ENERGY FLOW in INTERCONNECTED SYSTEMS

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Theme

How are open systems formalized?

How are systems interconnected?

How is energy transferred between systems?

Are energy transfer and interconnection related?

Theme

How are open systems formalized?

How are systems interconnected?

How is energy transferred between systems?

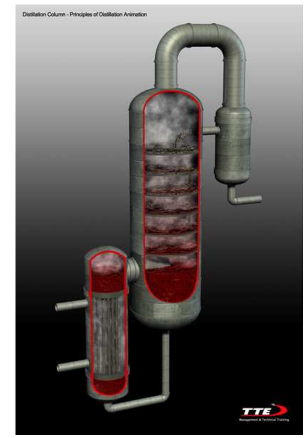
Are energy transfer and interconnection related?

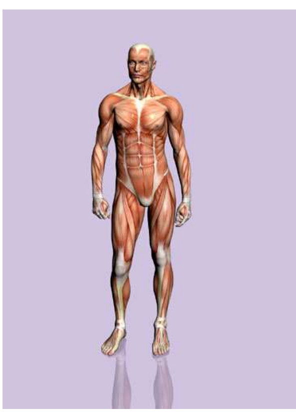
We deal only with electrical circuits and 1-dimensional mechanical systems.

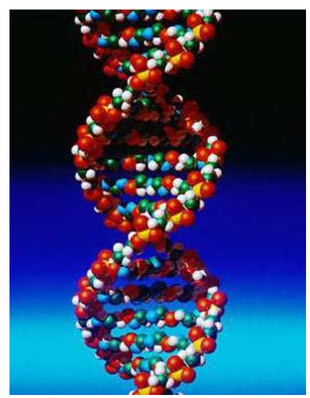
Other applications: hydraulic systems,
chemical systems,
thermal systems, ...

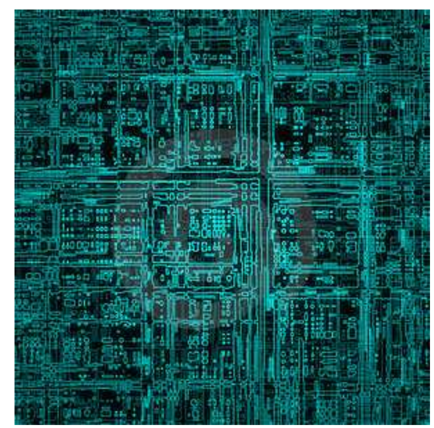
SYSTEMS


OIL REFINERY (GVG / PD)









Features

- **Open**
- **Interconnected**
- **Modular**
- **Dynamic**

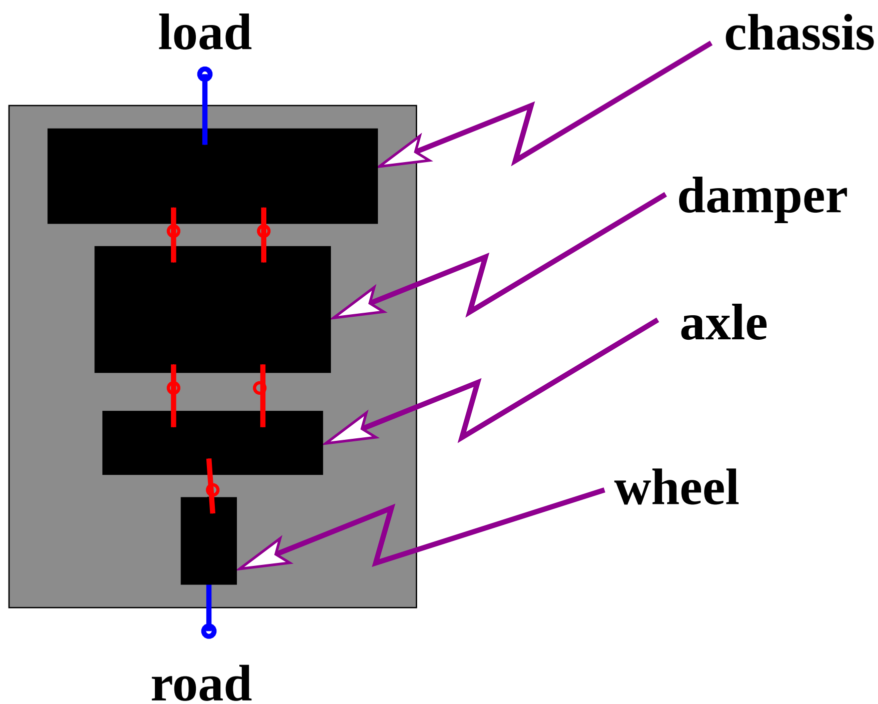
Features

- Open
- Interconnected
- Modular
- Dynamic

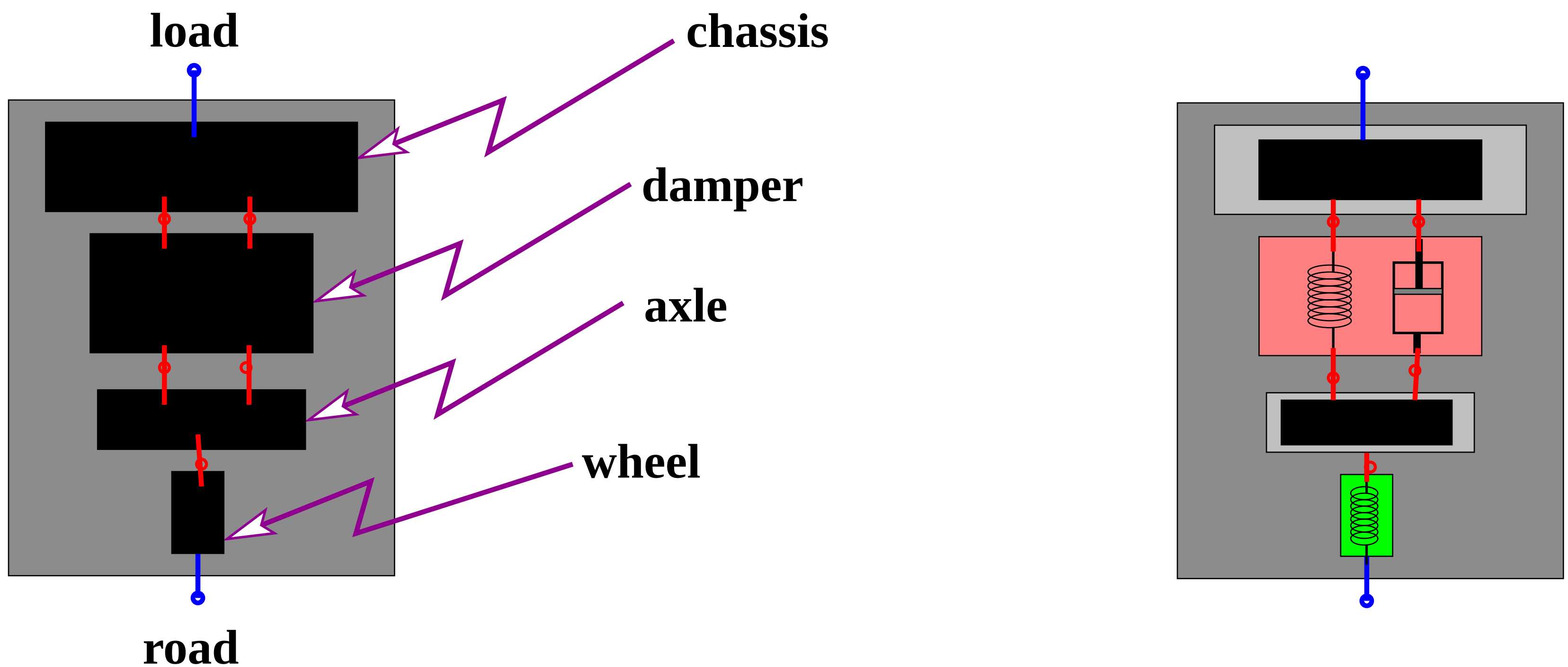
The ever-increasing computing power allows to model complex interconnected systems accurately by tearing, zooming, and linking.

$\leadsto$ **Simulation, model based design, ...**

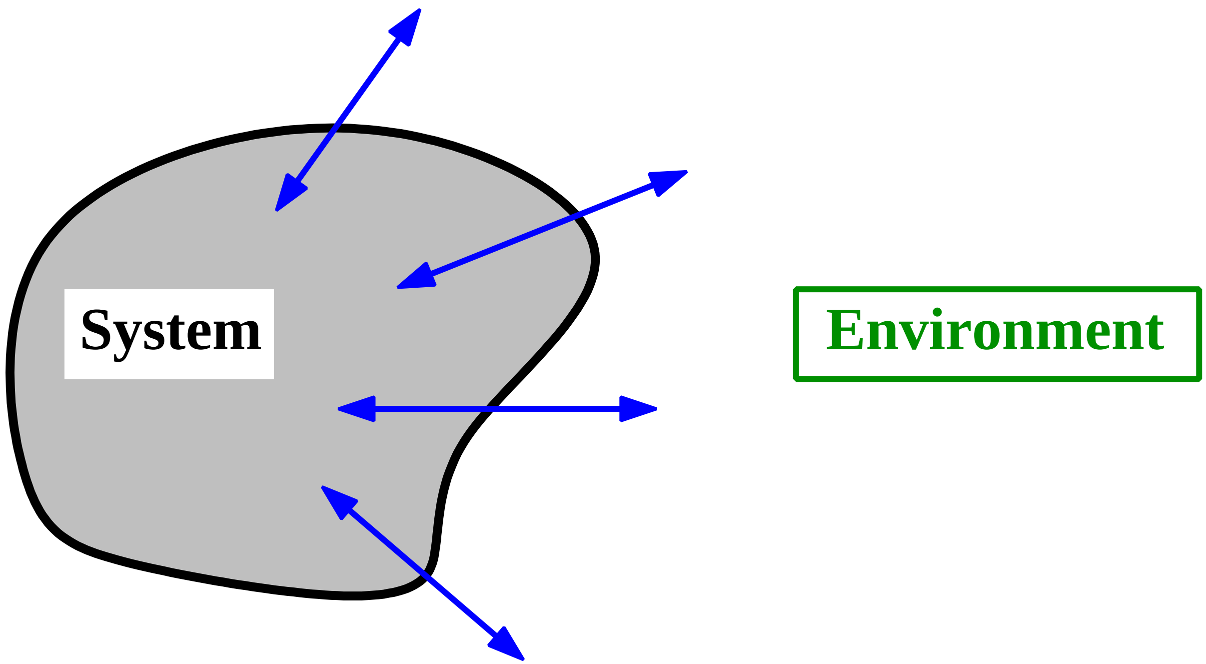
Example: A ‘quarter car’



Example: A ‘quarter car’

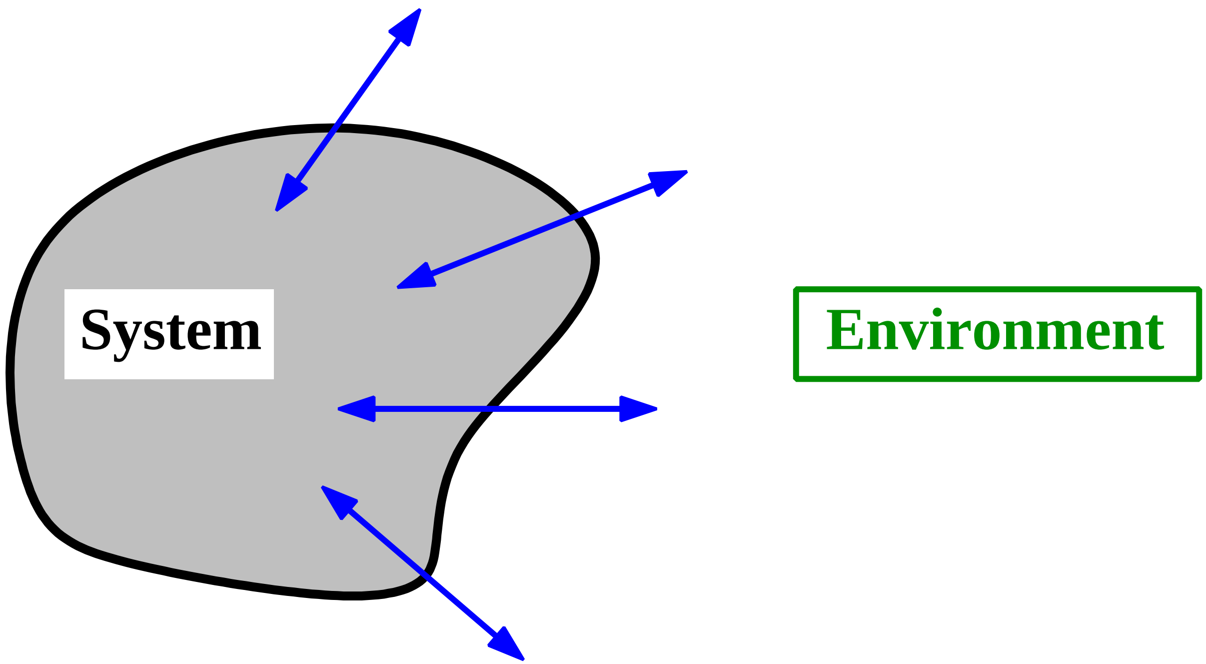


Open systems



Systems are ‘open’, they interact with their environment.

Open systems

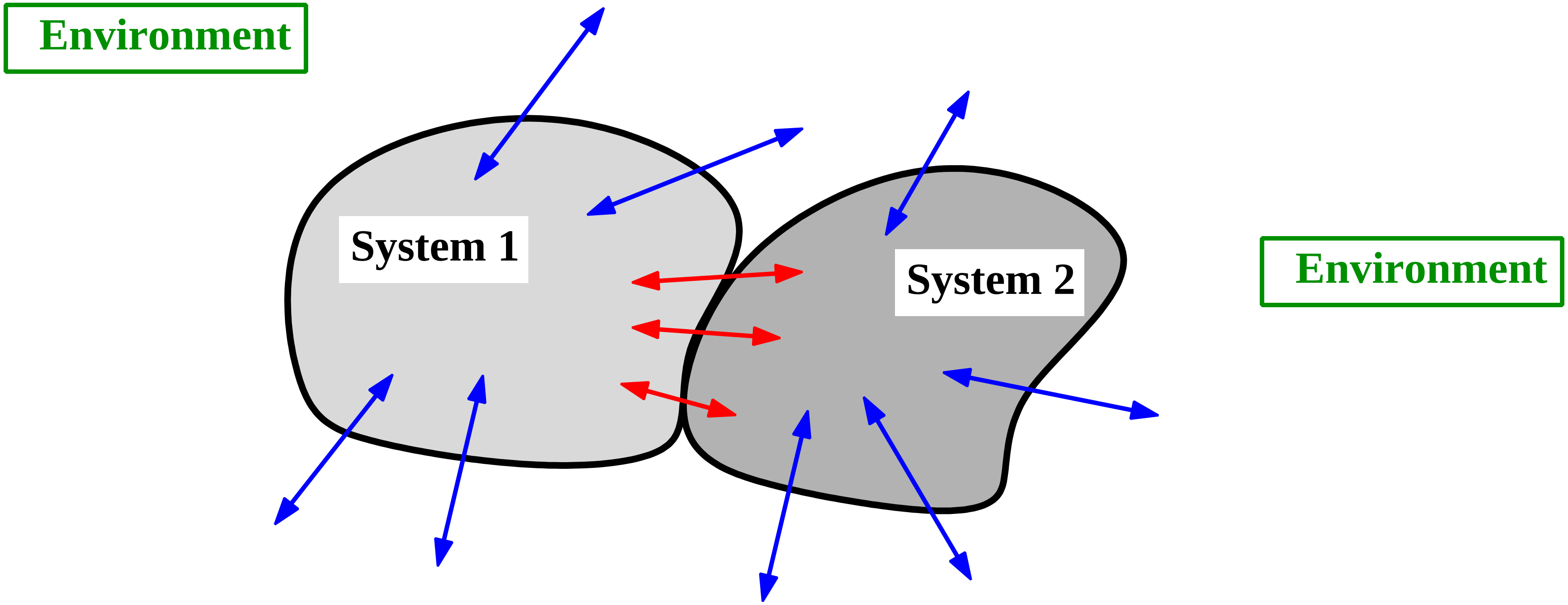


Systems are 'open', they interact with their environment.

How are such systems formalized?

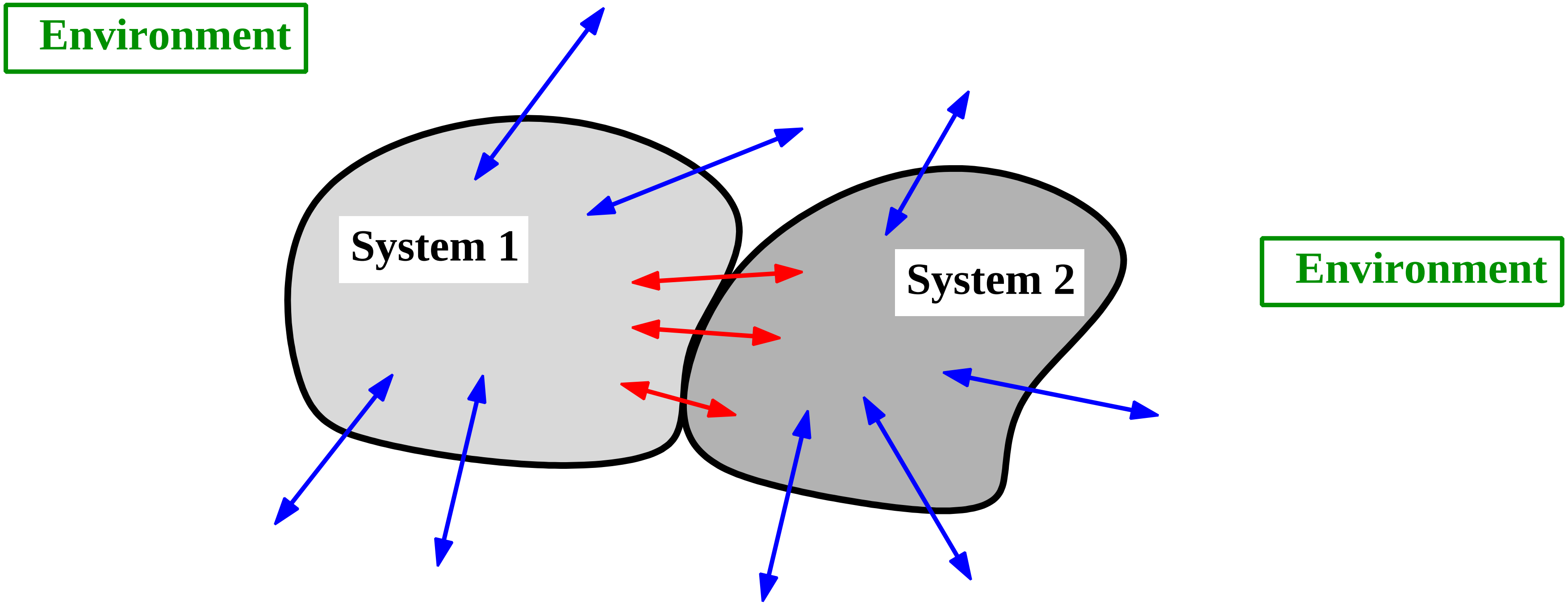
How is energy transferred from the environment to a system?

Interacting systems



Interconnected systems interact.

Interacting systems



Interconnected systems interact.

How is this interaction formalized?

How is energy transferred between systems?

Are energy transfer and interconnection related?

Motivation

The ever-increasing computing power allows to model complex interconnected systems accurately by tearing, zooming, and linking.

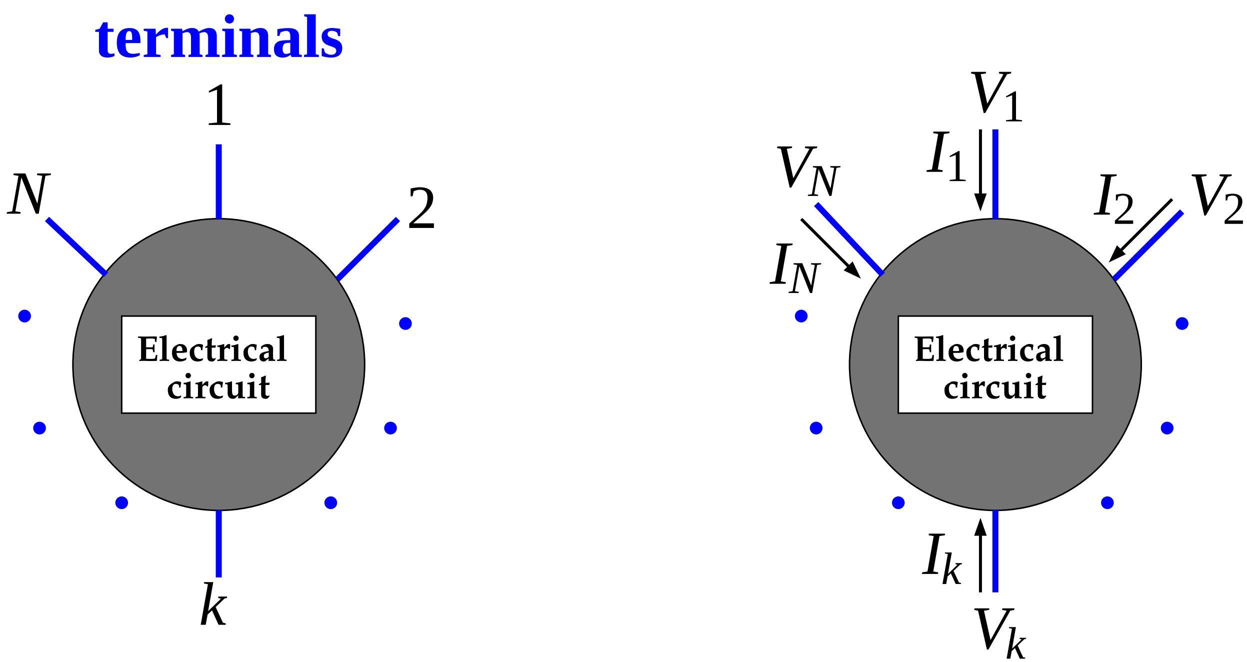
$\rightsquigarrow$ **Simulation, model based design, ...**

Requires the right mathematical concepts for

- **dynamical system,**
- **interconnection,**
- **interconnection architecture.**

SYSTEMS with TERMINALS

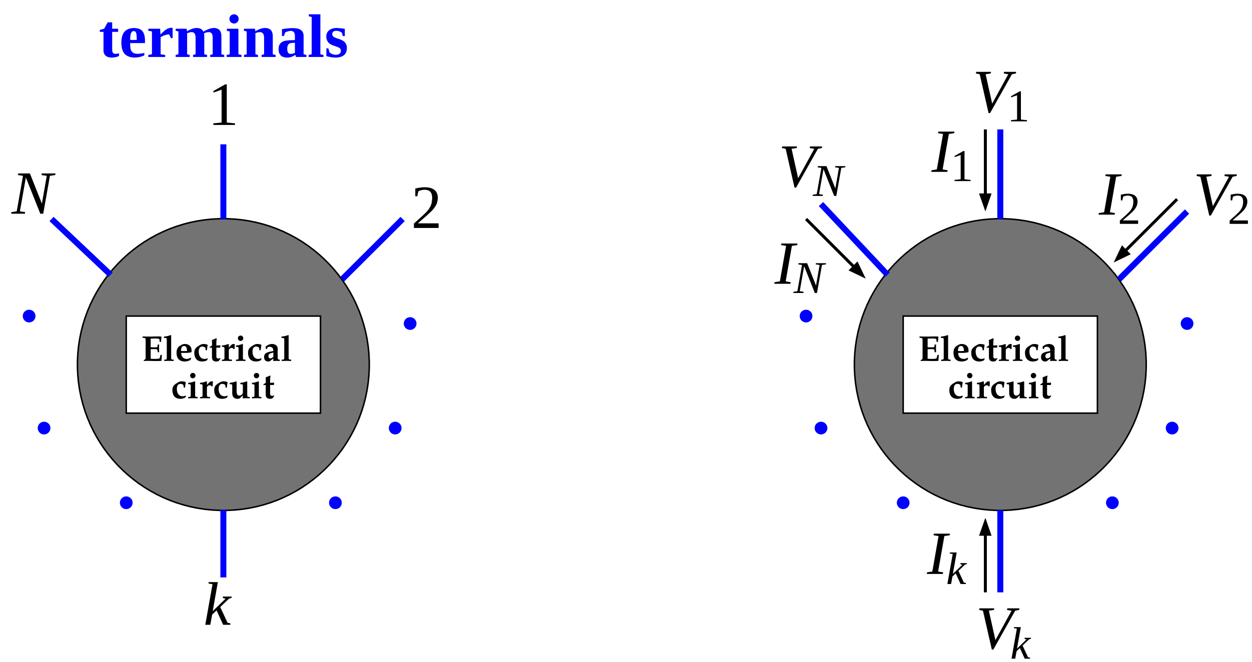
Electrical circuit



At each terminal:

a potential (!) and a current (counted > 0 into the circuit),

Electrical circuit



At each terminal:

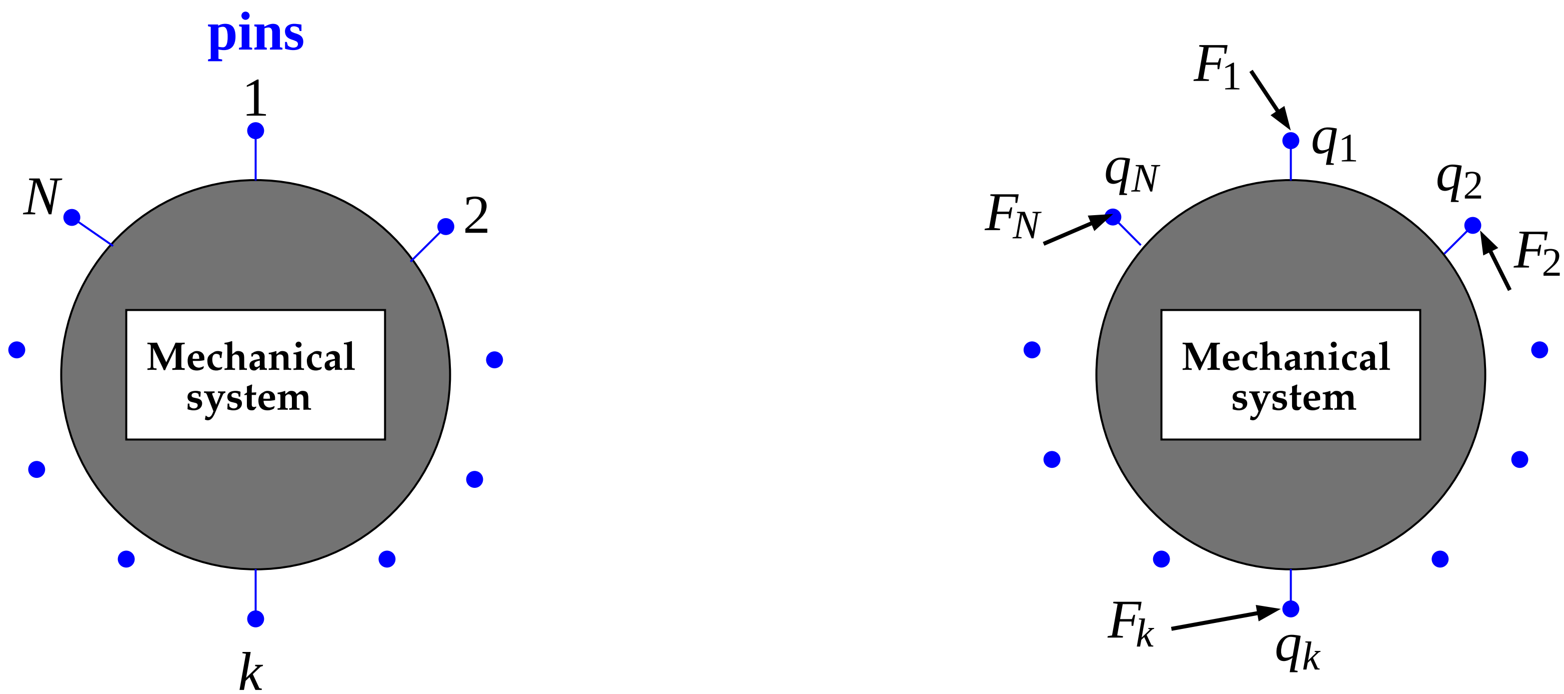
a potential (!) and a current (counted > 0 into the circuit),

$\rightsquigarrow$ **behavior** $\mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}$.

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

this potential/current trajectory is compatible with the circuit architecture and its element values.

Mechanical device



At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

More generally, a position, force, angle, and torque.

Other domains

- **Thermal systems:**

  At each terminal: a temperature and a heat flow.

- **Hydraulic systems:**

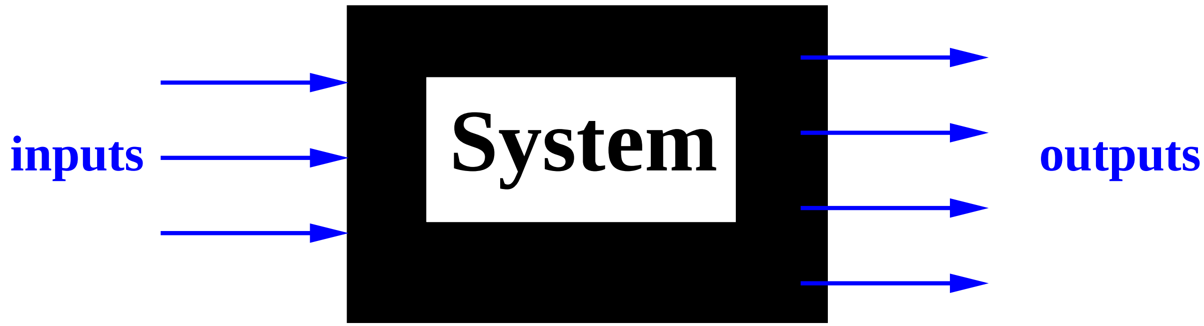
  At each terminal: a pressure and a mass flow.

- **Multidomain systems:**

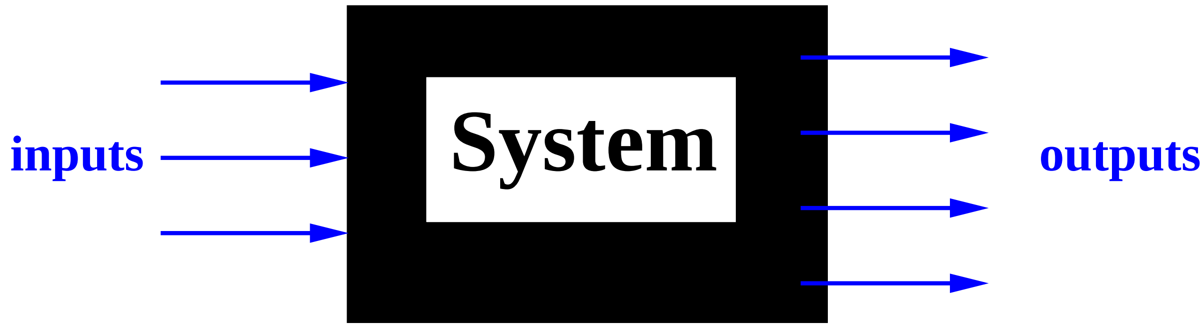
  Systems with terminals of different types, as motors, pumps, etc.

- **...**

Contrast with input/output systems



Contrast with input/output systems



Input/output thinking is *inappropriate* for physical systems.

A physical system is not a signal processor.

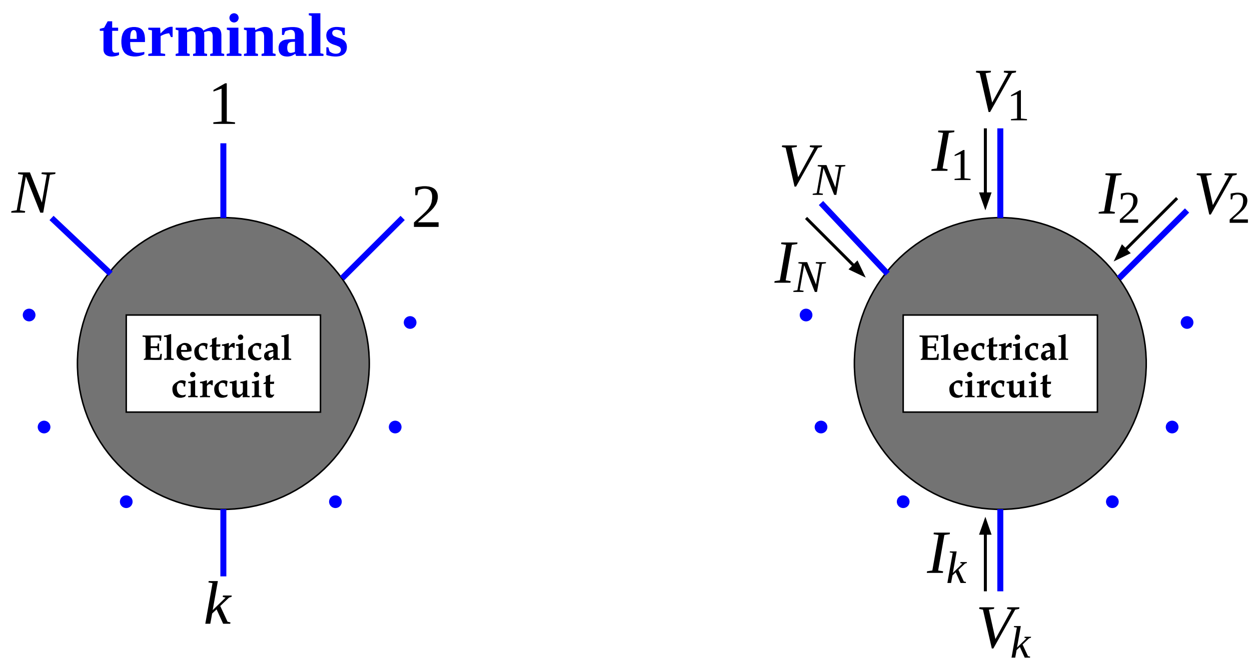
This observation $\rightsquigarrow$ behavioral approach with $\mathscr{B}$ central.

Early sources:

Brockway McMillan

Robert Newcomb

KVL and KCL

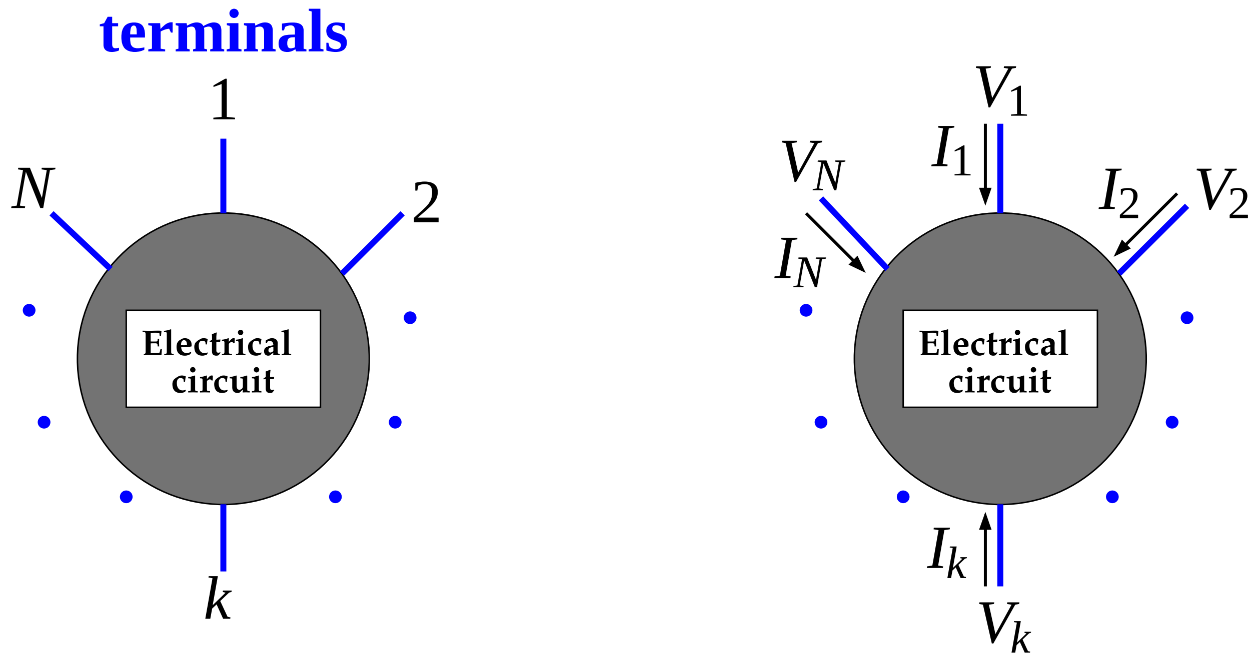


Kirchhoff's voltage law (KVL):

$$\llbracket (V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B} \textbf{ and } \alpha : \mathbb{R} \to \mathbb{R} \rrbracket$$
$$\Rightarrow \llbracket (V_1 + \alpha, V_2 + \alpha, \ldots, V_N + \alpha, I_1, I_2, \ldots, I_N) \in \mathscr{B} \rrbracket.$$

Equivalently, the behavioral equations contain the V_i's only through the potential differences $V_i - V_j$.

KVL and KCL



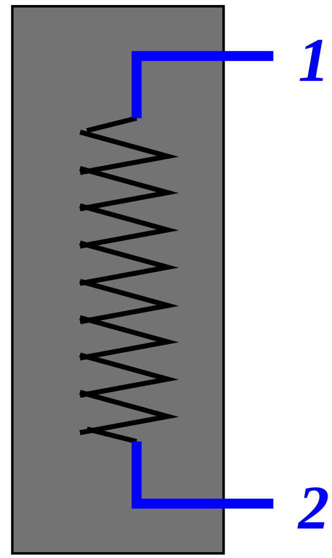
Kirchhoff's voltage law (KVL):

$$[\![(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B} \textbf{ and } \alpha : \mathbb{R} \to \mathbb{R}]\!]$$
$$\Rightarrow [\![(V_1 + \alpha, V_2 + \alpha, \ldots, V_N + \alpha, I_1, I_2, \ldots, I_N) \in \mathscr{B}]\!].$$

Kirchhoff's current law (KCL):

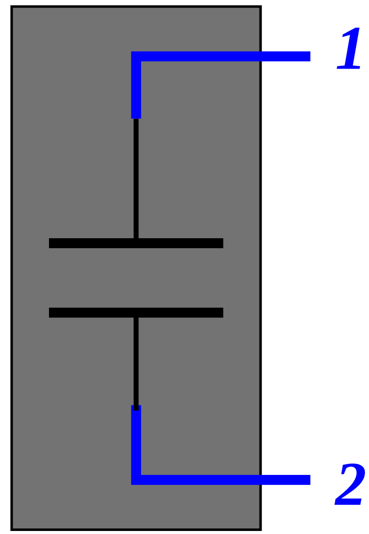
$$[\![(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}]\!] \Rightarrow [\![I_1 + I_2 + \cdots + I_N = 0]\!].$$

Examples



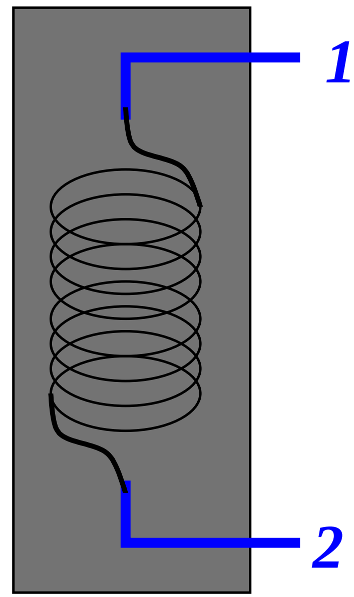
$$V_1 - V_2 = RI_1 \qquad I_1 + I_2 = 0$$

Examples



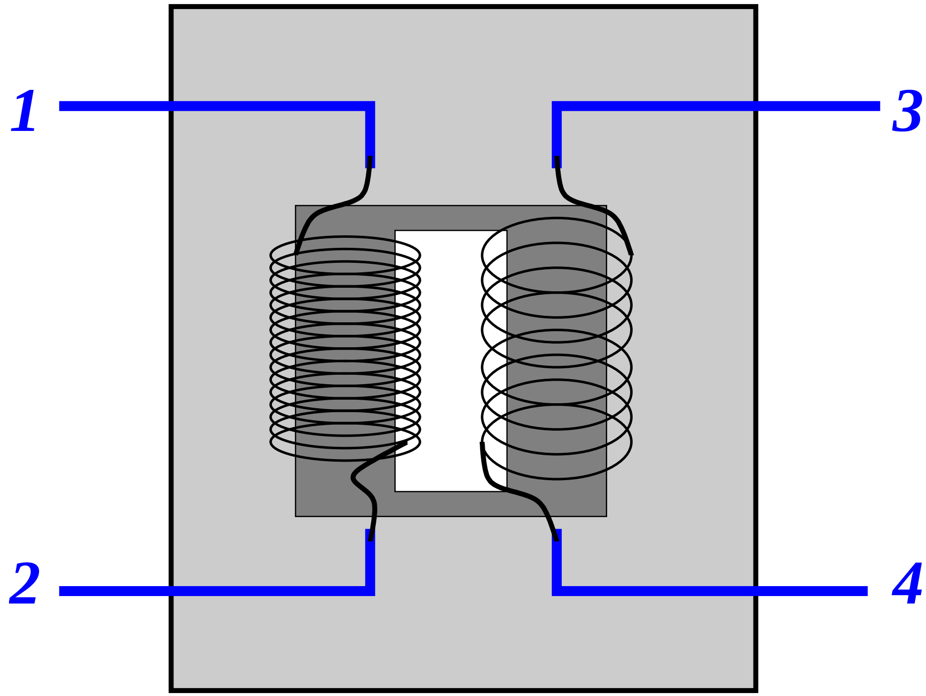
$$C\frac{d}{dt}\left(V_1 - V_2\right) = I_1 \qquad I_1 + I_2 = 0$$

Examples



$$L\frac{d}{dt}I_1 = V_1 - V_2 \qquad I_1 + I_2 = 0$$

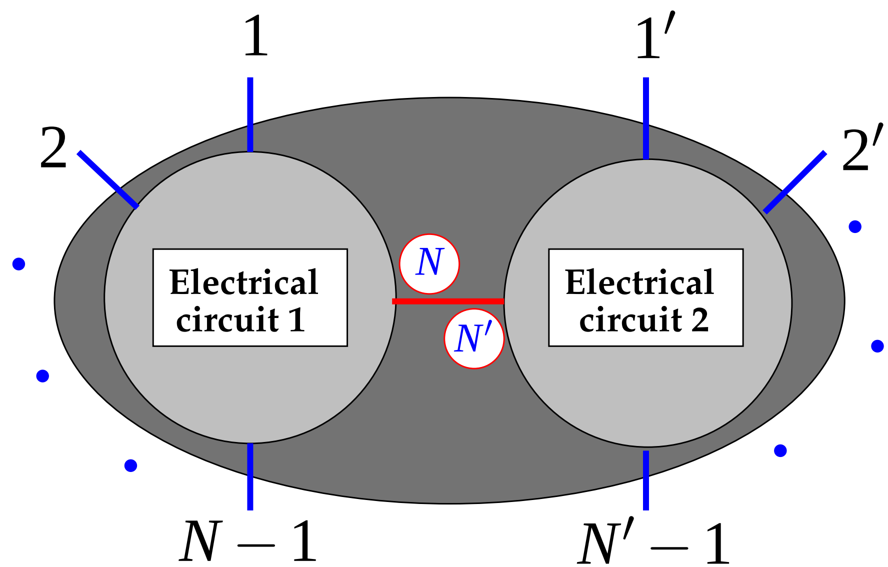
Examples



$$V_1 - V_2 = n\left(V_3 - V_4\right), -nI_1 = I_3 \qquad I_1 + I_2 = 0, I_3 + I_4 = 0$$

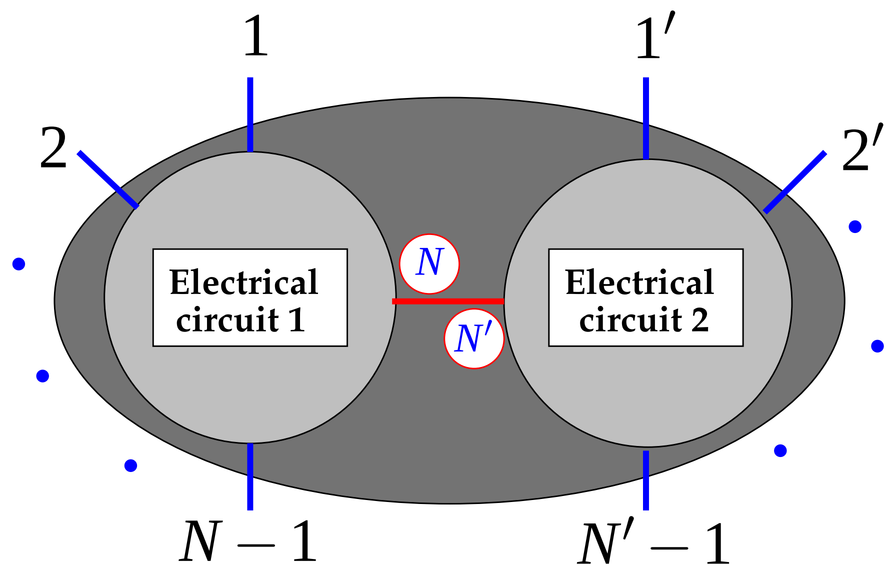
INTERCONNECTION

Interconnection of circuits



$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Interconnection of circuits



$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Behavior after interconnection:

$$\mathscr{B}_1 \sqcap \mathscr{B}_2$$

$$:= \{(V_1, \ldots, V_{N-1}, V_{1'}, \ldots, V_{N'-1}, I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}) \mid$$

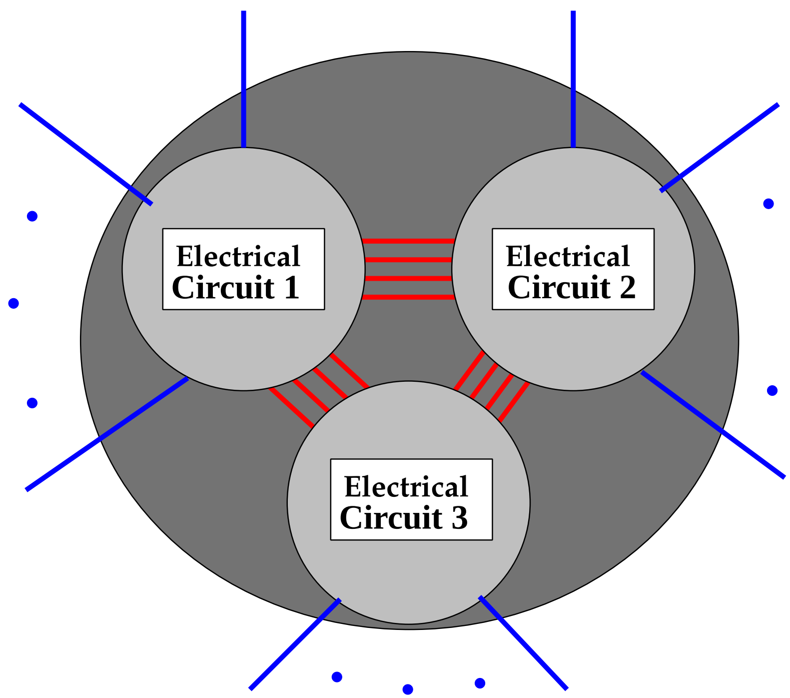
$$\exists V, I \textbf{ such that}$$

$$(V_1, \ldots, V_{N-1}, V, I_1, \ldots, I_{N-1}, I) \in \mathscr{B}_1 \textbf{ and}$$

$$(V_{1'}, \ldots, V_{N'-1}, V, I_{1'}, \ldots, I_{N'-1}, -I) \in \mathscr{B}_2\}.$$

Interconnection of circuits

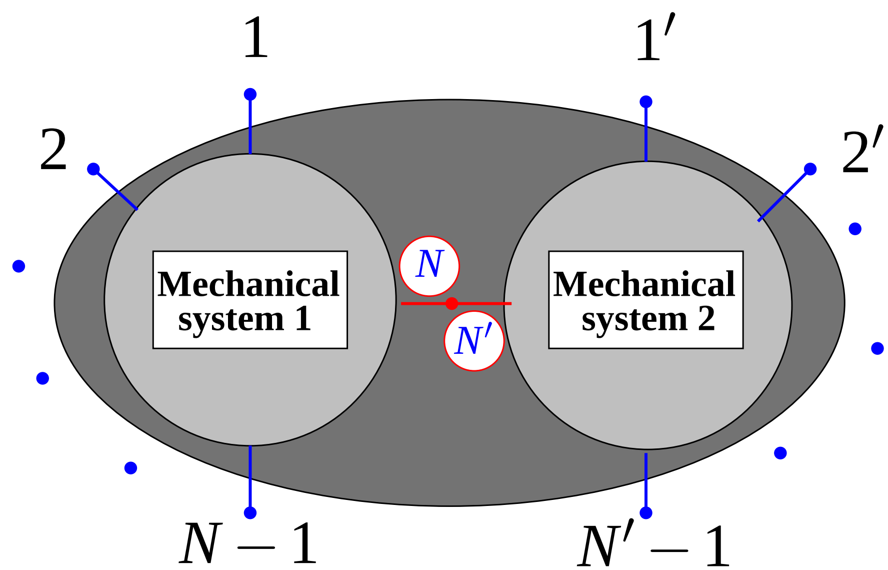
⇝ **more terminals and more circuits connected**



Preservation of properties

- $[\![\mathscr{B}_1, \mathscr{B}_2$ **satisfies KVL** $]\!] \Rightarrow [\![$**so does** $\mathscr{B}_1 \sqcap \mathscr{B}_2]\!]$
- $[\![\mathscr{B}_1, \mathscr{B}_2$ **satisfies KCL** $]\!] \Rightarrow [\![$**so does** $\mathscr{B}_1 \sqcap \mathscr{B}_2]\!]$
- $\cdots$

Interconnection of 1-D mechanical systems



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Other terminal types

- **Thermal systems:**
  At each terminal: a temperature and a heat flow.

$$T_N = T_{N'} \quad \textbf{and} \quad Q_N + Q_{N'} = 0.$$

- **Hydraulic systems:**
  At each terminal: a pressure and a mass flow.

$$p_N = p_{N'} \quad \textbf{and} \quad f_N + f_{N'} = 0.$$

- **...**

Sharing variables

$$
\begin{aligned}
V_N = \; & V_{N'} \quad \textbf{and} \quad & I_N + I_{N'} = \; & 0, \\
q_N = \; & q_{N'} \quad \textbf{and} \quad & F_N + F_{N'} = \; & 0, \\
T_N = \; & T_{N'} \quad \textbf{and} \quad & Q_N + Q_{N'} = \; & 0, \\
p_N = \; & p_{N'} \quad \textbf{and} \quad & f_N + f_{N'} = \; & 0, \\
& \vdots & &
\end{aligned}
$$

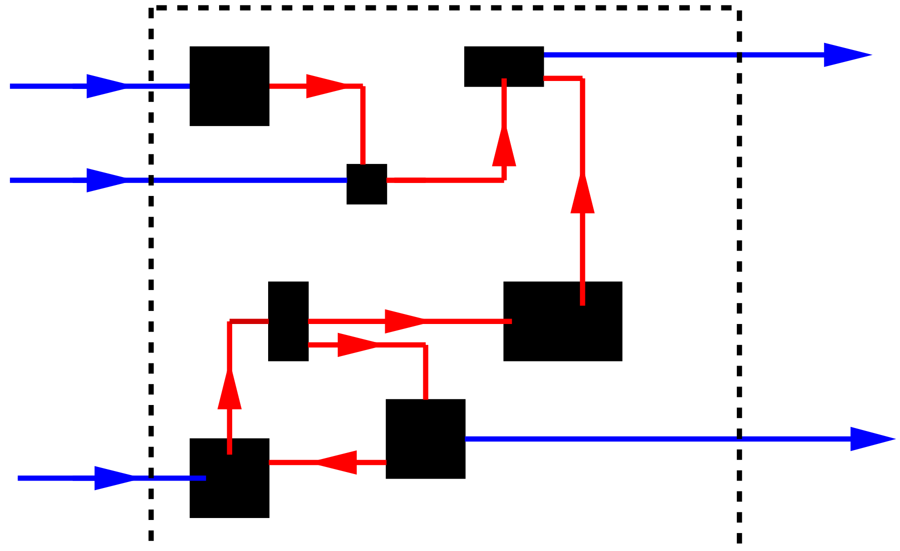
Interconnection means variable sharing.

Sharing variables

$$
\begin{aligned}
V_N = \ V_{N'} \quad &\textbf{and} \quad I_N + I_{N'} = \ 0, \\
q_N = \ q_{N'} \quad &\textbf{and} \quad F_N + F_{N'} = \ 0, \\
T_N = \ T_{N'} \quad &\textbf{and} \quad Q_N + Q_{N'} = \ 0, \\
p_N = \ p_{N'} \quad &\textbf{and} \quad f_N + f_{N'} = \ 0, \\
&\vdots
\end{aligned}
$$

‘through’ and ‘across’?
‘effort’ and ‘flow’?
product = power?

Contrast with signal flow graphs



Not appropriate for describing the interaction of physical systems.

A physical system is not a signal processor.

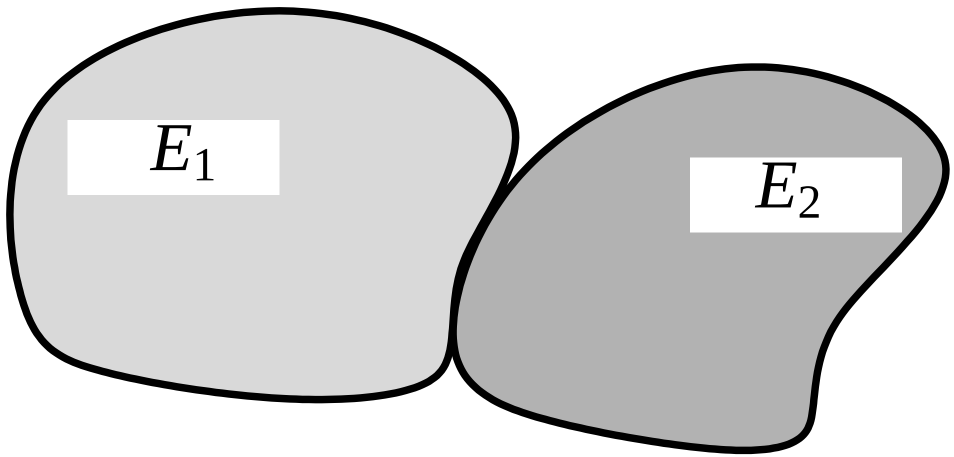
ENERGY TRANSFER

Energy as an extensive quantity

Our intuition has been built to think of energy as an extensive quantity,

Energy as an extensive quantity

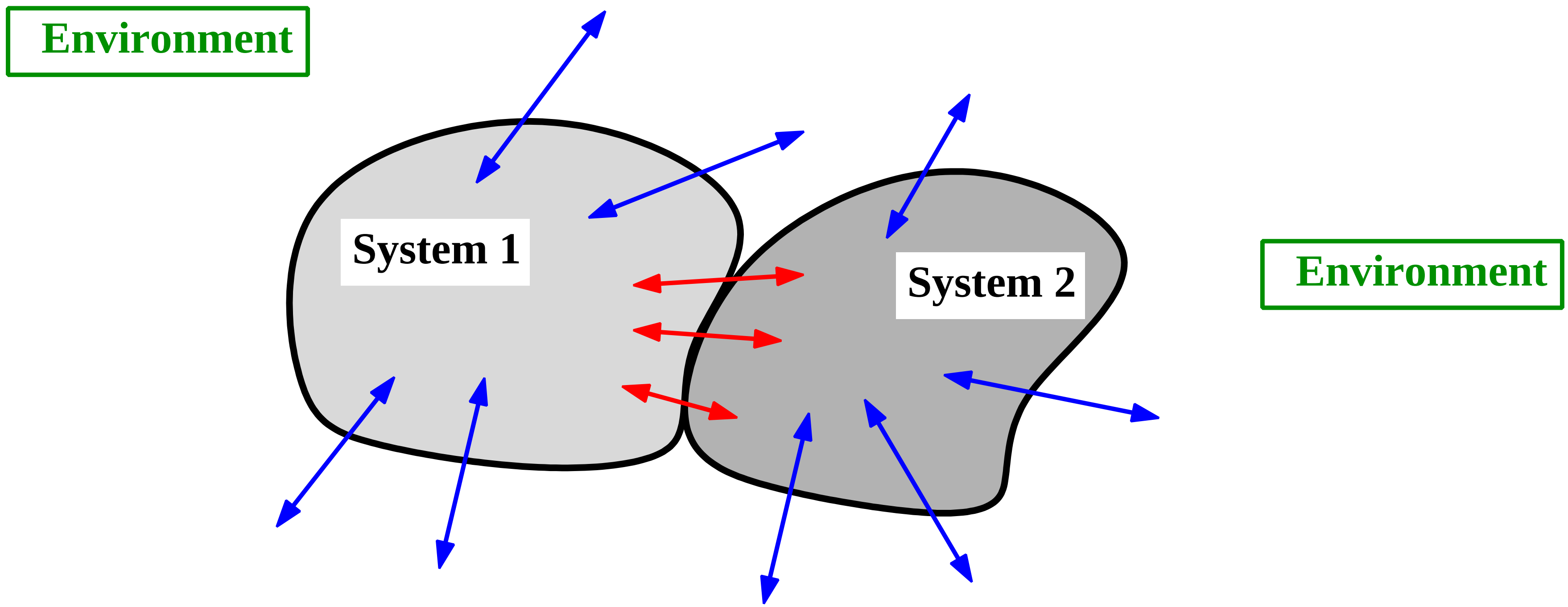
Our intuition has been built to think of energy as an extensive quantity, meaning that it is additive



$$E_{\text{total}} = E_1 + E_2.$$

Energy as an extensive quantity

Our intuition has been built to think of energy as an extensive quantity,

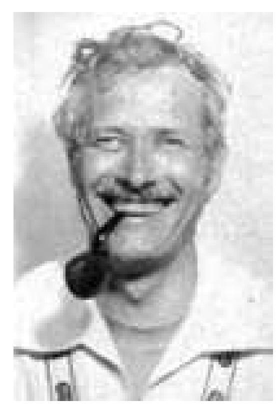


that flows in and out and between systems along the interconnected interfaces (terminals).

Energy as an extensive quantity

Our intuition has been built to think of energy as an extensive quantity, that flows in and out and between systems along the interconnected terminals).

Some methodologies for modeling interconnected systems, as bond-graph modeling and port-Hamiltonian systems, are based on this thinking.

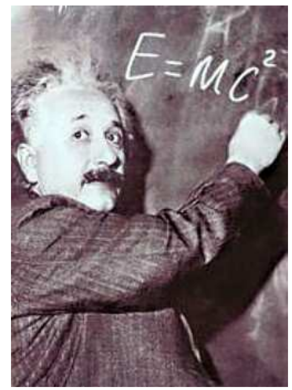


Henry Paynter

Arjan van der Schaft

Energy as an extensive quantity

In many situations, this view is correct. Mass, volume, and energy in the form of heat are extensive quantities.



Energy as an extensive quantity

In many situations, this view is correct. Mass, volume, and energy in the form of heat are extensive quantities.

However, energy is more subtle for other forms.

Kinetic energy is not additive.
Same with energy due to gravitational attraction,
due Coulomb forces, etc.
Heat is a special, extensive, form of energy.

Energy as an extensive quantity

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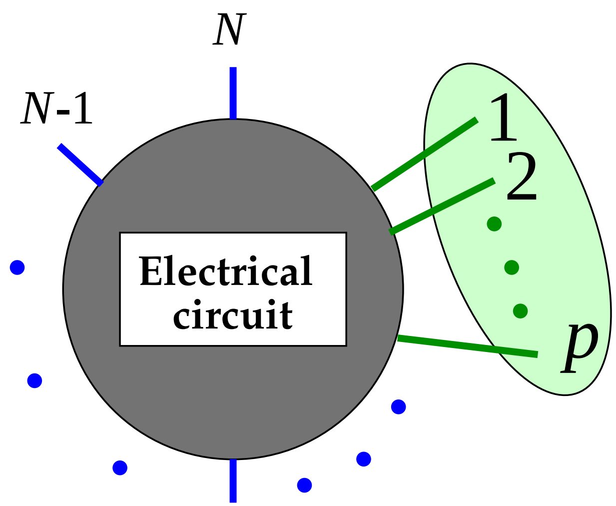
However, energy is more subtle for other forms.

Kinetic energy is not additive.
Same with energy due to gravitational attraction,
due Coulomb forces, etc.
Heat is a special, extensive, form of energy.

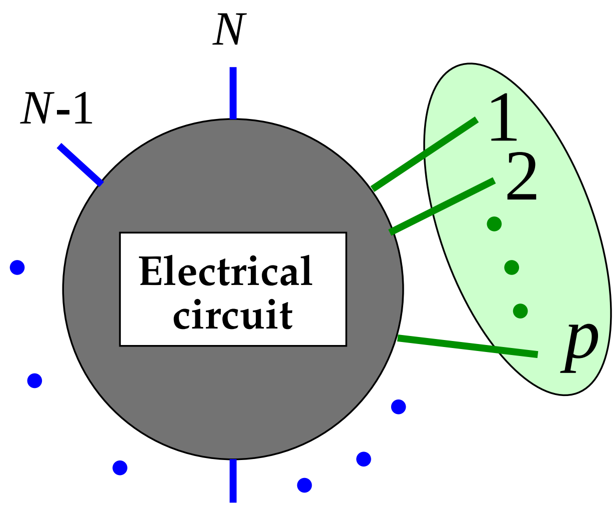
Energy and power are not a ‘local’ quantities.
They involve ‘action at a distance’.

PORTS

Ports



Ports



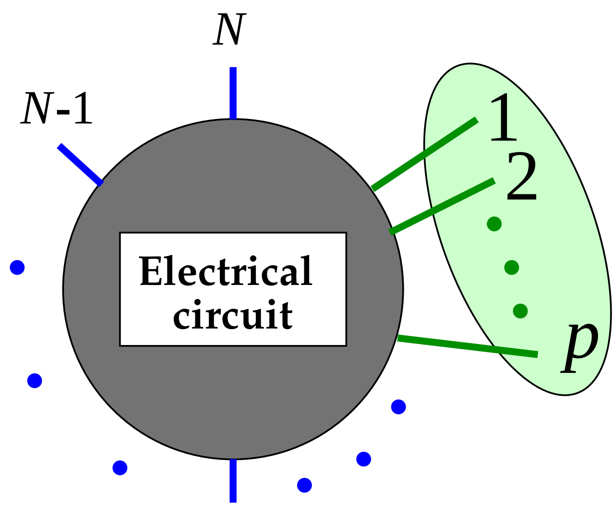
Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

$$(V_1, \ldots, V_p, V_{p+1}, \ldots, V_N, I_1, \ldots, I_p, I_{p+1}, \ldots, I_N) \in \mathscr{B}$$

$$\Rightarrow \quad I_1 + \cdots + I_p = 0.$$ ***'port KCL'***.

(KVL &) KCL $\Rightarrow$ **all terminals together form a port.**

Ports



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

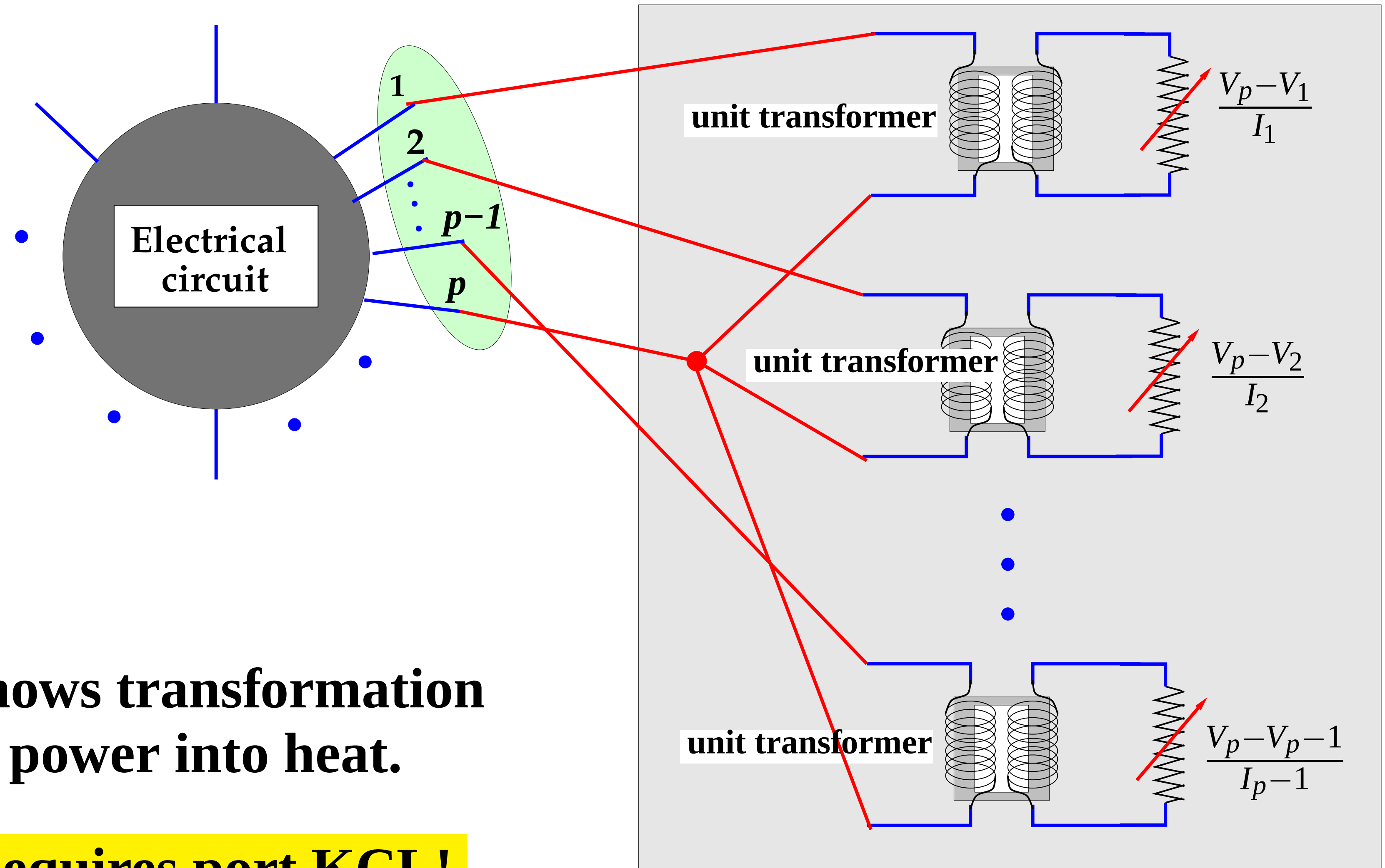
power in along these terminals = $V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)$,

energy in = $$\int_{t_1}^{t_2} (V_1(t)I_1(t) + \cdots + V_p(t)I_p(t))\, dt.$$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

Dissipation into heat

Justification:



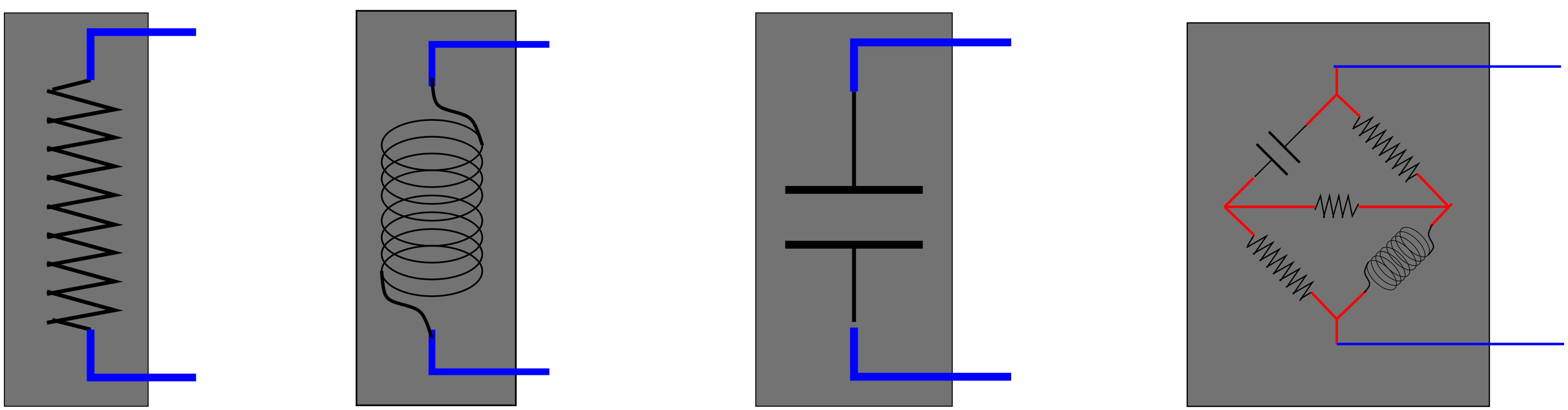
Shows transformation of power into heat.

Requires port KCL!

Examples

2-terminal 1-port devices:

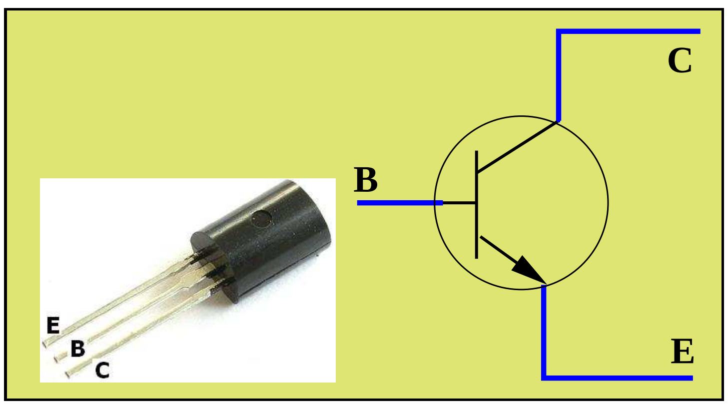
resistors, capacitors, inductors,
any 2-terminal circuit composed of these.

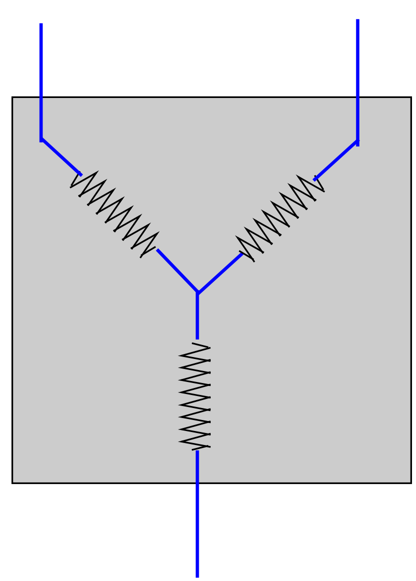


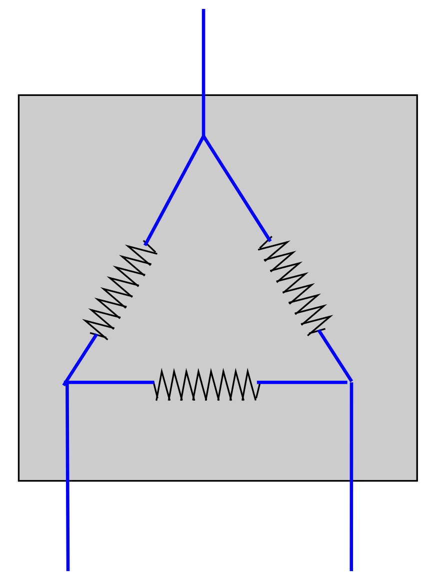
Examples

3-terminal 1-port devices:

transistors, Y's, Δ's.



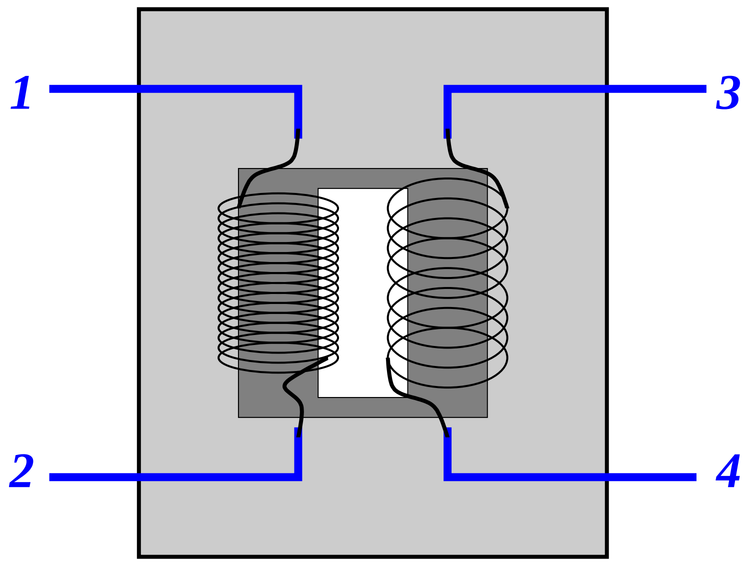




Examples

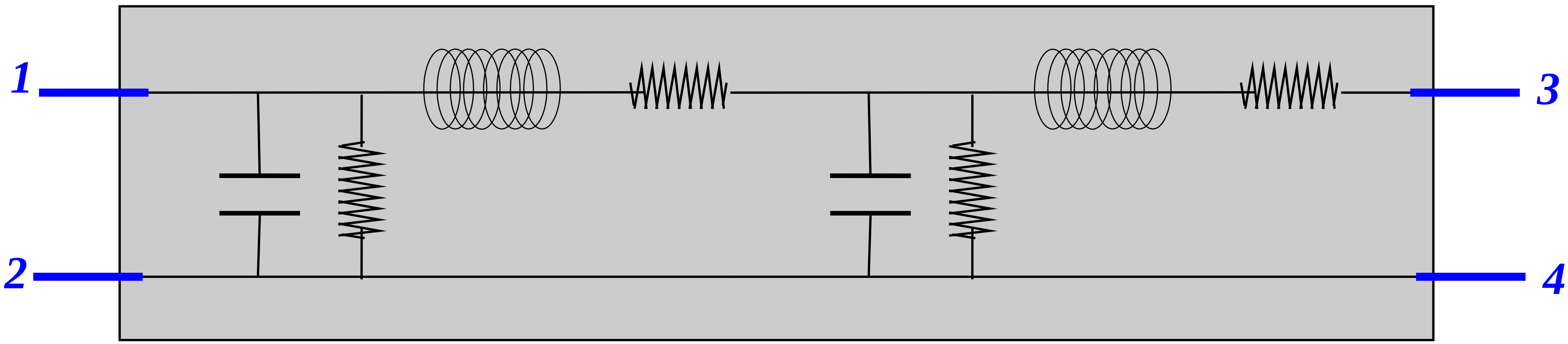
4-terminal 2-port devices:

Transformers, gyrators.



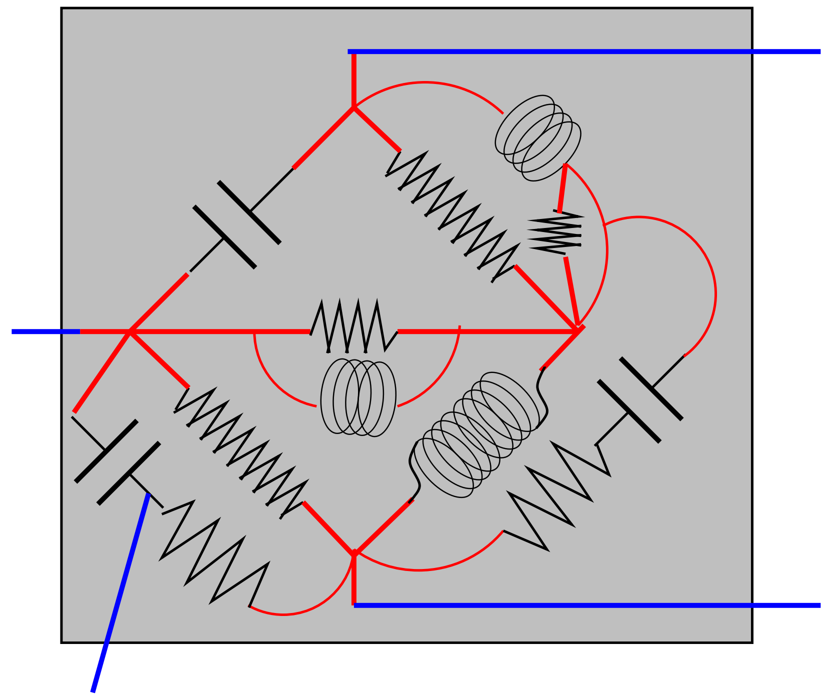
$$V_1 - V_2 = n\left(V_3 - V_4\right), -nI_1 = I_3 \qquad I_1 + I_2 = 0, I_3 + I_4 = 0$$

Examples



Terminals $\{1, 2, 3, 4\}$ form a port. But $\{1, 2\}$ and $\{3, 4\}$ do not.

Are ports common?



<u>Theorem</u>: **Consider an electrical circuit consisting of an interconnection of (linear passive) R's, L's, and C's. If every pair of terminals of the circuit graph is connected, then**

the only port is the one that consists of all the terminals.

Are ports common?

Corollary: Consider an electrical circuit consisting of an interconnection of (linear passive) 2-terminal 1-port impedances. If every pair of terminals of the circuit graph is connected, then

the only port is the one that consists of all the terminals.

Follows from the theorem, combined with Bott-Duffin (every positive real impedance can be viewed as an RLC circuit).

In order to have non-trivial ports, we need

2-port building blocks like transformers in the circuit.

Independence

$$(V_1,\ldots,V_p,V_{p+1},\ldots,V_N,I_1,\ldots,I_p,I_{p+1},\ldots,I_N) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$$

$$\Rightarrow \quad (V_1+\alpha,\ldots,V_p+\alpha,V_{p+1},\ldots,V_N,I_1,\ldots,I_p,I_{p+1},\ldots,I_N) \in \mathscr{B}.$$

'port KVL'

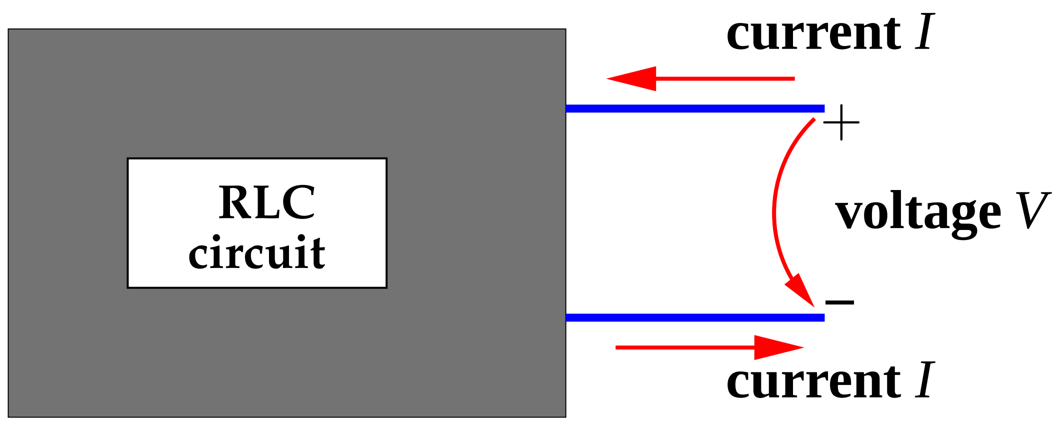
For linear passive circuits, there holds

port KVL $\Leftrightarrow$ **port KCL**.

We require port KCL $\quad I_1+I_2+\cdots+I_p=0.$

DIGRESSION: RLC SYNTHESIS

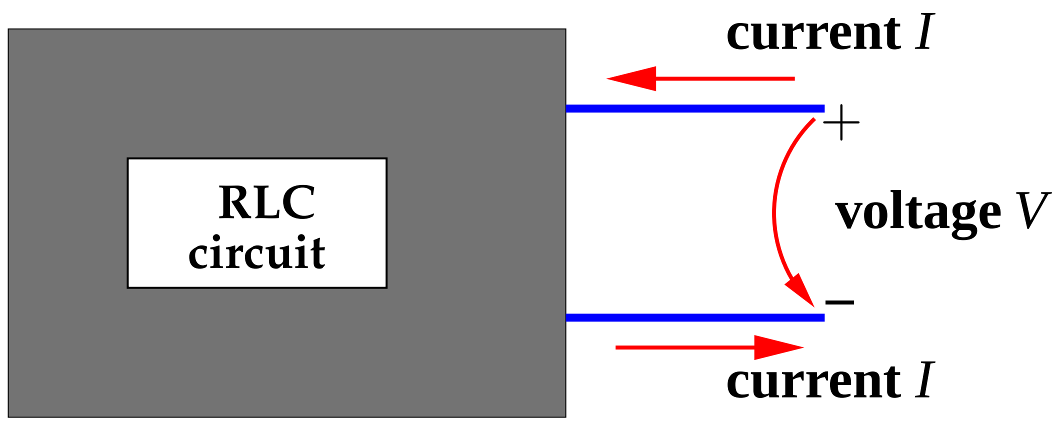
RLC circuits



Relationship between V and I

$$d\left(\frac{d}{dt}\right)V = n\left(\frac{d}{dt}\right)I \qquad n, d \quad \textbf{real polynomials}.$$

RLC circuits



Relationship between V and I

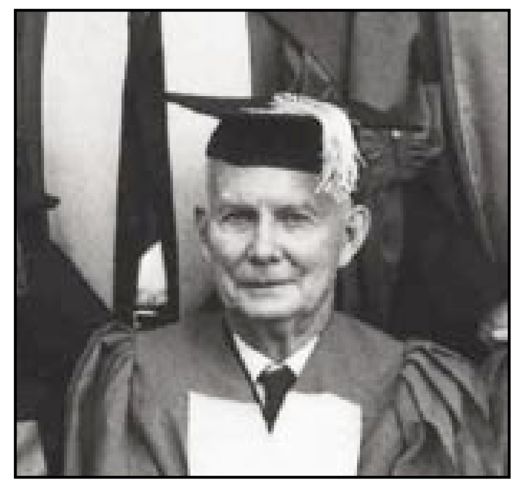
$$d\left(\frac{d}{dt}\right)V = n\left(\frac{d}{dt}\right)I \qquad n, d \quad \textbf{real polynomials}.$$

Which polynomial pairs (n, d) can occur?

Positive realness

<u>Theorem</u>: The following are equivalent

- Z **is realizable using (positive, linear) R,L,C's** ***and transformers.***
- $Z = \dfrac{n}{d}$ **is *positive real*,** **i.e., Real**$(s) \geq 0 \Rightarrow$ **Real**$(Z(s)) \geq 0$**.**
- $\cdots$



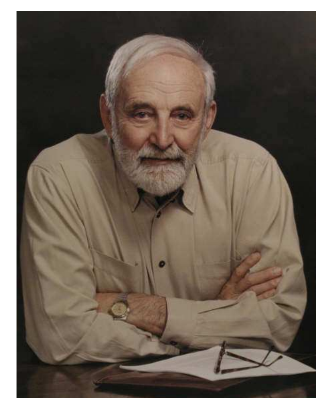
Otto Brune
1901-1982

Positive realness

In 1949 Raoul Bott and Richard Duffin dramatically improved Otto Brune's 1931 result.

<u>Theorem</u>: The following are equivalent

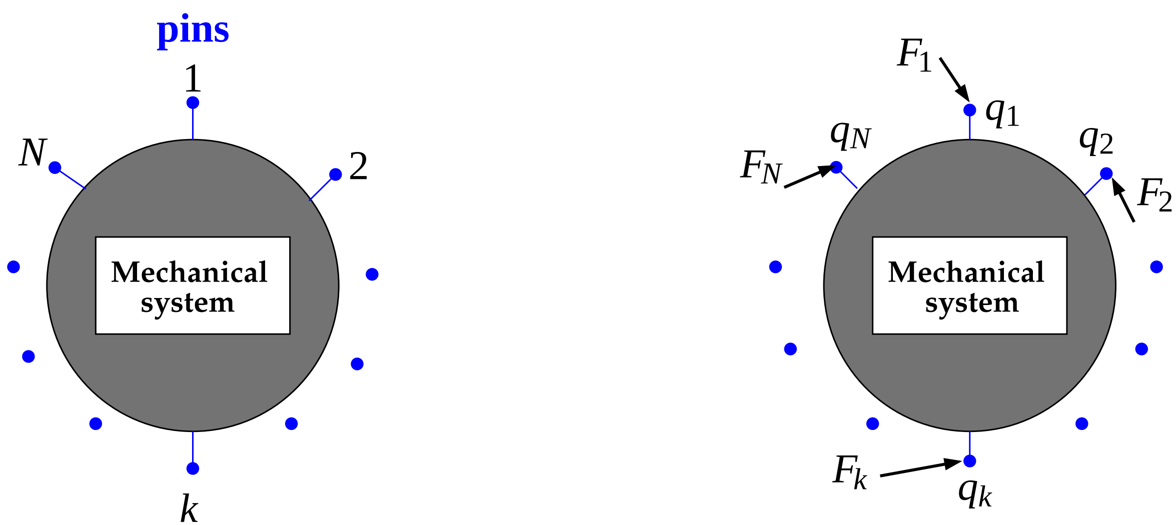
- Z **is realizable using (positive, linear) R,L,C's** ***without transformers.***
- $Z = \frac{n}{d}$ **is** ***positive real*****,** **i.e., Real**$(s) \geq 0 \Rightarrow$ **Real**$(Z(s)) \geq 0$**.**
- $\cdots$



Raoul Bott
1923-2005

(1-D) MECHANICAL SYSTEMS

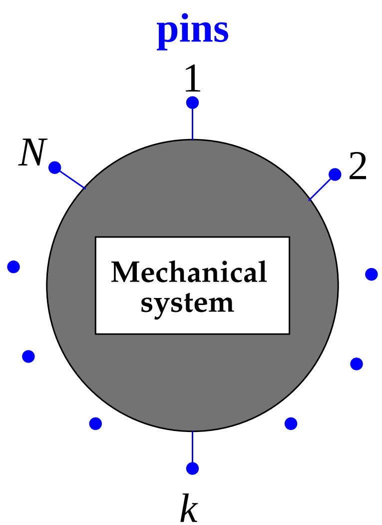
The behavior

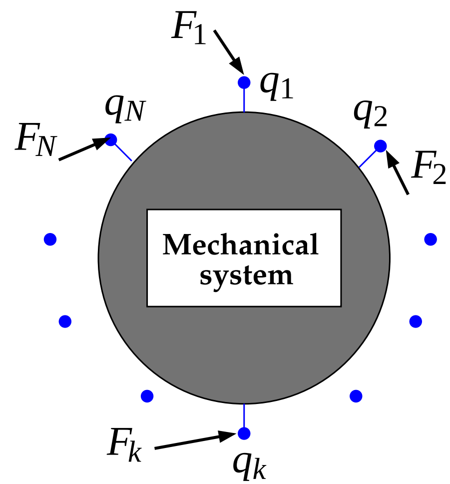


At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

The behavior



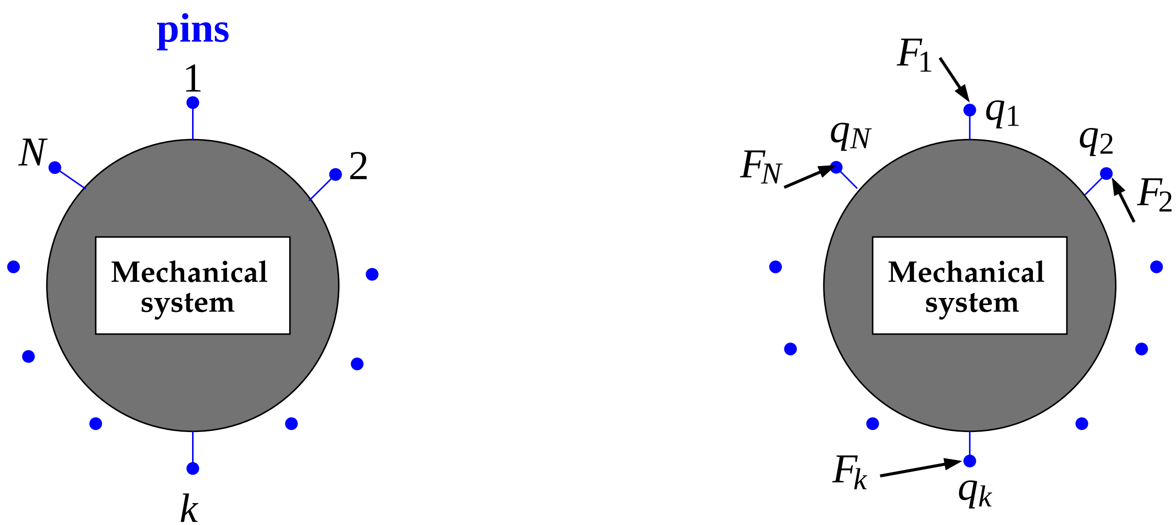


At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^{\bullet})^{2N})^{\mathbb{R}}$**.**

What are the analogues of KVL, KCL, of port?

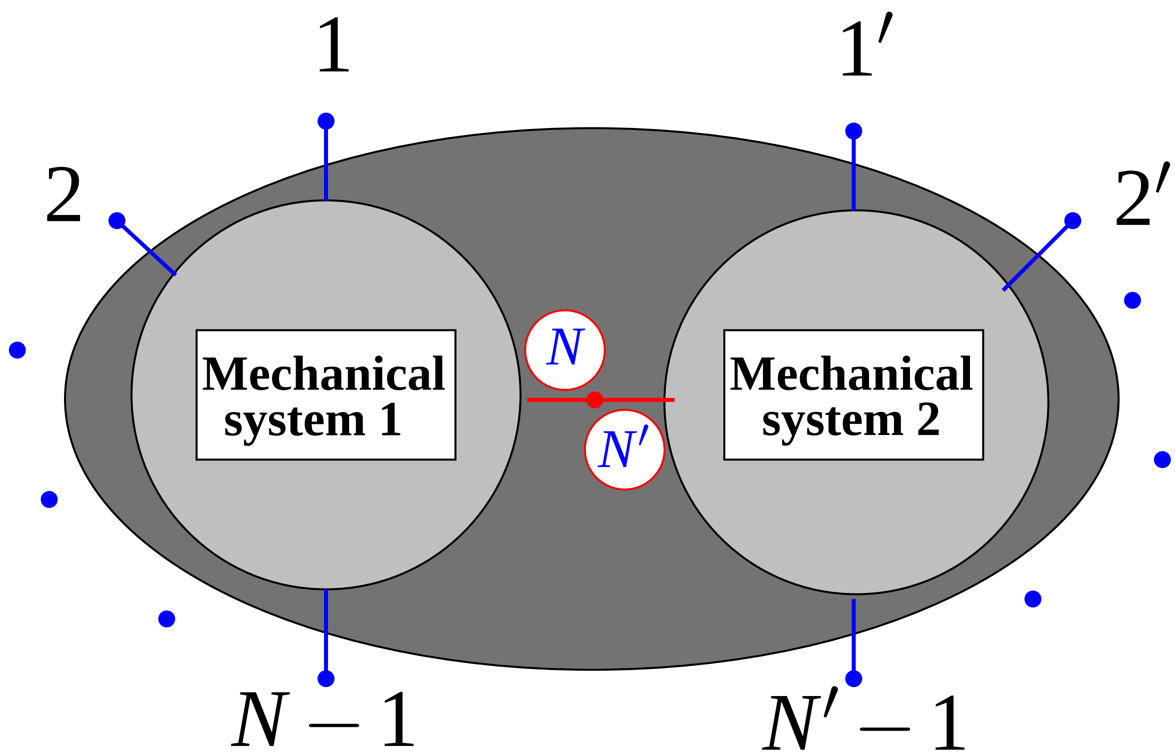
The behavior



- **invariance under uniform motion** $:\Leftrightarrow (q_1, q_2, \ldots, q_N, F_1, F_2, \ldots, F_N) \in \mathscr{B}$ **and** $v : t \in \mathbb{R} \mapsto (a + bt) \in \mathbb{R}^\bullet$**, imply** $(q_1 + v, q_2 + v, \ldots, q_N + v, F_1, F_2, \ldots, F_N) \in \mathscr{B}.$

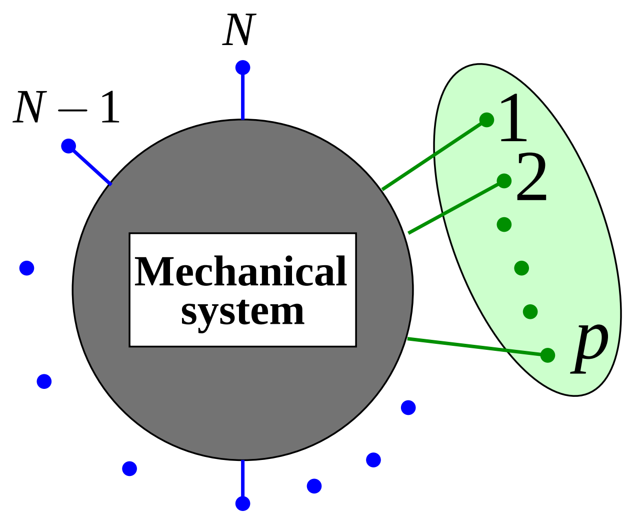
- **Kirchhoff's force law (KFL)** $:\Leftrightarrow (q_1, q_2, \ldots, q_N, F_1, F_2, \ldots, F_N) \in \mathscr{B}$ **implies** $F_1 + F_2 + \cdots + F_N = 0.$

Interconnection



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \qquad F_1 + F_2 + \cdots + F_p = 0.$$

Power and energy

If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

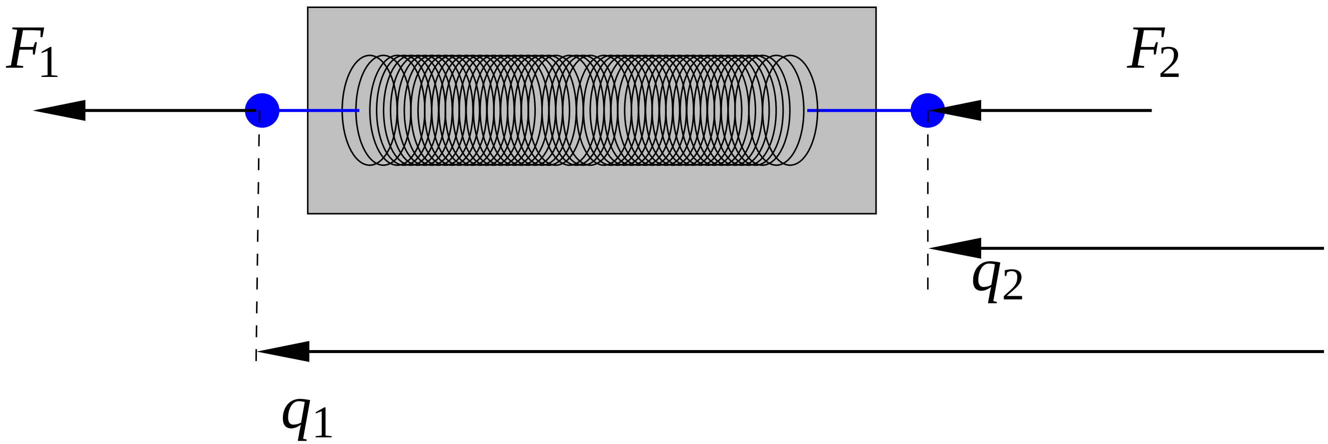
$$\textbf{power in} \quad = F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

and

$$\textbf{energy in} \quad = \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

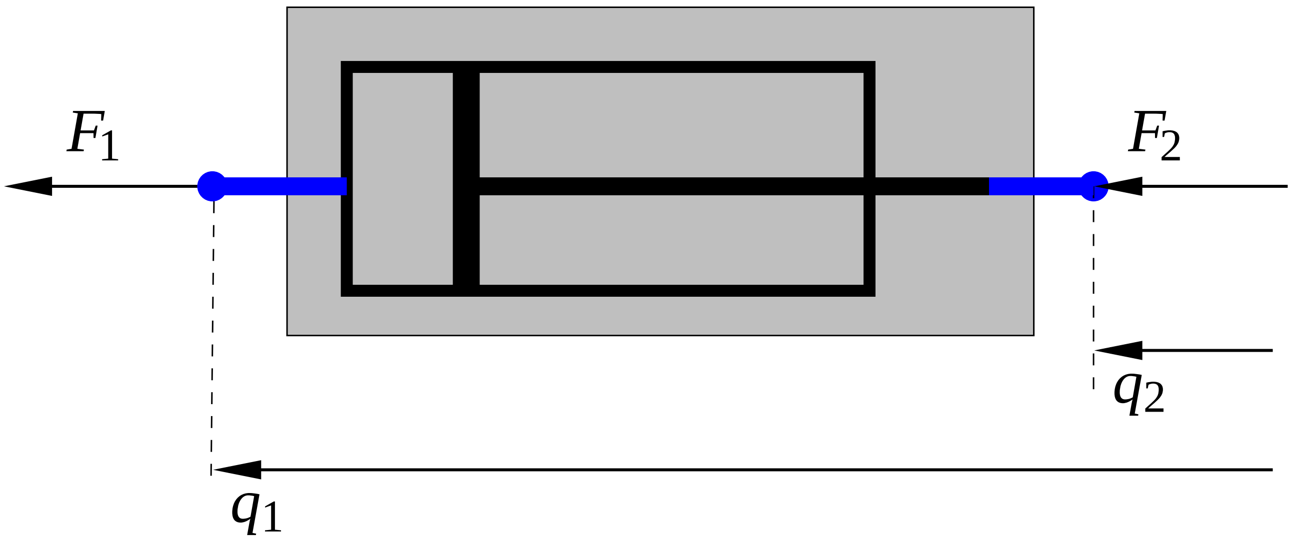
This interpretation in terms of power and energy is not valid unless these terminals form a port !

Examples



$$F_1 + F_2 = 0, \quad K(q_1 - q_2) = F_1.$$

Examples

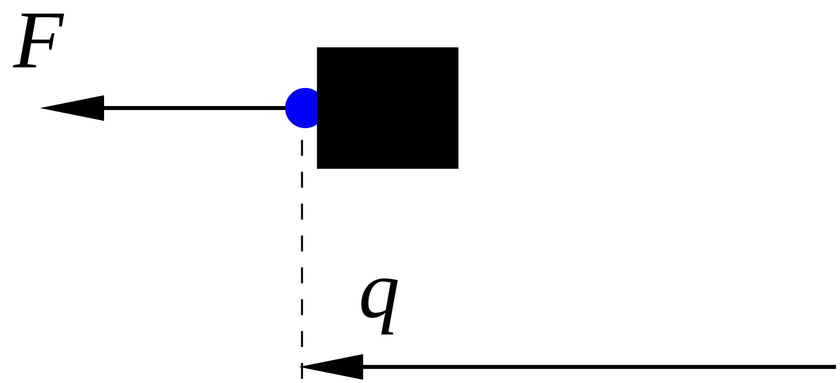


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1.$$

Springs and dampers,

and the interconnection of springs and dampers are ports.

Examples

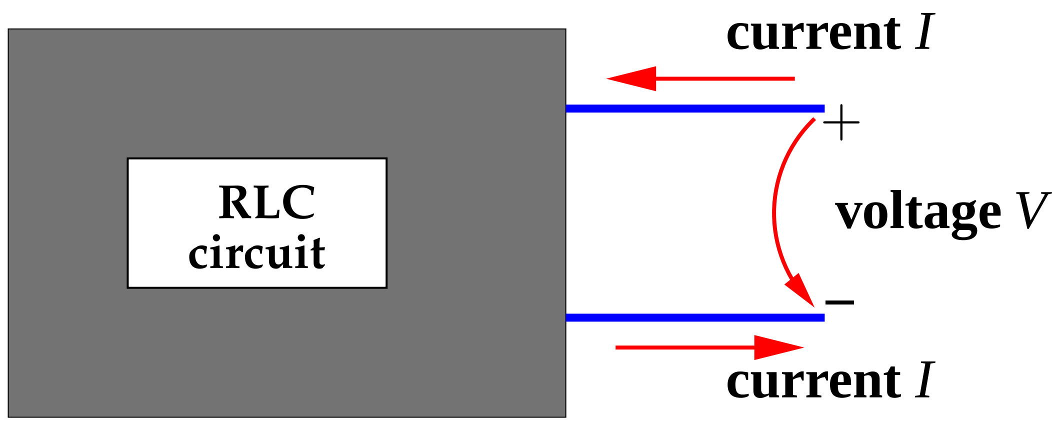


$$M\frac{d^2}{dt^2}q = F.$$

Not a port!!!

DIGRESSION: MECHANICAL SYNTHESIS

Electrical and mechanical synthesis



Relationship between V and I

$$d\left(\frac{d}{dt}\right)V = n\left(\frac{d}{dt}\right)I \quad n, d \ \textbf{ real polynomials}.$$

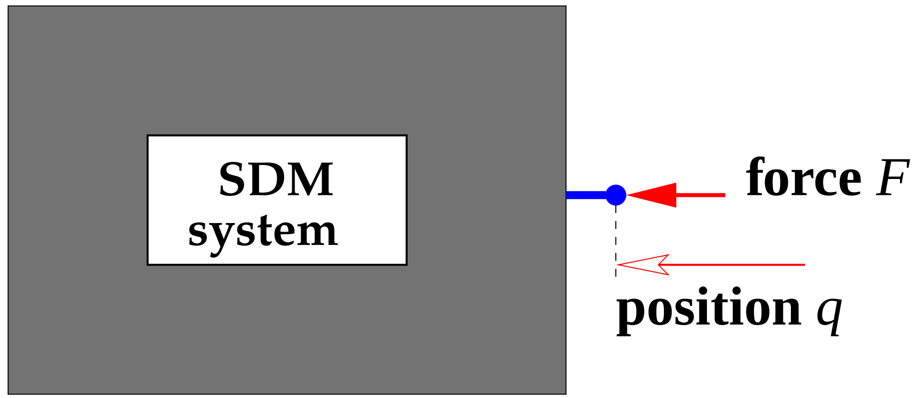
$$Z = \frac{n}{d} \quad \textbf{positive real}$$

Electrical and mechanical synthesis

What mechanical impedances are realizable using passive mechanical devices (dampers, springs, and masses)?

Is it possible to use RLC synthesis to obtain mechanical synthesis?

Electrical and mechanical synthesis



Relationship between F and q

$$d\left(\frac{d}{dt}\right)q = n\left(\frac{d}{dt}\right)F \qquad n, d \ \ \textbf{real polynomials}.$$

$$Z(s) = \frac{sn(s)}{d(s)} \quad \textbf{positive real ???}$$ **naive!**

Electrical-mechanical analogies

voltage $V \leftrightarrow v$ **velocity**

current $I \leftrightarrow F$ **force**

Resistor $\frac{1}{R}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	**Damper** $D(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$
Inductor $\frac{1}{L}(V_1 - V_2) = \frac{d}{dt} I_1, \;\; I_1 + I_2 = 0$	**Spring** $K(v_1 - v_2) = \frac{d}{dt} F_1, \;\; F_1 + F_2 = 0$
Capacitor $C \frac{d}{dt}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	**Mass** $M \frac{d}{dt} v = F$

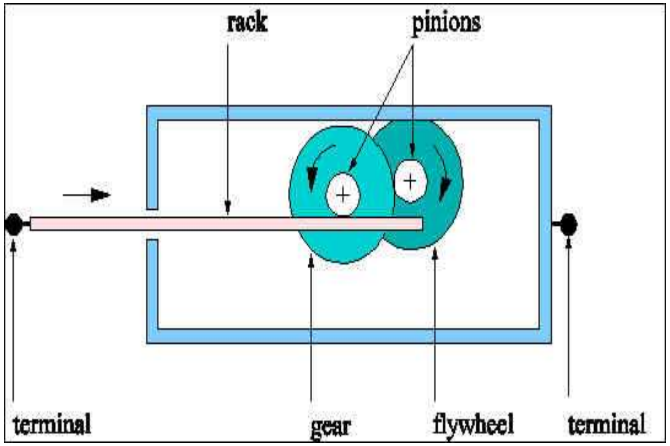
Electrical-mechanical analogies

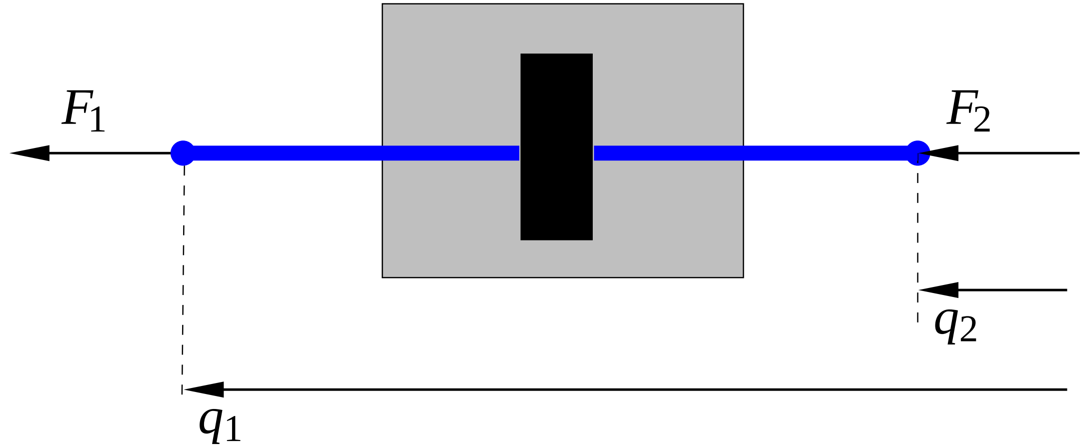
$V \leftrightarrow v$ $\qquad$ $I \leftrightarrow F$

The electrical analogue of a mass is a 'grounded' capacitor.

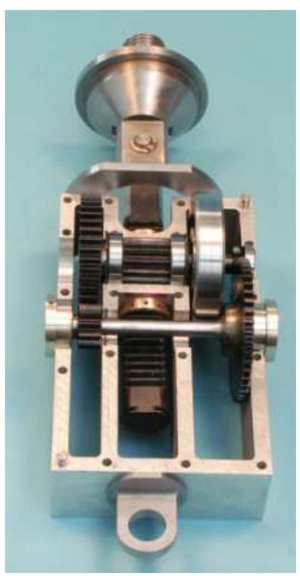
Electrical synthesis $\not\Rightarrow$ mechanical synthesis.

The inerter





$$B\frac{d^2}{dt^2}(q_1 - q_2) = F_1, \quad F_1 + F_2 = 0.$$



Malcolm Smith

Electrical-mechanical analogies

Resistor $\frac{1}{R}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	**Damper** $D(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$
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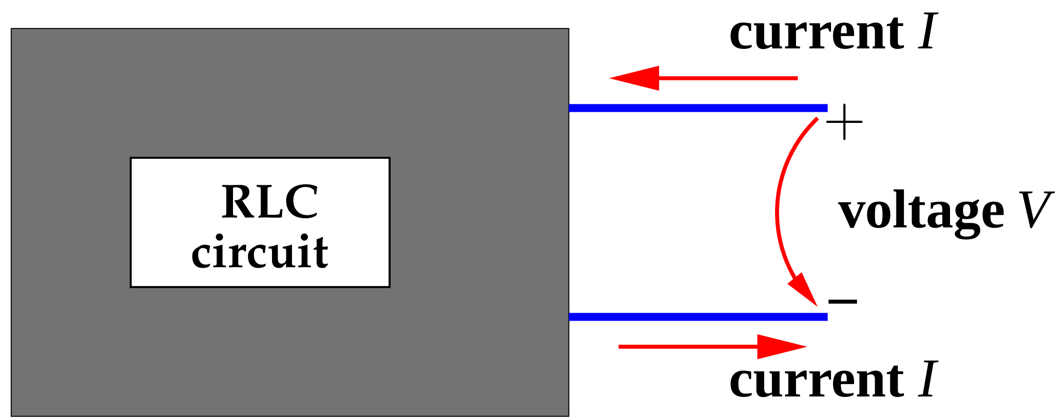
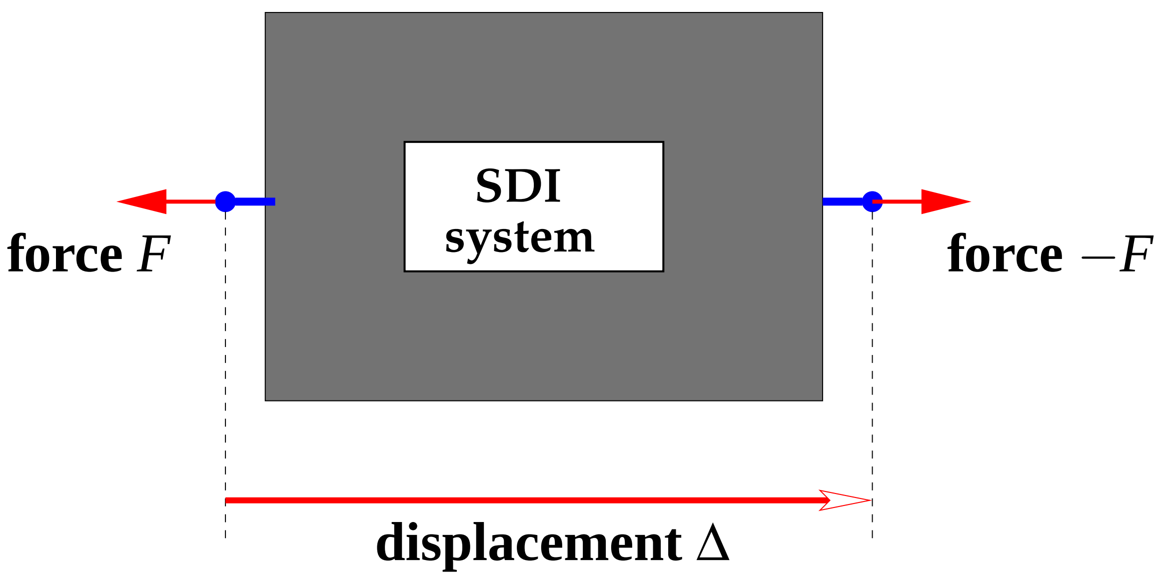
Capacitor $C\frac{d}{dt}(V_1 - V_2) = I_1, \;\; I_1 + I_2 = 0$	**Inerter** $B\frac{d}{dt}(v_1 - v_2) = F_1, \;\; F_1 + F_2 = 0$

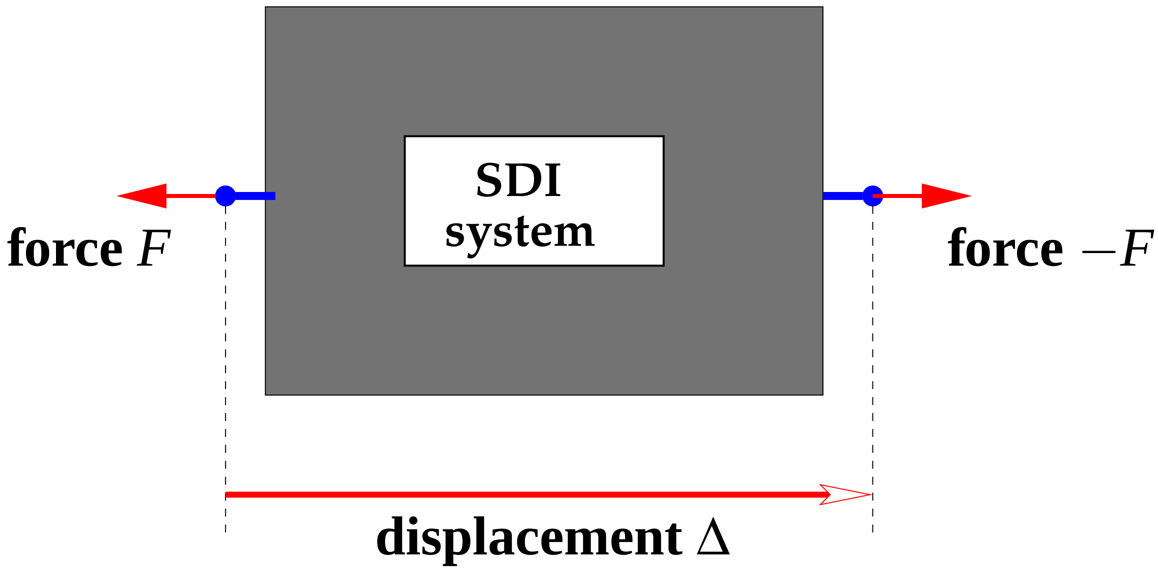
electrical RLC synthesis $\Leftrightarrow$ mechanical SDI synthesis.

RLC and SDI


current I
+
RLC
circuit
voltage V
−
current I


SDI
system
force F
force $-F$
displacement Δ

RLC and SDI



Relationship between F and Δ

$$d\left(\frac{d}{dt}\right)\Delta = n\left(\frac{d}{dt}\right)F \qquad n, d \text{ **real polynomials**.}$$

Realizable iff $\quad Z(s) = \dfrac{sn(s)}{d(s)} \quad$ **positive real**

Relevance of passive synthesis

Electrical domain:

Theoretical (electrical) engineering highlight ($<$1960's).

Until 1950's important for filter design.
Eclipsed by the introduction of solid state technology (transistors, etc.)

Relevance of passive synthesis

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Theoretical (electrical) engineering highlight ($<$1960's).

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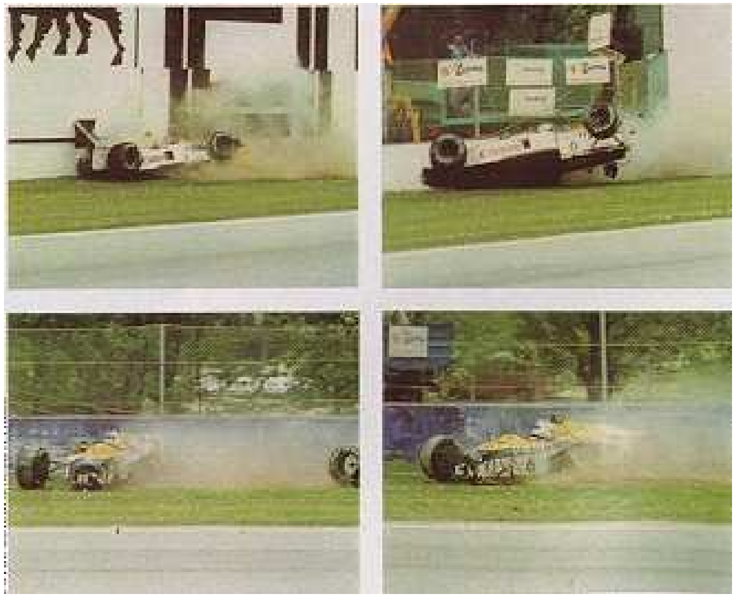
Mechanical domain: Recent interest.

- requires no energy supply
- simple design
- reliability
- safety

Hazards of active suspensions

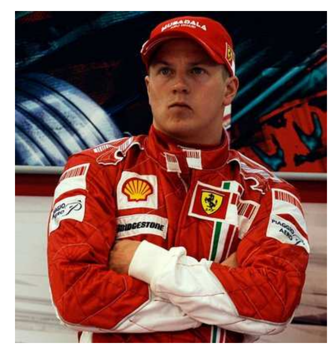


Nelson Piquet crash in the Indy 500 practice in 1992

Nelson Piquet

Success of passive suspensions

Raikkonen wins the 2005 Grand Prix in Spain with McLaren and Smith's 'J-damper'.



Kimi Raikkonen

Success of passive suspensions

F1 TEAMS FIND EXTRA GRIP WITH NEW DAMPER SET-UP

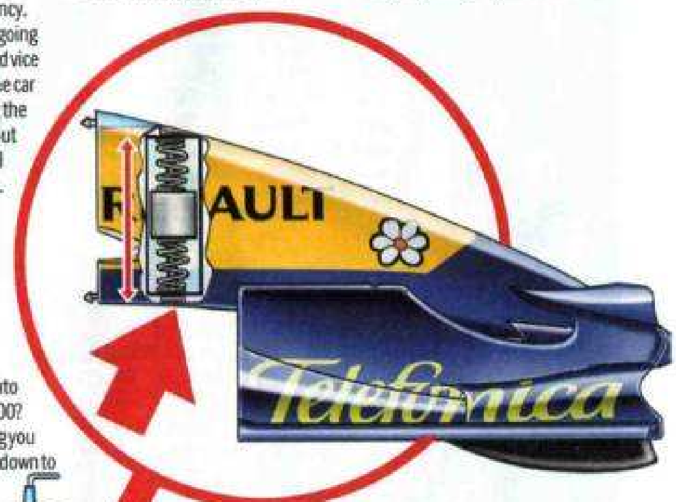
SEVERAL FORMULA 1 teams have been using a 'mass damper' this season in order to damp out tyre bouncing frequencies through corners and thereby keep the grip more consistent.

Renault and Red Bull are known to be using such a system and it's believed that Ferrari and Williams – and perhaps others too – also have it fitted. It's a simple mechanical device comprising a weight (believed to be of around 9kg) on a spring.

Anderson

Gary Anderson explained its purpose: "You see a car bouncing, such as when it hits a kerb. This will be at around 8-9Hz on the tyres. This device will get an equal frequency going in the opposite direction. It's a bit like a tall building designed to withstand an earthquake where they put the water tank on the roof – so that when the building sways, the water is going in the opposite direction at the same frequency.

"In this case, as the car is going up, the tyre is going down and vice versa. With this, the car will still bounce at the same frequency but the amplitude will be damped down. If you take an average tyre contact load and assume say 1000kg, you'd typically see a variation of 9-1100kg. But the driver needs to feel sure of what he's got as he goes into the corner. Is that 900 or 1100? With a weight of around 10kg you can maybe get the variation down to 950-1050kg, so he's got a more constant load. It's a very good idea. It would also be easy to set-up on a seven-post rig, balancing mass against damping. I reckon it would take no more than 15 minutes to set it up. On the downside it adds a bit of weight quite high up in the car."



GIORGIO PIOLA

Success of passive suspensions

Secrets of the inerter revealed

20 August 2008

A Cambridge University invention which was kept a closely-guarded secret because of the hidden advantage it offered to a Formula 1 racing team is finally being made available for widespread use.

For years, the mysterious "J-Damper", a vehicle suspension device described as the F1 technical innovation of the year, was carefully codenamed and concealed to prevent it from being copied by rivals.

McLaren agreed an exclusive right with the University to exploit the technology, but confidentiality restrictions ensured that other F1 teams were kept in the dark. Internet fan-sites and blogs began to buzz with speculation about what the device actually was.

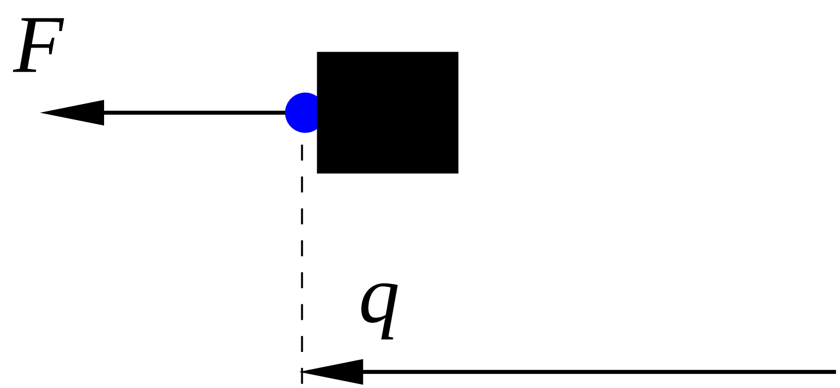
Now, with the lifting of the confidentiality agreement, the secret of the "J-Damper" can finally be revealed. Cambridge Enterprise, the University's commercialisation office, has signed a licence agreement with the American firm Penske Racing Shocks, enabling Penske to supply them to any team in F1.

In fact, the device was first conceived by its creator, Professor Malcolm Smith, as long ago as 1997 and raced for the first time by McLaren in 2005, when Kimi Raikkonen achieved a victory for the team at the Spanish Grand Prix.

The term "J-Damper" itself was merely a codename to keep the technology secret from potential competitors for as long as possible. Its proper name is

KINETIC ENERGY

Back to the mass

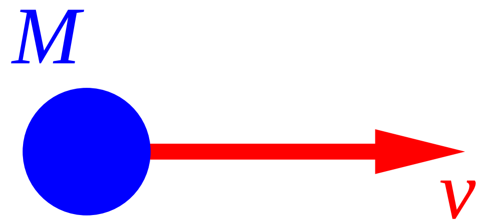


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M\left|\left|\frac{d}{dt}q\right|\right|^2 = F^\top \frac{d}{dt}q$$

If $F^\top v$ is not power,

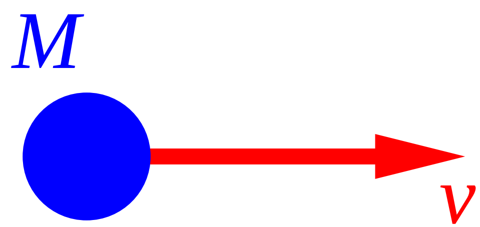
is then $\frac{1}{2}M||v||^2$ not the stored (kinetic) energy ???

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

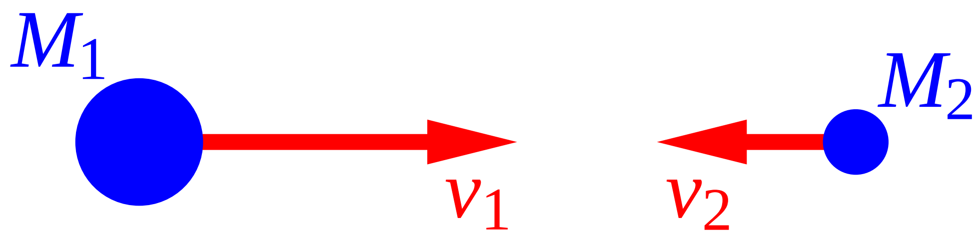
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

This formula is not invariant under uniform motion.

Kinetic energy and invariance under uniform motions



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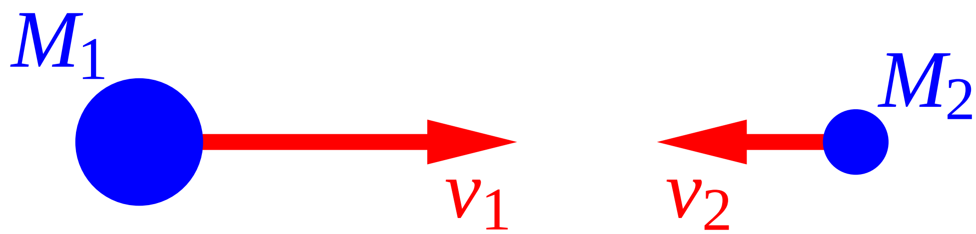
Kinetic energy and invariance under uniform motions



What is the kinetic energy?

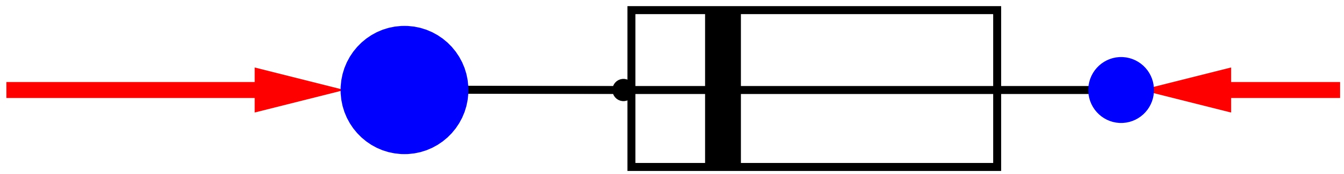
$$\mathcal{E}_{\textbf{kinetic}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1 - v_2||^2$$

Invariant under uniform motion.

Can be justified by mounting a damper or a spring between the masses.

Dissipation into heat

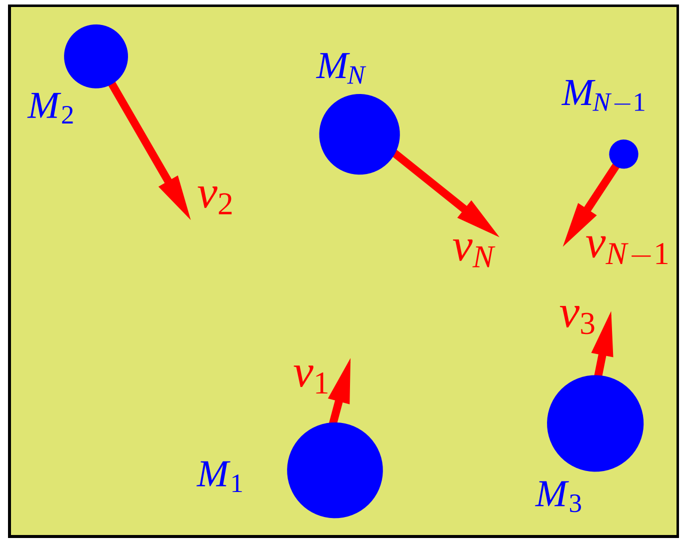
Justification



$$\frac{1}{2}\ \frac{M_1\ M_2}{M_1+M_2}\ ||v_1 - v_2||^2$$ **is the heat dissipated in the damper.**

Kinetic energy

Generalization to N masses.

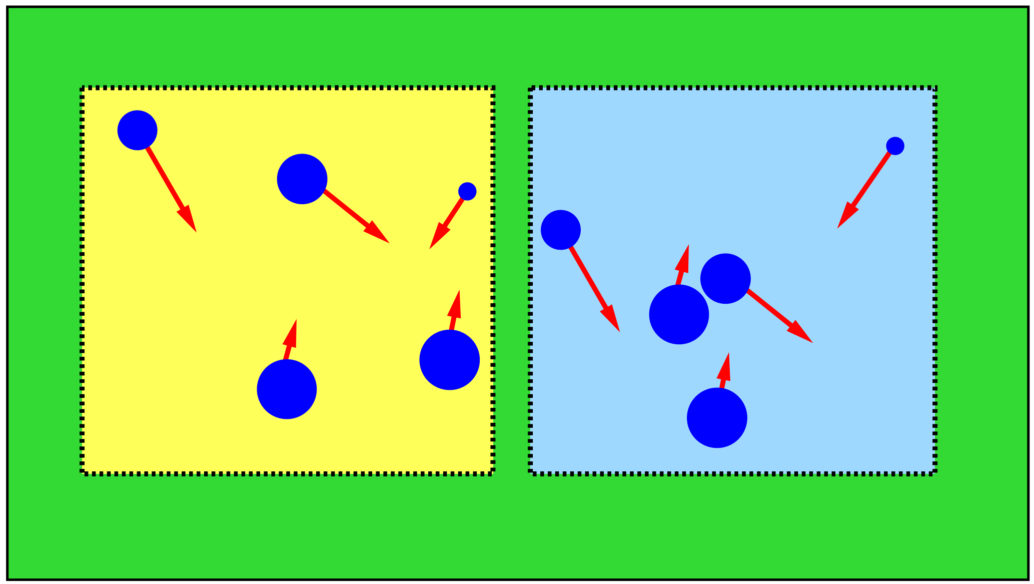


$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} \|v_i - v_j\|^2.$$

KFL $\Rightarrow$ $$\frac{d}{dt}\mathscr{E}_{\textbf{kinetic}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i.$$

Kinetic energy

Kinetic energy is not an extensive quantity, it is not additive.



Total kinetic energy $\neq$ sum of the parts.

Kinetic energy

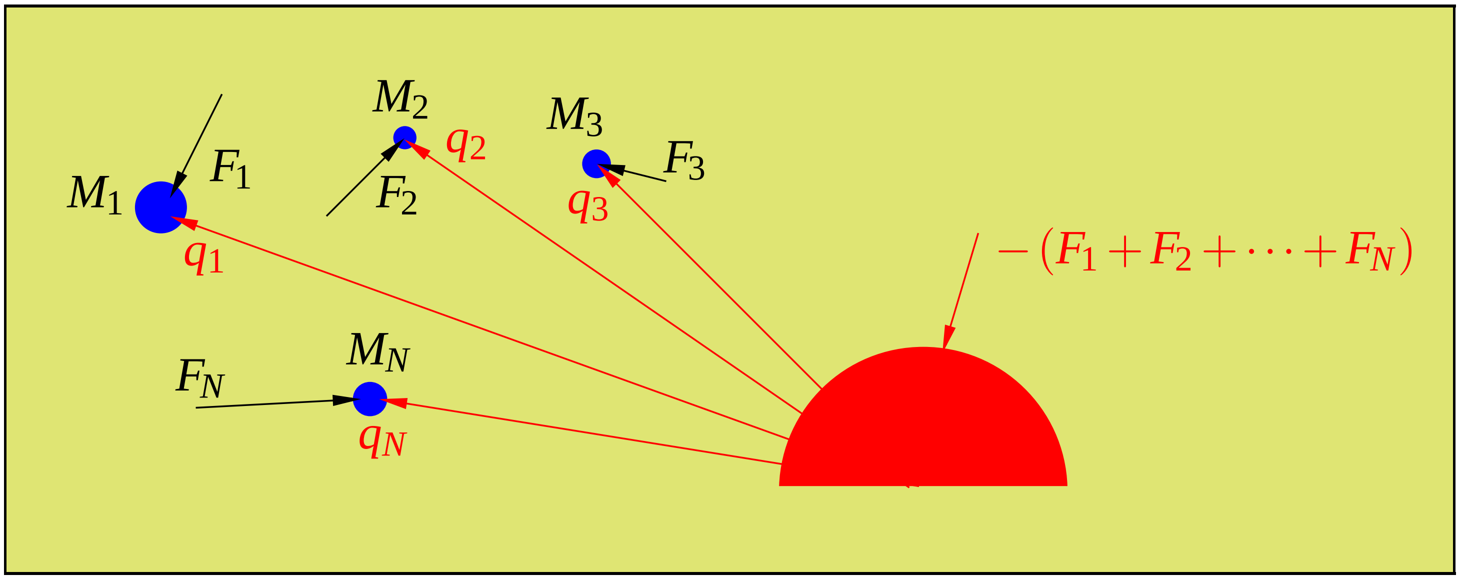
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2 .$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{classical}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2 .$$

Kinetic energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



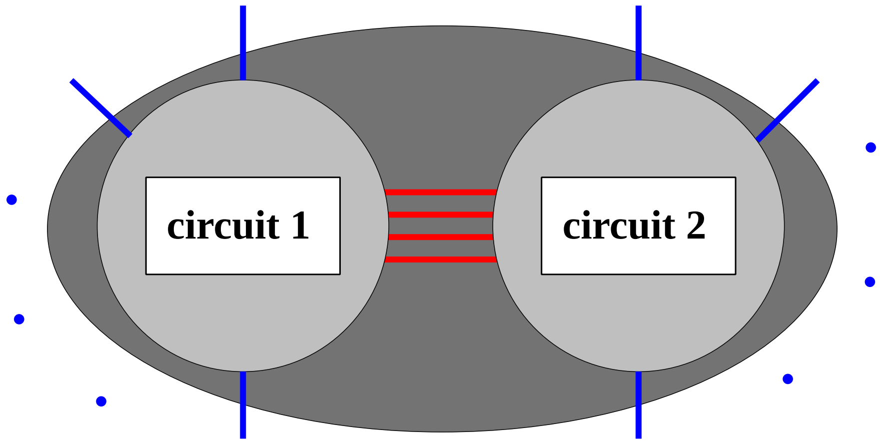
measure velocities w.r.t. this infinite mass ('ground'), then

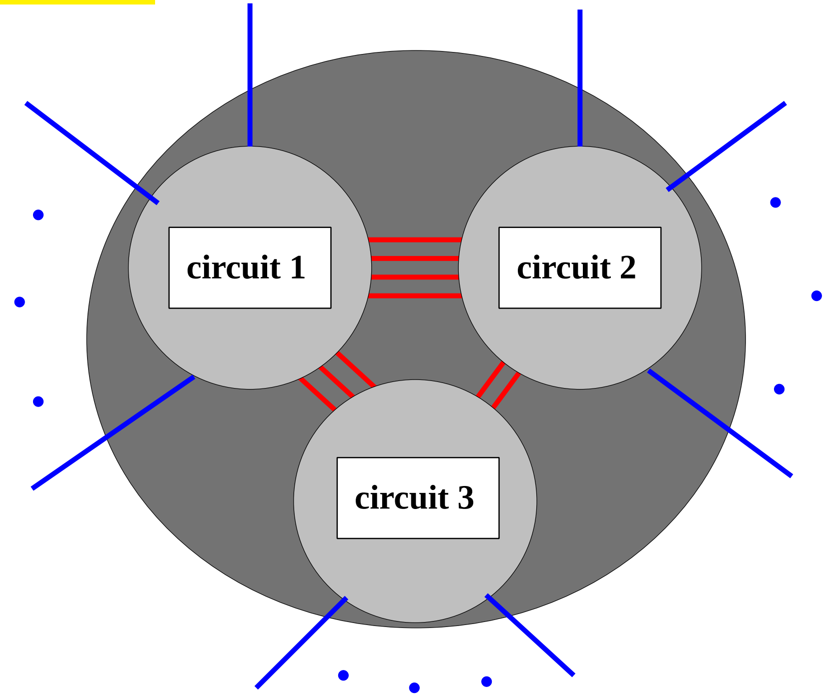
$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \qquad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

PORTS and TERMINALS

Energy transfer





One cannot speak about

“ the energy transferred from circuit 1 to circuit 2 ”
or *“ from the environment to circuit 1 ”*,

unless the relevant terminals form a port.

Analogously for mechanical systems, etc.

Conclusions

- **Dynamical system $\cong$ a behavior.**
- **Interconnection $\cong$ variable sharing.**
- **Energy transfer happens via ports,**
  hence it involves action at a distance.
- **Interconnection is 'local',**
  power and energy transfer involve 'action at a distance'.

Conclusions

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- **Interconnection is 'local',**
  power and energy transfer involve 'action at a distance'.
- **Electrical ports $\Leftrightarrow$ KCL.**
- **Mechanical ports $\Leftrightarrow$ KFL.**
- **New expression for kinetic energy, invariant under UM.**

Conclusions

- Dynamical system $\cong$ a behavior.
- Interconnection $\cong$ variable sharing.
- Energy transfer happens via ports,
  hence it involves action at a distance.
- Interconnection is 'local',
  power and energy transfer involve 'action at a distance'.
- Electrical ports $\Leftrightarrow$ KCL.
- Mechanical ports $\Leftrightarrow$ KFL.
- New expression for kinetic energy, invariant under UM.
- Terminals are for interconnection,
  ports are for energy transfer.

Copies of the lecture frames will be available from/at

`Jan.Willems@esat.kuleuven.be`

`http://www.esat.kuleuven.be/~jwillems`

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you