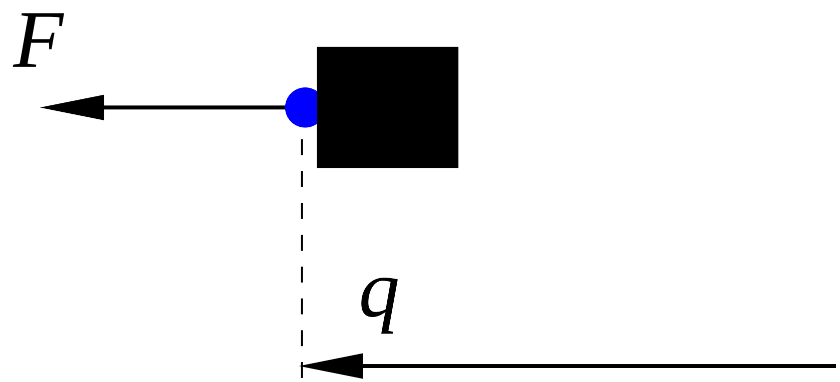


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1.$$

satisfies KFL $\rightsquigarrow$ a port

Springs and dampers, and mechanical devices formed by the interconnection of springs and dampers are ports.

A mass

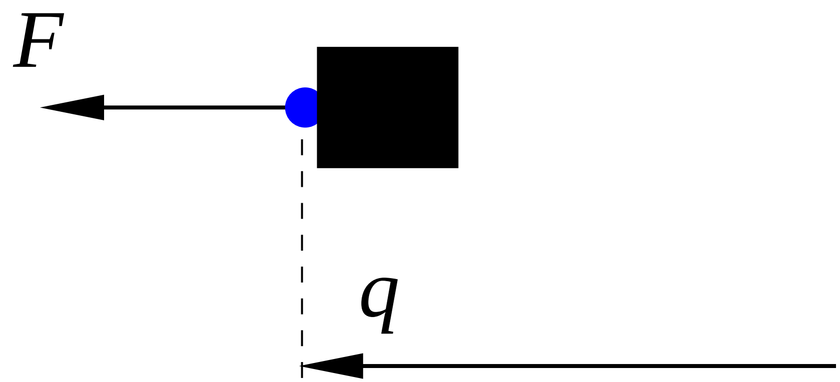


$$M\frac{d^2}{dt^2}q = F.$$

does not satisfy KFL

Not a port!!!

A mass



$$M\frac{d^2}{dt^2}q = F.$$

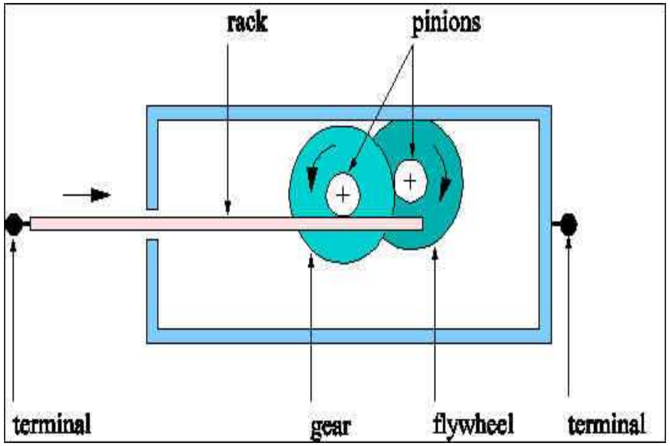
does not satisfy KFL

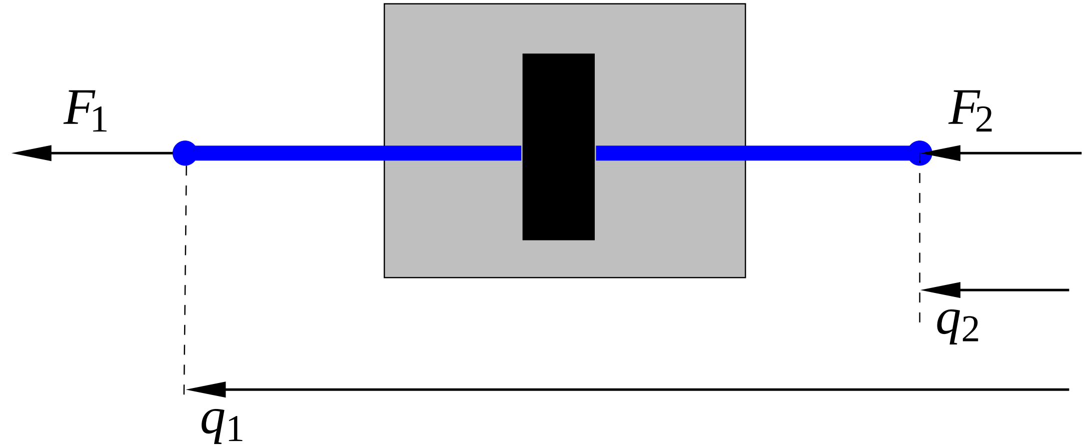
Not a port!!!

The mass is NOT the mechanical analogue of a capacitor.

RLC synthesis $\not\Leftrightarrow$ Damper-Spring-Mass synthesis

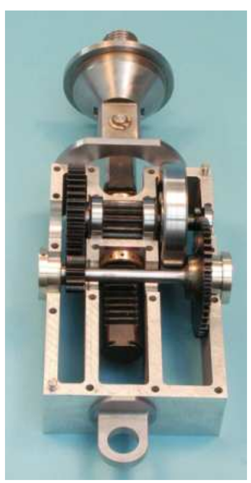
The inerter





$$B\frac{d^2}{dt^2}(q_1 - q_2) = F_1, \quad F_1 + F_2 = 0$$

satisfies KFL $\rightsquigarrow$ a port



Malcolm Smith

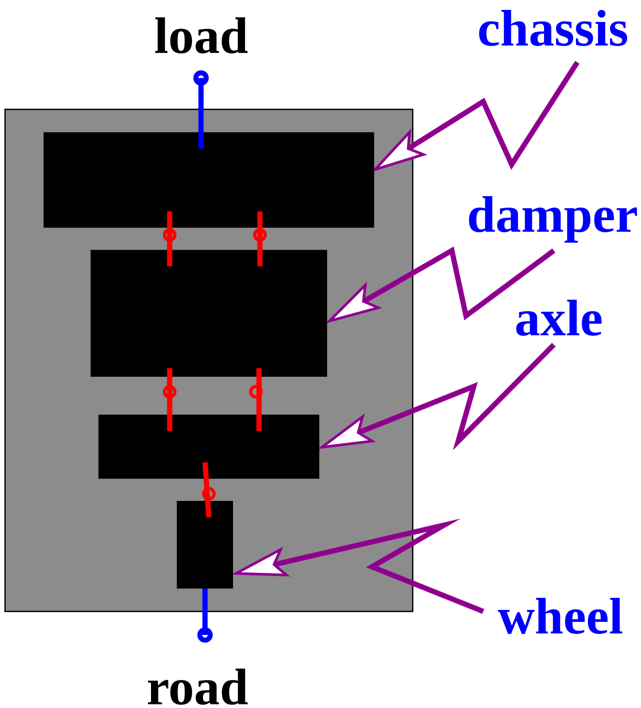
Electrical-mechanical analogies

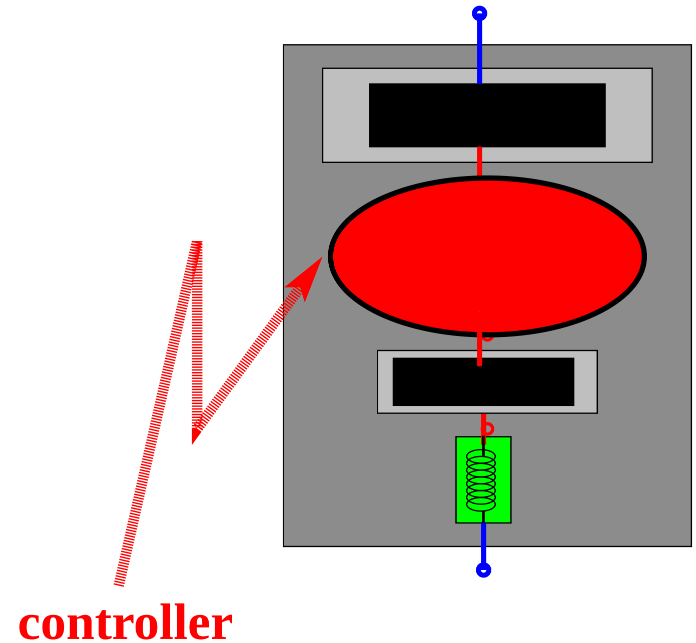
voltage $V \leftrightarrow v$ **velocity** **current** $I \leftrightarrow F$ **force**

Resistor	Damper
$\frac{1}{R}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	$D(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$
Inductor	**Spring**
$\frac{1}{L}(V_1 - V_2) = \frac{d}{dt} I_1, \;\; I_1 + I_2 = 0$	$K(v_1 - v_2) = \frac{d}{dt} F_1, \;\; F_1 + F_2 = 0$
Capacitor	**Inerter**
$C\frac{d}{dt}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	$B\frac{d}{dt}(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$

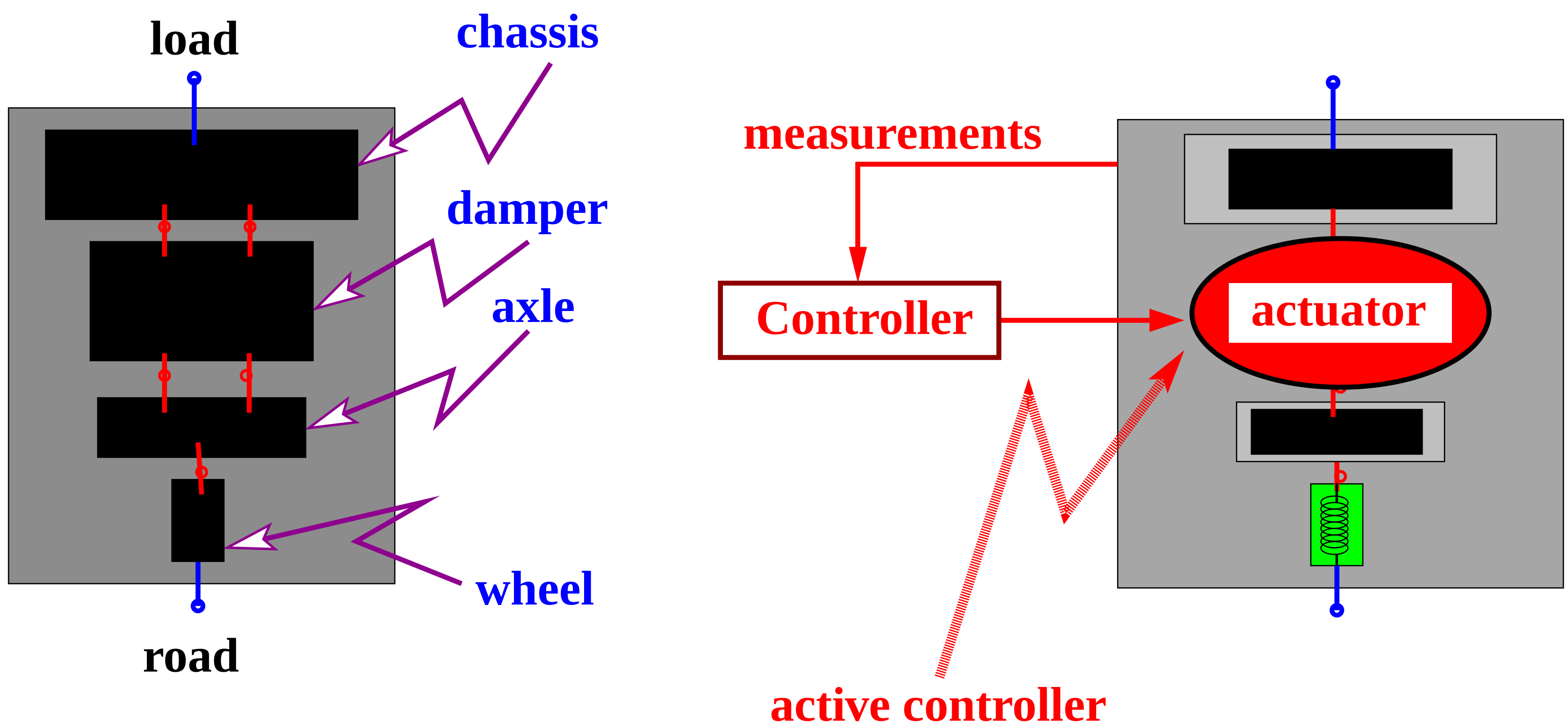
electrical RLC synthesis $\Leftrightarrow$ **mechanical DSI synthesis**

Example of behavioral control: A ‘quarter car’

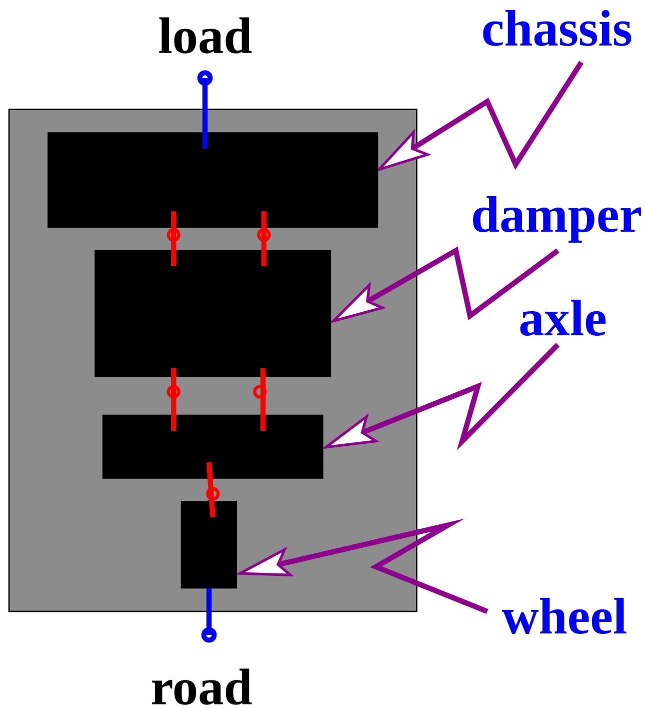


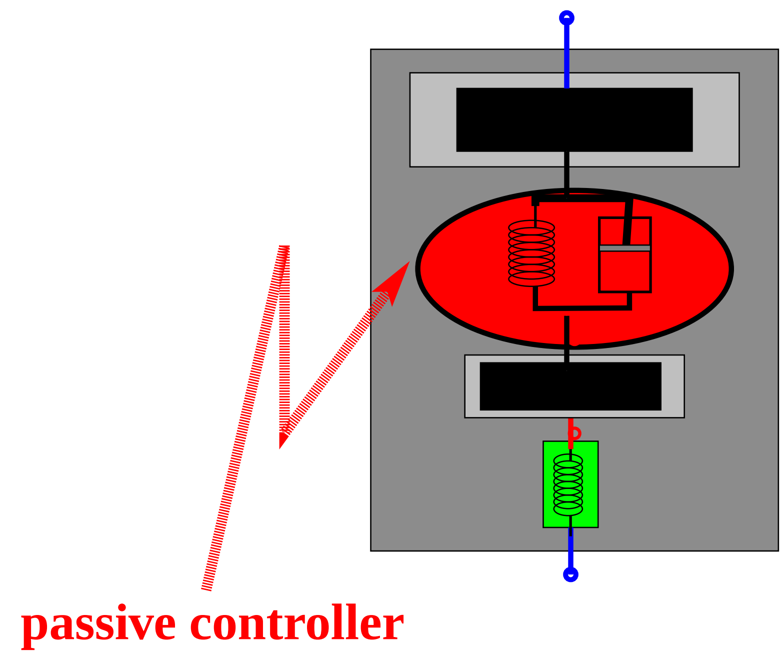


Example of behavioral control: A 'quarter car'



Example of behavioral control: A ‘quarter car’



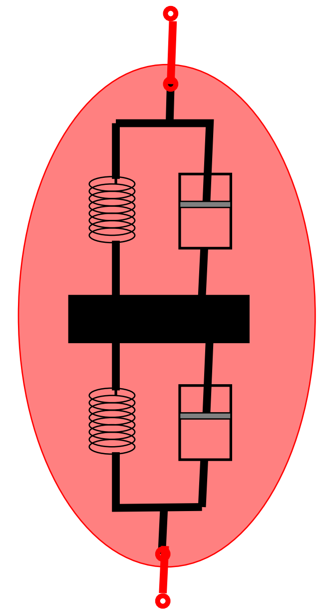


Suspension controller in Formula 1

Nigel Mansell victorious in 1992 with an active damper suspension.

Active dampers were banned in 1994 to break the dominance of the Williams team.

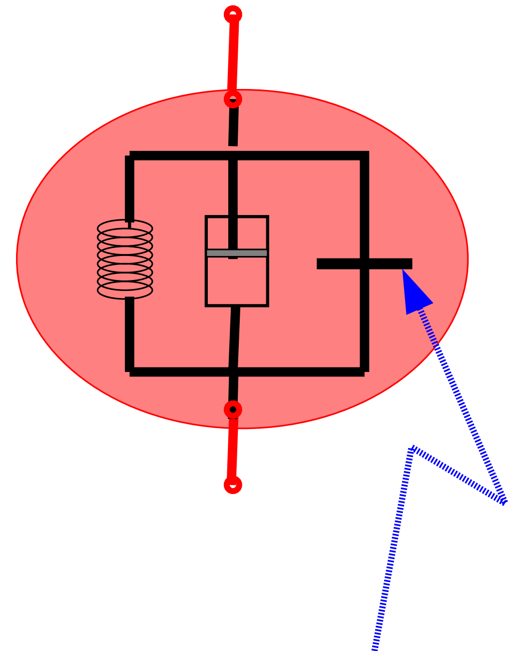
Suspension controller in Formula 1



Renault successfully use a passive ‘tuned mass damper’ in 2005/2006.

Banned in 2006, under the ‘movable aerodynamic devices’ clause.

Suspension controller in Formula 1



Kimi Räikkönen wins the 2005 Grand Prix in Spain with McLaren's 'J-damper', i.e., Smith's inerter.

![grandprix.com]

AUGUST 21, 2008

Ingenuity still brings success in Formula 1

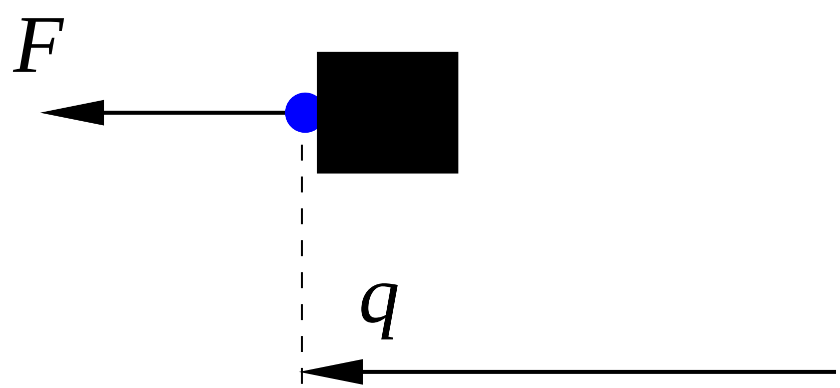
ShareThis

For years engineers have complained that the rules of Formula 1 mean that there is little room left for innovation but Cambridge University's engineering department has just revealed that this is not the case at all.

Professor Malcolm Smith, a fellow of Gonville and Caius College, created an innovative suspension system in the late 1990s and this was patented by the university. The first details were published in 2002 in the obscure Institute of Electrical and Electronics Engineers's publication called Transactions on Automatic Control. This was spotted by the boffins at McLaren and an exclusive deal was negotiated to allow the team to use the technology in F1. The new system was first used at the Spanish GP in 2005 and Kimi Raikkonen won.

The team used the name "J Damper" to describe the unit - in an effort to confuse the opposition - but it has now been revealed that it is actually called "an inerter". This is

Back to the mass

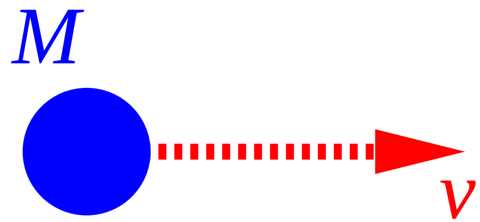


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \boxed{\frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q}$$

If $F^\top v$ is not power,

is $\frac{1}{2}M||\frac{d}{dt}q||^2$ not stored (kinetic, motion) energy ???

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

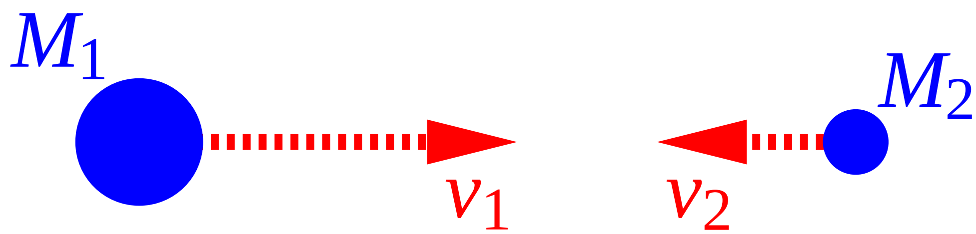
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem 's Gravesande
1688–1742

Émilie du Châtelet
1706–1749

This expression is not invariant under uniform motion.

Motion energy



What is the motion energy?

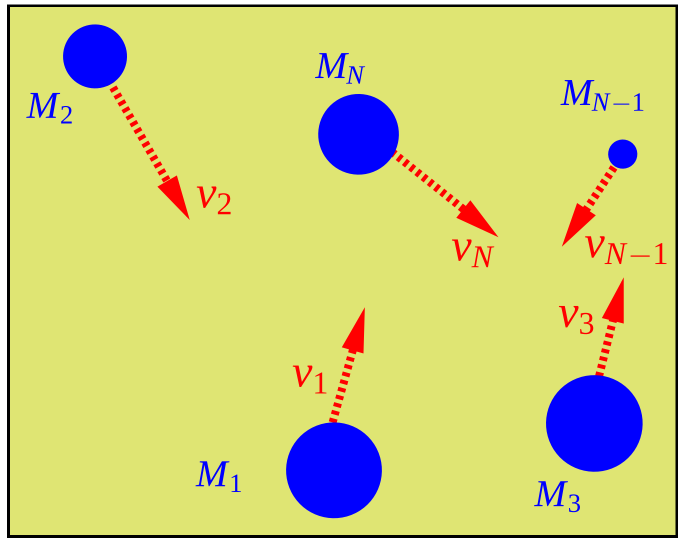
What quantity is transformable into heat?

$$\mathscr{E}_{\textbf{motion}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1 - v_2||^2$$

Invariant under uniform motion.

Motion energy

Generalization to N masses.

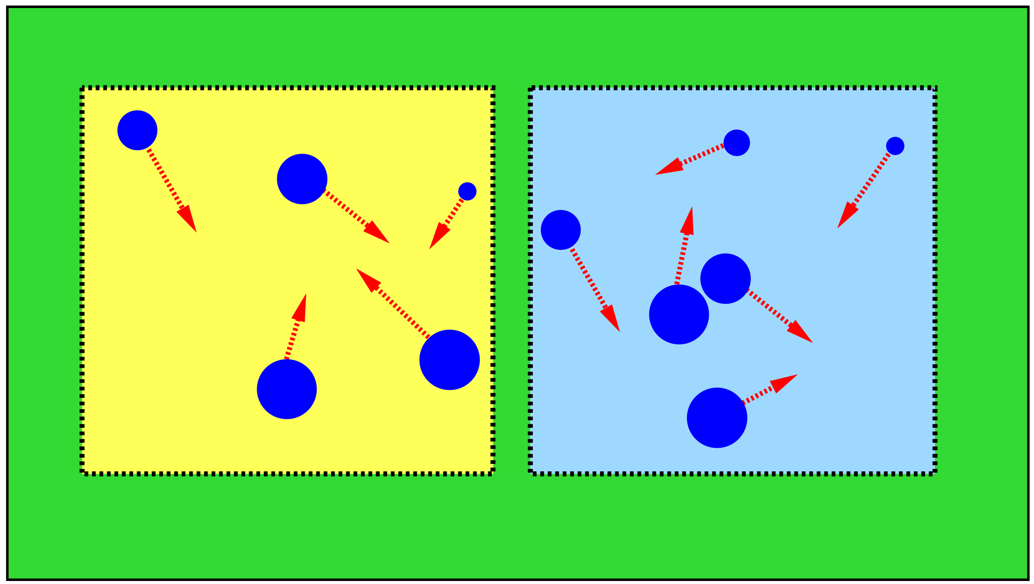


$$\mathscr{E}_{\mathbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

$$\textbf{KFL} \;\Rightarrow\; \frac{d}{dt}\mathscr{E}_{\mathbf{motion}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i.$$

Motion energy

Motion energy is not an extensive quantity, it is not additive.



Total motion energy $\neq$ sum of the parts.

Motion energy

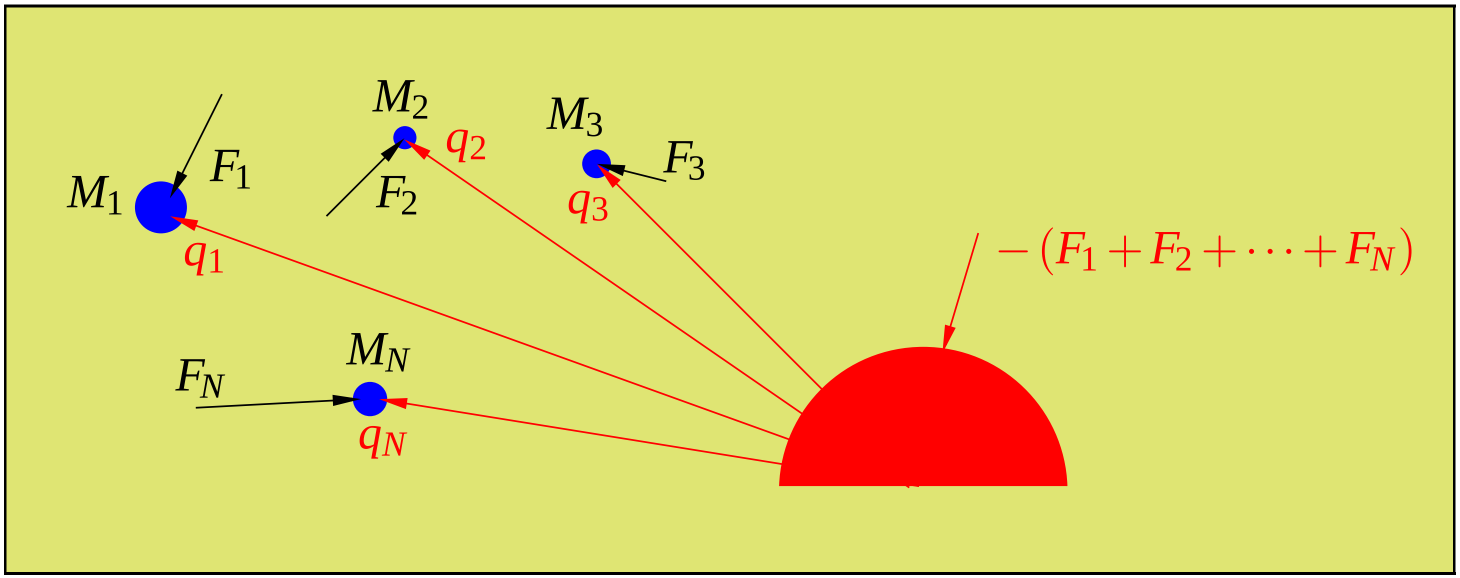
$$\mathcal{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2 .$$

Distinct from the classical expression of the kinetic energy,

$$\mathcal{E}_{\textbf{kinetic}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2 .$$

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



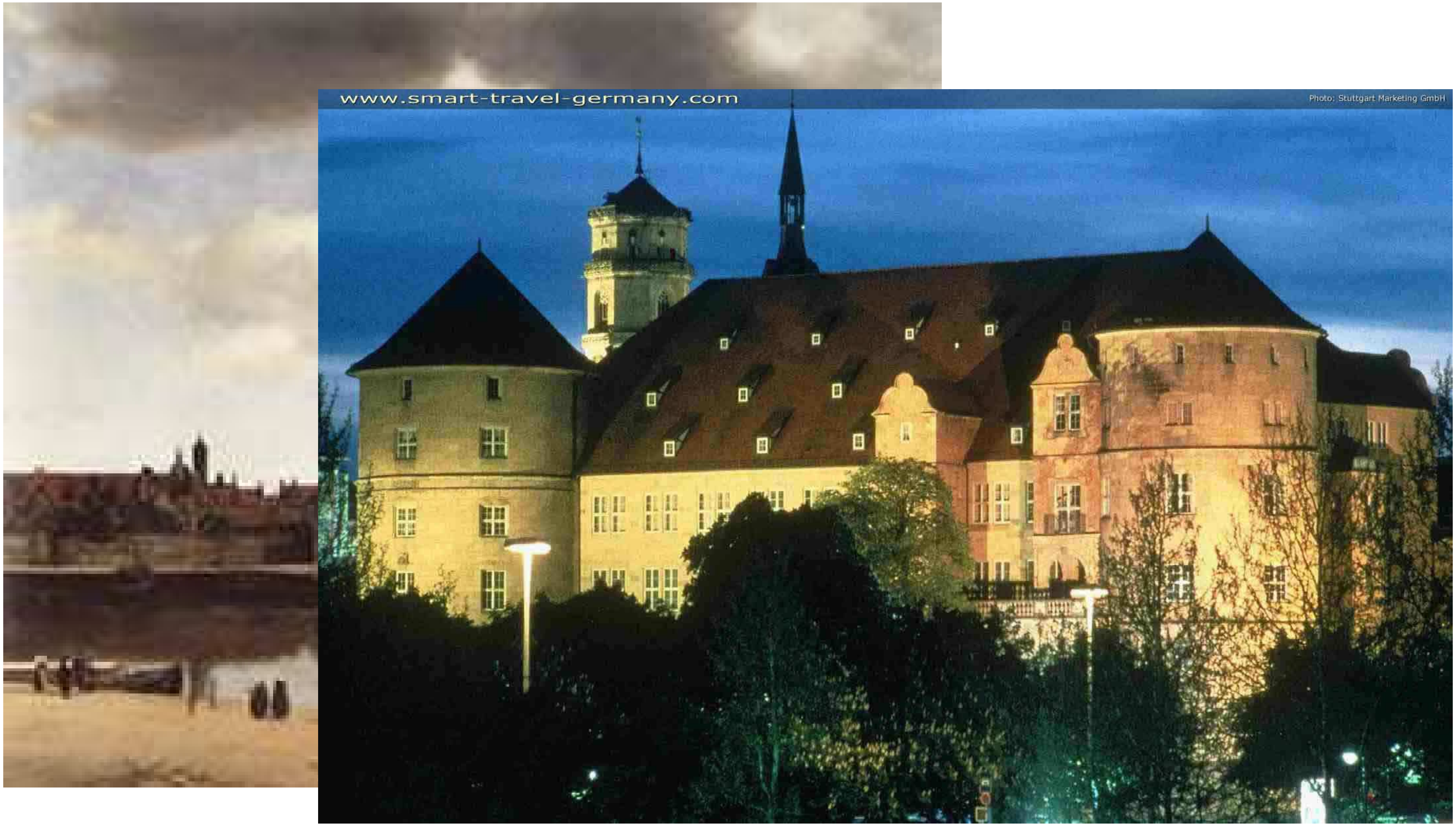
measure velocities w.r.t. this infinite mass ('ground'), then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \qquad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

Es geht alles vorüber, es geht alles vorbei ...

Es geht alles vorüber, es geht alles vorbei ...



Carsten, het ga je goed in Stuttgart!