ENERGY TRANSFER

IN

ELECTRICAL CIRCUITS

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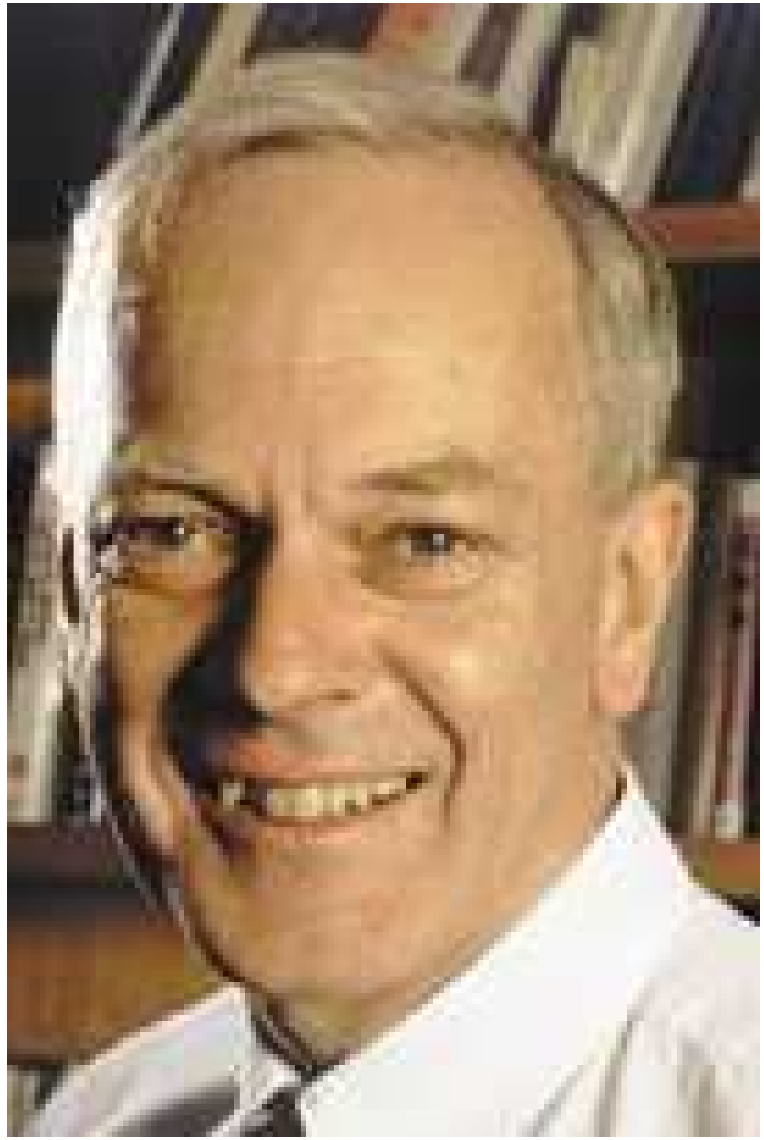
CDC workshop **December 14, 2010**



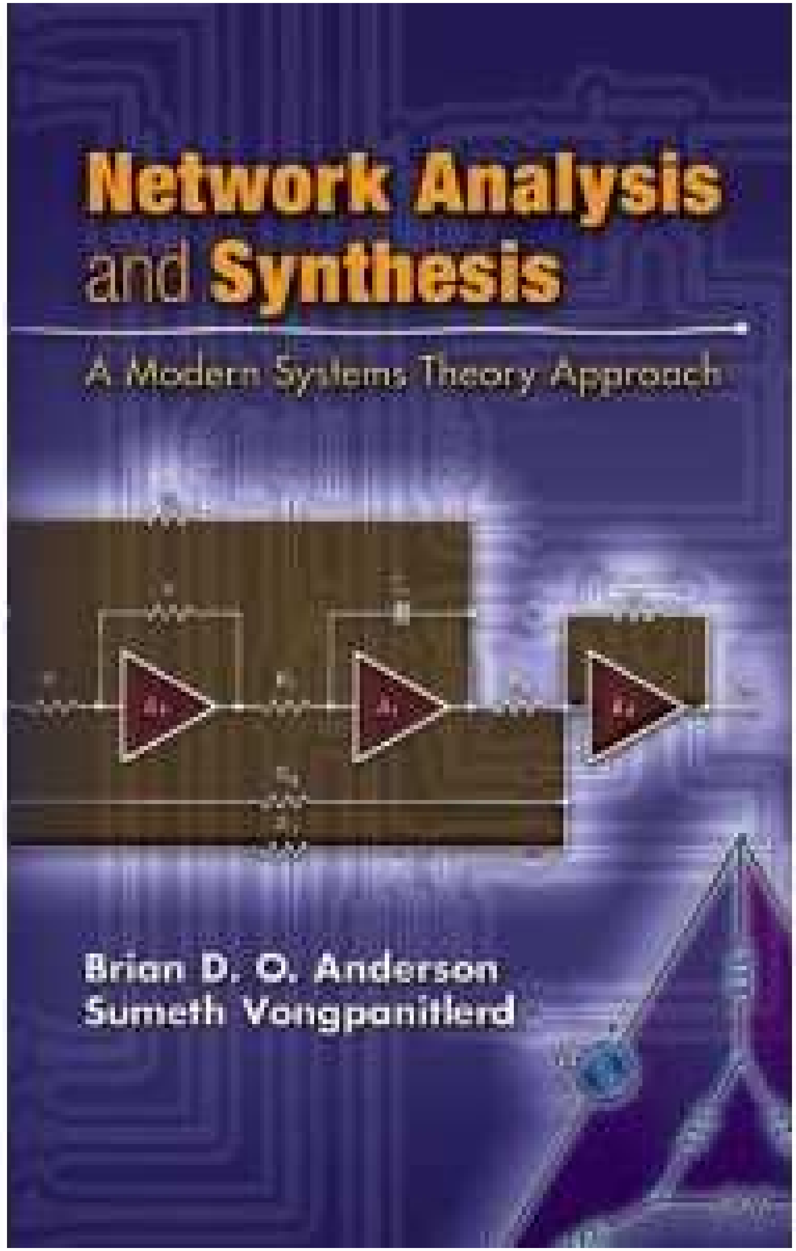
In honor of Brian Anderson
on the occasion of his seventieth birthday.

Circuits: Brian’s first love

Refereed Journal Papers

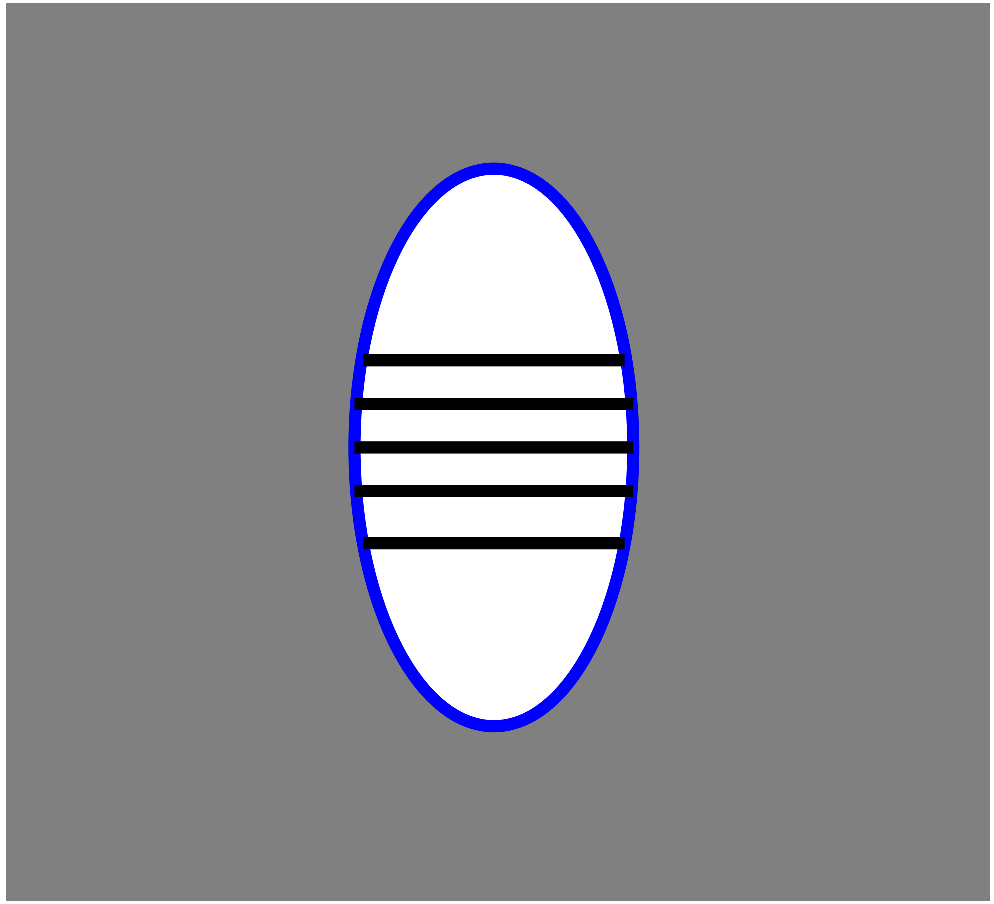
1965

- B. D. O. Anderson, D. A. Spaulding, and R. W. Newcomb. Useful time-variable circuit element equivalences. *Electronics Letters*, **1** (3): 56–57, May 1965.
- B. D. O. Anderson, D. A. Spaulding, and R. W. Newcomb. The time-variable transformer. *Proc. IEEE*, **53** (6): 634–635, June 1965.
- B. D. O. Anderson and R. W. Newcomb. On relations between series and shunt-augmented networks. *Proc. IEEE*, **53** (7): 725, July 1965.
- B. D. O. Anderson. Proof of the Manley-Rowe relations from quantum considerations. *Electronics Letters*, **1** (7): 199, September 1965.
- B. D. O. Anderson and R. W. Newcomb. On reciprocity and time-variable networks. *Proc. IEEE*, **53** (10): 1674, October 1965.
- B. D. O. Anderson and R. W. Newcomb. A capacitor-transformer gyrator realization. *Proc. IEEE*, **53** (10): 1640, October 1965.


Network Analysis
and Synthesis
A Modern Systems Theory Approach
Brian D. O. Anderson
Sumeth Vongpanitlerd

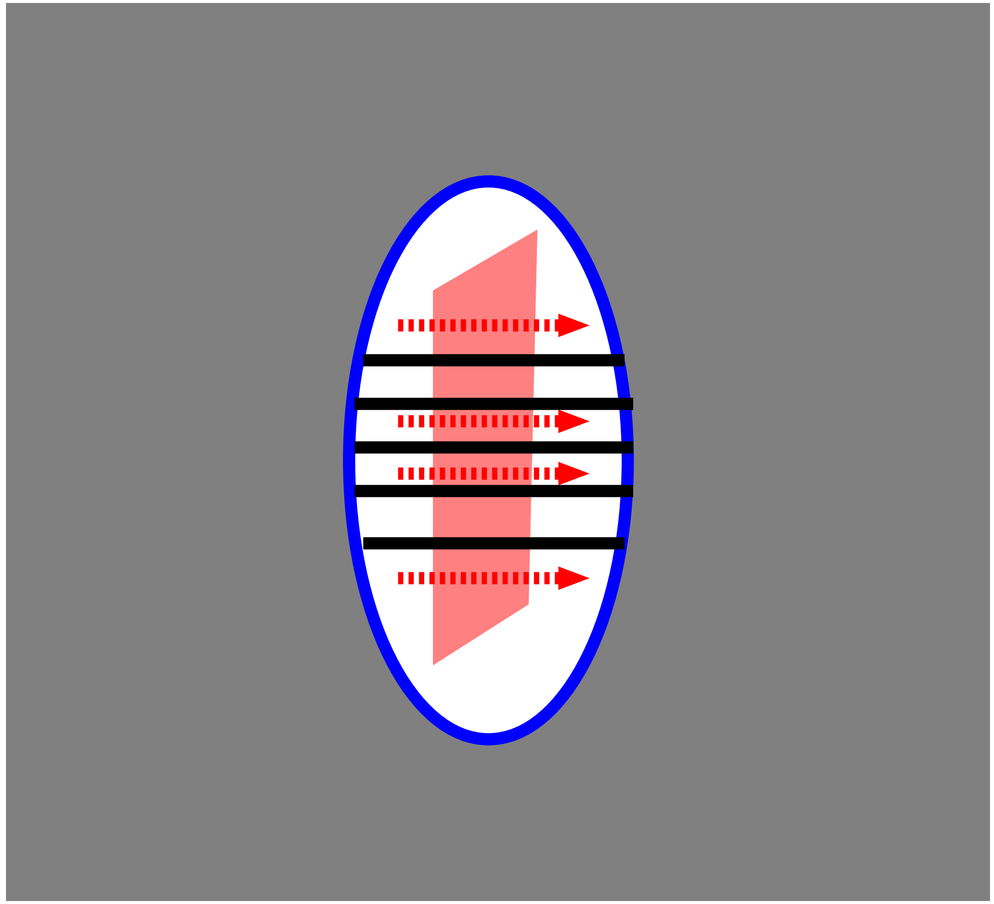
Theme

Electrical energy transfer



Observe the electrical variables on a set of wires.

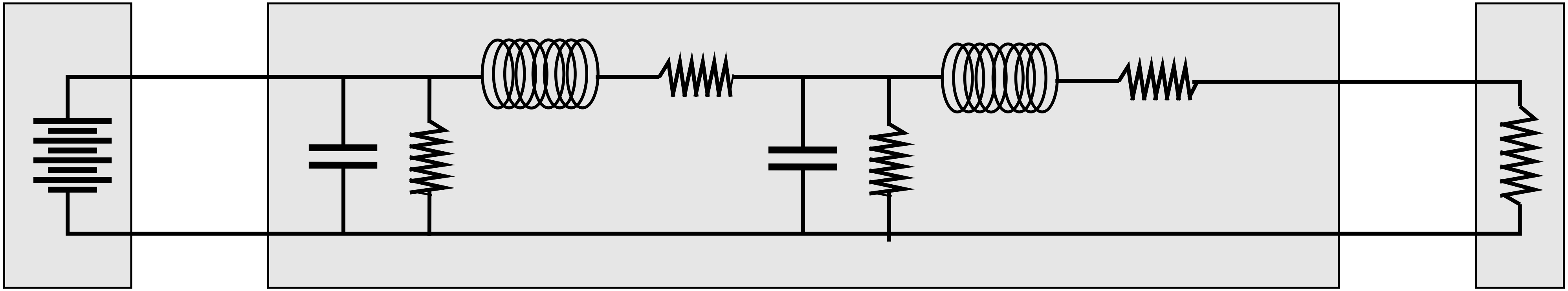
Electrical energy transfer



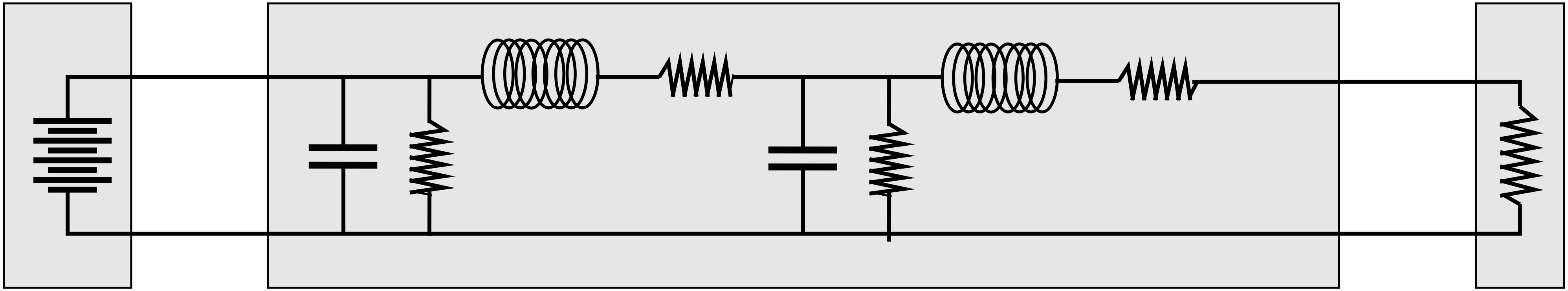
Observe the electrical variables on a set of wires.

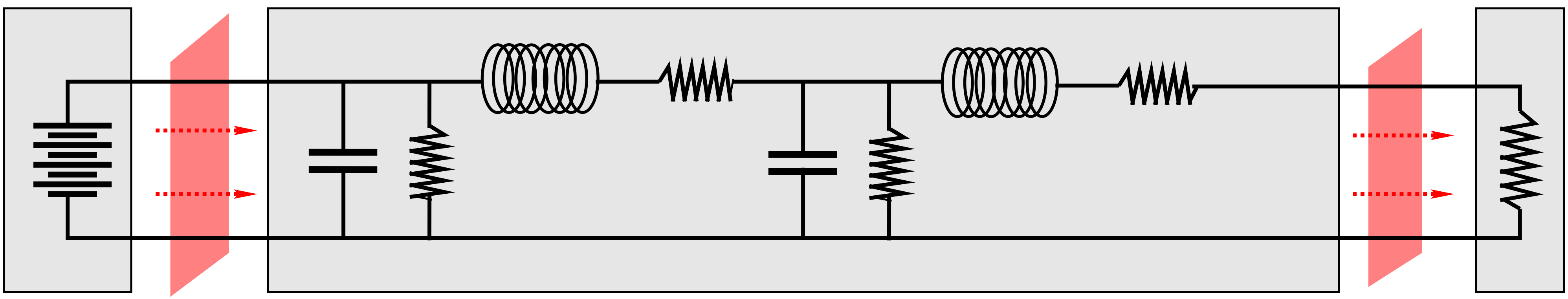
How much energy is transferred?

Examples

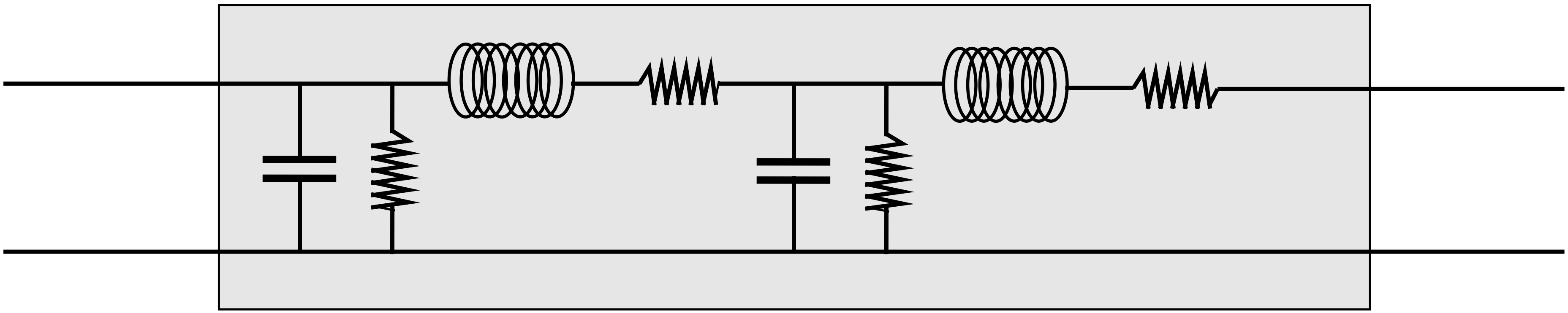


Examples

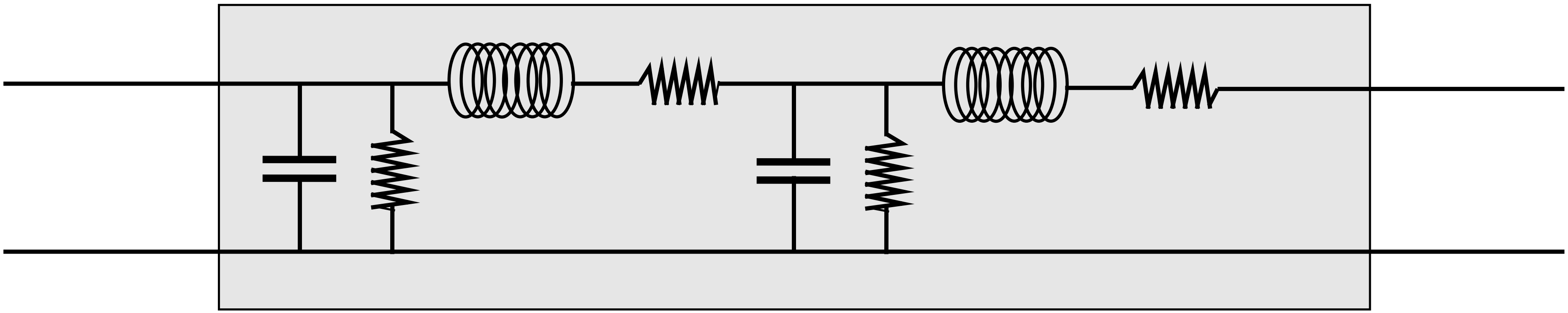


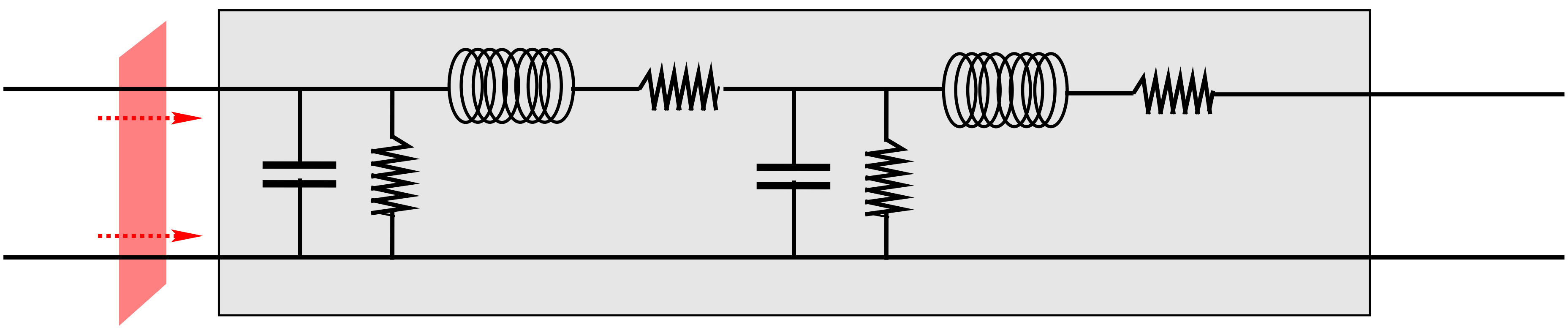


Examples

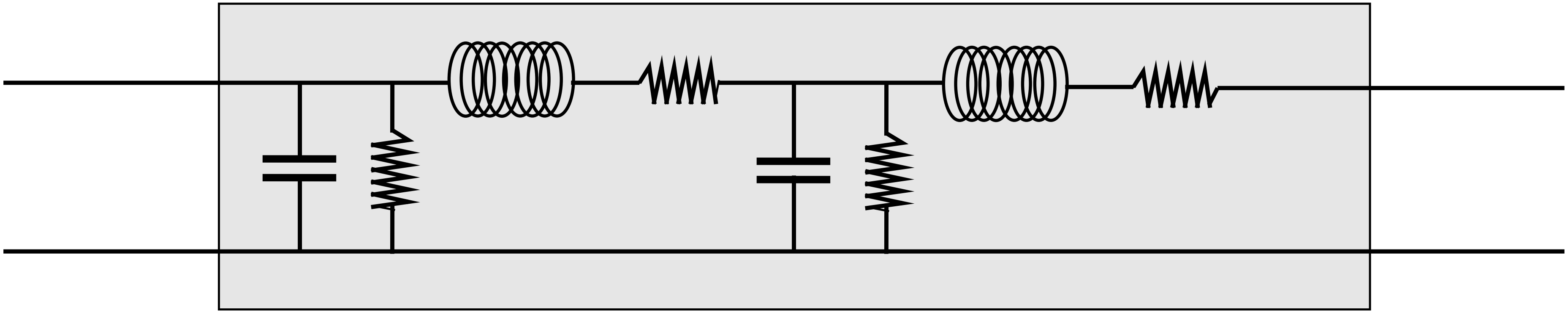


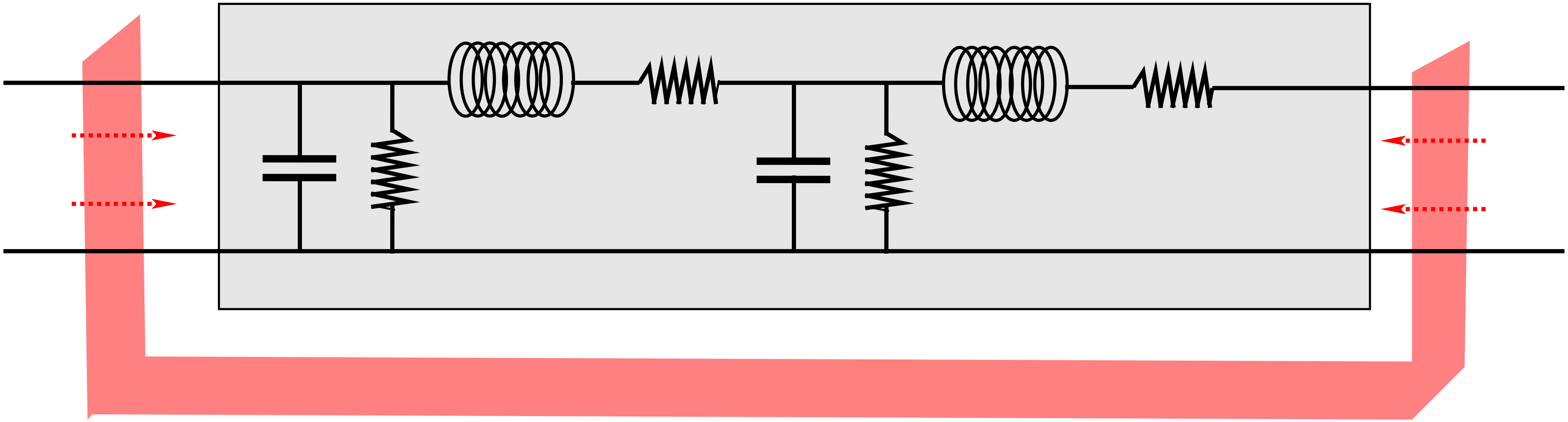
Examples



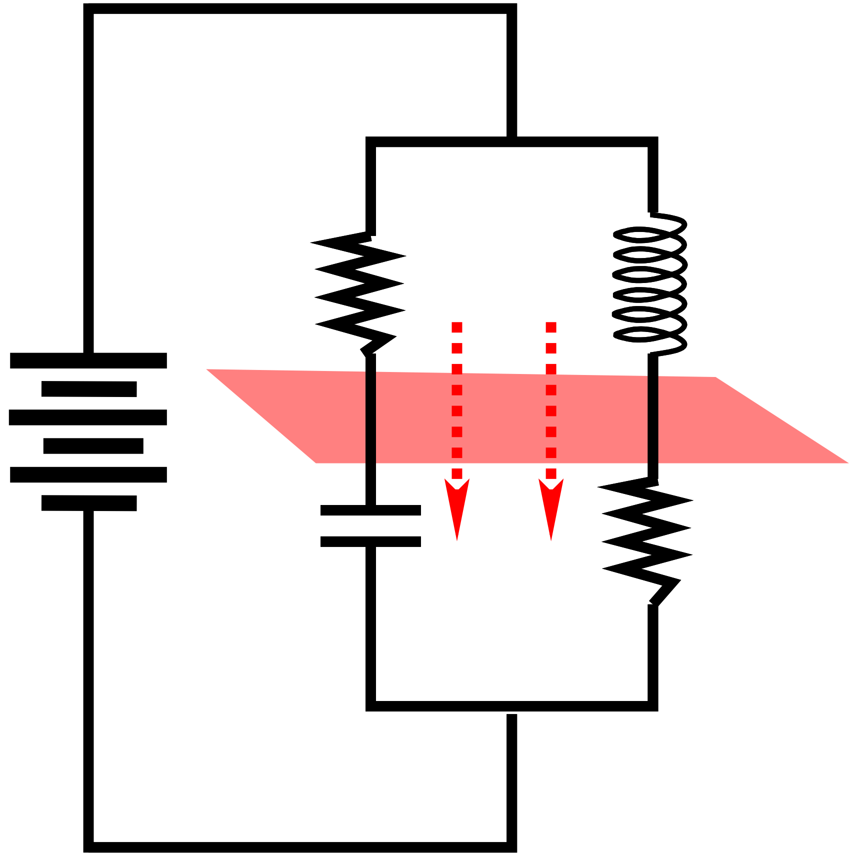


Examples





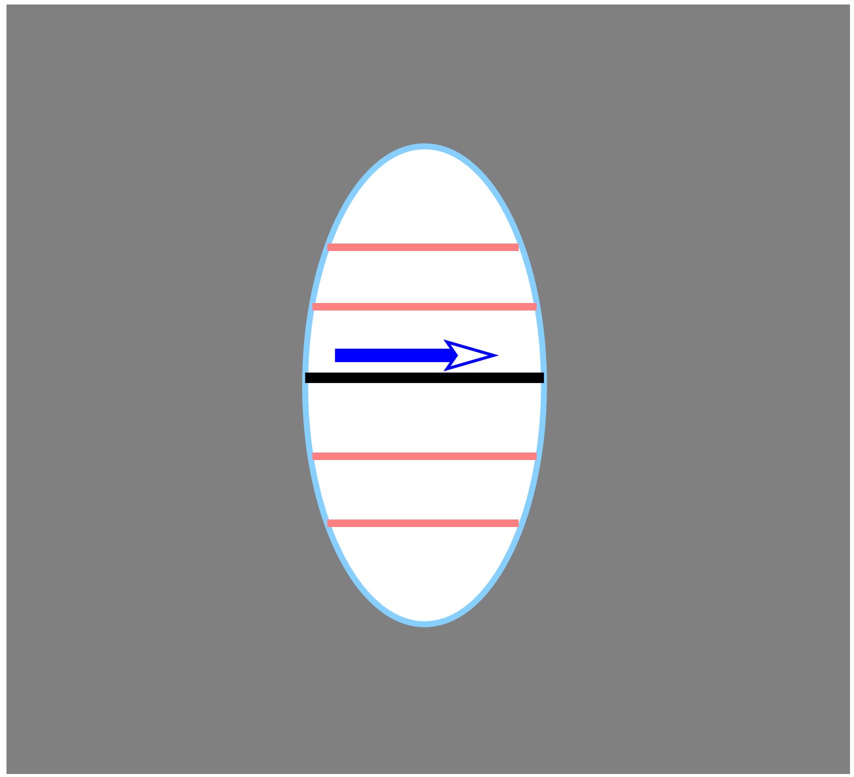
Examples



Electrical terminal variables

Currents and voltages

Assume no EM fields outside the wires.

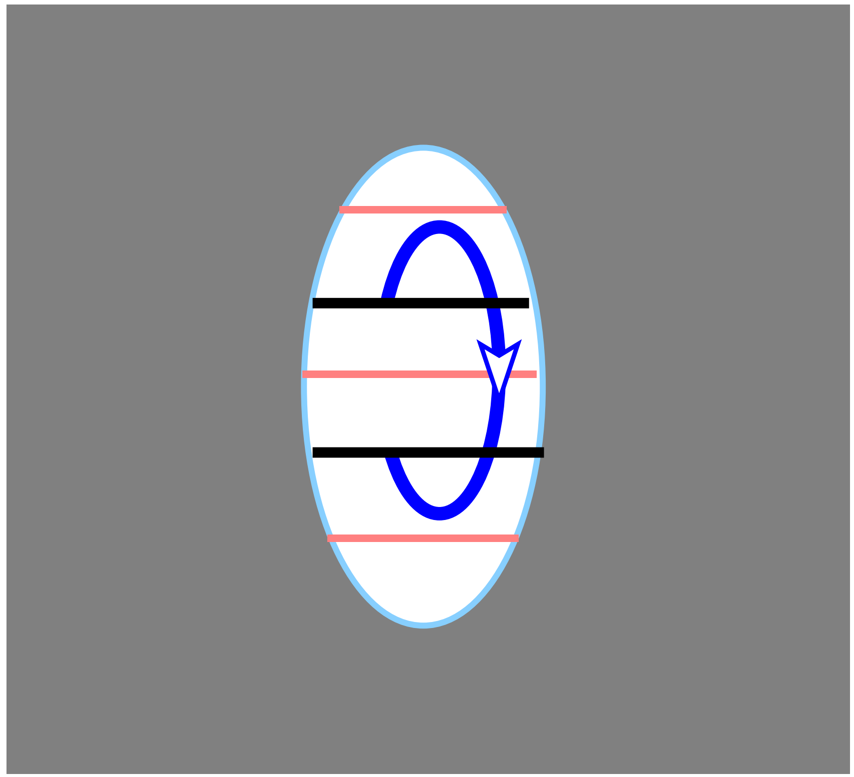


Through each wire, a current.

Measurable by ammeters.

Currents and voltages

Assume no EM fields outside the wires.



Across each pair of wires, a voltage.

Measurable by voltmeters.

Currents and voltages

Assume no EM fields outside the wires.

$$\rightsquigarrow \qquad I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \quad V = \begin{bmatrix} V_{1,1} & V_{1,2} & \cdots & V_{1,N} \\ V_{2,1} & V_{2,2} & \cdots & V_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N,1} & V_{N,2} & \cdots & V_{N,N} \end{bmatrix}.$$

Assume (I,V) given on a time interval $[t_0,t_1]$.

$$\rightsquigarrow \quad I : [t_0,t_1] \longrightarrow \mathbb{R}^N, \quad V : [t_0,t_1] \longrightarrow \mathbb{R}^{N\times N}.$$

How much energy is transferred?

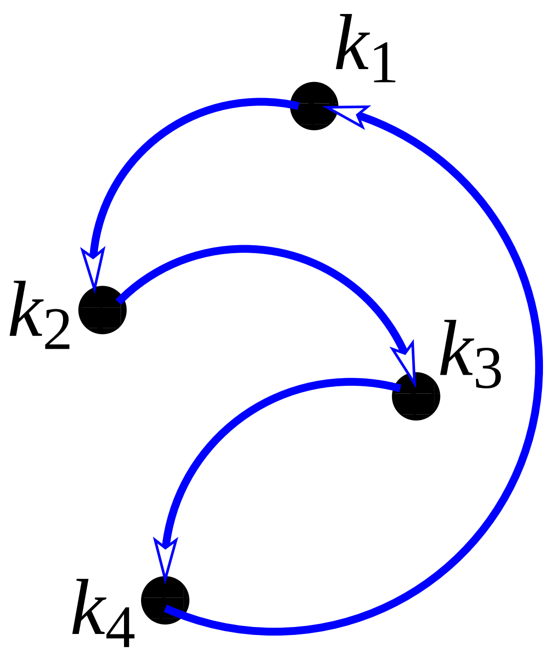
KVL & KCL

Kirchhoff's voltage law

KVL:

$$V_{k_1,k_2} + V_{k_2,k_3} + V_{k_3,k_4} + \cdots + V_{k_{n-1},k_n} + V_{k_n,k_1} = 0$$

for all $k_1, k_2, \ldots, k_n \in \{1, 2, \ldots, N\}$.



Physically, KVL is evident.

We henceforth assume it.

Potentials

<u>Thm</u>: $V : [t_0, t_1] \to \mathbb{R}^{N \times N}$ **satisfies KVL** $\Leftrightarrow$

$$\exists\, P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} : [t_0, t_1] \to \mathbb{R}^N \textbf{ such that } V_{k_1,k_2} = P_{k_1} - P_{k_2}.$$

Potentials

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$$P \textbf{ `potential'} \Rightarrow \begin{bmatrix} P_1 + \alpha \\ P_2 + \alpha \\ \vdots \\ P_N + \alpha \end{bmatrix} \textbf{ potential } \forall\, \alpha : [t_0, t_1] \longrightarrow \mathbb{R}.$$

Kirchhoff's current law

KCL: $$I_1 + I_2 + \cdots + I_N = 0.$$

Kirchhoff’s current law

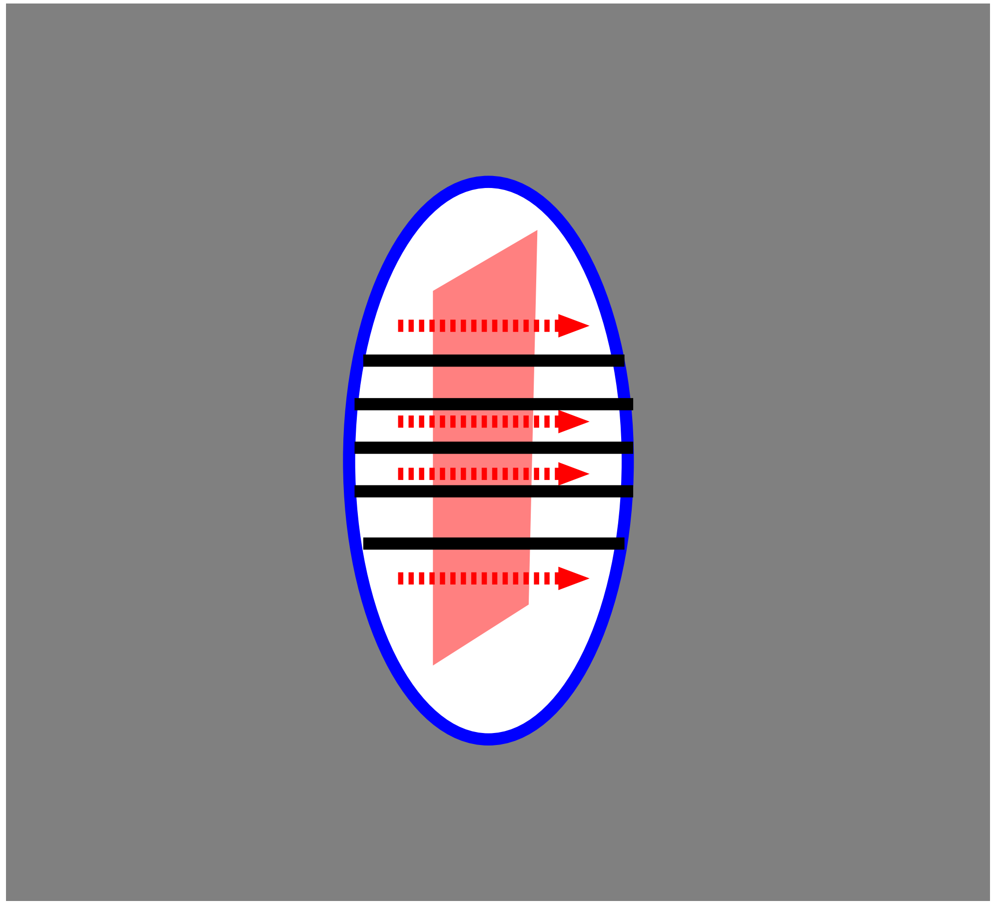
KCL: $$I_1 + I_2 + \cdots + I_N = 0.$$

KCL is a genuine restriction.

Often a consequence of the circuit architecture.
For example, the external terminals of an RLC circuit satisfy KCL.

Energy transfer

Energy



Observe $(I,V):[t_0,t_1]\to\mathbb{R}^N\times\mathbb{R}^{N\times N}$ **on a set of wires.**

How much energy is transferred?

Energy

Assume that KCL **(& KVL)** **holds. Then**

power across = $I_1(t)P_1(t) + \cdots + I_N(t)P_N(t),$

energy transmitted $=$ $$\boxed{\int_{t_1}^{t_2} [I_1(t)P_1(t) + \cdots + I_N(t)P_N(t)]\, dt.}$$

This interpretation in terms of energy is not valid

unless KCL is satisfied !

Energy

Assume that KCL (& KVL) holds. Then

energy transmitted $= \int_{t_1}^{t_2} [I_1(t)P_1(t) + \cdots + I_N(t)P_N(t)]\, dt.$

In terms of currents and voltages,

$$\int_{t_1}^{t_2} \left[I_1(t)\widehat{V_1}(t) + \cdots + I_N(t)\widehat{V_N}(t) \right] dt,$$

with

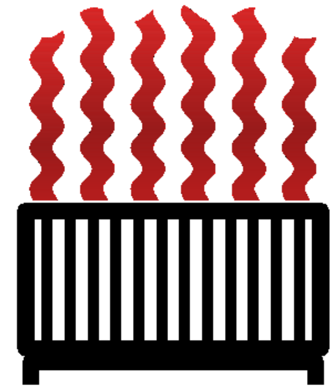
$$\widehat{V_k} := \frac{V_{k,1} + V_{k,2} + \cdots + V_{k,N}}{N}$$

Justification

Energy into heat

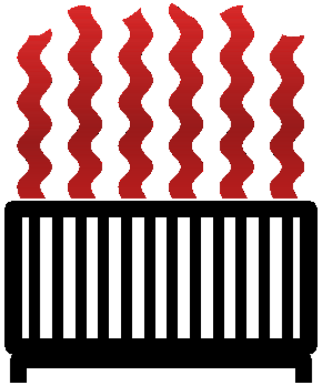
Energy :=

a physical quantity transformable into heat.



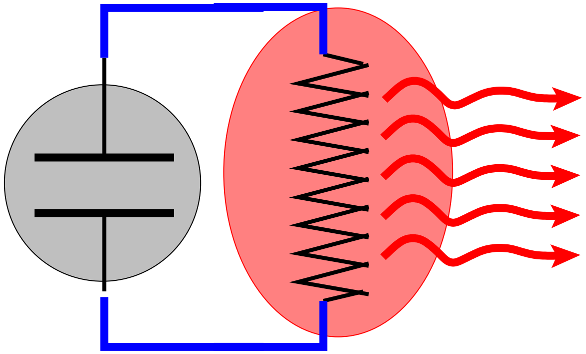
Energy **:=**

a physical quantity transformable into heat.

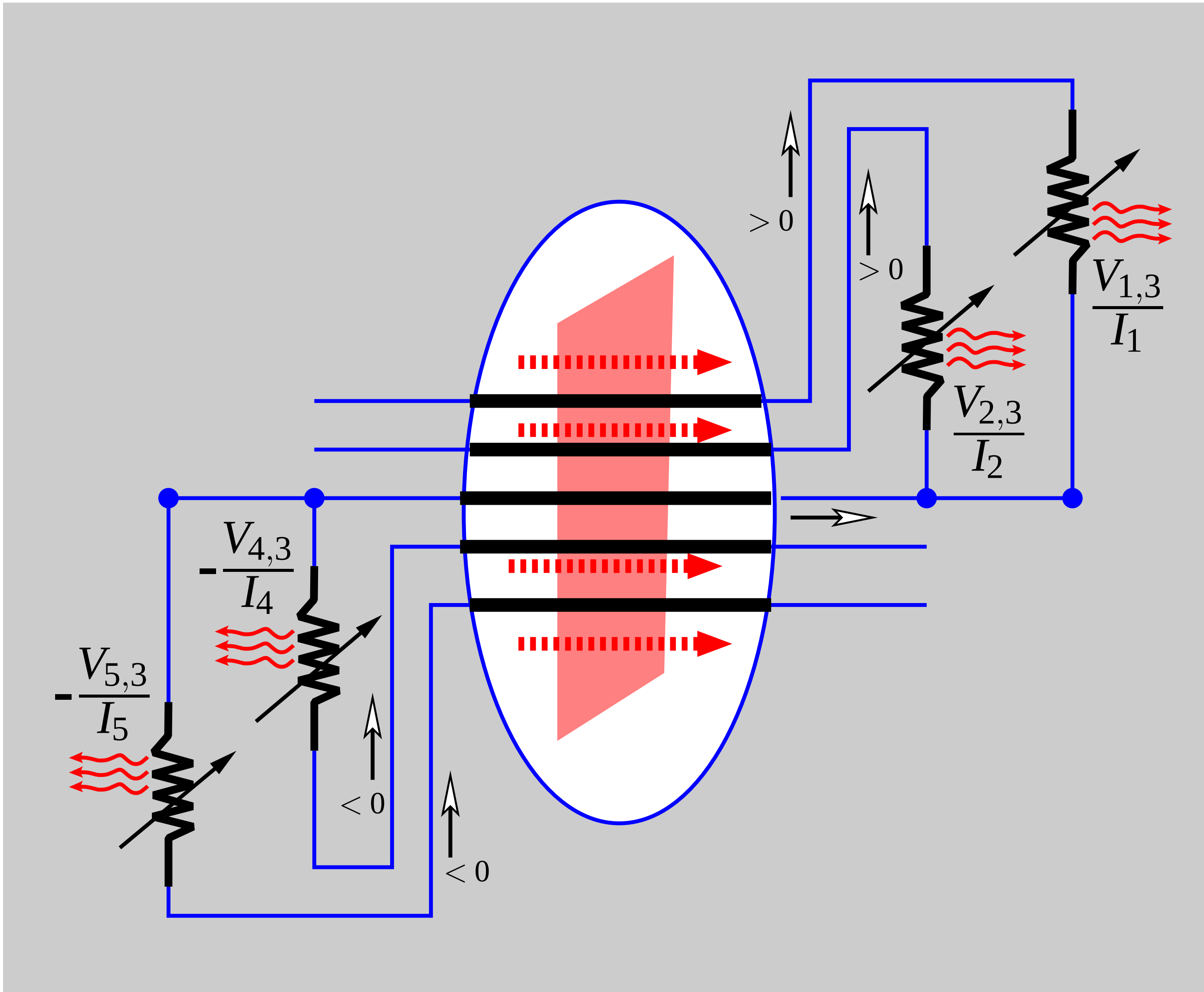


For example, capacitor $\mapsto$ resistor $\mapsto$ heat.

Energy on capacitor = $\frac{1}{2}CV^2$

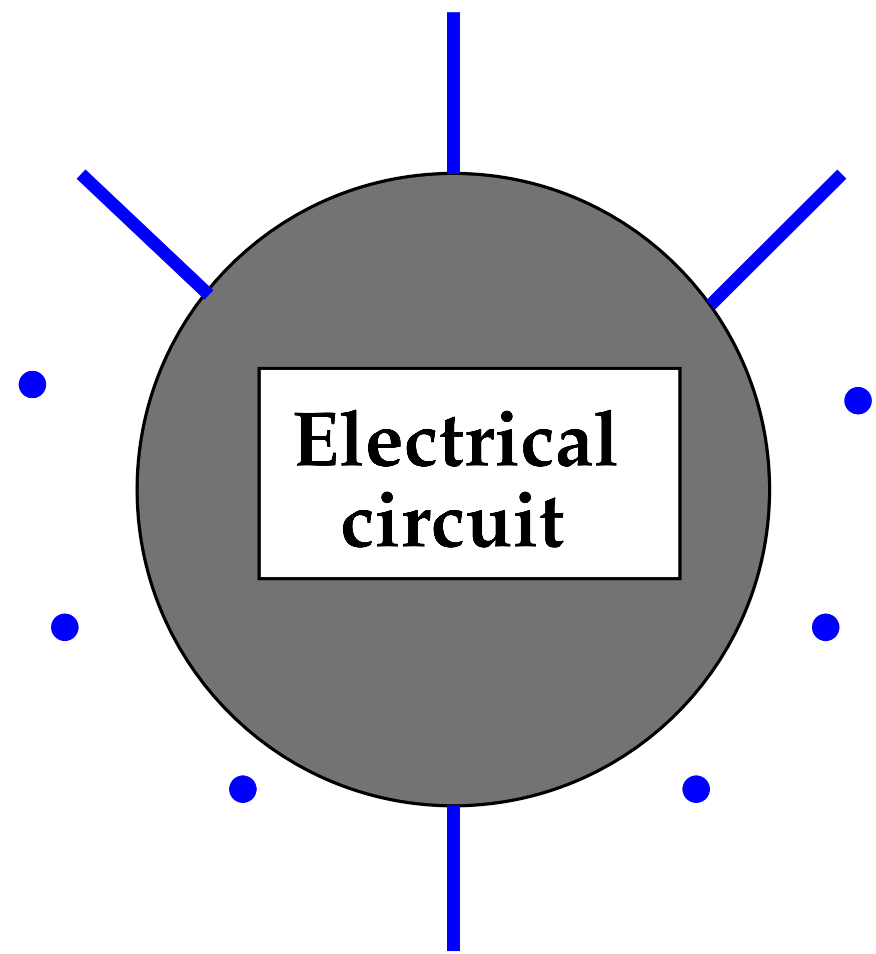


Currents into heat

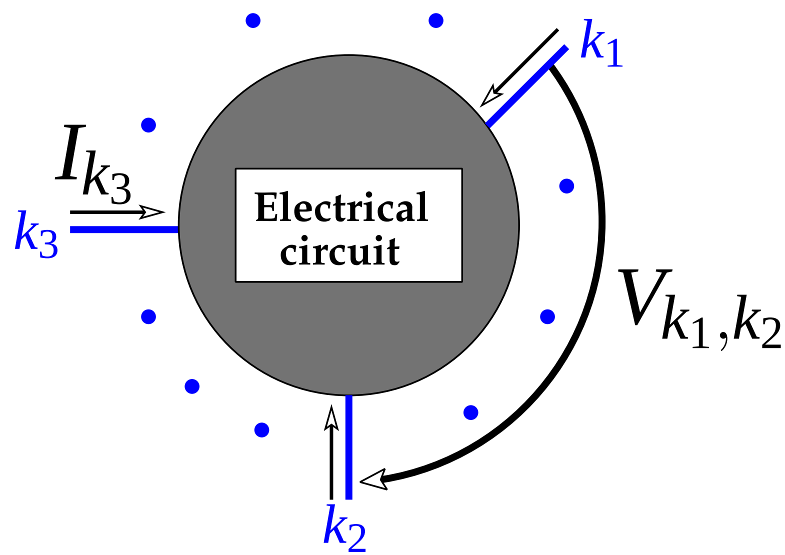


Terminal behavior

A circuit with external terminals



Currents and voltages



$$\rightsquigarrow \qquad I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}, \quad V = \begin{bmatrix} V_{1,1} & V_{1,2} & \cdots & V_{1,N} \\ V_{2,1} & V_{2,2} & \cdots & V_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N,1} & V_{N,2} & \cdots & V_{N,N} \end{bmatrix}.$$

Currents and voltages

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$(I,V) \in \mathscr{B}_{IV}$ **means**

$$\left(I_1, I_2, \ldots, I_k, \ldots, I_N, V_{1,1}, V_{1,2}, \ldots, V_{k_1,k_2}, \ldots, V_{N,N}\right) : \mathbb{R} \longrightarrow \mathbb{R}^N \times \mathbb{R}^{N \times N}$$

is compatible with circuit architecture and its element values.

These trajectories $(I,V) \in \mathscr{B}_{IV}$ **can conceivably occur.**

KVL & KCL

Kirchhoff voltage law:

$$[\![(I,V) \in \mathscr{B}_{IV}]\!]$$

$$\Rightarrow \quad [\![V_{k_1,k_2} + V_{k_2,k_3} + V_{k_3,k_4} + \cdots + V_{k_{n-1},k_n} + V_{k_n,k_1} = 0$$

for all $k_1, k_2, \ldots, k_n \in \{1, 2, \ldots, N\}]\!]$.

KVL leads to potentials $P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix}$ **and** $\mathscr{B}_{IP}$**.**

KVL & KCL

Kirchhoff voltage law:

$$[\![(I,V)\in\mathscr{B}_{IV}]\!]$$

$$\Rightarrow\ [\![V_{k_1,k_2}+V_{k_2,k_3}+V_{k_3,k_4}+\cdots+V_{k_{n-1},k_n}+V_{k_n,k_1}=0$$

for all $k_1,k_2,\ldots,k_n\in\{1,2,\ldots,N\}]\!].$

KVL leads to potentials and $\mathscr{B}_{IP}$**.**

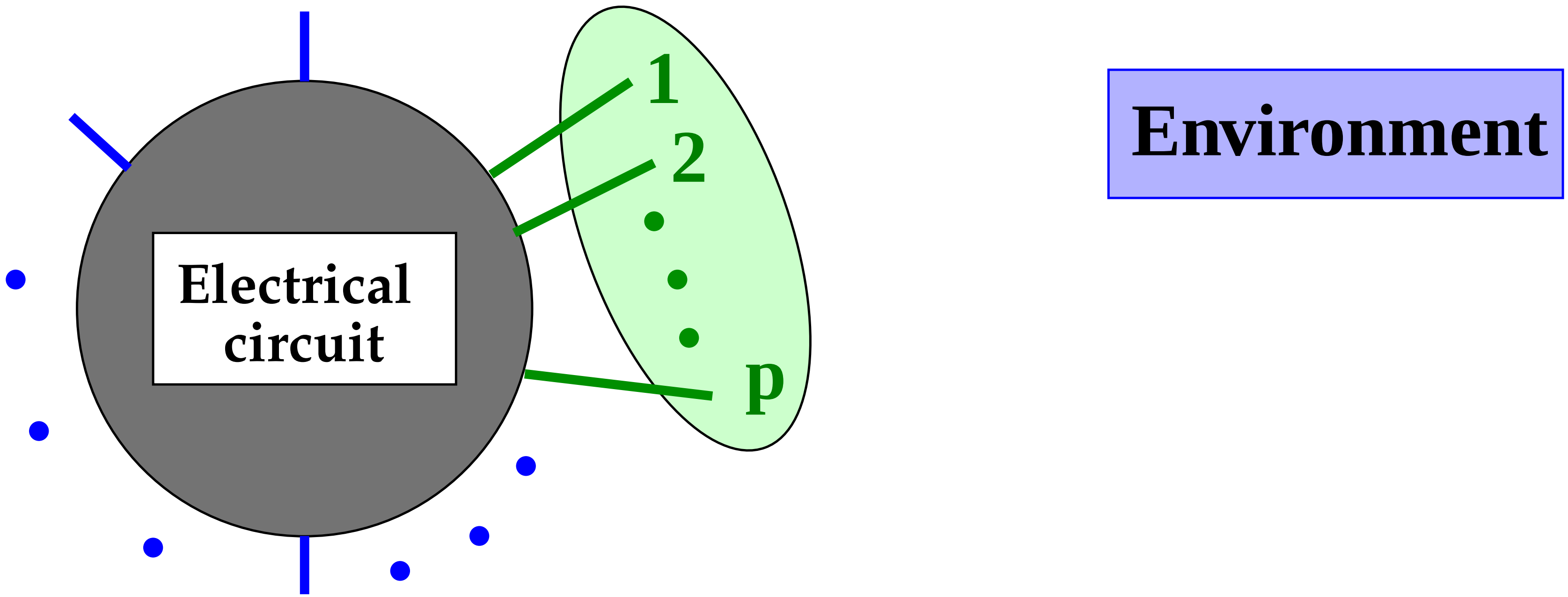
Kirchhoff current law:

$$[\![(I_1,I_2,\ldots,I_N,V_{1,1},V_{1,2},\ldots,V_{k_1,k_2},\ldots,V_{N,N})\in\mathscr{B}_{IV}]\!]$$

$$\Rightarrow[\![I_1+I_2+\cdots+I_N=0]\!].$$

Ports

Energy transfer

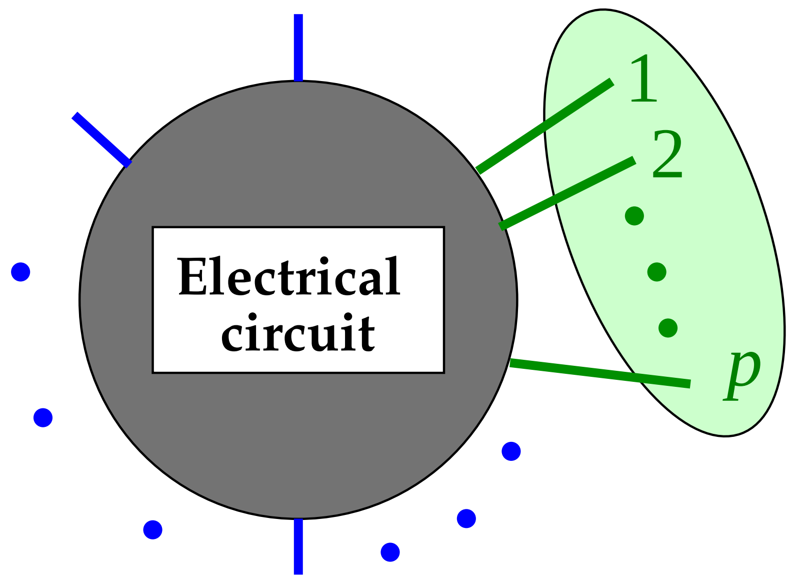


Can we speak about

the energy transferred from the environment to the circuit along these terminals?

Electrical ports

Assume KVL.



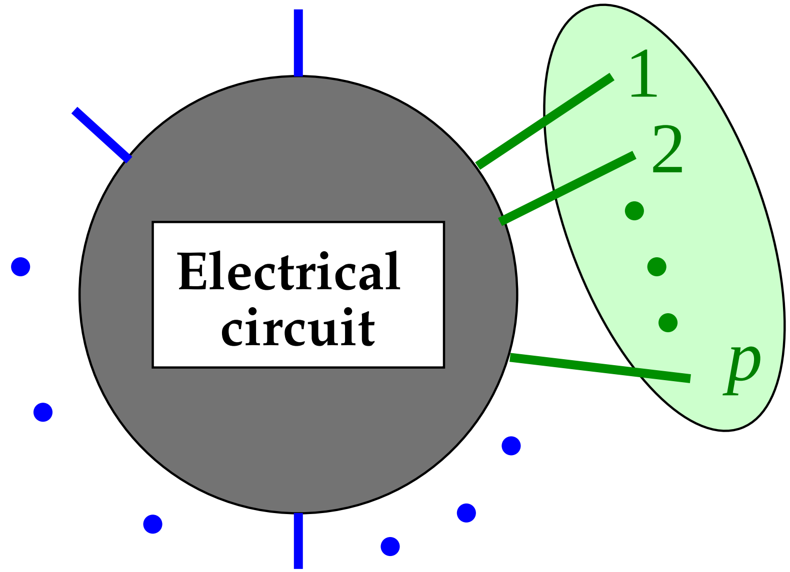
Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

$$[\![\left(I_1, \ldots, I_p, I_{p+1}, \ldots, I_N, V_{1,1}, \ldots, V_{k_1,k_2}, \ldots, V_{N,N}\right) \in \mathscr{B}_{IV}]\!]$$

$$\Rightarrow \quad [\![I_1 + I_2 + \cdots + I_p = 0]\!].$$

'port KCL'

Energy



If terminals $\{1,2,\ldots,p\}$ form a port, then

power in = $I_1(t)P_1(t)+\cdots+I_p(t)P_p(t)$

energy in = $\int_{t_1}^{t_2}\left[I_1(t)P_1(t)+\cdots+I_p(t)P_p(t)\right]dt$

This interpretation in terms of power and energy is not valid
unless these terminals form a port !

Internal ports

Analogous definition for internal terminals

$\rightsquigarrow$ **internal ports,**

combinations of external and internal terminals

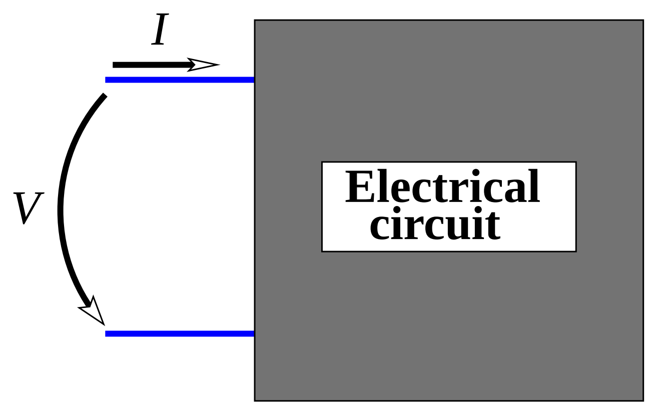
$\rightsquigarrow$ **mixed ports.**

Examples

2-terminal 1-port devices:

resistors, inductors, capacitors, memristors, etc.,

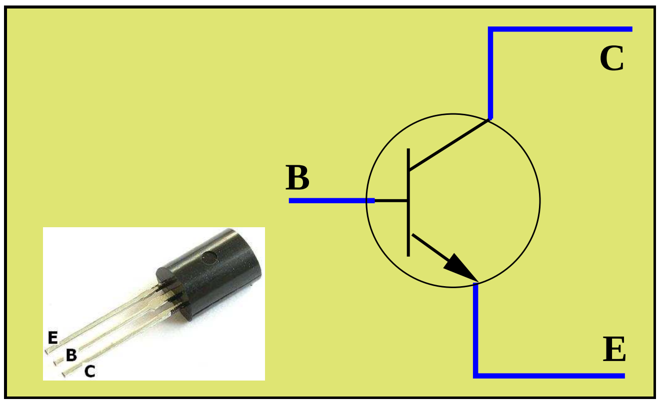
any 2-terminal circuit composed of these.

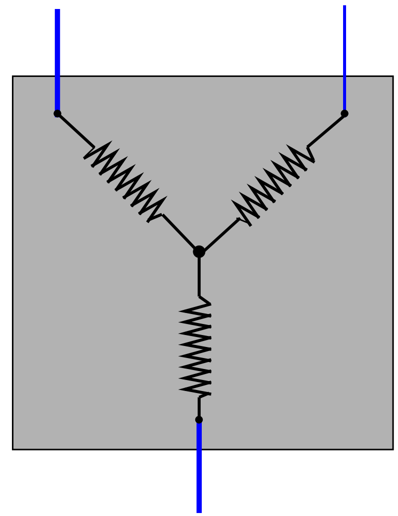


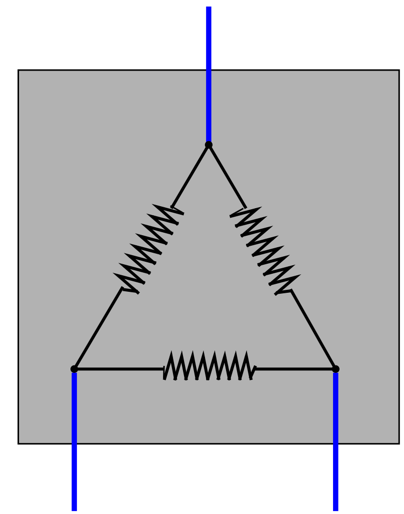
KVL $\Rightarrow$ **only** $V_{1,2} := V$ **matters,**

KCL $\Rightarrow$ $I_1 = -I_2 =: I$**.**

3-terminal circuits



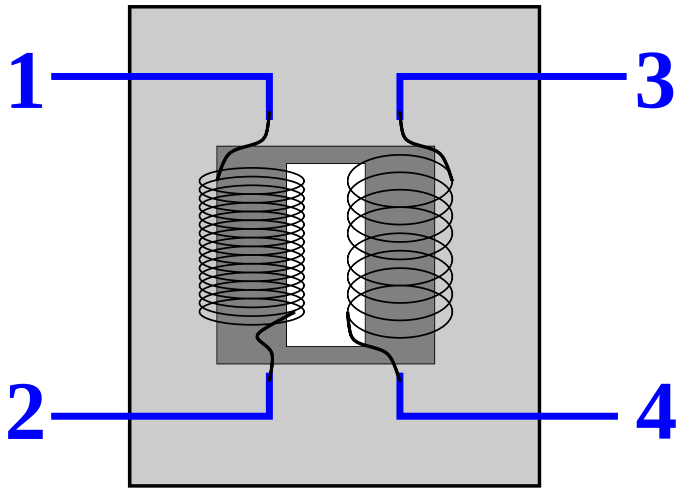




3-terminal 1-ports.

Transformer

A transformer:

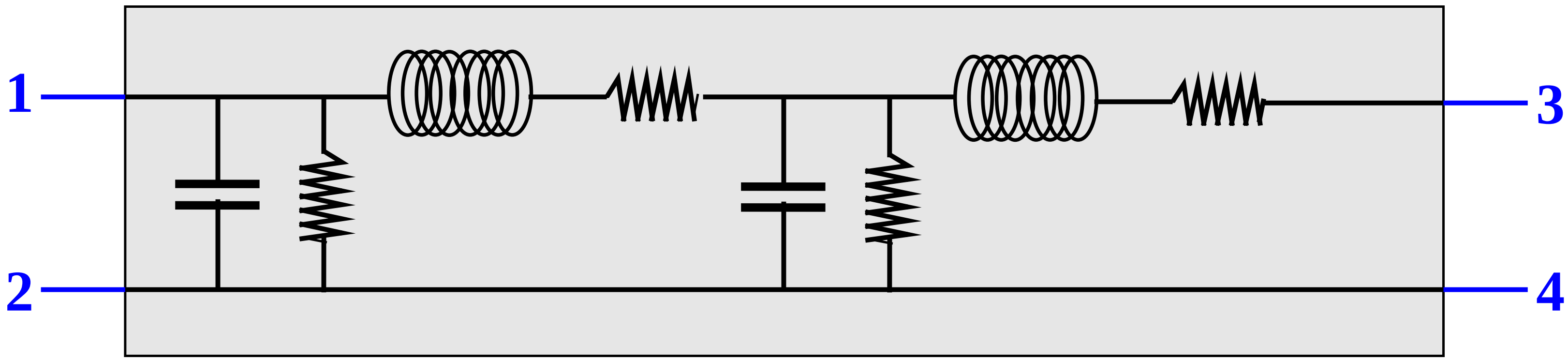


$$P_3 - P_4 = n\,(P_1 - P_2),$$
$$I_1 = -\,n\,I_3,$$
$$I_1 + I_2 = 0,\ I_3 + I_4 = 0.$$

$\{1,2\}$ and $\{3,4\}$ form ports.

A transformer = a 2-port with two 2-terminal ports.

Transmission line



Terminals $\{1,2,3,4\}$ **form a port;**

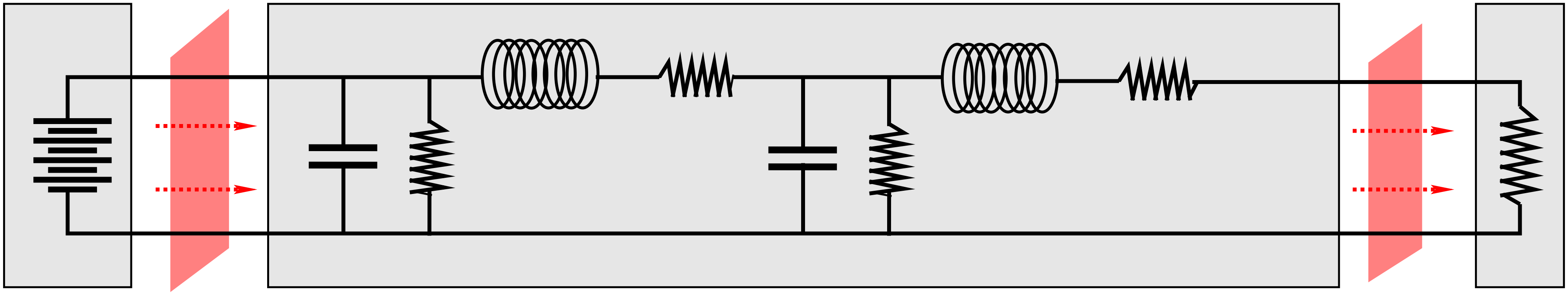
$\{1,2\}$ **and** $\{3,4\}$ **do not.**

We cannot speak about

"the energy transferred from terminals $\{1,2\}$ ***to*** $\{3,4\}$***"***,

or ***"from the environment to the circuit through*** $\{1,2\}$***"***.

Transmission line

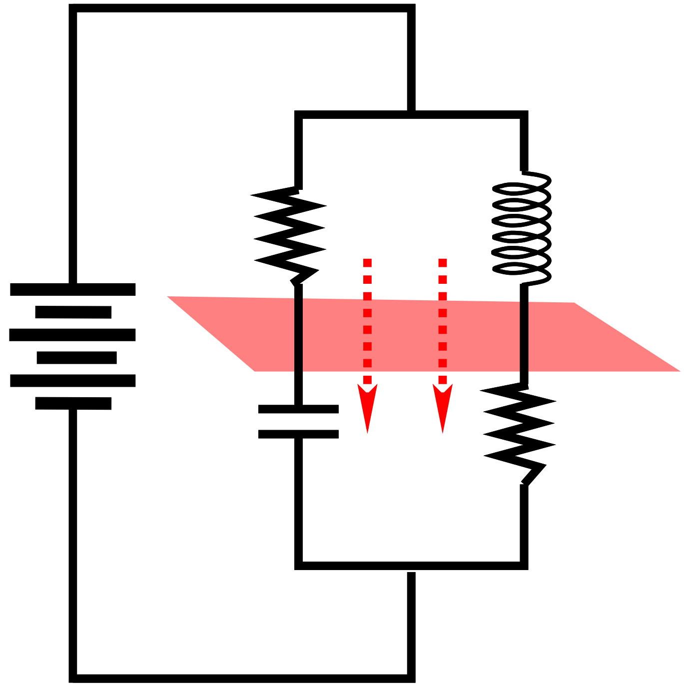


The energy flows from the source and to the load are well-defined, since the terminals form internal ports.

Therefore we can speak about

“the energy transferred from the source to the load”.

RLC circuit



Not an internal port: energy flow not well-defined.

Passivity

Definition of passivity

Assume KCL & KVL.

An N-terminal circuit said to be passive $:\Leftrightarrow$

$\forall\,(I,P) \in \mathscr{B}_{IP}, \forall\, t_0 \in \mathbb{R},\ \ \exists\, K \in \mathbb{R}$ **such that**

$$\int_{t_0}^{t_1} [I_1(t)P_1(t) + \cdots + I_N(t)P_N(t)]\, dt < K \qquad \textbf{for } t_1 \geq t_0.$$

Definition of passivity

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$\exists$ **an equivalent definition in terms of storages.**

For linear time-invariant differential circuits

$\leadsto$ **positive realness, LMI's, etc.**

Two theorems

Are ports common?

Theorem: Consider an electrical circuit consisting of an interconnection of (linear passive) R's, L's, C's.

Assume that every pair of terminals is connected by the circuit graph. Then

the only port is the one that consists of all the terminals.

Are ports common?

Theorem: Consider an electrical circuit consisting of an interconnection of (linear passive) R's, L's, C's.

Assume that every pair of terminals is connected by the circuit graph. Then

the only port is the one that consists of all the terminals.

For non-trivial ports, we need multi-port elements, as transformers or gyrators.

Port KVL

$$[\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1,\ldots,P_p,P_{p+1},\ldots,P_N)\in\mathscr{B}_{IP},\alpha:\mathbb{R}\to\mathbb{R}]\!]$$

$$\Rightarrow\ [\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1+\alpha,\ldots,P_p+\alpha,P_{p+1},\ldots,P_N)\in\mathscr{B}_{IP}]\!].$$

'port KVL'

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'port KVL'

Port KVL $\Leftrightarrow$

only $V_{k,\ell}$ **for** $k,\ell=1,2,\ldots,p$
and $V_{k',\ell'}$ **for** $k',\ell'=p+1,p+2,\ldots,N$
enter in the behavioral equations.

Port KVL

$$[\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1,\ldots,P_p,P_{p+1},\ldots,P_N)\in\mathscr{B}_{IP},\alpha:\mathbb{R}\to\mathbb{R}]\!]$$

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'port KVL'

Port KVL $\Leftrightarrow$

$$[\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1,\ldots,P_p,P_{p+1},\ldots,P_N)\in\mathscr{B}_{IP},\alpha,\beta:\mathbb{R}\to\mathbb{R}]\!]$$

$$\Rightarrow\ [\![\ (I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1+\alpha,\ldots,P_p+\alpha,P_{p+1}+\beta,\ldots,P_N+\beta)\in\mathscr{B}_{IP}\ ]\!].$$

Port KVL

$$[\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1,\ldots,P_p,P_{p+1},\ldots,P_N)\in\mathscr{B}_{IP},\alpha:\mathbb{R}\to\mathbb{R}]\!]$$

$$\Rightarrow\ [\![(I_1,\ldots,I_p,I_{p+1},\ldots,I_N,P_1+\alpha,\ldots,P_p+\alpha,P_{p+1},\ldots,P_N)\in\mathscr{B}_{IP}]\!].$$

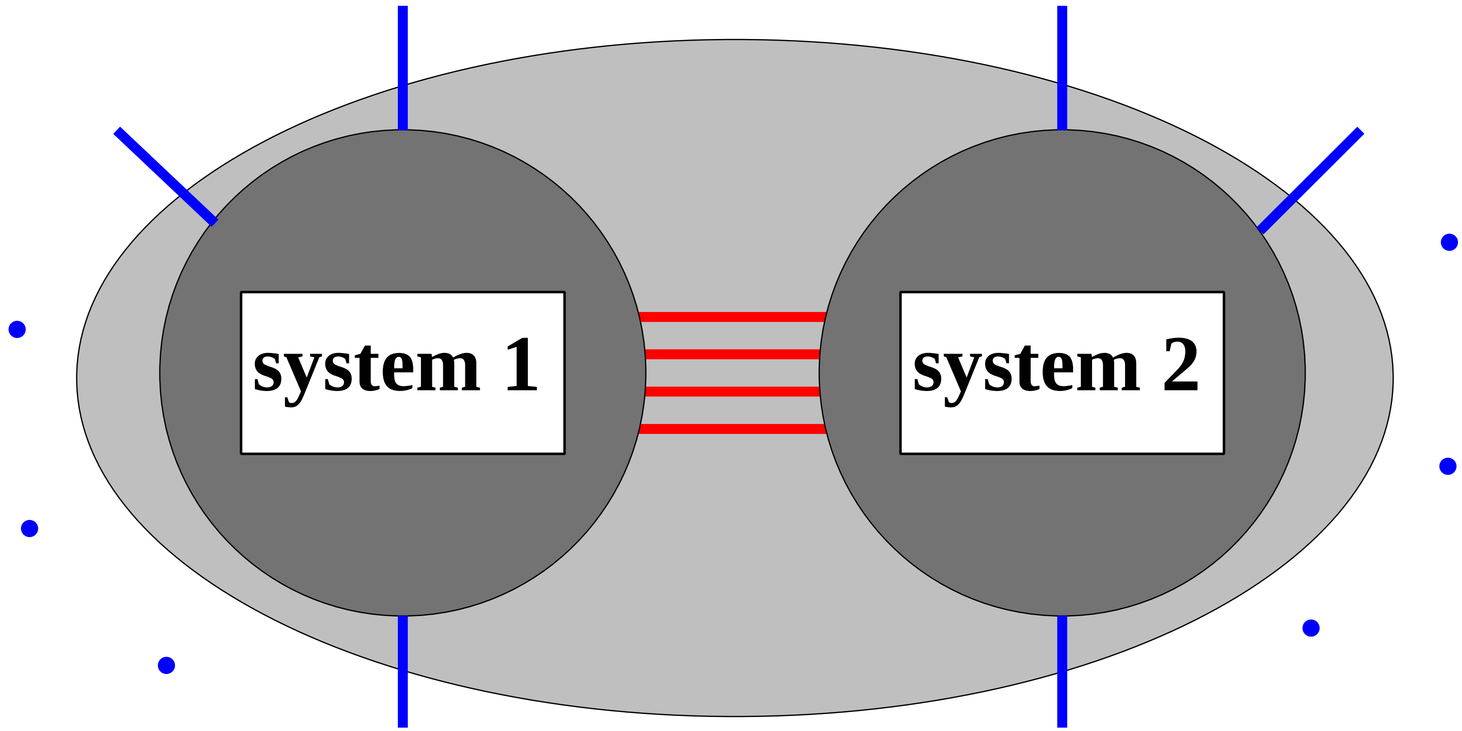
'port KVL'

Theorem: For linear passive controllable circuits

port KVL $\Leftrightarrow$ port KCL.

Conclusions

Energy transfer



One cannot speak about

"*the energy transferred from system 1 to system 2*"

or "*from the environment to system 1*",

unless the relevant terminals form a port.

Interconnection versus energy transfer

Terminals are for interconnection.

Ports are for energy transfer.

A ‘port’ is a set of terminals with a special property (port KCL).

Energy is NOT an ‘extensive’ quantity

Reference: ***Terminals and Ports*****, Circuits and Systems Magazine, vol. 10, issue 4, pages 8-16, Dec. 2010.**

Copies of the lecture frames available from/at

`http://www.esat.kuleuven.be/∼jwillems`

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

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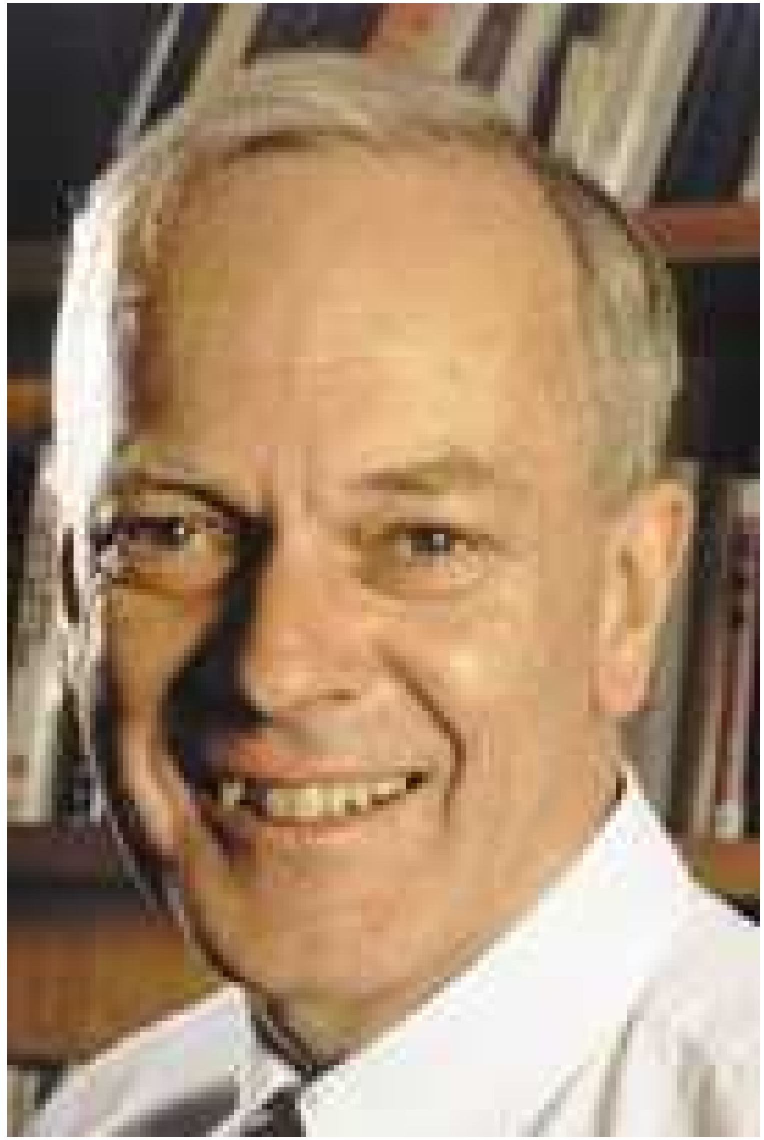
Thank you



Happy birthday, Brian!   Ad multos annos!