MODELING, INTERCONNECTION,

and ENERGY FLOW

for DYNAMICAL SYSTEMS

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Theme

How are open systems formalized?

How are systems interconnected?

How is energy transferred between systems?

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How are open systems formalized?

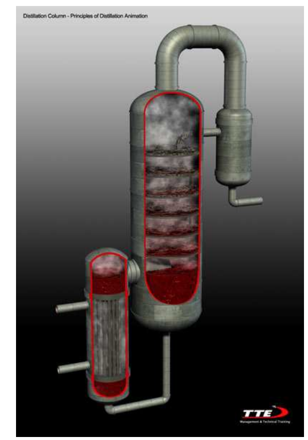
How are systems interconnected?

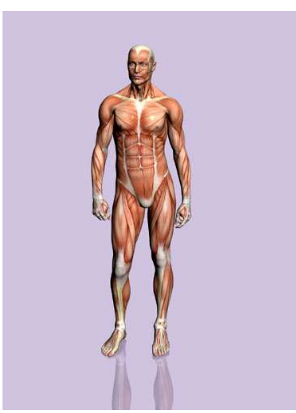
How is energy transferred between systems?

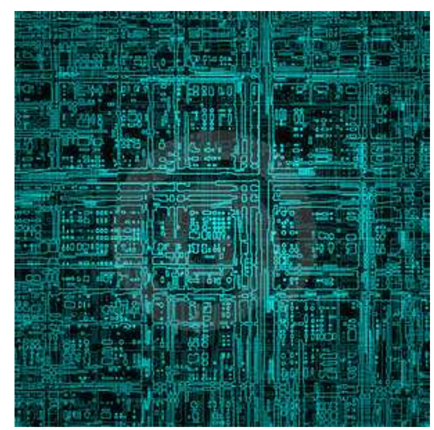
We deal with very simple examples,
mainly electrical circuits and
1-dimensional mechanical systems.

SYSTEMS


OIL REFINERY (GVG / PD)







Features

- **Open**
- **Interconnected**
- **Modular**

The ever-increasing computing power allows to model such complex interconnected systems accurately by

tearing, zooming, and linking.

⇝ **Simulation, model based design, ...**

Features

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The ever-increasing computing power allows to model such complex interconnected systems accurately by

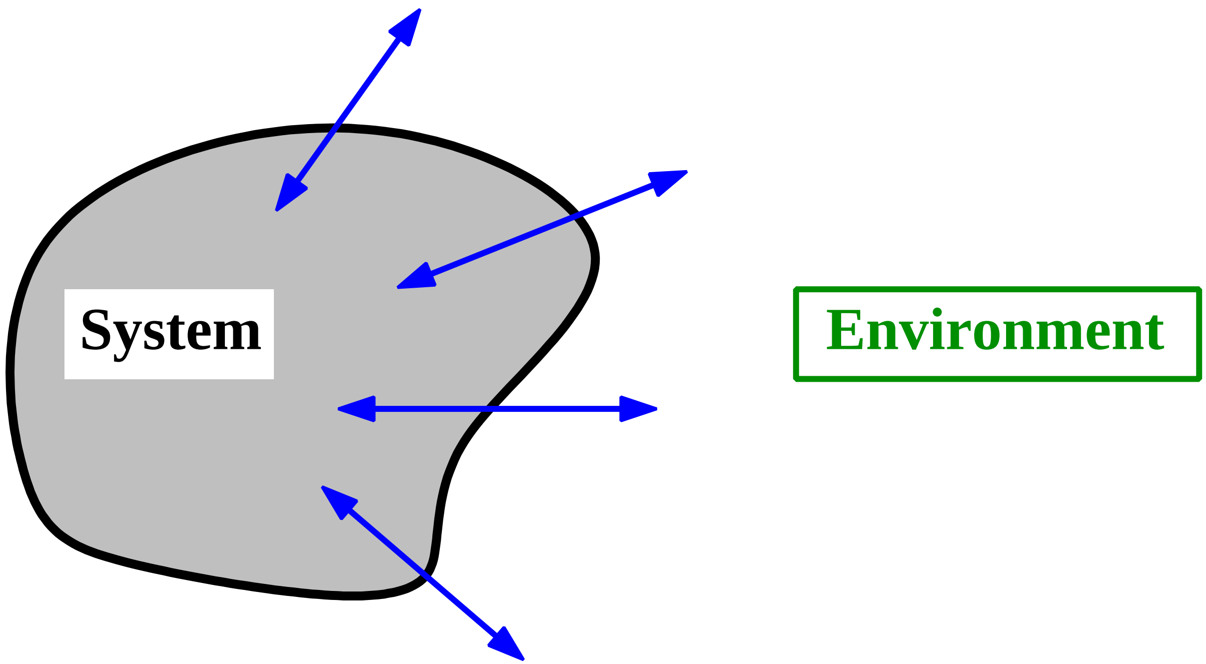
tearing, zooming, and linking.

$\leadsto$ **Simulation, model based design, ...**

Requires the right mathematical concepts

- **for dynamical system**
- **for interconnection**
- **for interconnection architecture**

Open systems

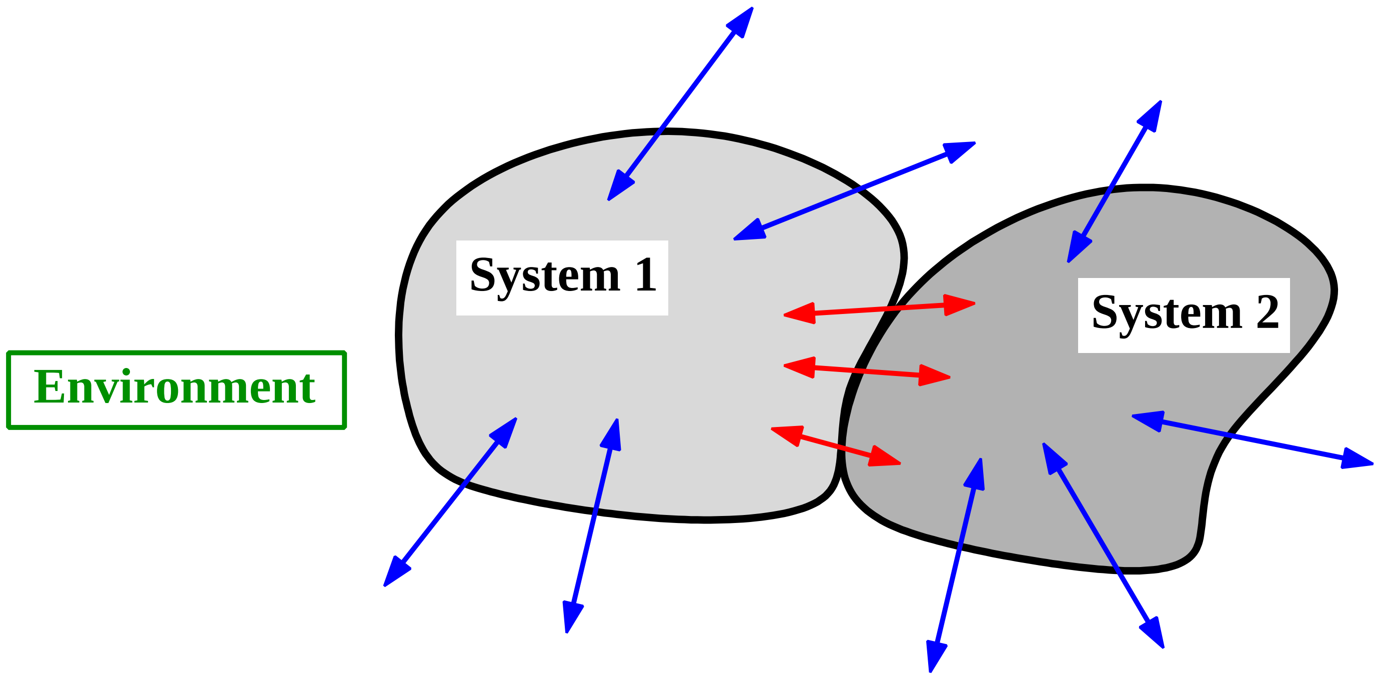


Systems are ‘open’, they interact with their environment.

How are such systems formalized?

How is energy transferred from the environment to a system?

Interacting systems



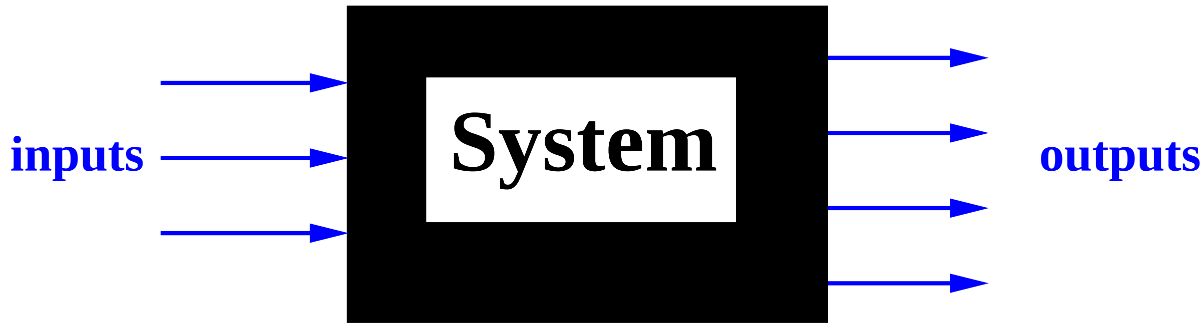
Interconnected systems interact.

How is this interaction formalized?

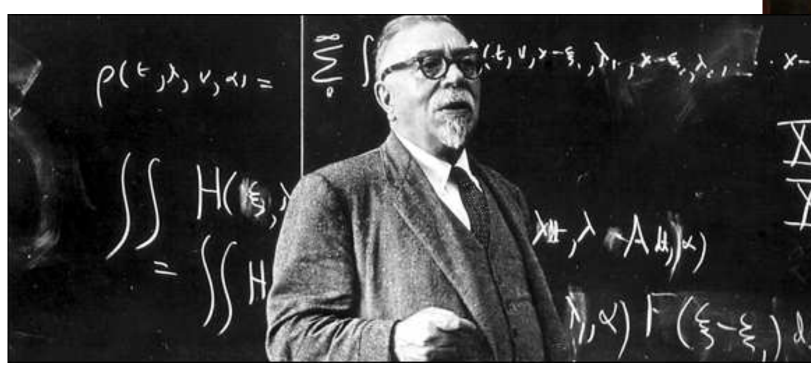
How is energy transferred between systems?

CLASSICAL VIEW

Input/output systems



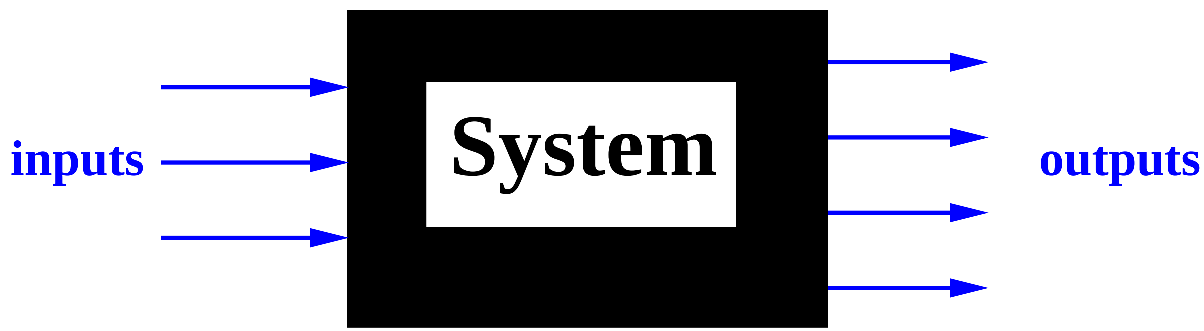
Oliver Heaviside



Norbert Wiener

Rudy Kalman

Input/output systems

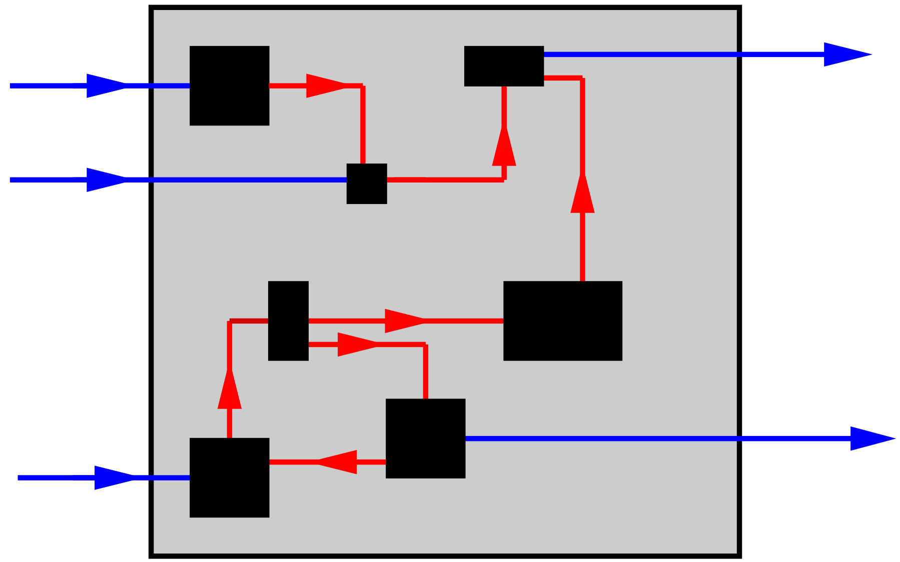


Input/output thinking is *inappropriate* for describing the functioning of physical systems.

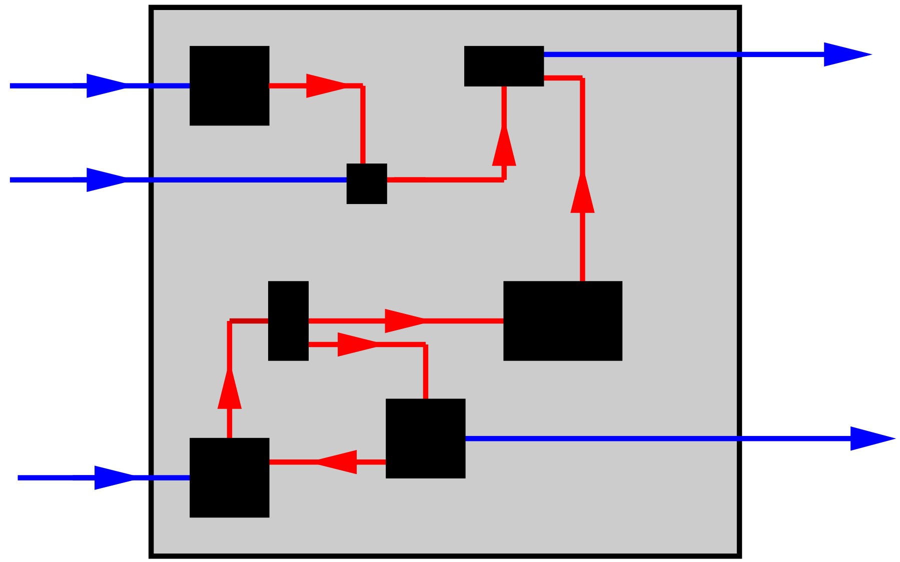
A physical system is not a signal processor.

Better concept: a behavior.

Signal flow graphs



Signal flow graphs



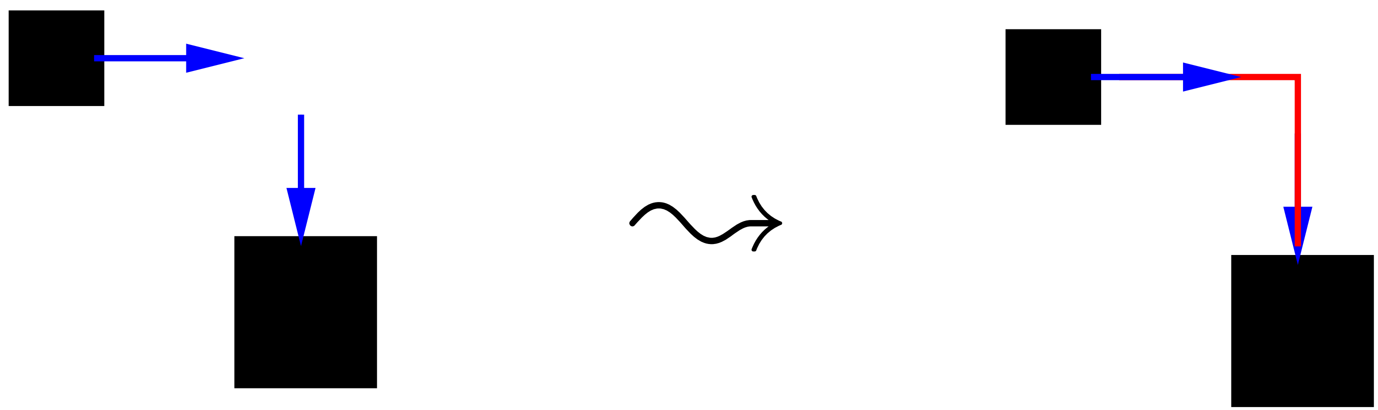
Signal flow graphs are *inappropriate* for describing the interaction physical systems.

A physical system is not a signal processor.

Better concept: a graph with leaves.

Interconnection

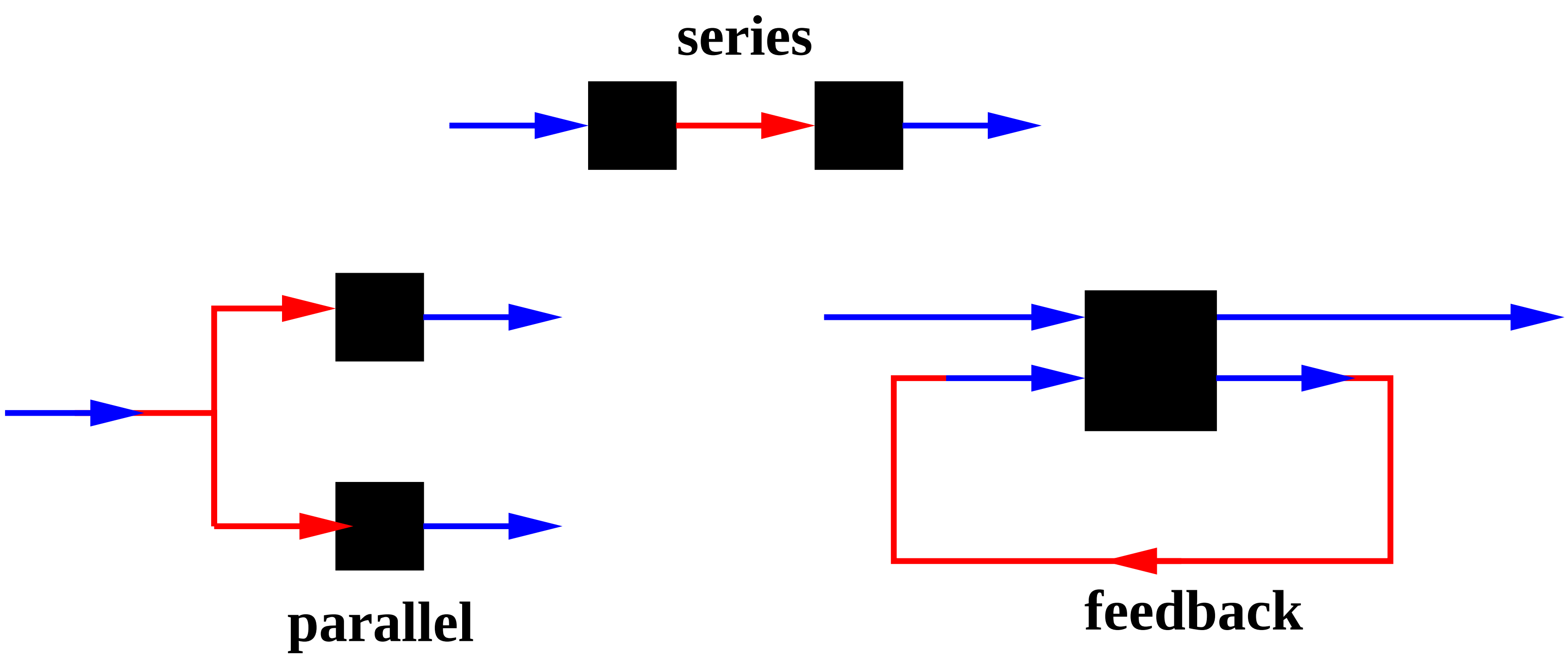
Interconnection as output-to-input assignment.



Interconnection

Interconnection as output-to-input assignment.

Examples:



Interconnection

Interconnection as output-to-input assignment.

Output-to-input assignment is *inappropriate* for describing the interconnection of physical systems.

A physical system is not a signal processor.

Better concept: variable sharing

The BEHAVIORAL APPROACH

The dynamic behavior

Definition: A *dynamical system* $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathscr{B})$**, with**

- $\mathbb{T} \subseteq \mathbb{R}$ **the time set,**
- $\mathbb{W}$ **the signal space,**
- $\mathscr{B} \subseteq (\mathbb{W})^{\mathbb{T}}$ **the behavior,**
  that is, $\mathscr{B}$ **is a family of maps from** $\mathbb{T}$ **to** $\mathbb{W}$**.**

$w : \mathbb{T} \to \mathbb{W} \in \mathscr{B}$ **means: the model allows the trajectory** w**,**

$w : \mathbb{T} \to \mathbb{W} \notin \mathscr{B}$ **means: the model forbids the trajectory** w**.**

Behavioral models

The behavior captures the essence of what a model is.

The behavior is all there is.
Equivalence of models, properties of models,
symmetries, system identification, etc.
must all refer to the behavior.

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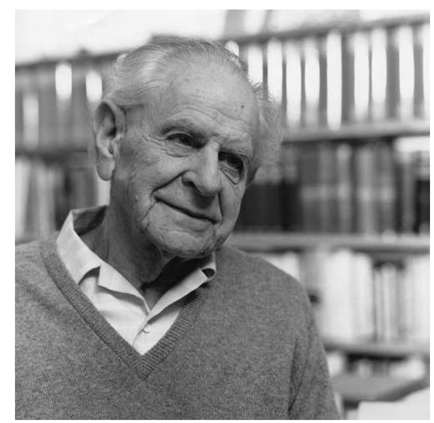
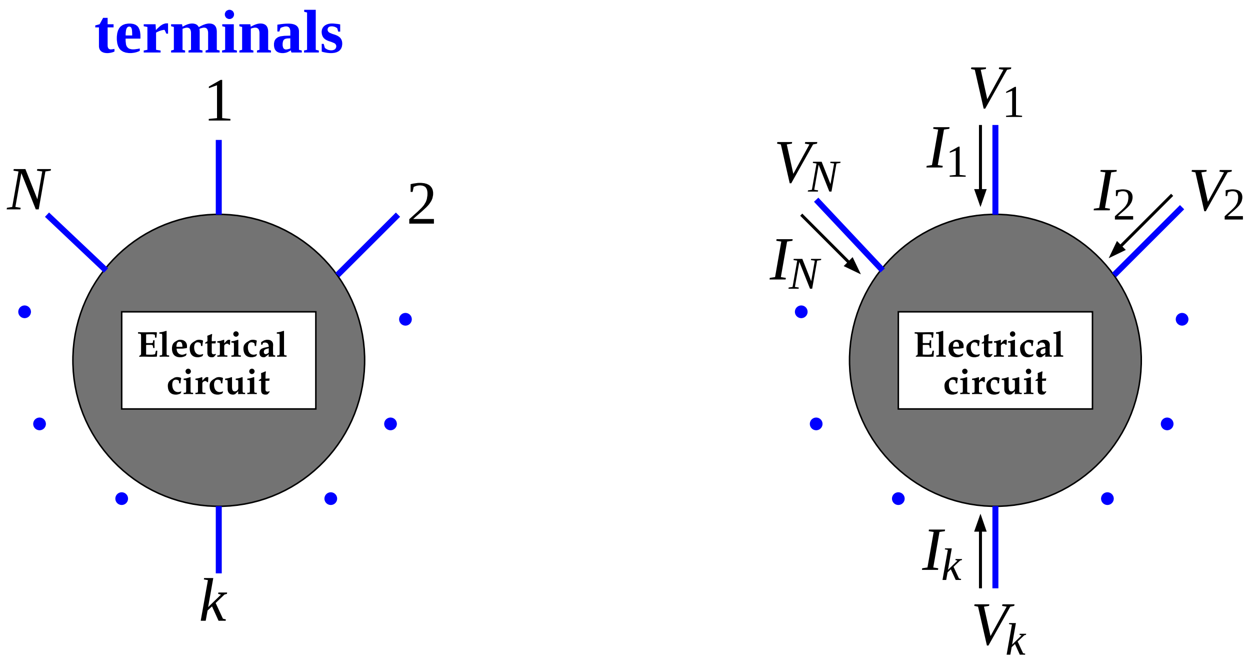
Every 'good' scientific theory is prohibition:
it forbids certain things to happen.
The more it forbids, the better it is.



Karl Popper (1902-1994)

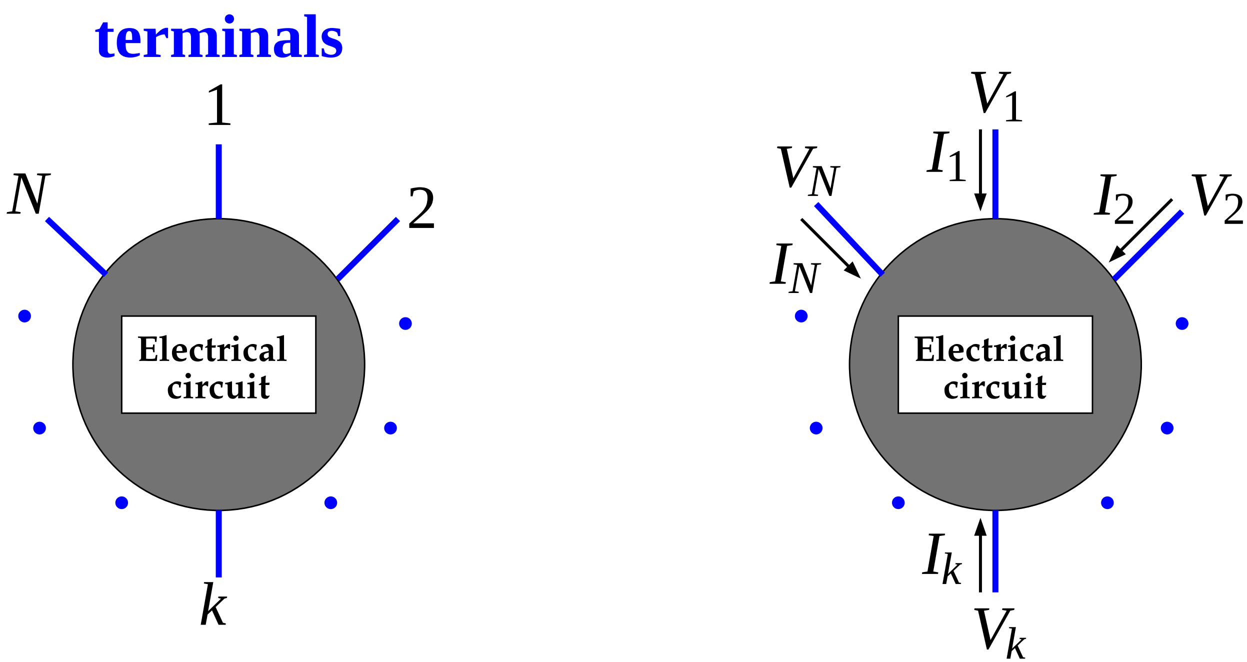
Electrical circuit



At each terminal:

a potential (!) and a current (counted > 0 into the circuit),

Electrical circuit



At each terminal:

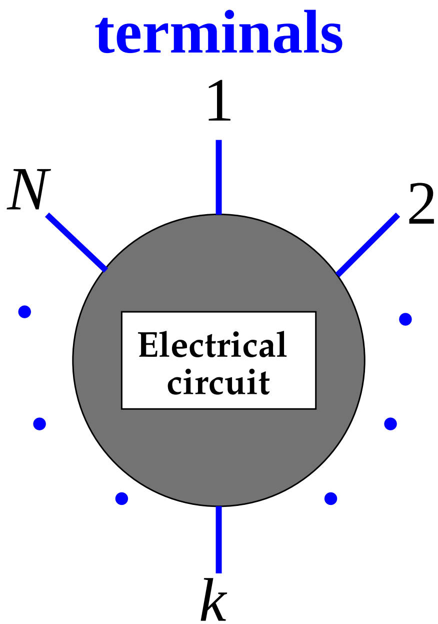
a potential (!) and a current (counted > 0 into the circuit),

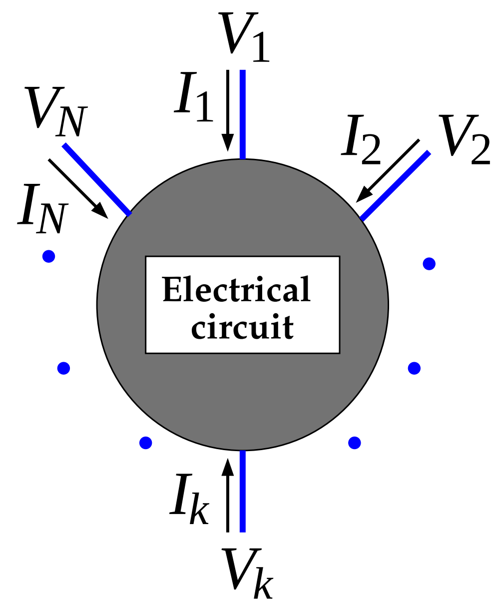
$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

this potential/current trajectory is compatible with the circuit architecture and its element values.

Electrical circuit

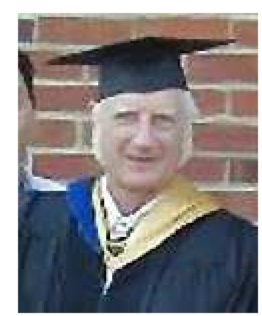




$\rightsquigarrow$ **behavior** $\mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}$.

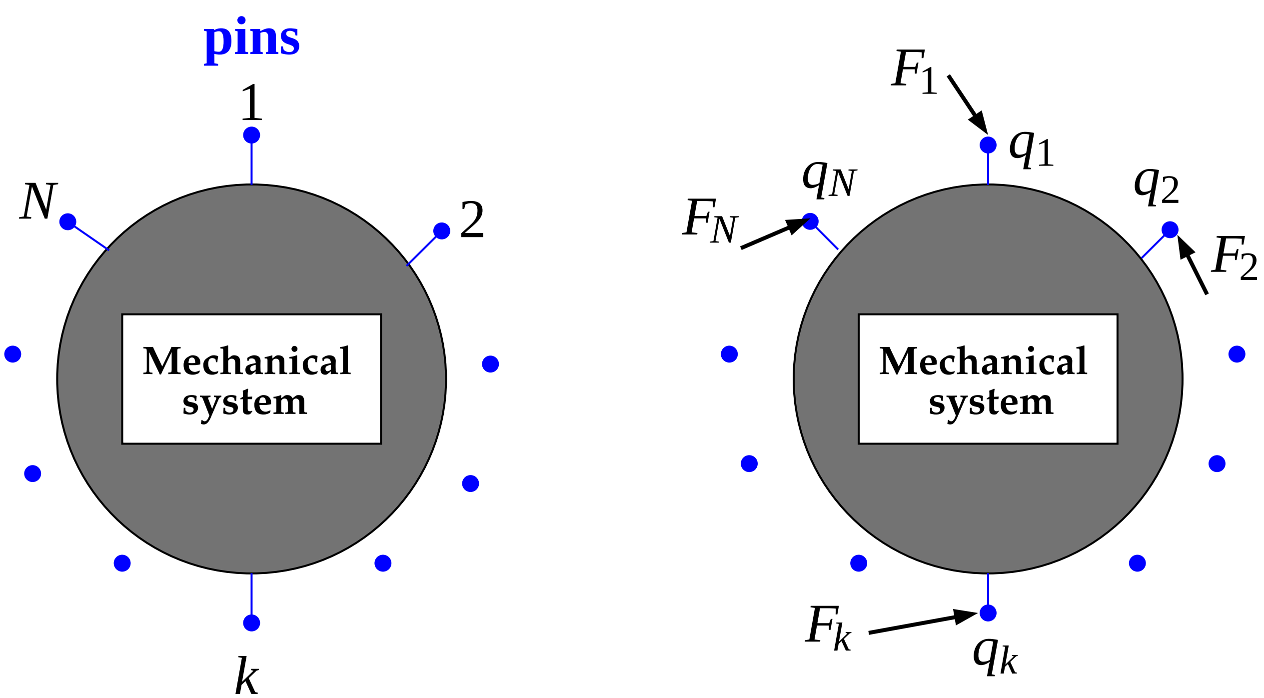
Early sources:

Brockway McMillan



Robert Newcomb

Mechanical device



At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q,F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$**.**

More generally, a position, force, angle, and torque.

Other domains

- **Thermal systems:**

  At each terminal: a temperature and a heat flow.

- **Hydraulic systems:**

  At each terminal: a pressure and a mass flow.

- **Multidomain systems:**

  Systems with terminals of different types, as motors, pumps, etc.

- **...**

The behavior

There has been an extensive development that deals with

system theory, control, system identification, etc.

from this point of view.

WHAT NEW DOES THIS BRING?

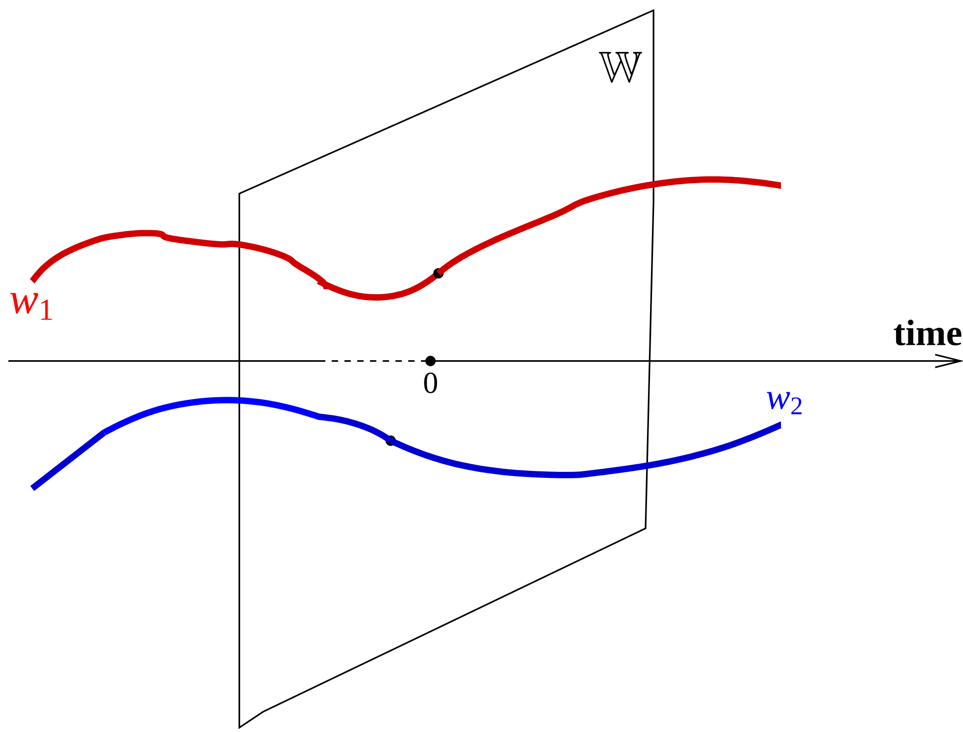
Controllability

The dynamical system $\Sigma = (\mathbb{T}, \mathbb{W}, \mathscr{B})$**, with** $\mathbb{T} = \mathbb{R}$ **or** $\mathbb{Z}$**, is said to be controllable** $:\Leftrightarrow$

for all $w_1, w_2 \in \mathscr{B}$**, there exist**
$T \in \mathbb{T}, T \geq 0$**, and** $w \in \mathscr{B}$**, such that**

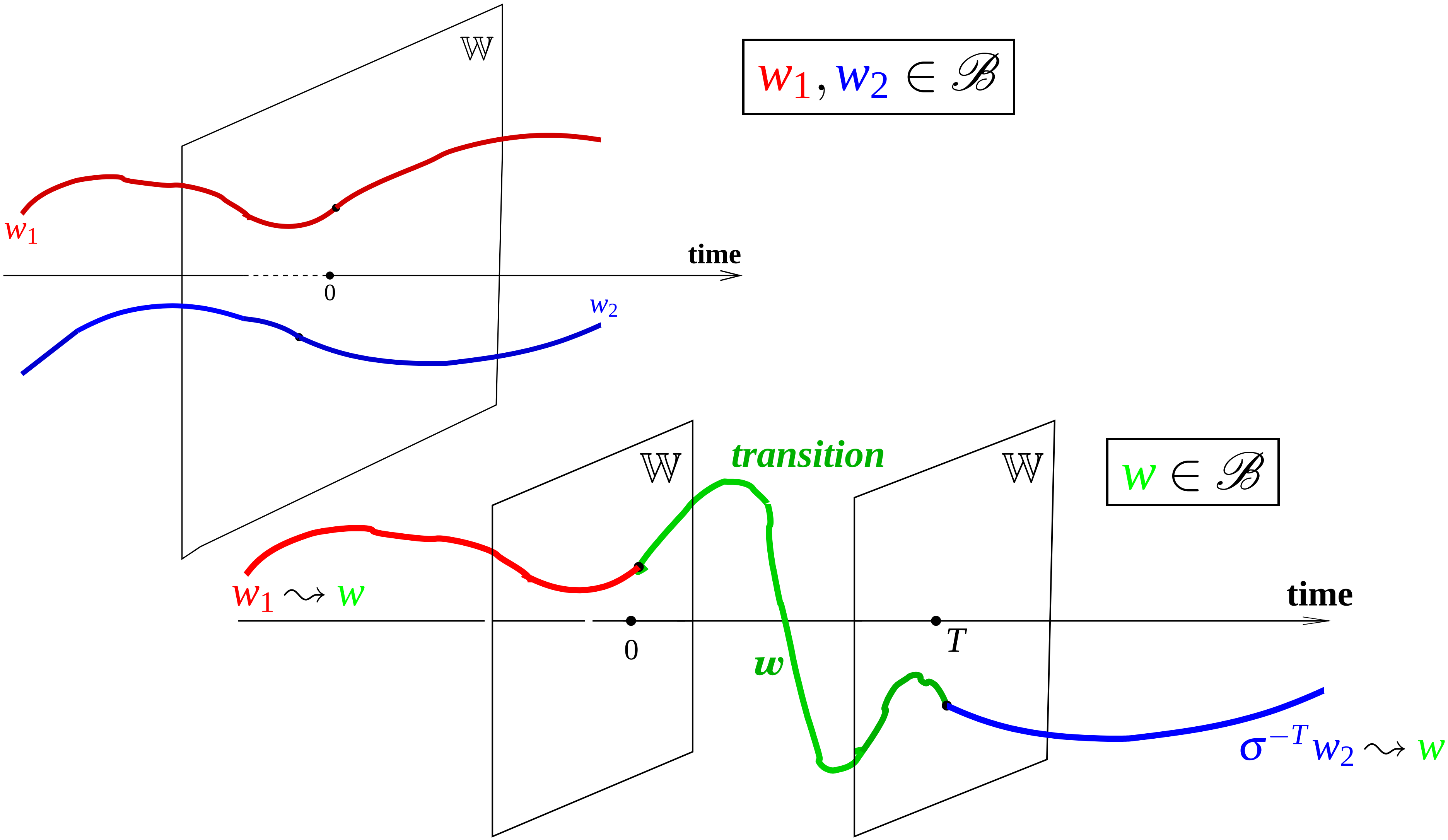
$$w(t) = \begin{cases} w_1(t) & \textbf{for } t < 0; \\ w_2(t - T) & \textbf{for } t \geq T. \end{cases}$$

Controllability in a picture



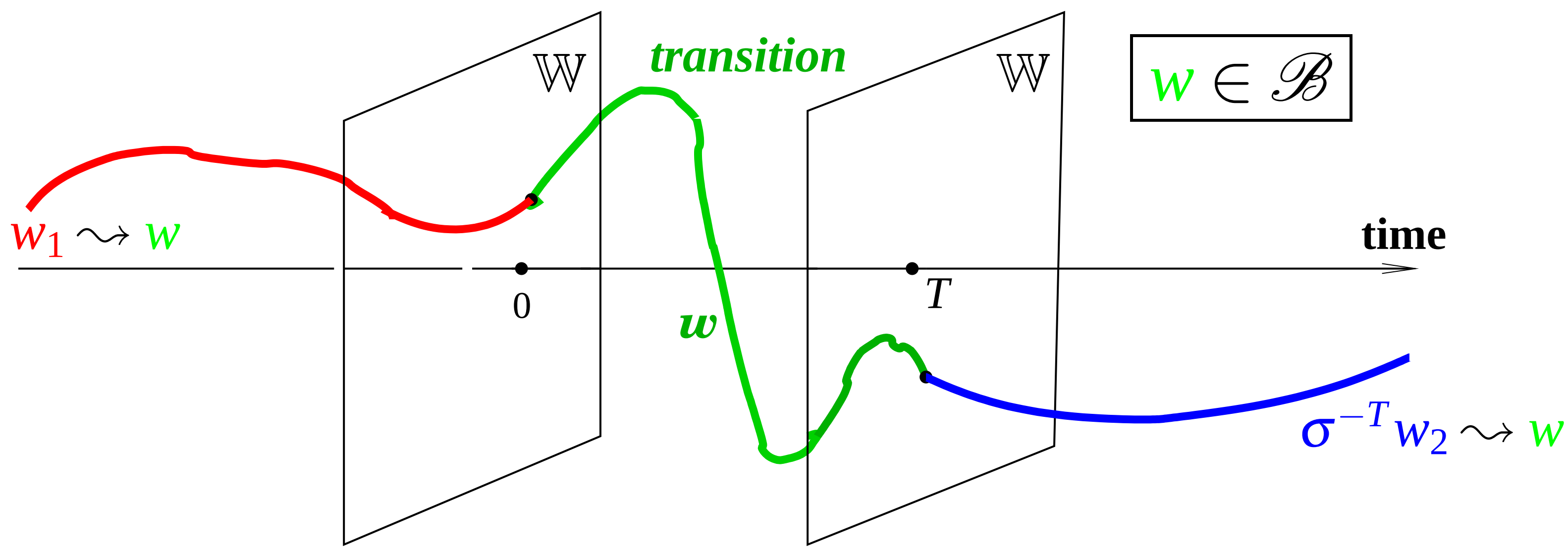
$w_1, w_2 \in \mathscr{B}$

Controllability in a picture



controllability :⇔ concatenability of trajectories after a delay

Controllability in a picture



controllability :⇔ concatenability of trajectories after a delay

Makes controllability into a genuine, an intrinsic property of a system, rather than merely a property of a state representation.

LTIDSs

A linear time-invariant differential system (LTIDS) $:\Leftrightarrow$

the behavior $\mathscr{B} \subseteq (\mathbb{R}^{\mathtt{w}})^{\mathbb{R}}$ **is the set of solutions of a system of linear constant-coefficient ODEs**

$$R_0 w + R_1 \frac{d}{dt} w + \cdots + R_{\mathtt{n}} \frac{d^{\mathtt{n}}}{dt^{\mathtt{n}}} w = 0,$$

with $R_0, R_1, \ldots, R_{\mathtt{n}} \in \mathbb{R}^{\bullet \times \mathtt{w}}$ **real matrices, and** $w : \mathbb{R} \to \mathbb{R}^{\mathtt{w}}$**.**

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with $R_0, R_1, \ldots, R_{\mathtt{n}} \in \mathbb{R}^{\bullet \times \mathtt{w}}$ **real matrices, and** $w : \mathbb{R} \to \mathbb{R}^{\mathtt{w}}$**.**
In polynomial matrix notation

$$R\left(\frac{d}{dt}\right) w = 0$$

with $R(\xi) = R_0 + R_1 \xi + \cdots + R_{\mathtt{n}} \xi^{\mathtt{n}} \in \mathbb{R}[\xi]^{\bullet \times \mathtt{w}}$**.**
$\mathscr{B}$ **= the set of solutions** $= \texttt{kernel}\left(R\left(\frac{d}{dt}\right)\right)$**.**

3 theorems for LTIDSs

1. **There exists a $1 \leftrightarrow 1$ relation between the LTIDSs and the $\mathbb{R}[\xi]$-submodules of $\mathbb{R}[\xi]^{\bullet}$.**

2. **In LTIDSs, variables can be eliminated:**

$$R\left(\frac{d}{dt}\right)w = M\left(\frac{d}{dt}\right)\ell \quad \Rightarrow \quad \tilde{R}\left(\frac{d}{dt}\right)w = 0.$$

   The projection of a kernel is a kernel.

3. **A LTIDS is controllable if and only if**

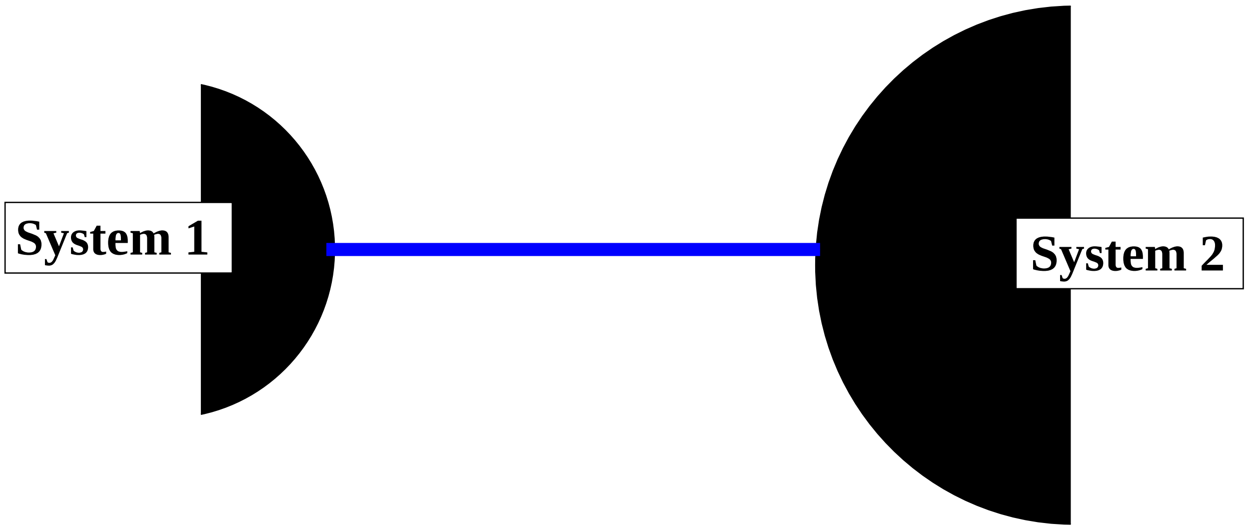
$$w = M\left(\tfrac{d}{dt}\right)\ell \qquad \mathscr{B} = \texttt{image}\left(M\left(\frac{d}{dt}\right)\right).$$

   Every image is a kernel. A kernel is an image if and only if the system is controllable.

Hold *mutatis mutandis* for discrete-time and for PDEs.

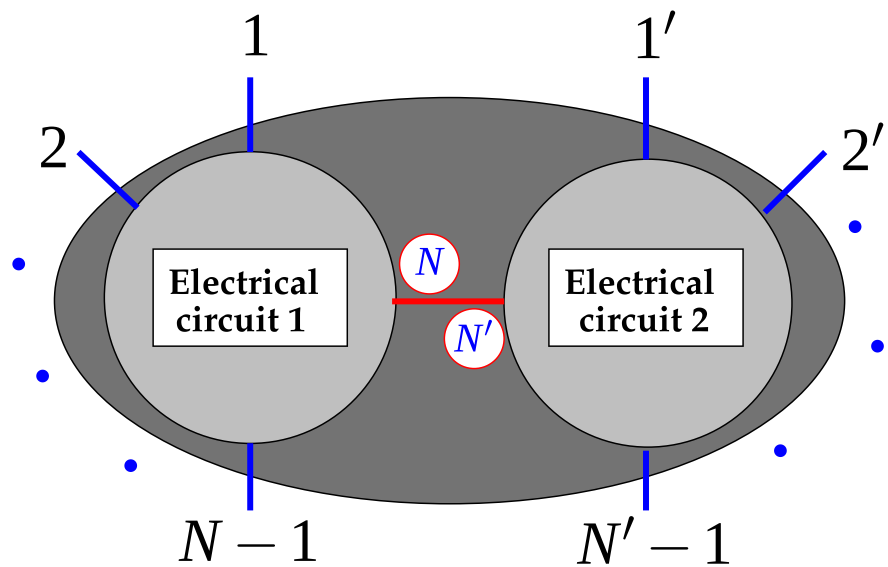
INTERCONNECTION

Connection of terminals



By interconnecting, the terminal variables are equated.

Interconnection of circuits



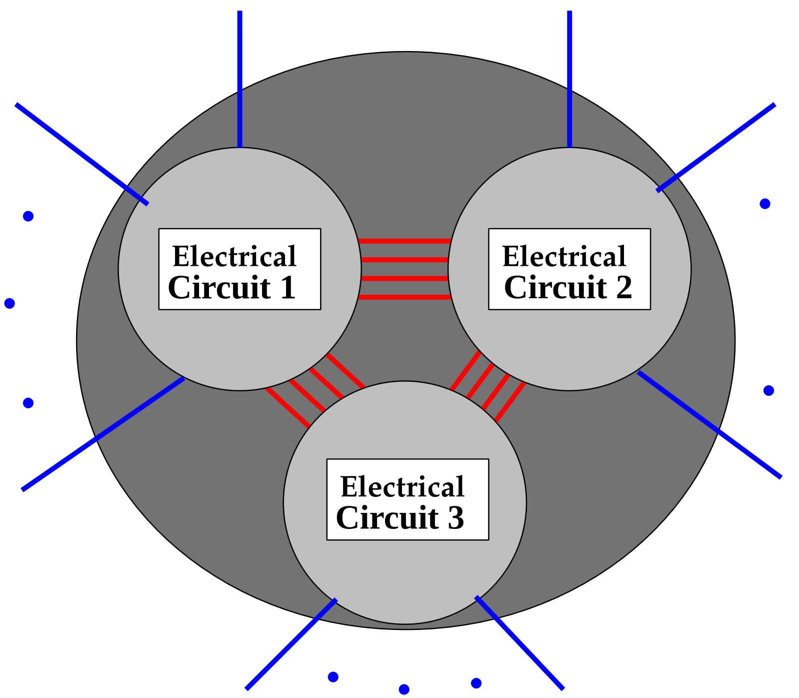
$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Behavior after interconnection:

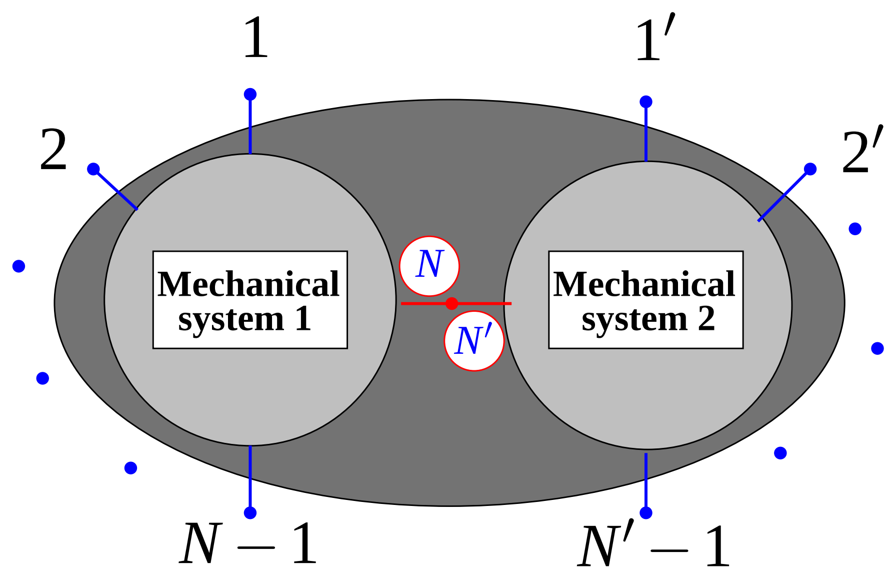
$$\mathscr{B}_1 \sqcap \mathscr{B}_2$$
$$:= \{(V_1, \ldots, V_{N-1}, V_{1'}, \ldots, V_{N'-1}, I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}) \mid$$
$$\exists\, V, I \textbf{ such that}$$
$$(V_1, \ldots, V_{N-1}, V, I_1, \ldots, I_{N-1}, I) \in \mathscr{B}_1 \textbf{ and}$$
$$(V_{1'}, \ldots, V_{N'-1}, V, I_{1'}, \ldots, I_{N'-1}, -I) \in \mathscr{B}_2\}.$$

Interconnection of circuits

⇝ **more terminals and more circuits connected**



Interconnection of 1-D mechanical systems



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Other terminal types

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **...**

Sharing variables

$$
\begin{aligned}
V_N = \ & V_{N'} && \textbf{and} & I_N + I_{N'} = \ & 0, \\
q_N = \ & q_{N'} && \textbf{and} & F_N + F_{N'} = \ & 0, \\
T_N = \ & T_{N'} && \textbf{and} & Q_N + Q_{N'} = \ & 0, \\
p_N = \ & p_{N'} && \textbf{and} & f_N + f_{N'} = \ & 0,
\end{aligned}
$$

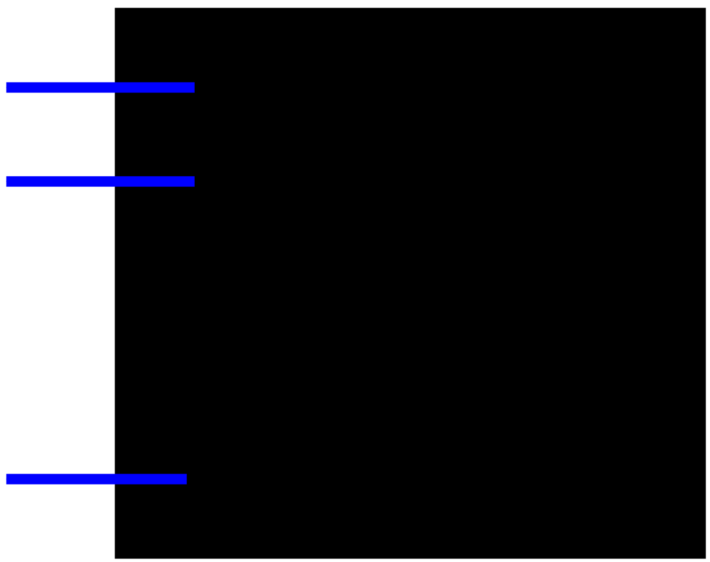
$$\vdots$$

Interconnection means variable sharing.

TEARING, ZOOMING, and LINKING

Tearing

¡¡ Model the behavior of selected variables !!



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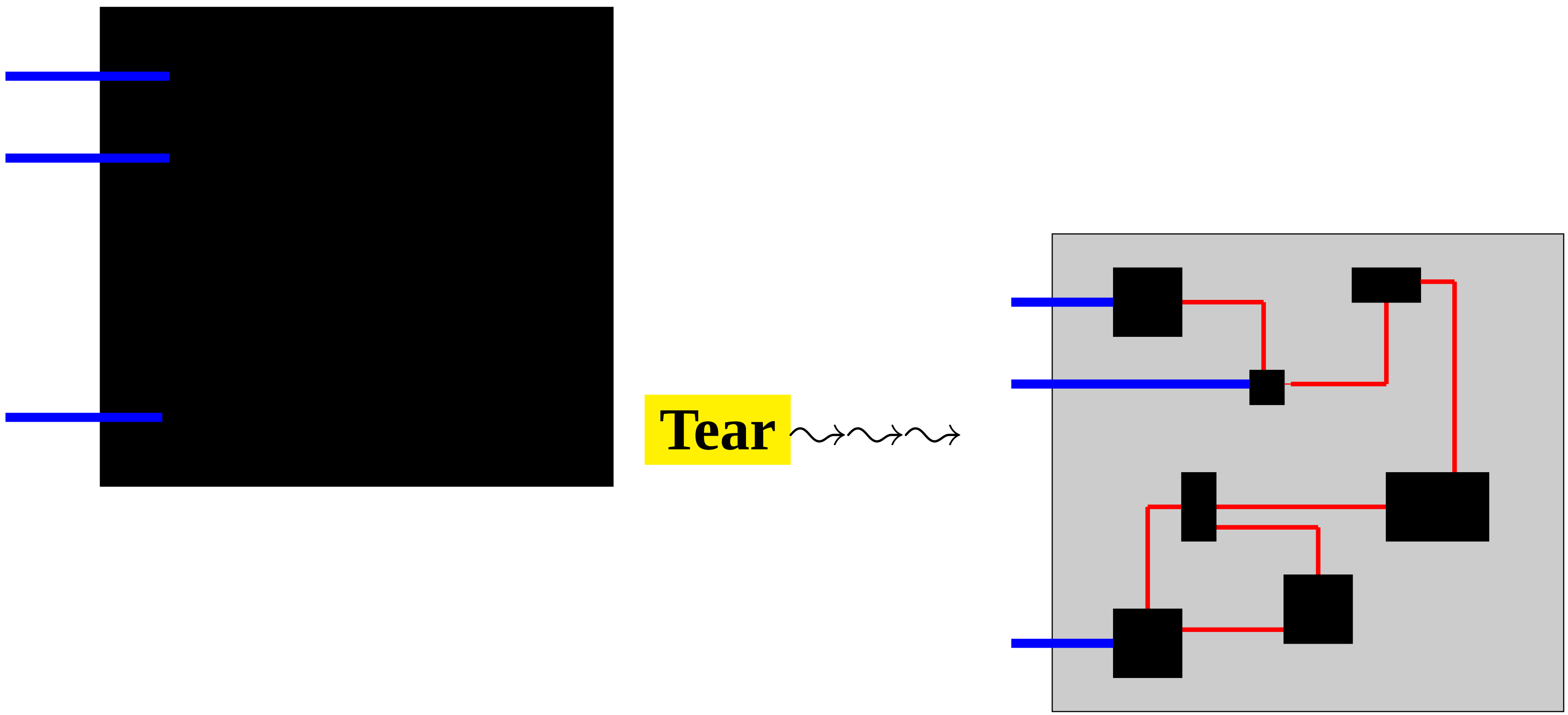
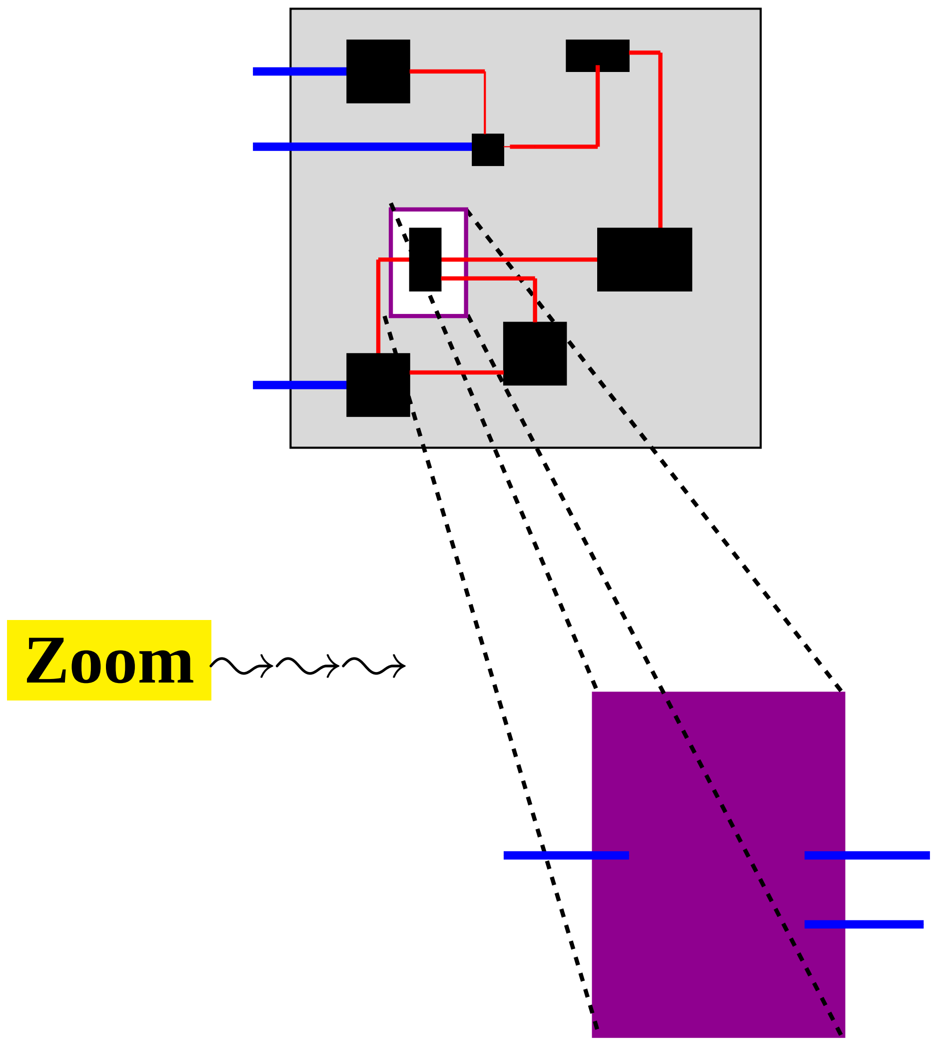
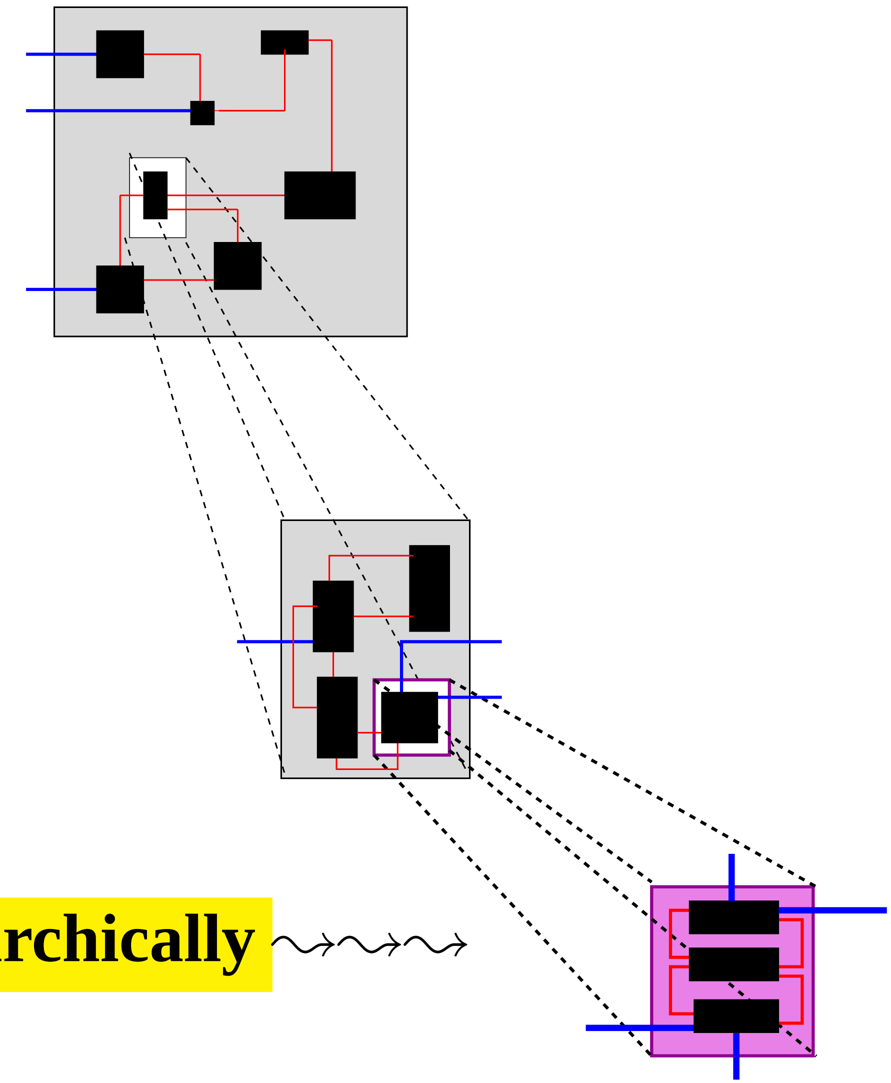


Zooming



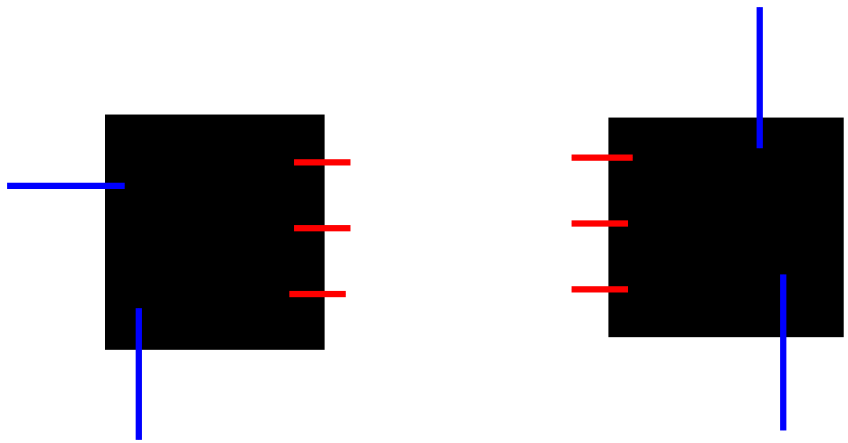
Zooming



Hierarchically ⇝⇝⇝

Proceed until subsystems (‘modularity’) are obtained whose model is known, from first principles, or stored in a database.

Linking



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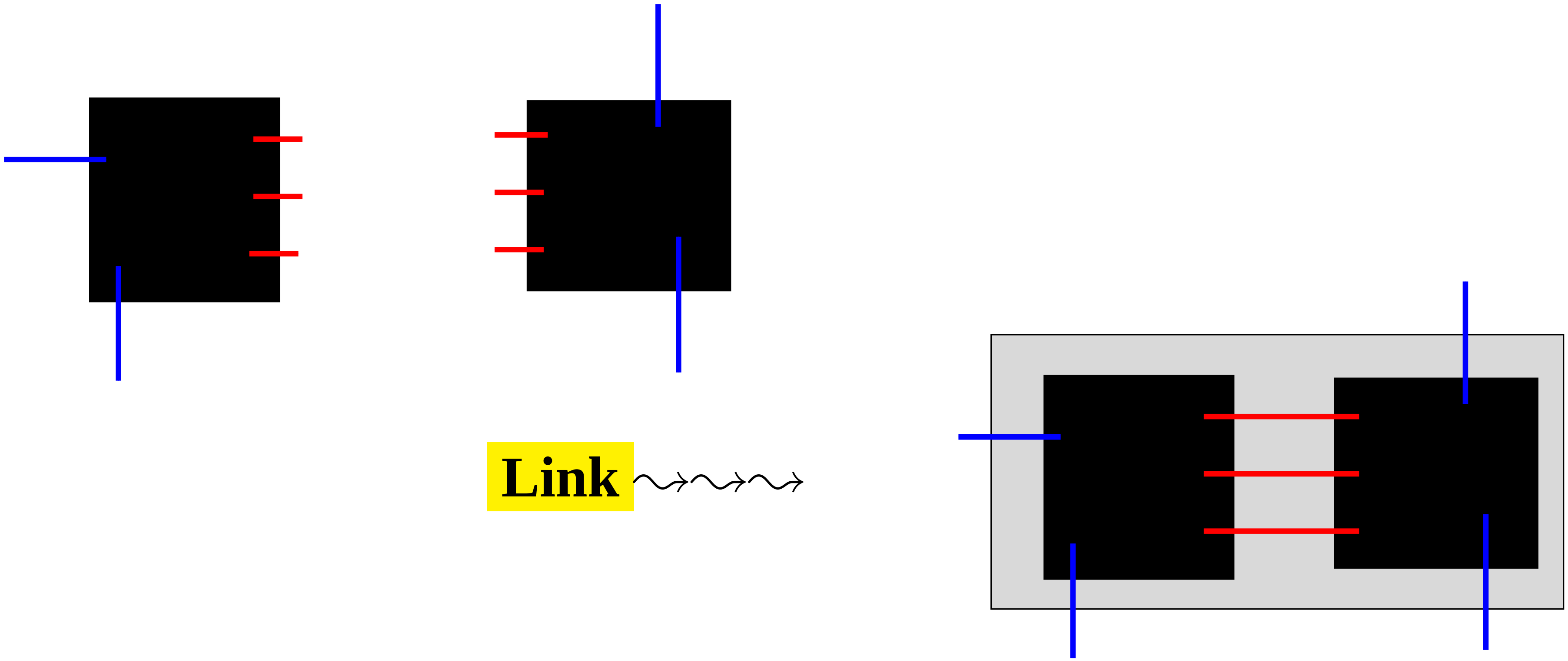

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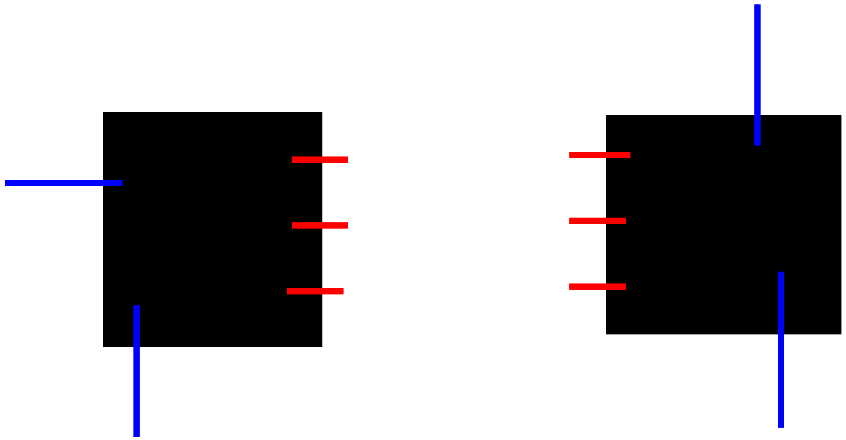
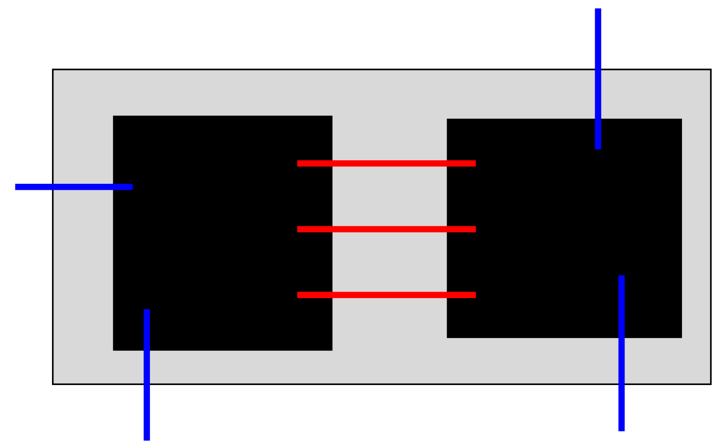


Link ⇝⇝⇝



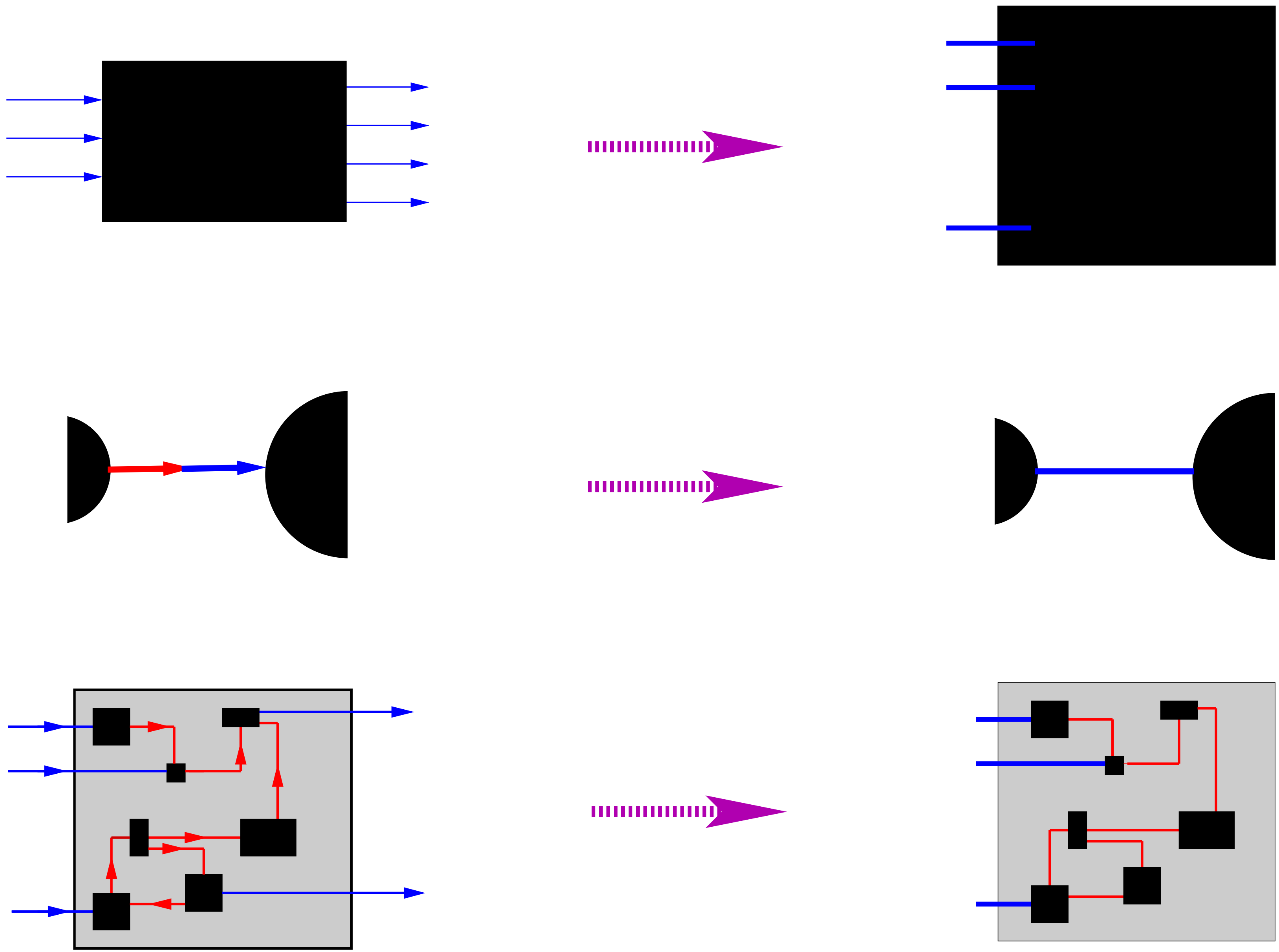
Model:

- **component behavior**
- **sharing equations**
- **elimination**

⇝ **behavior of the manifest variables.**

Tearing, zooming, and linking ⇝computer assisted modeling.

JUXTAPOSITION

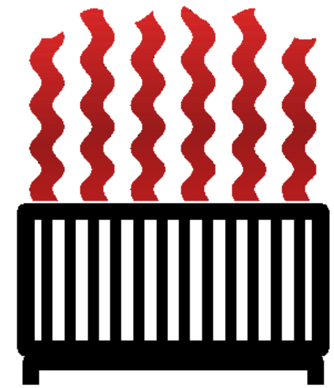


ENERGY TRANSFER

PORTS

Energy

Energy **:= a physical quantity transformable into heat.**



Energy

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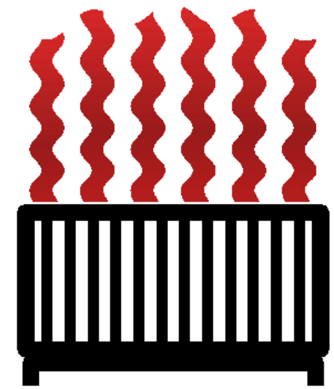
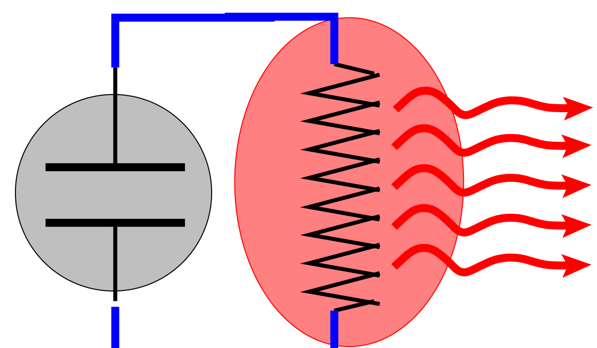
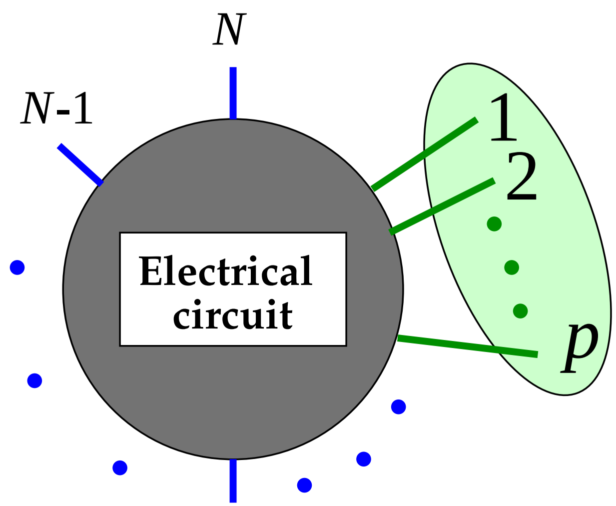


For example capacitor → resistor → heat.

Energy on capacitor = $\frac{1}{2}CV^2$



Electrical ports



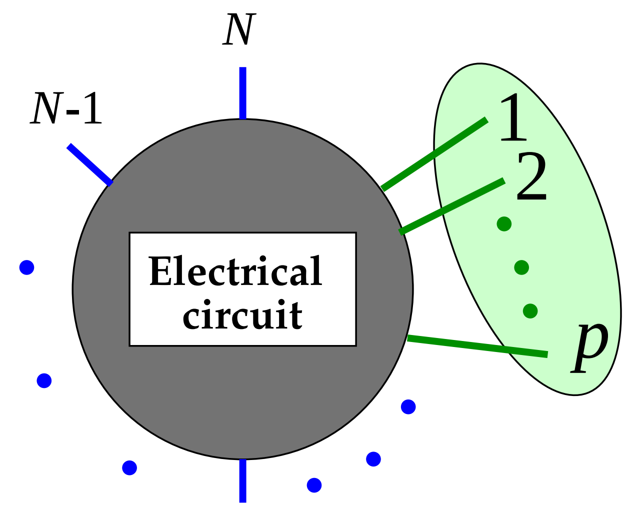
Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

$$(V_1, \ldots, V_p, V_{p+1}, \ldots, V_N, I_1, \ldots, I_p, I_{p+1}, \ldots, I_N) \in \mathscr{B}$$

$$\Rightarrow \quad I_1 + I_2 + \cdots + I_p = 0.$$ ***'port KCL'***.

KCL $\Rightarrow$ **all terminals together form a port.**

Electrical ports



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

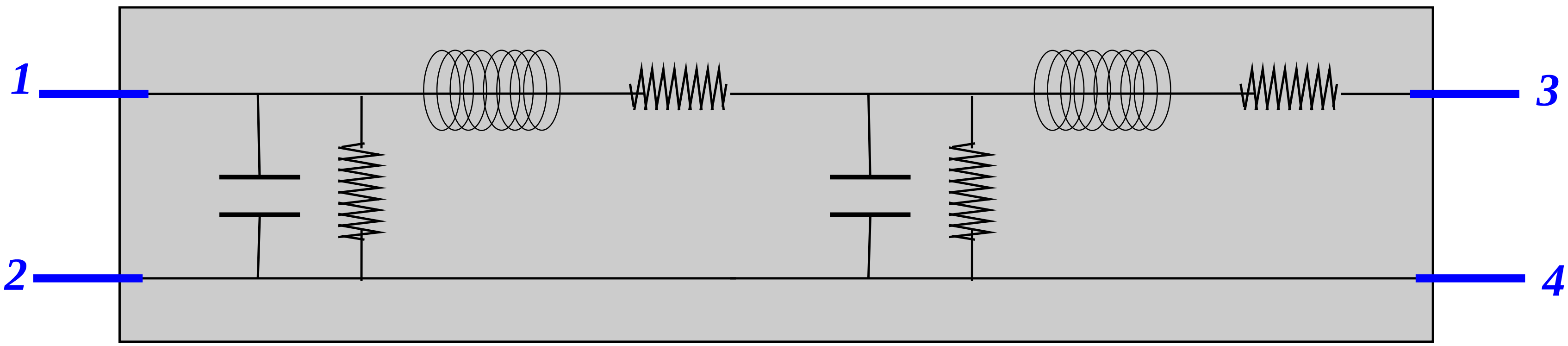
power in along these terminals = $V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)$,

energy in = $\displaystyle\int_{t_1}^{t_2} [V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)]\, dt.$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

Examples

R, L, C's, and their interconnection into a 2-terminal circuit form ports

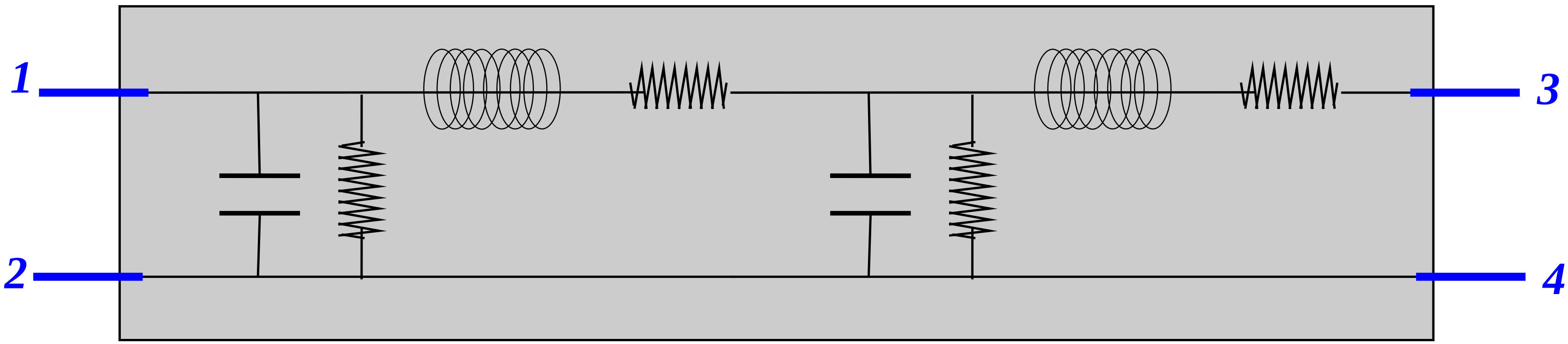


Terminals $\{1,2,3,4\}$ **form a port. But** $\{1,2\}$ **and** $\{3,4\}$ **do not.**

We cannot speak about

'the energy transferred from $\{1,2\}$ **to** $\{3,4\}$**'.**

Examples



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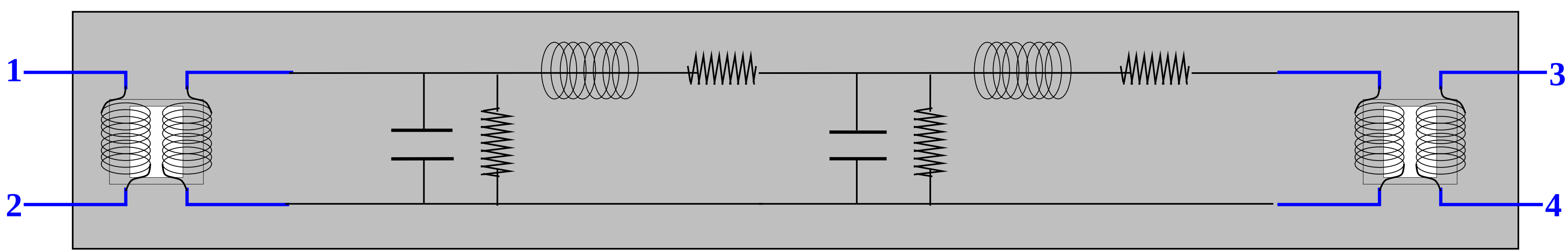
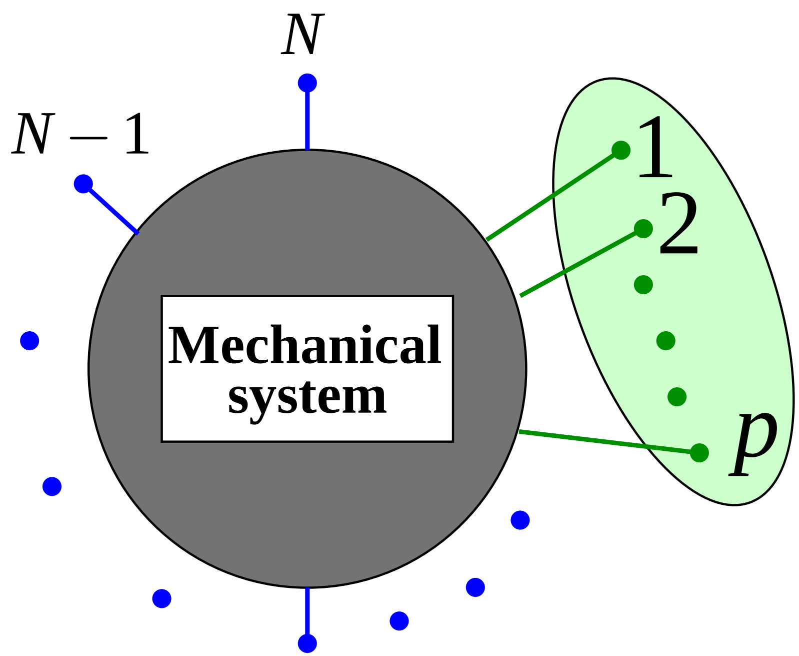


Terminals $\{1,2\}$ and $\{3,4\}$ form a port.

MECHANICAL PORTS

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \quad F_1 + F_2 + \cdots + F_p = 0.$$

'port KFL'

Power and energy

If terminals $\{1,2,\ldots,p\}$ form a port, then

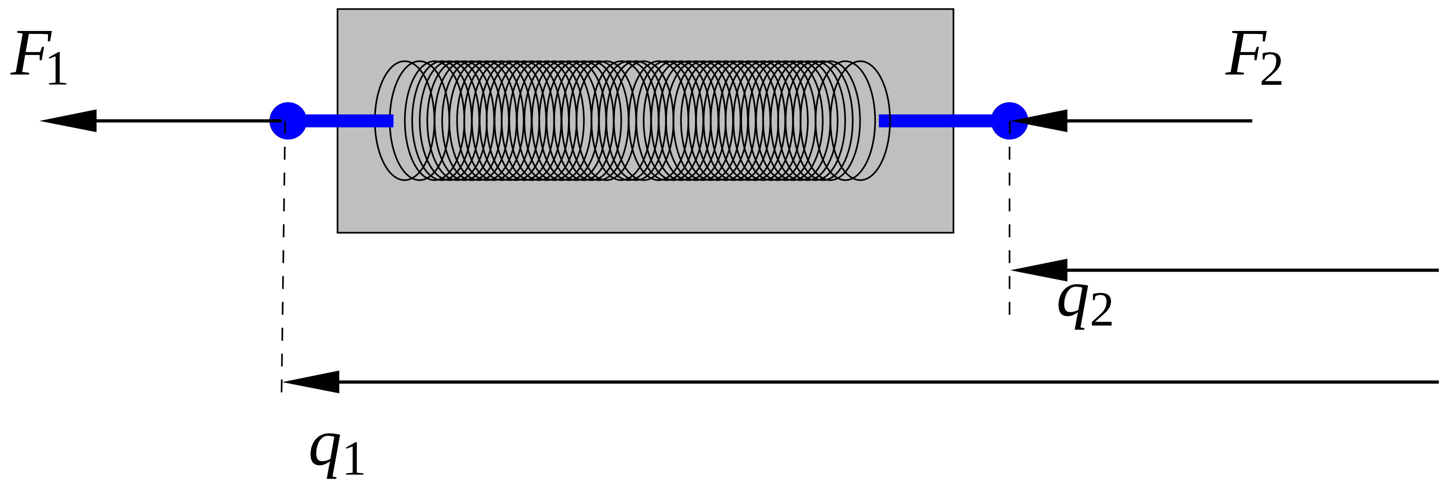
$$\textbf{power in} \;=\; F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

$$\textbf{energy in} \;=\; \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid
unless these terminals form a port !

Example

Spring

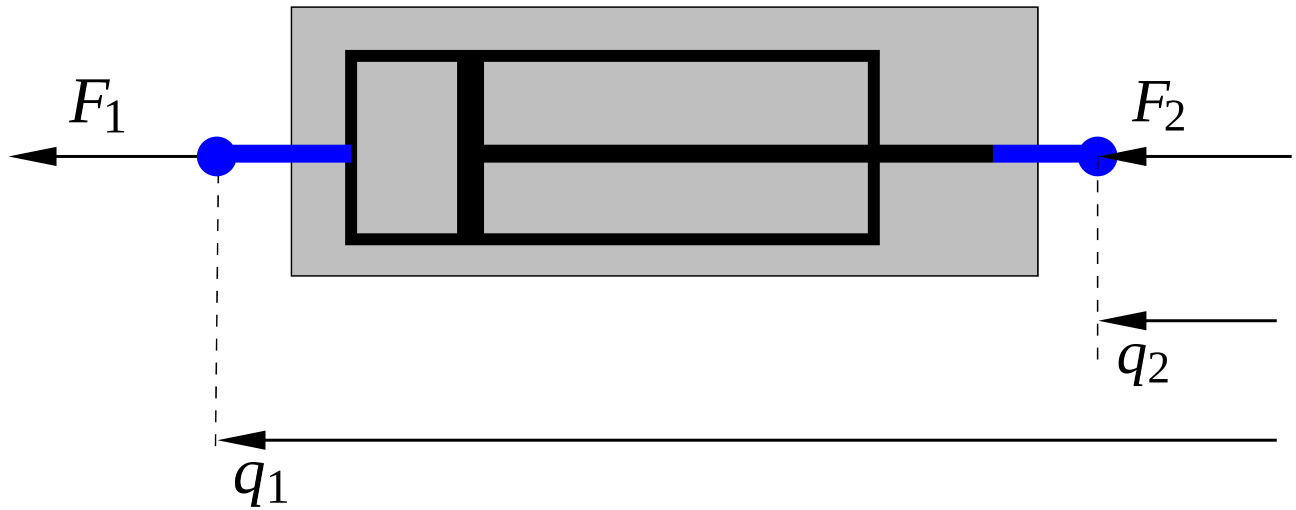


$$F_1 + F_2 = 0, \quad K(q_1 - q_2) = F_1$$

satisfies KFL

Example

Damper

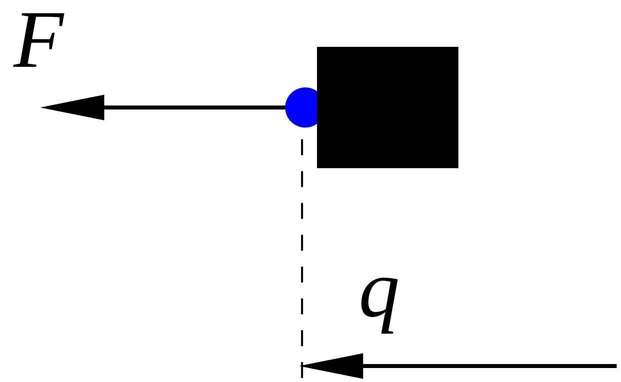


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1$$

satisfies KFL

Springs and dampers, and their interconnection form ports.

A mass



$$M\frac{d^2}{dt^2}q = F$$

does not satisfy KFL

Not a port!!!

Consequences

We discuss 3 consequences of the fact that a mass is not a port.

- **The inerter**
- **Motion energy**
- **Energy as an extensive quantity**

THE INERTER

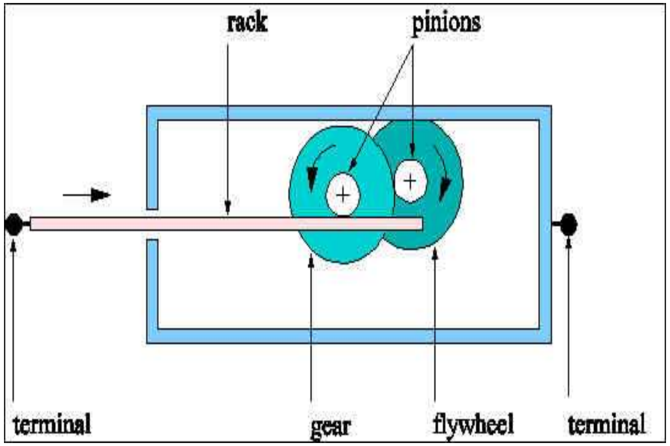
Mechanical synthesis

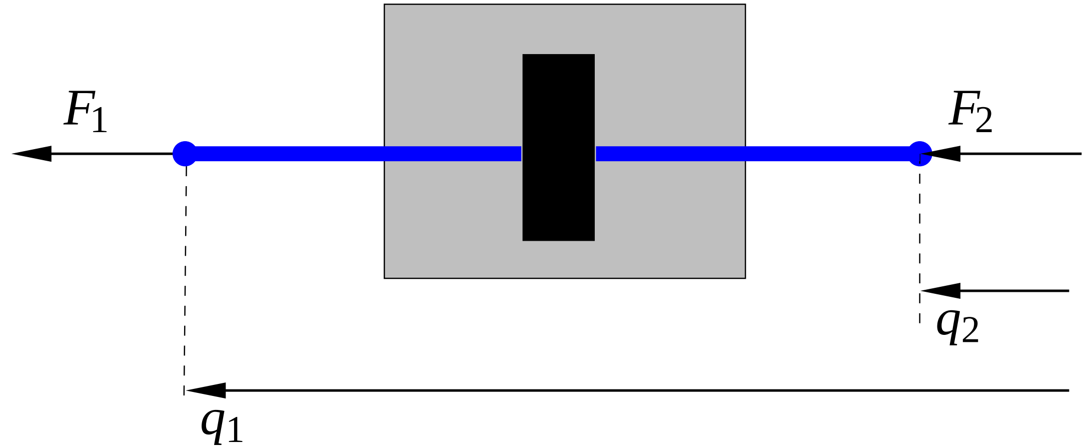
A mass (not a port) is NOT the mechanical analogue of a capacitor (a port).

RLC synthesis $\not\Leftrightarrow$ Damper-Spring-Mass synthesis

Is there a mechanical analogue of a capacitor ?

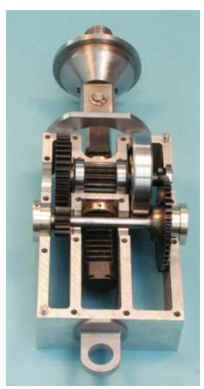
The inerter





$$B\frac{d^2}{dt^2}(q_1 - q_2) = F_1, \quad F_1 + F_2 = 0$$

satisfies KFL $\rightsquigarrow$ a port



Malcolm Smith

Springs, dampers, inerters, & their interconnections $\rightsquigarrow$ ports!

Electrical-mechanical analogies

voltage $V \leftrightarrow v$ **velocity**

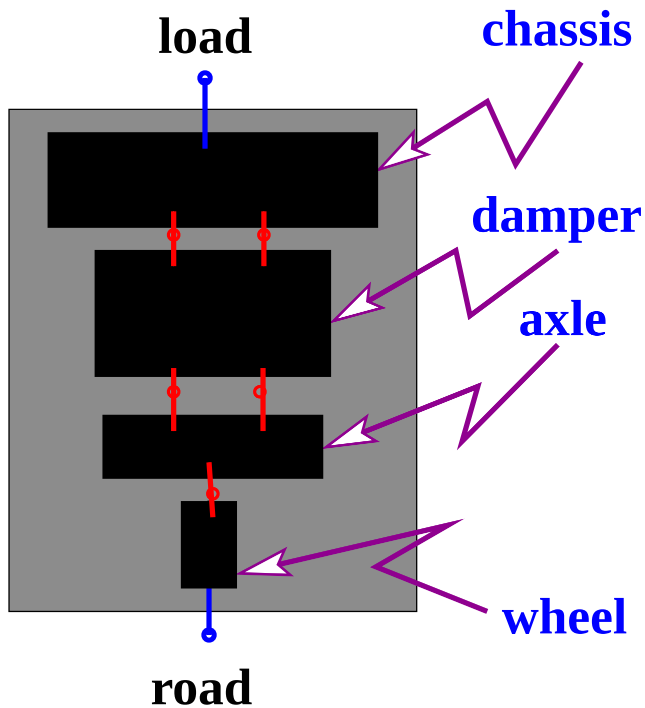
current $I \leftrightarrow F$ **force**

Resistor $\frac{1}{R}(V_1 - V_2) = I_1, \ \ I_1 + I_2 = 0$	**Damper** $D(v_1 - v_2) = F_1, \ \ F_1 + F_2 = 0$
Inductor $\frac{1}{L}(V_1 - V_2) = \frac{d}{dt} I_1, \ \ I_1 + I_2 = 0$	**Spring** $K(v_1 - v_2) = \frac{d}{dt} F_1, \ \ F_1 + F_2 = 0$
Capacitor $C\frac{d}{dt}(V_1 - V_2) = I_1, \ \ I_1 + I_2 = 0$	**Inerter** $B\frac{d}{dt}(v_1 - v_2) = F_1, \ \ F_1 + F_2 = 0$

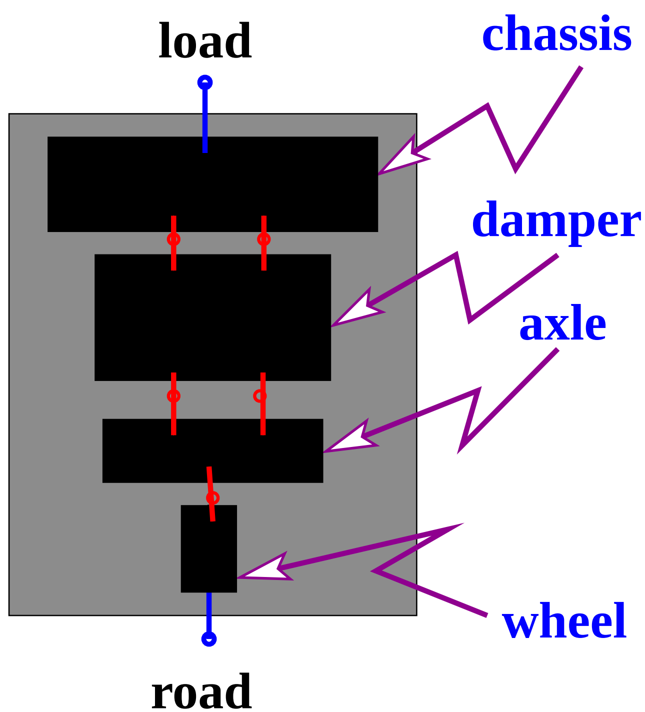
electrical RLC synthesis $\Leftrightarrow$ **mechanical DSI synthesis**

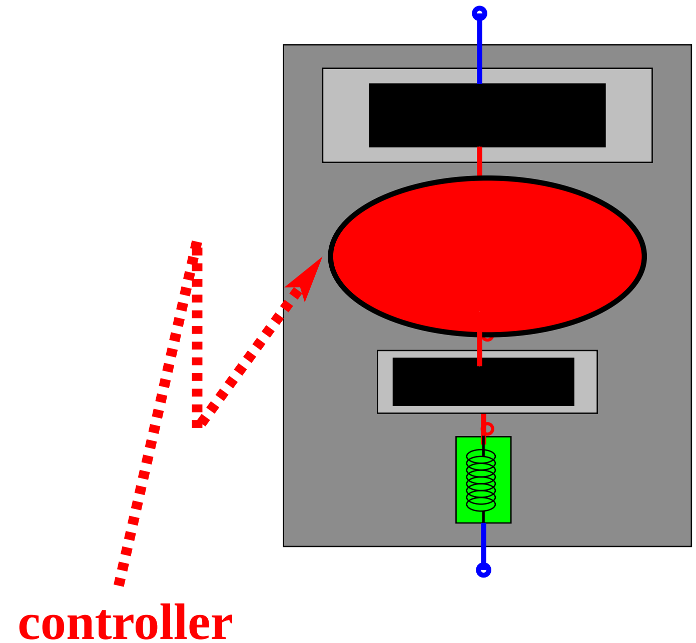
BEHAVIORAL CONTROL

Example of behavioral control: A ‘quarter car’

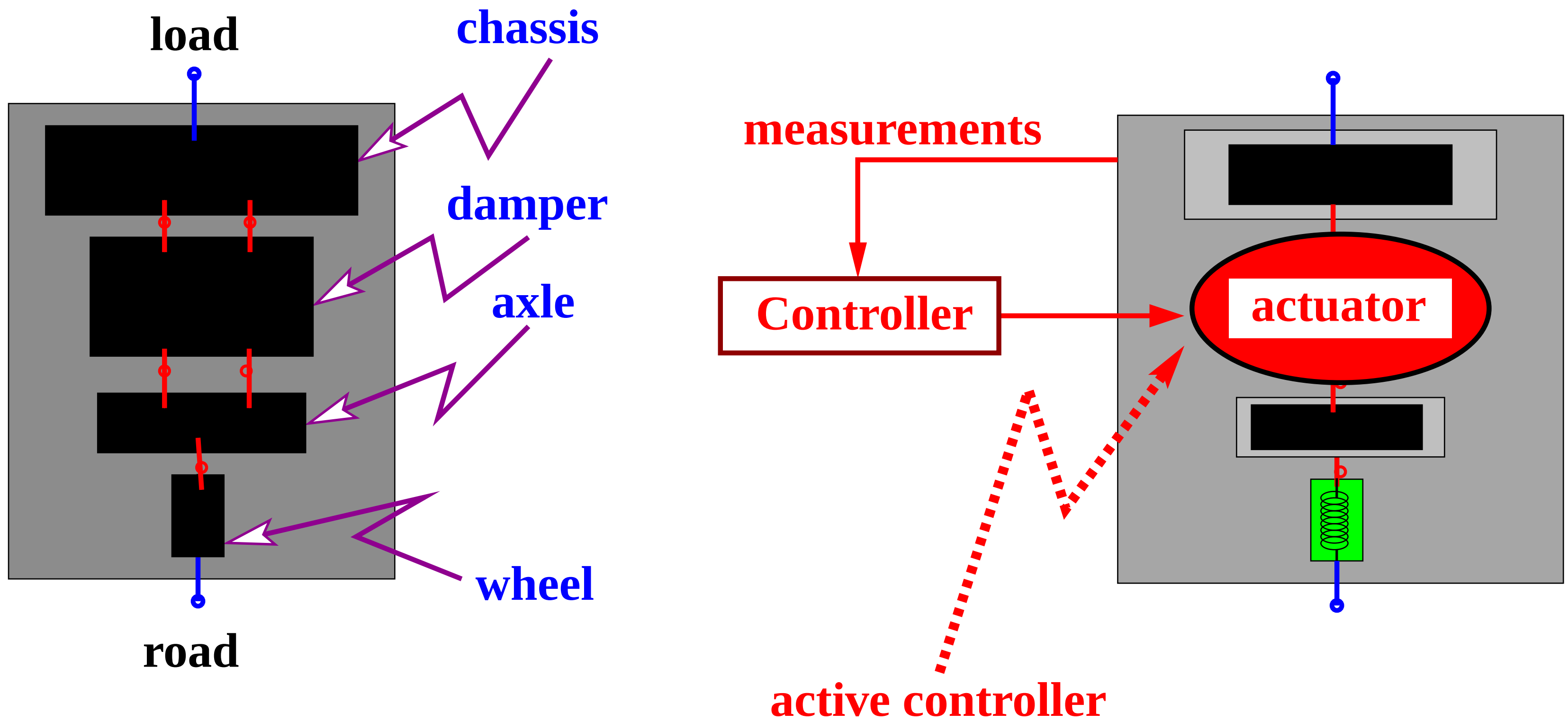


Example of behavioral control: A 'quarter car'

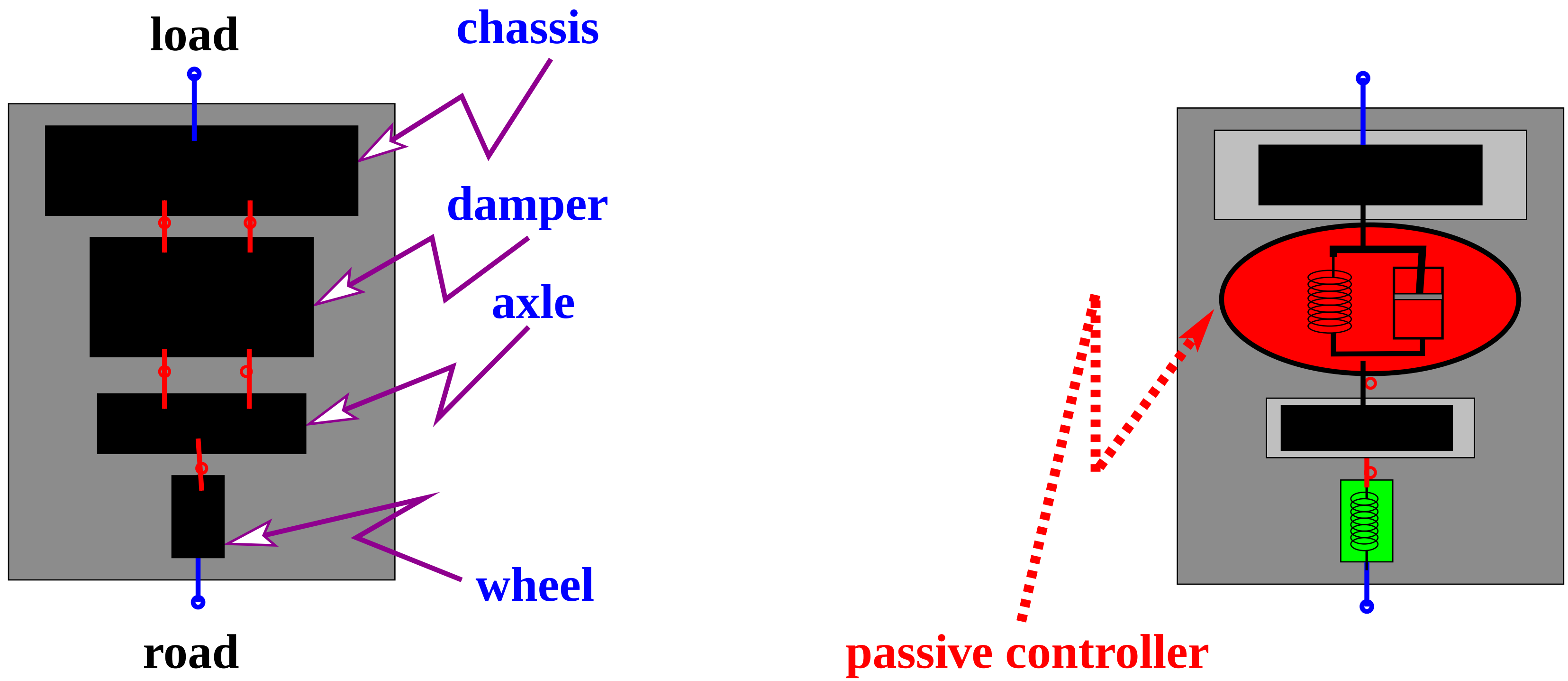




Example of behavioral control: A ‘quarter car’



Example of behavioral control: A ‘quarter car’

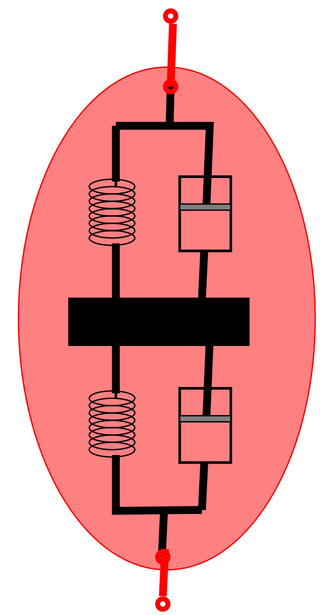


Suspension control in Formula 1

Nigel Mansell victorious in 1992
with an active damper suspension.

Active dampers were banned in 1994
to break the dominance of the Williams team.

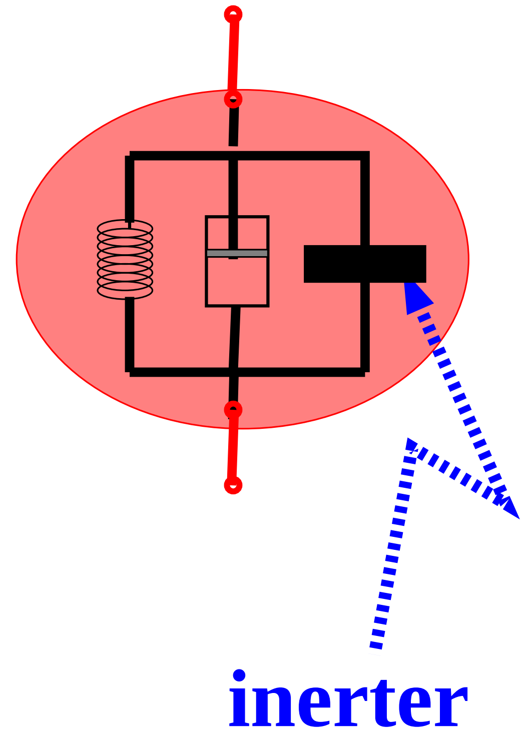
Suspension control in Formula 1



Renault successfully used a passive ‘tuned mass damper’ suspension in 2005/2006.

Tuned mass dampers were banned in 2006, under the ‘movable aerodynamic devices’ clause.

Suspension control in Formula 1



Kimi Räikkönen wins the 2005 Grand Prix in Spain with McLaren's 'J-damper', i.e., Smith's inerter.

grandprix.com

AUGUST 21, 2008

Ingenuity still brings success in Formula 1

ShareThis

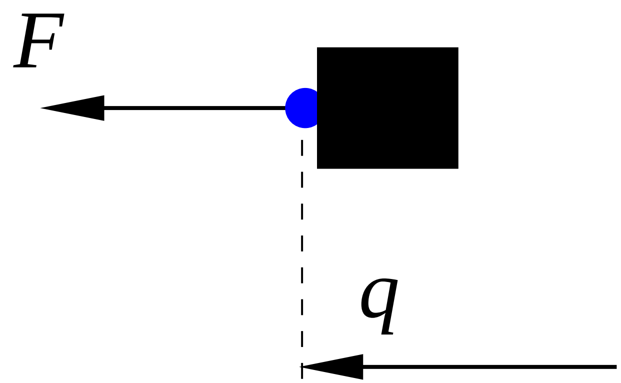
For years engineers have complained that the rules of Formula 1 mean that there is little room left for innovation but Cambridge University's engineering department has just revealed that this is not the case at all.

Professor Malcolm Smith, a fellow of Gonville and Caius College, created an innovative suspension system in the late 1990s and this was patented by the university. The first details were published in 2002 in the obscure Institute of Electrical and Electronics Engineers's publication called Transactions on Automatic Control. This was spotted by the boffins at McLaren and an exclusive deal was negotiated to allow the team to use the technology in F1. The new system was first used at the Spanish GP in 2005 and Kimi Raikkonen won.

The team used the name "J Damper" to describe the unit - in an effort to confuse the opposition - but it has now been revealed that it is actually called "an inerter". This is

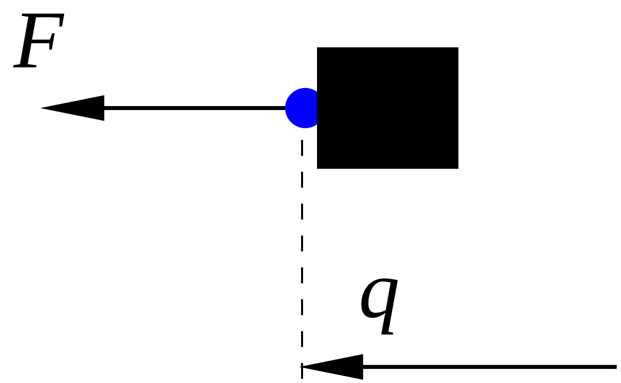
MOTION ENERGY

Back to the mass



$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q$$

Back to the mass

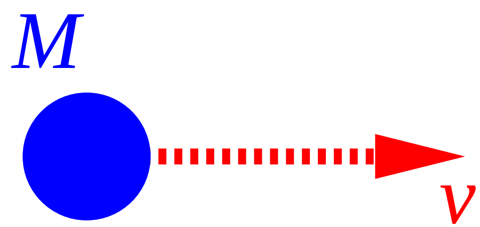


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q$$

If $F^\top \frac{d}{dt}q$ ***is not power,***

is $\frac{1}{2}M||\frac{d}{dt}q||^2$ ***not stored (kinetic, motion) energy ???***

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

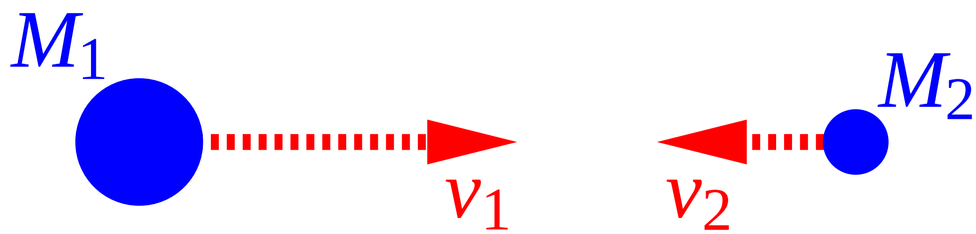
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2}\, M \,||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

This expression is not invariant under uniform motion.

Motion energy



What is the motion energy?

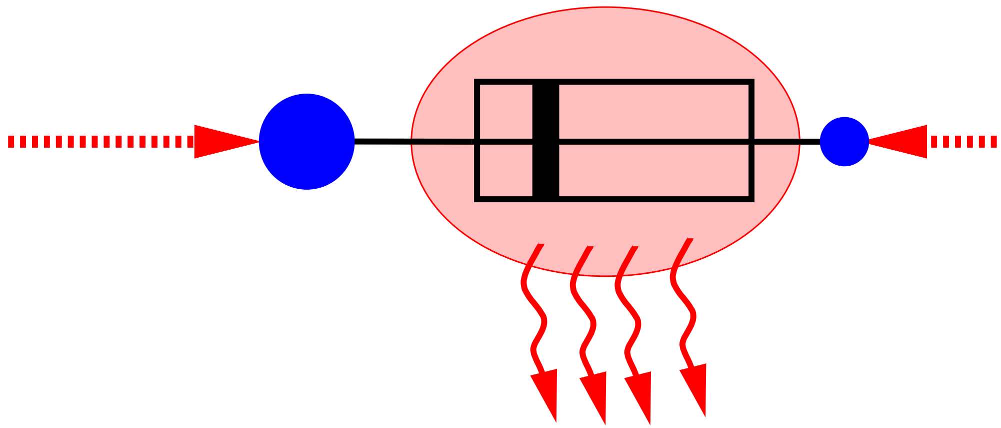
What quantity is transformable into heat?

$$\mathscr{E}_{\textbf{motion}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1 - v_2||^2$$

Invariant under uniform motion.

Dissipation into heat

Can be justified by mounting a damper between the masses.

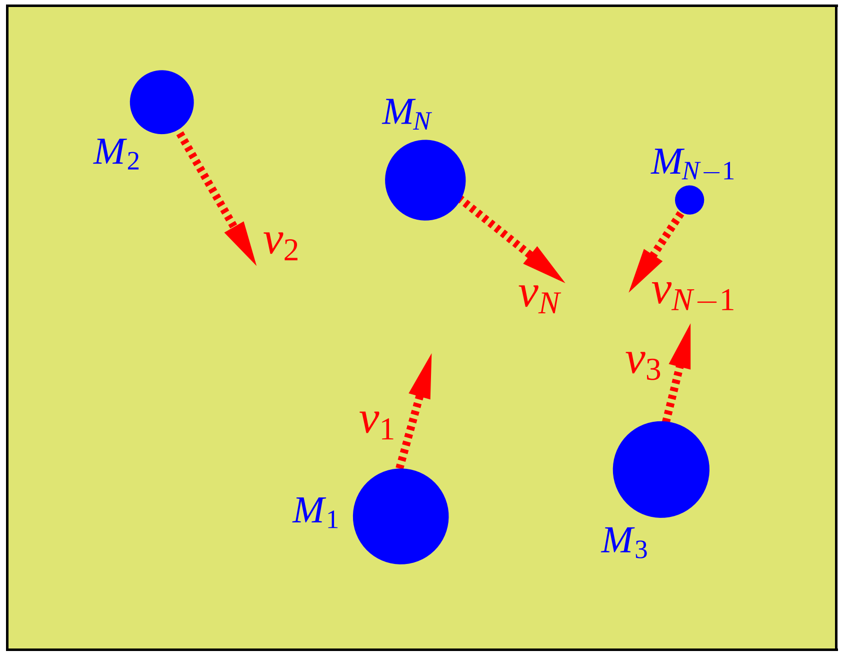


$$\frac{1}{2}\ \frac{M_1\,M_2}{M_1+M_2}\ ||v_1 - v_2||^2$$

is the heat dissipated in the damper.

Motion energy

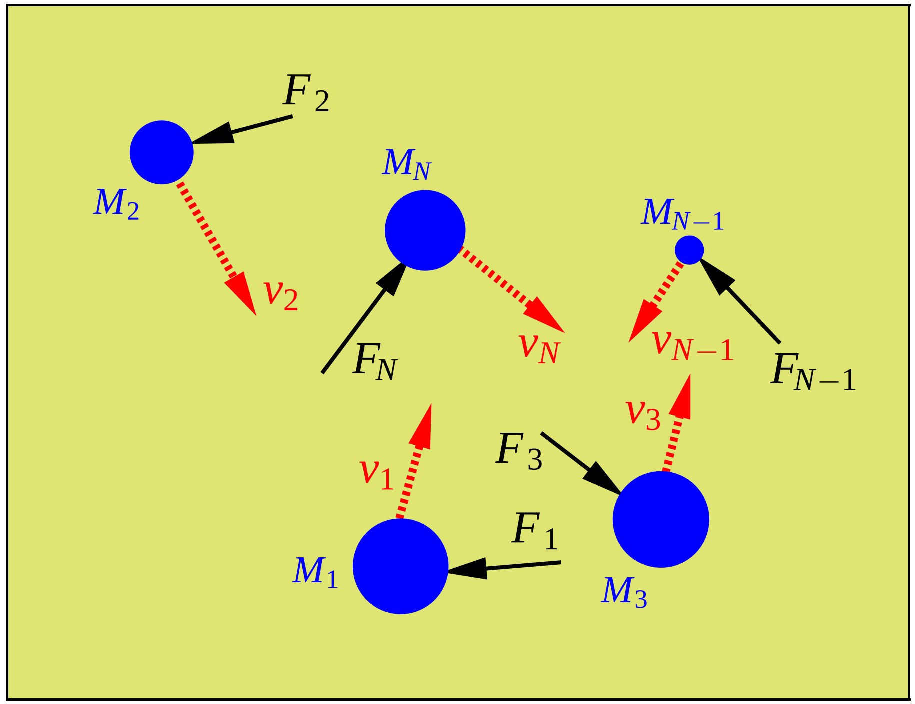
Generalization to N masses.



$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

Motion energy

With external forces.



$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

(KFL) $\sum_{i \in \{1,2,\ldots,N\}} F_i = 0 \quad \Rightarrow \quad \frac{d}{dt}\mathscr{E}_{\textbf{motion}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i.$

Motion energy

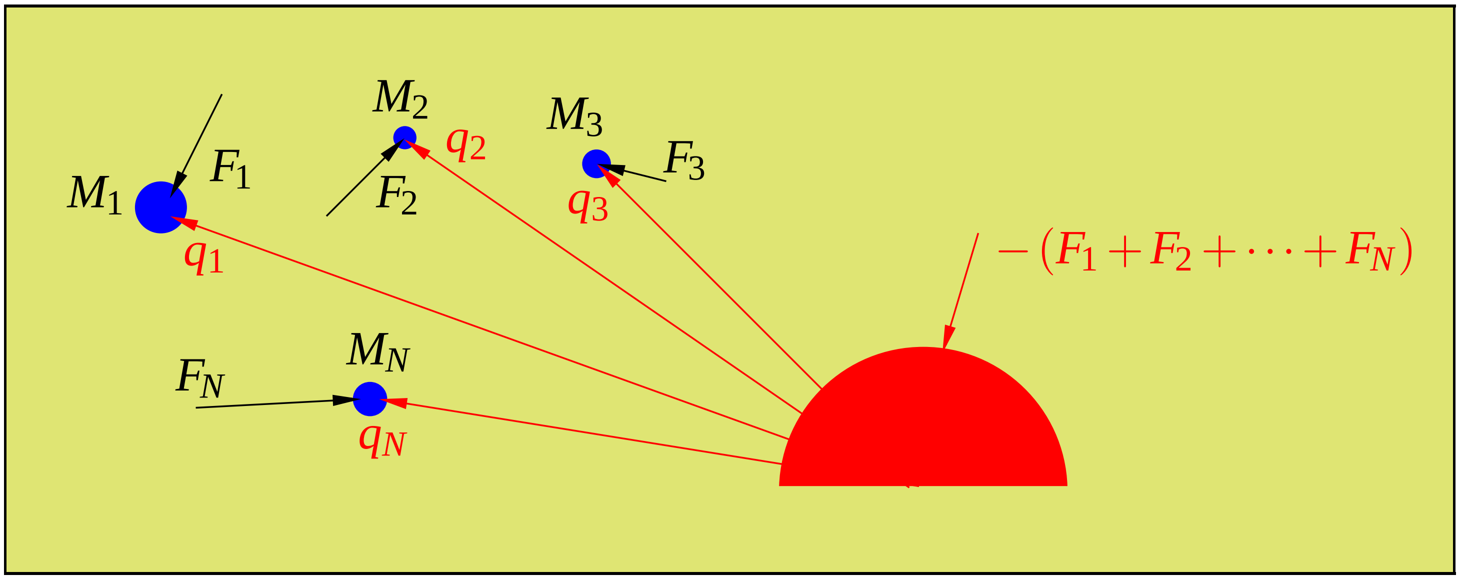
$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2.$$

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N),$



measure velocities w.r.t. this infinite mass ('ground'), then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

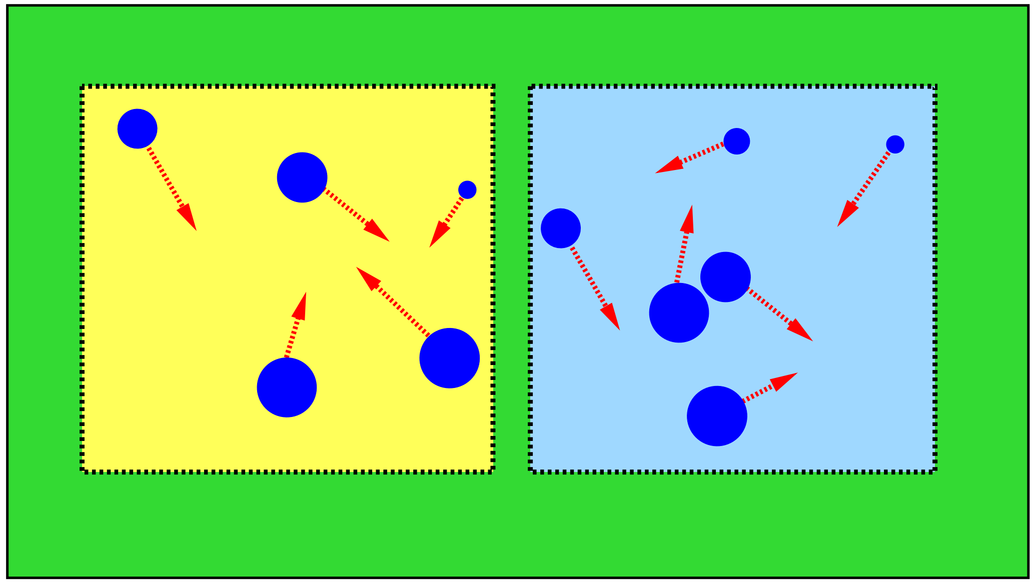
$$\longrightarrow \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

ENERGY as an

EXTENSIVE QUANTITY

Motion energy

Motion energy is not an extensive quantity, it is not additive.

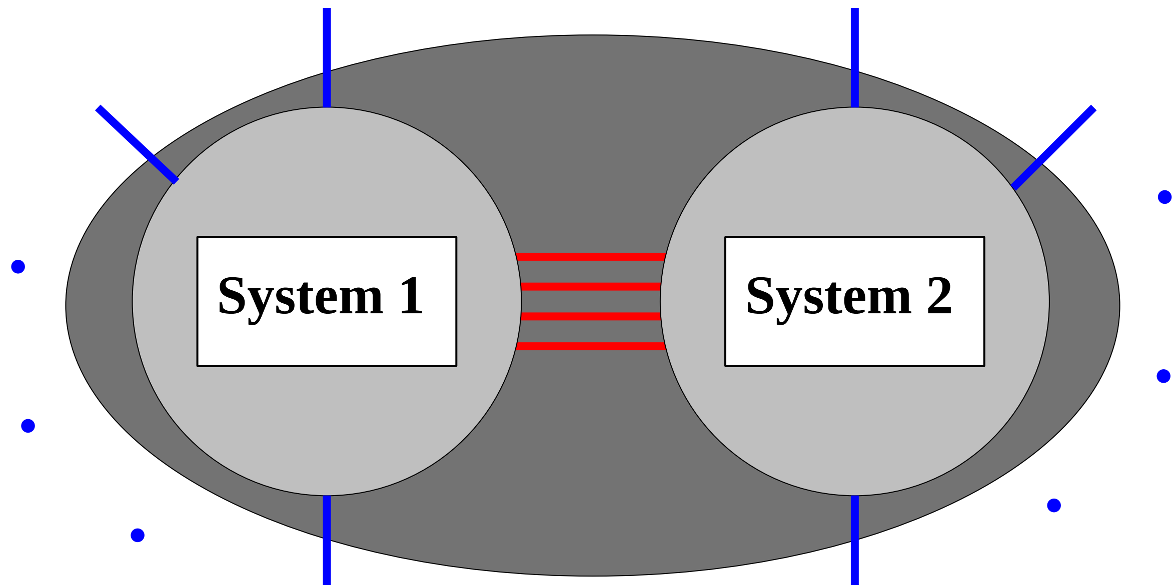


Total motion energy $\neq$ sum of the parts.

Power and energy involve ‘action at a distance’.

PORTS and TERMINALS

Energy transfer



One cannot speak about

“ the energy transferred from system 1 to system 2 ”
or ***“ from the environment to system 1 ”*****,**

unless the relevant terminals form a port.

Ports and terminals

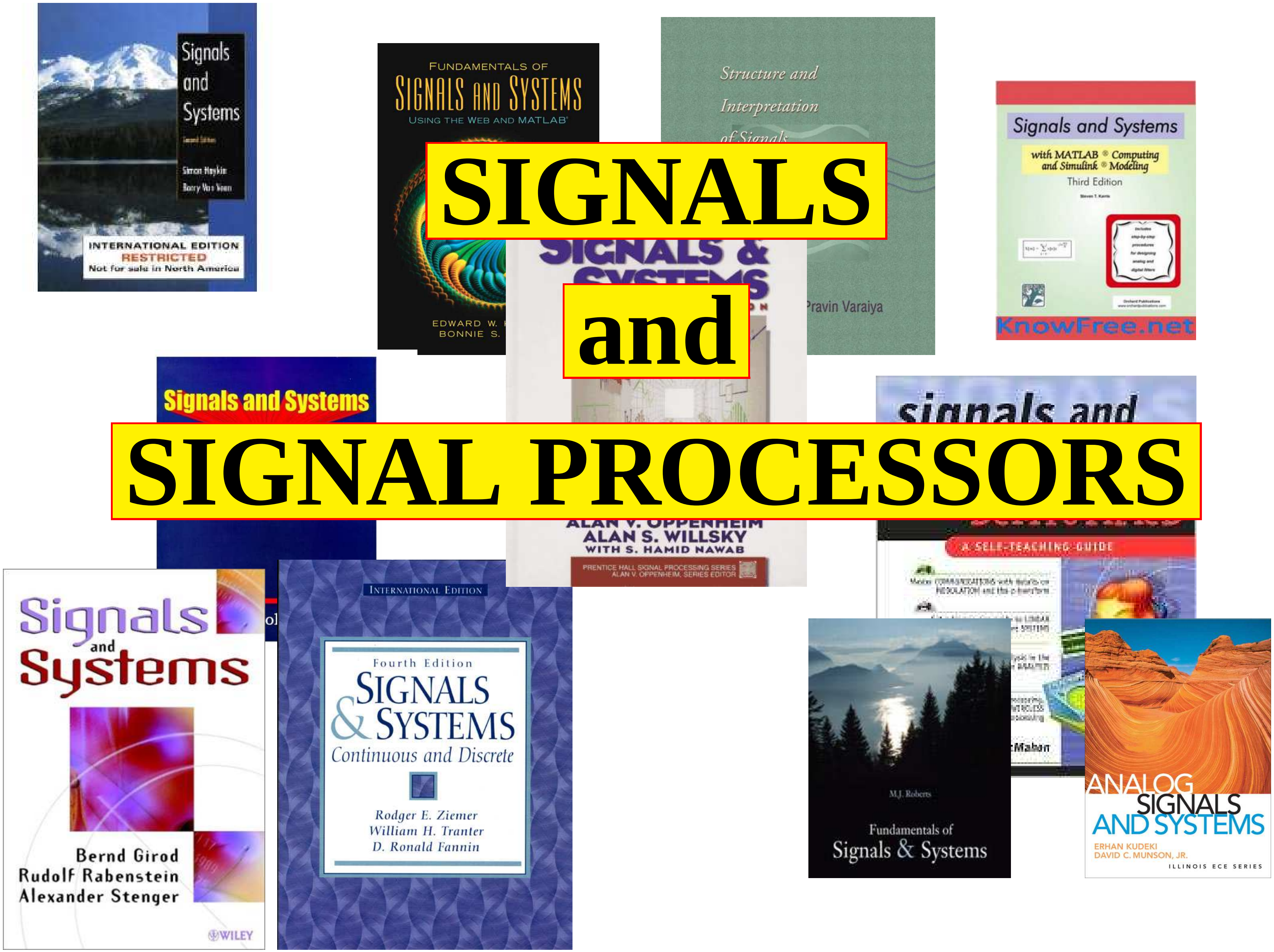
Terminals are for interconnection,

ports are for energy transfer.

CONCLUSION

Favorite textbooks

Favorite textbooks



Reference: The behavioral approach to open and interconnected systems, *Control Systems Magazine*, volume 27, pages 46-99, 2007.

Copies of the lecture frames will be available from/at

`Jan.Willems@esat.kuleuven.be`

`http://www.esat.kuleuven.be/∼jwillems`

Thank you

Thank you

Thank you

Thank you

Thank you

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Thank you