SYSTEM INTERCONNECTION

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Outline

- Open and connected
- Mathematical models, dynamical systems
- Latent variables
- Modeling by tearing, zooming, and linking
- Hierarchical features
- Terminals versus ports
- Passivity

Theme

Features of modern engineering systems

- open
- interconnected
- modular
- dynamic

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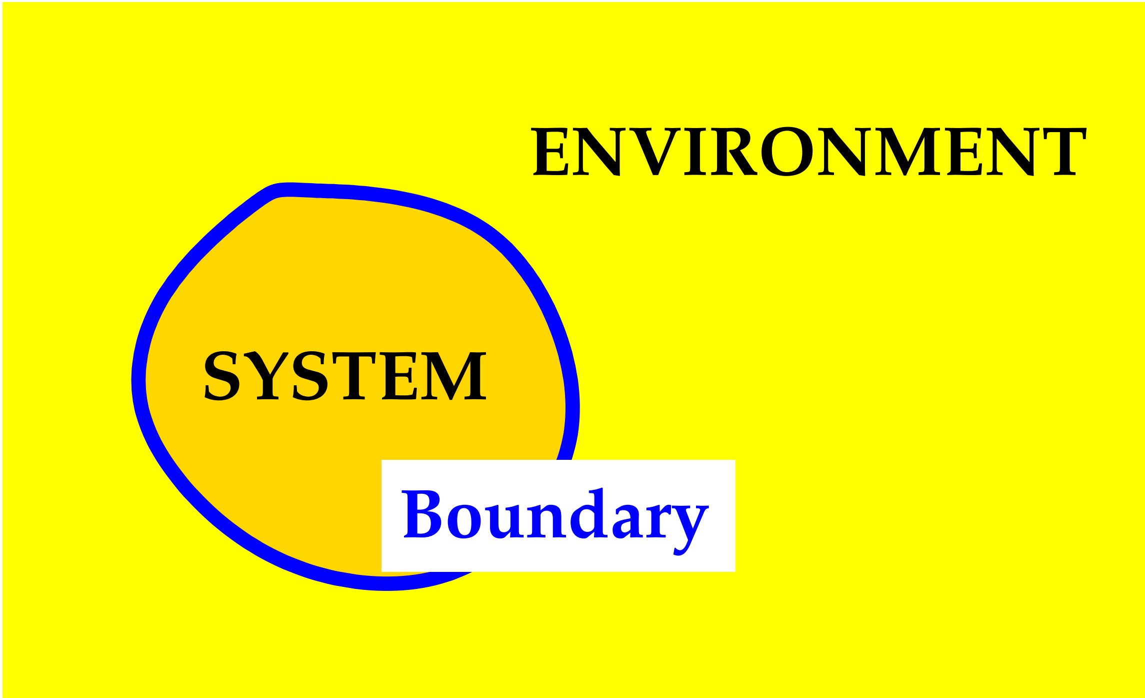
Aim of today's lecture:

develop a suitable mathematical language

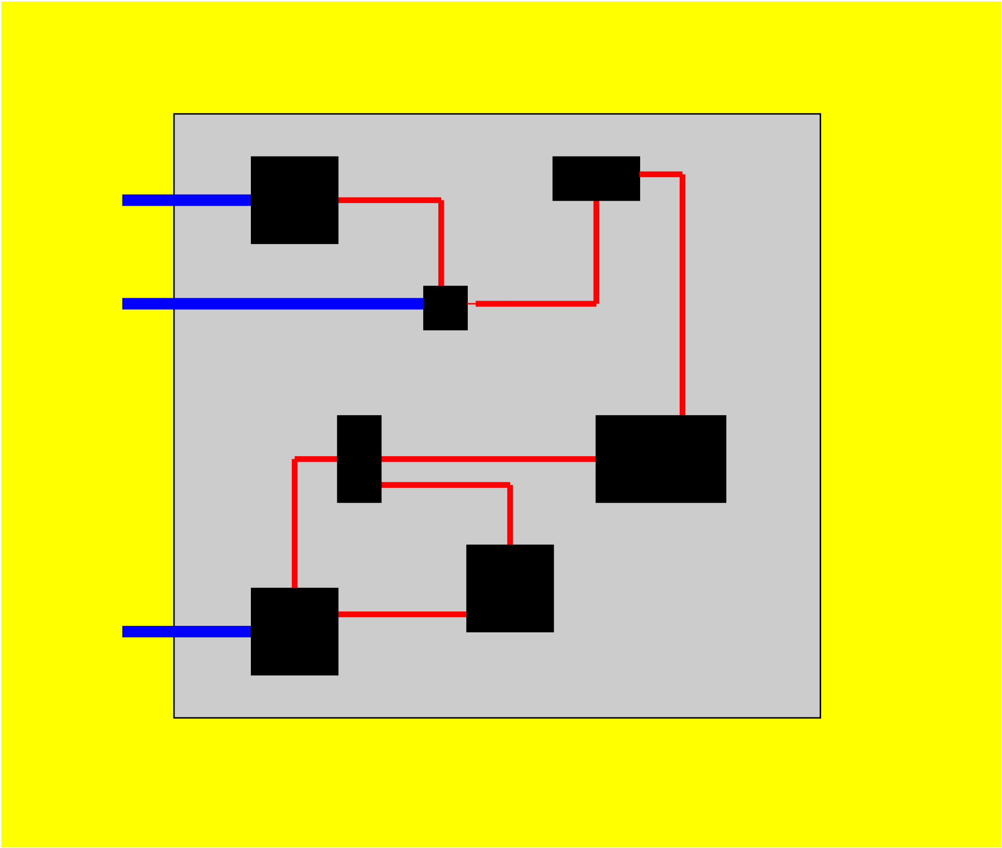
aimed at computer-assisted modeling.

Open



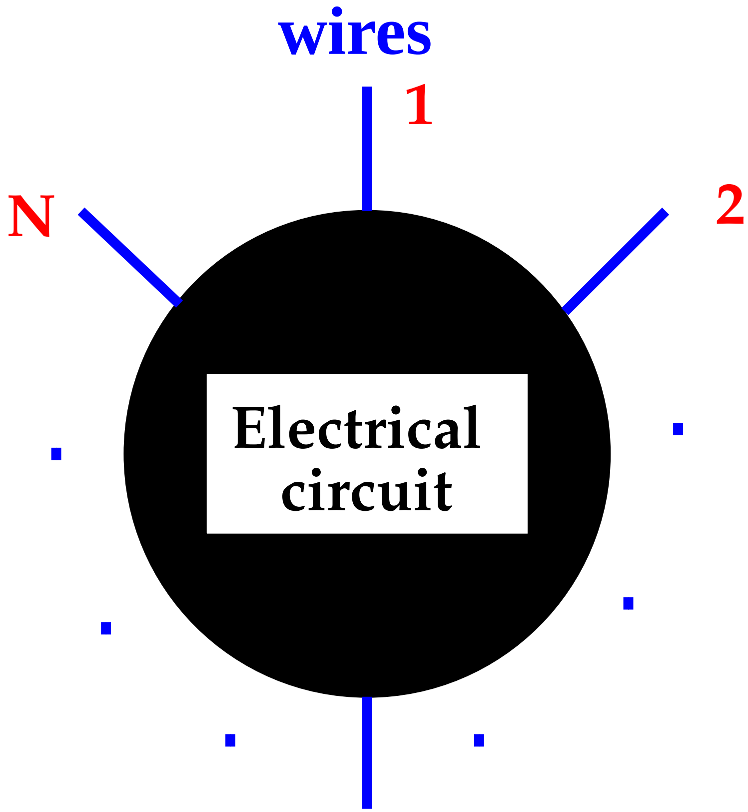
Systems interact with their environment

Connected



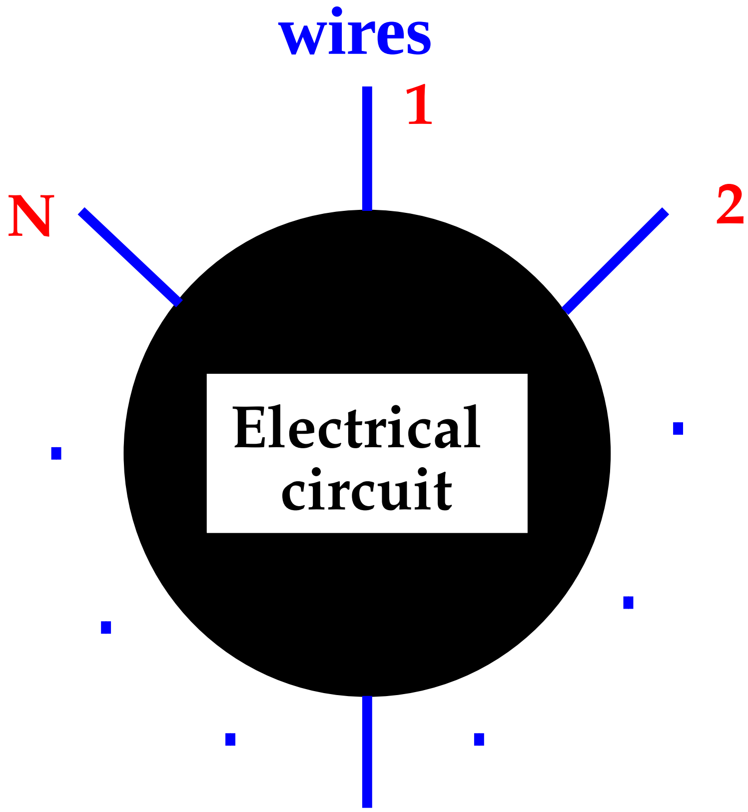
Systems consist of an architecture of interconnected subsystems

Prototypical example



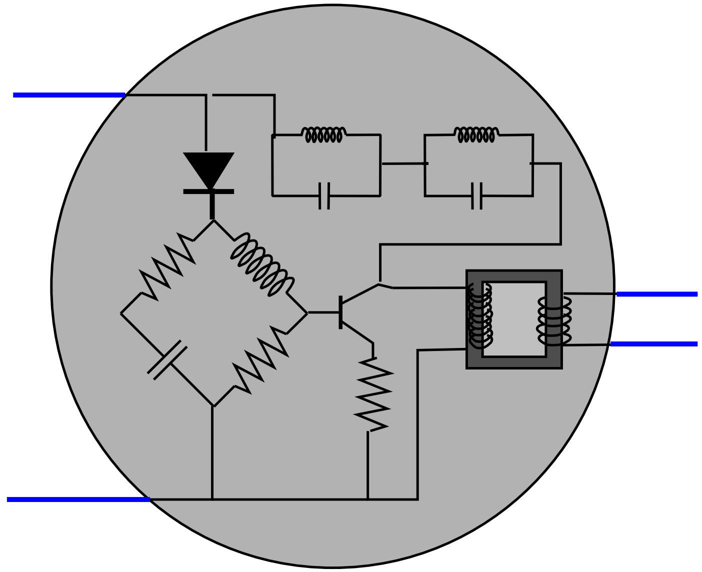
Open : **interaction through wires, 'terminals'**

Prototypical example



Open : **interaction through**

wires, 'terminals'



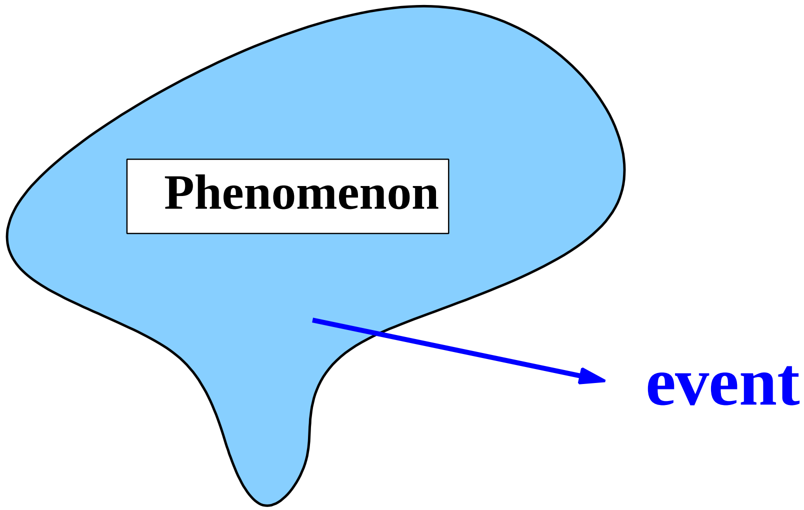
interconnection of standard modules

R'**s**, *L*'**s**, $C's$, **transistors, transformers, diodes,** ...

Mathematical models

The behavior

Assume that we have a phenomenon that produces *'events'*.



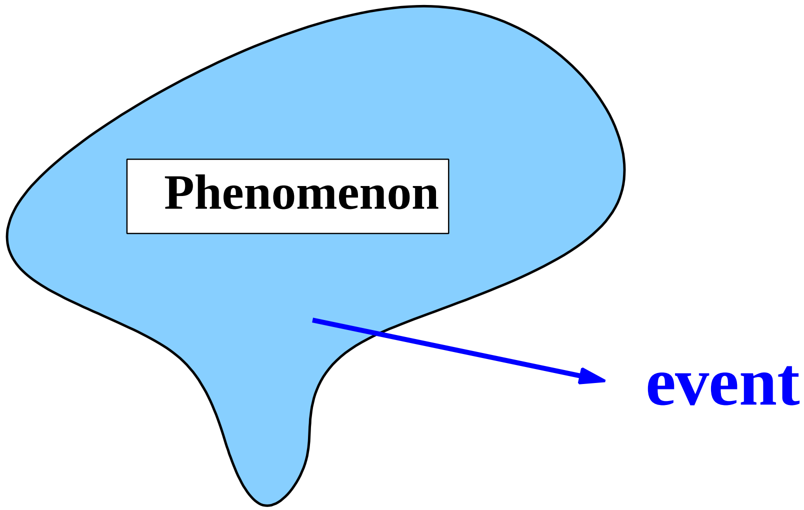
We view a deterministic model for a phenomenon as a prescription of which events can, and which cannot occur.

The behavior

Assume that we have a phenomenon that produces *'events'*.

universsum of events: $\mathscr{U}$

behavior of model: $\mathscr{B} \subseteq \mathscr{U}$

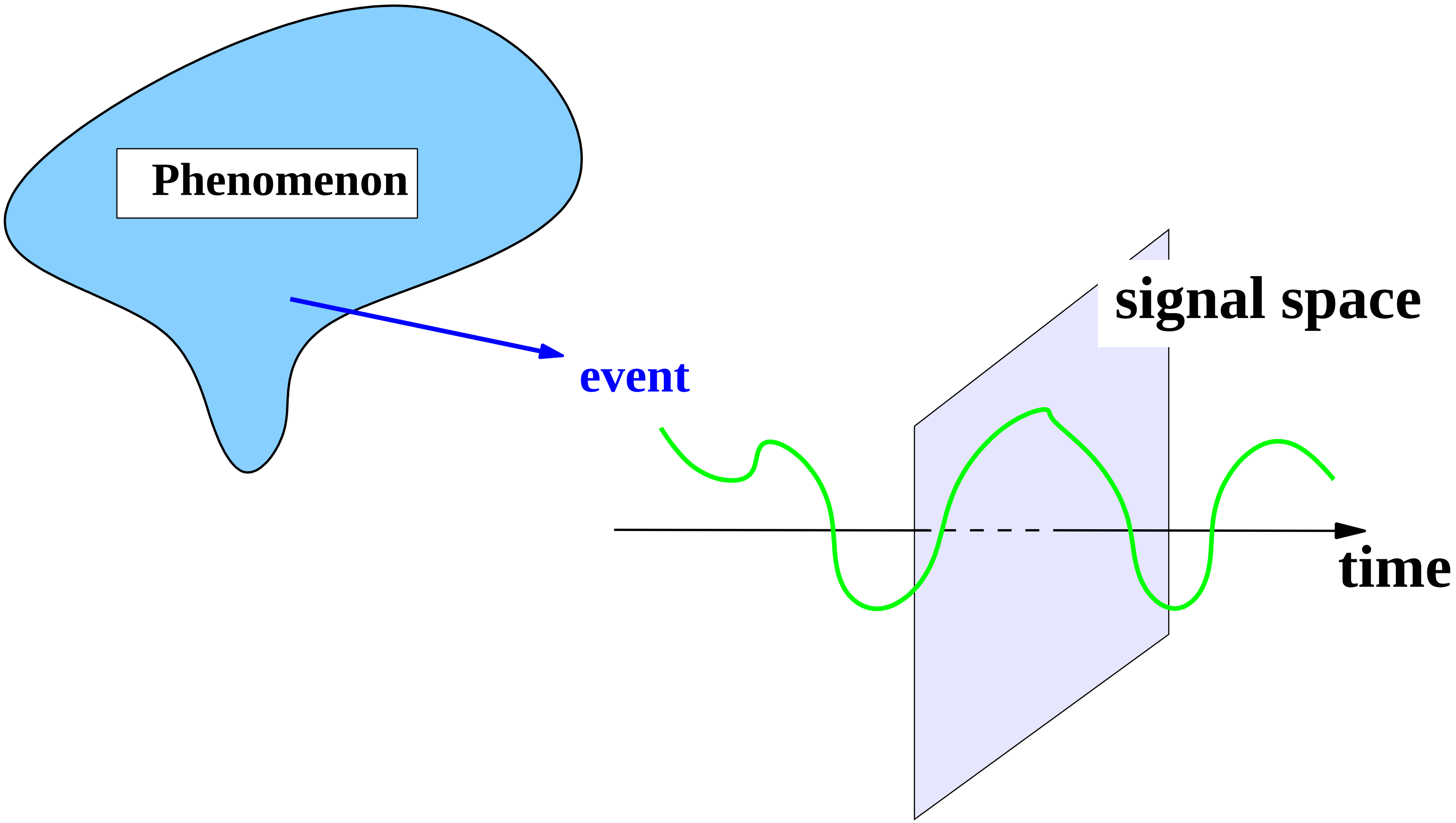


We view a deterministic model for a phenomenon as a prescription of which events can, and which cannot occur.

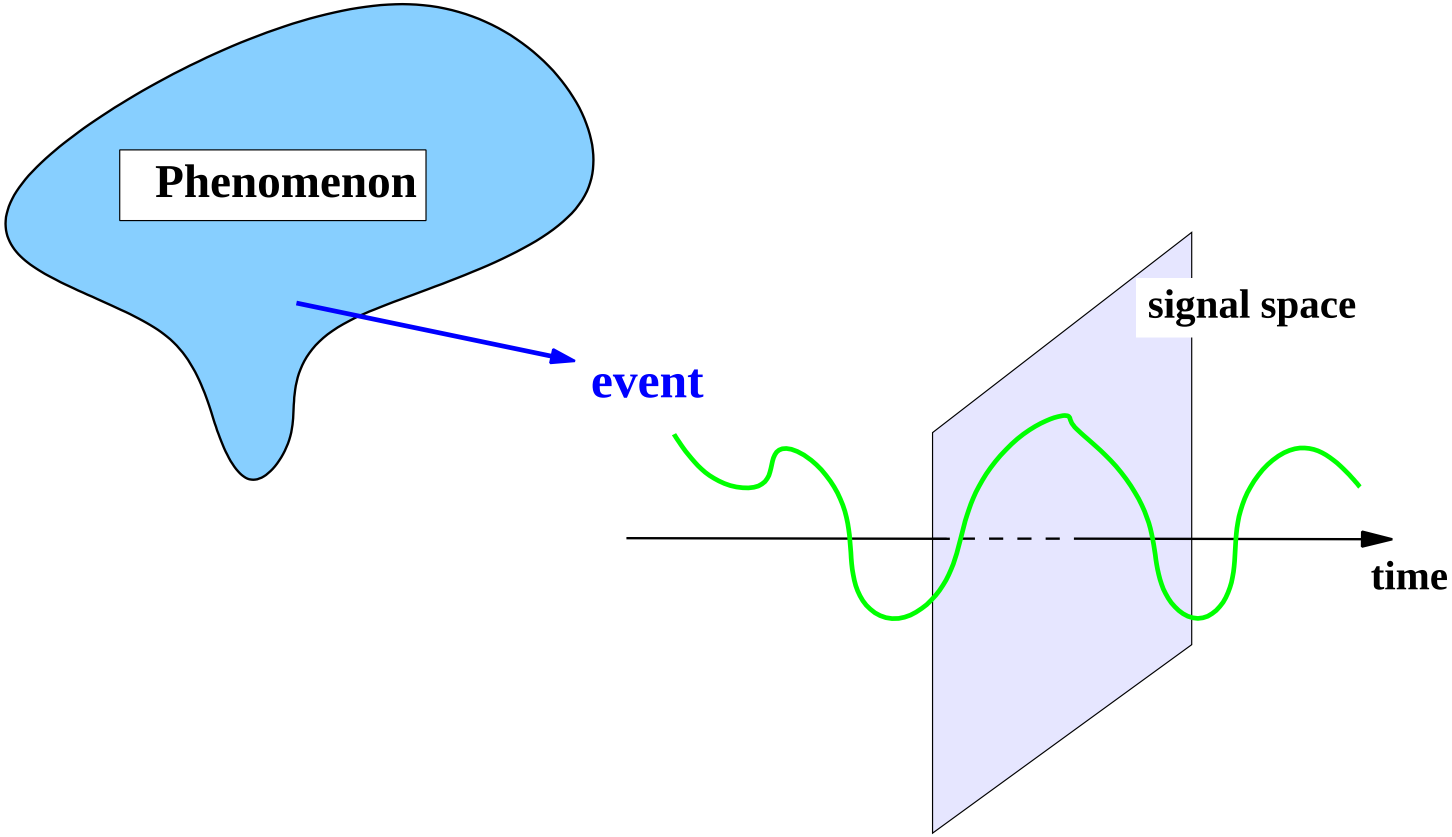
The set of events which, according to the model, are possible is called the behavior of the model, denoted by $\mathscr{B}$.

The dynamic behavior

In dynamical systems, the ‘events’ are maps,
with the time-axis as the domain,
and the signal space as the co-domain.
Hence events are functions of time.



The dynamic behavior



It is convenient to distinguish in the notation

the domain of the maps, the time set

and the codomain, the signal space

the set where the functions take on their values.

The dynamic behavior

Formal definition: A *dynamical system* $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathscr{B})$

$\mathbb{T} \subseteq \mathbb{R}$ **time set**

$\mathbb{W}$ **signal space**

$\mathscr{B} \subseteq \mathbb{W}^{\mathbb{T}}$ **the behavior**

a family of trajectories $\mathbb{T} \to \mathbb{W}$

$w : \mathbb{T} \to \mathbb{R}^{\mathtt{w}} \in \mathscr{B} \Leftrightarrow$ w **is compatible with the model**

$w : \mathbb{T} \to \mathbb{R}^{\mathtt{w}} \notin \mathscr{B} \Leftrightarrow$ **the model forbids** w

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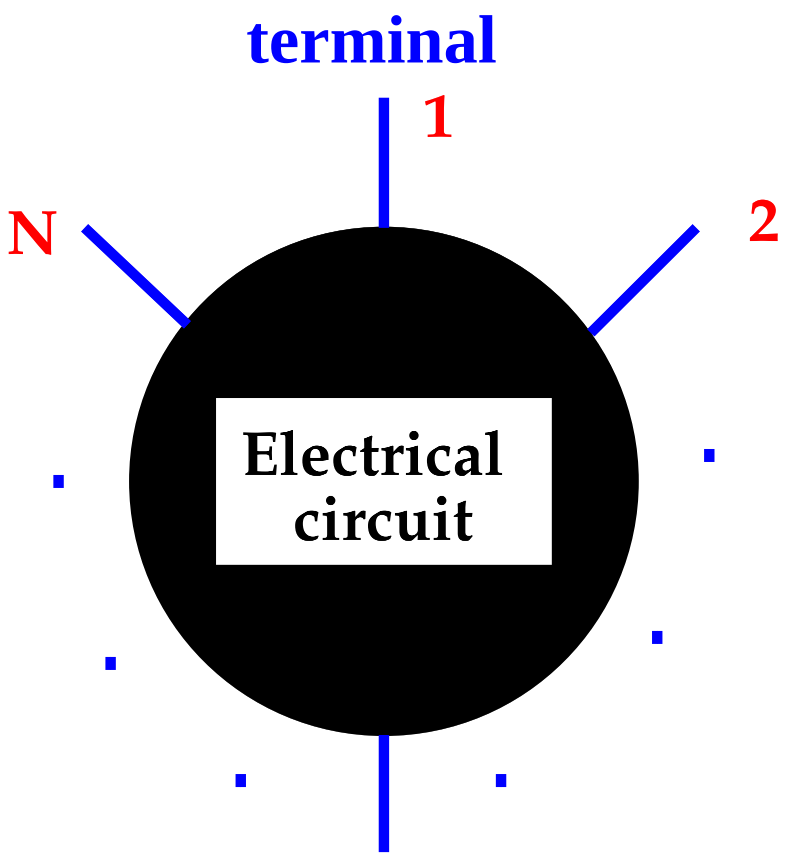
today, $\mathbb{T} = \mathbb{R}$, *continuous-time* **systems**

$\mathbb{W} = \mathbb{R}^{\mathtt{w}}$, **for some** $\mathtt{w} \in \mathbb{N}$

$\mathscr{B} \subseteq (\mathbb{R}^{\mathtt{w}})^{\mathbb{R}}$ **is a family of time trajectories taking values in a (finite-dimensional) vector space.**

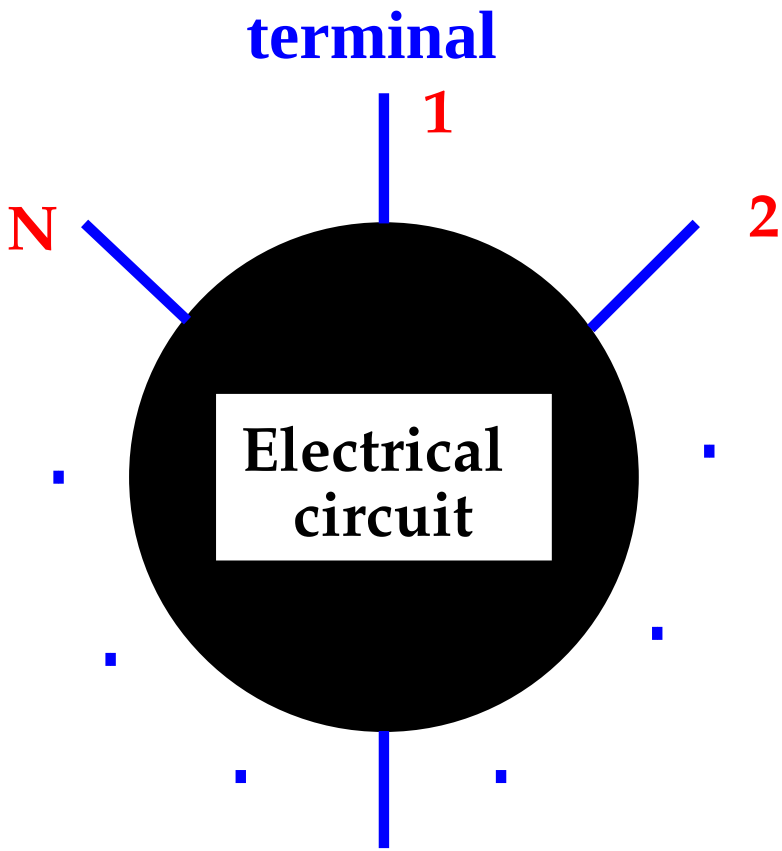
Example: terminal behavior of an electrical circuit



event = (**terminal potentials, terminal currents**) : $\mathbb{R} \rightarrow \mathbb{R}^{2N}$

Throughout: flow variables > 0 **<u>into</u> the system.**

Example: terminal behavior of an electrical circuit



$\mathbb{T} = \mathbb{R}$, $\mathbb{W} = \mathbb{R}^{2N}$

$\mathscr{B} =$ **all** $(V_1, I_1, \ldots V_N, I_N) : \mathbb{R} \rightarrow \mathbb{R}^{2N}$ **compatible with the circuit architecture and component values**

event = (**terminal potentials, terminal currents**) $: \mathbb{R} \rightarrow \mathbb{R}^{2N}$

Behavioral models

The behavior is all there is.
Equivalence of models, properties of models,
controllability, stabilizability,
symmetries, dissipativity, system identification, etc.,
must all refer to the behavior.

Controllability

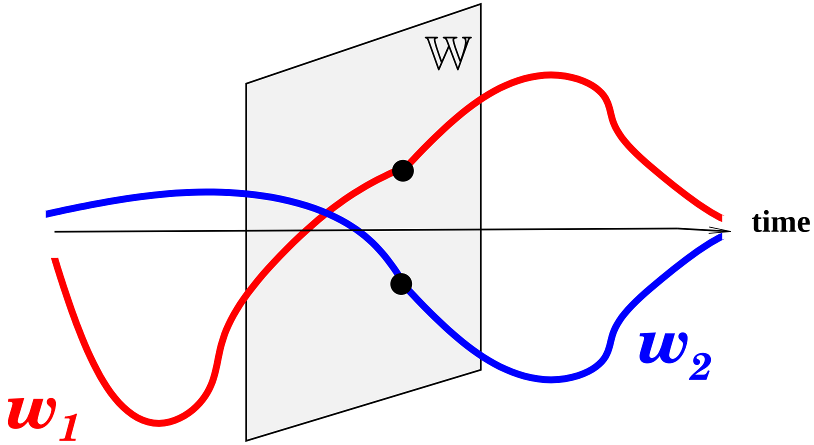
Controllability

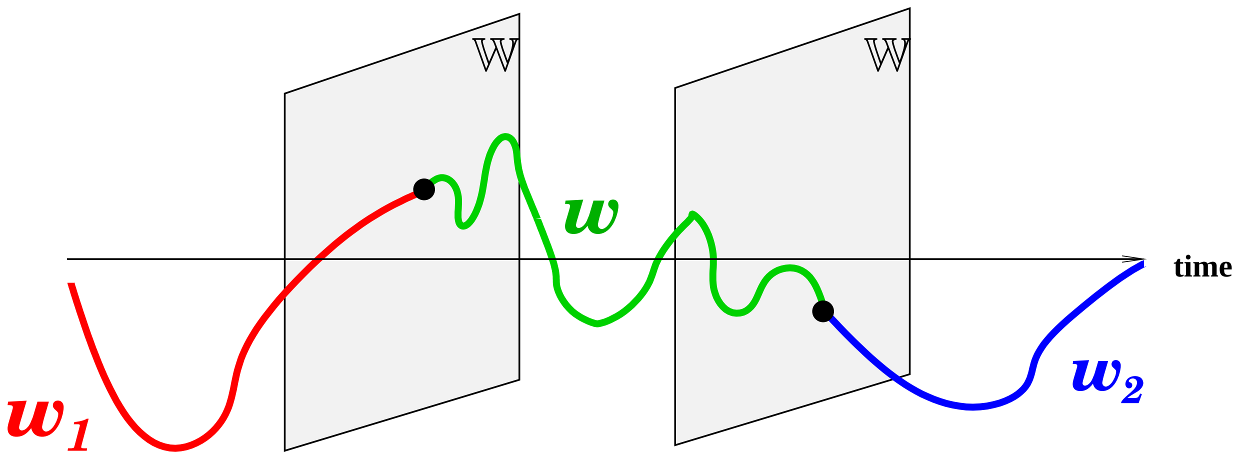
Assume that $\Sigma = (\mathbb{R}, \mathbb{W}, \mathscr{B})$ **is time-invariant**
(to avoid irrelevant complications)
and $\mathbb{T} = \mathbb{R}$ **(for the sake of concreteness)**

Σ **is said to be controllable** $:\Leftrightarrow$
for all $w_1, w_2 \in \mathscr{B}$**, there exists** $T \geq 0$ **and** $w \in \mathscr{B}$ **such that**

$$w(t) = \begin{cases} w_1(t) & \textbf{for} \quad t < 0 \\ w_2(t - T) & \textbf{for} \quad t \geq T \end{cases}$$

In pictures





controllability $\Leftrightarrow$ concatenability of trajectories after a delay

LTIDSs

The dynamical system $(\mathbb{R}, \mathbb{R}^{\mathtt{w}}, \mathscr{B})$ **is**

a linear time-invariant differential system (LTIDS) $:\Leftrightarrow$
the behavior consists of the set of solutions of a system of linear constant coefficient ODEs

$$R_0 w + R_1 \frac{d}{dt} w + \cdots + R_{\mathtt{n}} \frac{d^{\mathtt{n}}}{dt^{\mathtt{n}}} w = 0.$$

$R_0, R_1, \cdots, R_{\mathtt{n}} \in \mathbb{R}^{\bullet \times \mathtt{w}}$ **real matrices that parametrize the system, and** $w : \mathbb{R} \to \mathbb{R}^{\mathtt{w}}$**.**

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$$R\left(\frac{d}{dt}\right) w = 0$$

with $R(\xi) = R_0 + R_1 \xi + \cdots + R_{\mathtt{n}} \xi^{\mathtt{n}} \in \mathbb{R}[\xi]^{\bullet \times \mathtt{w}}$
a polynomial matrix.

Controllability tests

Controllability theorem

- **The behavior** $\mathscr{B}$ **of** $R\left(\frac{d}{dt}\right)w = 0$ **is controllable;**
- $\texttt{rank}(R(\lambda))$ **is the same for all** $\lambda \in \mathbb{C}$**;**
- $\mathscr{B}$ **allows an image representations** $w = M\left(\frac{d}{dt}\right)\ell$**;**

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- $\vdots$
- the $\mathbb{R}[\xi]$-module $\langle R \rangle$ is closed;
- $\mathbb{R}[\xi]^{1\times \mathtt{w}} / \langle R \rangle$ is torsion free.

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- $\mathbb{R}[\xi]^{1\times\mathtt{w}}/\langle R \rangle$ **is torsion free.**

- **There exist computer-algebra based tests.**
- **Explains the notorious common factor problem for**

$$p\left(\frac{d}{dt}\right)y = q\left(\frac{d}{dt}\right)u$$

Manifest and latent variables

First principles models

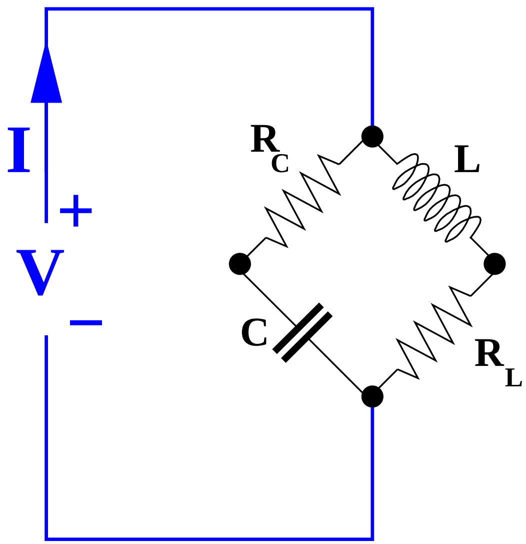
First principles models invariably contain auxiliary variables in addition to the variables whose behavior we intend to model.

manifest variables: the variables the model aims at.

latent variables:
auxiliary variables introduced during the modeling process.

Example: an RLC circuit

Model the port behavior of



$$\mathbb{T} = \mathbb{R}, \mathbb{W} = \mathbb{R}^2, w = \begin{bmatrix} V \\ I \end{bmatrix}$$

$V =$ **port voltage**

$I =$ **port current**

Example: an RLC circuit

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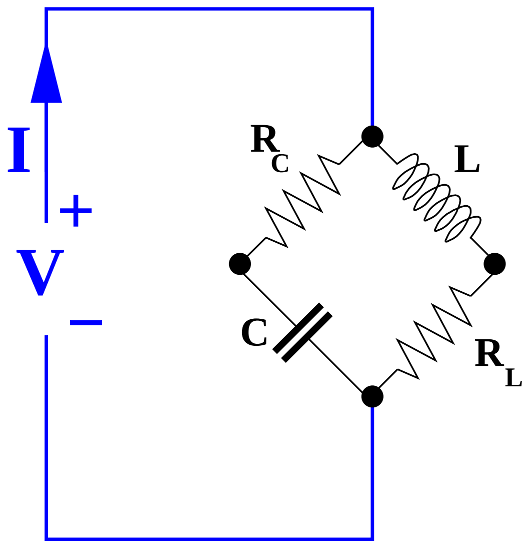

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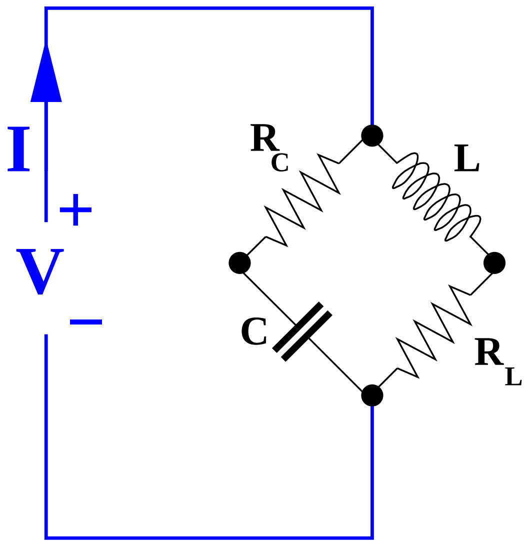
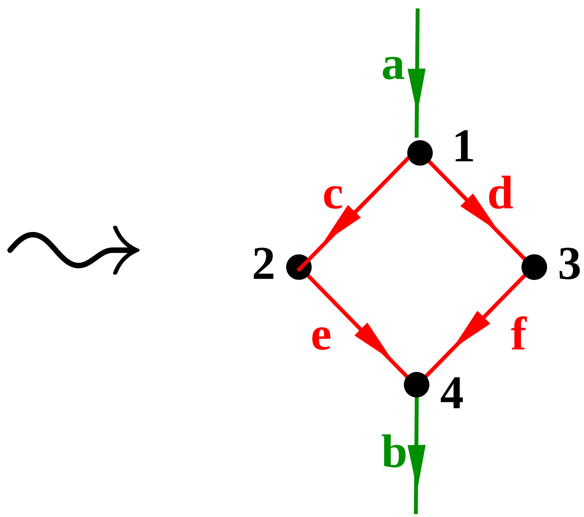
$V =$ **port voltage**
$I =$ **port current**

This example involves 2-terminal electrical components. Many methods for modeling such circuits have been developed.

Choice of latent variables

Here we follow modified nodal analysis (MNA). We associate with the circuit a digraph, and choose as latent variables the potentials of the vertices and the currents in the edges





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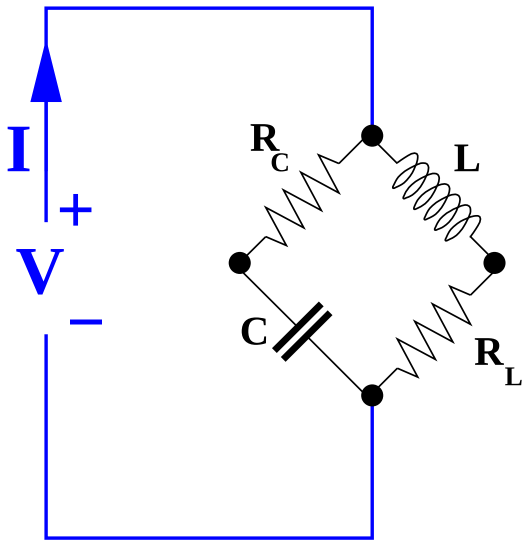
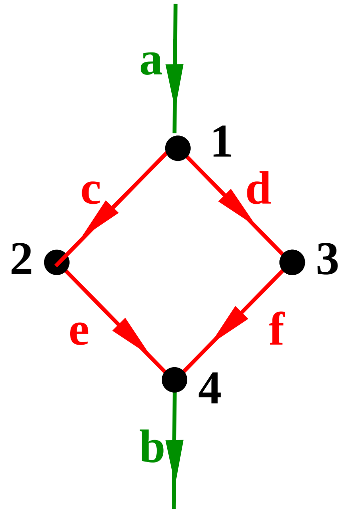




manifest variables: V, I

latent variables: $(V_1, V_2, V_3, V_4); \ \ (I_a, I_b, I_c, I_d, I_e, I_f)$

Behavioral equations tableau

KCL :	**vertex 1** :	I_a	$=$	$I_c + I_d$
	vertex 2 :	I_c	$=$	I_e
	vertex 3 :	I_d	$=$	I_f
	vertex 4 :	I_b	$=$	$I_g + I_h$
Constitutive	**edge c** :	$V_1 - V_2$	$=$	$R_C I_c$
equations:	**edge d** :	$V_1 - V_3$	$=$	$L \frac{d}{dt} I_d$
	edge e :	$C \frac{d}{dt}(V_2 - V_4)$	$=$	I_e
	edge f :	$V_3 - V_4$	$=$	$R_L I_f$
Manifest	**port voltage** :	V	$=$	$V_1 - V_4$
variables:	**port current** :	I	$=$	I_a

Behavioral equations

In total 10 latent variables: $(V_1, V_2, V_3, V_4); \quad (I_a, I_b, I_c, I_d, I_e, I_f)$

2 manifest variables: (V, I)

10 equations.

Which equation(s) govern(s) (V, I)**?**

For the case at hand, a simple calculation leads to:

The port equation

The port behavior $\mathscr{B}$ consists of the solutions of:

Case 1: $CR_C \neq \frac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + \left(1 + \frac{R_C}{R_L}\right) CR_C \frac{d}{dt} + CR_C \frac{L}{R_L} \frac{d^2}{dt^2}\right) V$$
$$= \left(1 + CR_C \frac{d}{dt}\right) \left(1 + \frac{L}{R_L} \frac{d}{dt}\right) R_C I$$

Case 2: $CR_C = \frac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + CR_C \frac{d}{dt}\right) V = \left(1 + CR_C \frac{d}{dt}\right) R_C I$$

The elimination problem

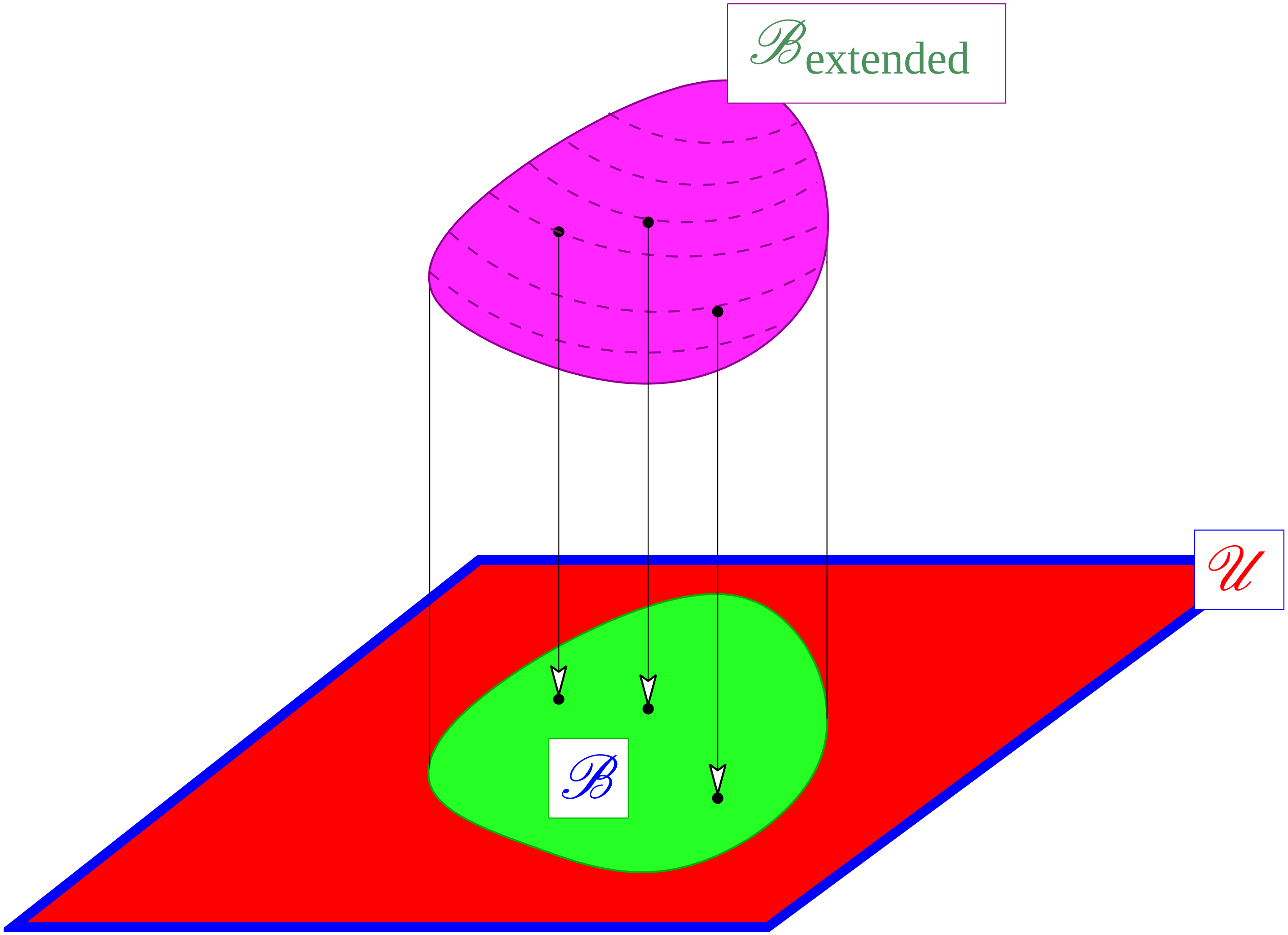
Assume that the behavior of the manifest and latent variables jointly, $\mathscr{B}_{\text{extended}}$, has a certain structure.

Does the manifest behavior $\mathscr{B}$ retain this structure?

'Structure': linearity, open, closed, (semi-)algebraic variety, polyhedron, governed by LMIs, solution set of a system of ODEs, linear constant coefficients ODEs, PDEs ...

Important question, from a system theoretic, modeling, and practical point of view.

Projection



Elimination theorem

The elimination theorem for LTIDSs

**The projection of the set of solutions
of a system of linear constant coefficient ODEs
is again the set of solutions
of a system of linear constant coefficient ODEs .**

Elimination theorem

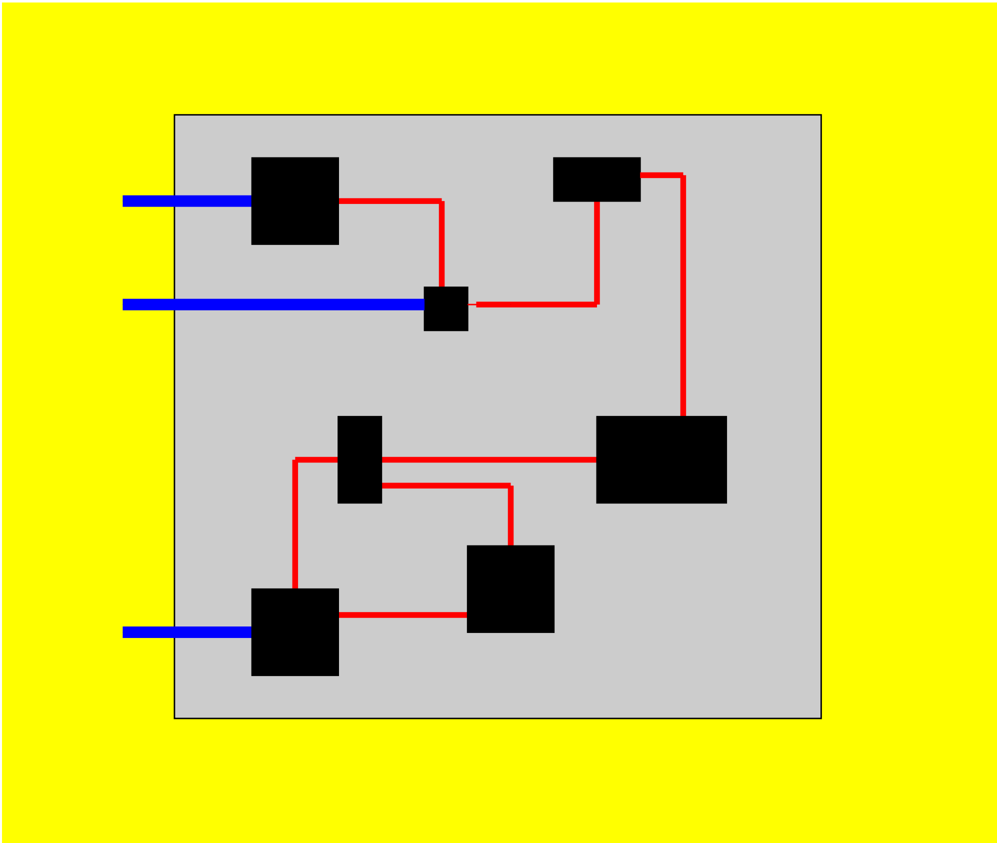
The elimination theorem for LTIDSs

The projection of the set of solutions of a system of linear constant coefficient ODEs is again the set of solutions of a system of linear constant coefficient ODEs .

- **There exist computer-algebra based algorithms for elimination for LTIDSs.**
- **There is no nonlinear elimination theorem.**

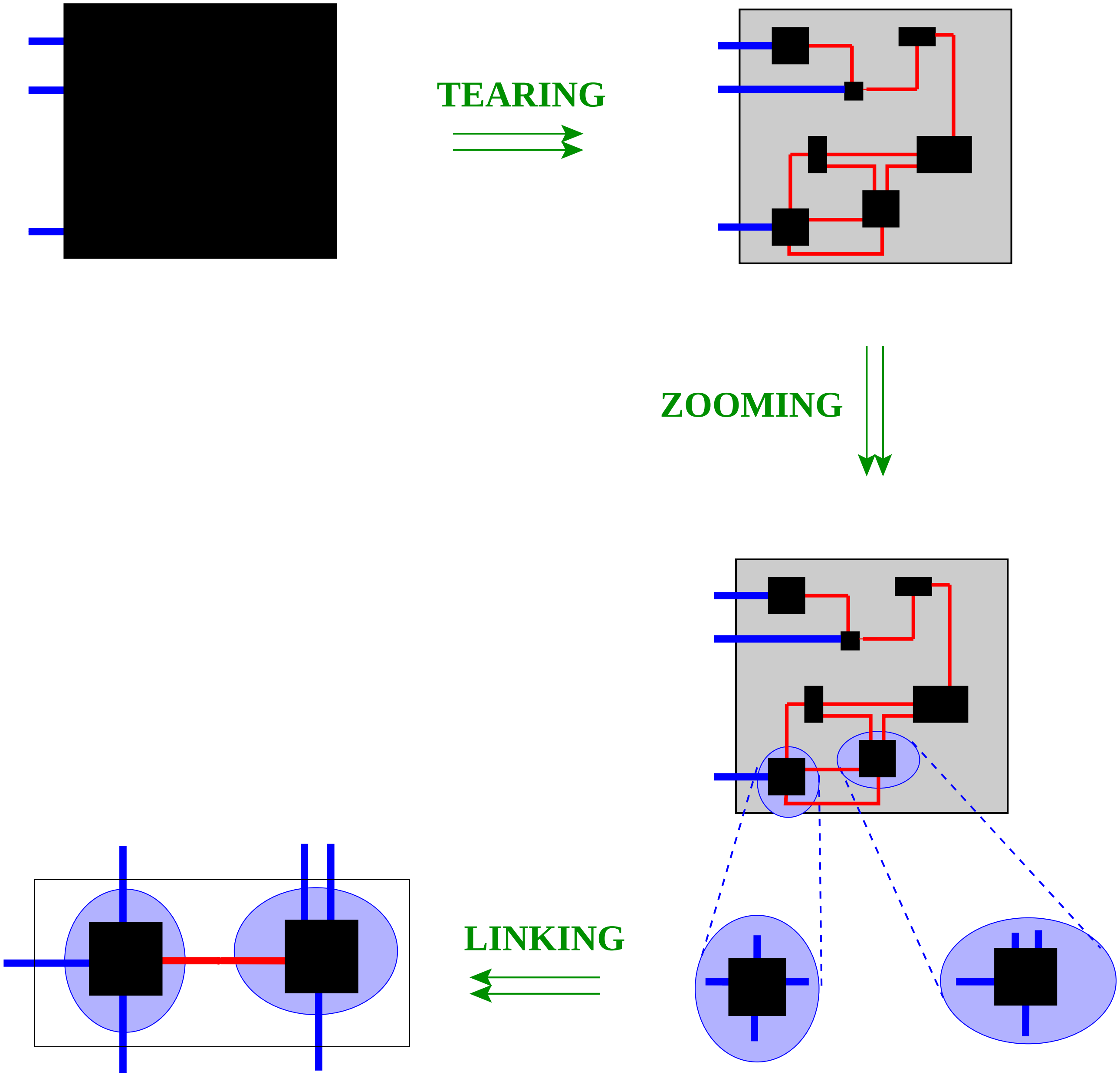
Interconnection architecture

Objective



Formalize modeling of interconnected systems.

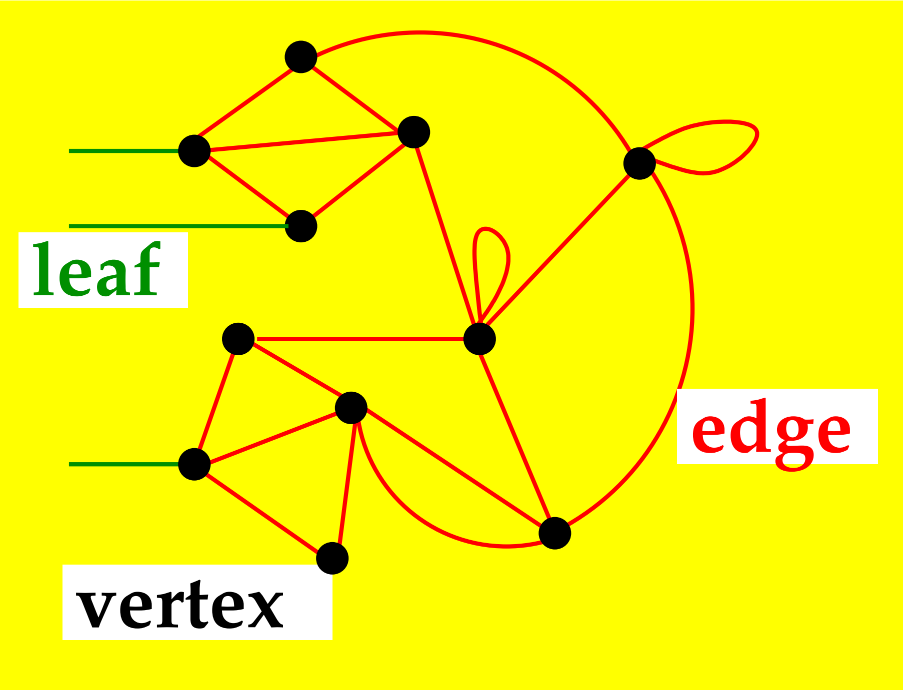
Modeling by tearing, zooming, and linking



Formailization

Architecture:

graph with leaves



vertices	$\rightsquigarrow$	**systems with terminals**
edges	$\rightsquigarrow$	**connected terminals**
leaves	$\rightsquigarrow$	**interaction with environment**
terminals	$\rightsquigarrow$	**system variables**

Behavioral equations

1. **Module specification** for each vertex.
   Relation among the variables on the terminals.

2. **Interconnection equations** for each edge.
   Equating the variables on the terminals associated with the same edge.

3. **Manifest variable assignment**
   Specifies the variables of interest.

Behavioral equations

1. **Module specification** for each vertex.

   Relation among the variables on the terminals.

   A specification of the behavior of the terminal variables of the subsystems stored as (parametrized) modules in a data-base.

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

   Interconnection laws stored in a data-base.
   Laws depend on terminal type:
   electrical / mechanical / hydraulic / etc.

3. **Manifest variable assignment**

   Specifies the variables of interest.

Behavioral equations

1. **Module specification** for each vertex.

   Relation among the variables on the terminals.

   Terminal behavior of subcircuits.

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

   $V_1 = V_2, I_1 + I_2 = 0$

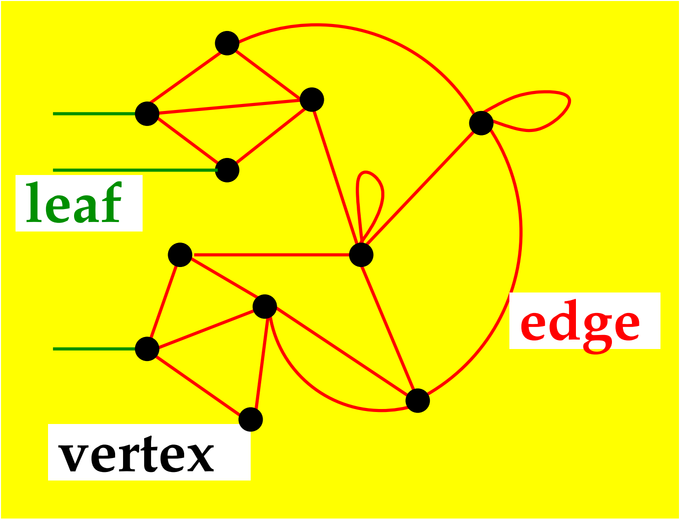
3. **Manifest variable assignment**

   Potentials and currents on the external terminals.

Hierarchy

New modules from old ones

Tearing, zooming, linking is hierarchical:



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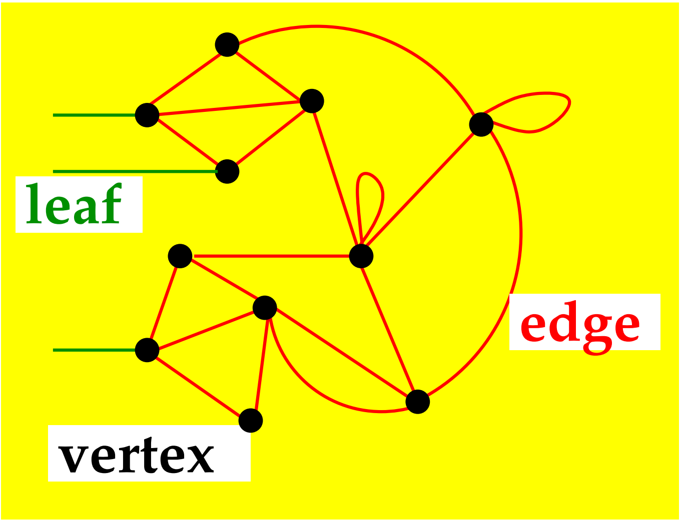

Embed modules in vertices, obtain behavioral equations for the interconnected system, eliminate the latent variables,

New modules from old ones

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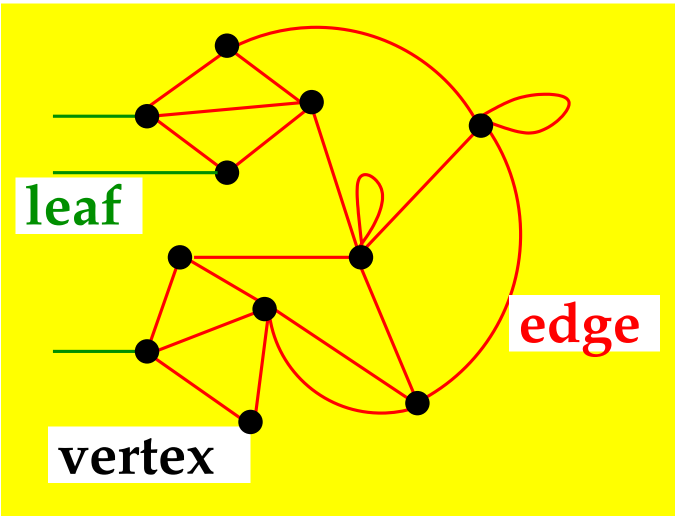
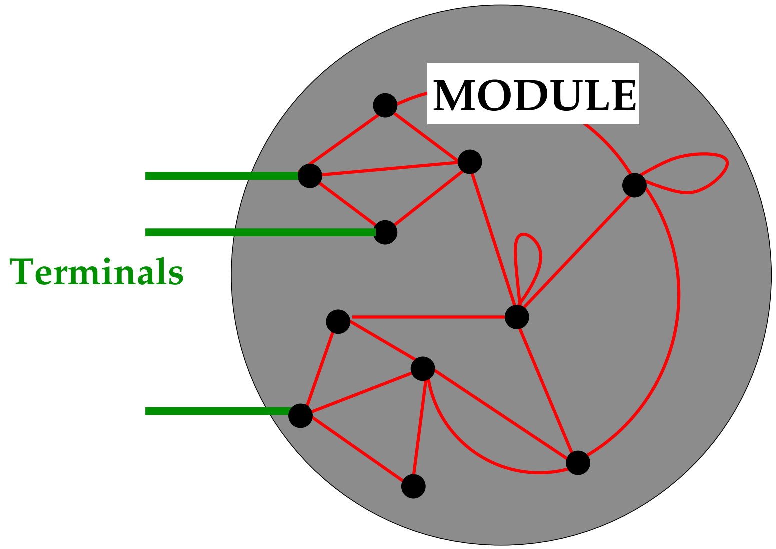
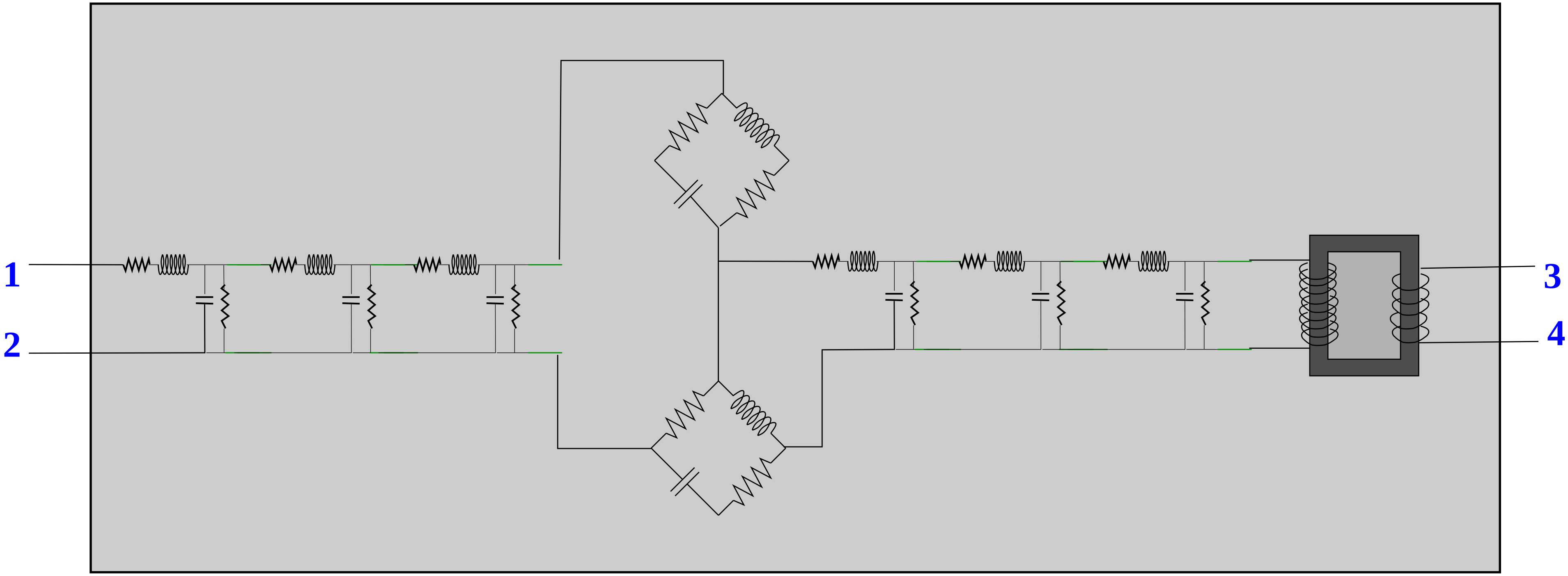




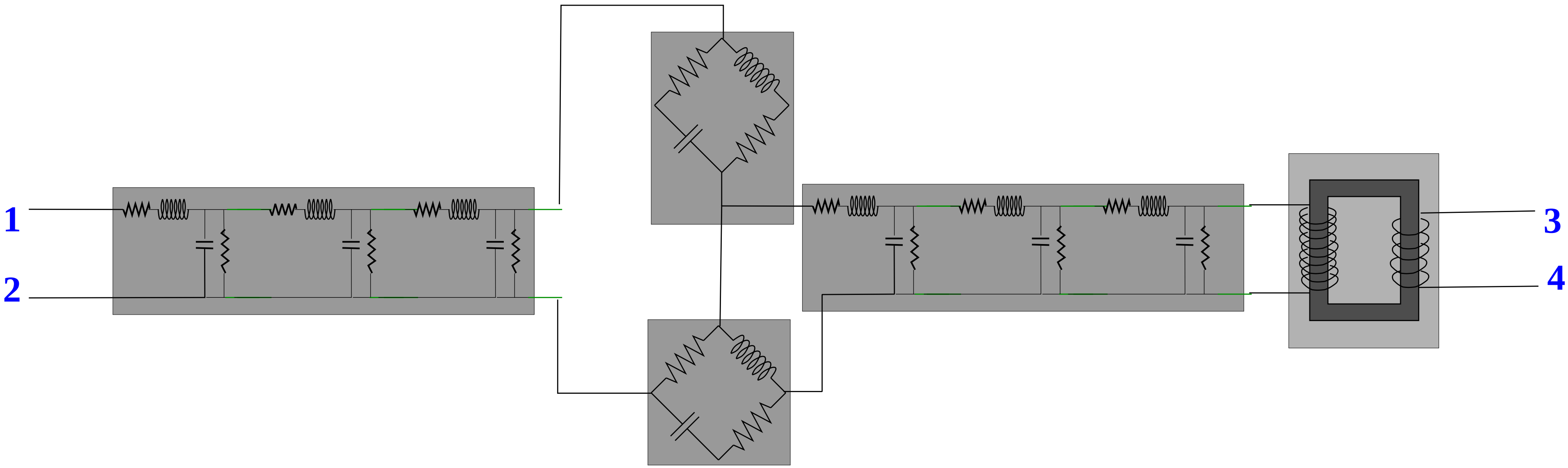
Embed modules in vertices, obtain behavioral equations for the interconnected system, eliminate the latent variables, and use interconnected system as a module with terminals in a new interconnection architecture.

Example



!! **Model relation between** V_1, I_1, V_2, I_2 !!

Tearing

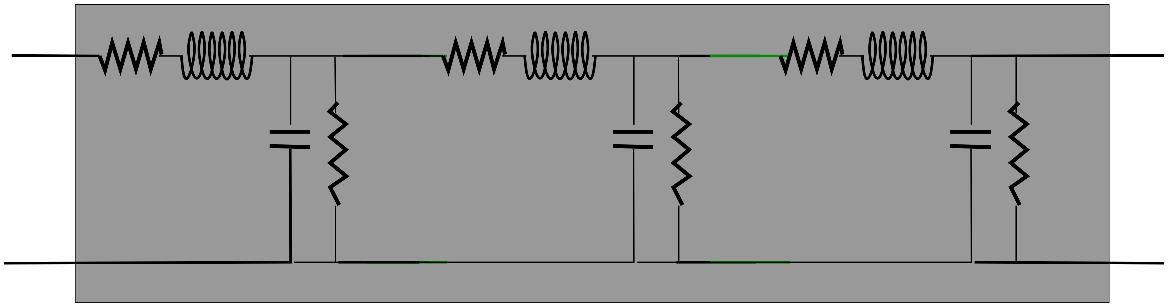


View as interconnection of 5 subsystems:
one trafo,
two 4-terminal RRLC ladders,
two 3-terminal RLC circuits.

Model the subsystems one-by-one.

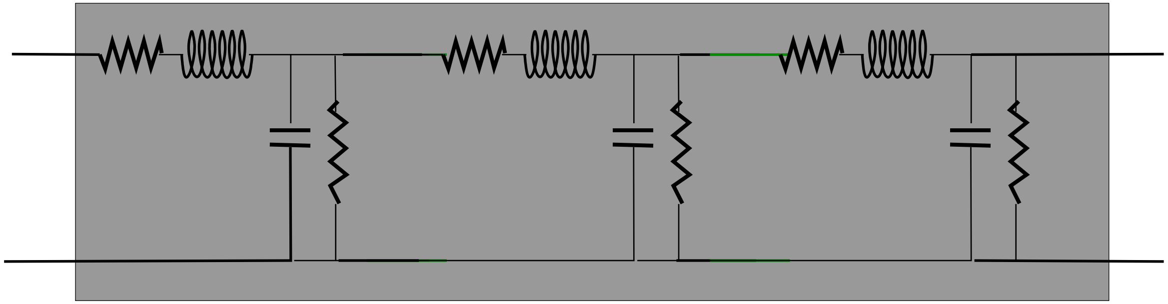
Hierarchy

Subsystems 1 and 4

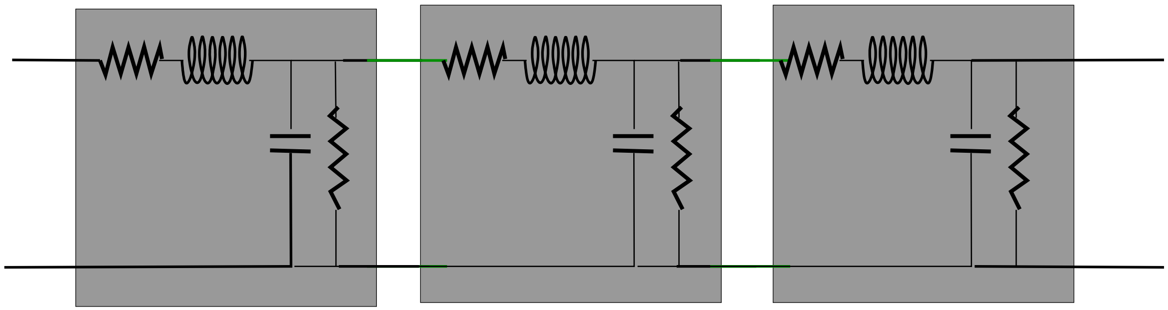


Hierarchy

Subsystems 1 and 4



Tearing



Hierarchy

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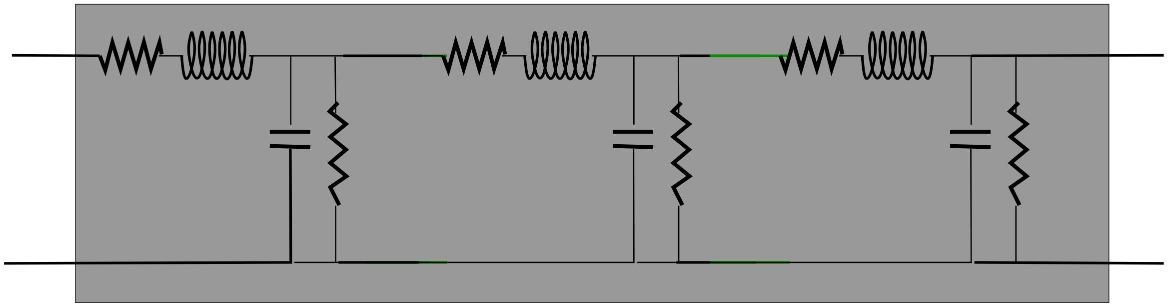
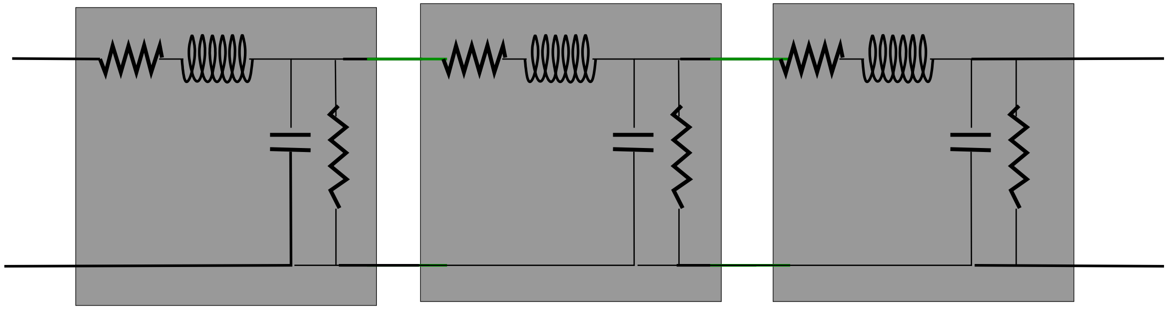
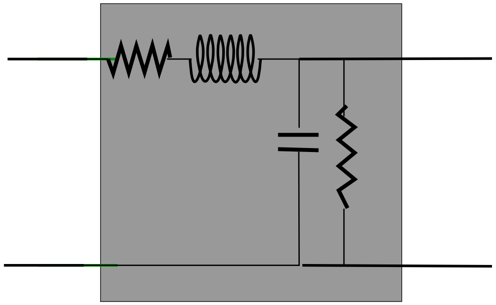


Tearing

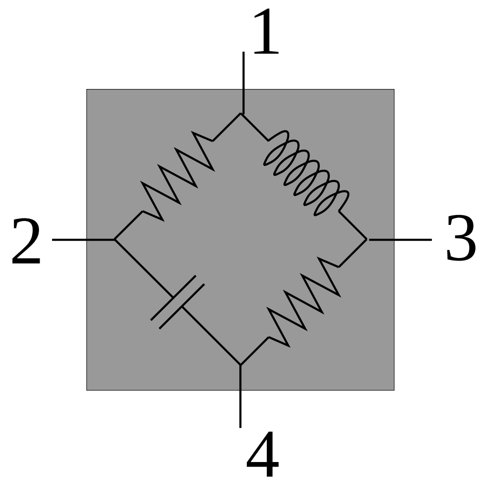


Zooming



Subsystems 2 and 3

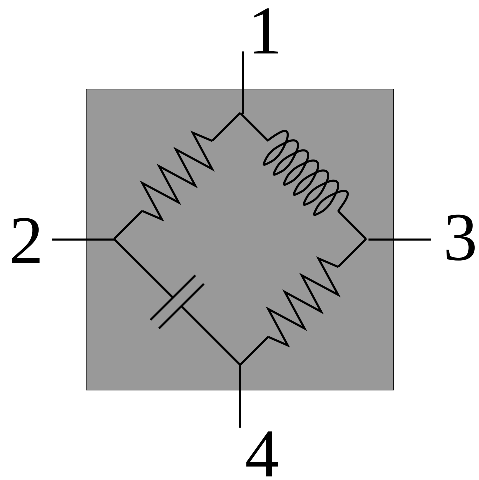
Model 4-terminal circuit



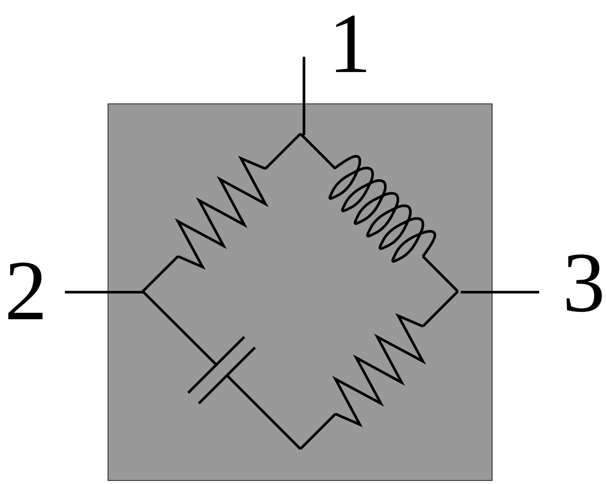
Hereditary

Subsystems 2 and 3

Model 4-terminal circuit



Specialize: $I_4 = 0$**,**
eliminate V_4



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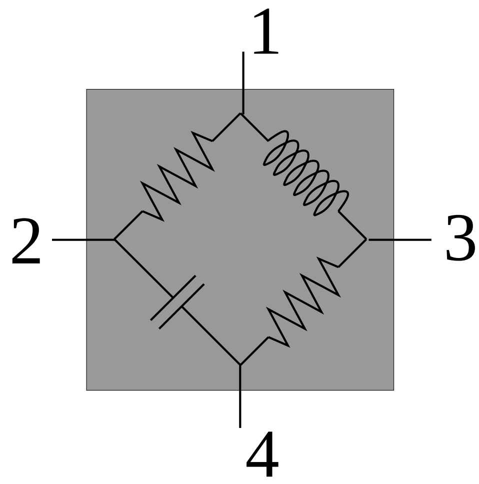

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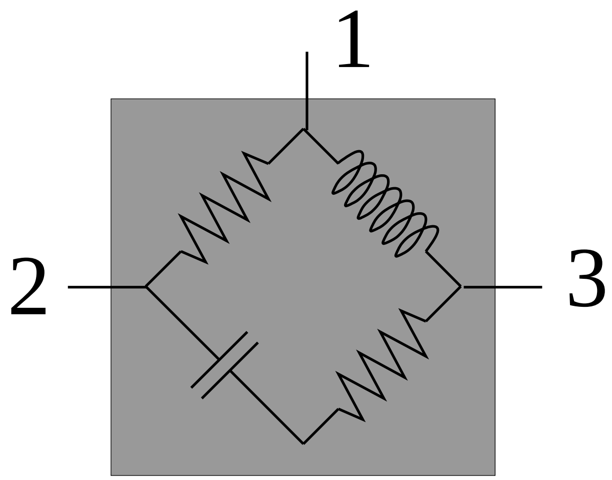
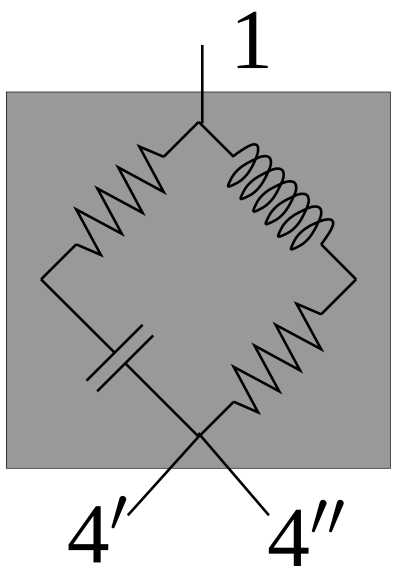
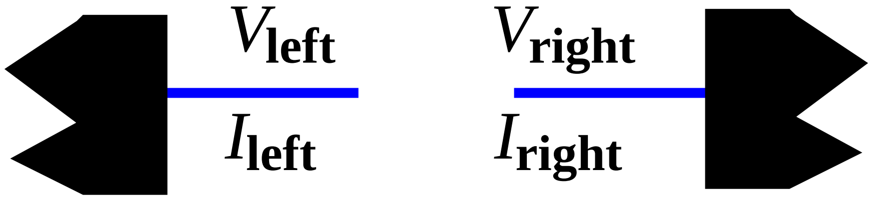


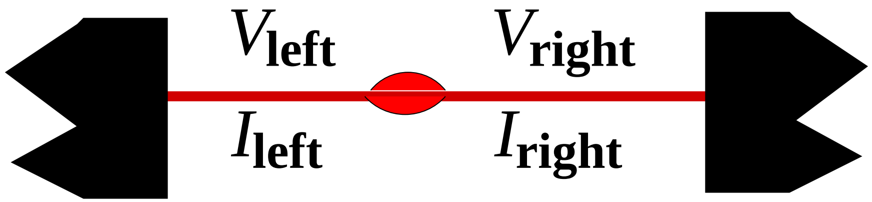


Set $I_2 = I_3 = 0$**,**
eliminate V_2, V_3**,**
set $V_4' = V_4'' = V_4$,
$I_4' + I_4'' = I_4$**,**
eliminate V_4, I_4**.**

Linking

All interconnections are of electrical type





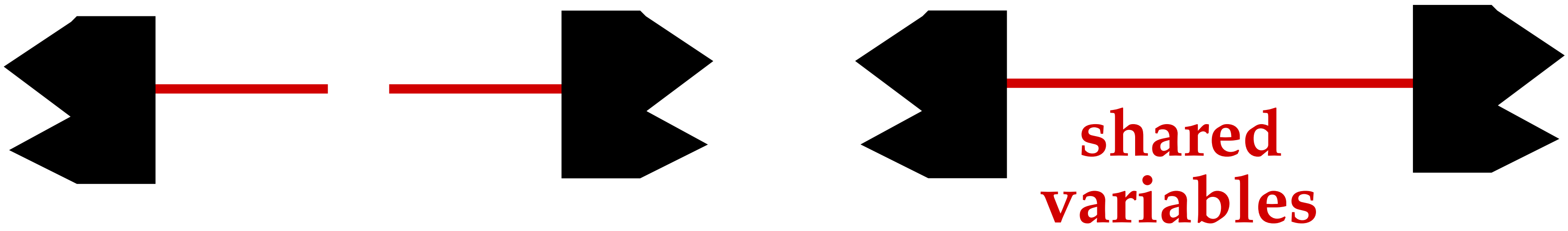
<u>**Interconnection equations:**</u>

potential left $=$ **potential right** $\rightsquigarrow$ $V_{\text{left}} = V_{\text{right}}$

current left $+$ **current right** $= 0$ $\rightsquigarrow$ $I_{\text{left}} + I_{\text{right}} = 0$

Terminals versus ports

Bond graphs



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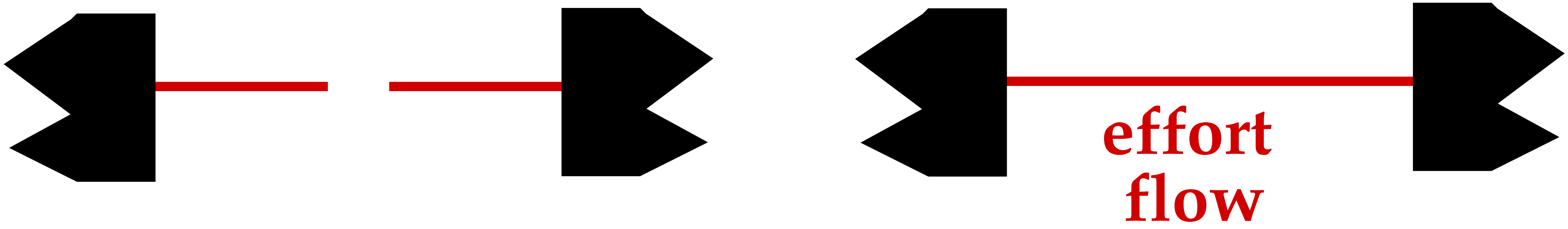

Premise: Interconnection variables consist of

an effort and a flow effort × **flow = power**

Interconnection ⇔
[efforts equal] & [flows sum to 0]
⇒ **power equal**

'Power is the universal currency of physical systems'

Effort times flow

Interconnection variables:

- **voltage & current**
- **force & velocity**
- **pressure & mass flow**
- **temperature &** $\dfrac{\text{heat flow}}{\text{temperature}}$
- **...**

Effort times flow

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Do interconnections really equate efforts and flows, with

effort × flow = power?

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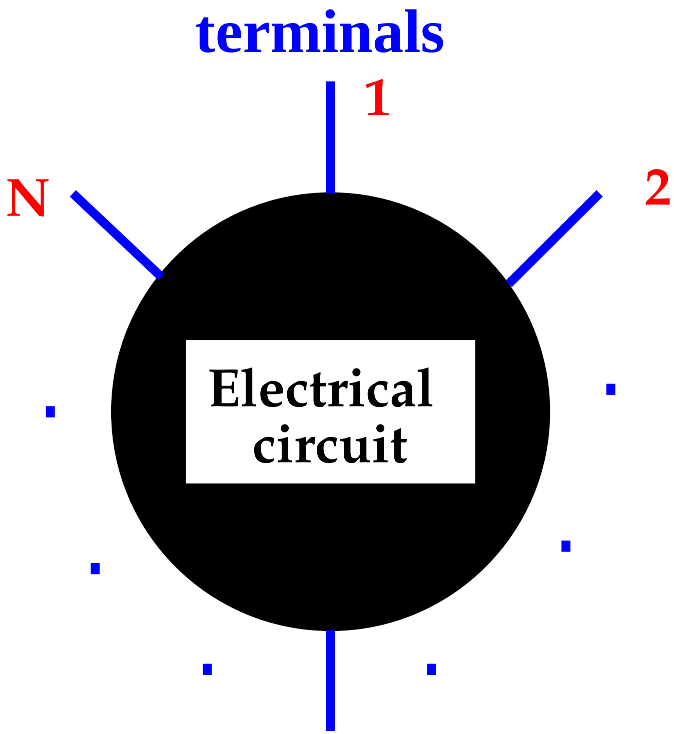
Do interconnections really equate efforts and flows, with

effort $\times$ flow = power?

Terminals are for interconnection, *ports* are for energy transfer

We illustrate this, for electrical interconnections only.

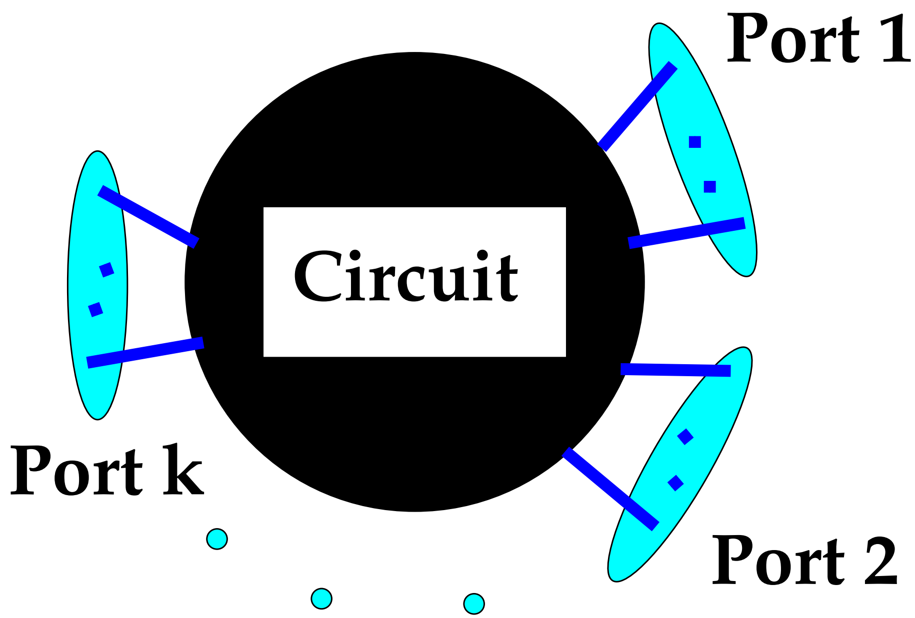
Terminals versus ports



Terminal variables and behavior (N terminals, $2N$ real variables in total – a potential and a current for each terminal):

$$(V_1, I_1, V_2, I_2, \ldots, V_N, I_N) \rightsquigarrow \textbf{ behavior } \mathscr{B} \subseteq \left(\mathbb{R}^{2N}\right)^{\mathbb{R}}$$

Definition of a port

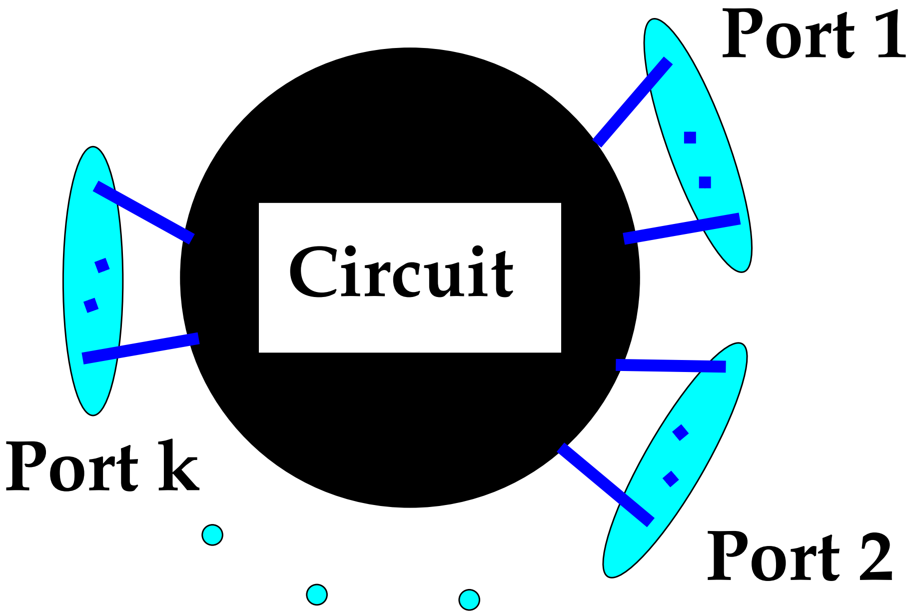


A subset of the terminals forms a port :⇔

sum currents on port terminals = 0

adding any, but the same, function of time
to each of the port terminal potentials,
but not to the other terminal potentials
⇒ a new set of legal potentials.

Definition of a port

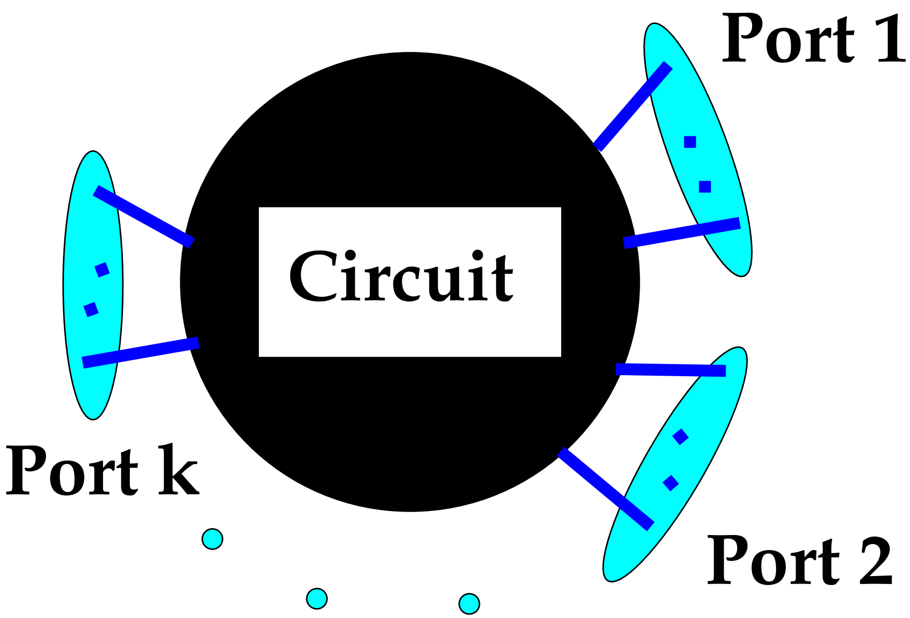


$$\left(\boxed{V_1, I_1 \ldots, V_p, I_p}, V_{p+1}, \ldots, I_n\right) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$$

$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_p + \alpha, I_p}, V_{p+1}, \ldots, I_n\right) \in \mathscr{B} \qquad \boxed{I_1 + \cdots + I_p} = 0$$

Definition of a port



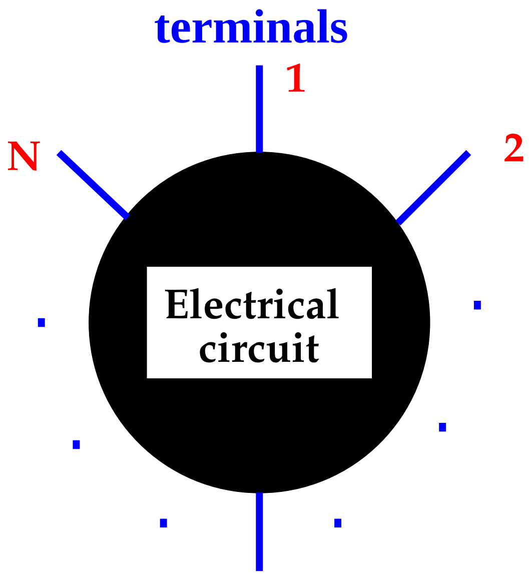
Equivalenty: the behavioral equations contain the variables $V_1, V_2 \ldots, V_p$ **only as the differences**

$$V_i - V_j \qquad \textbf{for } i, j = 1, \ldots p$$

and contain as a 'consequence' the equation

$$I_1 + I_2 + \cdots + I_p = 0$$

Kirchhoff's laws



All the terminals together form a port

$$\left(\boxed{V_1, I_1 \ldots, V_N, I_N}\right) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$$

$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_N + \alpha, I_N}\right) \in \mathscr{B}$$

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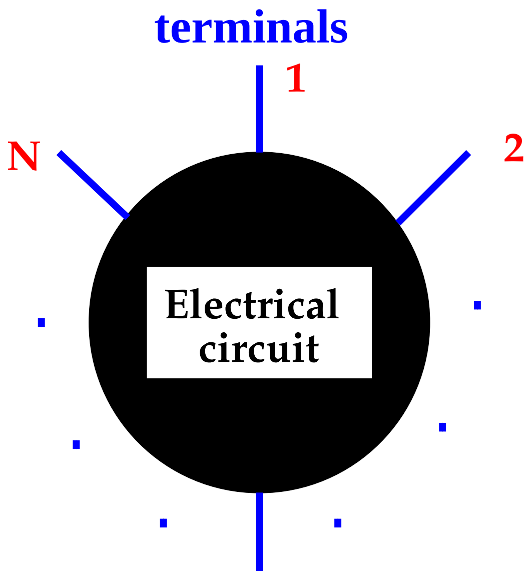

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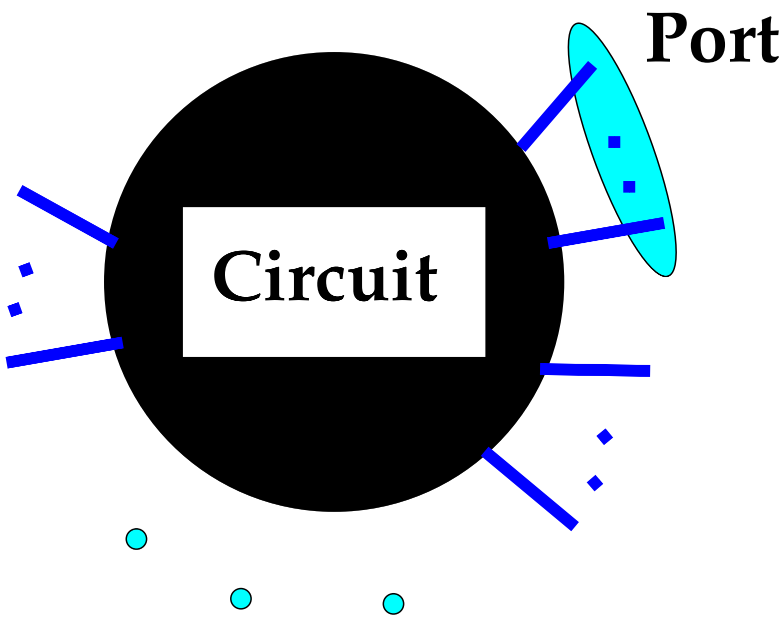
Viewed as 'laws' governing electrical circuits, these may be thought of as the KVL & KCL.

Power and energy

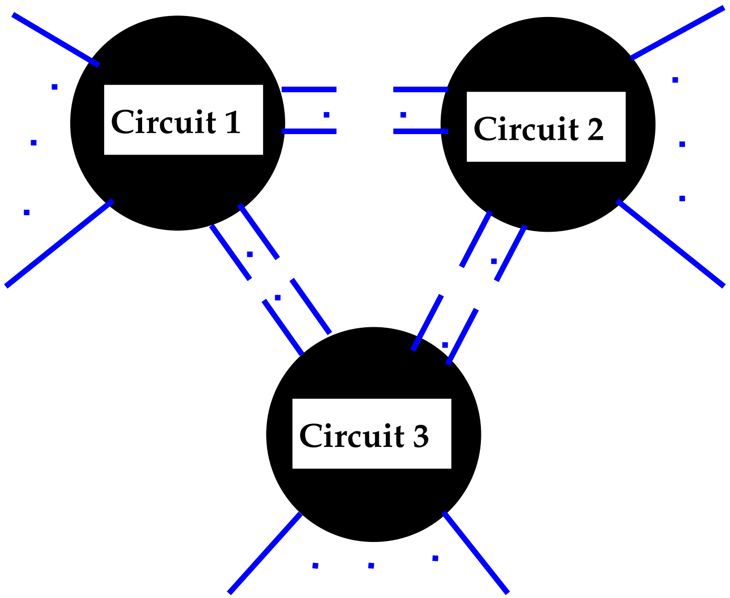


The energy that flows into the circuit along the terminals $1, 2, \ldots, \mathrm{p}$ **during the interval** $[t_1, t_2]$ **equals**

$$\int_{t_1}^{t_2} \Sigma_{\mathrm{k}=1,\ldots,\mathrm{p}} V_{\mathrm{k}}(t)\, I_{\mathrm{k}}(t)\, dt$$

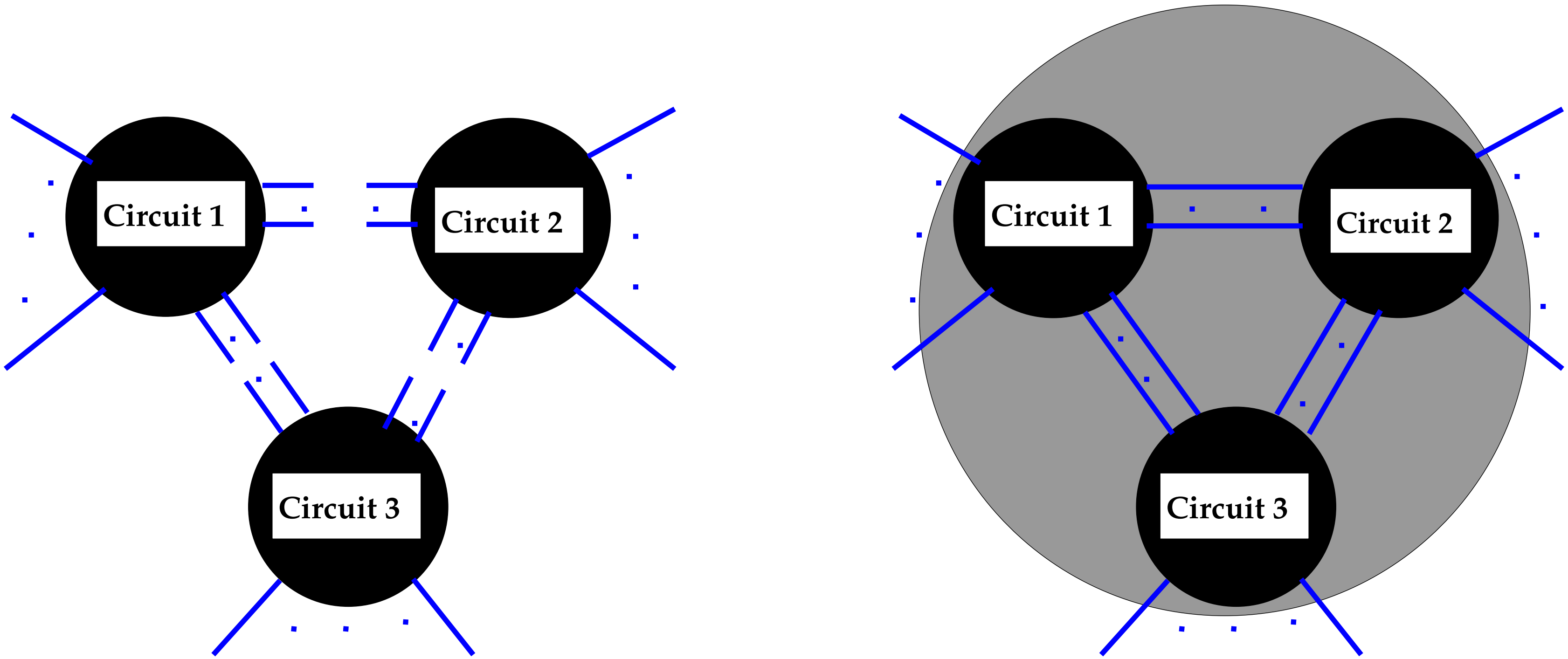
provided these terminals form a port!

Terminals versus ports



Start with 3 circuits, to be interconnected along the indicated terminals.

Terminals versus ports



Interconnection through terminals, energy transfer through ports. **One cannot speak about**

"the energy transferred from circuit 1 to circuit 2"

unless their interconnected terminals form a port.

Inherited properties

Properties of behaviors

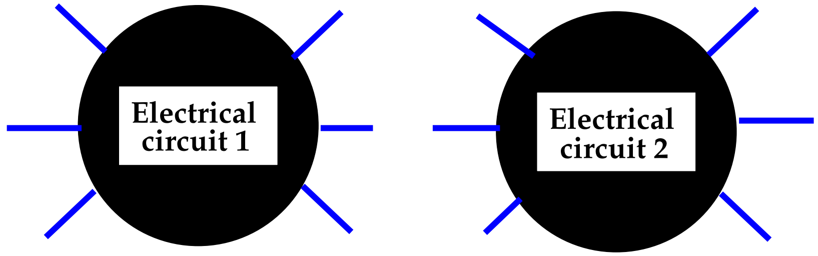
The view of a system as a behavior allows to deduce that important properties are preserved under interconnection, as

- **KVL & KCL**
- **passivity**
- **reciprocity**
- **linearity, time-invariance**
- $\mathscr{B}$ **is the kernel of a system of constant coefficient ODEs**

Properties of behaviors

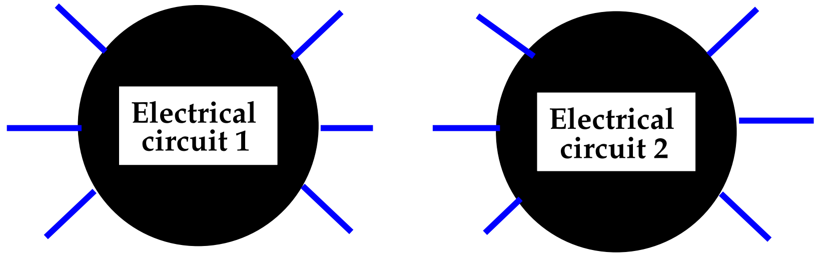
- **KVL & KCL**
- **passivity**

Kirchhoff's laws

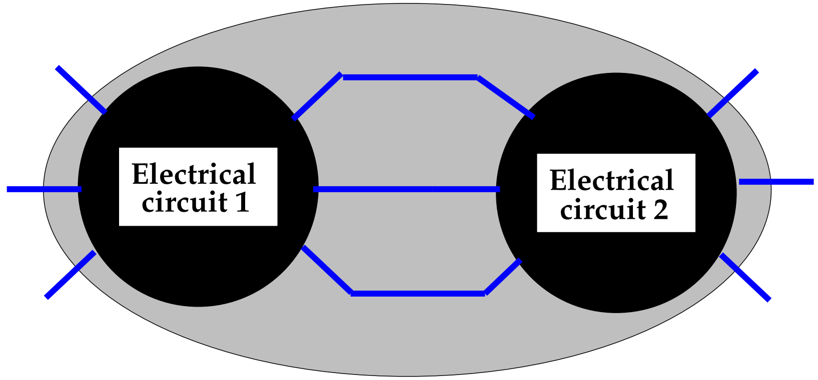


Assume the individual circuits satisfy KVL and KCL (that is, the terminals form a port),

Kirchhoff's laws



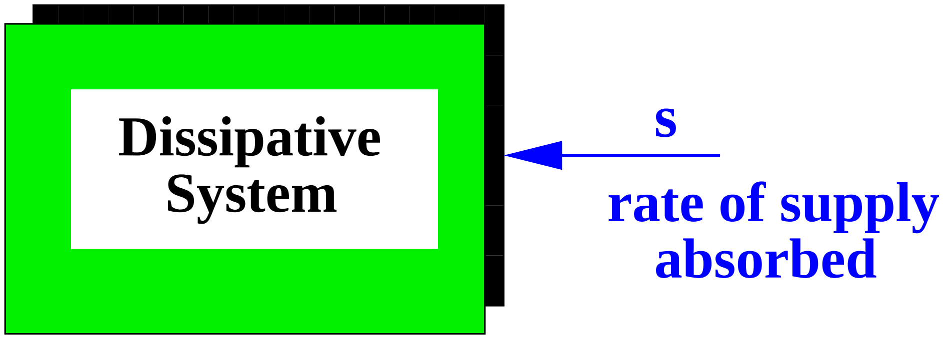
Assume the individual circuits satisfy KVL and KCL (that is, the terminals form a port), then so do the external terminals of the interconnection



$\Rightarrow$ Any interconnection of electrical components satisfies KVL and KCL.

Dissipativity

How should one define dissipativity?



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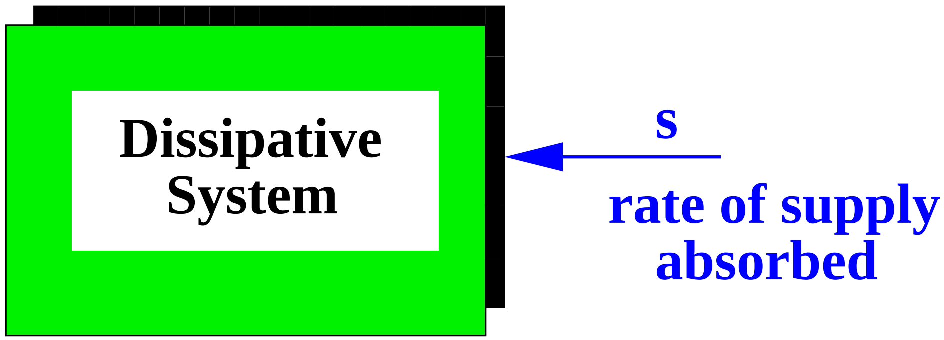
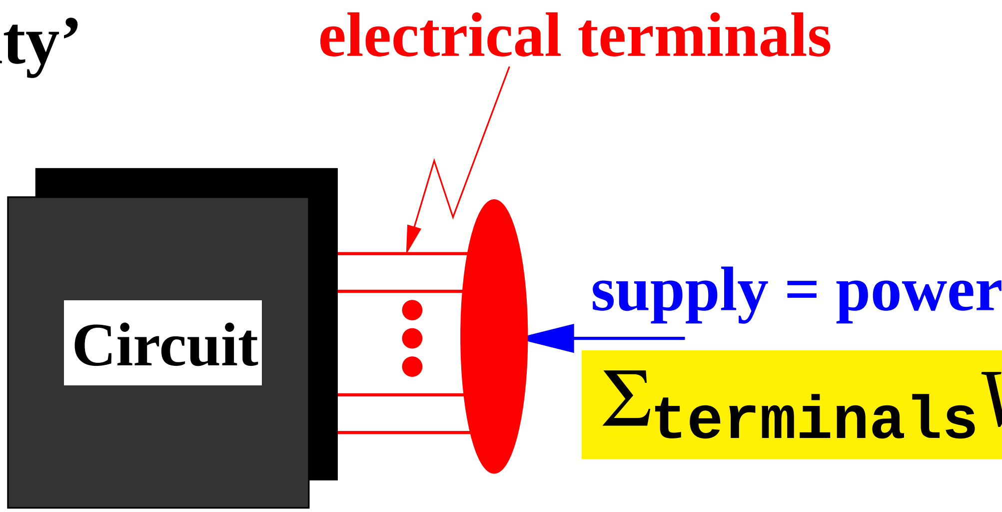


For example, 'passivity'



$$\Sigma_{\mathtt{terminals}} V_{\mathtt{k}} I_{\mathtt{k}}$$

Storage

Consider the system $(\mathbb{R}, \mathbb{R}, \mathscr{S})$

And $(\mathbb{R}, \mathbb{R} \times \mathbb{R}, \mathscr{S}_{\texttt{extended}})$

Assume $(\mathbb{R}, \mathbb{R} \times \mathbb{R}, \mathscr{S}_{\texttt{extended}}) \leadsto (\mathbb{R}, \mathbb{R}, \mathscr{S})$ **after projection.**

Storage

Consider the system $(\mathbb{R}, \mathbb{R}, \mathscr{S})$ $\qquad s \in \mathscr{S}$ **means**
$s : \mathbb{R} \to \mathbb{R}$ **is the supply rate as a function of time.**

And $(\mathbb{R}, \mathbb{R} \times \mathbb{R}, \mathscr{S}_{\texttt{extended}})$ $\qquad (s, V) \in \mathscr{S}_{\texttt{extended}}$ **means**
$s : \mathbb{R} \to \mathbb{R}$, **is the supply rate,**
$V : \mathbb{R} \to \mathbb{R}$ **is the storage as a function of time.**

Definition: Call V **a storage** $:\Leftrightarrow$

$$V(t_2) \leq V(t_1) + \int_{t_1}^{t_2} s(t)\, dt$$

for all $(s, V) \in \mathscr{S}_{\texttt{extended}}$ **and** $t_1 \leq t_2$**.**

Equivalently, $\frac{d}{dt} V \leq s$

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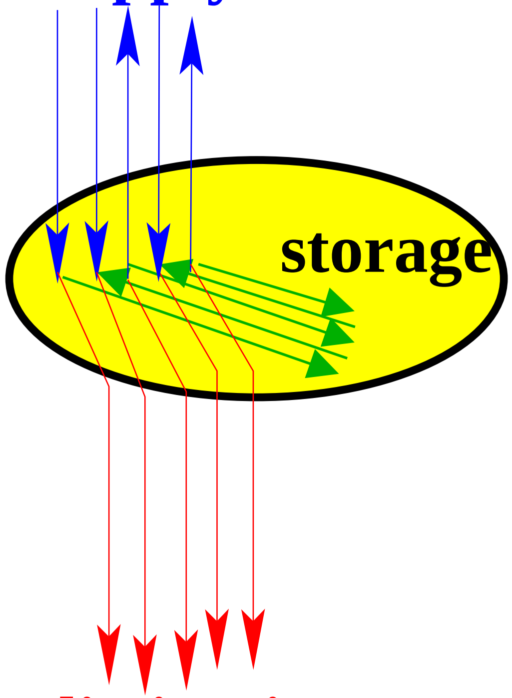

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Equivalently, $\frac{d}{dt} V \leq s$

Definition of dissipativity

Theorem: The following two conditions are equivalent:

- **For all** $s \in \mathscr{S}$, **there exists** $K \in \mathbb{R}$ **such that**

$$-\int_0^T s(t)\,dt < K \text{ for all } T > 0.$$

- **There exists a non-negative storage**

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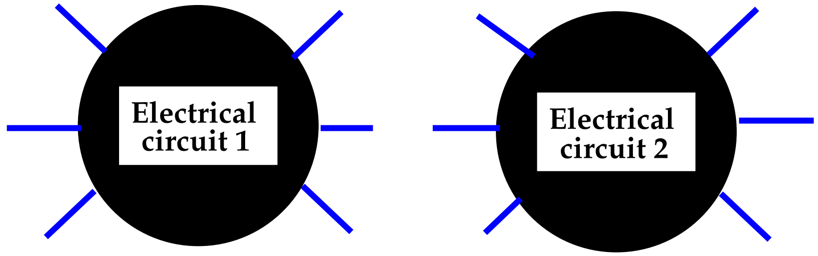
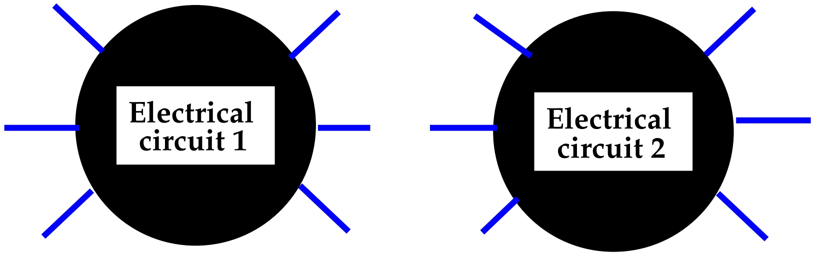
With $s = \Sigma_{\texttt{terminals}} V_k I_k$**, either of these two equivalent conditions leads to a good definition of passivity for circuits.**

Passivity of electrical circuits

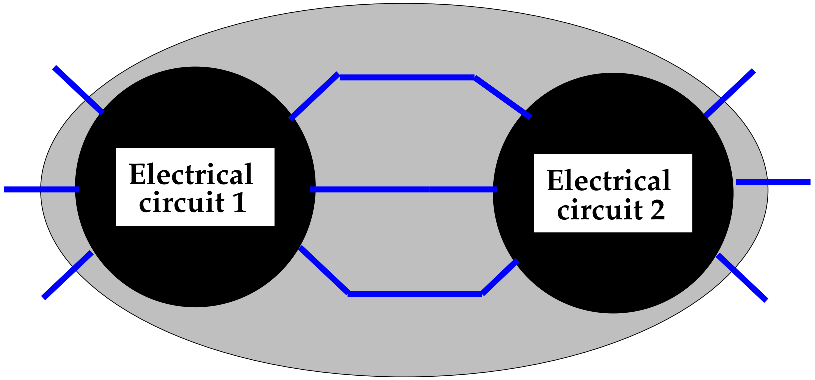


Assume the individual circuits are passive,

Passivity of electrical circuits



Assume the individual circuits are passive, then so is the interconnection

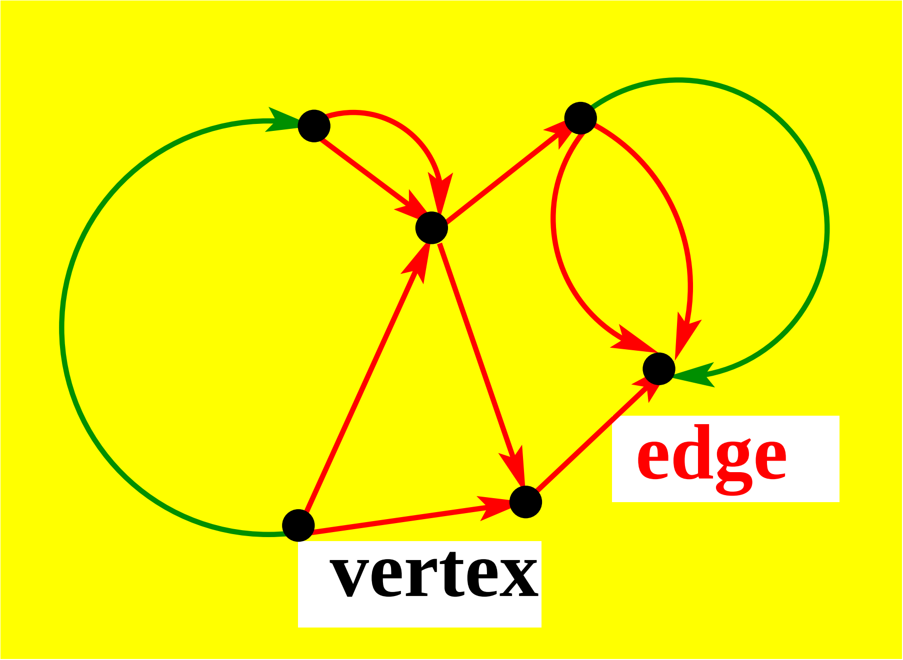


$\Rightarrow$ An interconnection of passive electrical components is passive.

Reflections

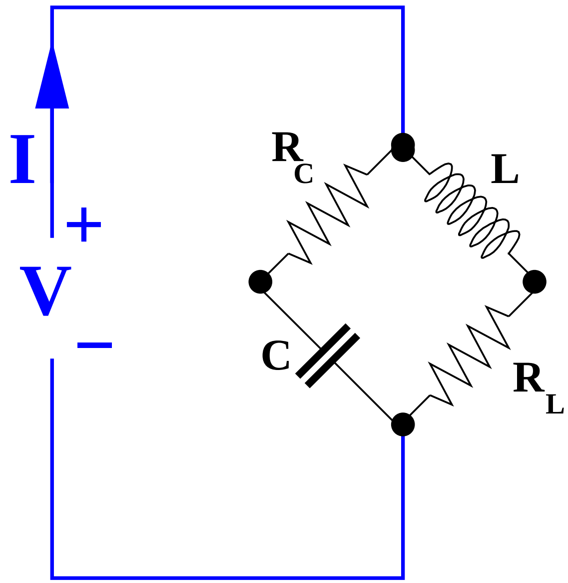
Classical circuit theory

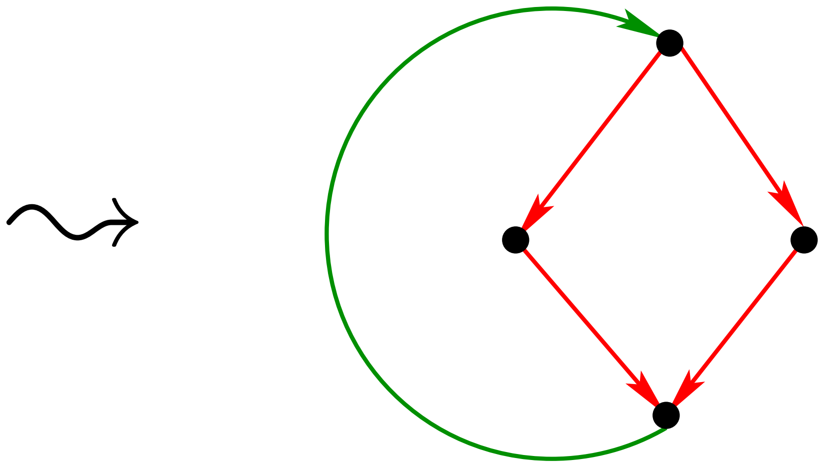
Classical circuit theory evolves around a digraph with 2-terminal elements or external ports in the edges and connections in the vertices.



Classical circuit theory

Classical circuit theory evolves around a digraph with 2-terminal elements or external ports in the edges and connections in the vertices. For example,





Limitations

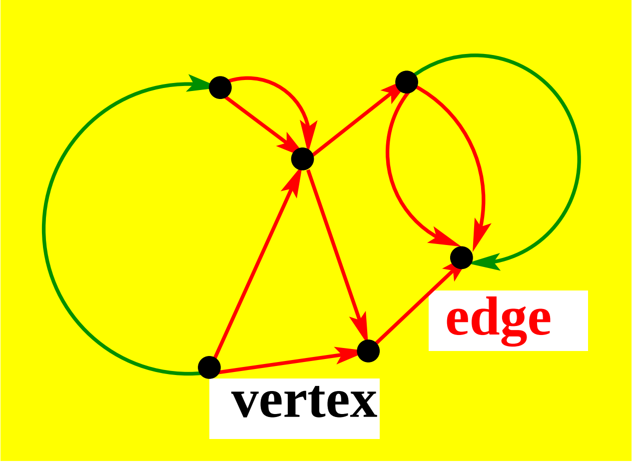
- Deals with 2-terminal ports (mainly with 2-terminal elements) and with 2-terminal external ports.
- Is port oriented, and does not articulate that terminals, not ports, make the interconnections.
- The external ports are especially bothersome: how do we know what the environment will be?
- It is not hierarchical.

The key is to use a (di)graph with leaves rather than a digraph.

Vertices and edges

In circuit graphs,

subsystems are in the edges , connections are in the vertices



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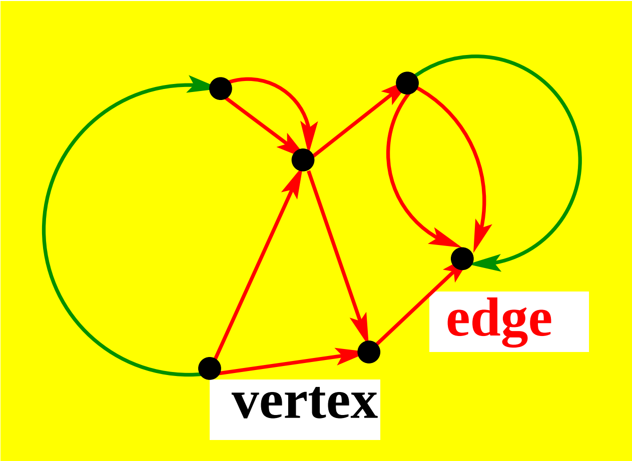

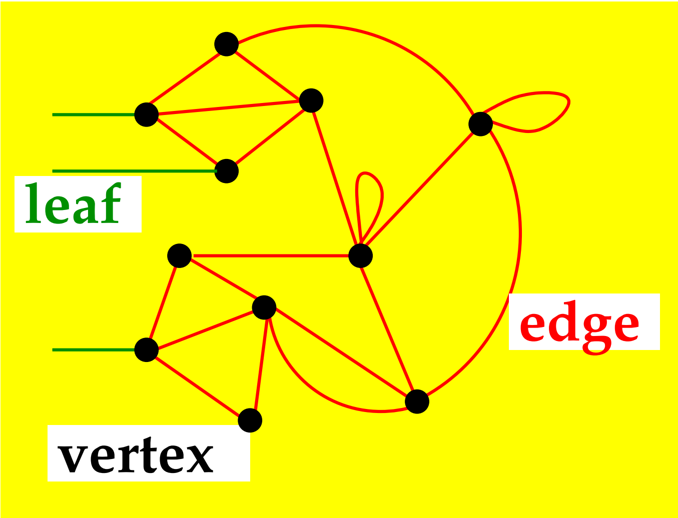

Contrast with tearing, zooming, linking:

subsystems are in the vertices , connections are in the edges

Ceterum censeo

The input/output approach as the primary and universal view of open systems is a misconception.

Physical systems are not signal processors !

Ceterum censeo

The input/output approach as the primary and universal view of open systems is a misconception.

Physical systems are not signal processors !

Signals and Systems $\rightsquigarrow$ **Signals and Signal Processors!**

Three thoughts to take home

1. **A dynamical system = a family of trajectories.**
2. **Interconnection = variable sharing**
3. **Control = interconnection**

Want to read about it? See

The behavioral approach to open and interconnected systems,

***Control Systems Magazine*, volume 27, pages 46-99, 2007.**

The lecture frames are available from/at

`http://www.esat.kuleuven.be/∼jwillems`

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