MODELING

INTERCONNECTED SYSTEMS

JAN C. WILLEMS

K.U. Leuven, Flanders, Belgium

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Theme

How are open systems formalized?

How are systems interconnected?

How is energy transferred between systems?

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How are open systems formalized?

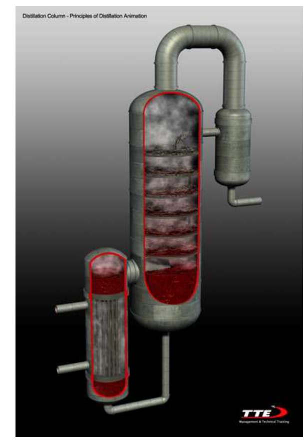
How are systems interconnected?

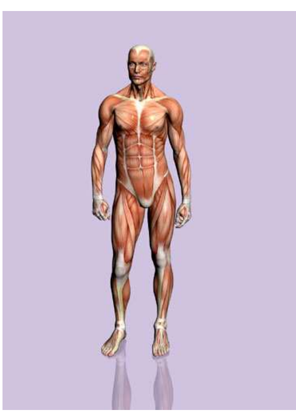
How is energy transferred between systems?

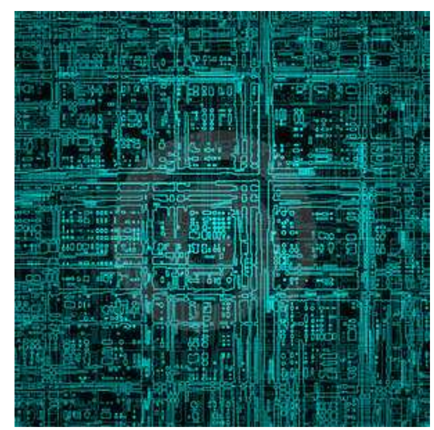
We deal with very simple examples,
mainly electrical circuits and
1-dimensional mechanical systems.

SYSTEMS


OIL REFINERY (GVG / PD)







Features

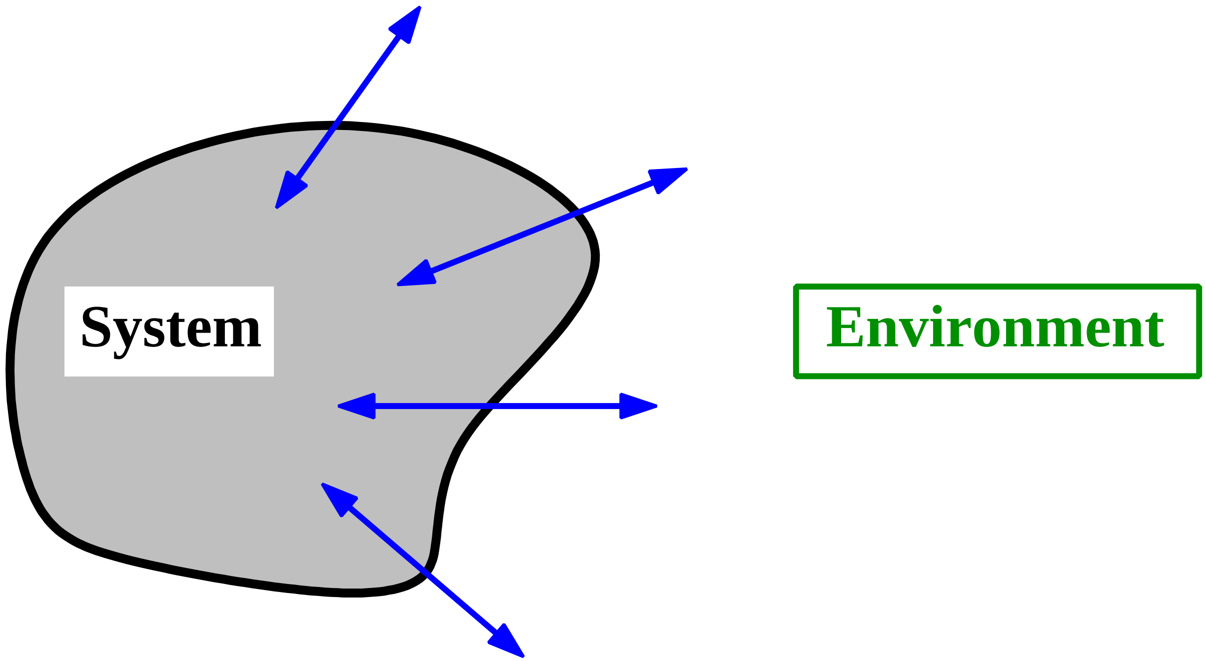
- **Open**
- **Interconnected**
- **Modular**
- **Dynamic**

The ever-increasing computing power allows to model such complex interconnected systems accurately by

tearing, zooming, and linking.

⇝ **Simulation, model based design, ...**

Open systems

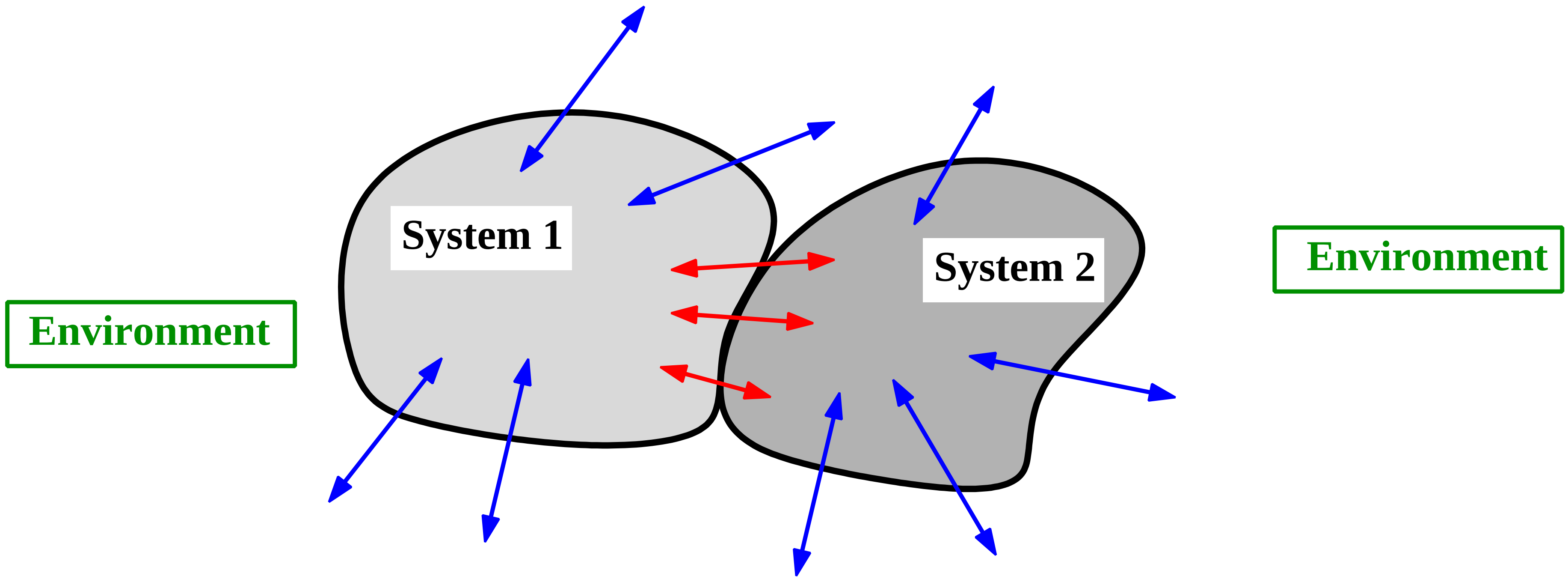


Systems are 'open', they interact with their environment.

How are such systems formalized?

How is energy transferred from the environment to a system?

Interacting systems



Interconnected systems interact.

How is this interaction formalized?

How is energy transferred between systems?

Motivation

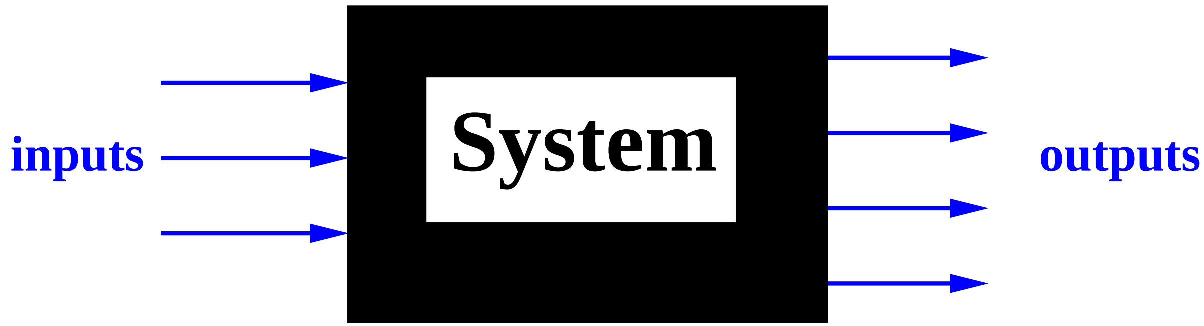
The ever-increasing computing power allows to model complex interconnected systems accurately.

Requires the right mathematical concepts

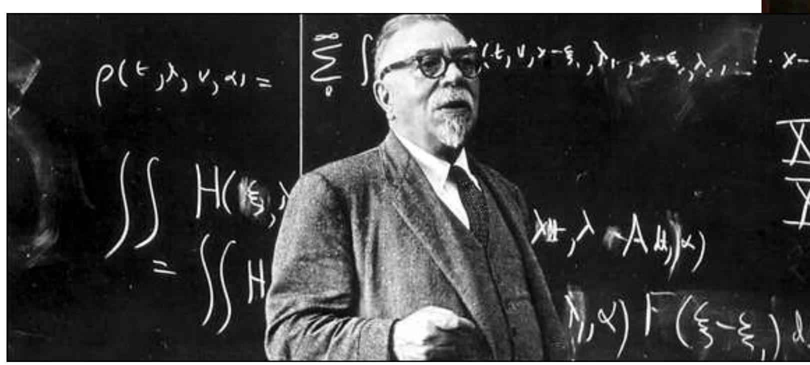
- **for dynamical system,**
- **for interconnection,**
- **for interconnection architecture.**

CLASSICAL VIEW

Input/output systems



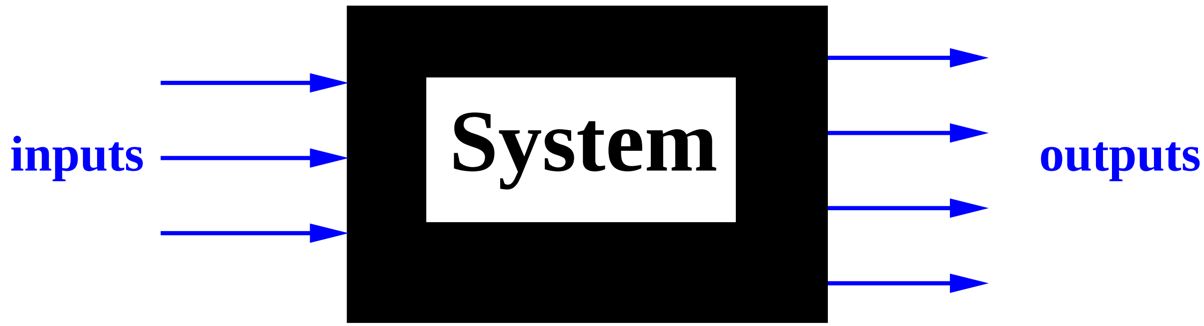
Oliver Heaviside



Norbert Wiener

Rudy Kalman

Input/output systems

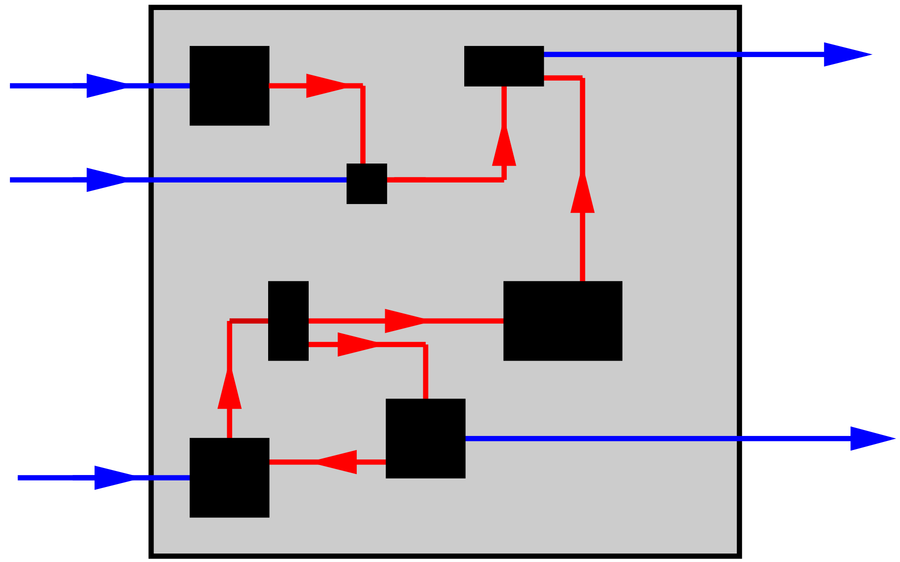


Input/output thinking is *inappropriate* for describing the functioning of physical systems.

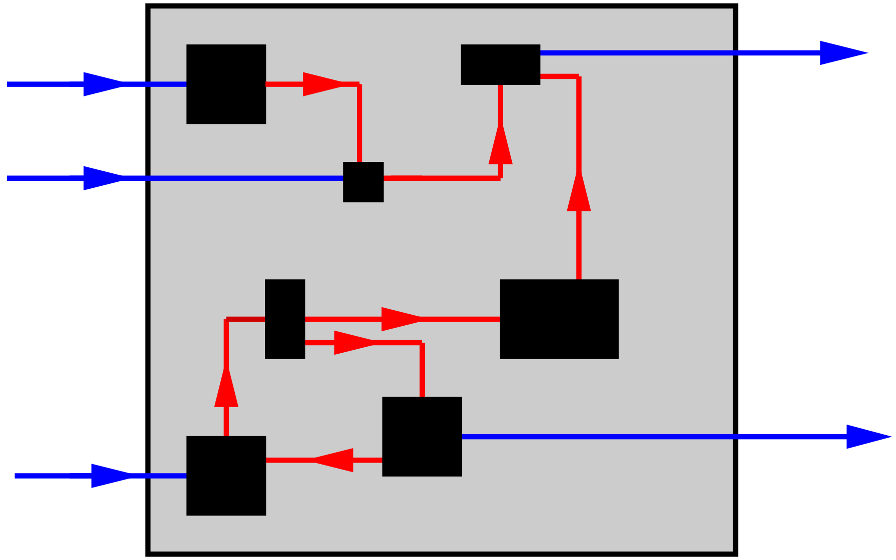
A physical system is not a signal processor.

Better concept: a behavior.

Signal flow graphs



Signal flow graphs



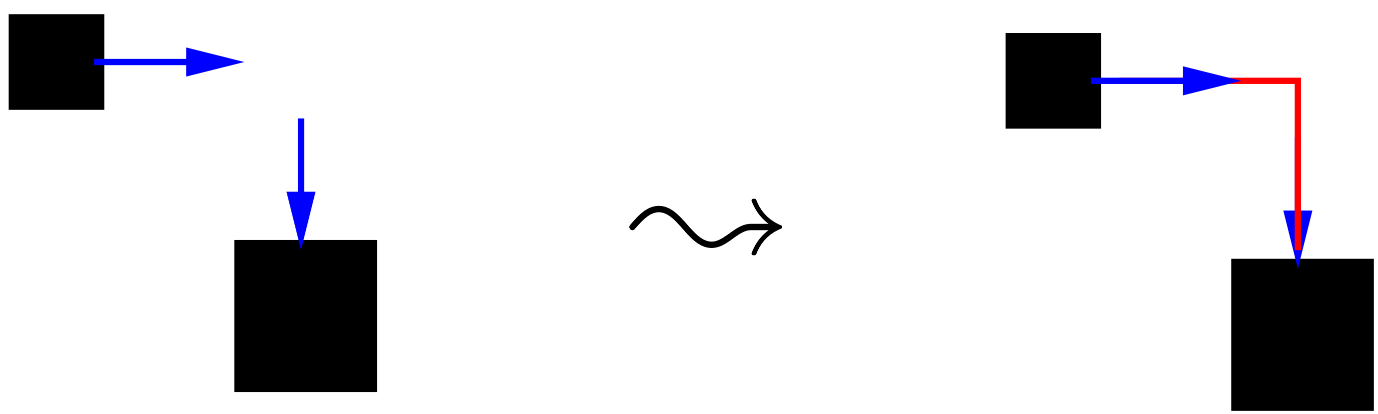
Signal flow graphs are *inappropriate* for describing the interaction physical systems.

A physical system is not a signal processor.

Better concept: a graph with leaves.

Interconnection

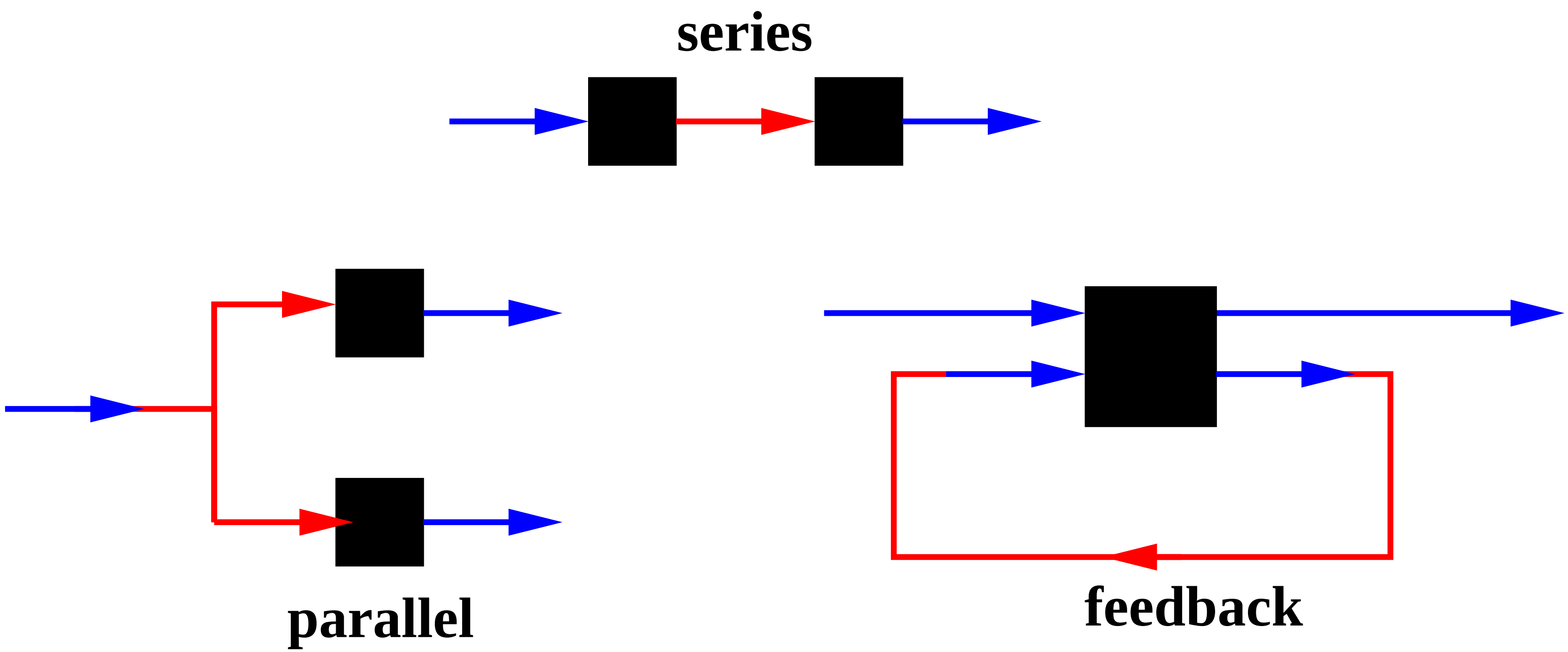
Interconnection as output-to-input assignment.



Interconnection

Interconnection as <u>output-to-input assignment.</u>

Examples:



Interconnection

Interconnection as output-to-input assignment.

Output-to-input assignment is *inappropriate* for describing the interconnection of physical systems.

A physical system is not a signal processor.

Better concept: variable sharing

The BEHAVIORAL APPROACH

The dynamic behavior

Definition: **A** ***dynamical system*** $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathscr{B})$**, with**

- $\mathbb{T} \subseteq \mathbb{R}$ **the time set,**
- $\mathbb{W}$ **the signal space,**
- $\mathscr{B} \subseteq (\mathbb{W})^{\mathbb{T}}$ **the behavior,**
  that is, $\mathscr{B}$ **is a family of maps from** $\mathbb{T}$ **to** $\mathbb{W}$**.**

$w : \mathbb{T} \to \mathbb{W} \in \mathscr{B}$ **means: the model allows the trajectory** w**,**

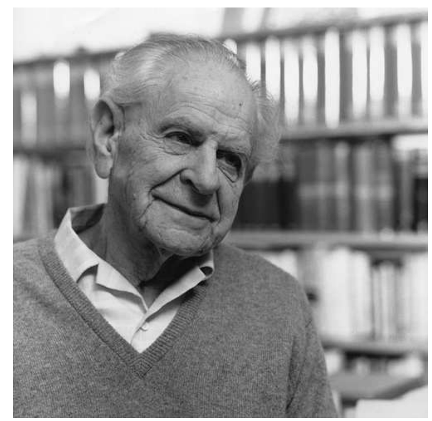
$w : \mathbb{T} \to \mathbb{W} \notin \mathscr{B}$ **means: the model forbids the trajectory** w**.**

Behavioral models

The behavior captures the essence of what a model is.

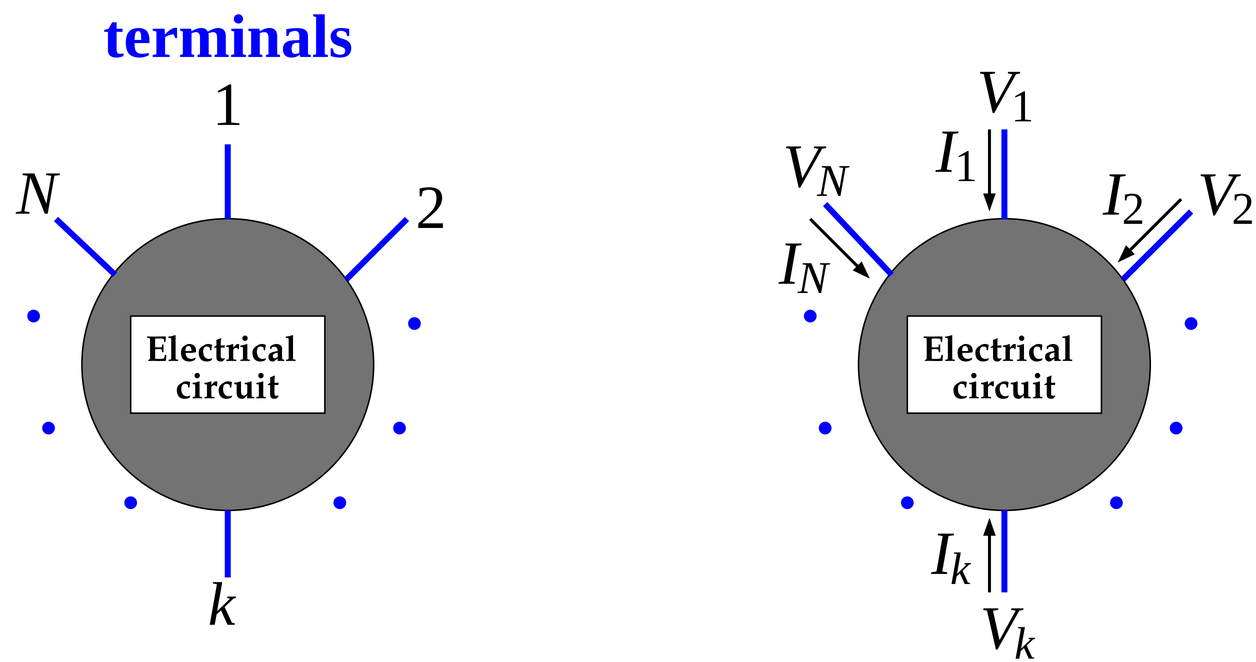
The behavior is all there is.
Equivalence of models, properties of models,
symmetries, system identification, etc.
must all refer to the behavior.

Every 'good' scientific theory is prohibition:
it forbids certain things to happen.
The more it forbids, the better it is.



Karl Popper (1902-1994)

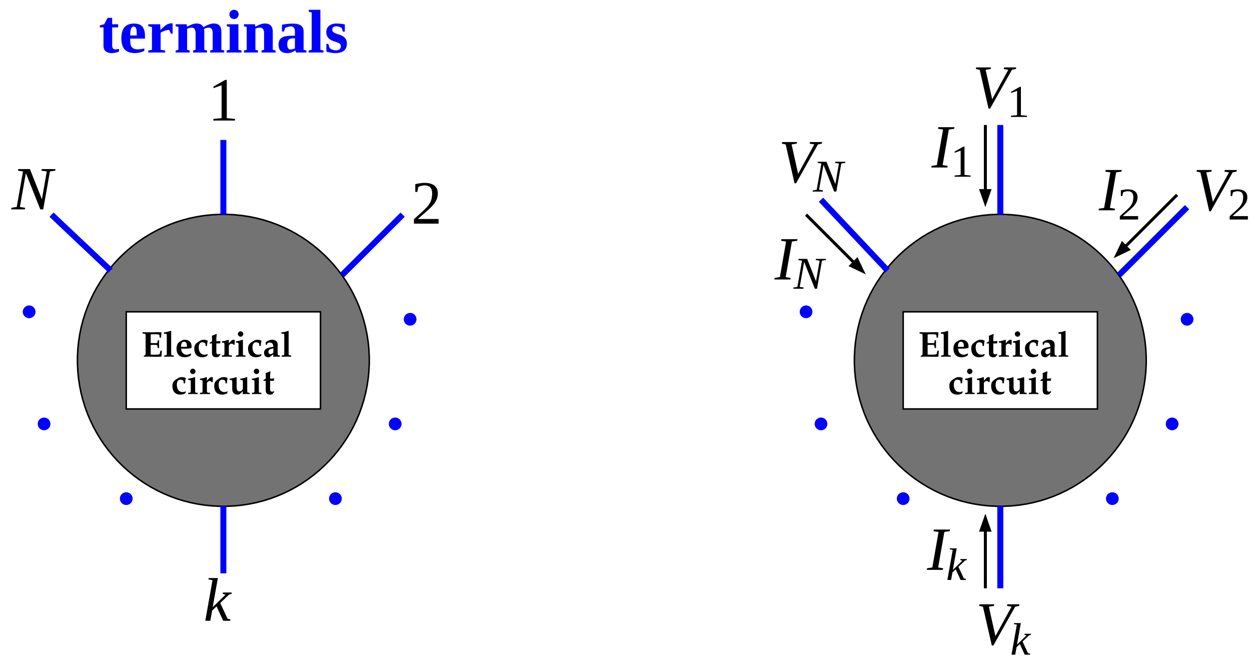
Electrical circuit



At each terminal:

a potential (!) and a current (counted > 0 into the circuit),

Electrical circuit



At each terminal:

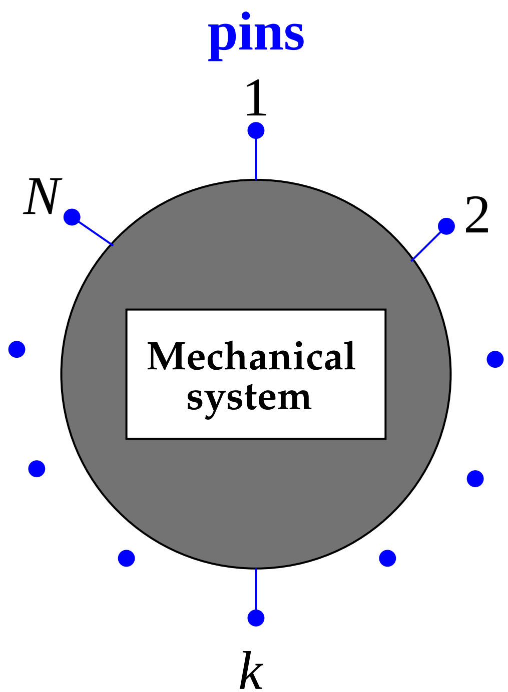
a potential (!) and a current (counted > 0 into the circuit),

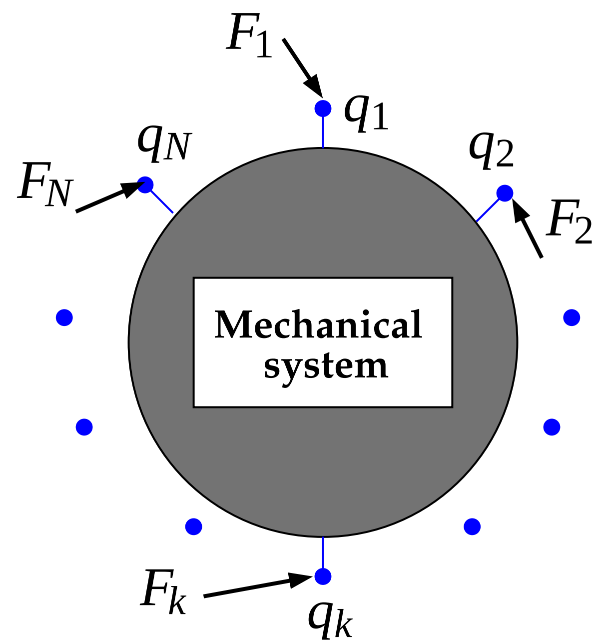
$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

this potential/current trajectory is compatible with the circuit architecture and its element values.

Mechanical device





At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^{\bullet})^{2N})^{\mathbb{R}}$.

More generally, a position, force, angle, and torque.

Other domains

- **Thermal systems:**

  At each terminal: a temperature and a heat flow.

- **Hydraulic systems:**

  At each terminal: a pressure and a mass flow.

- **Multidomain systems:**

  Systems with terminals of different types, as motors, pumps, etc.

- **...**

The behavior

There has been an extensive development that deals with

system theory, control, system identification, etc.

from this point of view.

WHAT NEW DOES THIS BRING?

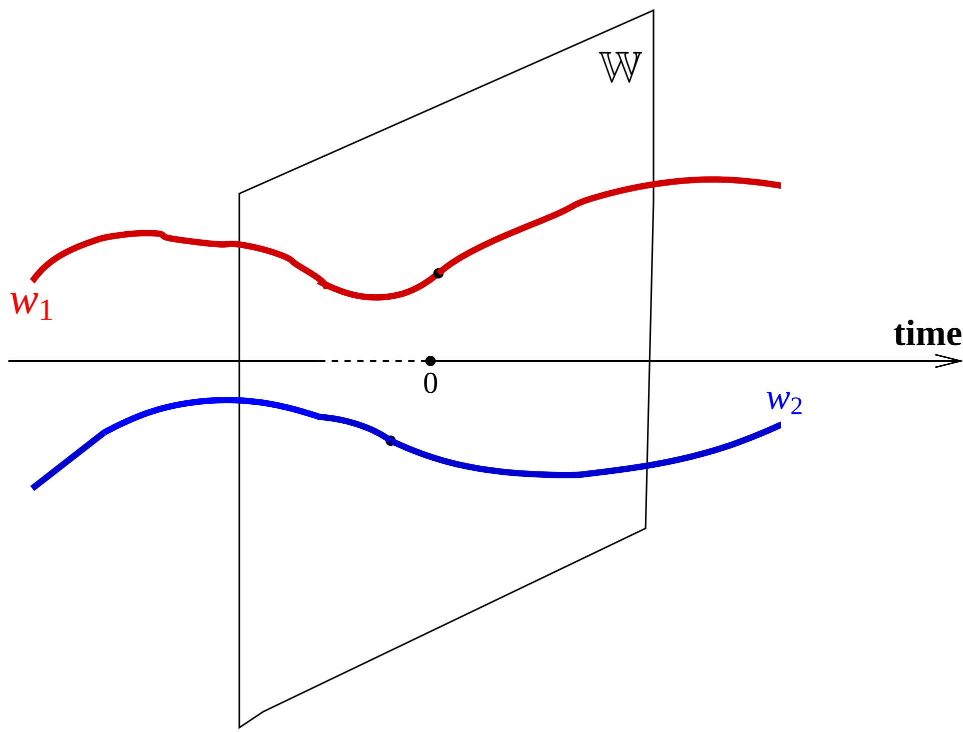
Controllability

The dynamical system $\Sigma = (\mathbb{T}, \mathbb{W}, \mathscr{B})$, with $\mathbb{T} = \mathbb{R}$ or $\mathbb{Z}$, is said to be controllable :$\Leftrightarrow$

for all $w_1, w_2 \in \mathscr{B}$, there exist
$T \in \mathbb{T}, T \geq 0$, and $w \in \mathscr{B}$, such that

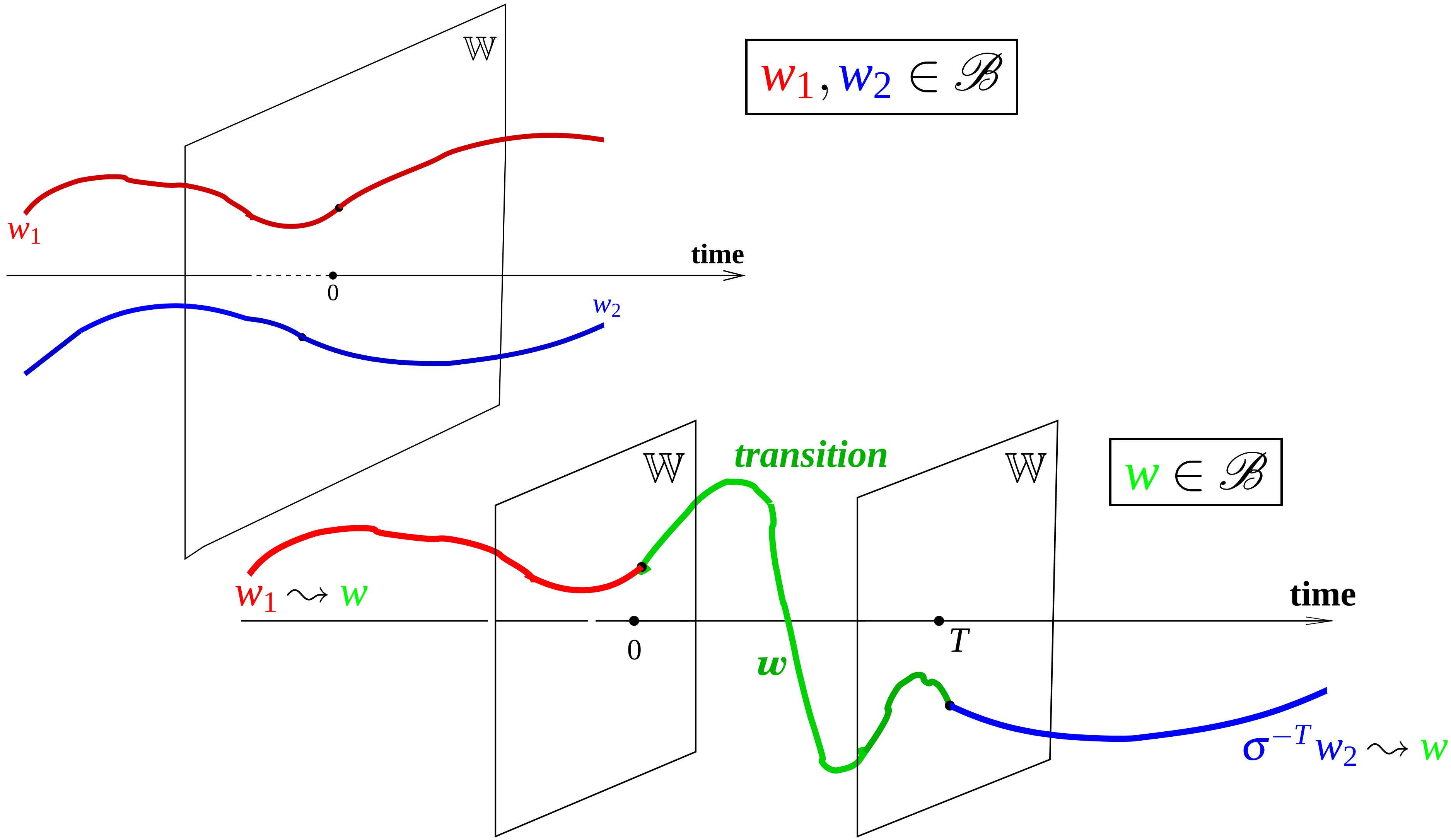
$$w(t) = \begin{cases} w_1(t) & \textbf{for } t < 0; \\ w_2(t-T) & \textbf{for } t \geq T. \end{cases}$$

Controllability in a picture



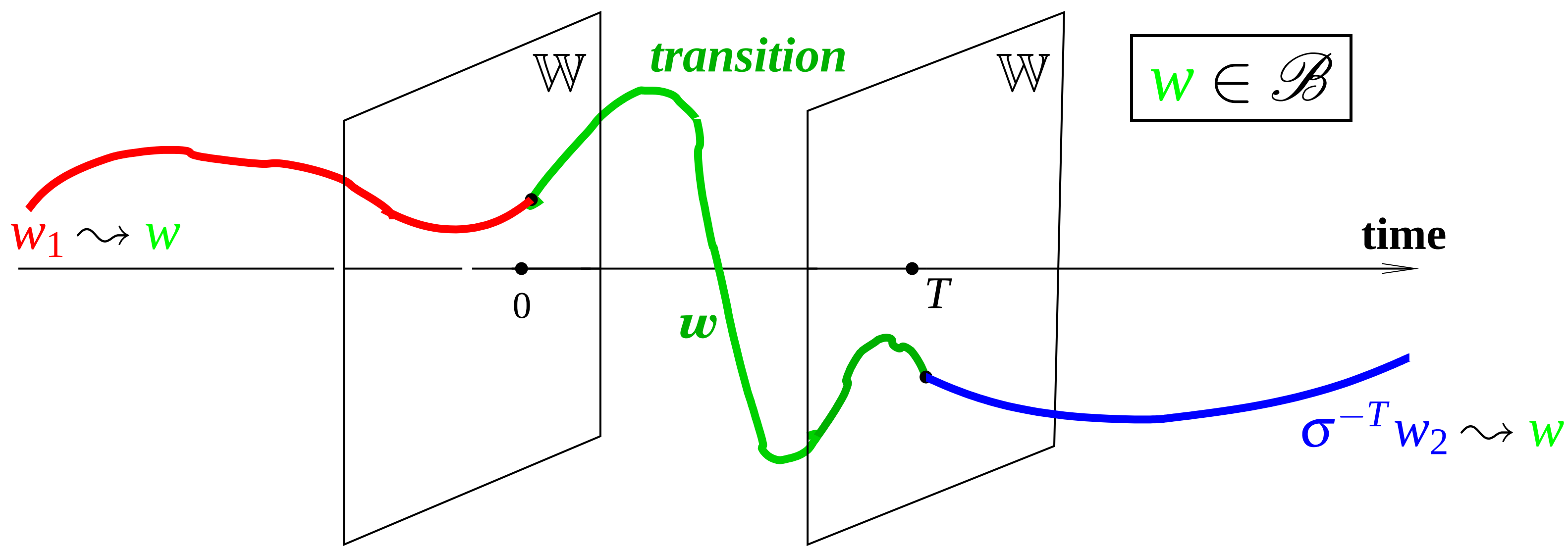
$w_1, w_2 \in \mathscr{B}$

Controllability in a picture



controllability :$\Leftrightarrow$ concatenability of trajectories after a delay

Controllability in a picture



controllability :⇔ concatenability of trajectories after a delay

This makes controllability into an intrinsic property of a system, rather than a property of a state representation.

LTIDSs

A linear time-invariant differential system (LTIDS) :⇔
the behavior $\mathscr{B} \subseteq (\mathbb{R}^{\mathtt{w}})^{\mathbb{R}}$ **is the set of solutions of a system of linear constant-coefficient ODEs**

$$R_0 w + R_1 \frac{d}{dt} w + \cdots + R_{\mathtt{n}} \frac{d^{\mathtt{n}}}{dt^{\mathtt{n}}} w = 0,$$

with $R_0, R_1, \ldots, R_{\mathtt{n}} \in \mathbb{R}^{\bullet \times \mathtt{w}}$ **real matrices that parametrize the system, and** $w : \mathbb{R} \to \mathbb{R}^{\mathtt{w}}$**.**

In polynomial matrix notation

$$R\left(\tfrac{d}{dt}\right) w = 0$$

with $R(\xi) = R_0 + R_1 \xi + \cdots + R_{\mathtt{n}} \xi^{\mathtt{n}} \in \mathbb{R}[\xi]^{\bullet \times \mathtt{w}}$**.**

3 theorems for LTIDSs

1. **There exists a** $1 \leftrightarrow 1$ **relation between the LTIDSs and the** $\mathbb{R}[\xi]$**-submodules of** $\mathbb{R}[\xi]^{\bullet}$**.**

3 theorems for LTIDSs

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2. **In LTIDSs, variables can be eliminated:**

$$R\left(\frac{d}{dt}\right)w = M\left(\frac{d}{dt}\right)\ell \qquad \Rightarrow \qquad \tilde{R}\left(\frac{d}{dt}\right)w = 0.$$

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3. **A LTIDS is controllable if and only if its behavior can be expressed as**

$$w = M\left(\tfrac{d}{dt}\right)\ell.$$

Every image is a kernel. A kernel is an image if and only if the system is controllable.

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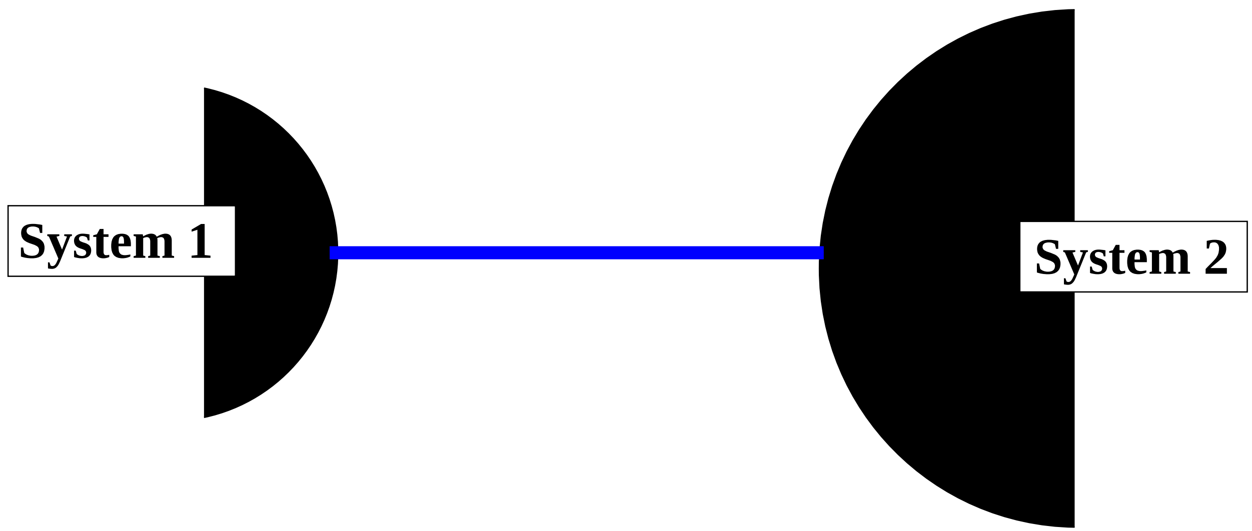
These theorems hold *mutatis mutandis* for discrete-time LTIDSs and for systems described by linear PDEs.

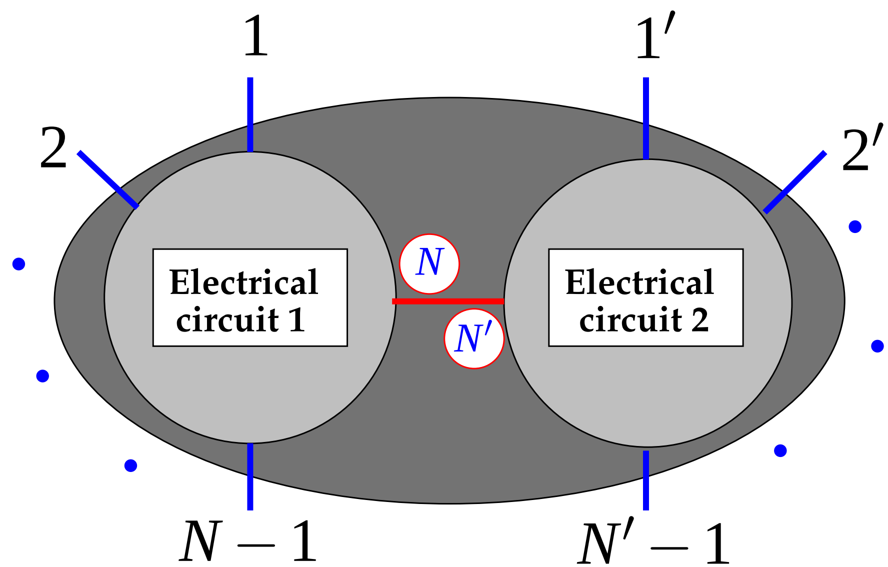
INTERCONNECTION

Connection of terminals



By interconnecting, the terminal variables are equated.

Interconnection of circuits



$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Behavior after interconnection:

$\mathscr{B}_1 \sqcap \mathscr{B}_2$

$$:= \{(V_1, \ldots, V_{N-1}, V_{1'}, \ldots, V_{N'-1}, I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}) \mid$$

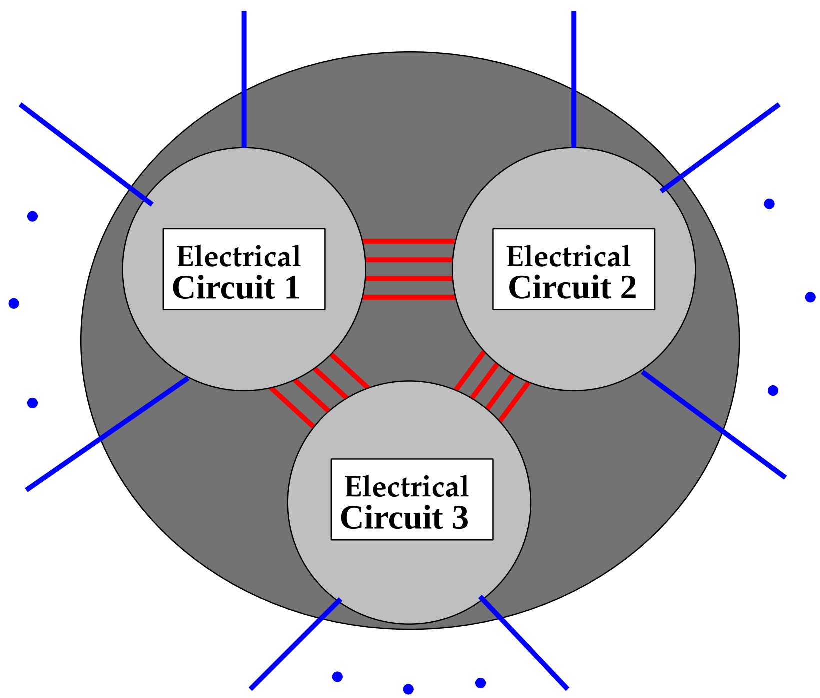
$\exists\, V, I$ **such that**

$$(V_1, \ldots, V_{N-1}, V, I_1, \ldots, I_{N-1}, I) \in \mathscr{B}_1 \textbf{ and}$$

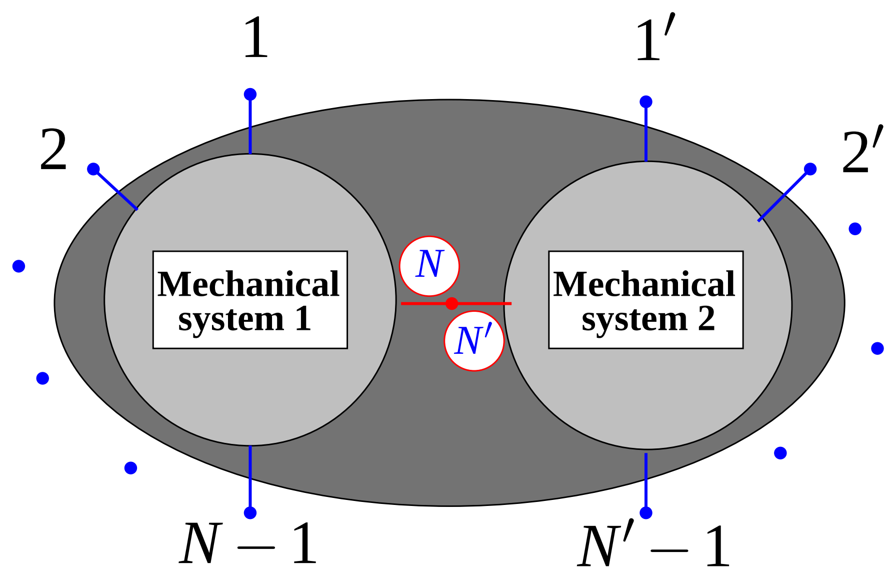
$$(V_{1'}, \ldots, V_{N'-1}, V, I_{1'}, \ldots, I_{N'-1}, -I) \in \mathscr{B}_2\}.$$

Interconnection of circuits

⇝ **more terminals and more circuits connected**



Interconnection of 1-D mechanical systems



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Other terminal types

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **...**

Sharing variables

$$
\begin{aligned}
V_N = \ & V_{N'} \quad \textbf{and} \quad & I_N + I_{N'} = \ & 0, \\
q_N = \ & q_{N'} \quad \textbf{and} \quad & F_N + F_{N'} = \ & 0, \\
T_N = \ & T_{N'} \quad \textbf{and} \quad & Q_N + Q_{N'} = \ & 0, \\
p_N = \ & p_{N'} \quad \textbf{and} \quad & f_N + f_{N'} = \ & 0,
\end{aligned}
$$

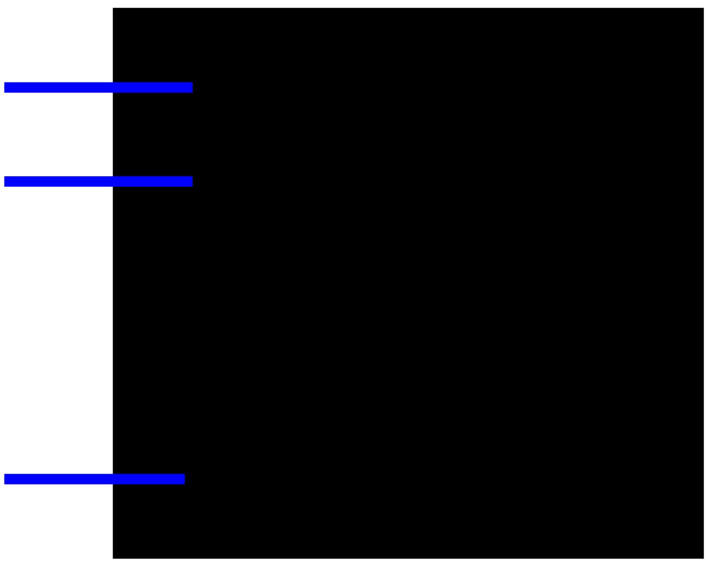
$$\vdots$$

Interconnection means variable sharing.

TEARING, ZOOMING, and LINKING

Tearing

¡¡ Model the behavior of selected variables !!



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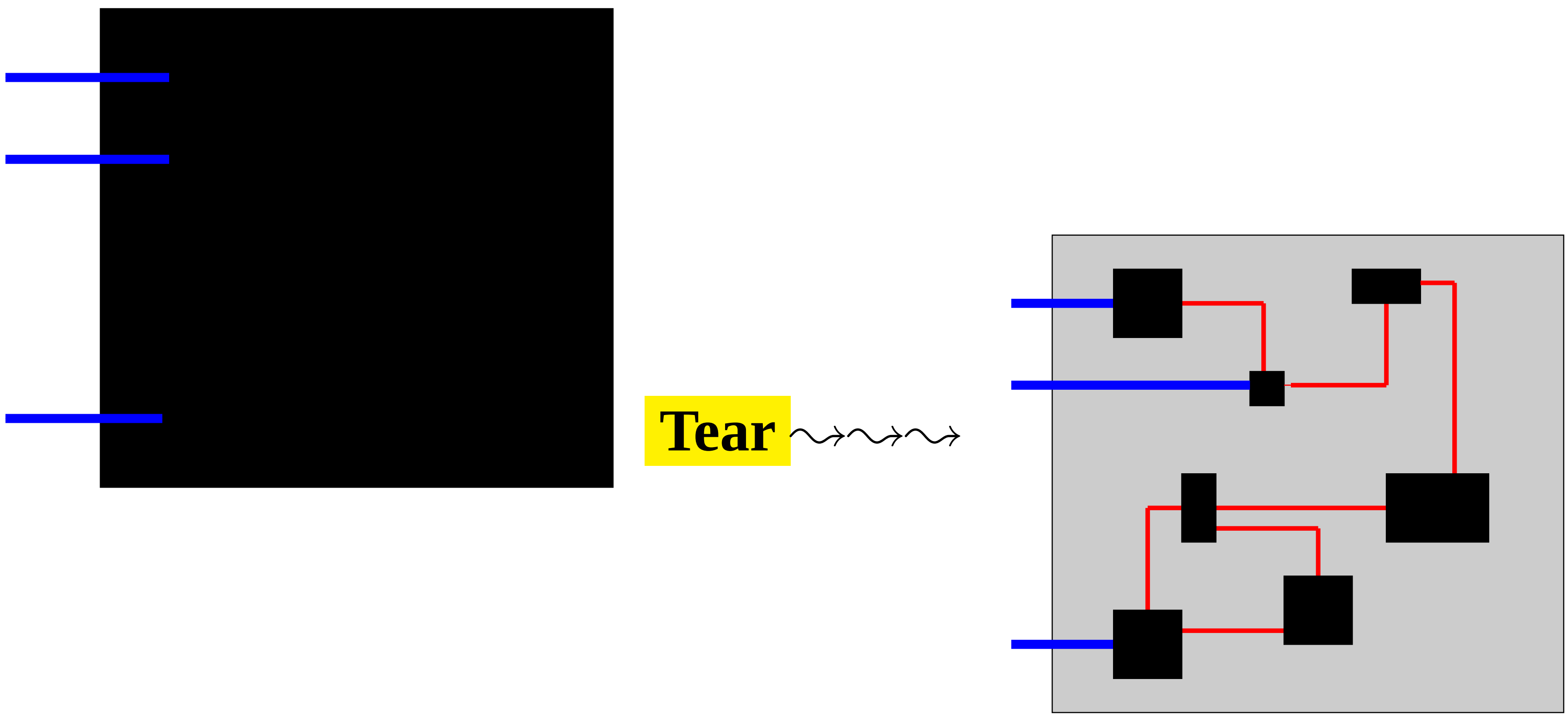
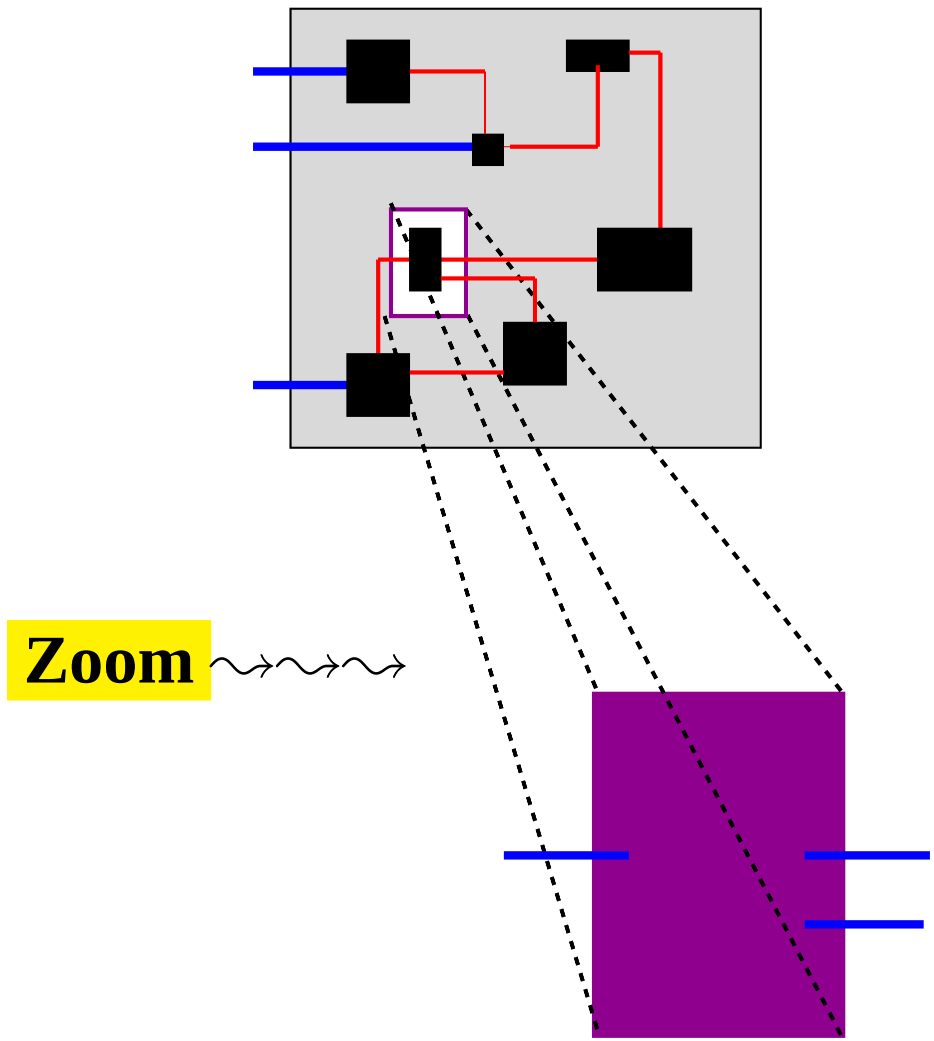
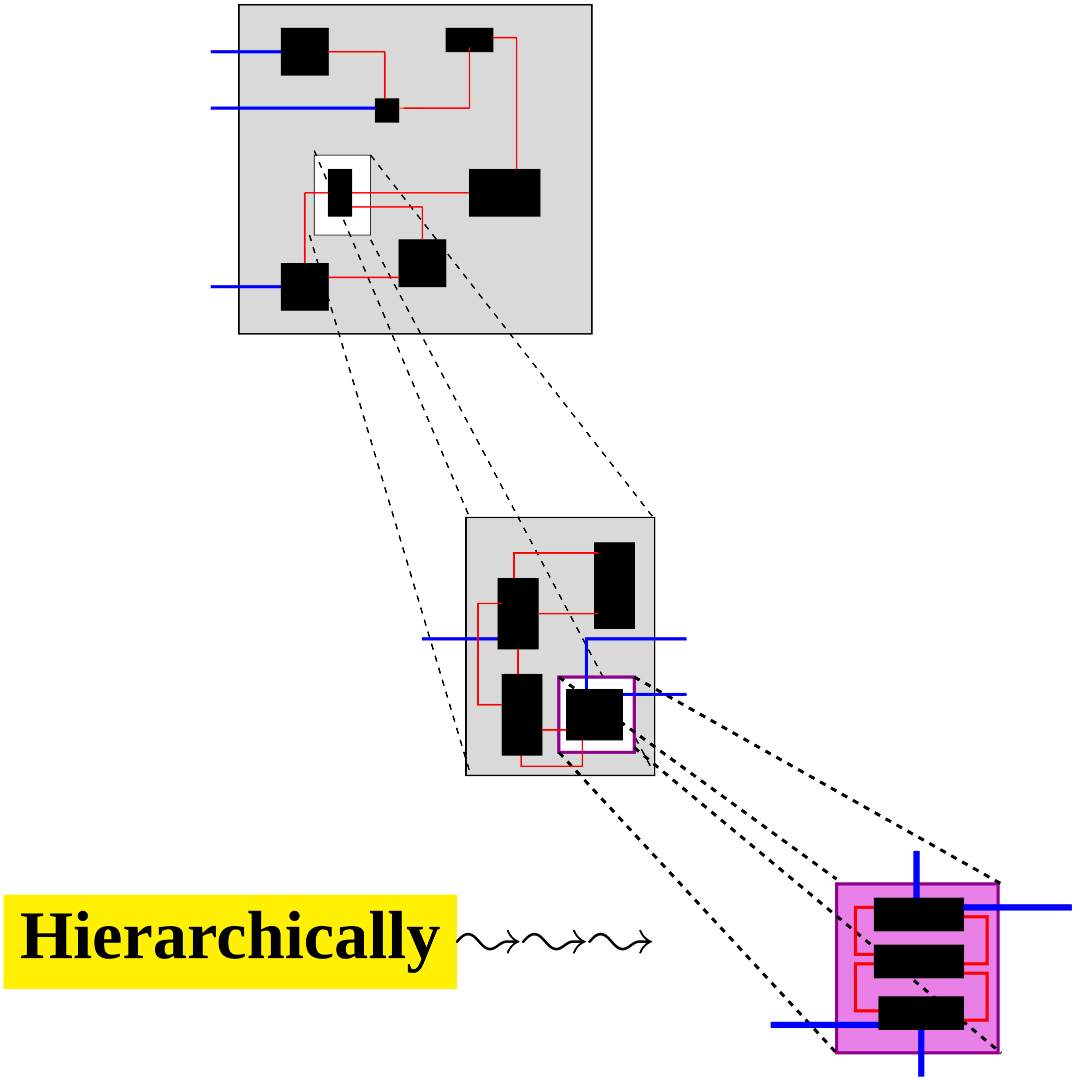


Zooming

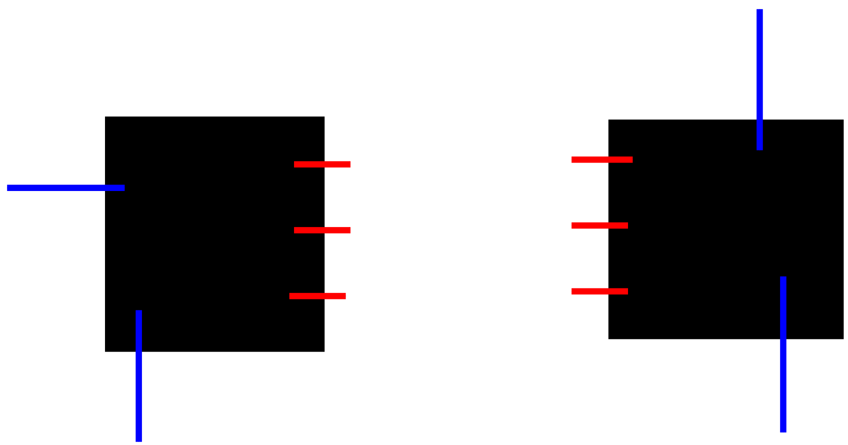


Zooming



Proceed until subsystems (‘modularity’) are obtained whose model is known, from first principles, or stored in a database.

Linking



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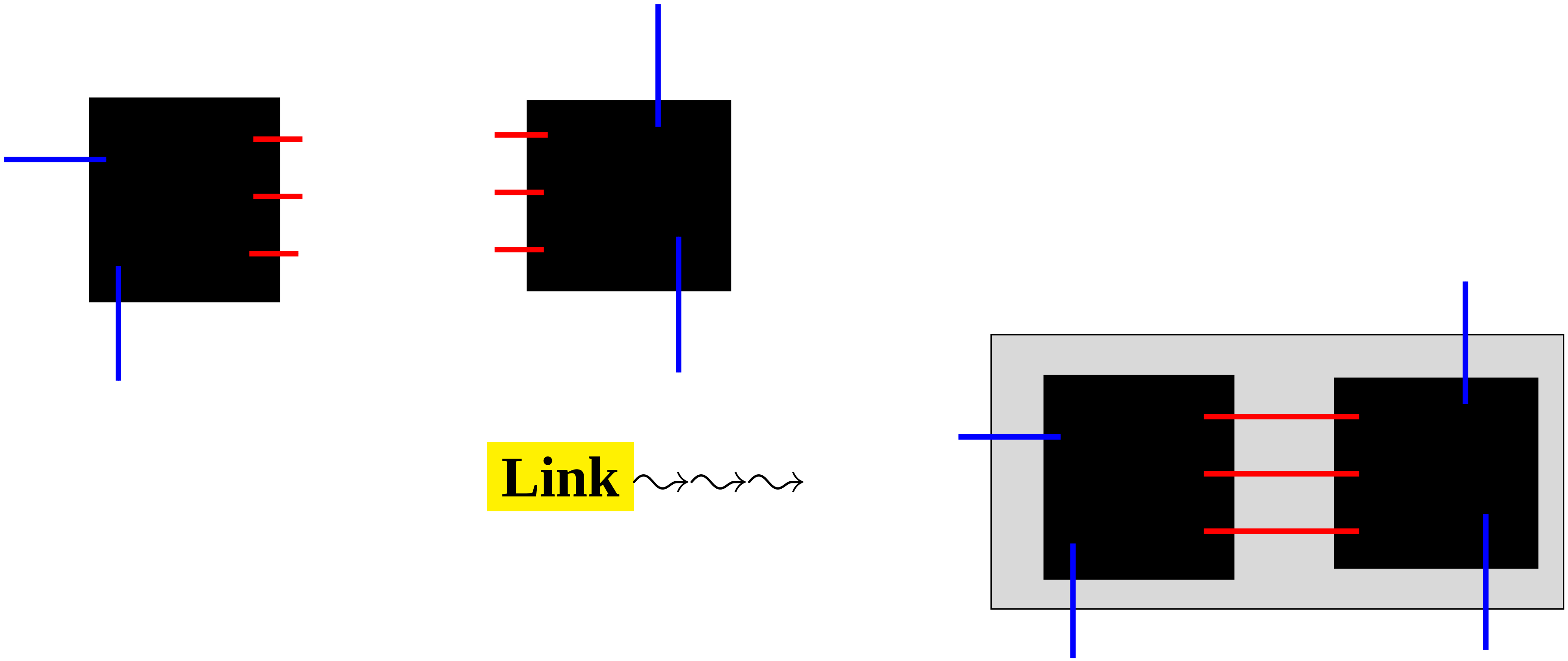
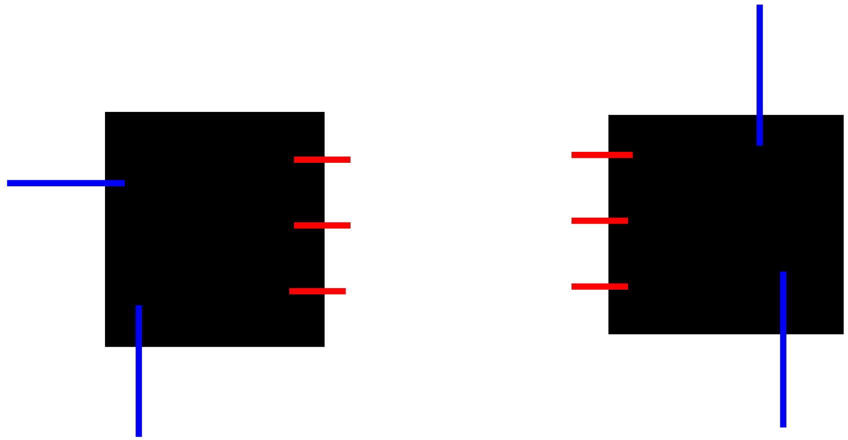
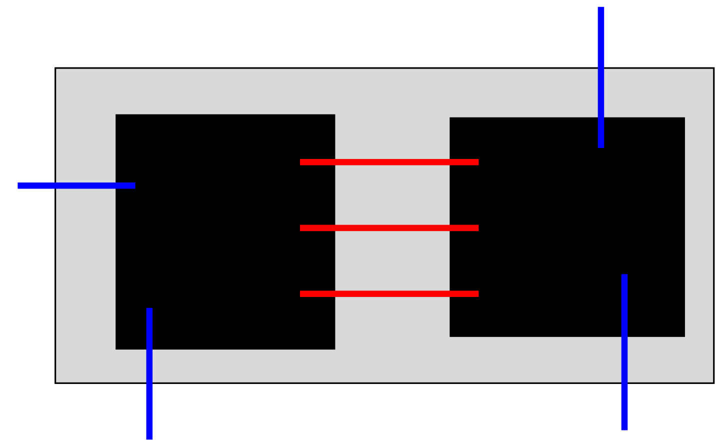


Linking



Link ⇝⇝⇝



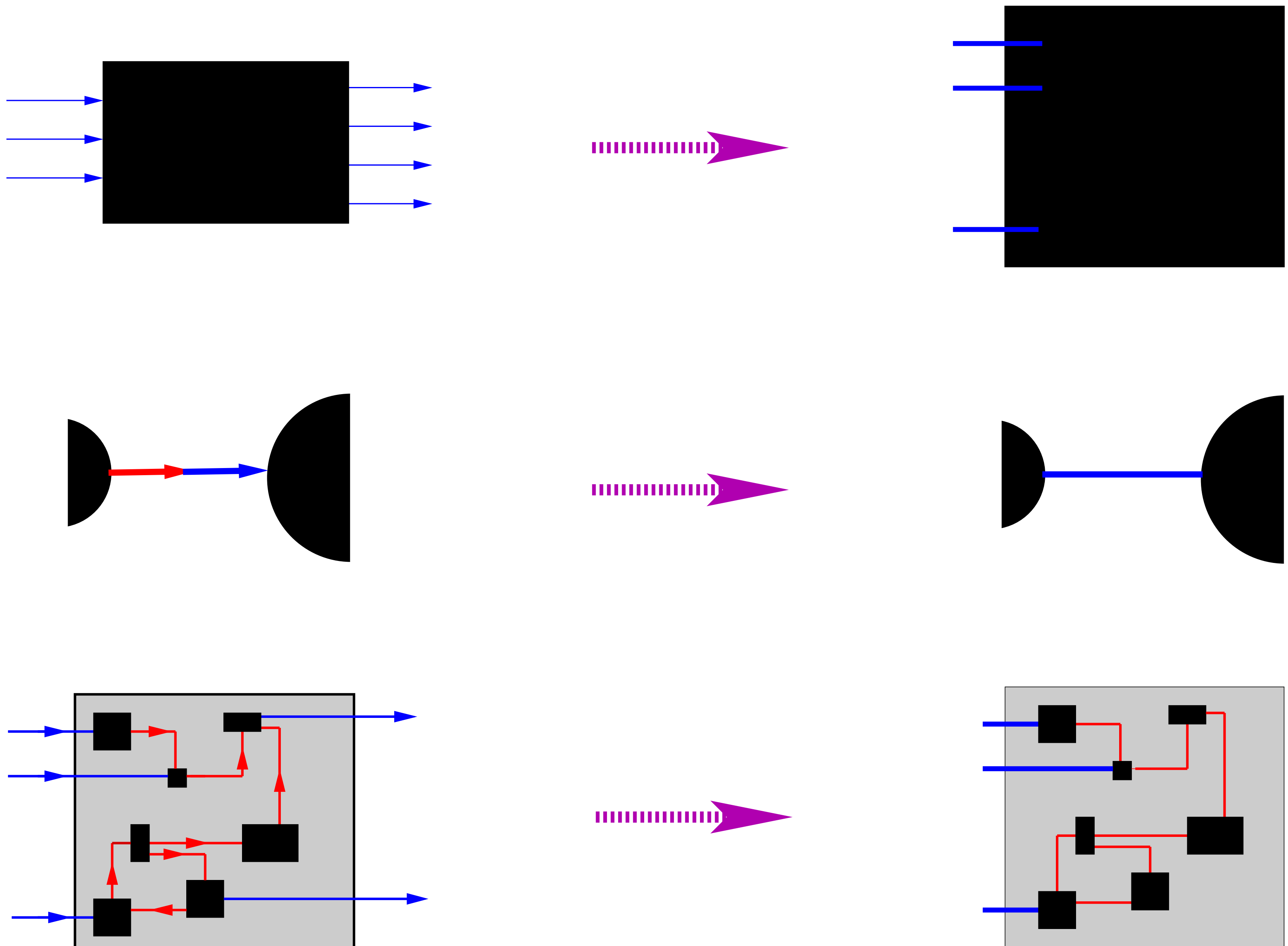
Model:

- **component behavior**
- **sharing equations**
- **elimination**

⇝ **behavior of the manifest variables.**

Tearing, zooming, and linking ⇝ computer assisted modeling.

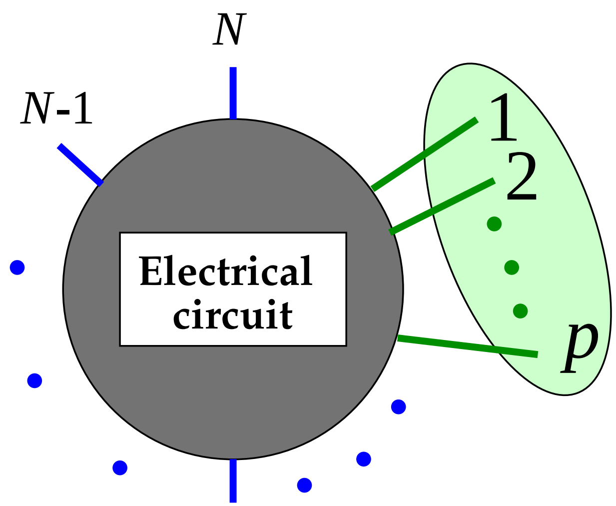
JUXTAPOSITION



ENERGY TRANSFER

PORTS

Ports



Terminals $\{1,2,\ldots,p\}$ **form a port** $:\Leftrightarrow$

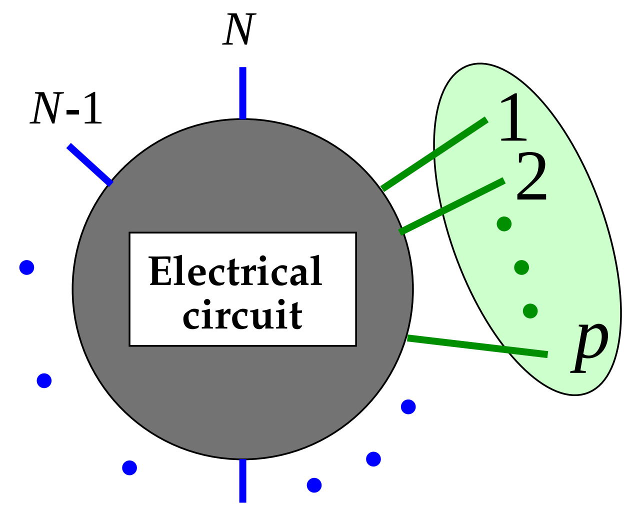
$$(V_1,\ldots,V_p,V_{p+1},\ldots,V_N,I_1,\ldots,I_p,I_{p+1},\ldots,I_N) \in \mathscr{B}$$

$$\Rightarrow \quad I_1 + \cdots + I_p = 0.$$

'port KCL'.

KCL $\Rightarrow$ **all terminals together form a port.**

Electrical ports



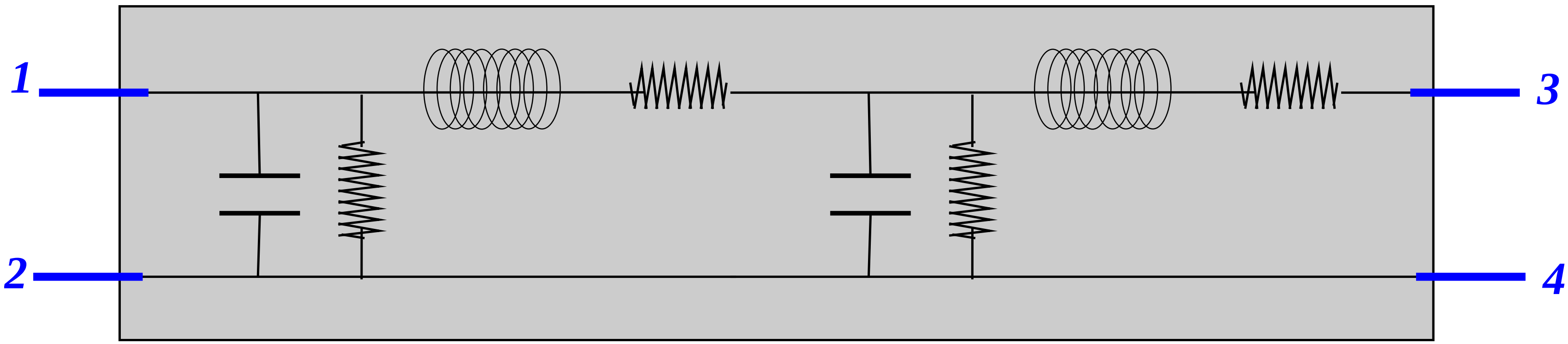
If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

power in along these terminals = $V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)$,

energy in = $\displaystyle\int_{t_1}^{t_2} [V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)]\, dt.$

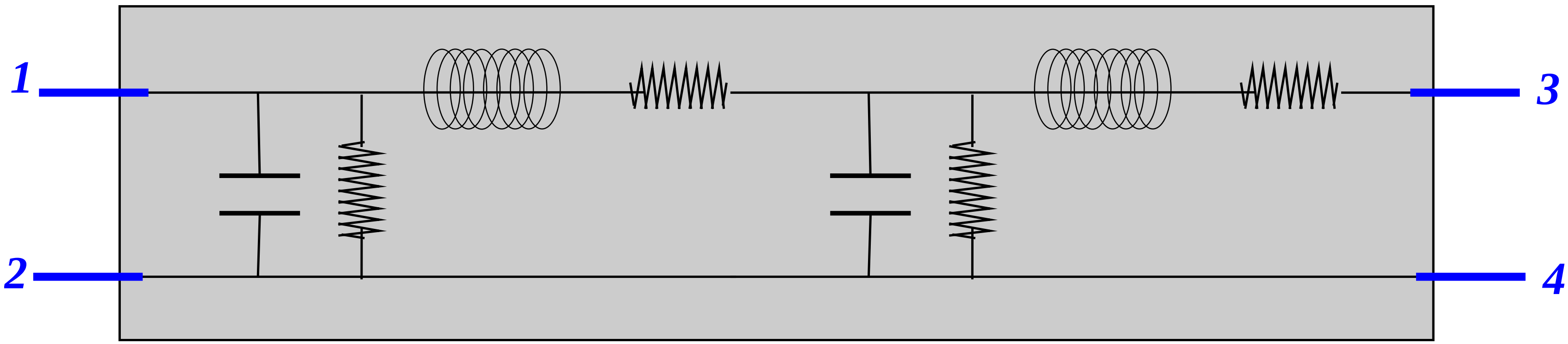
This interpretation in terms of power and energy is not valid unless these terminals form a port !

Examples



Terminals $\{1,2,3,4\}$ **form a port. But** $\{1,2\}$ **and** $\{3,4\}$ **do not.**

Examples



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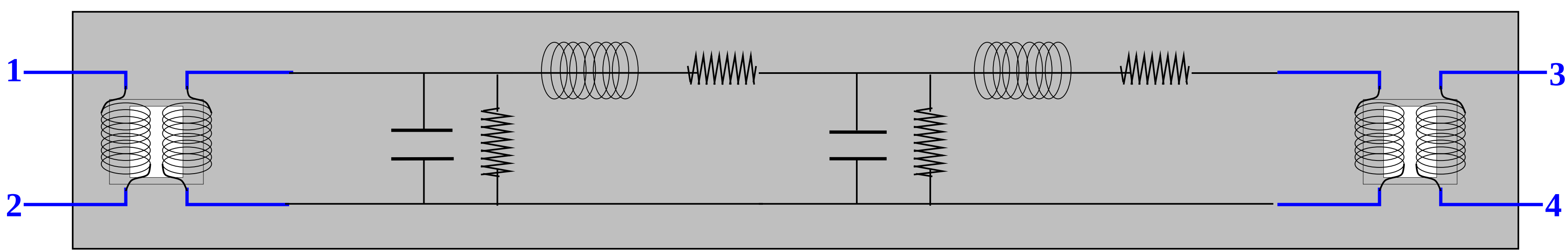
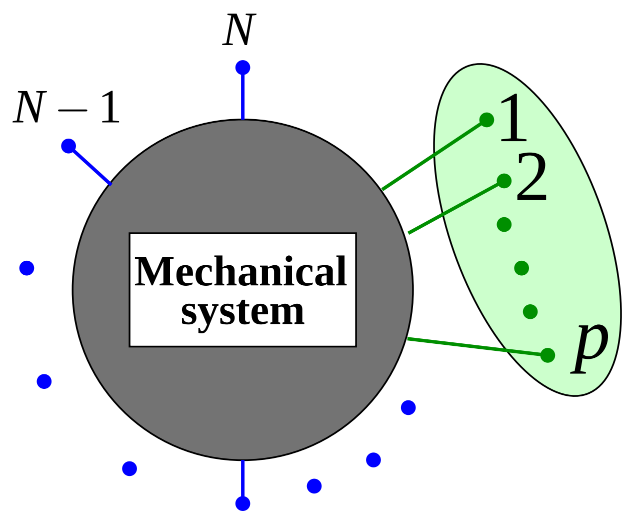


Terminals $\{1, 2\}$ **and** $\{3, 4\}$ **form a port.**

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \quad F_1 + F_2 + \cdots + F_p = 0.$$

'port KFL'

Power and energy

If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

$$\textbf{power in} \;=\; F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

and

$$\textbf{energy in} \;=\; \int_{t_1}^{t_2} \left[F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right] dt.$$

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unless these terminals form a port !

Examples

Resistors, capacitors, inductors, transistors, ..., 2-terminal circuits formed by their interconnection, form ports.

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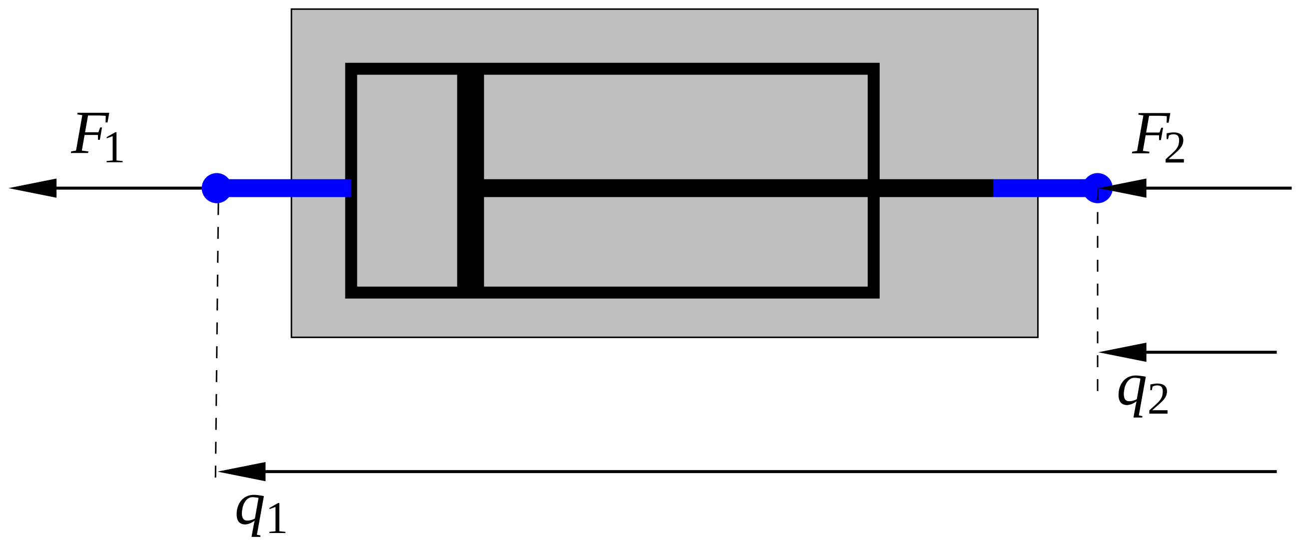
A damper

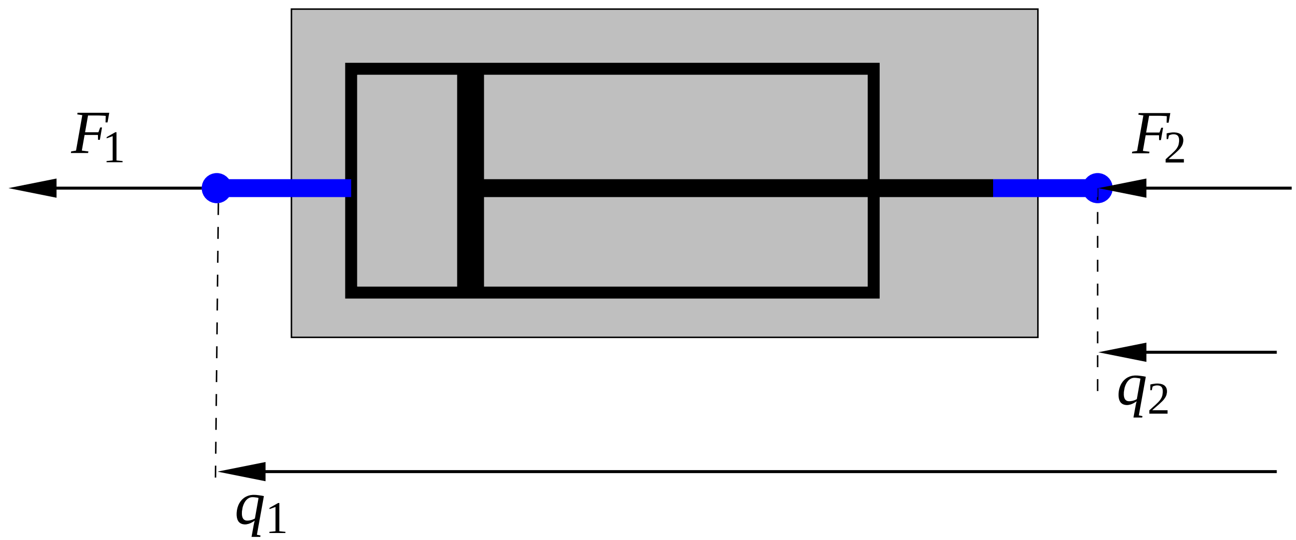


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1$$ **satisfies KFL** $\rightsquigarrow$ **a port**

Examples

Resistors, capacitors, inductors, transistors, ..., 2-terminal circuits formed by their interconnection, form ports.

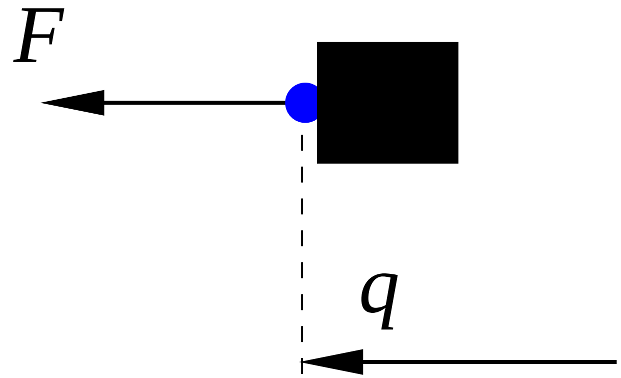
A damper



$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1$$ **satisfies KFL $\rightsquigarrow$ a port**

Springs and dampers, and mechanical devices formed by the interconnection of springs and dampers form ports.

A mass

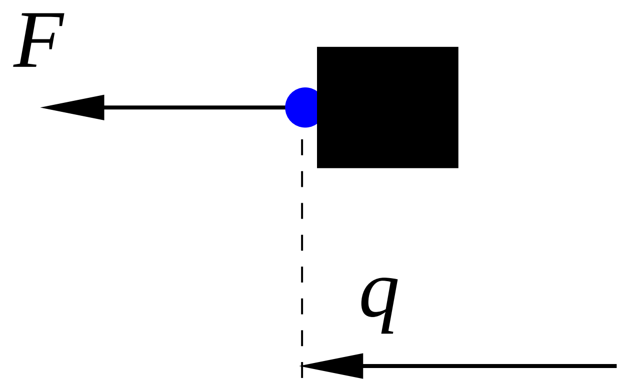


$$M\frac{d^2}{dt^2}q = F.$$

does not satisfy KFL

Not a port!!!

A mass



$$M\frac{d^2}{dt^2}q = F.$$

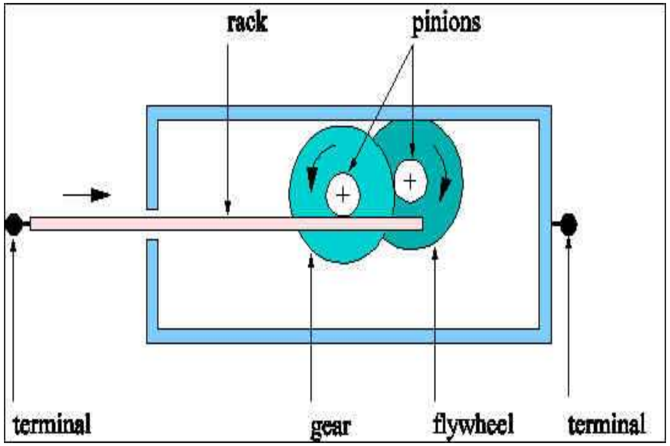
does not satisfy KFL

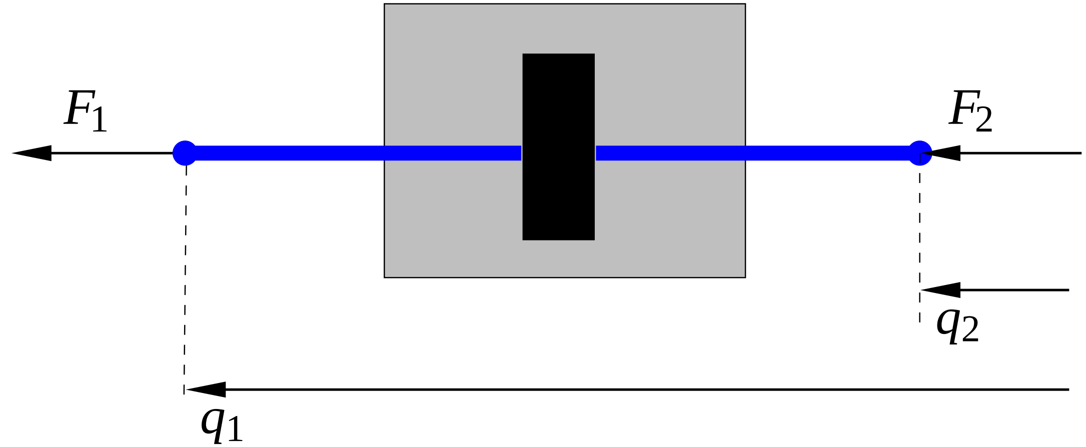
Not a port!!!

The mass is NOT the mechanical analogue of a capacitor.

RLC synthesis $\not\Leftrightarrow$ **Damper-Spring-Mass synthesis**

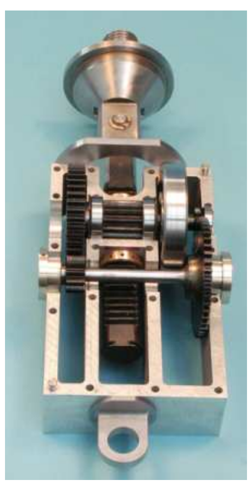
The inerter





$$B\frac{d^2}{dt^2}(q_1 - q_2) = F_1, \quad F_1 + F_2 = 0$$

satisfies KFL $\rightsquigarrow$ a port



Malcolm Smith

Electrical-mechanical analogies

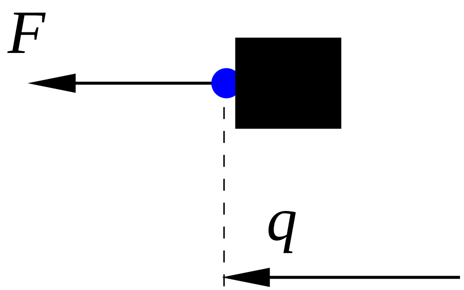
voltage $V \leftrightarrow v$ **velocity** **current** $I \leftrightarrow F$ **force**

Resistor $\frac{1}{R}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	**Damper** $D(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$
Inductor $\frac{1}{L}(V_1 - V_2) = \frac{d}{dt} I_1, \;\; I_1 + I_2 = 0$	**Spring** $K(v_1 - v_2) = \frac{d}{dt} F_1, \;\; F_1 + F_2 = 0$
Capacitor $C\frac{d}{dt}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	**Inerter** $B\frac{d}{dt}(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$

electrical RLC synthesis $\Leftrightarrow$ **mechanical DSI synthesis**

KINETIC ENERGY

Back to the mass

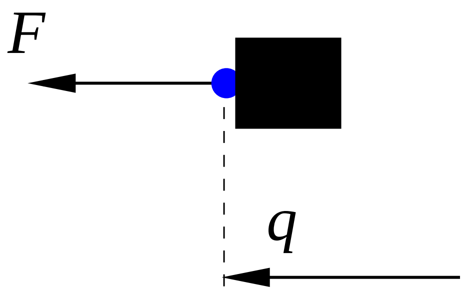


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q$$

$F^\top v$ **not power** $\Rightarrow$ $\frac{1}{2}M||\frac{d}{dt}q||^2$ **not kinetic energy ?**

Willem 's Gravesande
1688–1742

Émilie du Châtelet
1706–1749



Back to the mass

$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top \frac{d}{dt}q$$

$F^\top v$ **not power** $\Rightarrow$ $\frac{1}{2}M||\frac{d}{dt}q||^2$ **not kinetic energy ?**

Willem 'S Gravesande
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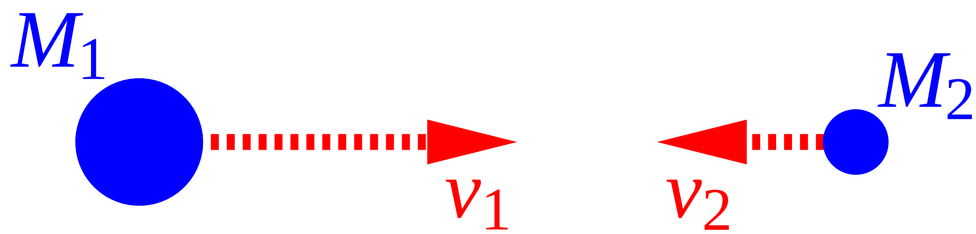
$E_{\text{kinetic}} = \frac{1}{2}M||\frac{d}{dt}q||^2$ **is not invariant under uniform motion.**

Motion energy



What is the motion energy?

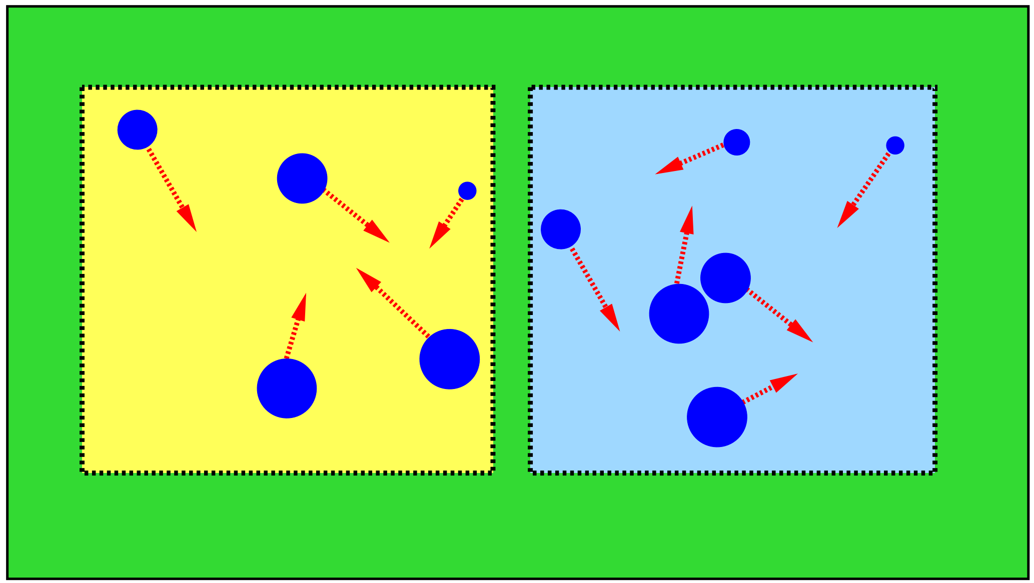
What quantity is transformable into heat?

$$\mathscr{E}_{\textbf{motion}} = \frac{1}{2} \frac{M_1 \, M_2}{M_1 + M_2} \, ||v_1 - v_2||^2$$

Invariant under uniform motion.

Motion energy

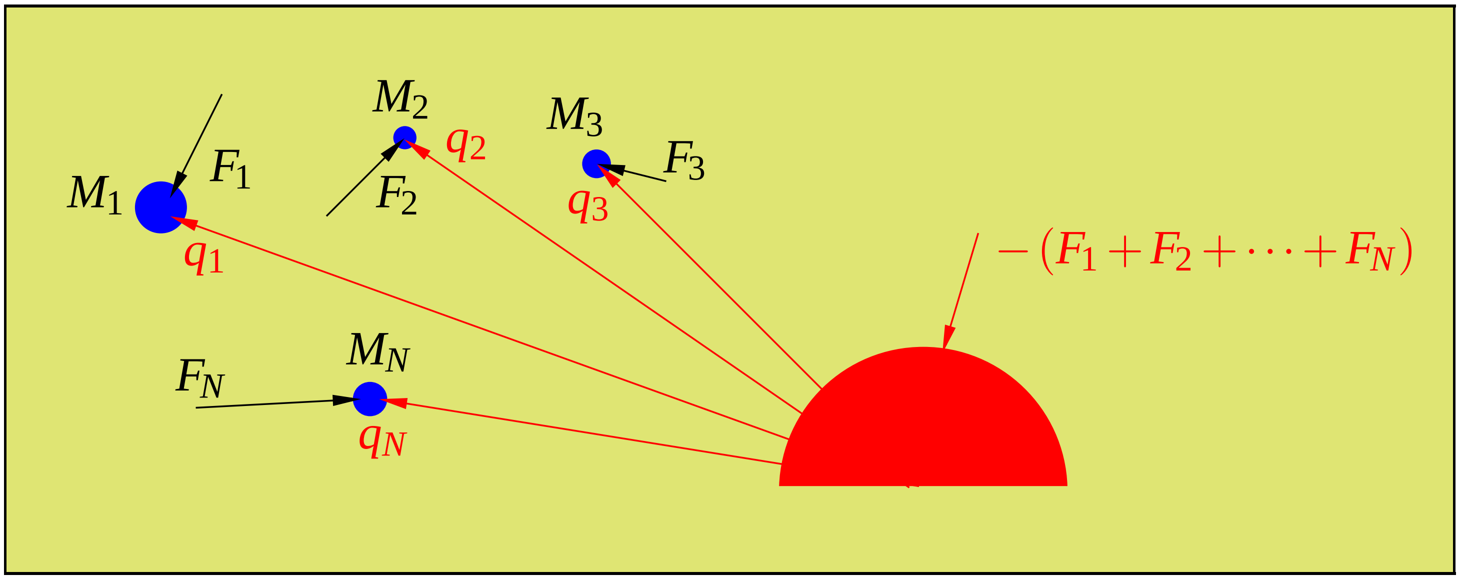
Motion energy is not an extensive quantity, it is not additive.



Total motion energy $\neq$ sum of the parts.

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



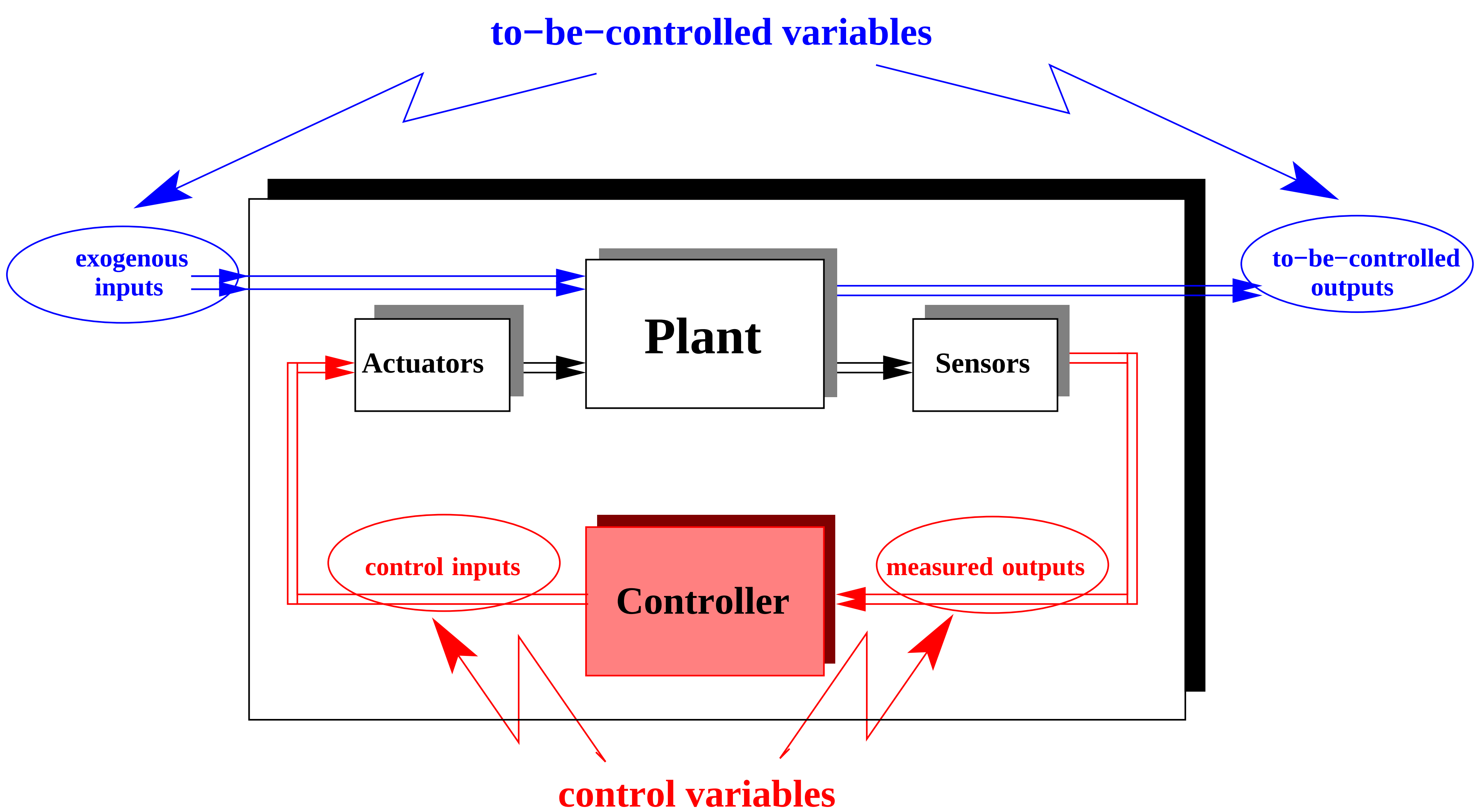
measure velocities w.r.t. this infinite mass ('ground'), then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

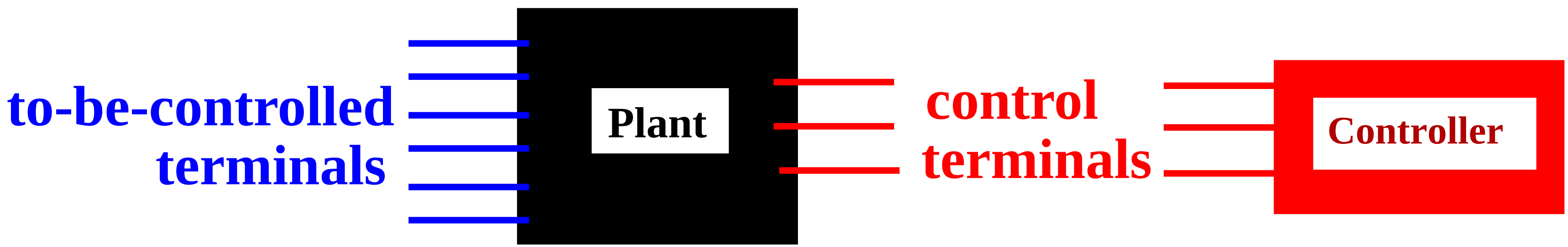
$$\longrightarrow \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2.$$

CONTROL as INTERCONNECTION

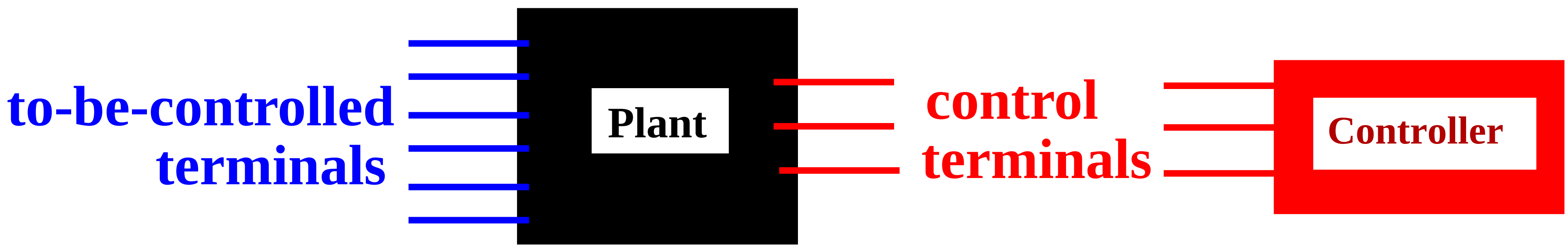
Feedback control



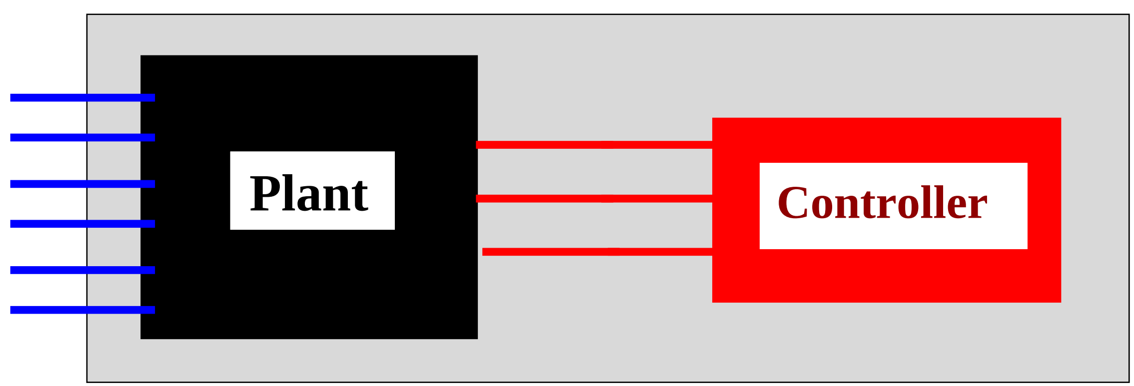
Behavioral control



Behavioral control

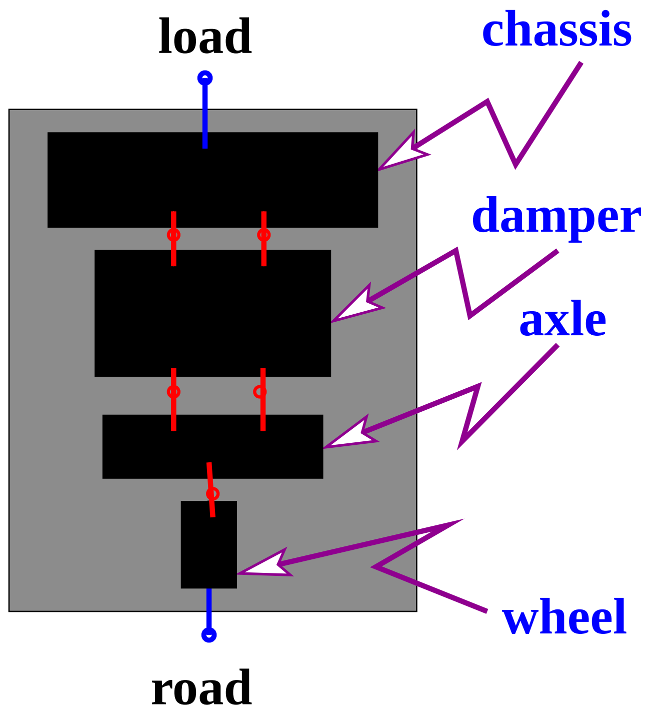


control = interconnection.

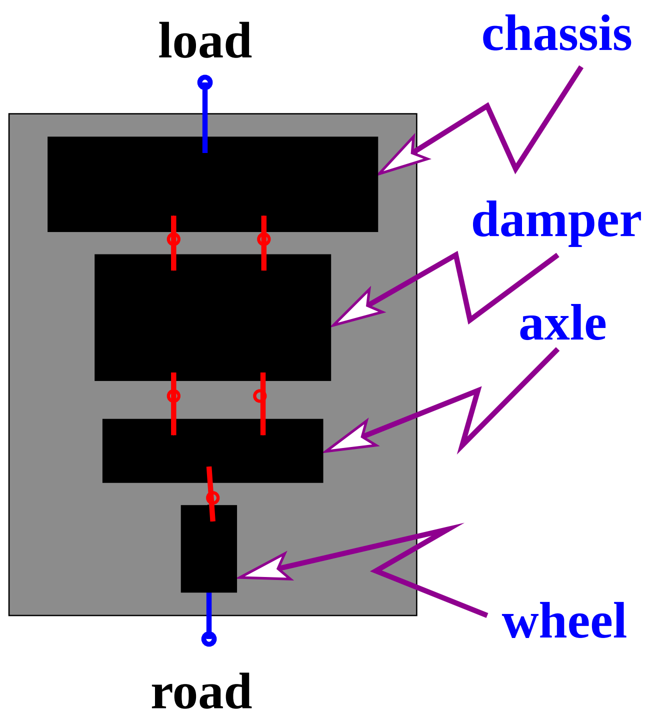


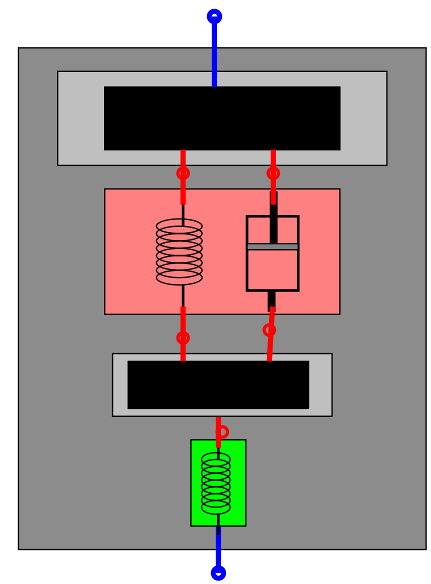
controlled system

Example: A ‘quarter car’



Example: A ‘quarter car’





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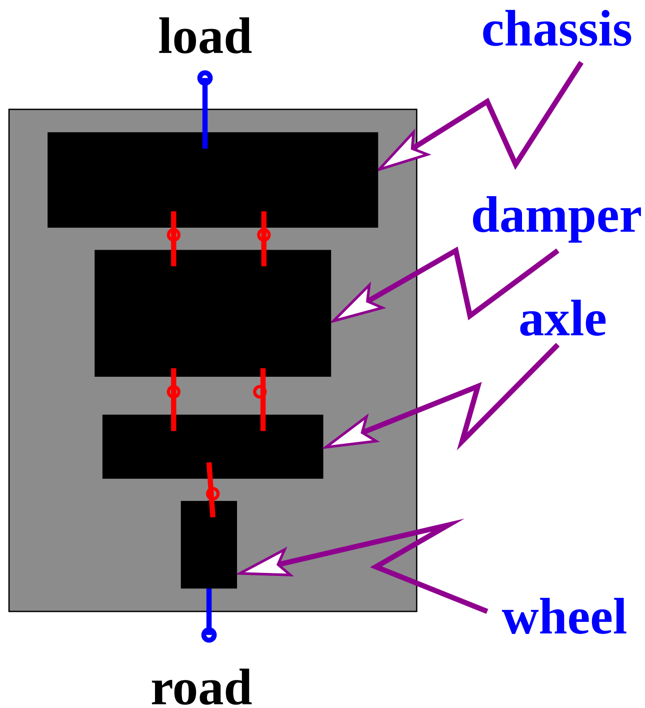

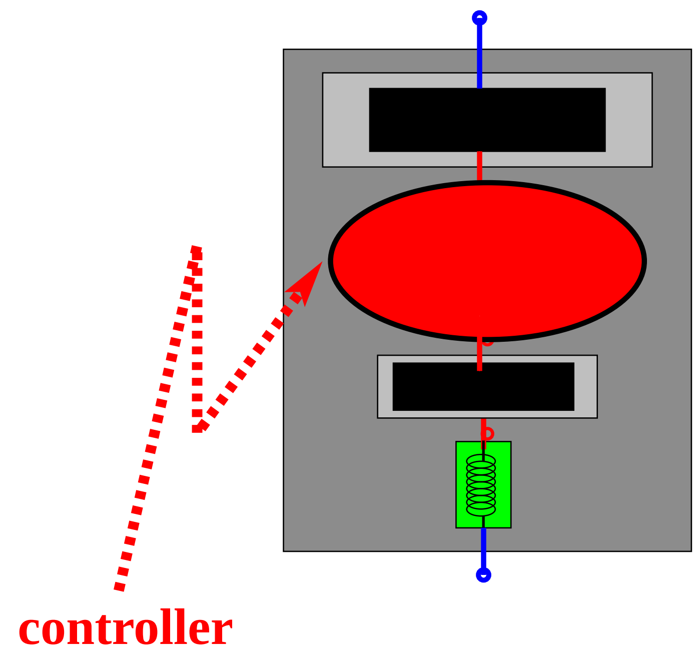

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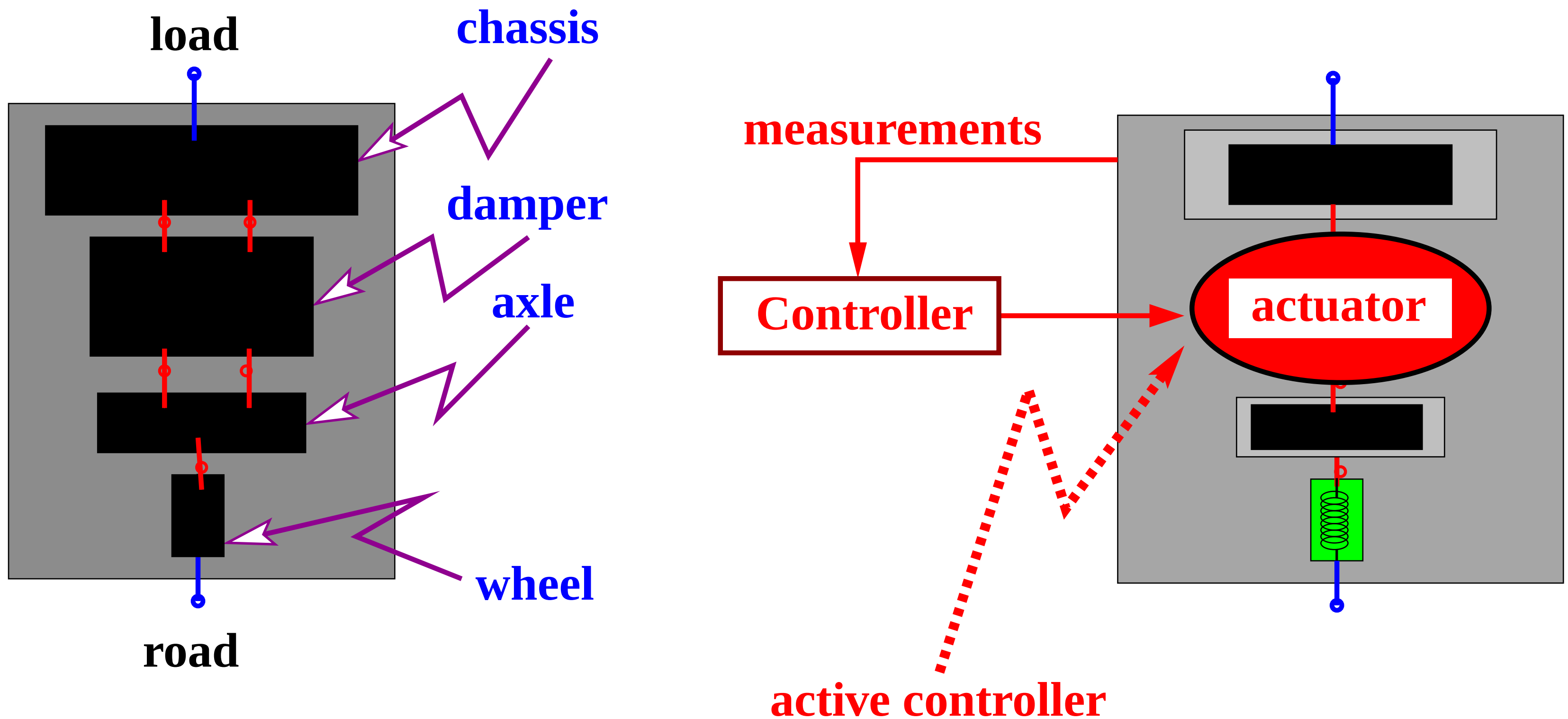

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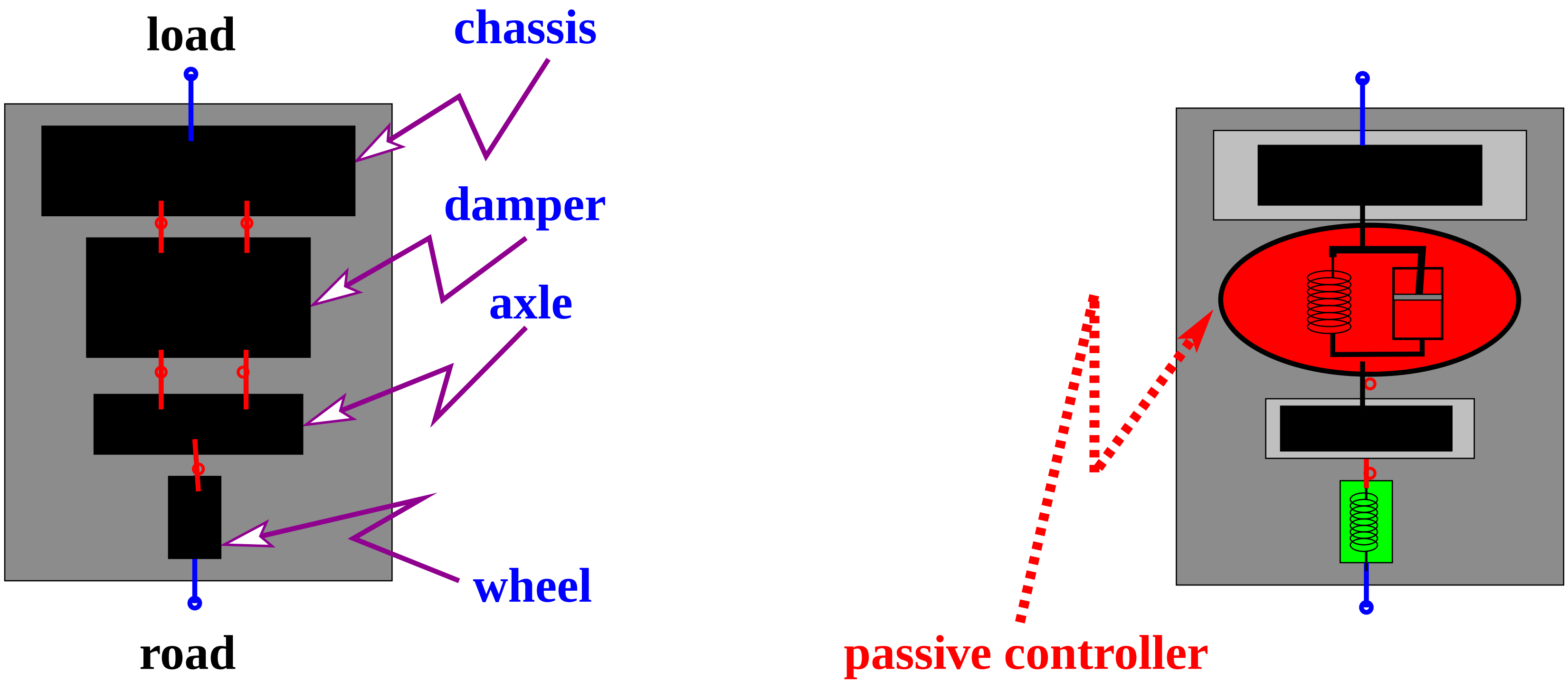
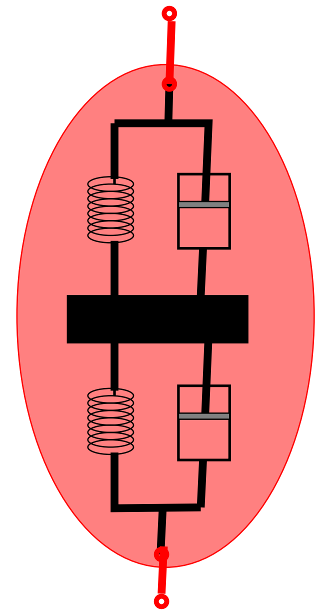


Suspension control in Formula 1

Nigel Mansell victorious in 1992 with an active damper suspension.

Active dampers were banned in 1994 to break the dominance of the Williams team.

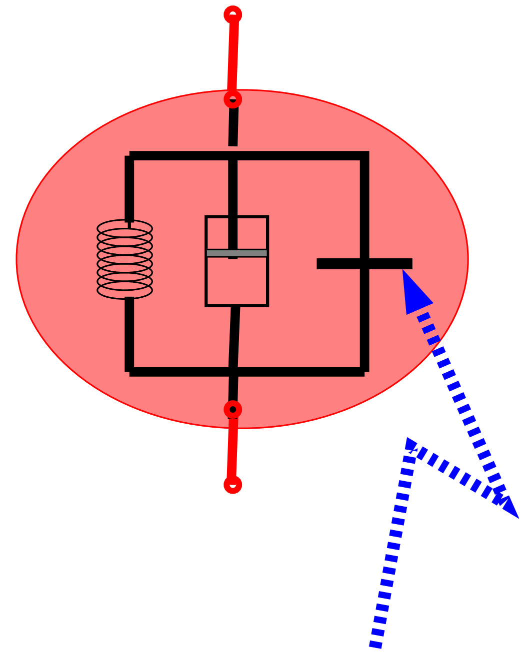
Suspension control in Formula 1



Renault successfully use a passive ‘tuned mass damper’ in 2005/2006.

Banned in 2006, under the ‘movable aerodynamic devices’ clause.

Suspension control in Formula 1



Kimi Räikkönen wins the 2005 Grand Prix in Spain with McLaren's 'J-damper', i.e., Smith's inerter.

grandprix.com

AUGUST 21, 2008

Ingenuity still brings success in Formula 1

ShareThis

For years engineers have complained that the rules of Formula 1 mean that there is little room left for innovation but Cambridge University's engineering department has just revealed that this is not the case at all.

Professor Malcolm Smith, a fellow of Gonville and Caius College, created an innovative suspension system in the late 1990s and this was patented by the university. The first details were published in 2002 in the obscure Institute of Electrical and Electronics Engineers's publication called Transactions on Automatic Control. This was spotted by the boffins at McLaren and an exclusive deal was negotiated to allow the team to use the technology in F1. The new system was first used at the Spanish GP in 2005 and Kimi Raikkonen won.

The team used the name "J Damper" to describe the unit - in an effort to confuse the opposition - but it has now been revealed that it is actually called "an inerter". This is grandprix.com

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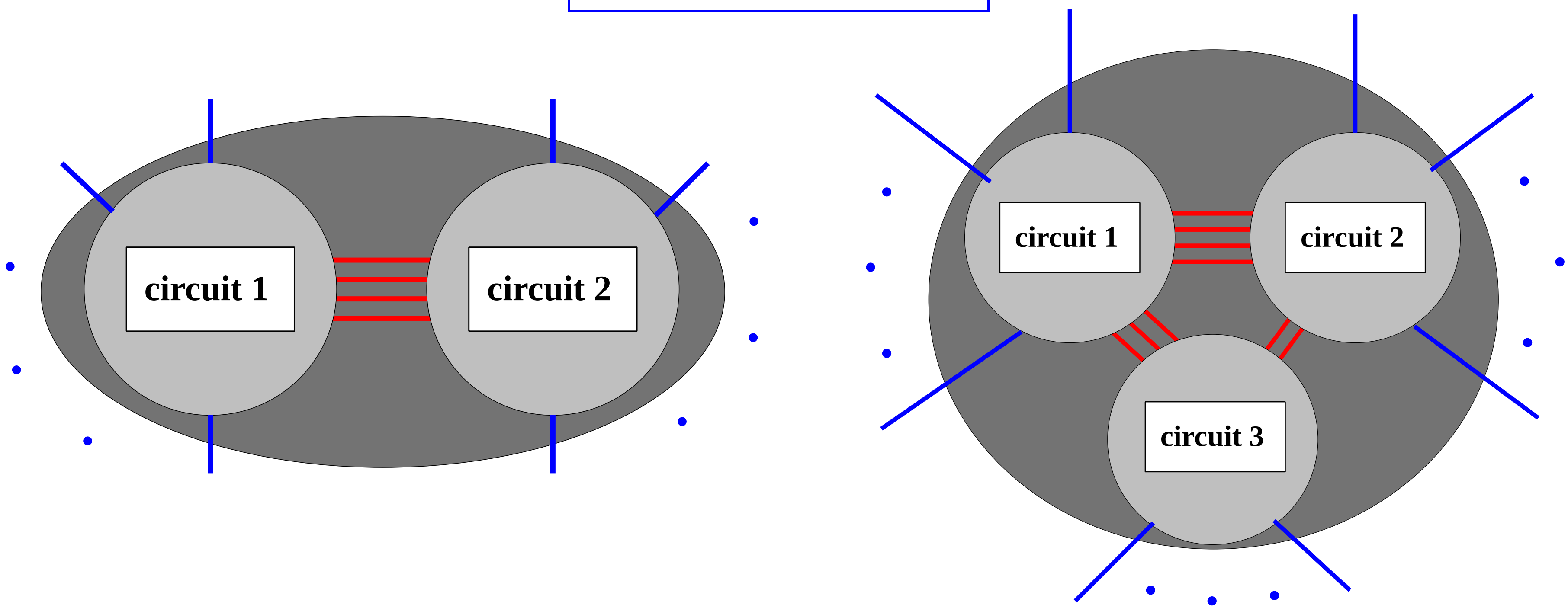
PORTS and TERMINALS

Interconnection versus energy transfer

Terminals are for interconnection.

Ports are for energy transfer

Energy transfer



One cannot speak about

"the energy transferred from circuit 1 to circuit 2"
or *"from the environment to circuit 1"*,

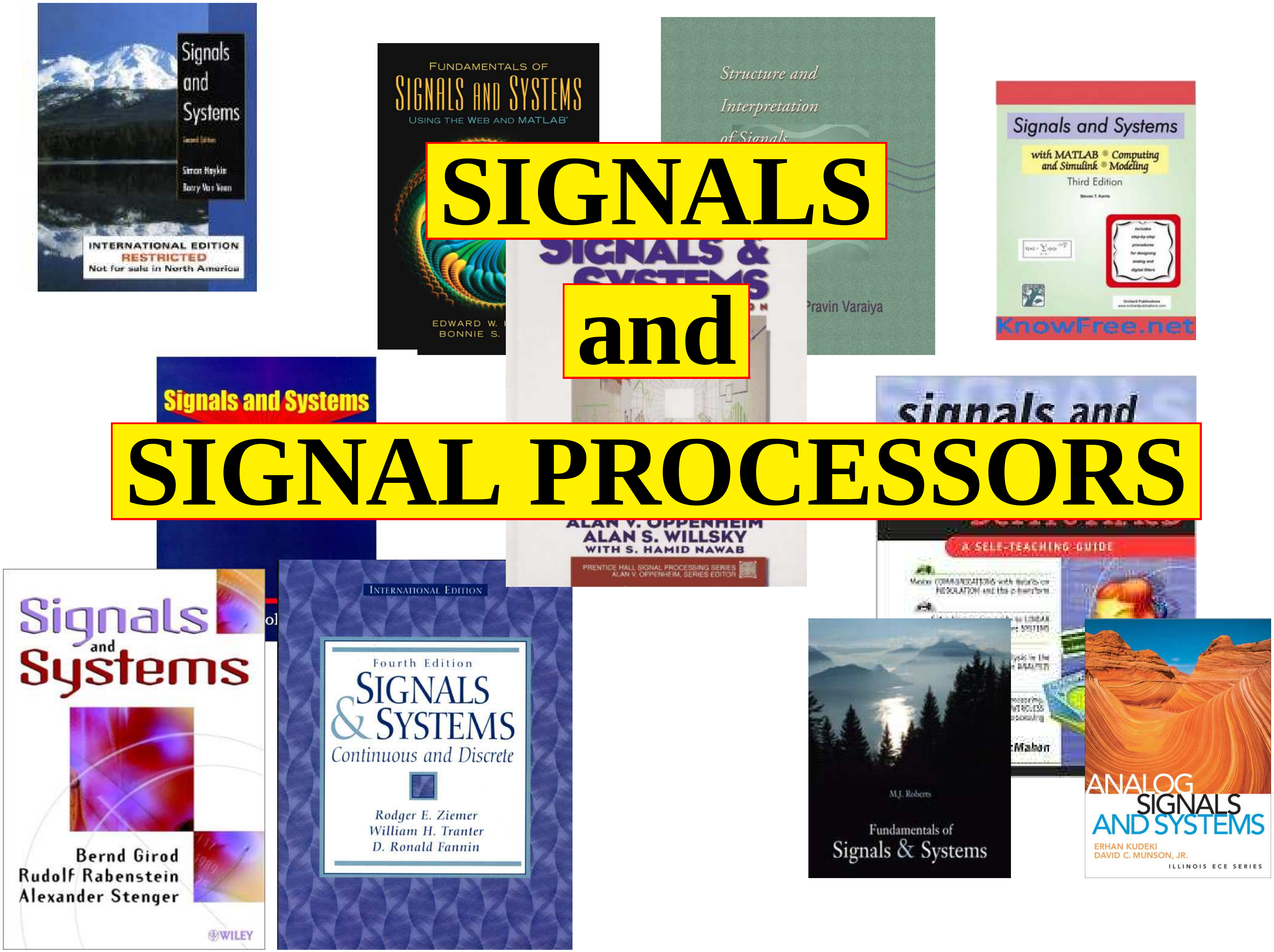
unless the relevant terminals form a port.

Analogously for mechanical systems, etc.

CONCLUSION

Favorite textbooks

Favorite textbooks



SIGNALS

and

SIGNAL PROCESSORS

Reference: The behavioral approach to open and interconnected systems, *Control Systems Magazine*, volume 27, pages 46-99, 2007.

Copies of the lecture frames will be available from/at

`Jan.Willems@esat.kuleuven.be`

`http://www.esat.kuleuven.be/~jwillems`

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you