TERMINALS and PORTS

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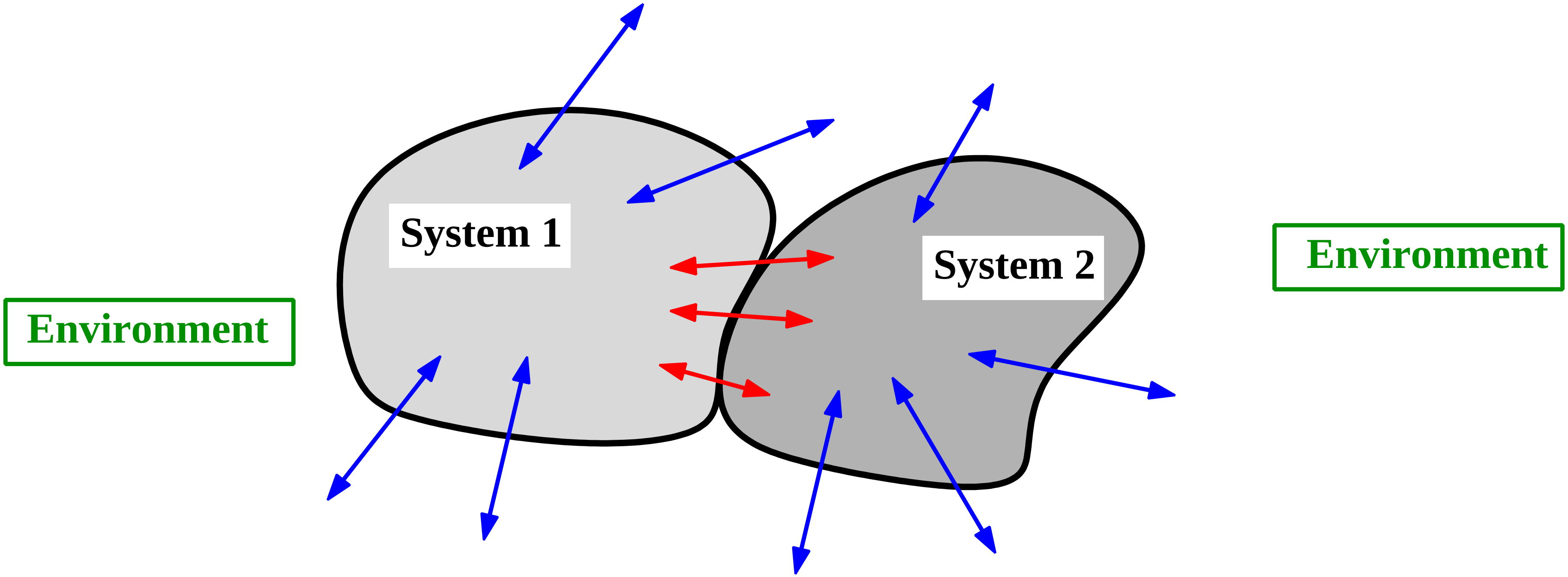
Back to Basics **colloquium**

Universidade do Porto, October 27, 2010

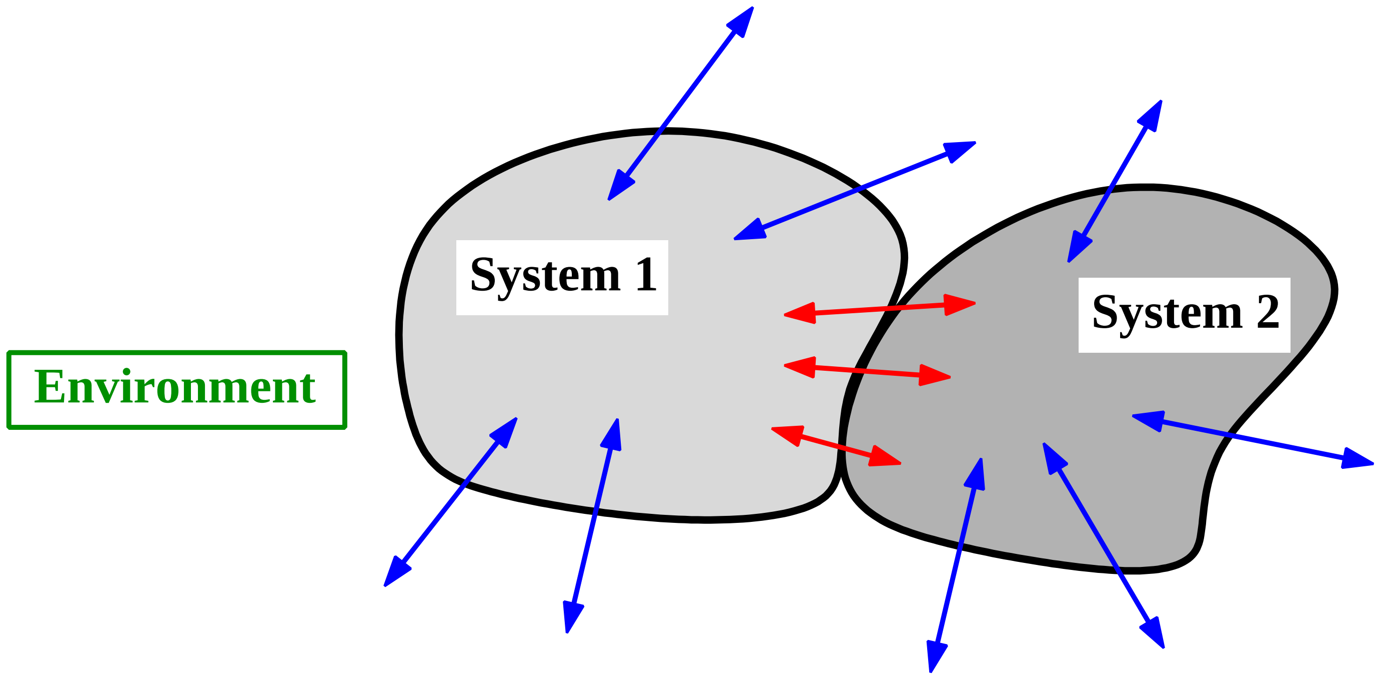
Theme



How does energy flow from the environment into a system?

How is energy transferred between systems?

Theme

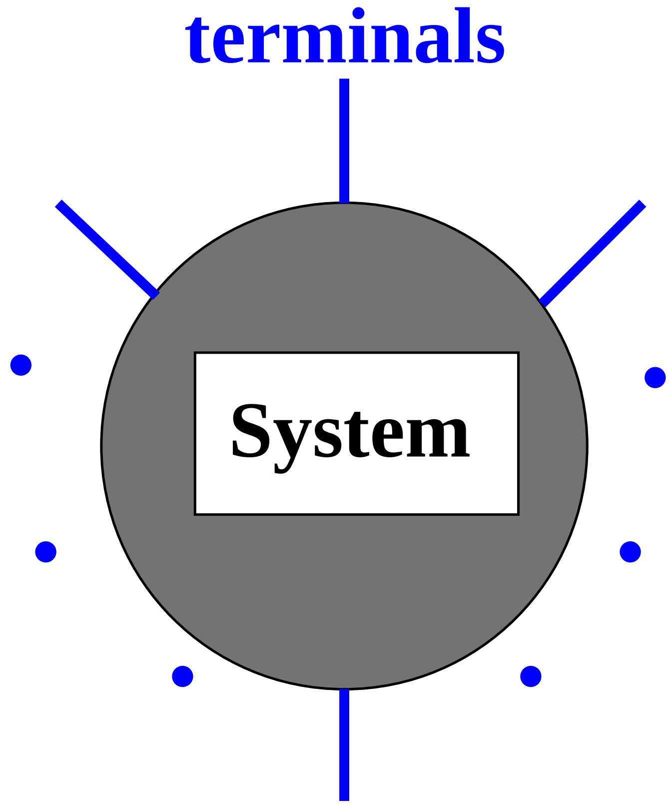


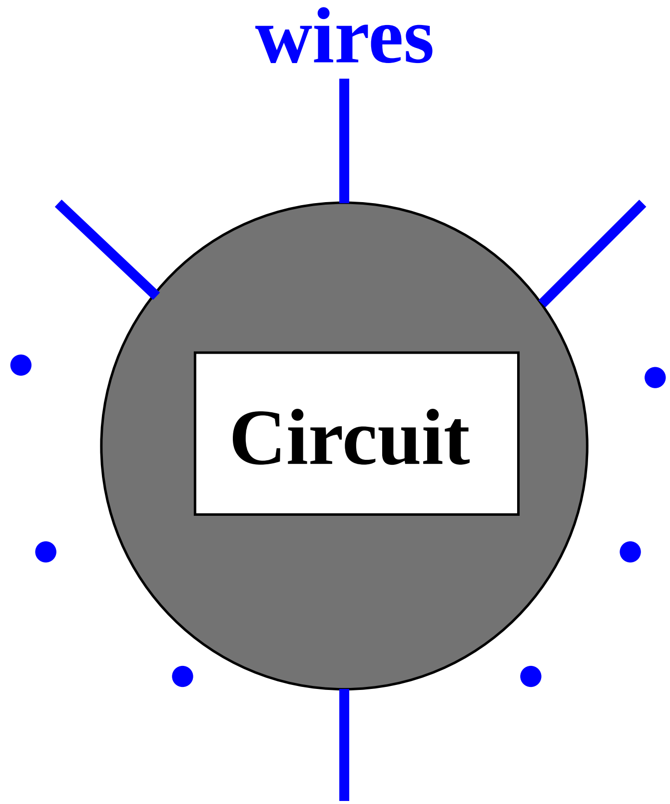
How does energy flow from the environment into a system?

How is energy transferred between systems?

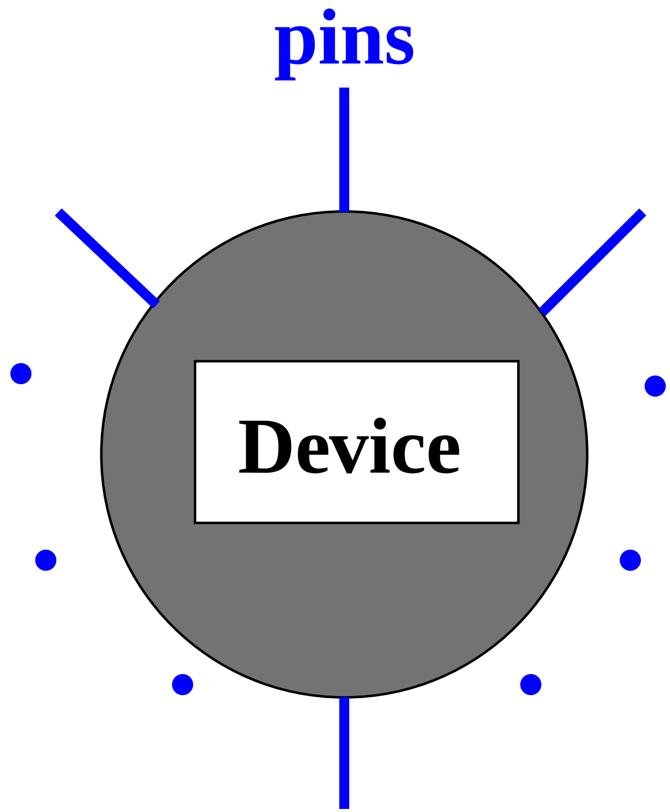
Today: **electrical circuits** **(&** **mechanical systems).**

Systems with terminals


terminals
System

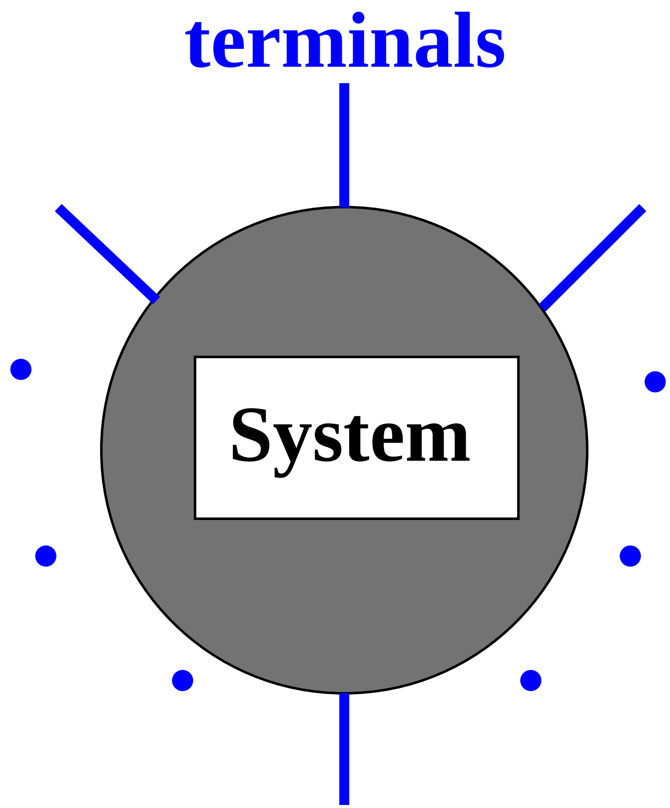


For example, an electrical circuit with wires,



For example, an electrical circuit with wires,

a mechanical device with pins,



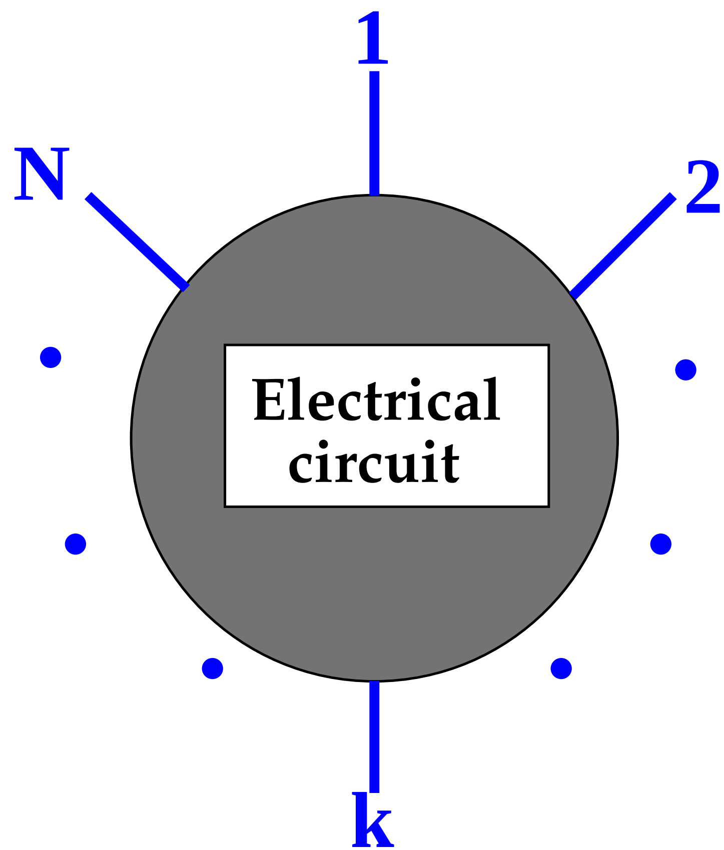
For example, an electrical circuit with wires,

a mechanical device with pins,

hydraulics with pipes, heat exchange with ducts, etc.

Interaction variables

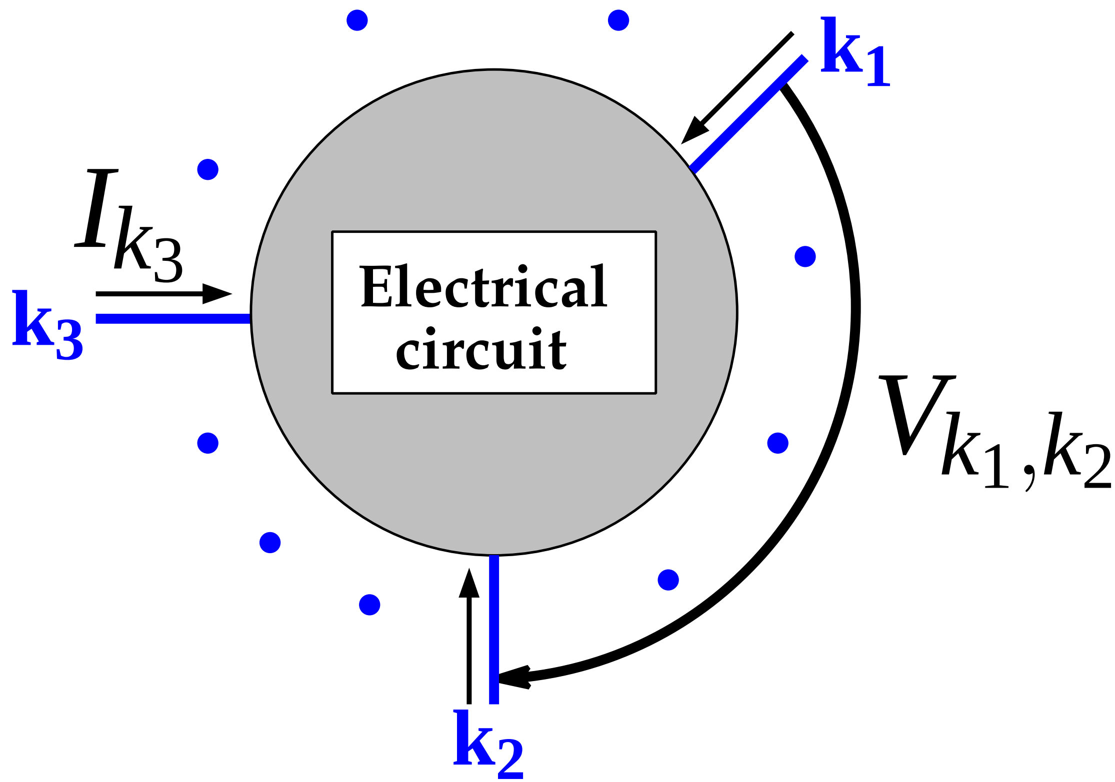
Electrical circuits



¡¡ Describe electrical interaction with environment !!

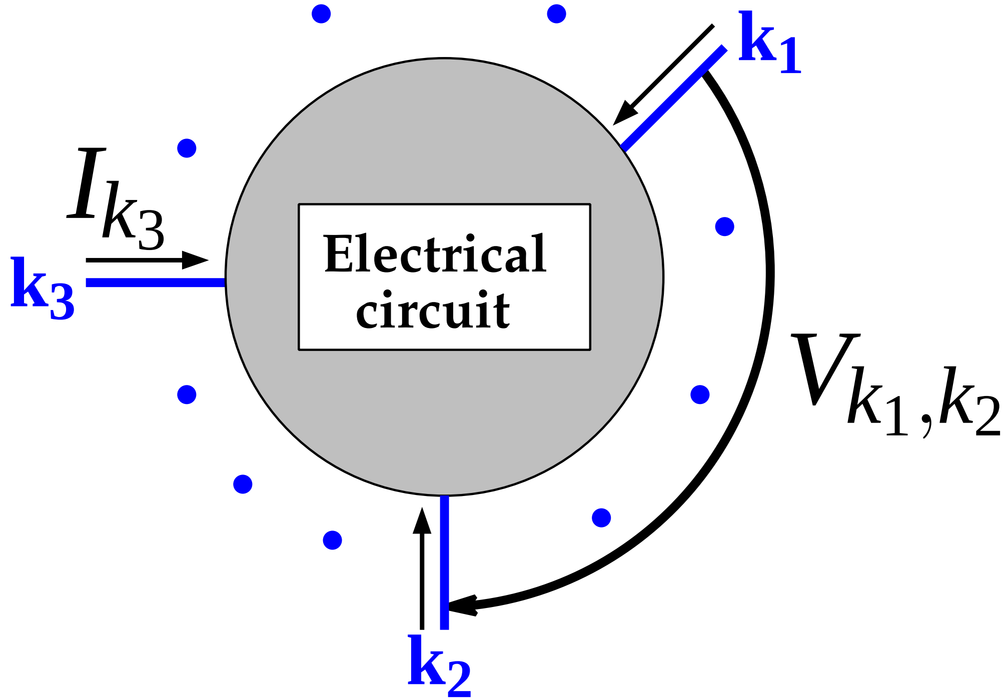
What are the interaction variables?

Currents and voltages



interaction variables: **currents in** & **voltages across.**

Measurable by ammeters and voltmeters.



$$\rightsquigarrow \qquad I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \quad V = \begin{bmatrix} V_{1,1} & V_{1,2} & \cdots & V_{1,N} \\ V_{2,1} & V_{2,2} & \cdots & V_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N,1} & V_{N,2} & \cdots & V_{N,N} \end{bmatrix}.$$

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$\rightsquigarrow \Sigma_{IV} = \left(\mathbb{R}, \mathbb{R}^N \times \mathbb{R}^{N\times N}, \mathscr{B}_{IV}\right), \quad \mathscr{B}_{IV} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^{N\times N}\right)^{\mathbb{R}}$**.**

$(I,V) \in \mathscr{B}_{IV}$ **means**

$\left(I_1, I_2, \ldots, I_k, \ldots, I_N, V_{1,1}, V_{1,2}, \ldots, V_{k_1,k_2}, \ldots, V_{N,N}\right) : \mathbb{R} \longrightarrow \mathbb{R}^N \times \mathbb{R}^{N\times N}$

is compatible with circuit architecture and element values.

Exactly the trajectories $(I,V) \in \mathscr{B}_{IV}$ **can conceivably occur.**

$$\rightsquigarrow \qquad I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \quad V = \begin{bmatrix} V_{1,1} & V_{1,2} & \cdots & V_{1,N} \\ V_{2,1} & V_{2,2} & \cdots & V_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N,1} & V_{N,2} & \cdots & V_{N,N} \end{bmatrix}.$$

$$\rightsquigarrow \Sigma_{IV} = \left(\mathbb{R}, \mathbb{R}^N \times \mathbb{R}^{N \times N}, \mathscr{B}_{IV}\right), \quad \mathscr{B}_{IV} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^{N \times N}\right)^{\mathbb{R}}.$$

‘Behavioral approach’

$\Leftrightarrow$ a dynamical system = a set of trajectories.

KVL

Kirchhoff voltage law (KVL):

$$[\![(I,V) \in \mathscr{B}_{IV}]\!]$$

$$\Rightarrow \quad [\![V_{k_1,k_2} + V_{k_2,k_3} + V_{k_3,k_4} + \cdots + V_{k_{n-1},k_n} + V_{k_n,k_1} = 0$$

$$\textbf{for all } k_1, k_2, \ldots, k_n \in \{1, 2, \ldots, N\}]\!].$$

KVL

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$$\textbf{KVL} \quad \Leftrightarrow \quad V_{k_1,k_2} + V_{k_2,k_3} + V_{k_3,k_1} = 0$$

$$\forall\, k_1, k_2, k_3 \in \{1, 2, \ldots, N\}.$$

$$\Rightarrow \quad V_{k_1,k_2} = -V_{k_2,k_1} \quad \forall\, k_1, k_2 \in \{1, 2, \ldots, N\}.$$

Potentials

Thm: $V : \mathbb{R} \to \mathbb{R}^{N \times N}$ **satisfies KVL** $\quad \Leftrightarrow$

$$\exists\, P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} : \mathbb{R} \to \mathbb{R}^N \text{ such that } V_{k_1,k_2} = P_{k_1} - P_{k_2}.$$

Potentials

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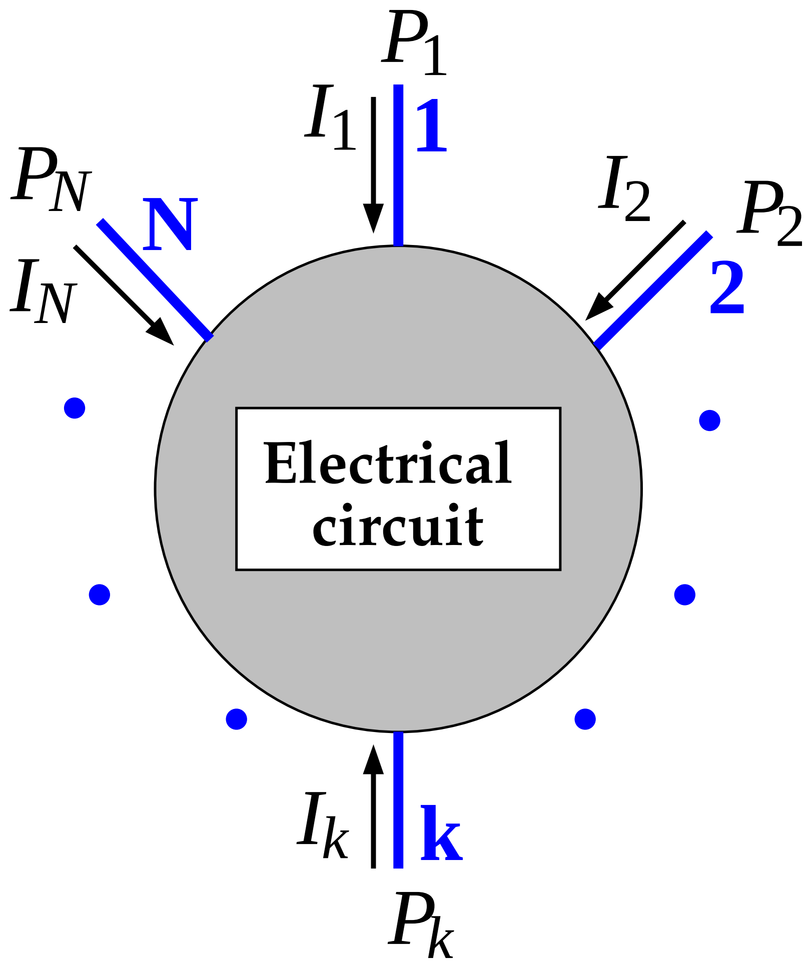
$$P \text{ 'potential'} \Rightarrow \begin{bmatrix} P_1+\alpha \\ P_2+\alpha \\ \vdots \\ P_N+\alpha \end{bmatrix} \text{ potential } \quad \forall\, \alpha : \mathbb{R} \to \mathbb{R}.$$

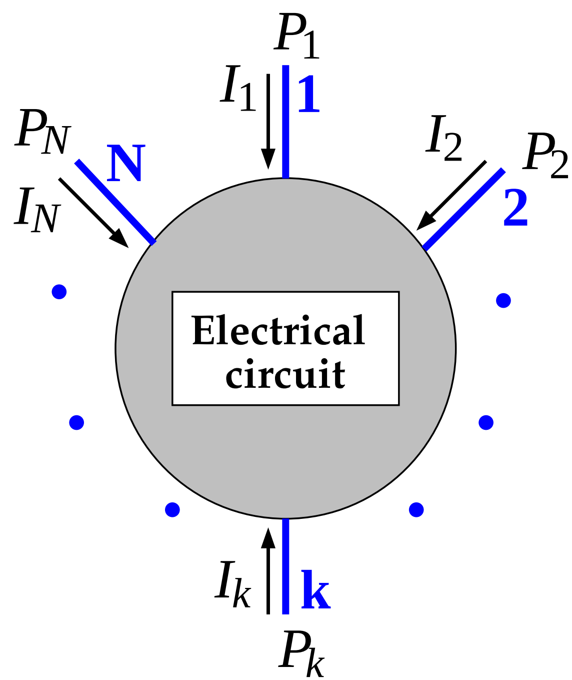
Potentials '<u>unobservable</u>' from currents & voltages.

Currents and potentials



KVL $\Rightarrow$ at each terminal: a current and a potential

$$\rightsquigarrow \Sigma_{IP} = \left(\mathbb{R}, \mathbb{R}^N \times \mathbb{R}^N, \mathscr{B}_{IP}\right), \quad \mathscr{B}_{IP} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$



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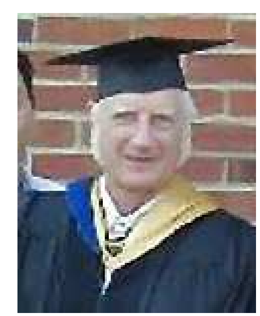
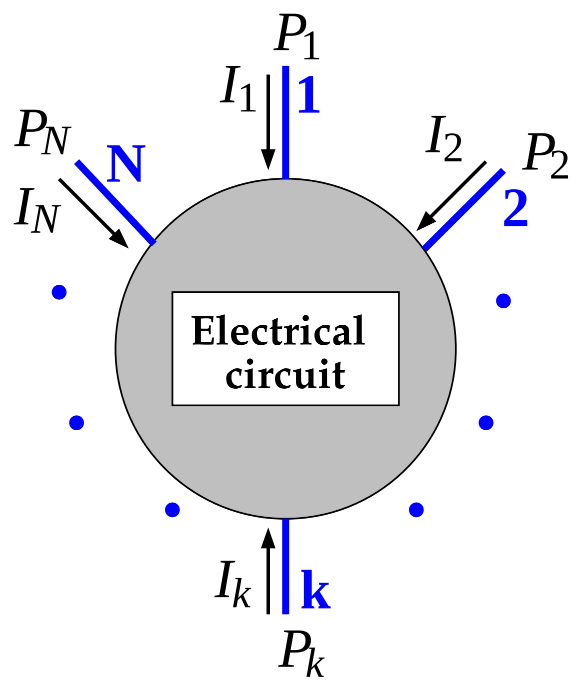
Early sources:

Brockway McMillan



Robert Newcomb

KVL for potentials

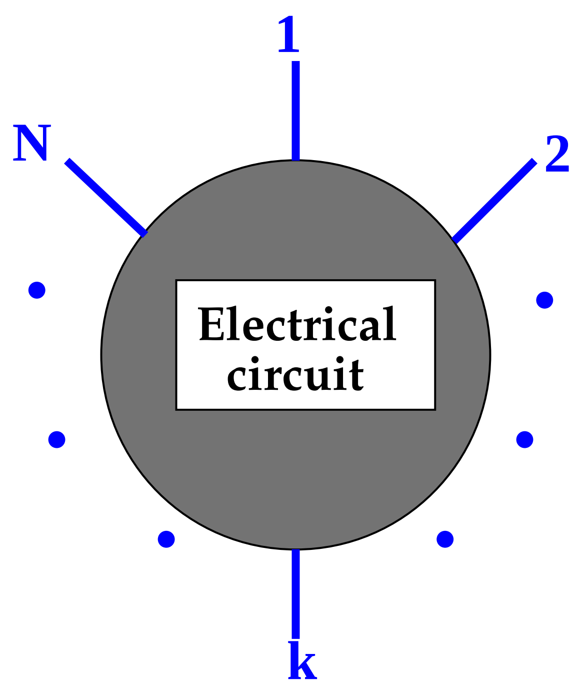


Kirchhoff voltage law (KVL):

$$[\![(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) \in \mathscr{B}_{IP} \text{ and } \alpha : \mathbb{R} \to \mathbb{R}]\!]$$
$$\Rightarrow [\![(I_1, I_2, \ldots, I_N, P_1 + \alpha, P_2 + \alpha, \ldots, P_N + \alpha) \in \mathscr{B}_{IP}]\!].$$

Only $P_{k_1} - P_{k_2}$ appears in the behavioral equations.

KCL



Kirchhoff current law (KCL):

$$[\![(I_1, I_2, \ldots, I_N, V_{1,1}, V_{1,2}, \ldots, V_{k_1,k_2}, \ldots, V_{N,N}) \in \mathscr{B}_{IV}]\!]$$

$$\Rightarrow [\![I_1 + I_2 + \cdots + I_N = 0]\!].$$

Assuming KVL, (KCL):

$$[\![(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) \in \mathscr{B}_{IP}]\!] \Rightarrow [\![I_1 + I_2 + \cdots + I_N = 0]\!].$$

Modeling problem

Given an electrical circuit,

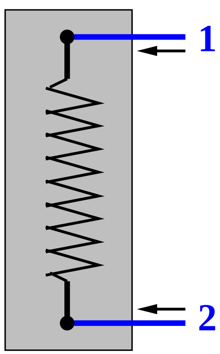
specify the current/voltage behavior

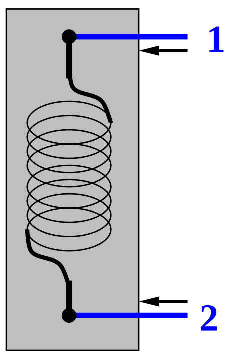
$$\mathscr{B}_{IV} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^{N\times N}\right)^{\mathbb{R}}$$

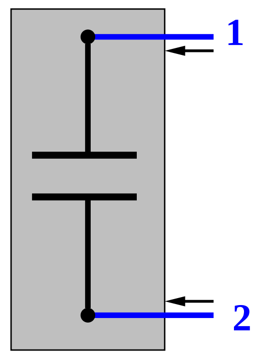
or, assuming KVL, the current/potential behavior

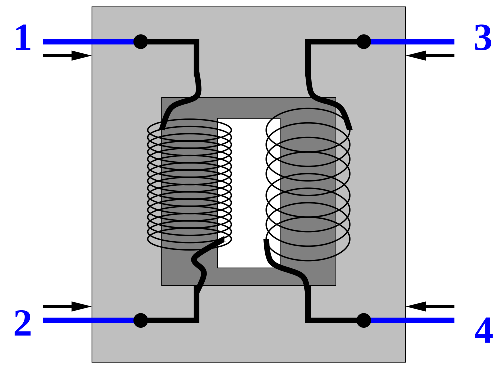
$$\mathscr{B}_{IP} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

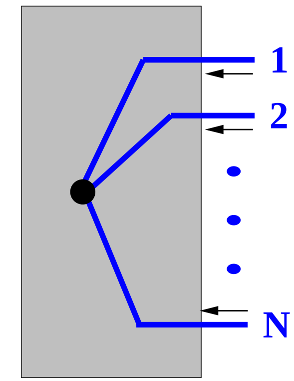
Standard elements





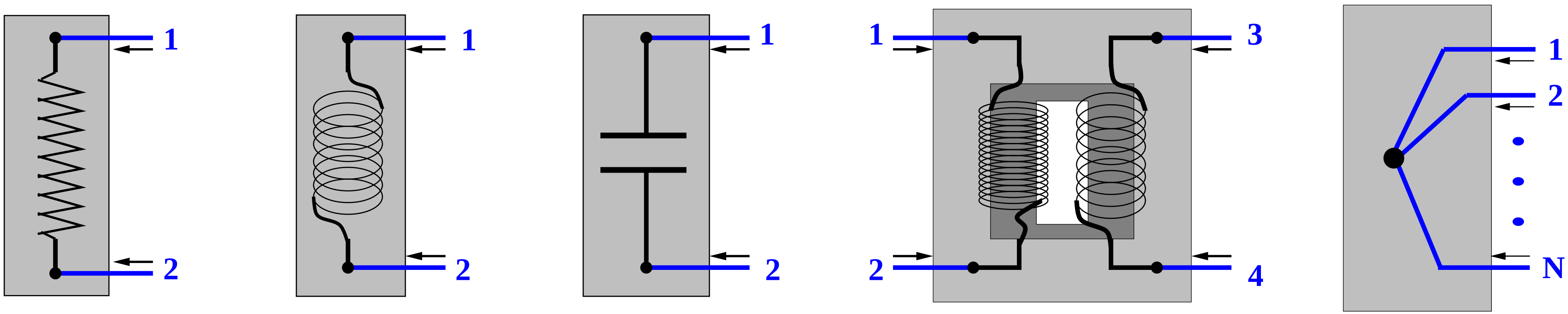






transistors, gyrators, current sources, voltage sources, OPAMPs, ...

Standard elements



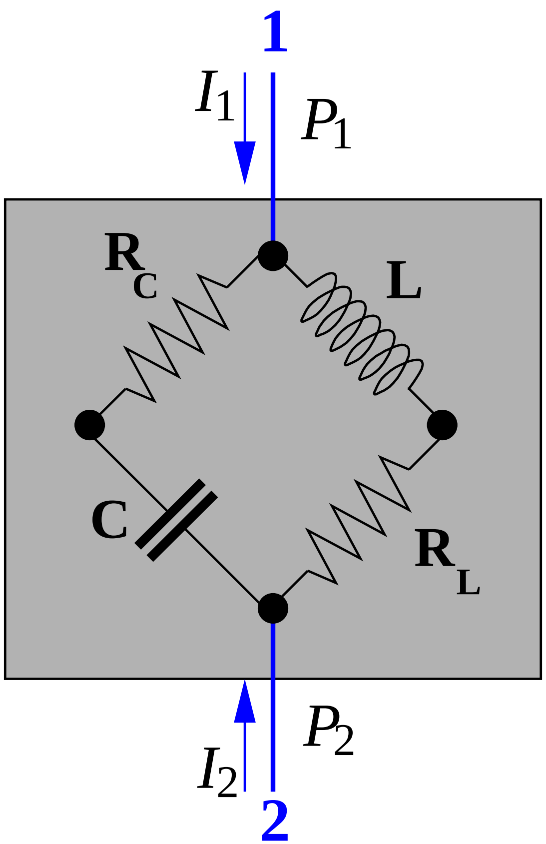
transistors, gyrators, current sources, voltage sources, OPAMPs, ...

$$
\begin{array}{lrcll}
\textbf{resistor:} & P_1 - P_2 & = & R\, I_1, & I_1 + I_2 = 0, \\
\textbf{inductor:} & P_1 - P_2 & = & L\, \frac{d}{dt} I_1, & I_1 + I_2 = 0, \\
\textbf{capacitor:} & C\, \frac{d}{dt}(P_1 - P_2) & = & I_1, & I_1 + I_2 = 0,
\end{array}
$$

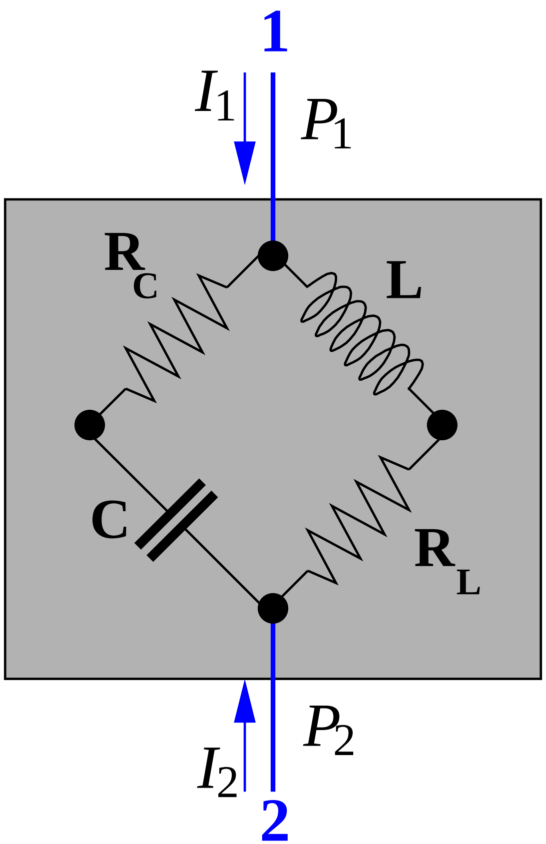
trafo: $P_3 - P_4 = n\,(P_1 - P_2),\ I_1 = -n\, I_3, I_1 + I_2 = 0,\ I_3 + I_4 = 0,$

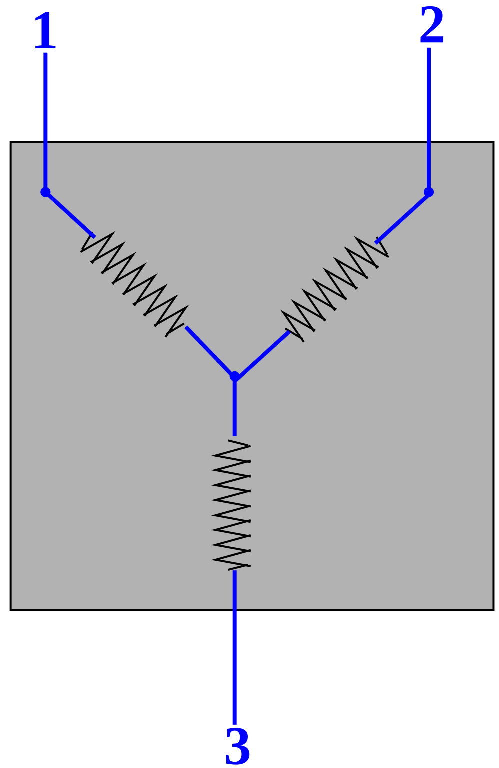
connector: $I_1 + I_2 + \cdots + I_N = 0, \quad P_1 = P_2 = \cdots = P_N.$

Examples

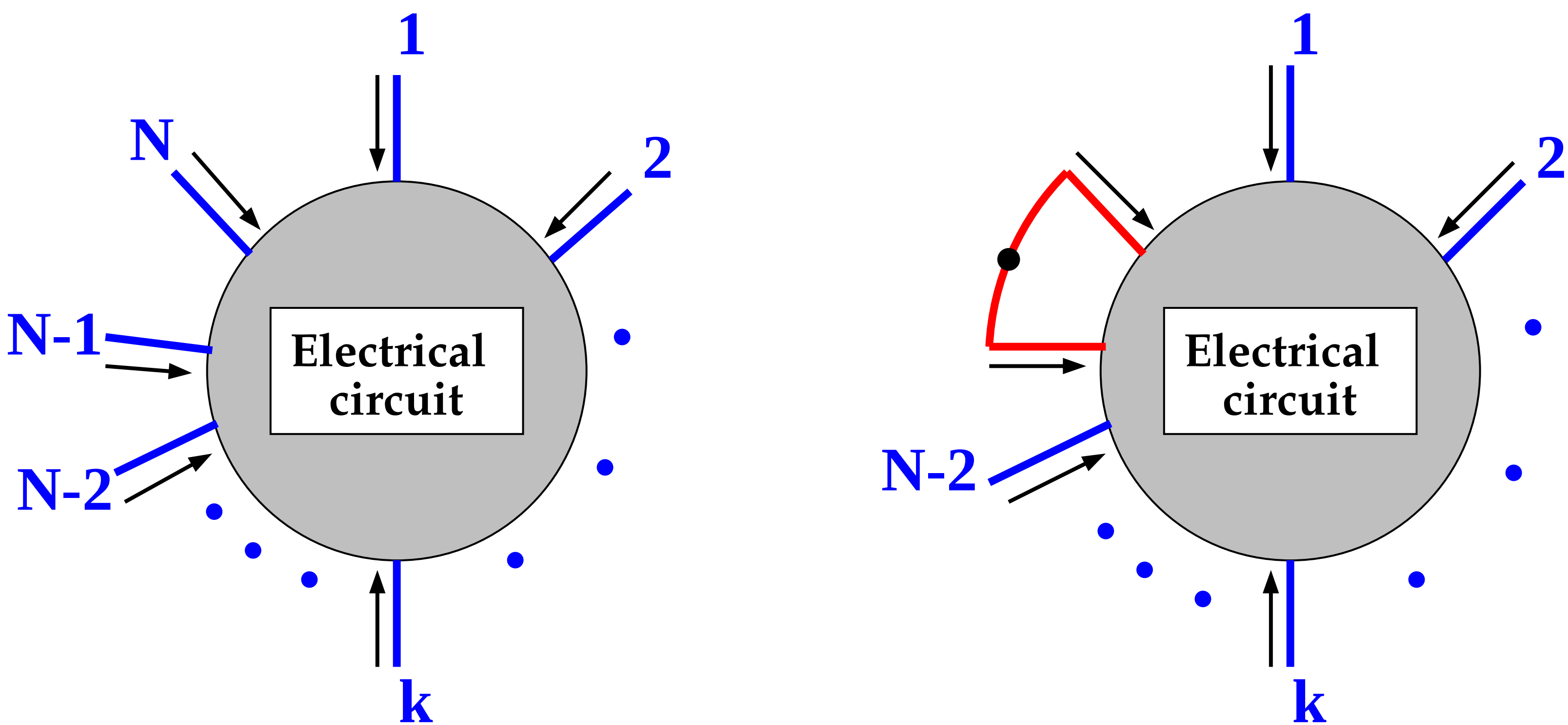


Examples





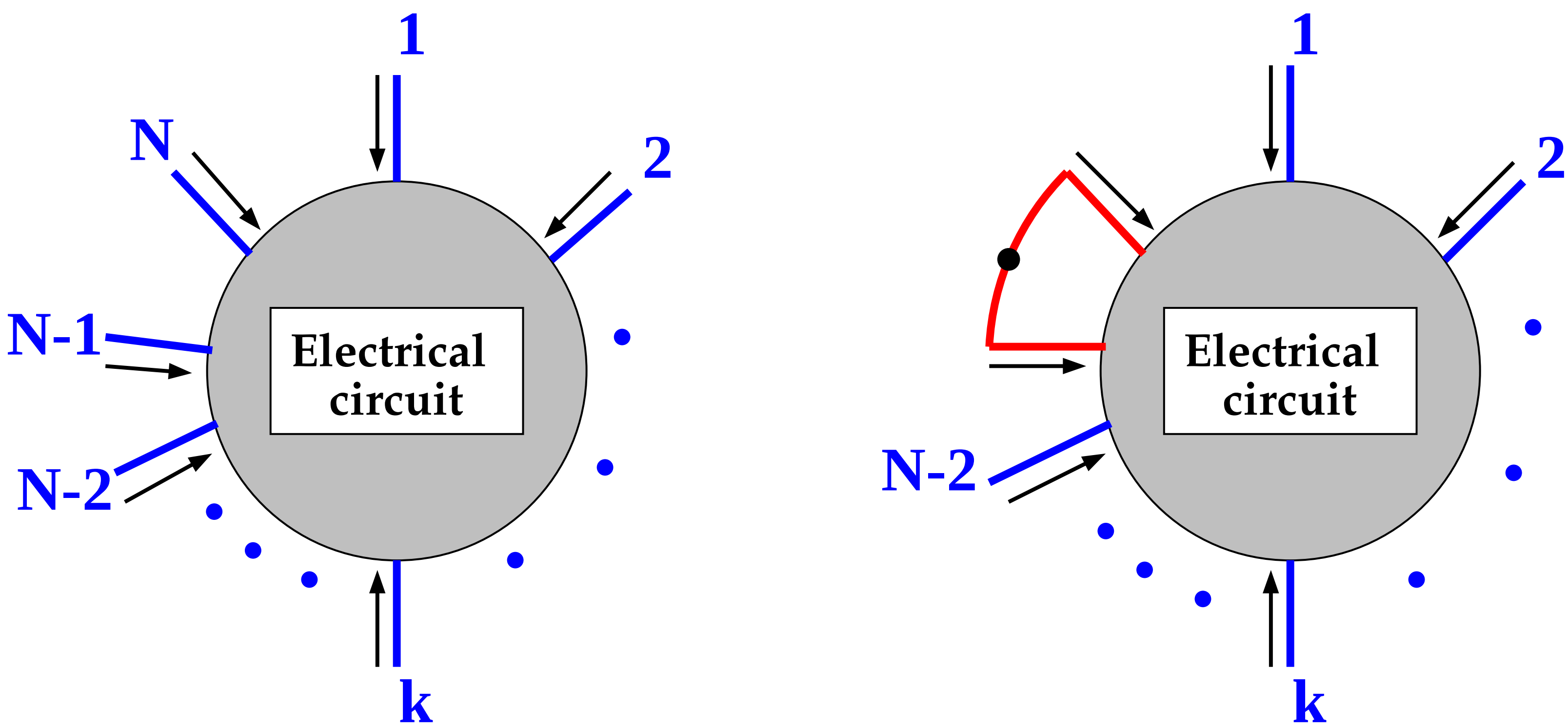
Interconnection



Imposes, in addition to behavioral equations,

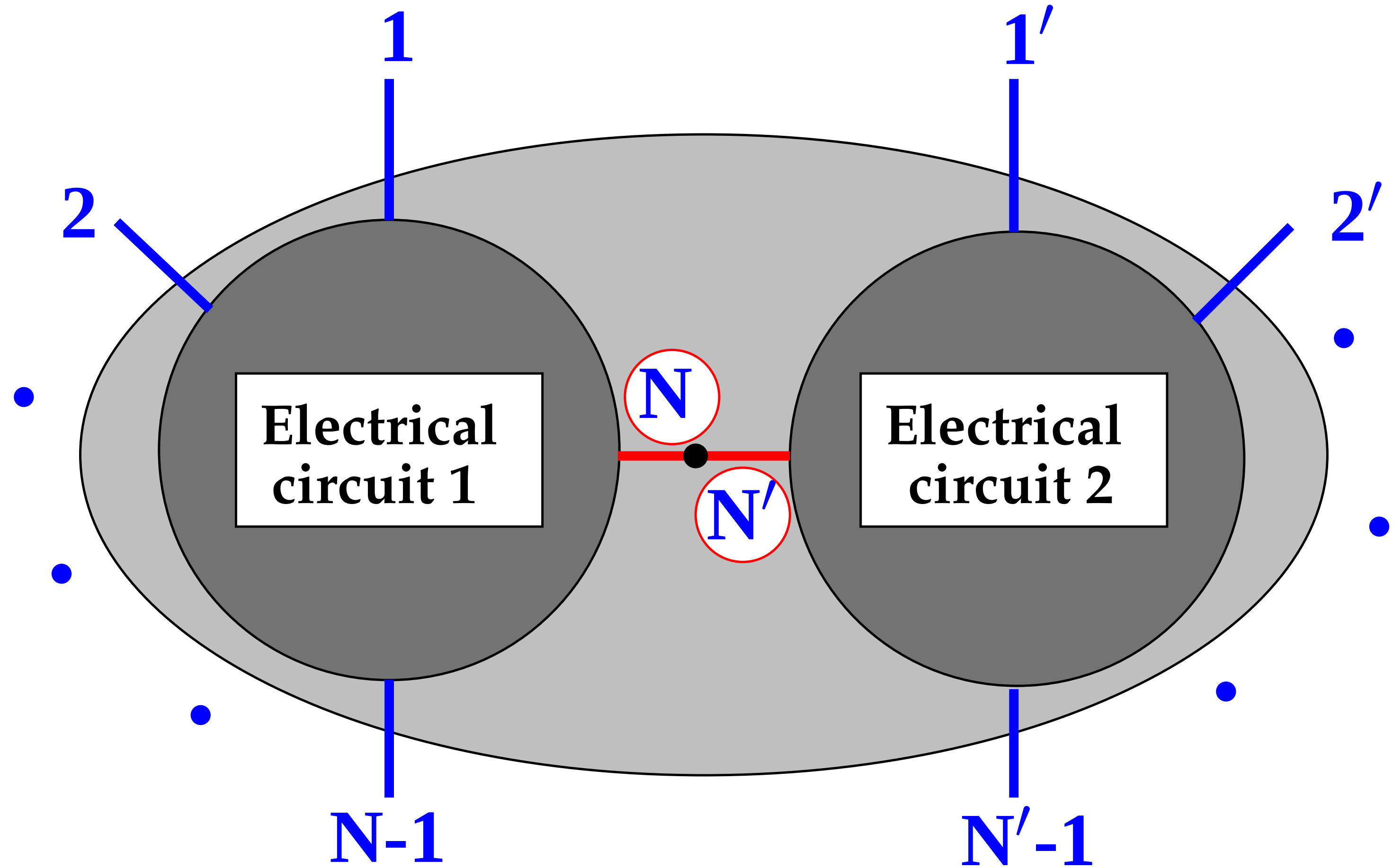
$V_{N-1,k} = V_{N,k}$ $k = 1, 2, \ldots, N$ **and** $I_{N-1} + I_N = 0.$

$\rightsquigarrow N - 2$ **terminals. Preserves KVL and KCL.**



Imposes, in addition to behavioral equations, assuming KVL,

$$P_{N-1} = P_N \quad \textbf{and} \quad I_{N-1} + I_N = 0.$$

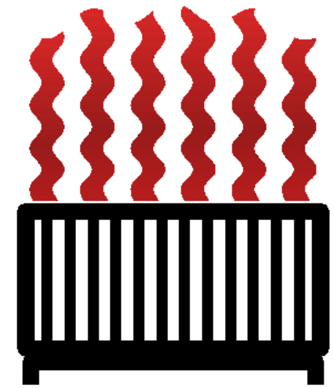

1
1′
2
2′
Electrical
circuit 1
N
N′
Electrical
circuit 2
N-1
N′-1

Energy Transfer

Energy

Energy

:= physical quantity transformable into heat.



Energy

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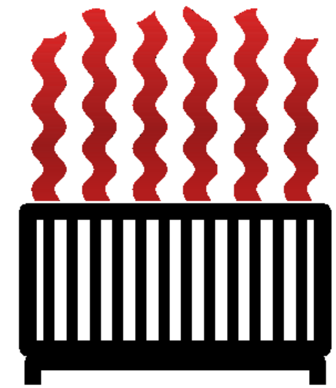
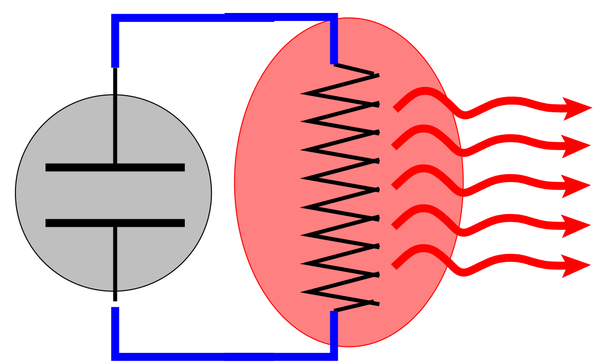


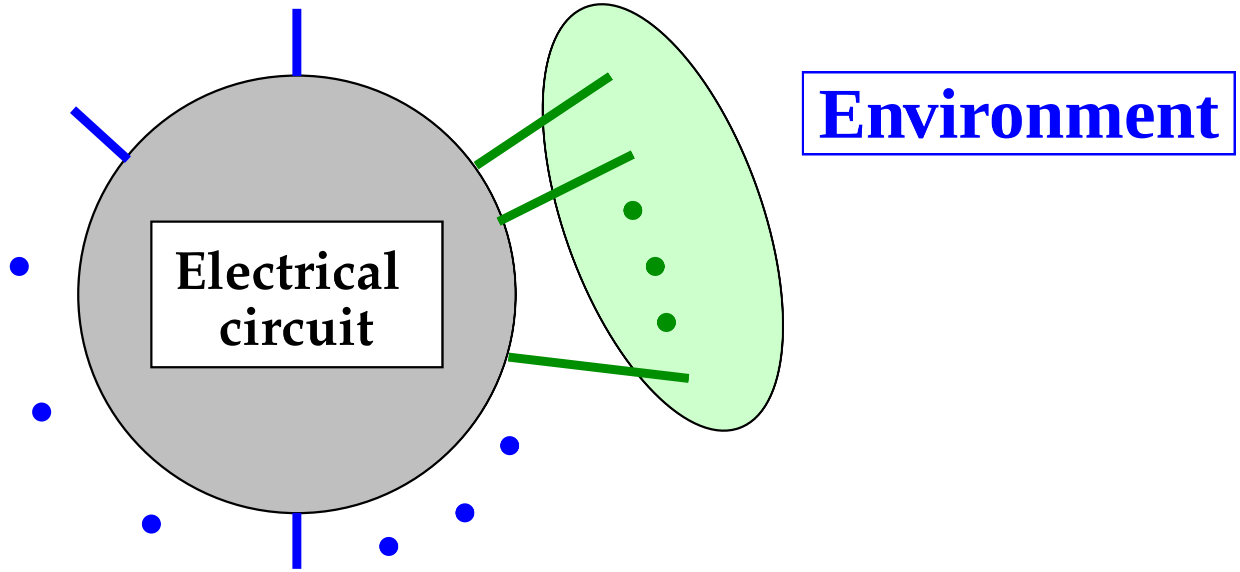
For example, capacitor $\mapsto$ resistor $\mapsto$ heat.

Energy on capacitor = $\frac{1}{2}CV^2$



Electrical ports

Energy transfer

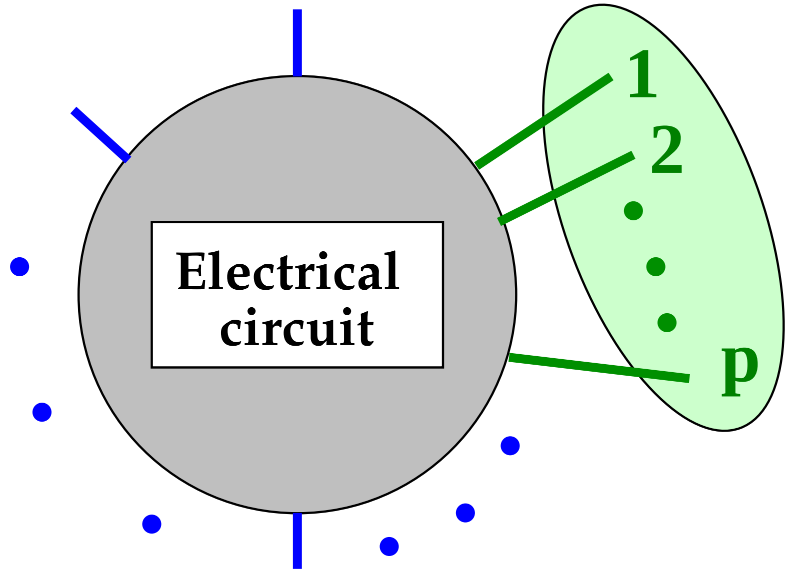


Monitor the current/potential on a set of terminals.

Can we speak about *the energy transferred from the environment to the circuit along these terminals*?

Ports

Assume KVL.



Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

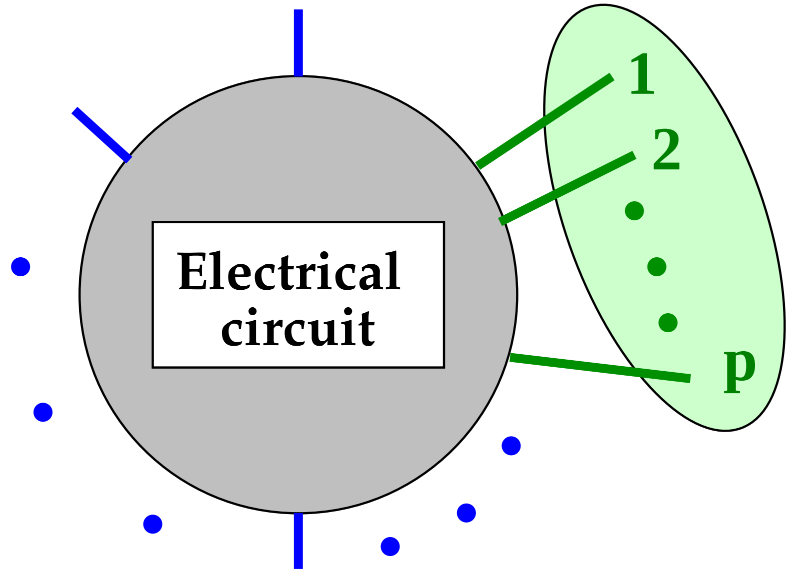
$$[\![\left(I_1, \ldots, I_p, I_{p+1}, \ldots, I_N, P_1, \ldots, P_p, P_{p+1}, \ldots, P_N\right) \in \mathscr{B}_{IP}]\!]$$

$$\Rightarrow \quad [\![I_1 + I_2 + \cdots + I_p = 0]\!].$$

'port KCL'

KCL $\Leftrightarrow$ **all terminals together form a port.**

Ports



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

power in = $P_1(t)I_1(t) + P_2(t)I_2(t) + \cdots + P_p(t)I_p(t)$

energy in = $\int_{t_1}^{t_2} [P_1(t)I_1(t) + P_2(t)I_2(t) + \cdots + P_p(t)I_p(t)]\, dt$

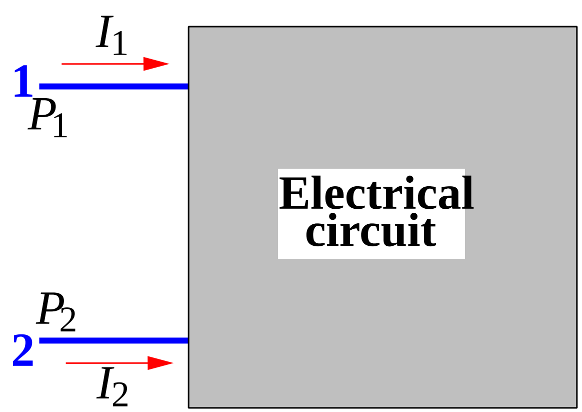
This interpretation in terms of power and energy is not valid
unless these terminals form a port !

Examples

2-terminal 1-port devices:

resistors, inductors, capacitors, memristors, etc.,

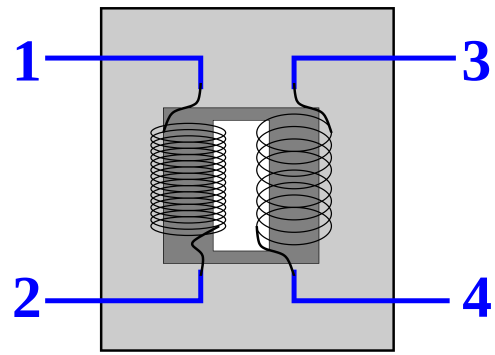
any 2-terminal circuit composed of these.



KCL $\Rightarrow$ **a port** $(I_1 = -I_2)$.

Example

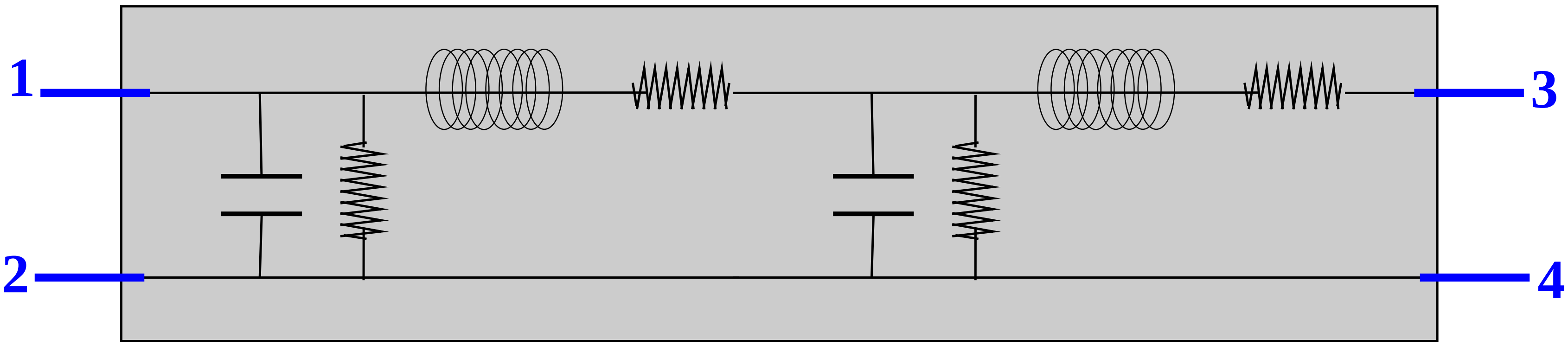
2-port: a transformer.



$$P_3 - P_4 = n\,(P_1 - P_2),\ I_1 = -n\,I_3, I_1 + I_2 = 0,\ I_3 + I_4 = 0.$$

$\{1,2\}$ **and** $\{3,4\}$ **form ports.**

Examples



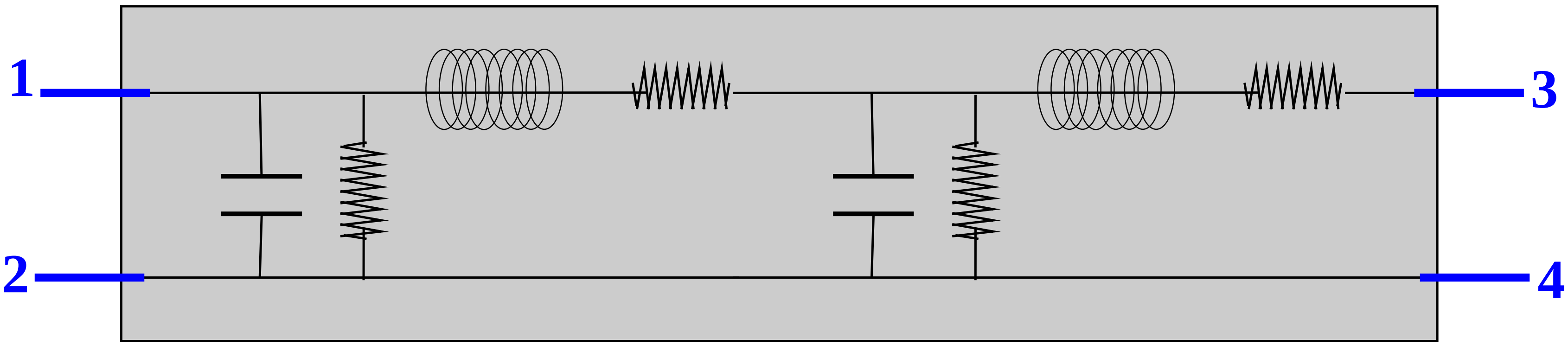
Terminals $\{1, 2, 3, 4\}$ **form a port.**

$\{1, 2\}$ **and** $\{3, 4\}$ **do not.**

We cannot speak about

the energy transferred from terminals $\{1, 2\}$ ***to*** $\{3, 4\}$**.**

Examples



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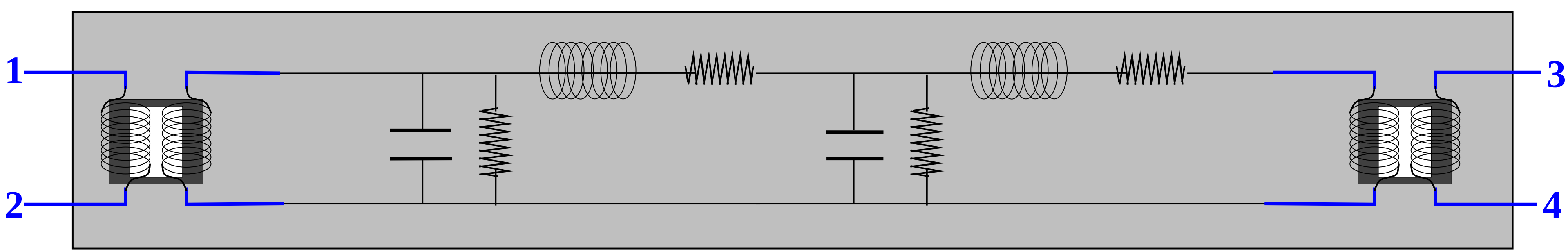
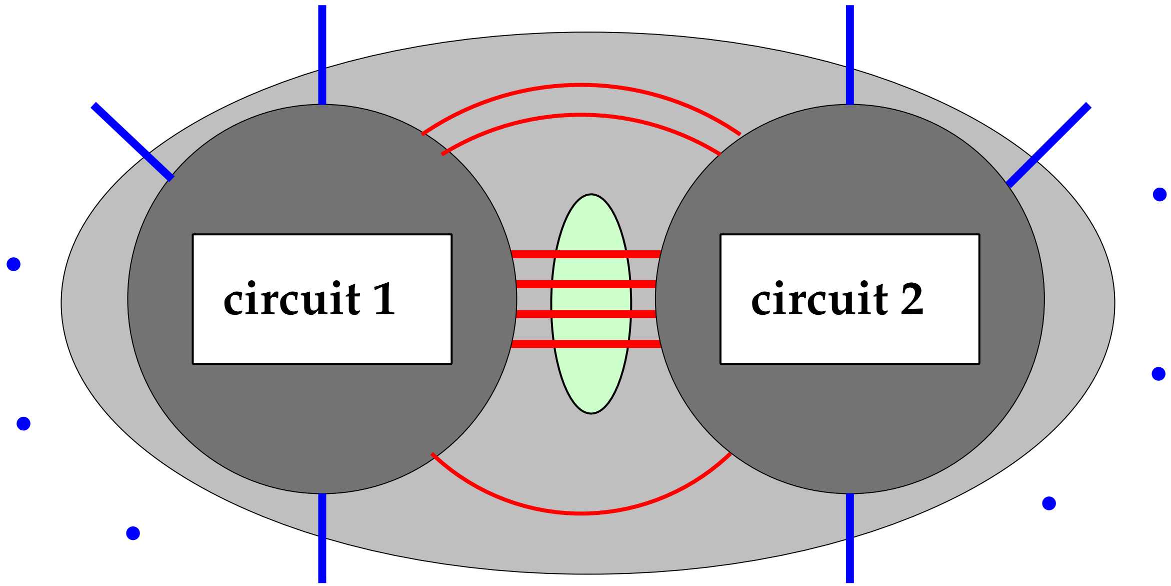


Terminals $\{1, 2\}$ **and** $\{3, 4\}$ **form ports.**

Energy transfer between circuits

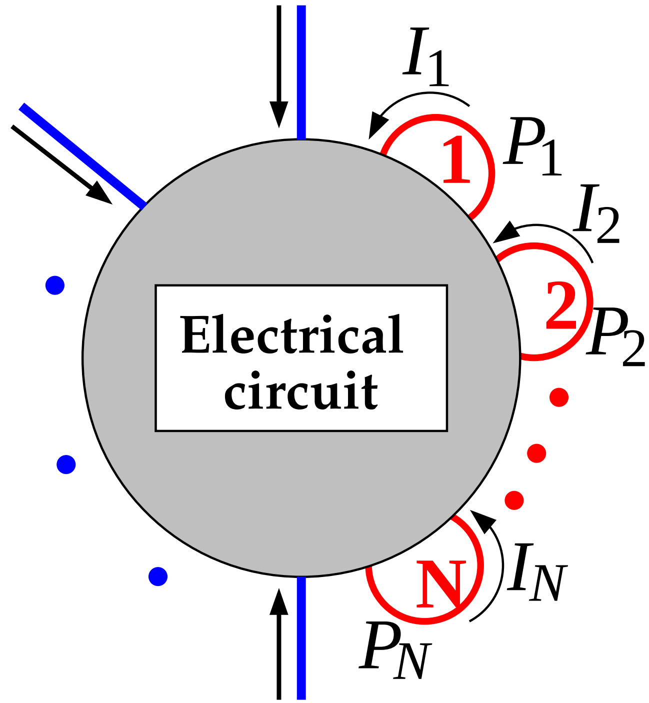


Assume that we monitor the current/potential on a set of terminals between circuits or within a circuit.

Can we speak about

***the energy transferred along these terminals*?**

Internal ports



Terminals $\{1, 2, \ldots, N\}$ **form an internal port** $:\Leftrightarrow$

$$[\![(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) \in \mathscr{B}_{IP}]\!]$$

$$\Rightarrow [\![I_1 + I_2 + \cdots + I_N = 0]\!].$$ ***'internal port-KCL'***

Power and energy

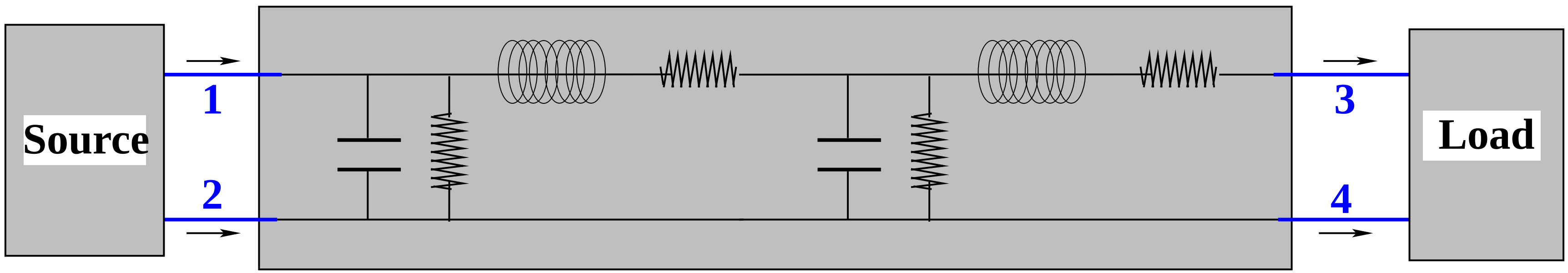
Flow through the terminals ***from one side to the other*** **in the direction of the arrows:**

power = $I_1(t)P_1(t) + I_2(t)P_2(t) + \cdots + I_N(t)P_N(t)$

energy = $\int_{t_1}^{t_2} [I_1(t)P_1(t) + I_2(t)P_2(t) + \cdots + I_N(t)P_N(t)]\, dt$

This physical interpretation of power and energy is valid only if the terminals form an internal port.

Example



The source and the load are 2-terminal 1-ports

$\Rightarrow$ **terminals** $\{1,2\}$ **and** $\{3,4\}$ **form internal ports.**

Therefore, we can speak of

***the energy transferred from the source to the load*.**

Energy is NOT an ‘extensive’ quantity

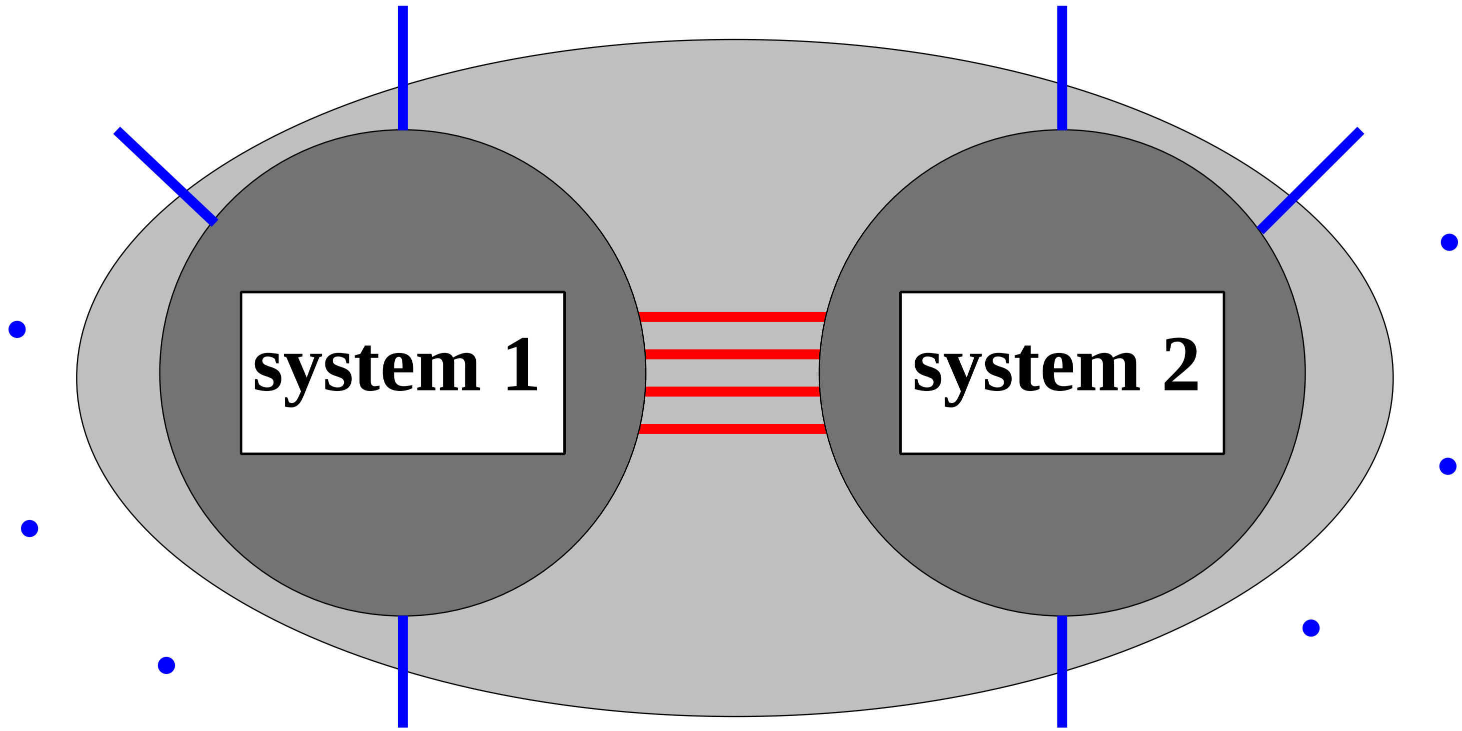
Interconnection versus energy transfer

Terminals are for interconnection.

Ports are for energy transfer.

A ‘port’ is a set of terminals with a special property (Kirchhoff’s port current law).

Energy transfer



One cannot speak about

"the energy transferred from system 1 to system 2"

or *"from the environment to system 1"*,

unless the relevant terminals form a port.

Are ports common?

Thm: Consider an electrical circuit consisting of an interconnection of (linear passive) R's, L's, and C's.

Assume that every pair of terminals of the circuit graph is connected.

Then

the only port is the one consisting of all the terminals.

Port KVL

$$[\![(I_1, \ldots, I_p, I_{p+1}, \ldots, I_N, P_1, \ldots, P_p, P_{p+1}, \ldots, P_N,) \in \mathscr{B}_{IP}, \alpha : \mathbb{R} \to \mathbb{R}]\!]$$

$$\Rightarrow \; [\![(I_1, \ldots, I_p, I_{p+1}, \ldots, I_N, P_1 + \alpha, \ldots, P_p + \alpha, P_{p+1}, \ldots, P_N,) \in \mathscr{B}_{IP}]\!] .$$

'port KVL'

Only $P_k - P_\ell$ **for** $k, \ell = 1, 2, \ldots, p$ **and**

$$P_{k'} - P_{\ell'} \text{ for } k', \ell' = p+1, p+2, \ldots, N$$

enter behavioral equations.

Port KVL

$$[\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1,\ldots,P_p,P_{p+1},\ldots,P_N,) \in \mathscr{B}_{IP}, \alpha : \mathbb{R} \to \mathbb{R}]\!]$$

$$\Rightarrow \; [\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1+\alpha,\ldots,P_p+\alpha,P_{p+1},\ldots,P_N,) \in \mathscr{B}_{IP}]\!] \,.$$

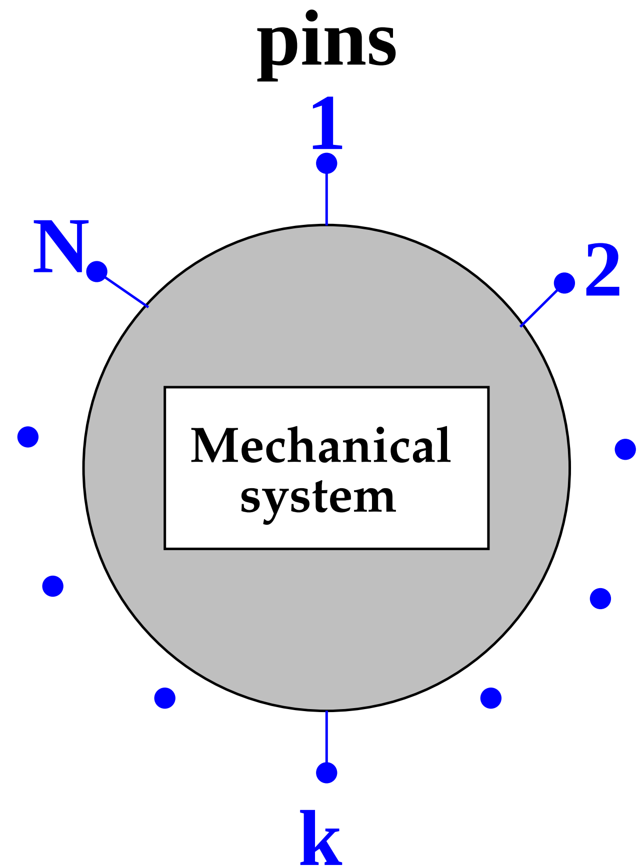
'port KVL'

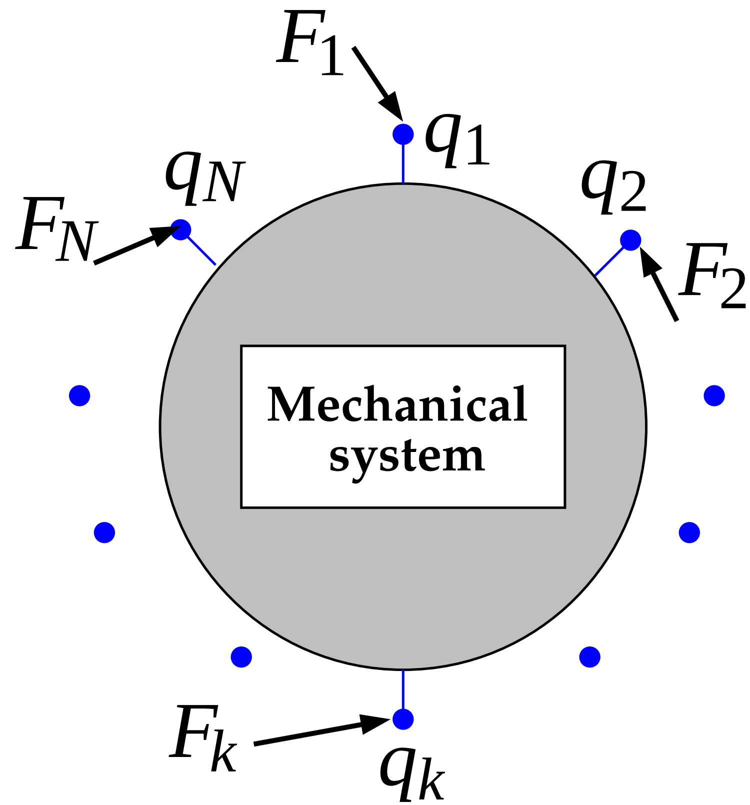
For linear passive controllable circuits, there holds

port KVL $\Leftrightarrow$ port KCL.

Mechanical systems

The behavior

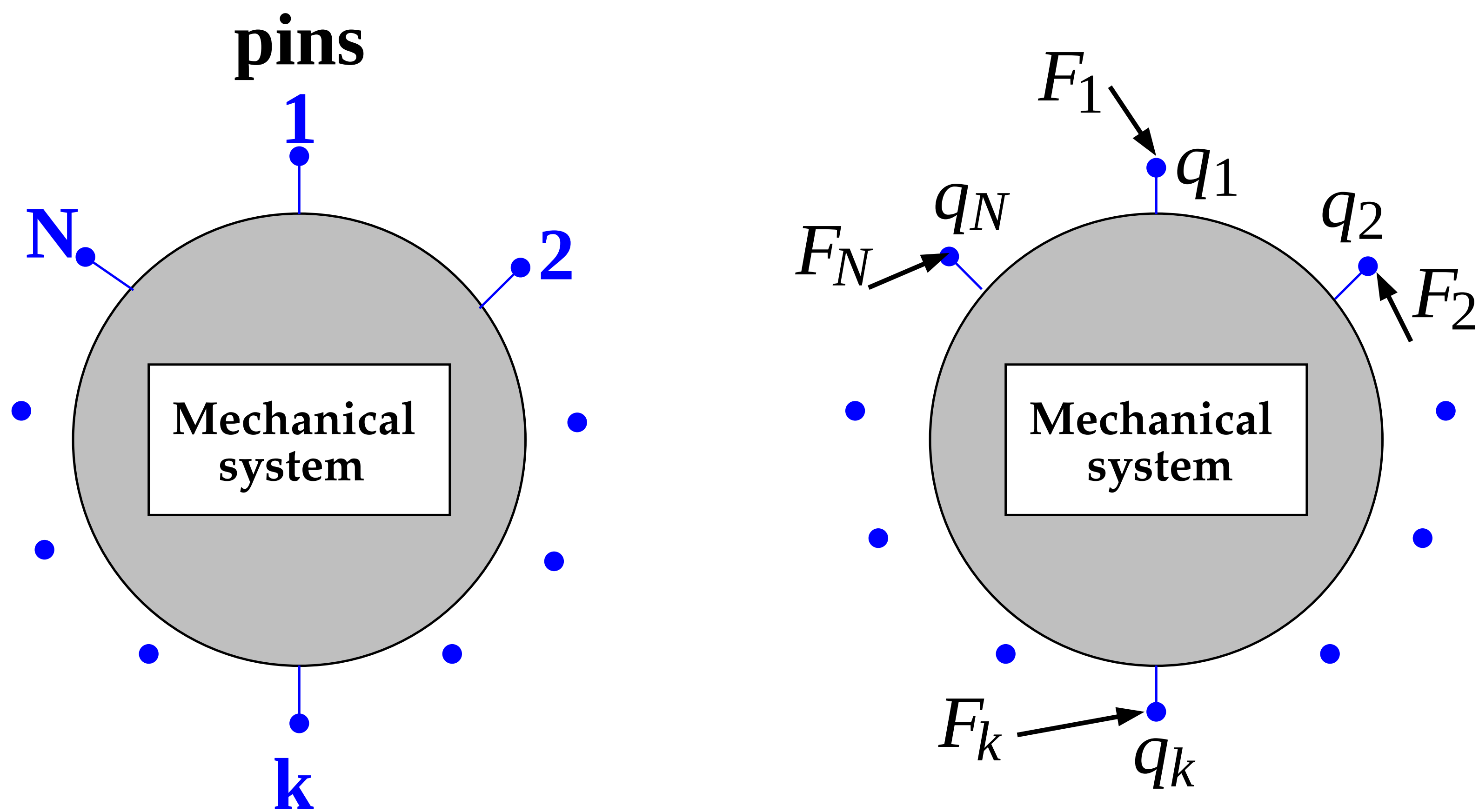




At each terminal: a position and a force.

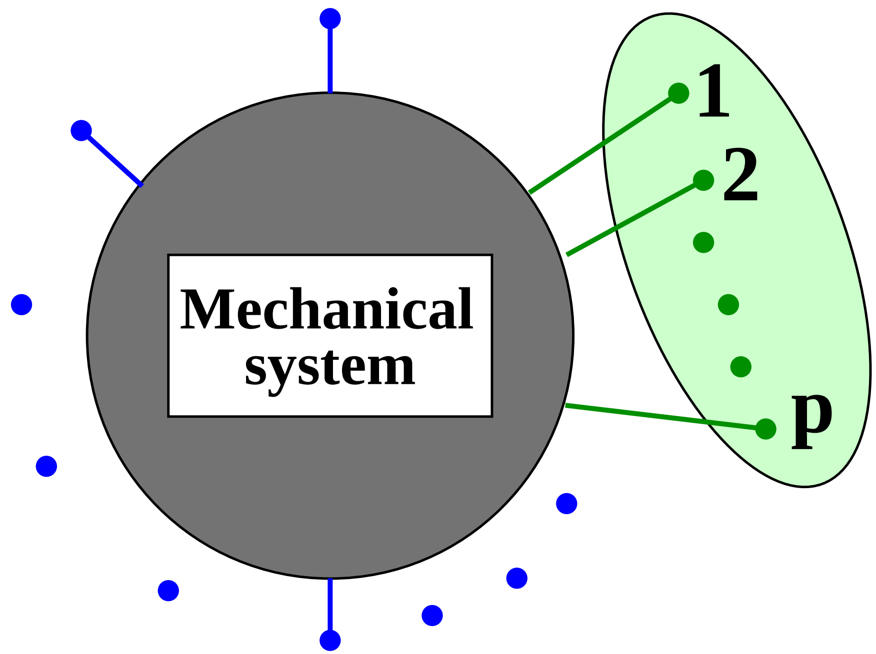
$$\rightsquigarrow \quad (q, F) \in \mathscr{B}_{qF}.$$

The behavior



What are the analogues of KVL, KCL, of port?

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$\left(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N\right) \in \mathscr{B}_{qF},$$

$$\Rightarrow \quad F_1 + F_2 + \cdots + F_p = 0.$$

'port KFL'

Power and energy

If terminals $\{1, 2, \ldots, p\}$ form a port, then

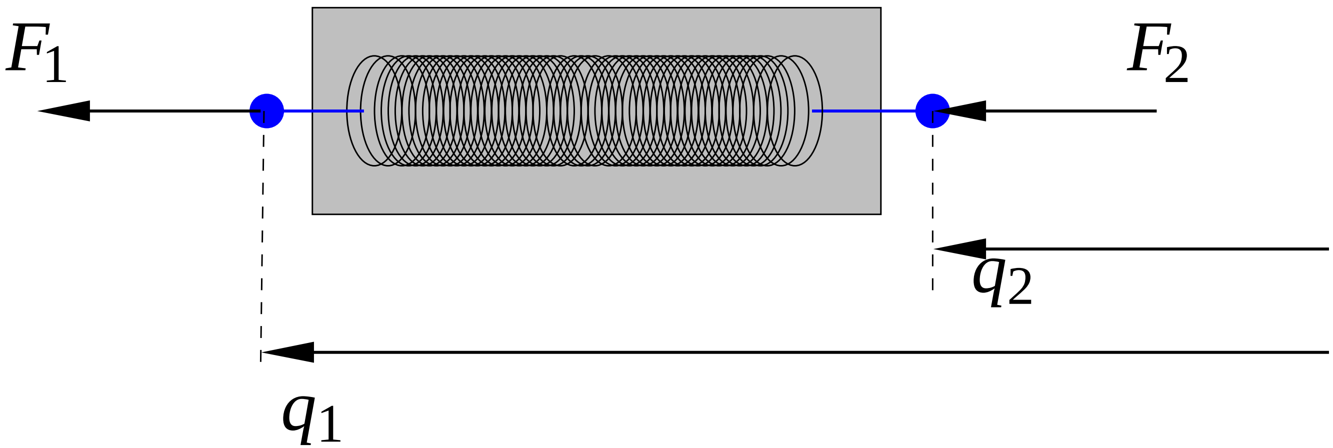
$$\textbf{power in} \quad = F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t),$$

$$\textbf{energy in} \quad = \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

Examples

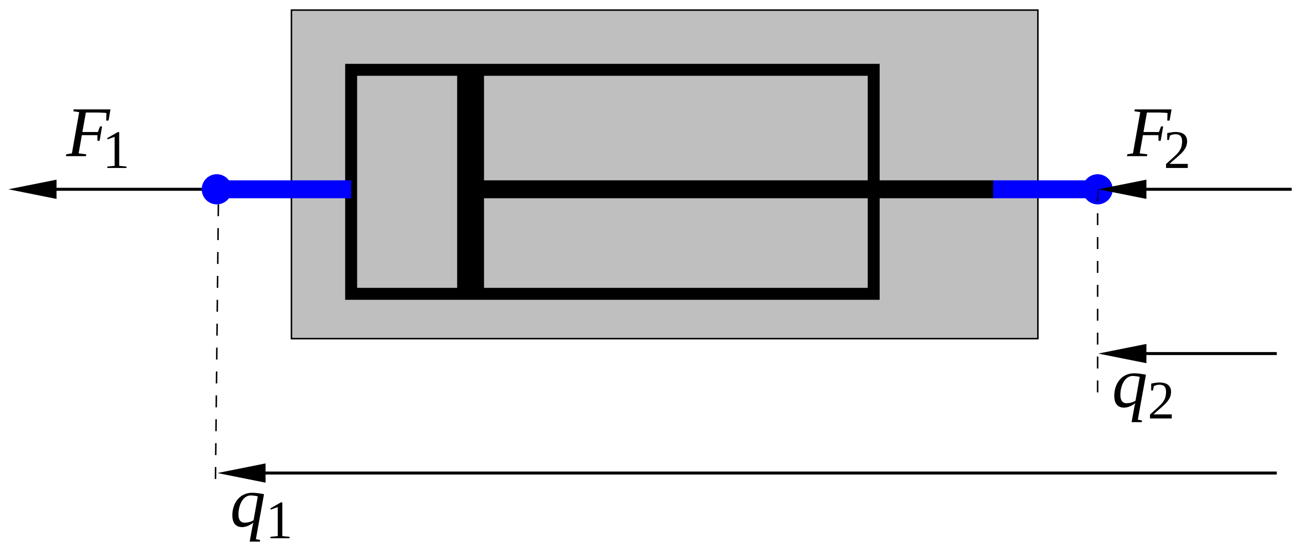
Spring



$F_1 + F_2 = 0, \quad K(q_1 - q_2) = F_1,$ **a port.**

Examples

Damper

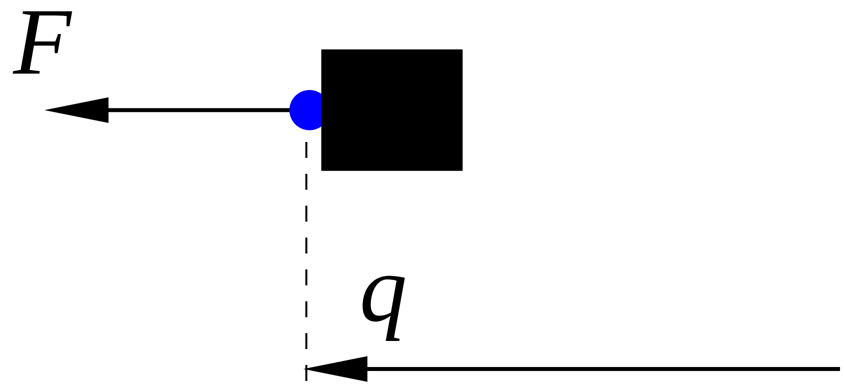


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1,$$ **a port.**

Springs and dampers, and the interconnection of springs and dampers are ports.

Examples

Mass

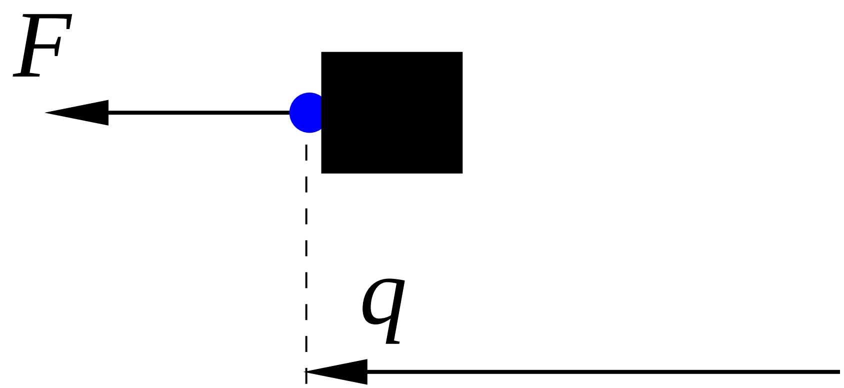


$$M\frac{d^2}{dt^2}q = F.$$

Not a port!!!

Motion energy

Back to the mass

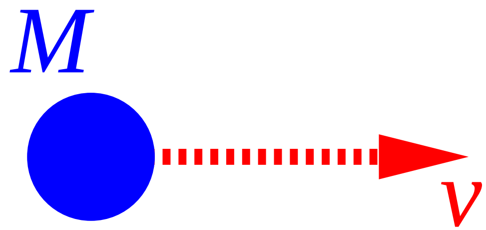


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q$$

Since $F^\top \frac{d}{dt}q$ is not power,

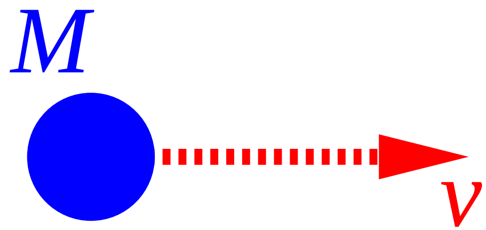
is $\frac{1}{2}M||\frac{d}{dt}q||^2$ not stored (kinetic, motion) energy ???

Kinetic energy



What is the kinetic energy?

Kinetic energy



What is the kinetic energy?

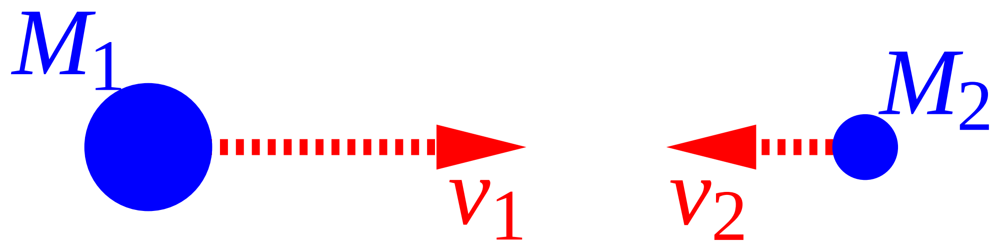
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

Not invariant under uniform motion.

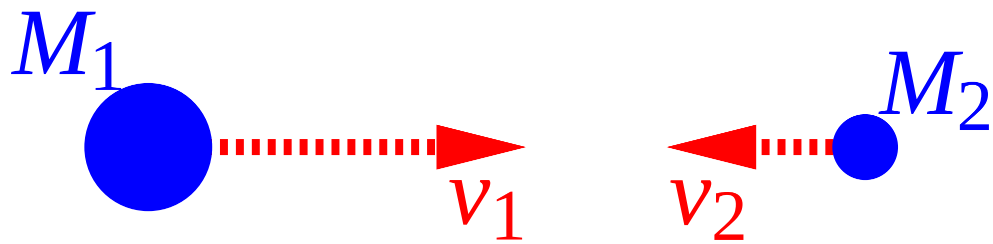
Motion energy



What is the motion energy?

What quantity is transformable into heat?

Motion energy



What is the motion energy?

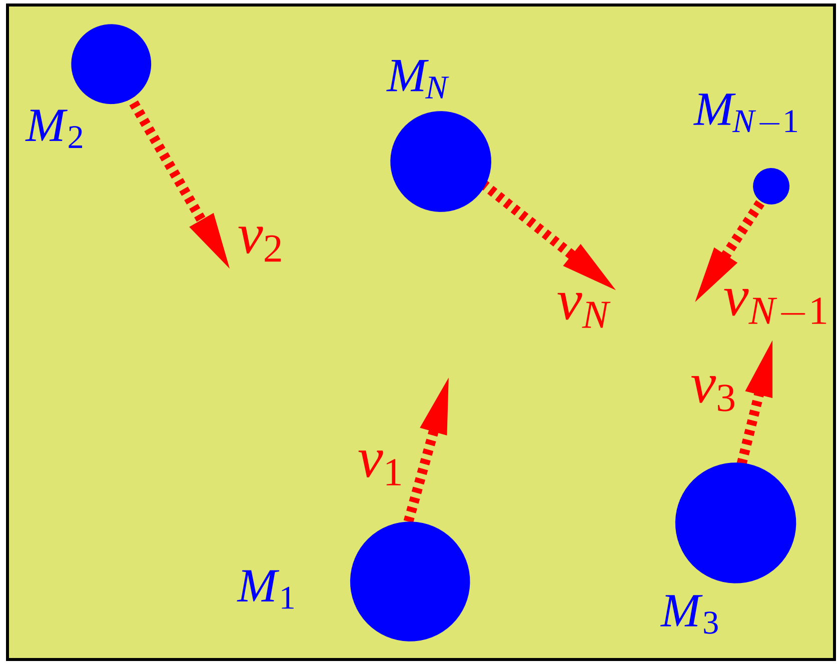
What quantity is transformable into heat?

$$\mathscr{E}_{\textbf{motion}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1 - v_2||^2$$

Invariant under uniform motion.

Motion energy

Generalization to N masses.

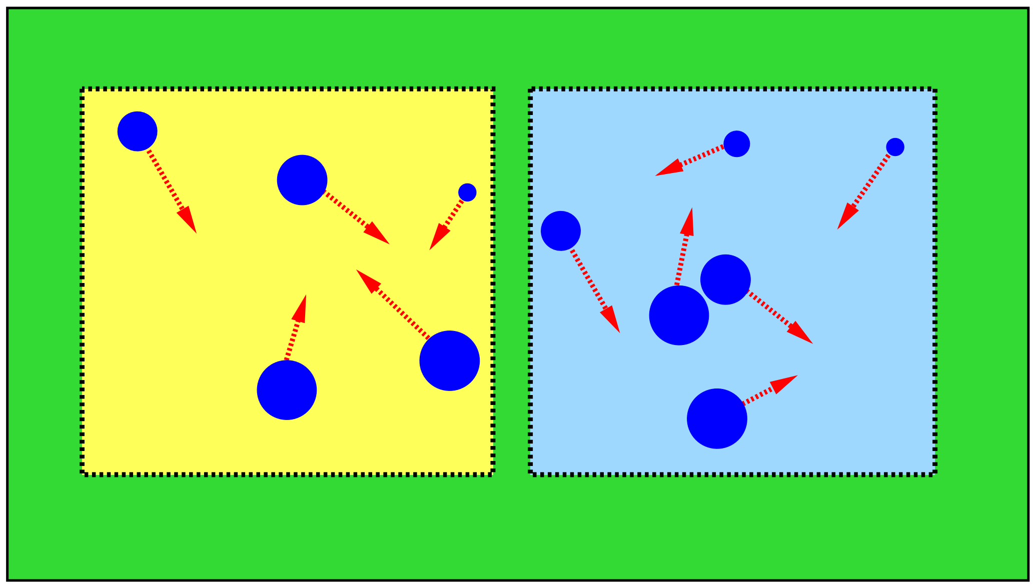


$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

$$\textbf{KFL} \;\Rightarrow\; \frac{d}{dt}\mathscr{E}_{\textbf{motion}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i.$$

Motion energy

Motion energy is not an extensive quantity, it is not additive.



Total motion energy $\neq$ sum of the parts.

Motion energy

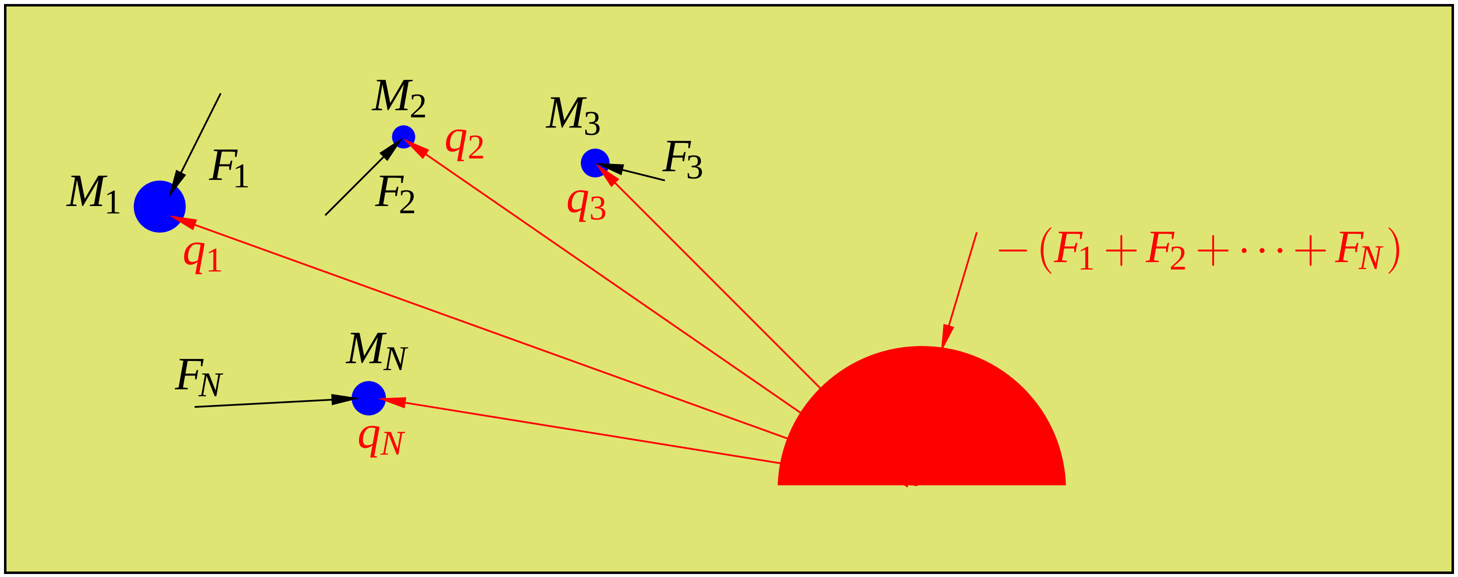
$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



measure velocities w.r.t. this infinite mass ('ground'), then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

Conclusions

- **Dynamical system $\cong$ a behavior.**
- **Interconnection $\cong$ variable sharing.**
- **Energy transfer happens via ports,**
  hence it involves action at a distance.
- **Interconnection is ‘local’,**
  power and energy transfer involve ‘action at a distance’.

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- **Electrical ports $:\Leftrightarrow$ port KCL.**
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- **New expression for motion energy, invariant under UM.**

- **Terminals are for interconnection,**
  ports are for energy transfer.

Reference: IEEE Circuits and Systems Magazine, Dec. 2010.

Copies of the lecture frames will be available from/at

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