# OPEN DYNAMICAL SYSTEMS

# Their aims and origins

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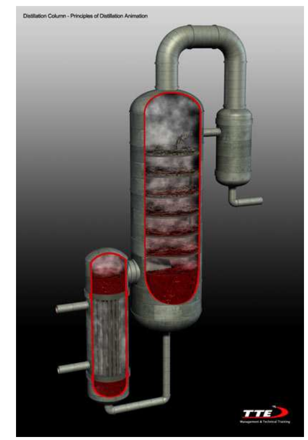
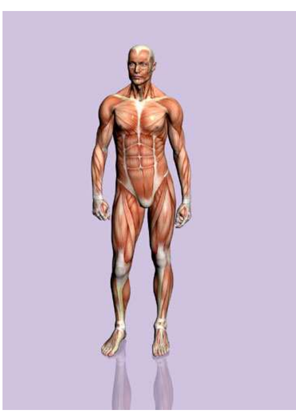
# Theme

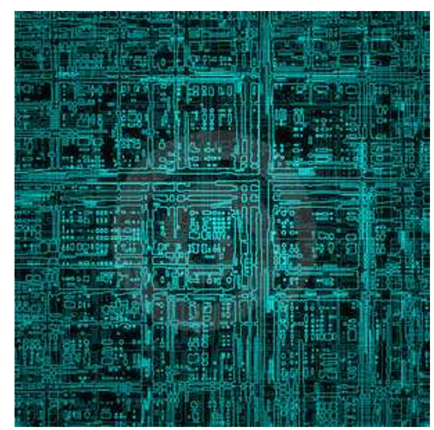
- How are open systems formalized?
- How are systems interconnected?
- How is energy transferred between systems?

# SYSTEMS


OIL REFINERY (GVG / PD)







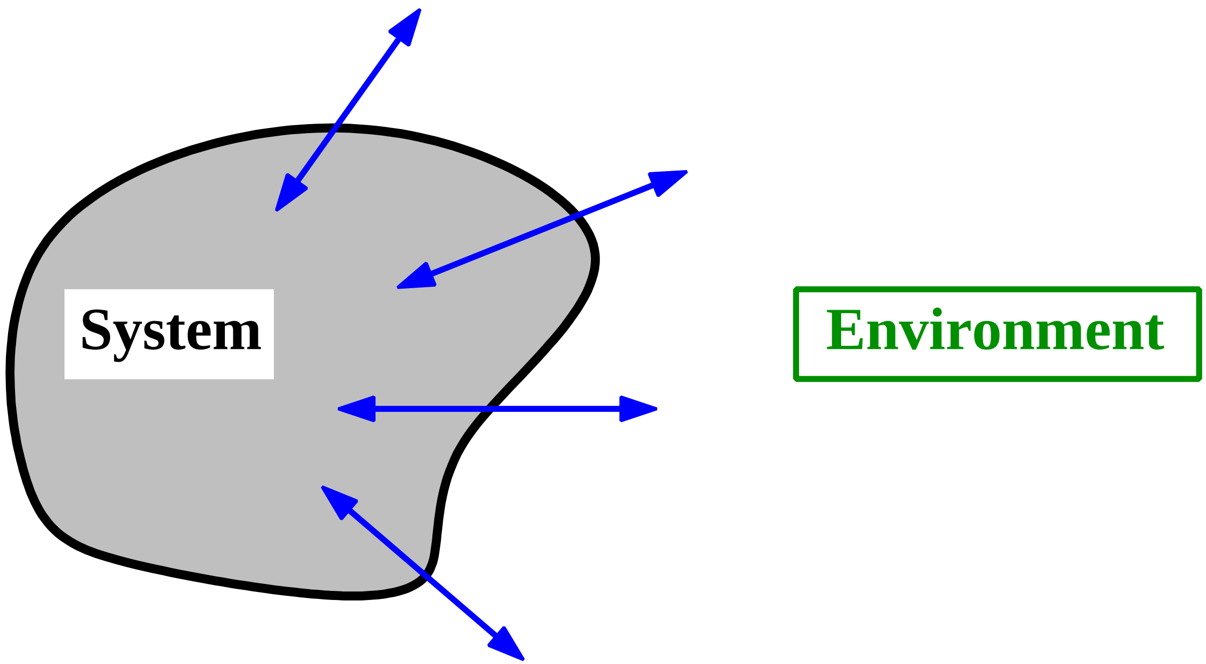
# Features

- **Open**
- **Interconnected**
- **Modular**
- **Dynamic**

**The ever-increasing computing power allows to model complex interconnected systems accurately by tearing, zooming, and linking.**

$\leadsto$ **Simulation, model based design, ...**

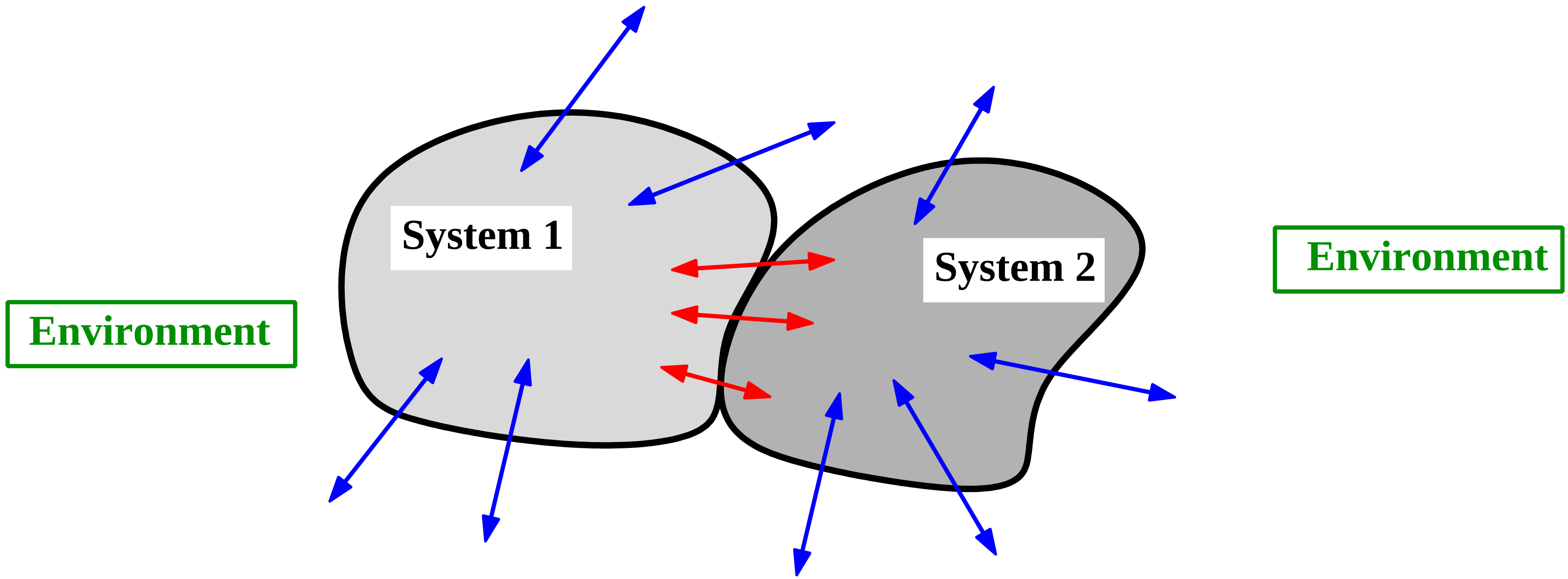
# Open systems




**Systems are ‘open’, they interact with their environment.**

**How are such systems formalized?**

# Interacting systems




**Interconnected systems interact.**

**How is this interaction formalized?**

# Modularity

**Systems consist of the interconnection of**

**repeated building blocks.**

**Essential for computer-assisted modeling.**

# Modularity

**Systems consist of the interconnection of repeated building blocks.**

**Essential for computer-assisted modeling.**

**Examples:**

**electrical circuits $\rightsquigarrow$**

**resistors, capacitors, inductors,**

**transistors, diodes, sources, etc.**

**mechanical devices $\rightsquigarrow$**

**masses, springs, dampers,**

**connecting bars, joints, etc.**

**etc.**

# Dynamical

**Main interest: the evolution over time.**

**How do the variables evolve in the long-term?**

**Are there excessive transients?**

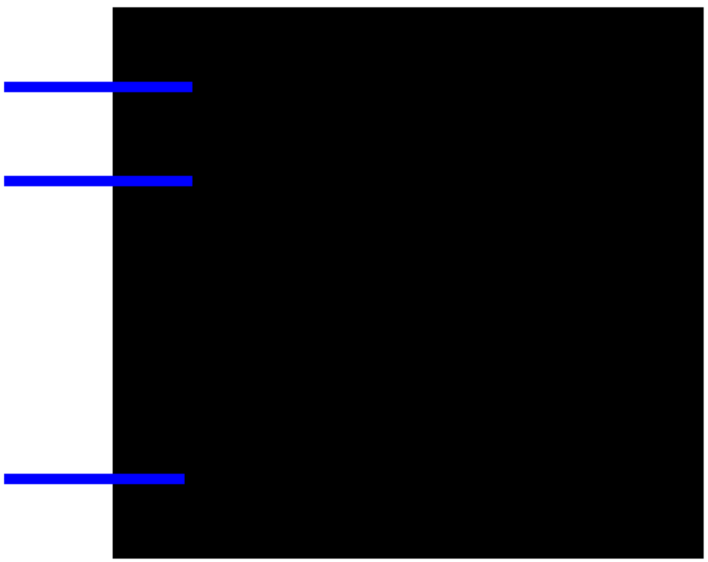
**Do small variations drastically change the future?**

**etc.**

# TEARING, ZOOMING, LINKING

# Tearing

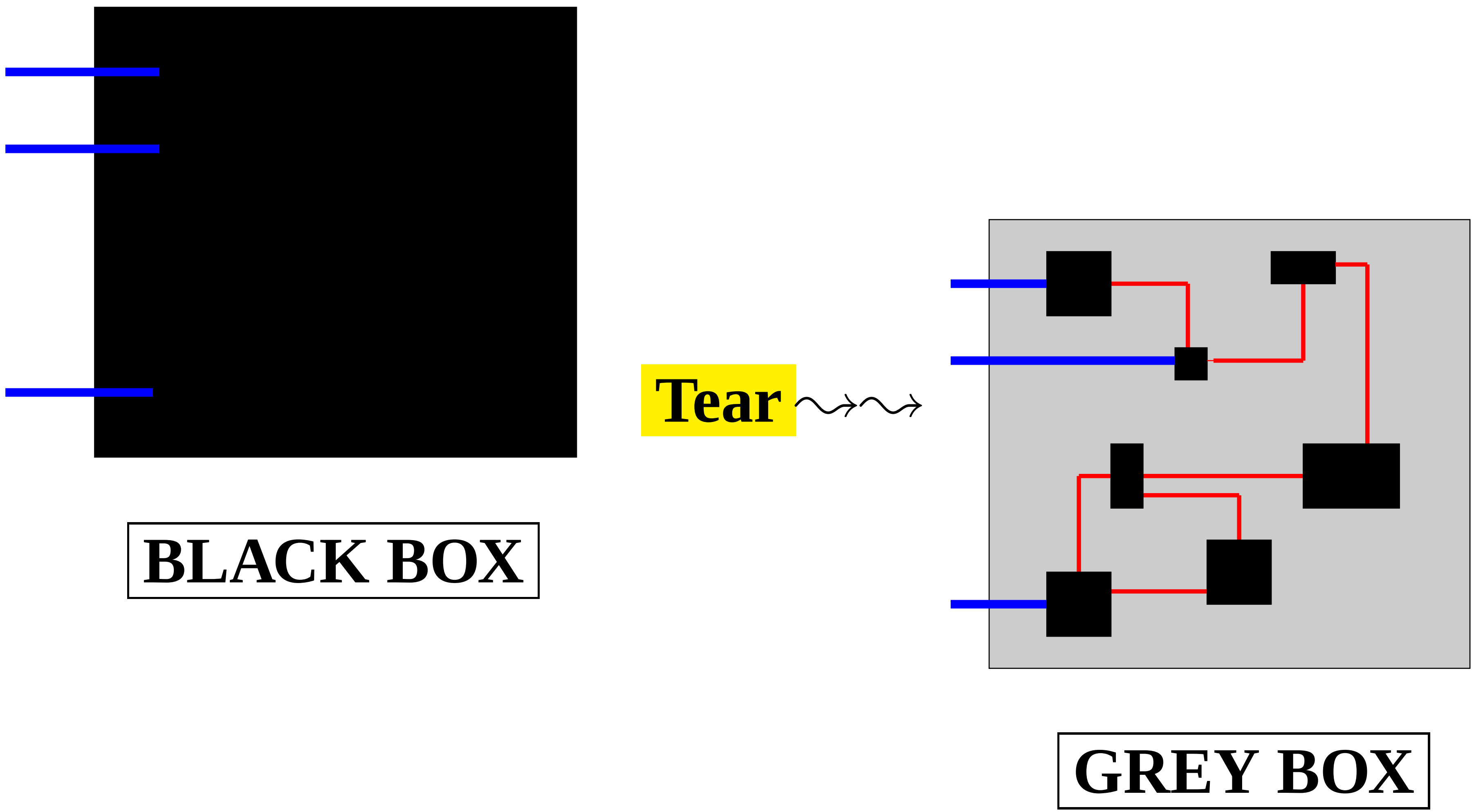
**¡¡ Model the behavior of selected variables !!**



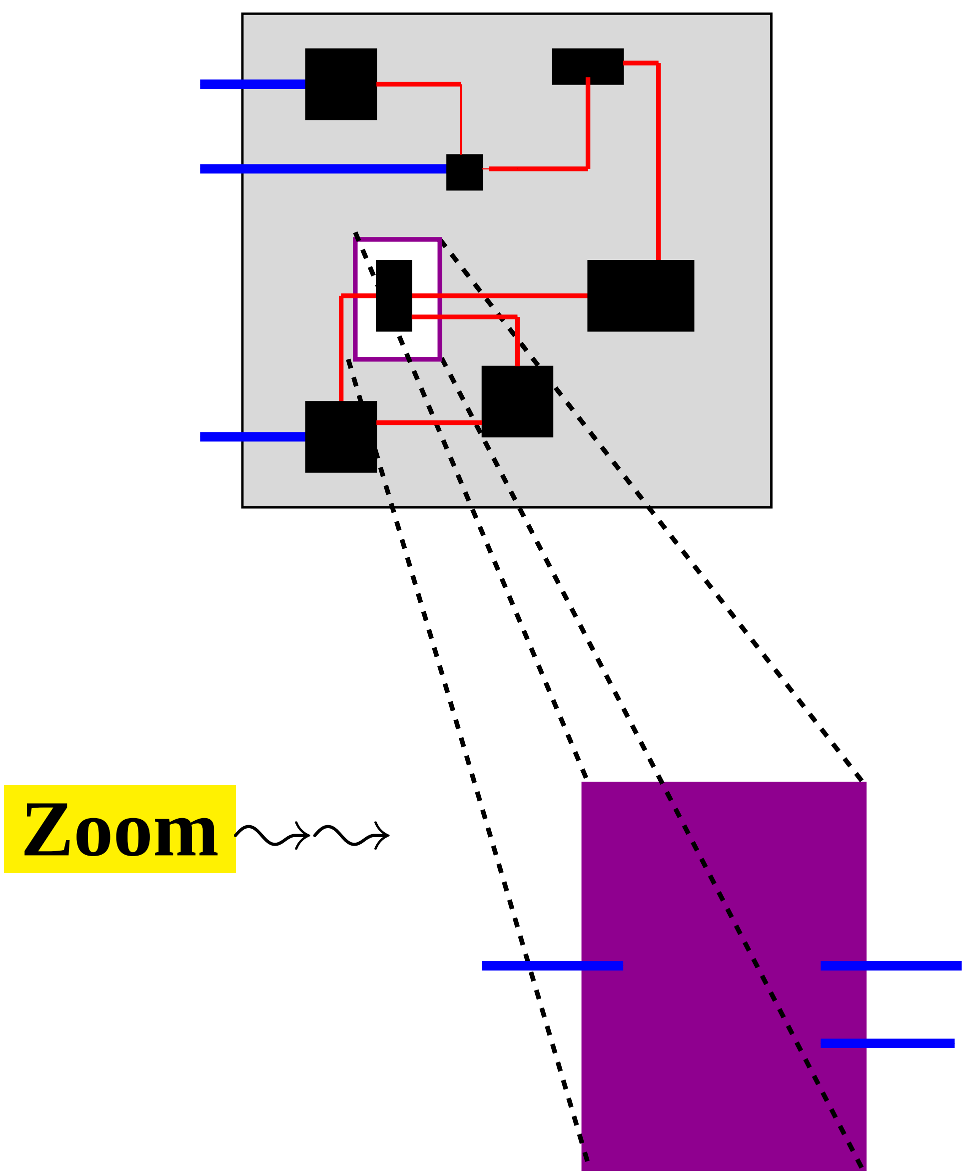
**BLACK BOX**

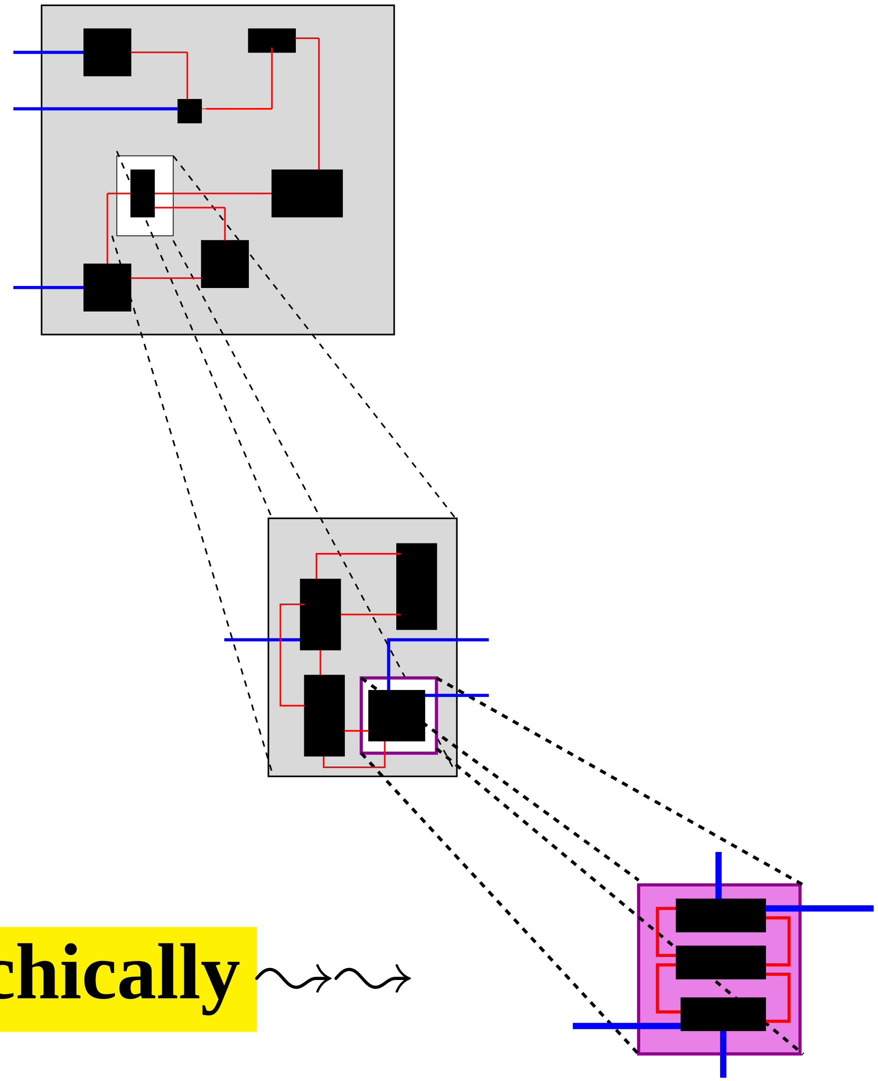
# Tearing

**¡¡ Model the behavior of selected variables !!**



# Zooming

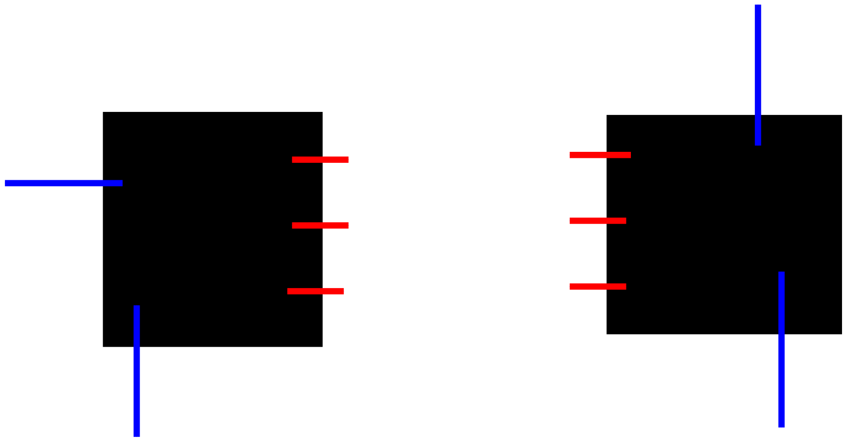




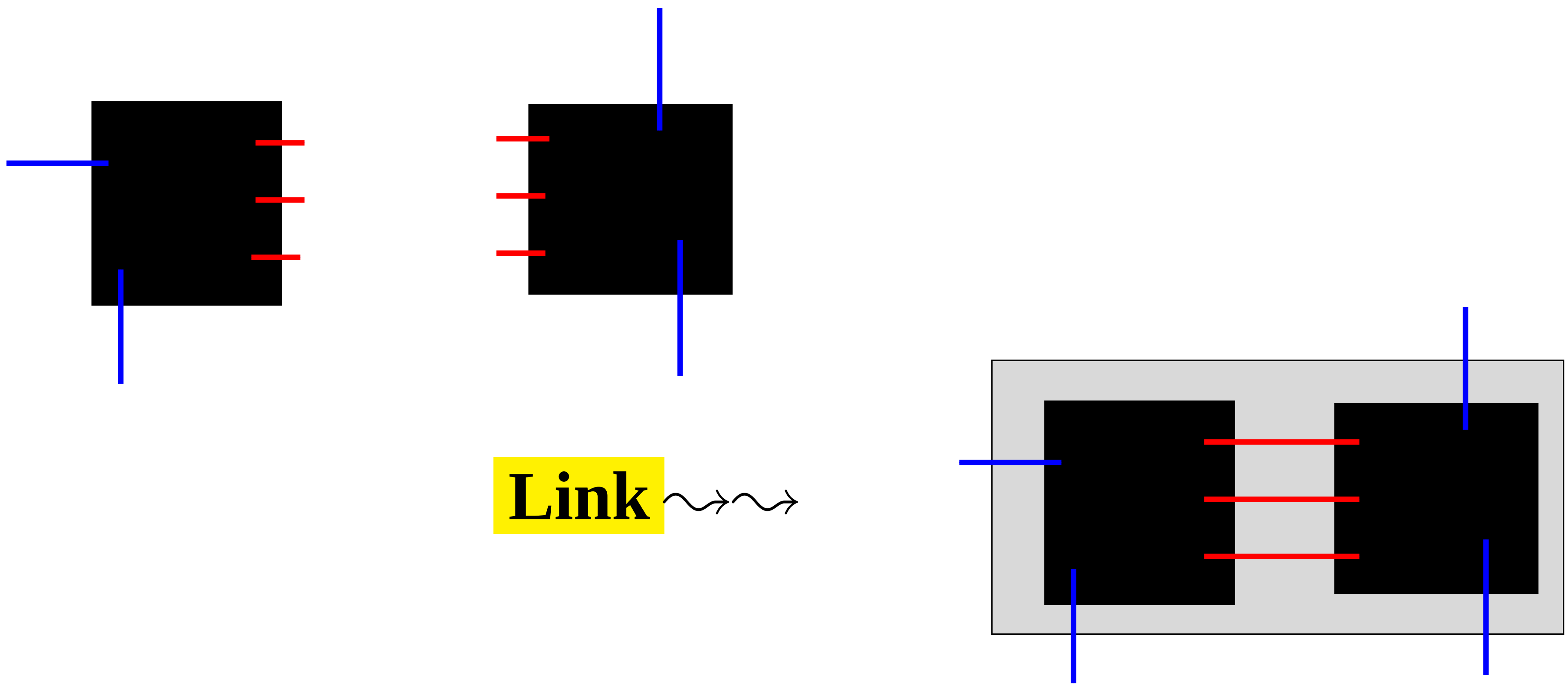
**Zoom hierarchically** ⇝⇝

**Proceed until subsystems (‘modularity’) are obtained whose model is known from first principles, or stored in a database.**

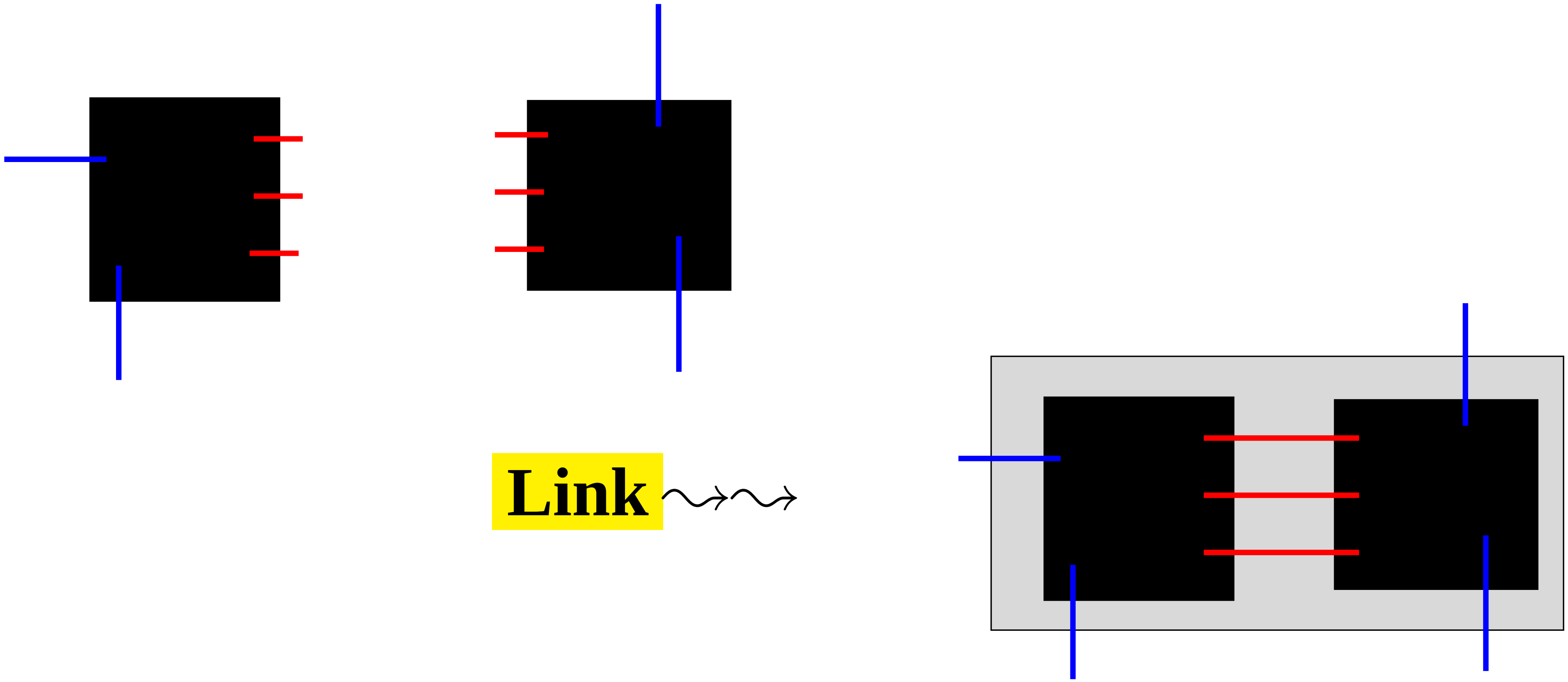
# Linking



# Linking



# Linking




**model for component variables + linking equations**

**⇝ model of behavior of the black box variables.**

**Tearing, zooming, & linking ⇔ computer assisted modeling.**

# Aims

**The ever-increasing computing power allows to model complex interconnected systems accurately.**

**But requires the right mathematical concepts**

- **for dynamical system,**
- **for interconnection,**
- **for interconnection architecture.**

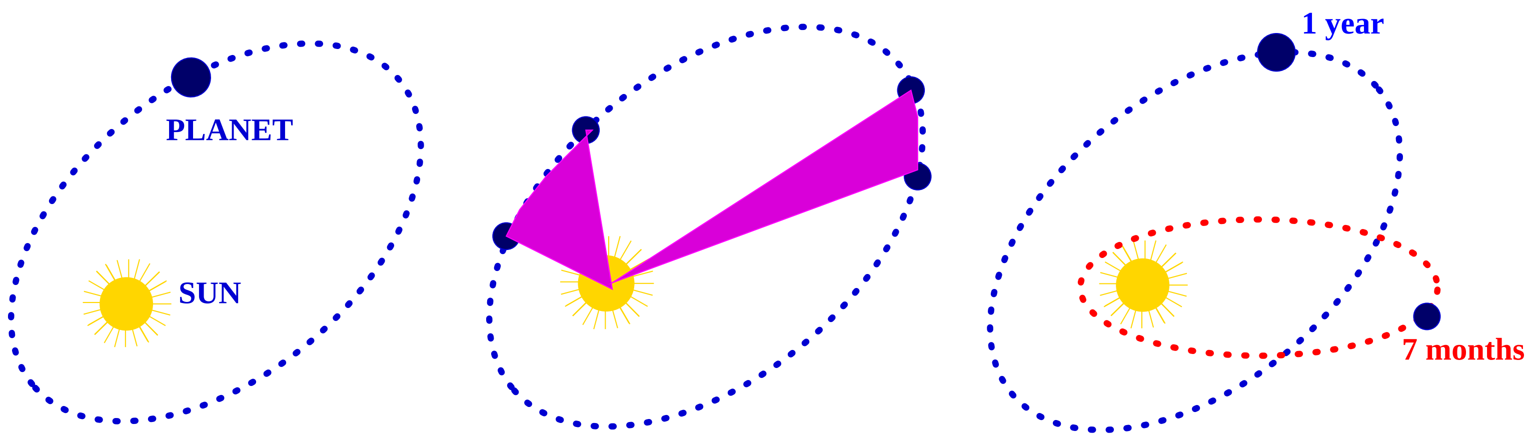
# HOW IT ALL BEGAN ...

# Planetary motion


**How, for heaven's sake, does it move?**

# Kepler's laws

## Variable: the position as a function of time.




- **K1: ellipse, sun in focus,**
- **K2: = areas in = times,**
- **K3: square of the period = third power of major axis.**


Johannes Kepler
(1571–1630)

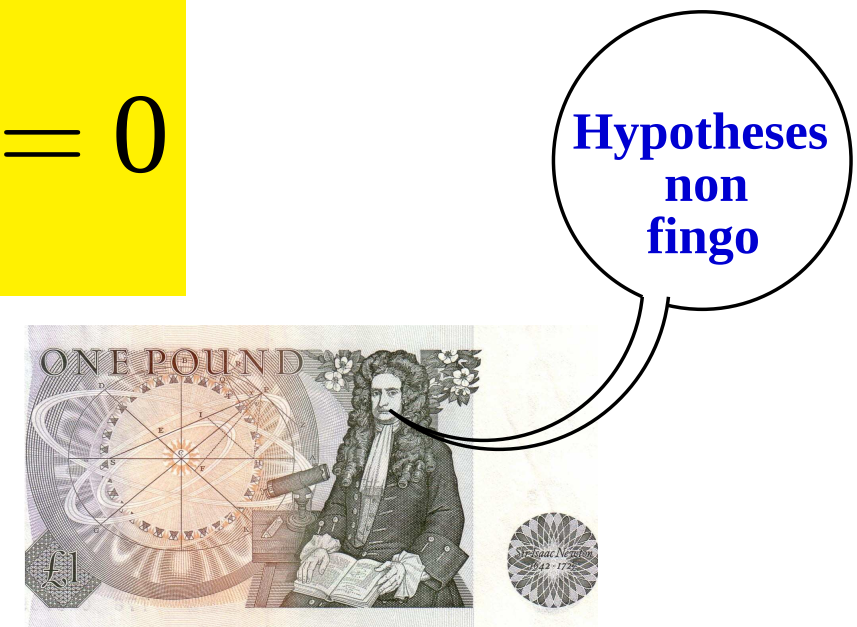
# Newton's version

**Acceleration = function of position and velocity** $\rightsquigarrow$

$$\frac{d^2}{dt^2}w(t) = A(w(t), \frac{d}{dt}w(t)).$$

**Via calculus and calculations: K1, K2, & K3** $\Leftrightarrow$

$$\frac{d^2}{dt^2}w(t) + \frac{\vec{1}_{w(t)}}{||w(t)||^2} = 0$$




**Isaac Newton (1643–1727)**

# Newton's version

$$\frac{d^2}{dt^2}w(t) + \frac{\vec{1}_{w(t)}}{||w(t)||^2} = 0$$

**with** $x = \begin{bmatrix} w \\ \frac{d}{dt}w \end{bmatrix} \rightsquigarrow \frac{d}{dt}x(t) = f(x(t))$

$$\boxed{x(0) \Rightarrow x(\cdot)}$$

**The motion of a planet is completely determined by its initial position and initial velocity.**

**This led to the idea of a (deterministic) closed system.**

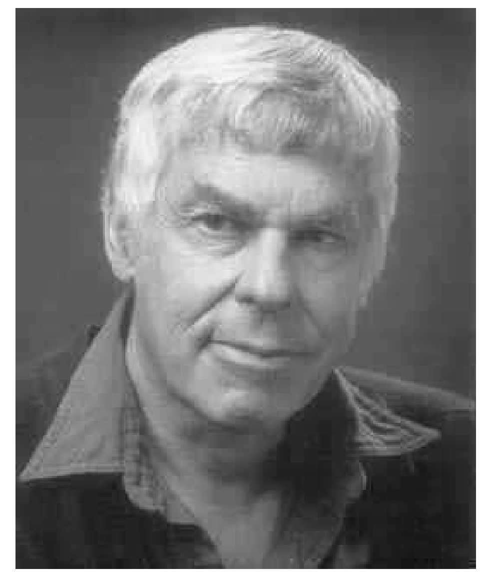
# The paradigm of closed systems

**Motion completely determined by initial conditions.**

**No environmental influences.**

**Henri Poincaré (1854-1912)**

**George Birkhoff (1884-1944)**



**Stephen Smale (1930- )**

$\leadsto$ **differential equations, chaos, cellular automata, etc.**

# The paradigm of closed systems

**Motion completely determined by initial conditions.**

**No environmental influences.**

**Inadequate for modeling:**

**How could they forget**

**about Newton's second law,**
**about Maxwell's equations,**
**about thermodynamics,**
**about tearing, zooming & linking?**

# Newton's laws

**Gravitation:** $F_1(t) = mM \frac{\vec{1}_{w(t)}}{||w(t)||^2}$

**Second law:** $F_2(t) = m \frac{d^2}{dt^2} w(t)$

**Third law:** $F_1(t) + F_2(t) = 0$

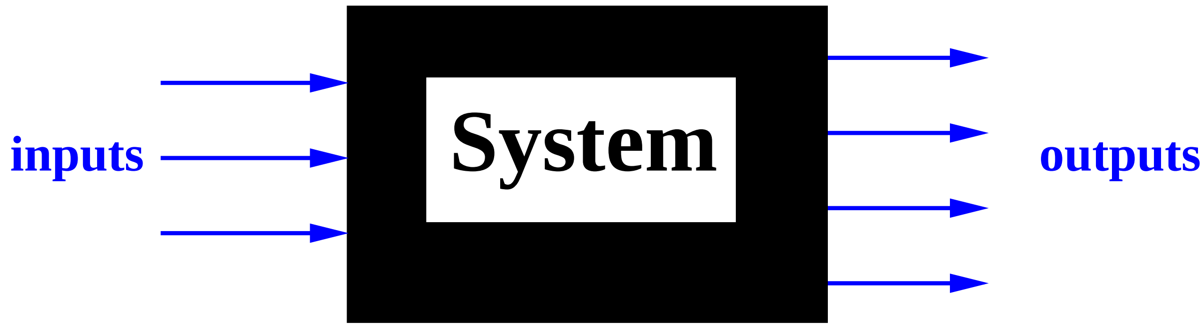
$$\Downarrow$$

$$\frac{d^2}{dt^2} w(t) + \frac{\vec{1}_{w(t)}}{||w(t)||^2} = 0$$

**Newton painted by William Blake**

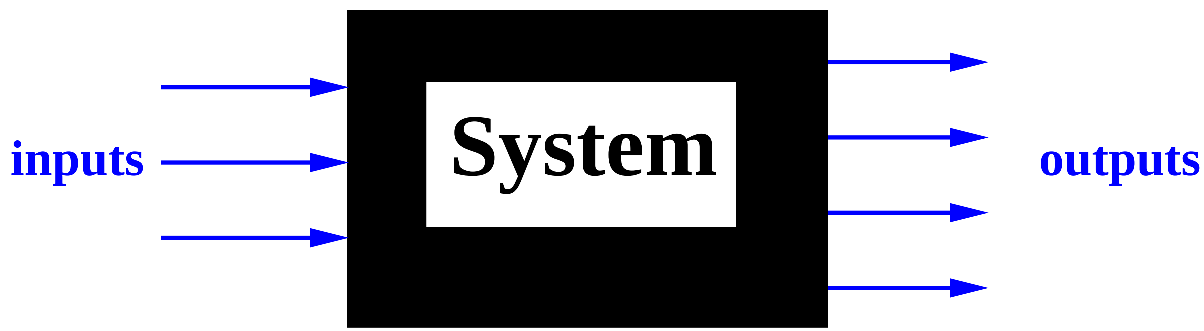
# INPUT/OUTPUT VIEW

# Input/output systems




**Appealing: cause & effect, stimulus & response, etc.**

# Input/output systems

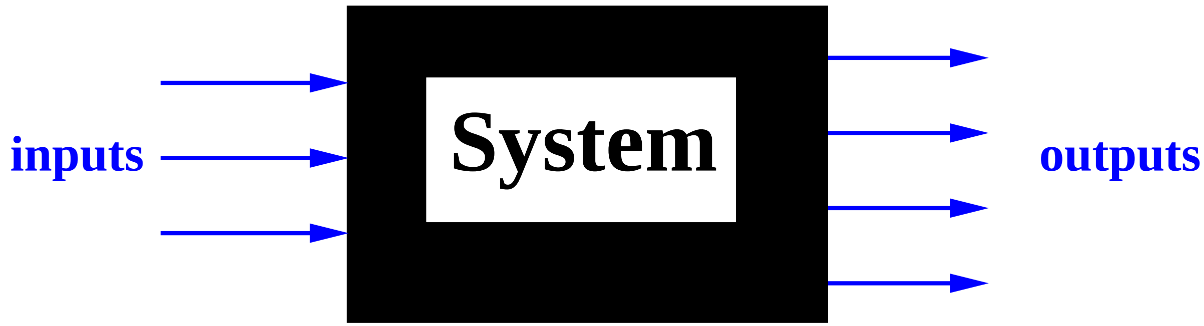



**Appealing: cause & effect, stimulus & response, etc.**

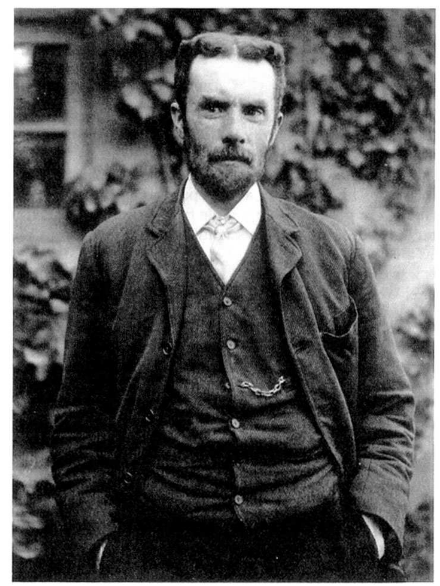
**Developed mainly in electrical engineering since $\pm$ 1920, for circuit analysis and synthesis, and in control engineering.**

**These models do not cope well with initial conditions, very awkward framework for nonlinear models.**

# Input/output systems

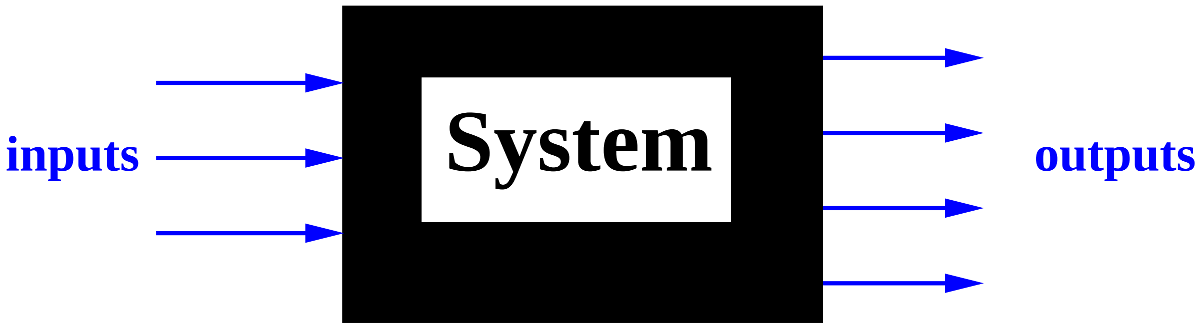


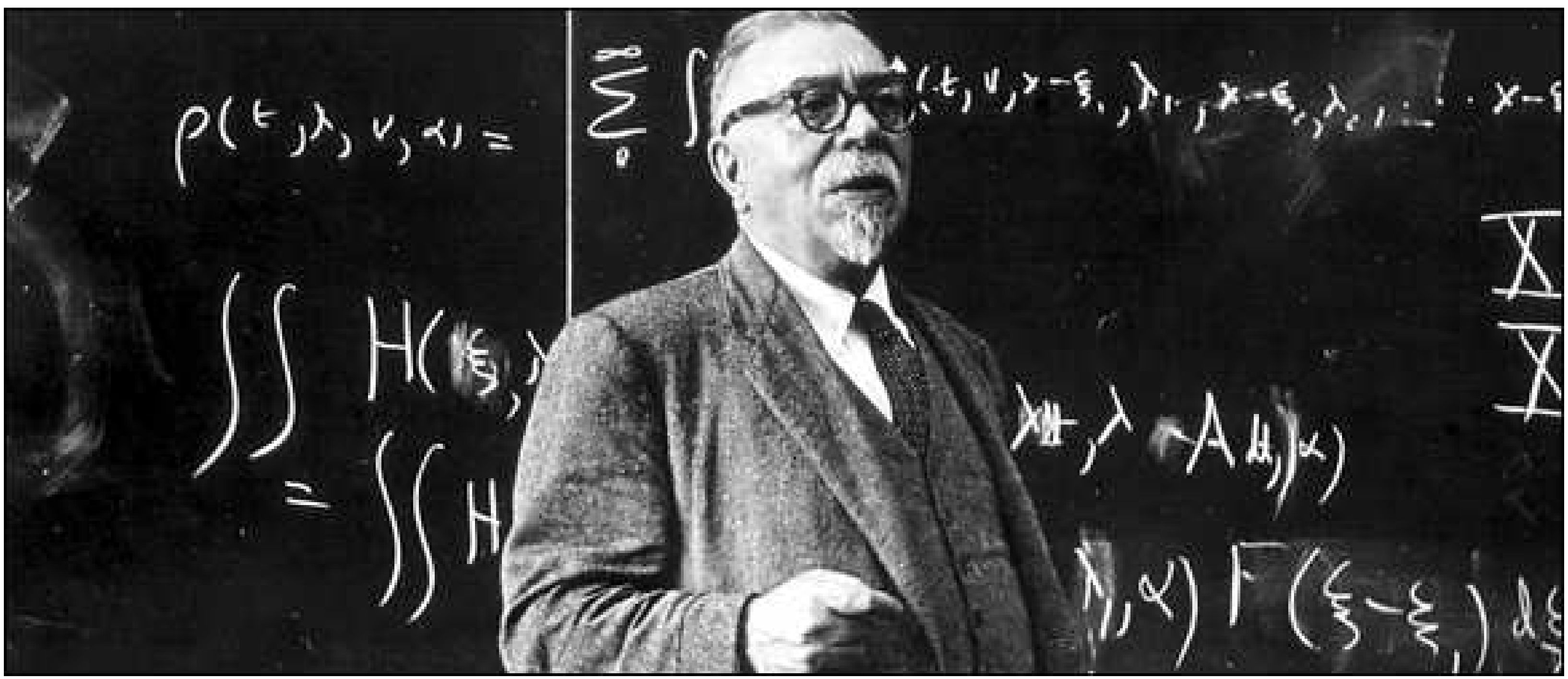

Lord Rayleigh (1842-1919)



Oliver Heaviside
(1850-1925)

# Input/output systems



**Norbert Wiener (1894-1964)**

# Paradigm shift

**Around 1960, the model class shifted to**

$$\frac{d}{dt}x(t) = f(x(t), u(t)), \qquad y(t) = h(x(t), u(t)).$$

**The generation of outputs from inputs is viewed as follows**

$x(0)$ **and** $u(\cdot)$ **lead to** $x(\cdot)$ **through** $\quad \frac{d}{dt}x(t) = f(x(t), u(t))$

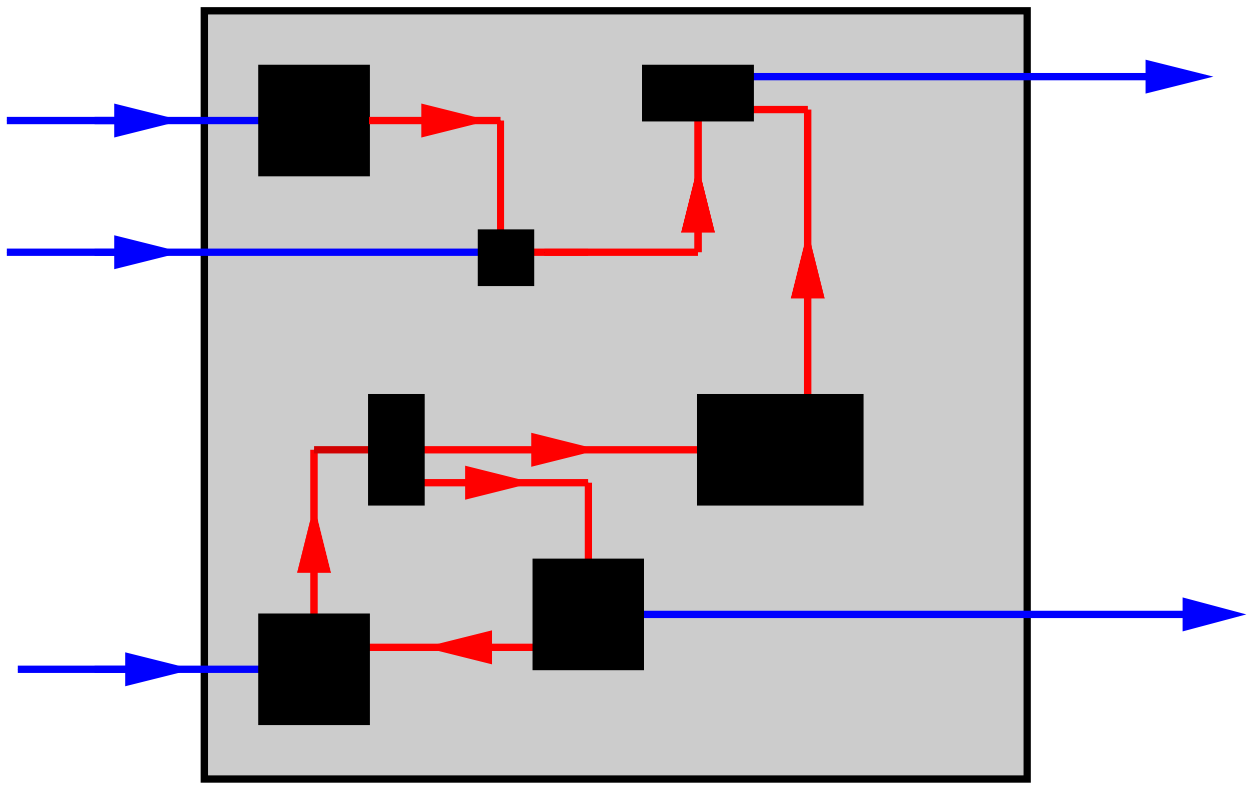
$x(\cdot)$ **and** $u(\cdot)$ **lead to** $y(\cdot)$ **through** $\quad y(t) = h(x(t), u(t))$**.**

$\leadsto$ **a vigorous research program, encompassing all aspects of dynamical modeling, signal processing, and control...**

**Rudolf Kalman (1930- )**

# INTERCONNECTION

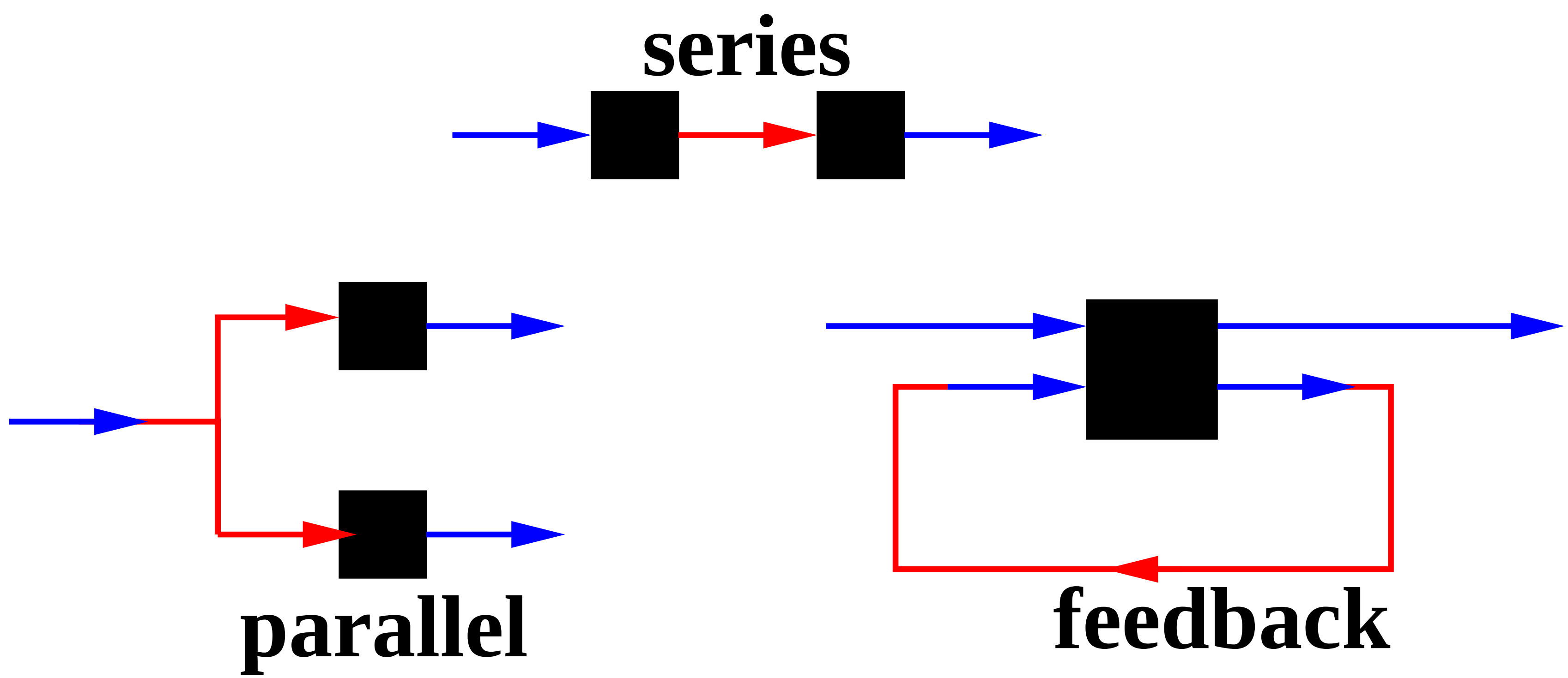
# Signal flow graphs



**‘Pathways’.**

# Signal flows graphs

**Examples:** combinations of

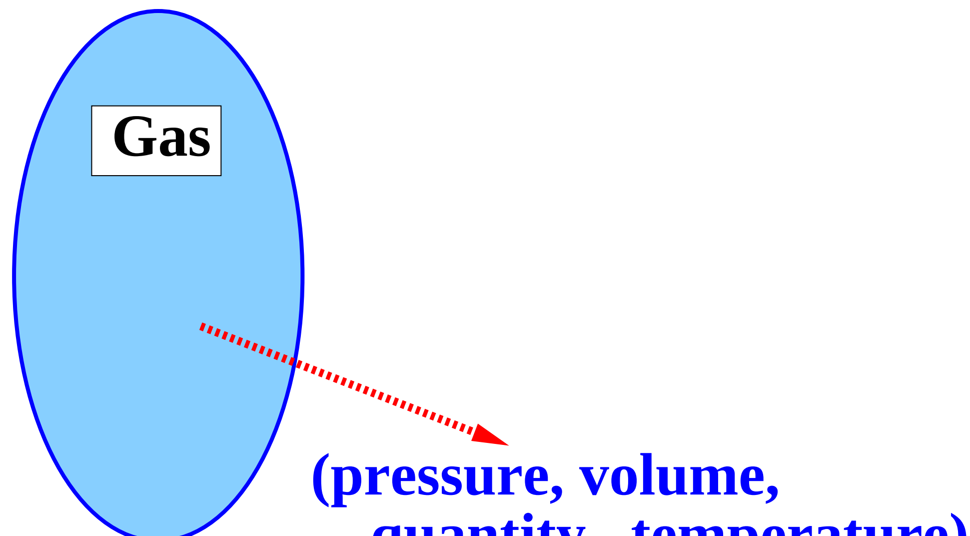


# INADEQUACIES of I/O THINKING

# Problems with I/O

- **Physical laws dictate the simultaneous occurrence of events.**
  **No cause/effect is implied.**

  **E.g., the gas law**




$$PV = NT$$

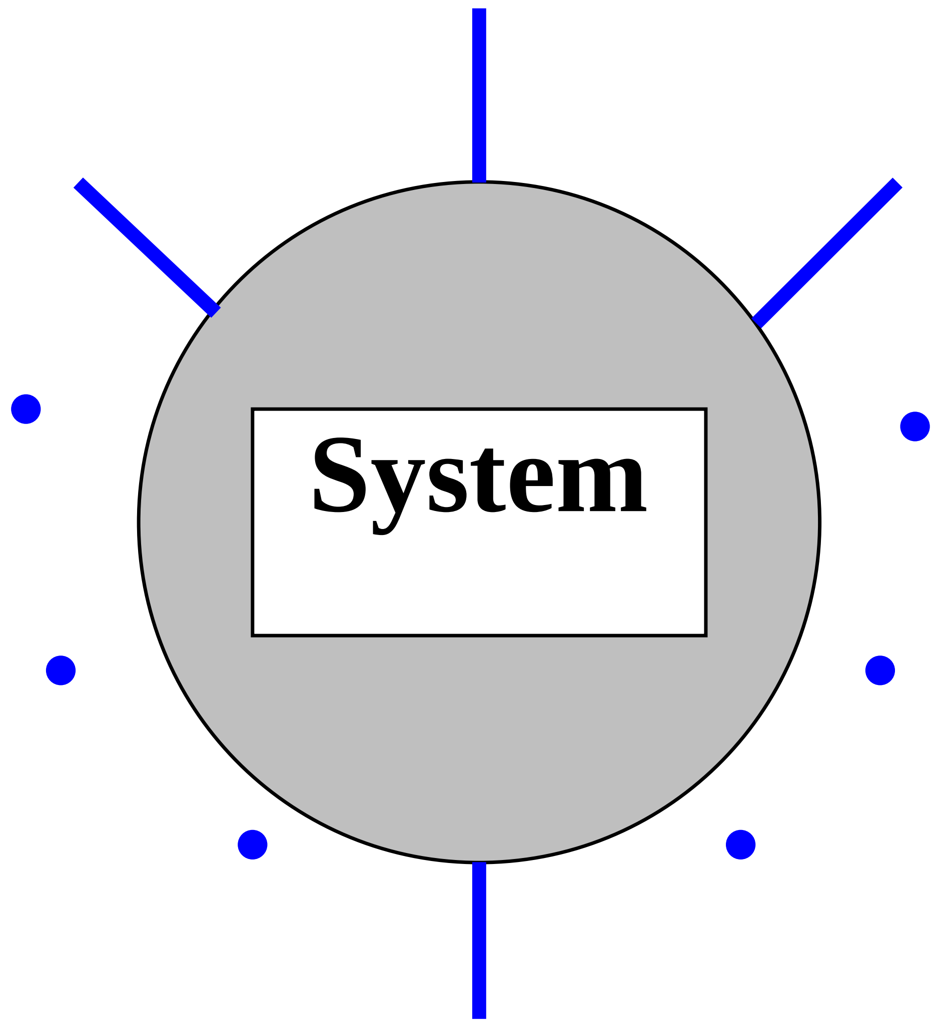
# Problems with I/O

- **Physical laws dictate the simultaneous occurrence of events.**
  **No cause/effect is implied.**

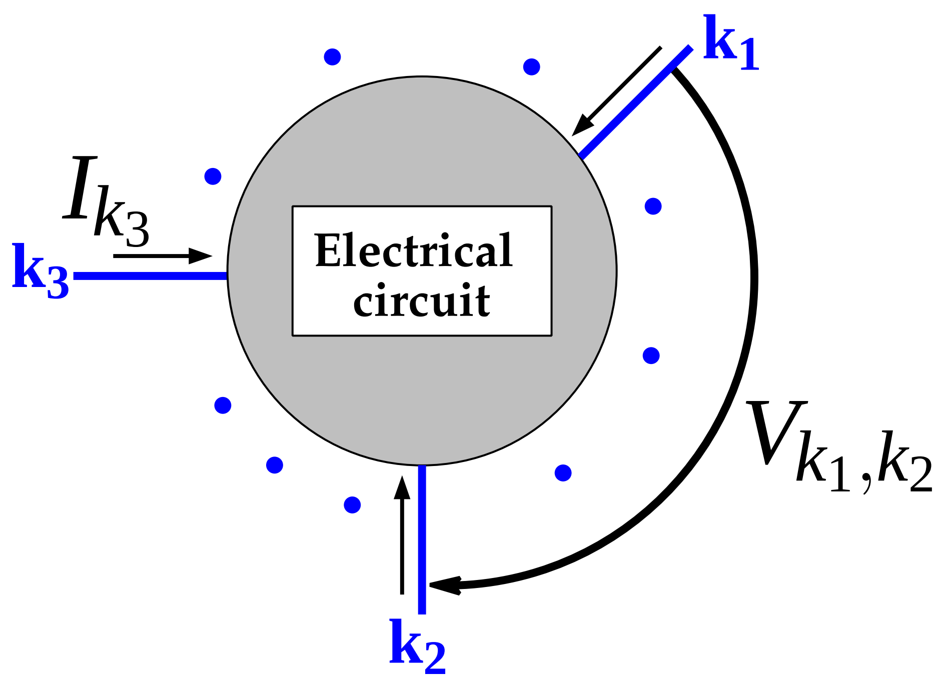
- **Interconnection of physical systems leads to variable sharing.**
  **Not signal transmission.**

**A physical system is not a signal processor.**

# Systems with terminals



# Systems with terminals



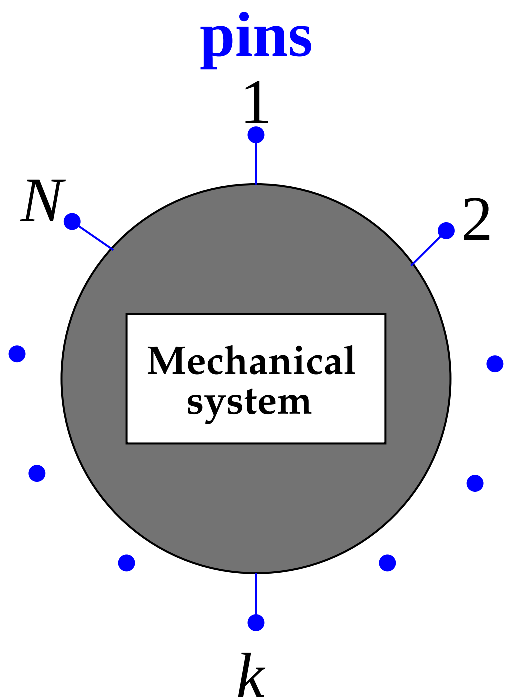

<u>interaction variables</u>: currents & voltages.

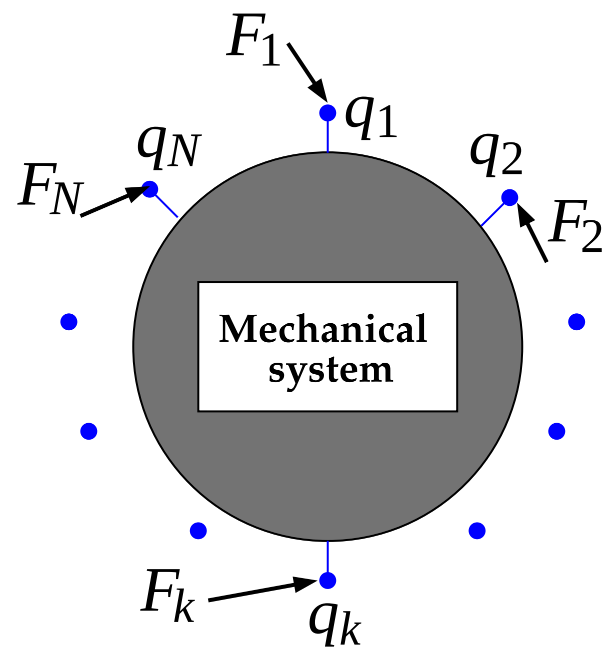
**measurable by ammeters and voltmeters.**

**What is the cause and what is the effect?**

**What is the stimulus and what is the response?**

# Systems with terminals




**At each terminal: a position and a force.**

**More generally, position, force, angle, torque.**

**What is the cause and what is the effect?**

**What is the stimulus and what is the response?**

# Other domains

- Thermal systems:

  At each terminal: a temperature and a heat flow.

- Hydraulic systems:

  At each terminal: a pressure and a mass flow.

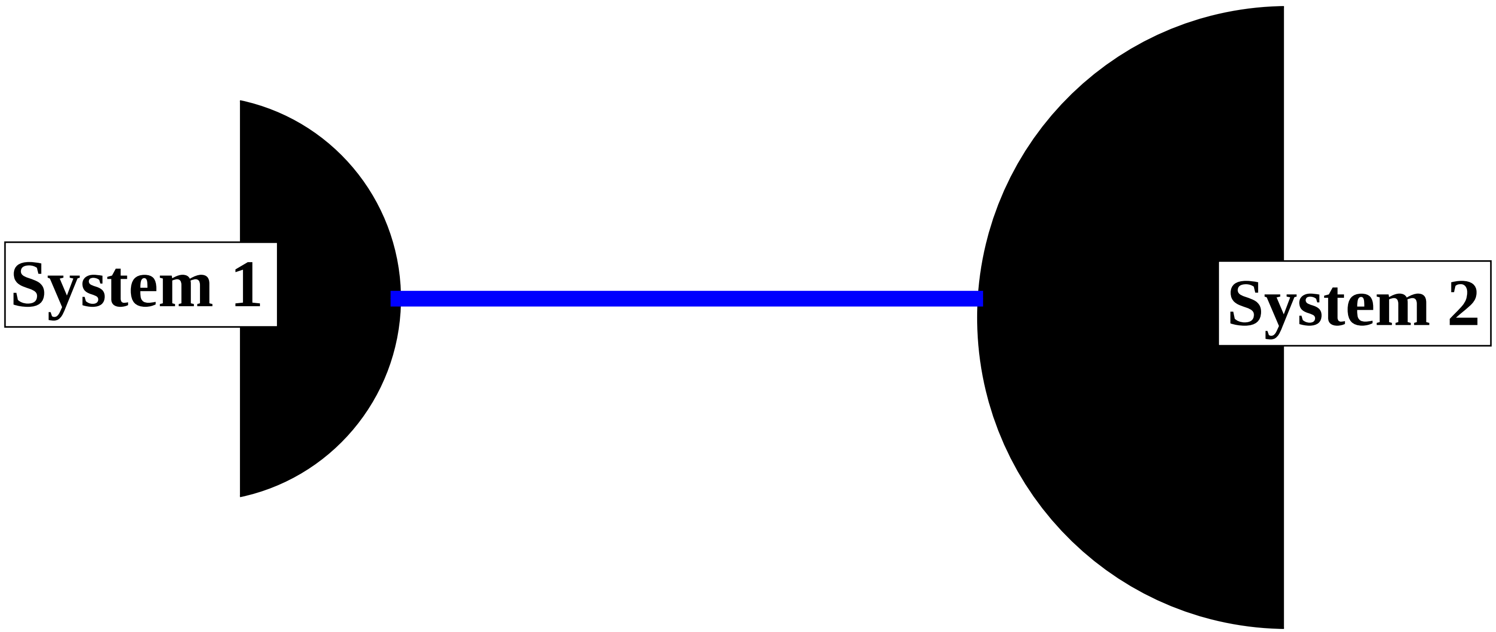
- Multidomain systems:

  Systems with terminals of different types,
  as motors, pumps, etc.

At each terminal, there are many simultaneous variables.

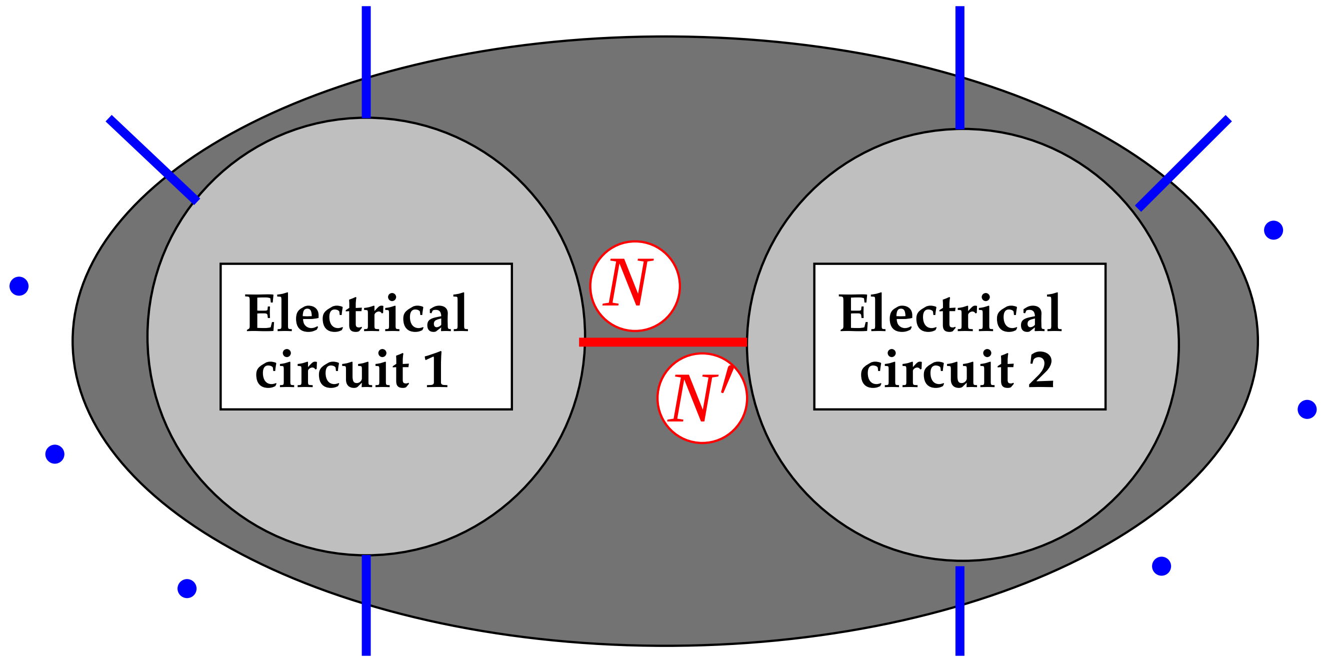
Why and how should we separate these in stimulus and response?

# Connection of terminals




**By interconnecting, the terminal variables are equated.**

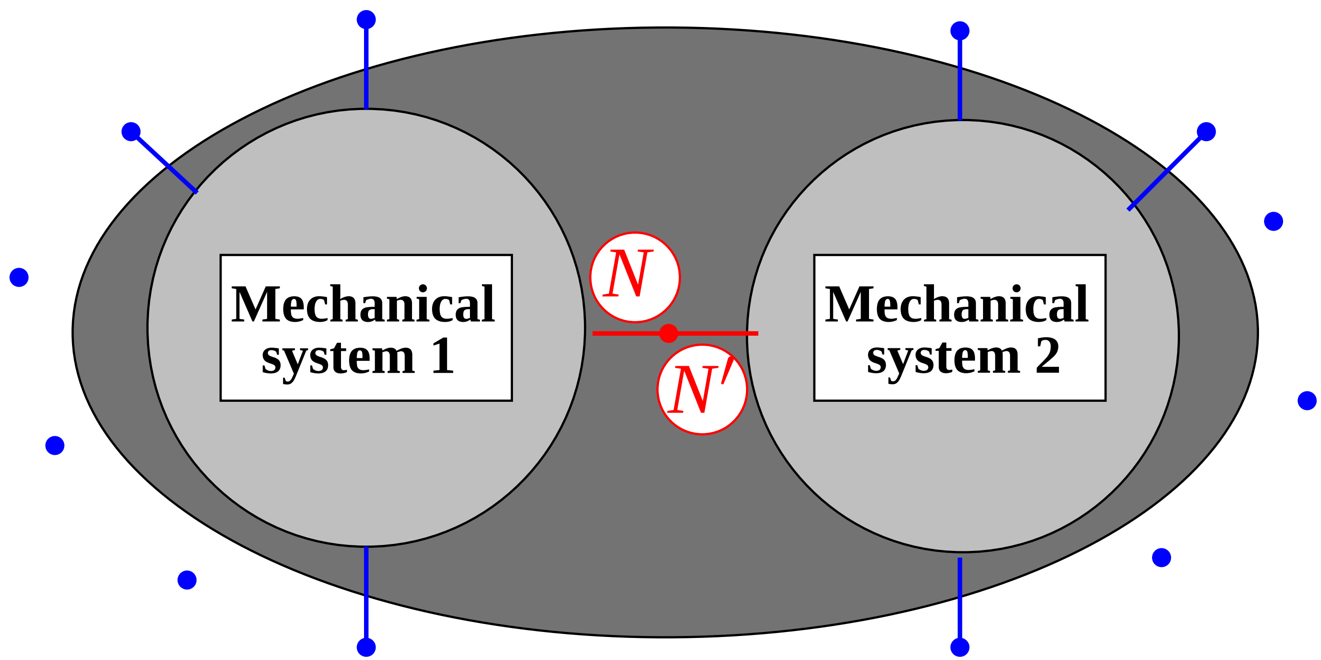
# Interconnection of electrical circuits




$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

**The $V$'s are potentials. We silently used Kirchhoff's voltage law.**

# Interconnection of mechanical systems




$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

# Other domains

- **Thermal systems:**
  **At each terminal: a temperature and a heat flow.**

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  **At each terminal: a pressure and a mass flow.**

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **etc.**

# Sharing variables

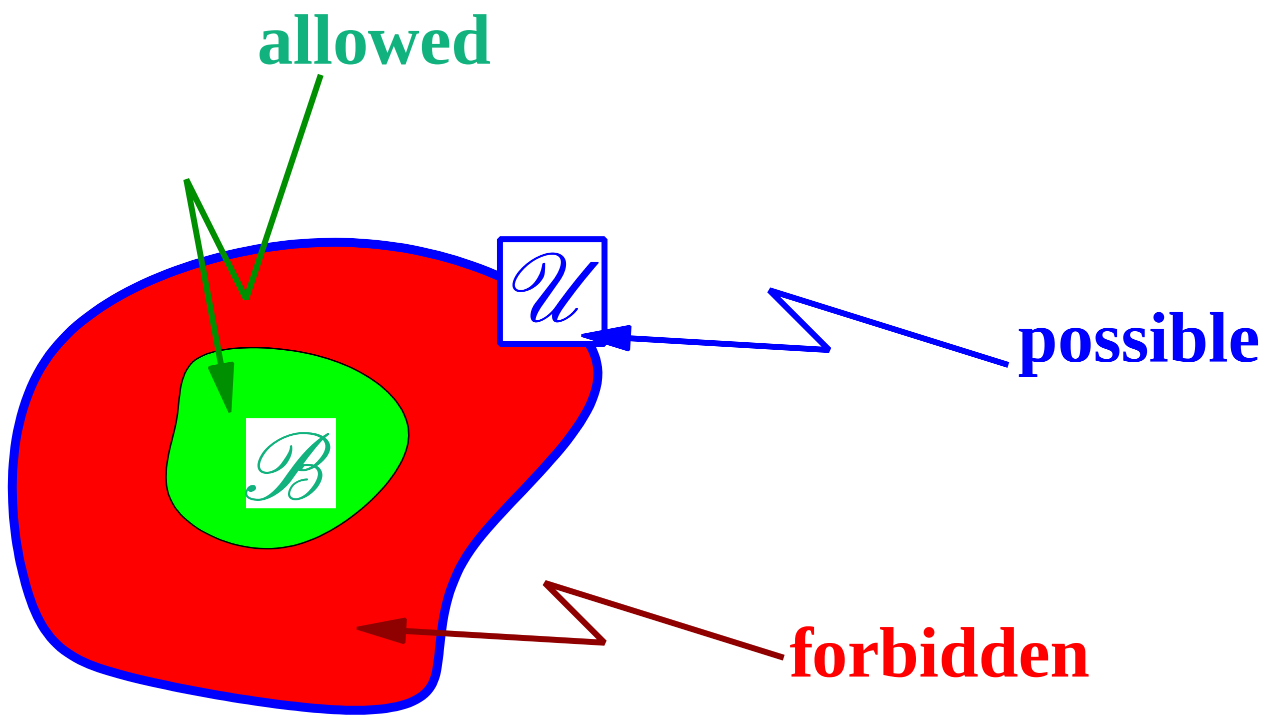
$$
\begin{aligned}
V_N = {} & V_{N'} && \textbf{and} & I_N + I_{N'} = {} & 0, \\
q_N = {} & q_{N'} && \textbf{and} & F_N + F_{N'} = {} & 0, \\
T_N = {} & T_{N'} && \textbf{and} & Q_N + Q_{N'} = {} & 0, \\
p_N = {} & p_{N'} && \textbf{and} & f_N + f_{N'} = {} & 0, \\
& \vdots
\end{aligned}
$$

**Interconnection $\Leftrightarrow$ variable sharing.**

**An interconnection usually involves *more than one* variable. Signal flow graphs with pathways involving *a single* variable between two systems should be scrutinized with skepticism.**

# The BEHAVIORAL APPROACH

# The behavior




**A model tells which events are possible.**

**It does not articulate a cause/effect,**

**stimulus/response relation.**

# The dynamic behavior

**Definition:** **A** ***dynamical system*** $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathscr{B})$**, with**

- $\mathbb{T} \subseteq \mathbb{R}$ **the time set,**
- $\mathbb{W}$ **the signal space,**
- $\mathscr{B} \subseteq \mathbb{W}^{\mathbb{T}}$ **the behavior.**

$w \in \mathscr{B}$ **means: the model allows the trajectory** $w$**,**

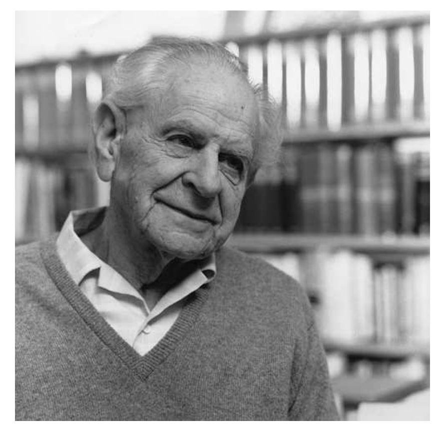
$w \notin \mathscr{B}$ **means: the model forbids the trajectory** $w$**.**

# Behavioral models

**The behavior captures the essence of what a model is.**

**The behavior is all there is.**
**Equivalence of models, properties of models,**
**symmetries, system identification, etc.**
**must all refer to the behavior.**

***Every 'good' scientific theory is prohibition:***
***it forbids certain things to happen.***
***The more it forbids, the better it is.***



**Karl Popper (1902-1994)**

# Technical development

There has been an extensive development that deals with

system theory, control, system identification, etc.

from this point of view,

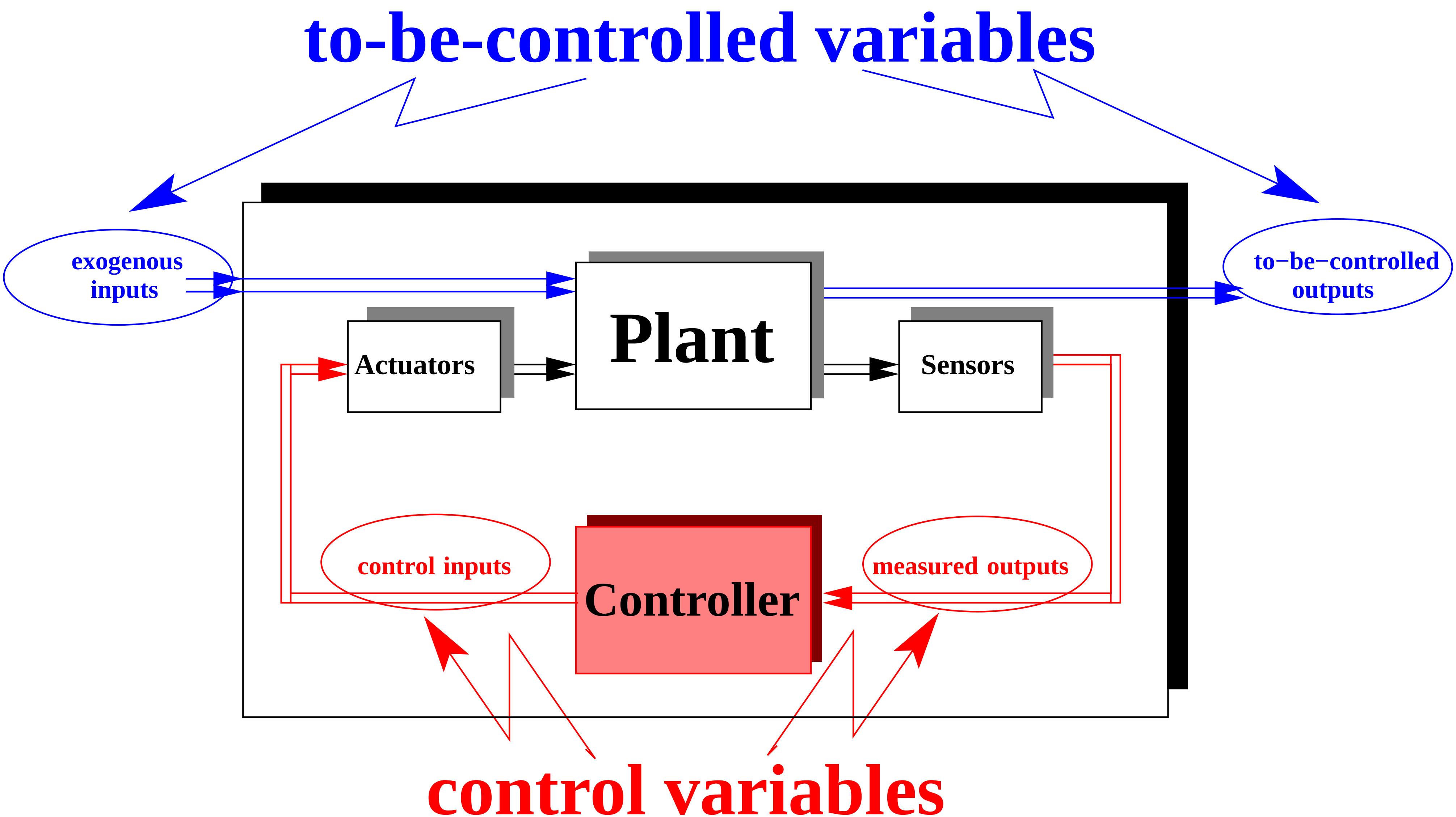
with systems 'behaviors' and

interconnection 'variable sharing'.

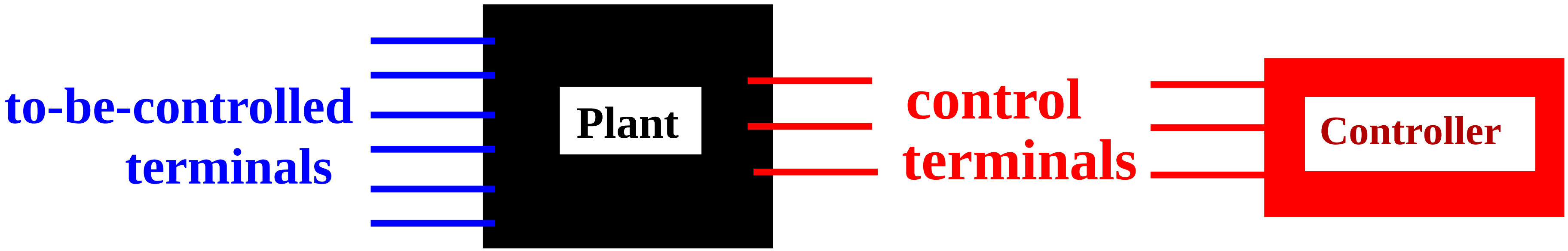
# WHAT NEW DOES THIS BRING?

# CONTROL as INTERCONNECTION

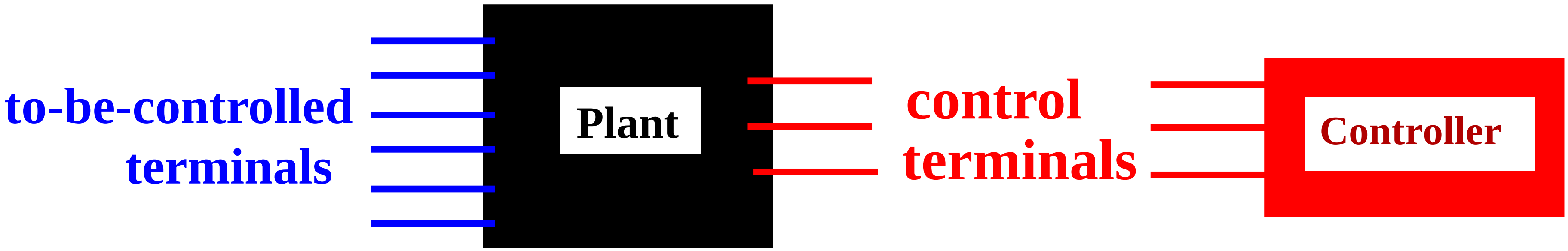
# Feedback control



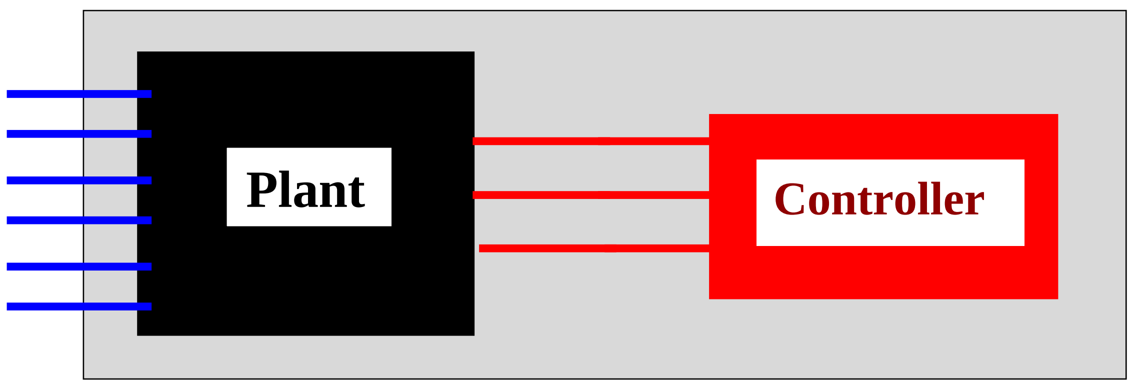
# Behavioral control



# Behavioral control



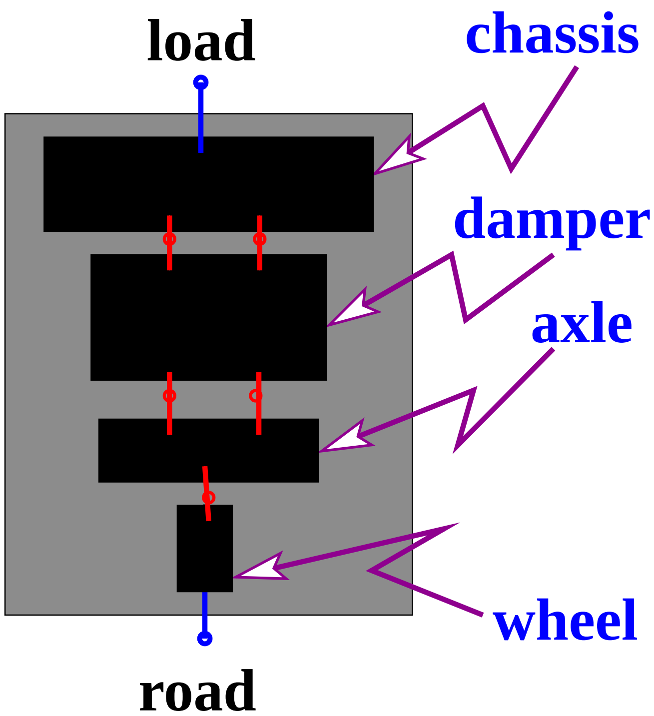

**control = interconnection.**




**controlled system**

**control = integrated system design.**

# Example: A ‘quarter car’



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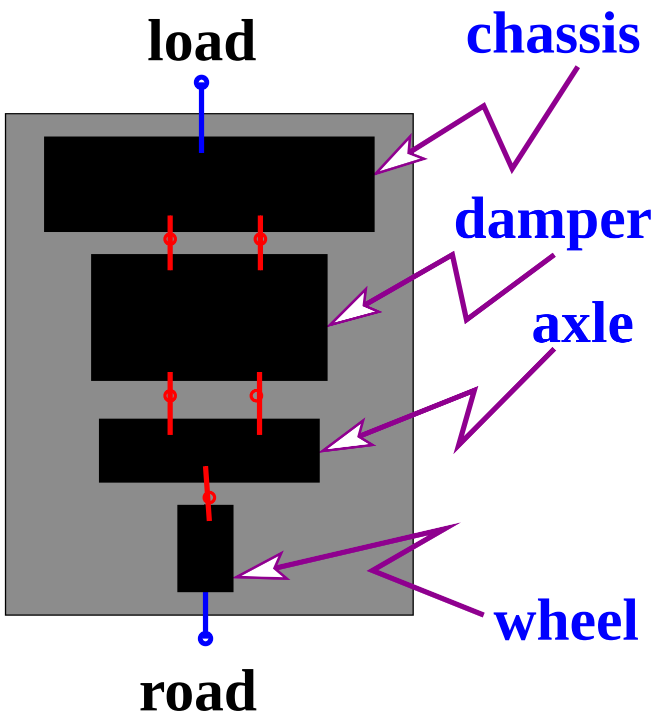


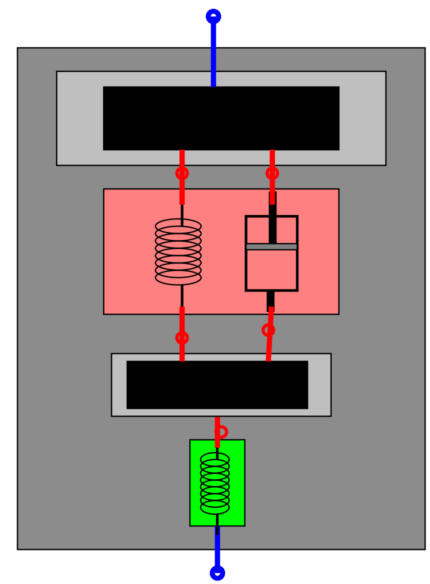

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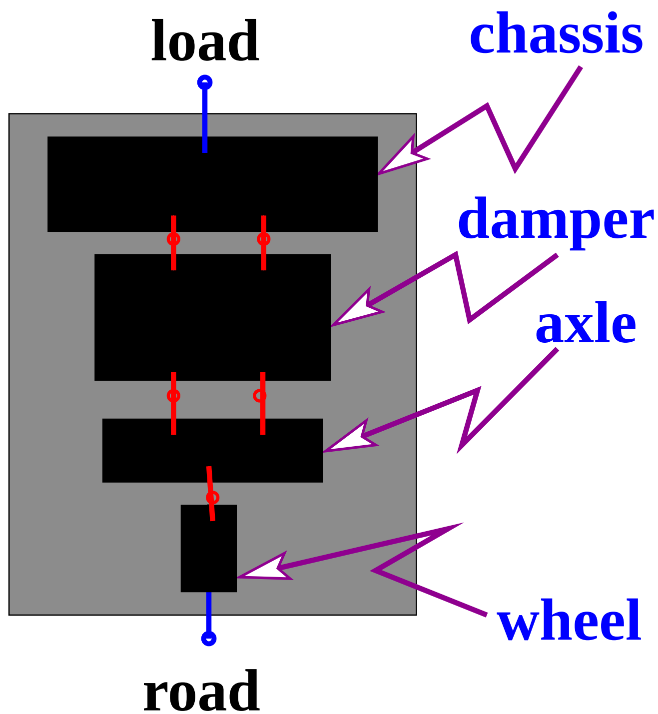


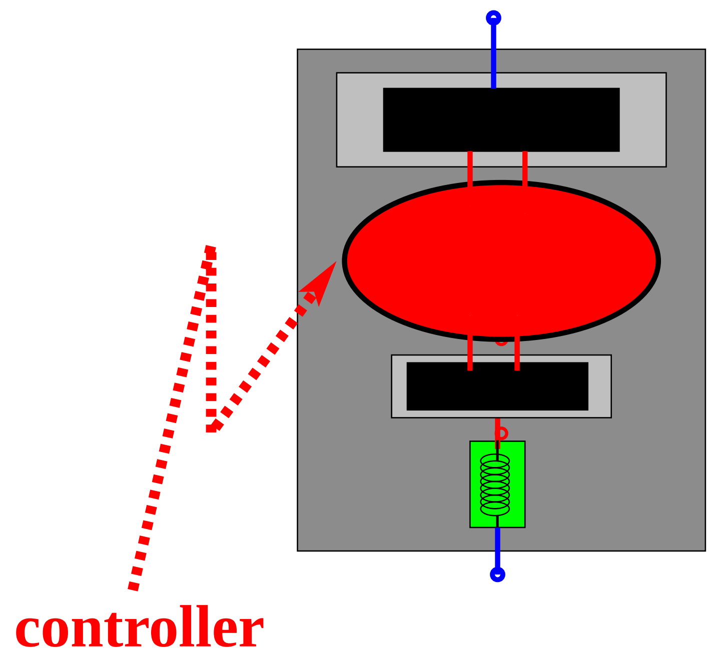

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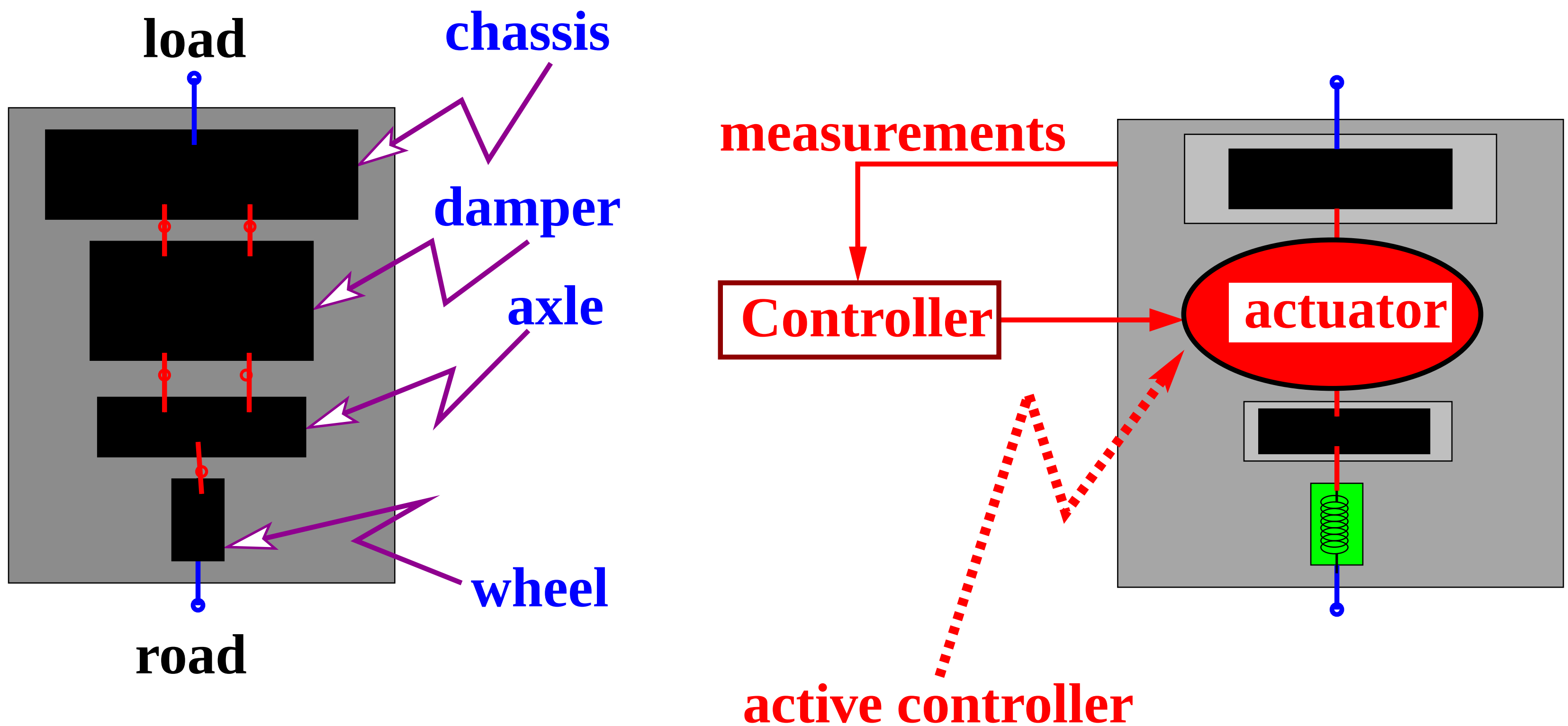

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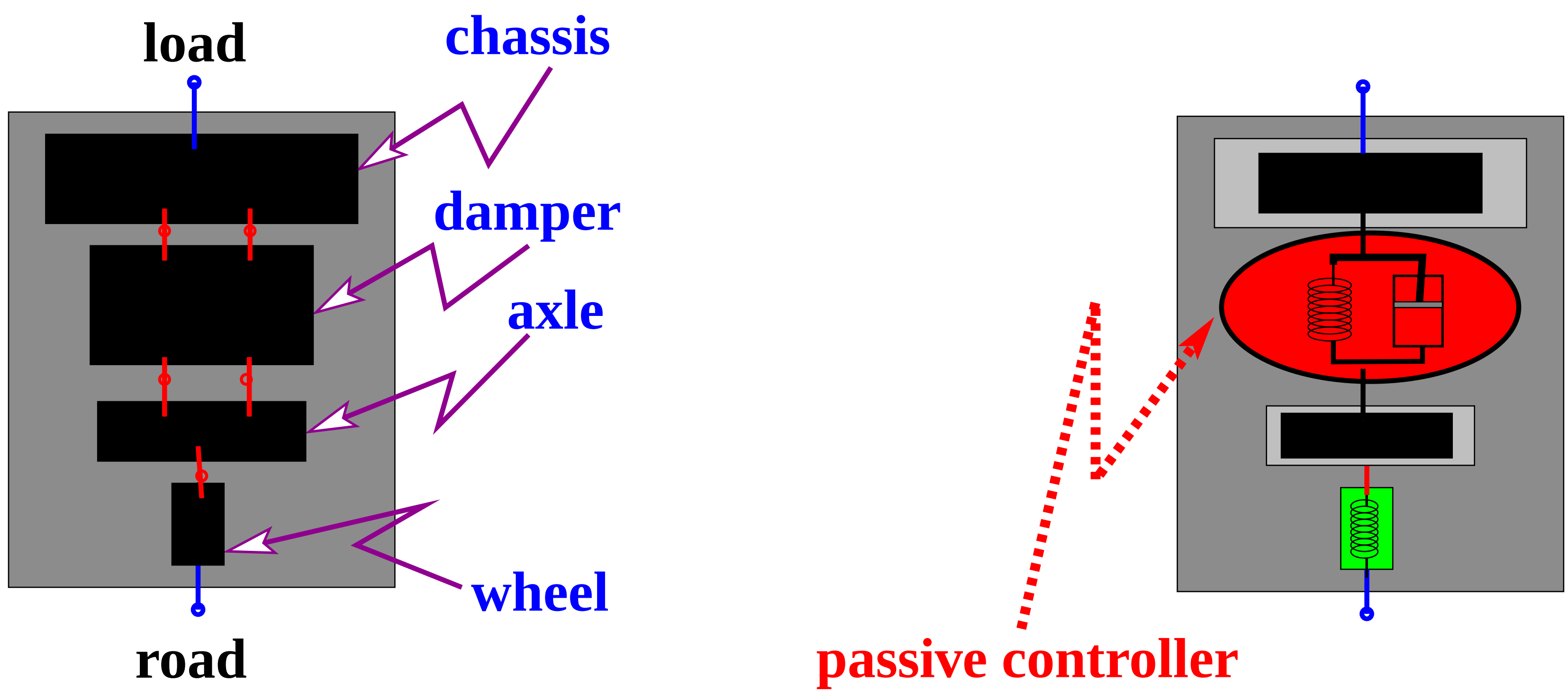
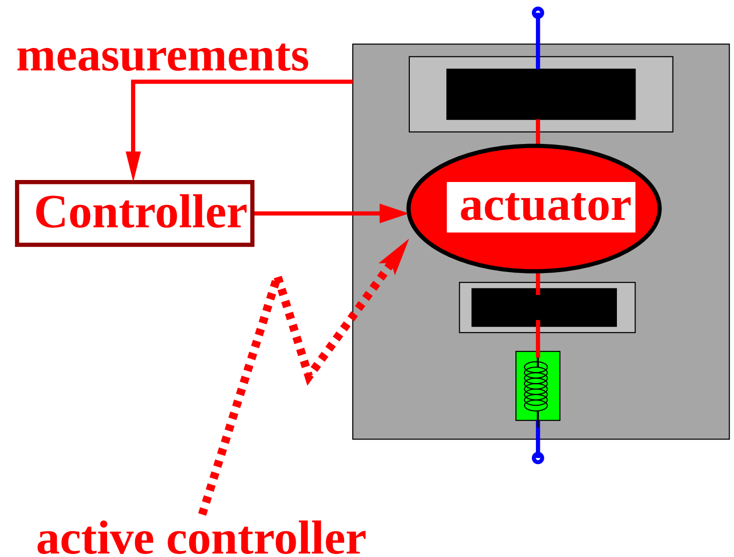


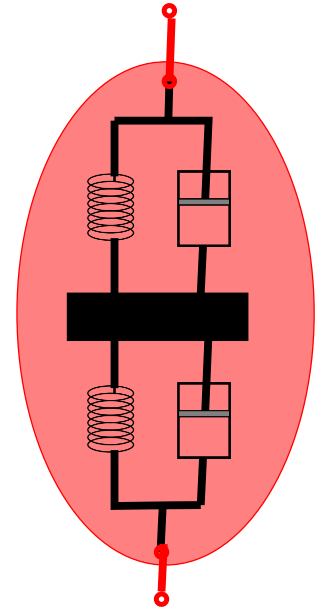
# Suspension control in Formula 1


**Nigel Mansell victorious in 1992 with an active damper.**

**Active dampers were banned in 1994 to break the dominance of the Williams team.**

# Suspension control in Formula 1

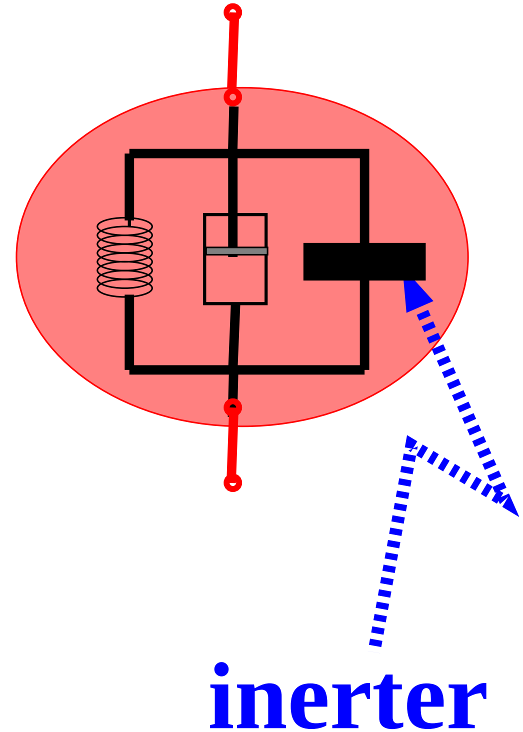


**Later, Renault successfully used a passive 'tuned mass damper'.**

**Banned in 2006,**

**under the 'movable aerodynamic devices' clause.**

# Suspension control in Formula 1

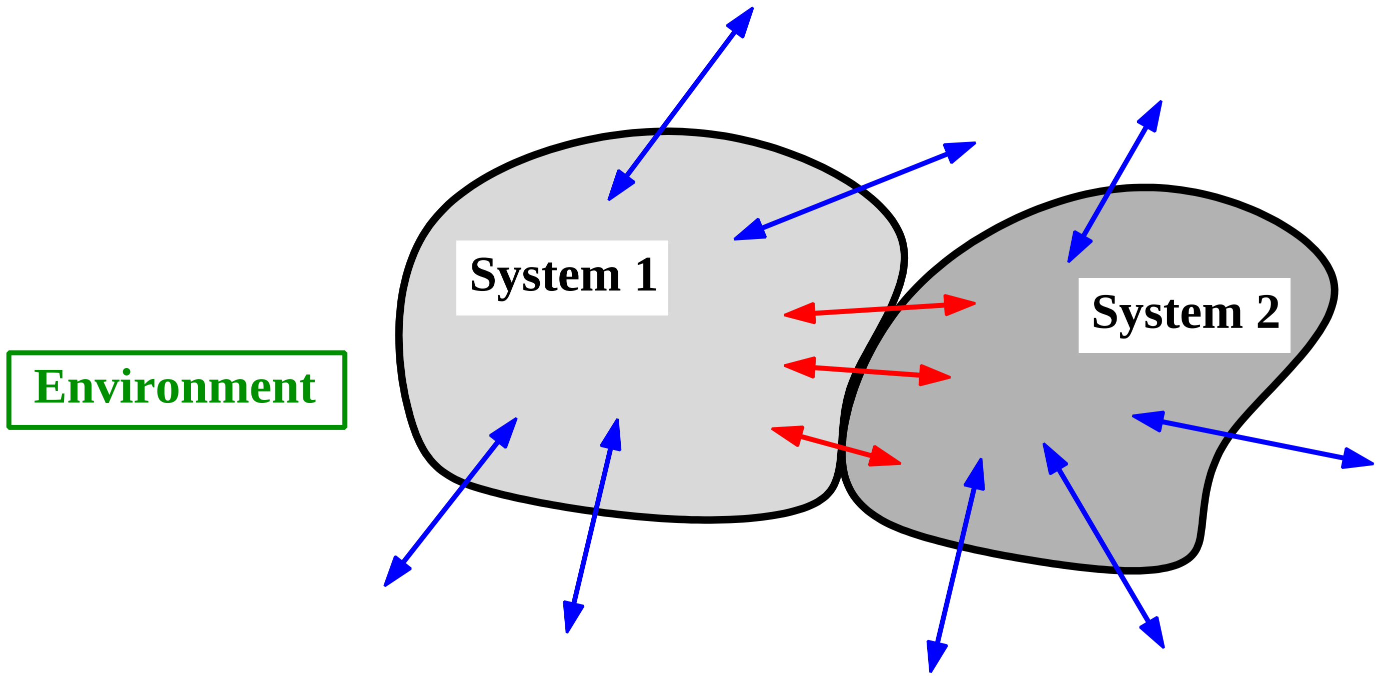

**Kimi Räikkönen wins the 2005 Grand Prix in Spain**

**with McLaren's 'J-damper', i.e., an inerter.**

**A passive suspension with no ($\sim$ small) mass.**

# ENERGY TRANSFER

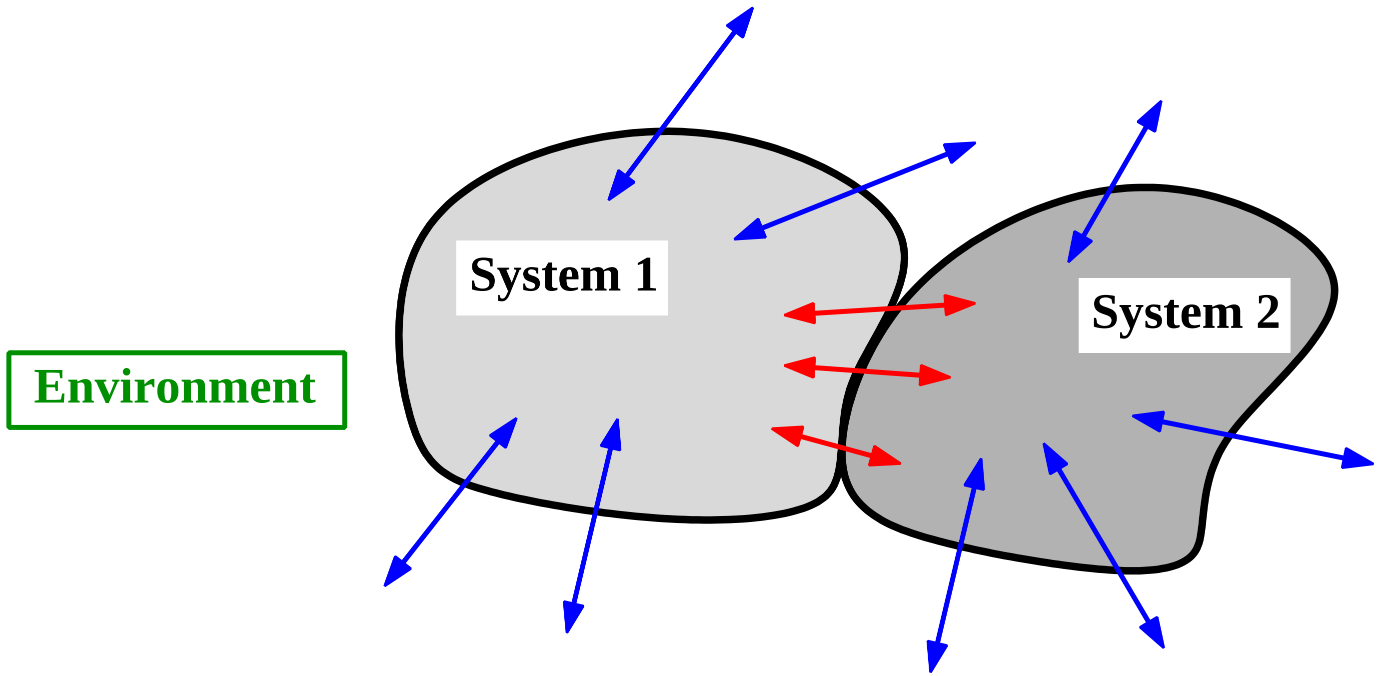
# Interacting systems




**How does energy flow from the environment to a system?**

**How is energy transferred between systems?**

# Interacting systems




**How does energy flow from the environment to a system?**

**How is energy transferred between systems?**

**Energy is NOT an ‘extensive’ quantity.**

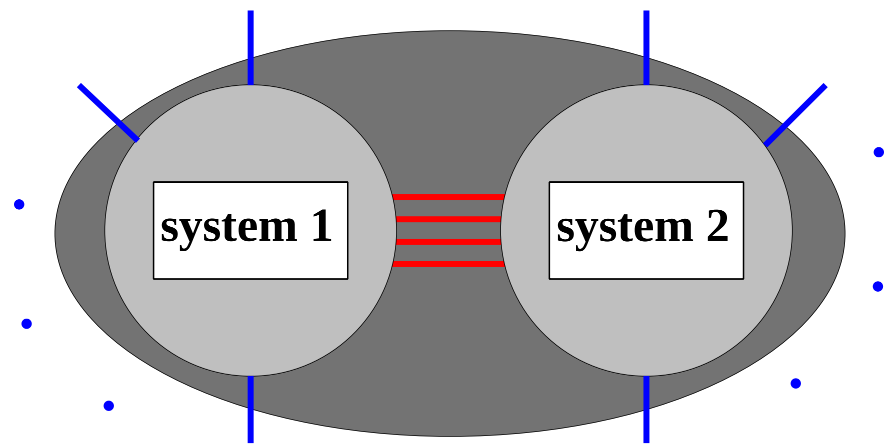
# Interconnection versus energy transfer

**Terminals are for interconnection.**

**Ports are for energy transfer.**

**A ‘port’ is a set of terminals with a special property (related to Kirchhoff’s current law).**

# Energy transfer



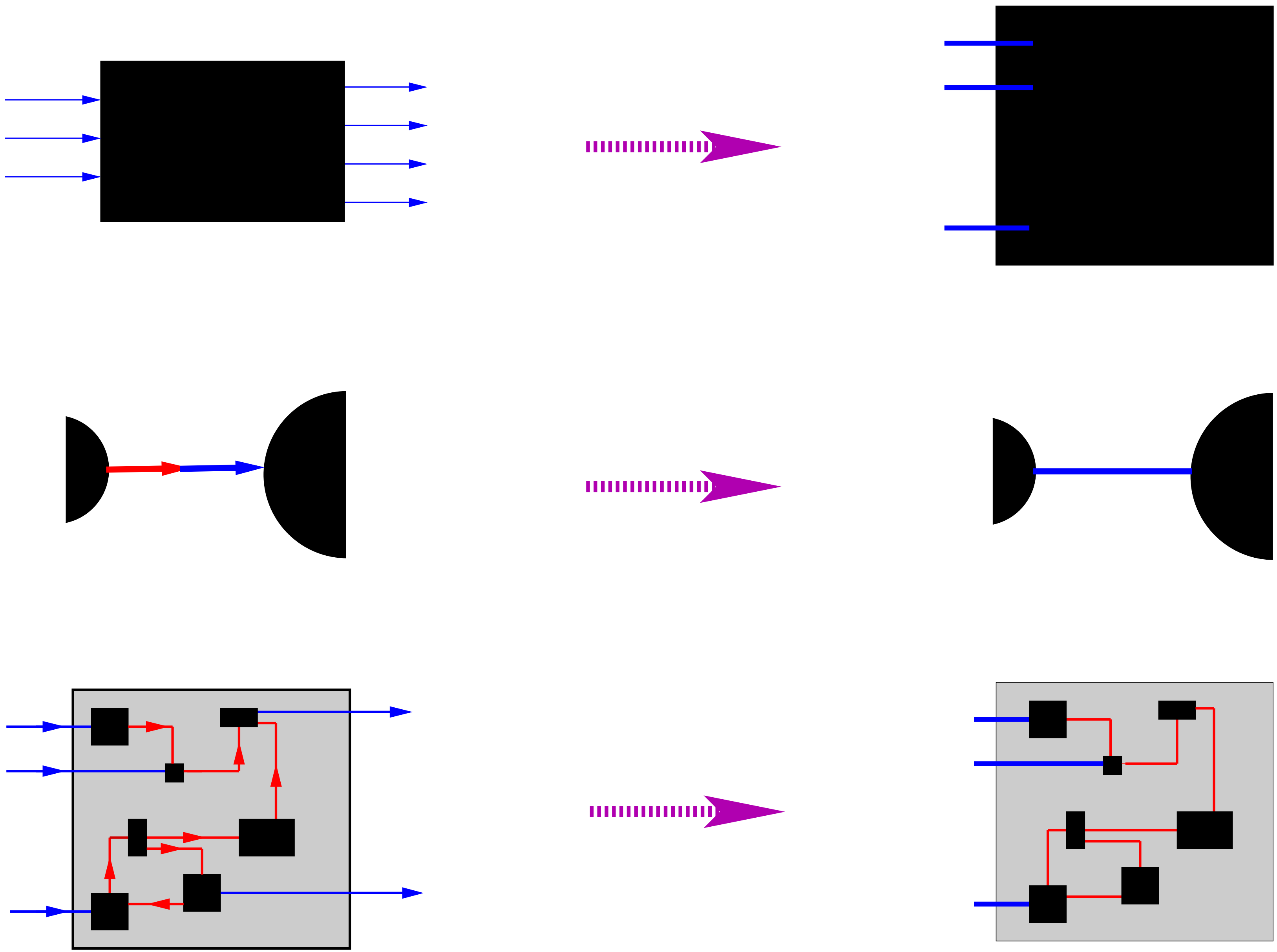

**One cannot speak about**

***“ the energy transferred from system 1 to system 2 ”***

**or** ***“ from the environment to system 1 ”*****,**

**unless the relevant terminals form a port.**

# CONCLUSION



**Reference: The behavioral approach to open and interconnected systems, *Control Systems Magazine*, volume 27, pages 46-99, 2007.**

**Copies of the lecture frames will be available from/at**

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