PORTS and TERMINALS

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 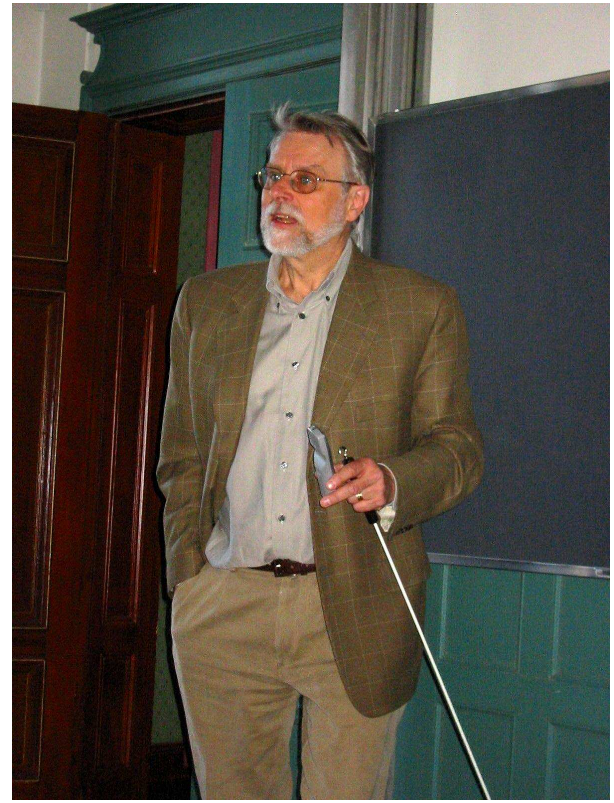

In honor of

Chris Byrnes and Anders Lindquist.

Theme

How are systems interconnected?

How is energy transferred between systems?

Are energy transfer and interconnection related?

Theme

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How is energy transferred between systems?

Are energy transfer and interconnection related?

We deal only with electrical circuits and mechanical systems.
Other applications: hydraulic systems,
chemical systems, ...

ENERGY

Energy as an extensive quantity

Our intuition has been built to think of energy as an extensive quantity,

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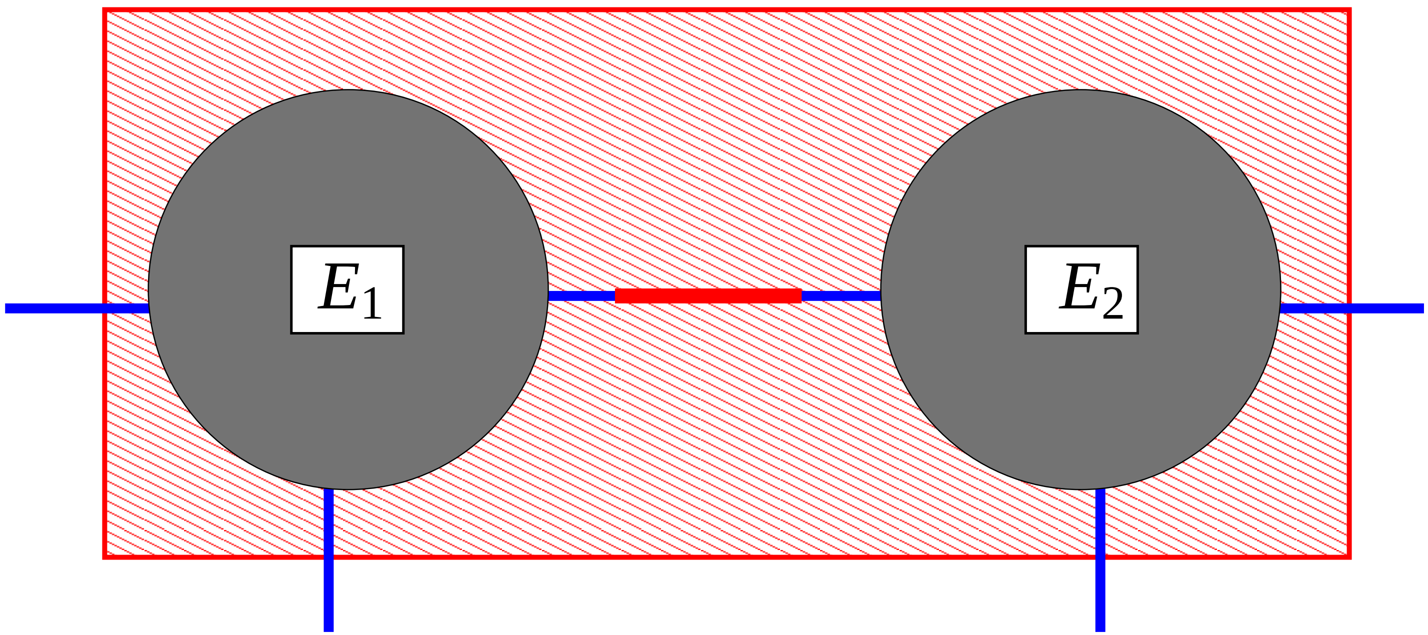
meaning that it is additive



$$E_{\text{total}} = E_1 + E_2 .$$

Energy as an extensive quantity

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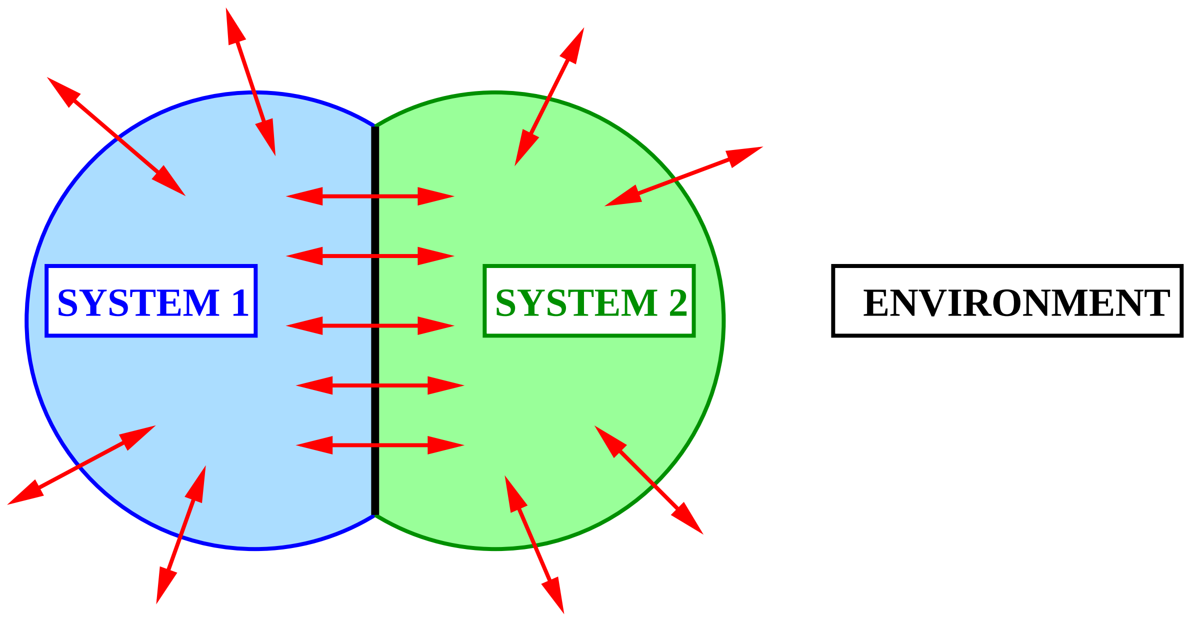

that flows in and out and between systems along the interconnected interfaces.

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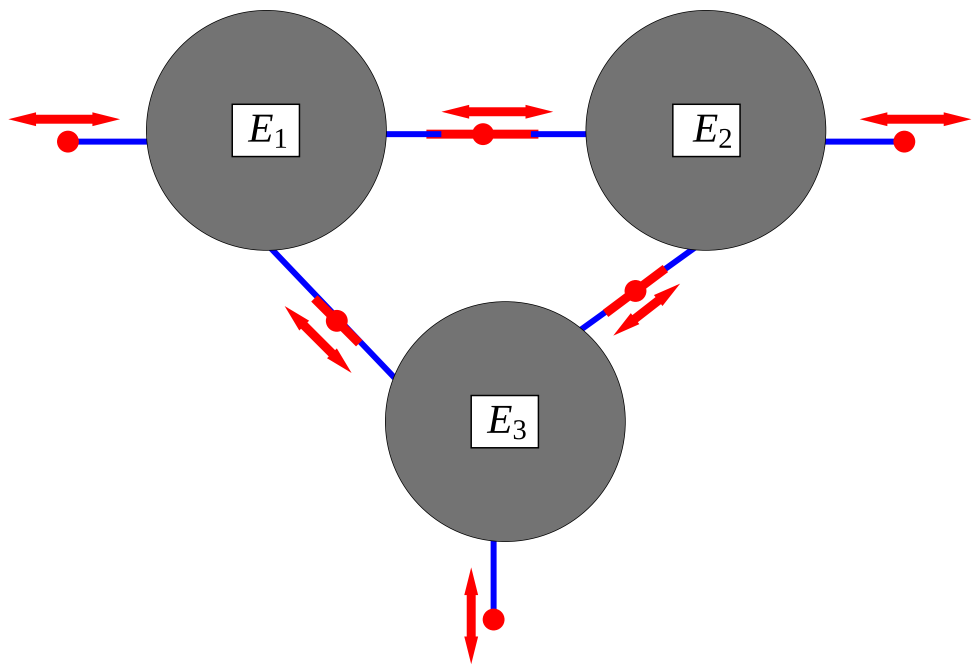


that flows in and out and between systems along the interconnection terminals.

Energy as an extensive quantity

Our intuition has been built to think of energy as an extensive quantity,
that flows in and out and between systems
along the interconnection terminals.

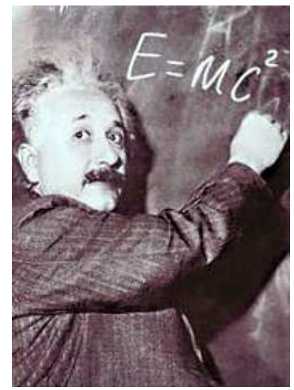
Some methodologies for modeling interconnected systems, as bond-graph modeling and port-Hamiltonian systems, are based on this thinking.

'Power is the universal currency of physical systems'

P.J. Gawthrop and G.P. Bevan, *Bond-graph modeling*, IEEE Control Systems Magazine, vol. 27, pp. 2445, 2007.

Energy as an extensive quantity

In many situations, this view is correct. Mass, volume, and energy in the form of heat are extensive quantities.



Albert and Jan in De Haan

Energy as an extensive quantity

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However, energy is more subtle for other forms.

Kinetic energy is not additive.
Same with energy due to gravitational attraction,
due Coulomb forces, etc.

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In many situations, this view is correct. Mass, volume, and energy in the form of heat are extensive quantities.

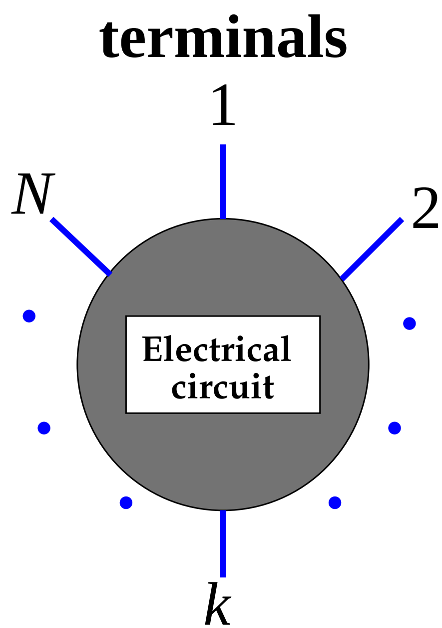
However, energy is more subtle for other forms.

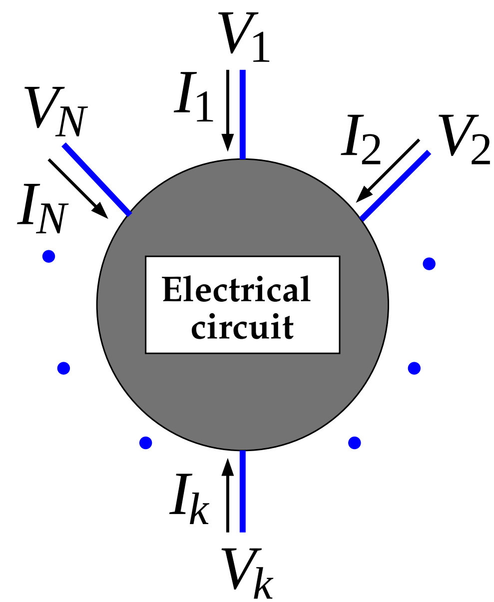
Kinetic energy is not additive.
Same with energy due to gravitational attraction,
due Coulomb forces, etc.

Energy and power are not a ‘local’ quantities.
They involve ‘action at a distance’.

BEHAVIORAL CIRCUIT THEORY

N-terminal circuit

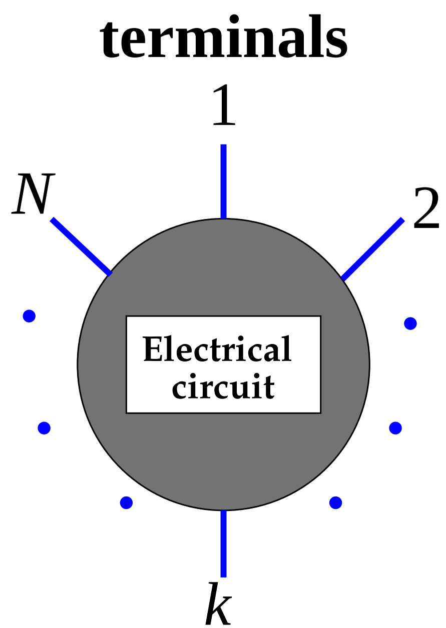


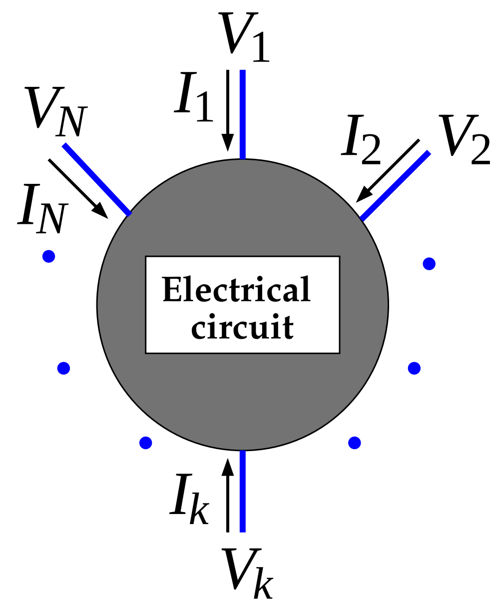


At each terminal:

a potential (!) and a current (counted > 0 into the circuit),

N-terminal circuit





At each terminal:

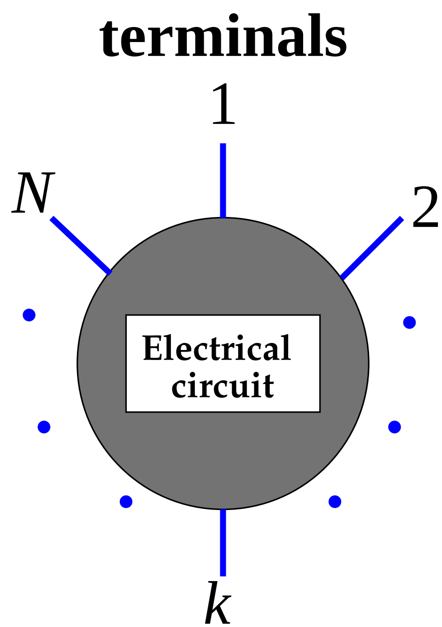
a potential (!) and a current (counted > 0 into the circuit),

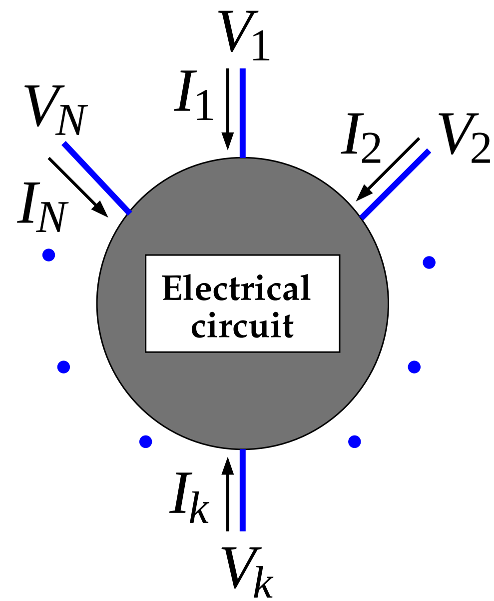
$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}.$$

$(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B}$ **means:**

this potential/current trajectory is compatible with the circuit architecture and its element values.

KVL and KCL



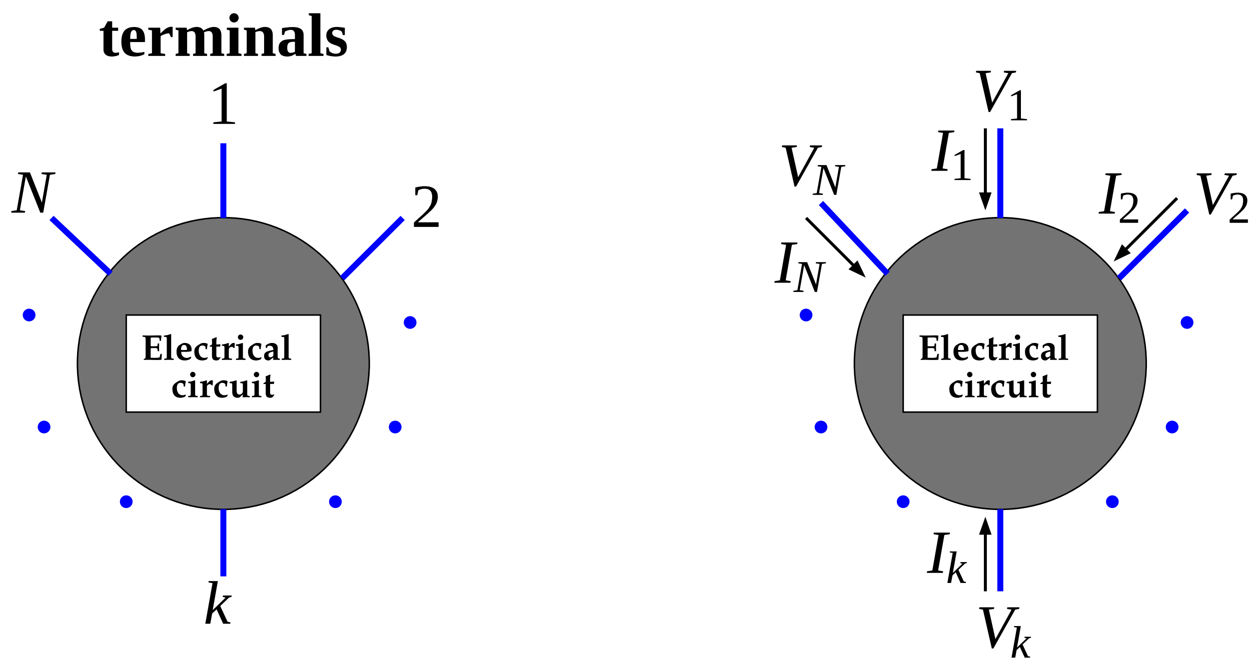


Kirchhoff's voltage law (KVL):

$$[\![(V_1, V_2, \ldots, V_N, I_1, I_2, \ldots, I_N) \in \mathscr{B} \text{ and } \alpha : \mathbb{R} \to \mathbb{R}]\!]$$
$$\Rightarrow [\![(V_1 + \alpha, V_2 + \alpha, \ldots, V_N + \alpha, I_1, I_2, \ldots, I_N) \in \mathscr{B}]\!].$$

Equivalently, the behavioral equations contain the V_i's only through the potential differences $V_i - V_j$.

KVL and KCL



Kirchhoff's voltage law (KVL):

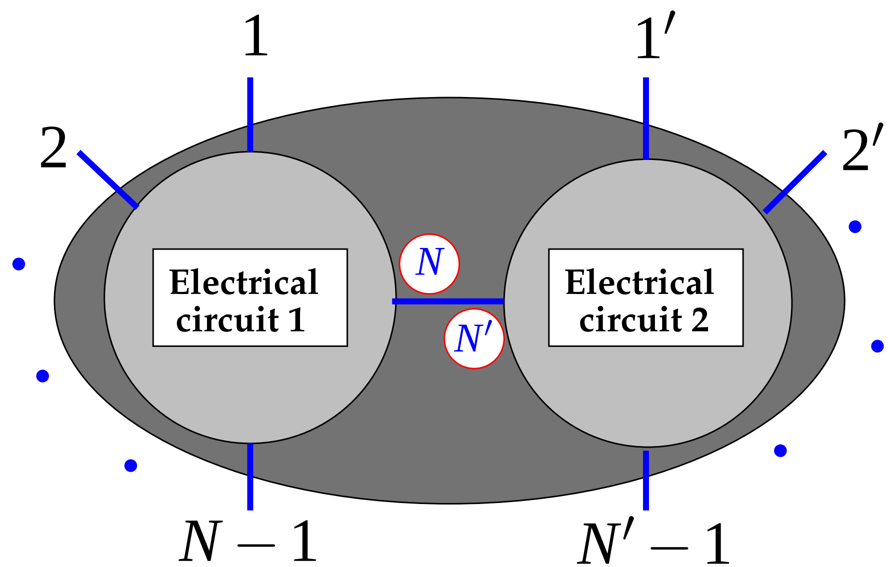
$$[\![(V_1,V_2,\ldots,V_N,I_1,I_2,\ldots,I_N)\in\mathscr{B} \textbf{ and } \alpha:\mathbb{R}\to\mathbb{R}]\!] \Rightarrow [\![(V_1+\alpha,V_2+\alpha,\ldots,V_N+\alpha,I_1,I_2,\ldots,I_N)\in\mathscr{B}]\!].$$

Kirchhoff's current law (KCL):

$$[\![(V_1,V_2,\ldots,V_N,I_1,I_2,\ldots,I_N)\in\mathscr{B}]\!] \Rightarrow [\![I_1+I_2+\cdots+I_N=0]\!].$$

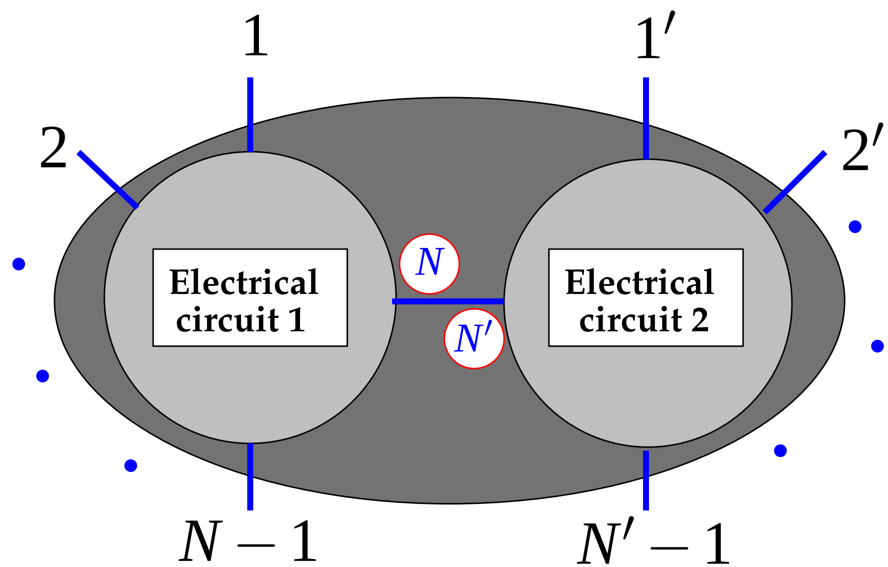
INTERCONNECTION

Interconnection of circuits



$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Interconnection of circuits



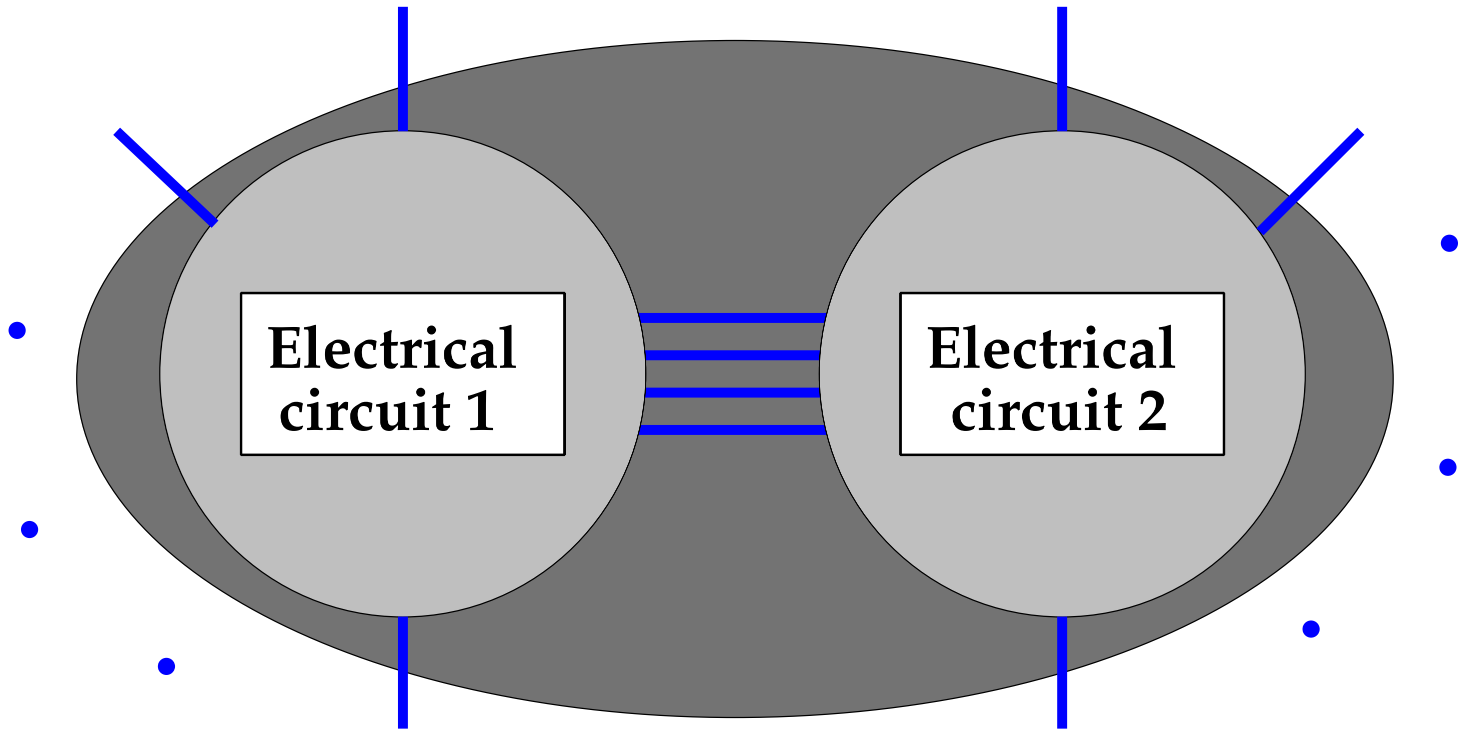
$$V_N = V_{N'} \quad \textbf{and} \quad I_N + I_{N'} = 0.$$

Behavior after interconnection:

$$\mathscr{B}_1 \sqcap \mathscr{B}_2$$
$$:= \{ (V_1, \ldots, V_{N-1}, V_{1'}, \ldots, V_{N'-1}, I_1, \ldots, I_{N-1}, I_{1'}, \ldots, I_{N'-1}) \mid$$
$$\exists V, I \text{ such that}$$
$$(V_1, \ldots, V_{N-1}, V, I_1, \ldots, I_{N-1}, I) \in \mathscr{B}_1 \text{ and}$$
$$(V_{1'}, \ldots, V_{N'-1}, V, I_{1'}, \ldots, I_{N'-1}, -I) \in \mathscr{B}_2 \}.$$

Interconnection of circuits

⇝ **more terminals connected**



Interconnection of circuits

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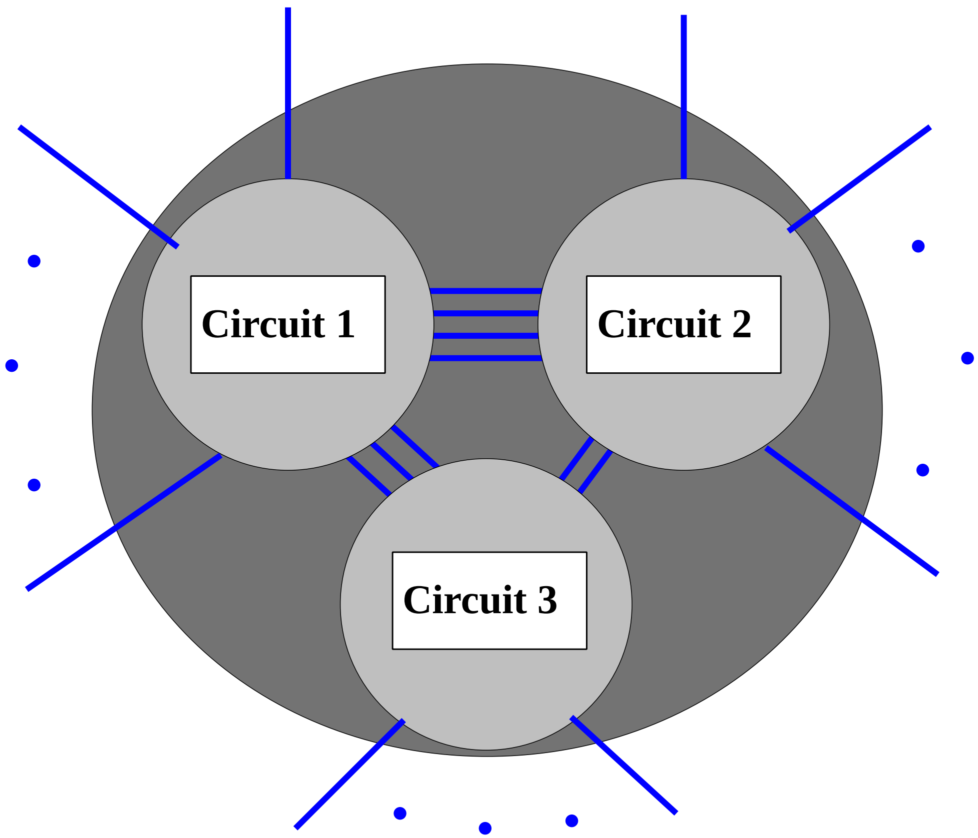
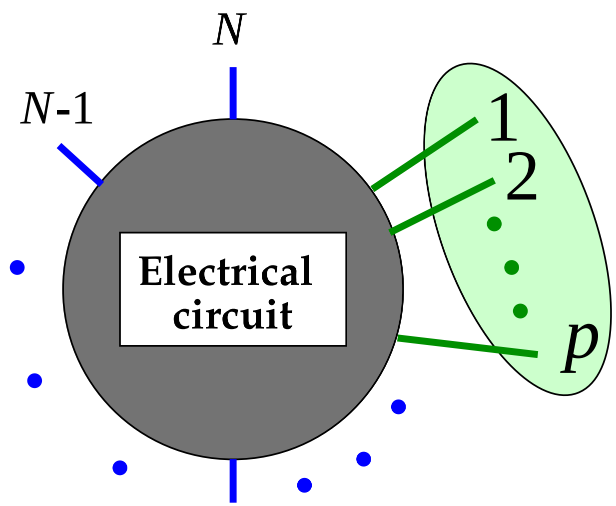


Preservation of properties

- $[\![\mathscr{B}_1, \mathscr{B}_2$ **satisfy KVL & KCL** $]\!] \Rightarrow [\![$**so does** $\mathscr{B}_1 \sqcap \mathscr{B}_2]\!]$
- $[\![\mathscr{B}_1, \mathscr{B}_2$ **linear** $]\!] \Rightarrow [\![\mathscr{B}_1 \sqcap \mathscr{B}_2$ **linear** $]\!]$
- $[\![\mathscr{B}_1, \mathscr{B}_2$ **passive** $]\!] \Rightarrow [\![\mathscr{B}_1 \sqcap \mathscr{B}_2$ **passive** $]\!]$
- $\cdots$

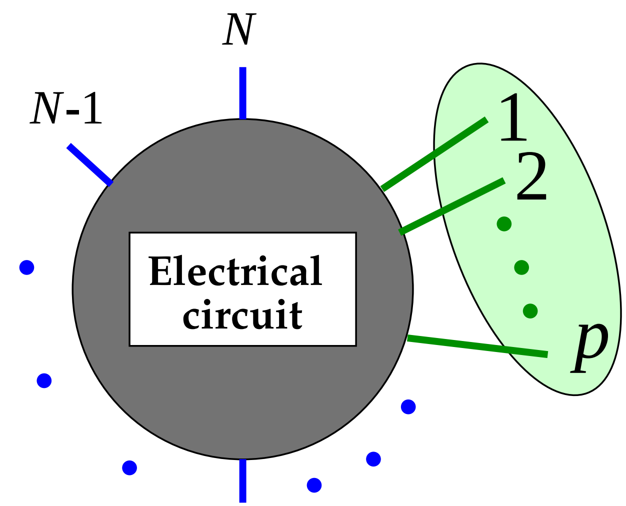
ENERGY TRANSFER

Ports



Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

Ports



Terminals $\{1,2,\ldots,p\}$ **form a port** $:\Leftrightarrow$

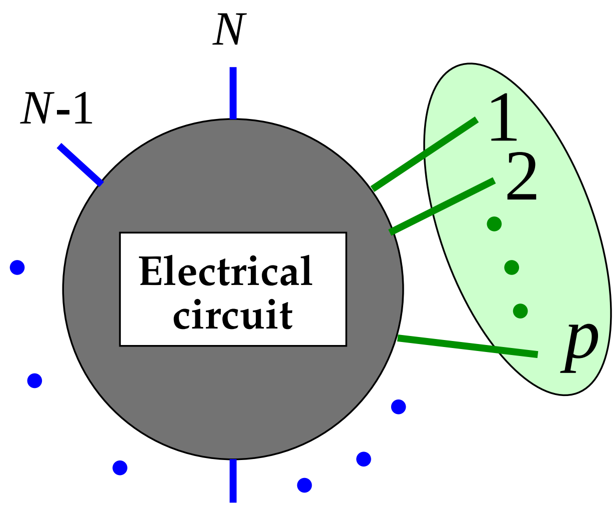
$$[\![(V_1,\ldots,V_p,V_{p+1},\ldots,V_N,I_1,\ldots,I_p,I_{p+1},\ldots,I_N)\in\mathscr{B},\alpha:\mathbb{R}\to\mathbb{R}]\!]$$

$$\Rightarrow\ [\![\,(V_1+\alpha,\ldots,V_p+\alpha,V_{p+1},\ldots,V_N,I_1,\ldots,I_p,I_{p+1},\ldots,I_N)\in\mathscr{B}$$

and $I_1+\cdots+I_p=0\,]\!]$.

Equivalently, the behavioral eq'ns contain the variables $V_i, i\in\{1,2,\ldots,p\}$**, only through** V_i-V_j **for** $i,j\in\{1,2,\ldots,p\}$.

Ports



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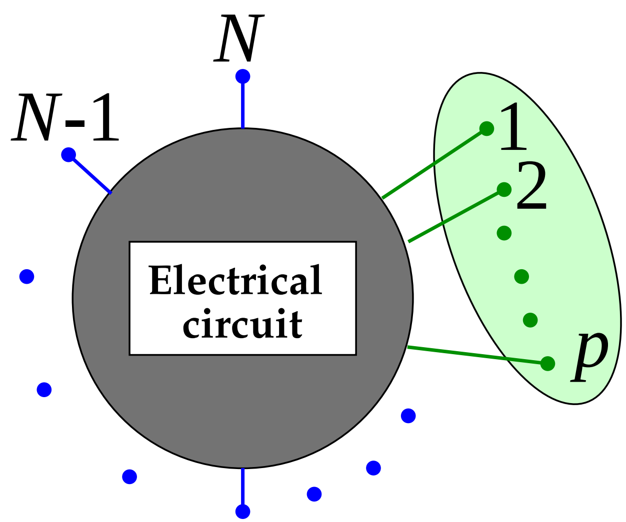
$$\Rightarrow\ [\![(V_1+\alpha,\ldots,V_p+\alpha,V_{p+1},\ldots,V_N,I_1,\ldots,I_p,I_{p+1},\ldots,I_N)\in\mathscr{B}$$

and $I_1+\cdots+I_p=0]\!]$.

KVL & KCL $\Leftrightarrow$ **all terminals combined form a port.**

Ports



If terminals $1, 2, \ldots, p$ **form a port, then**

power in along these terminals = $V_1(t)I_1(t) + \cdots + V_p(t)I_p(t)$,

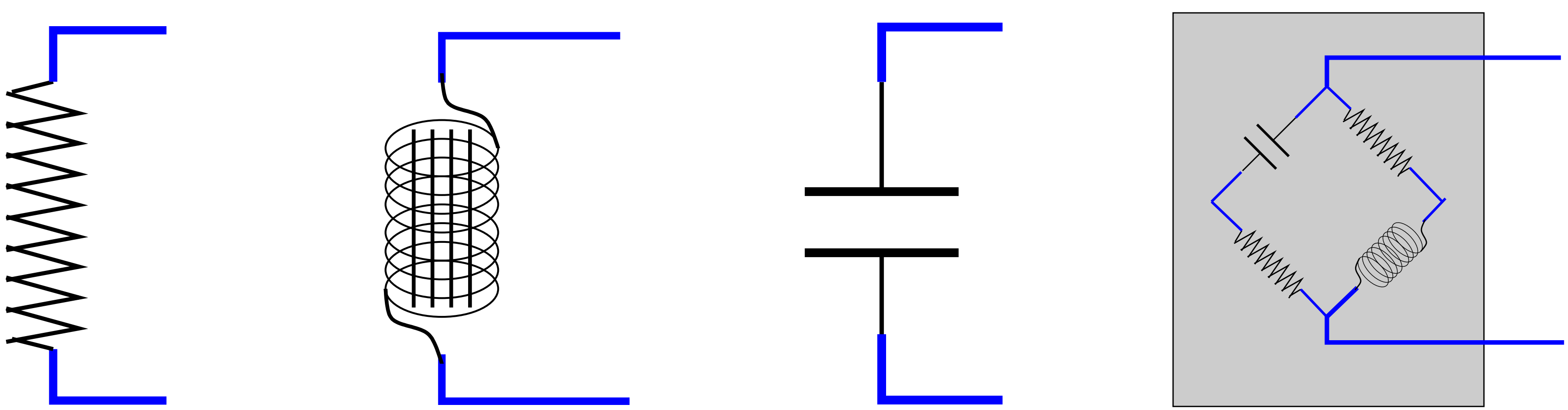
energy in = $$\int_{t_1}^{t_2} (V_1(t)I_1(t) + \cdots + V_p(t)I_p(t))\, dt.$$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

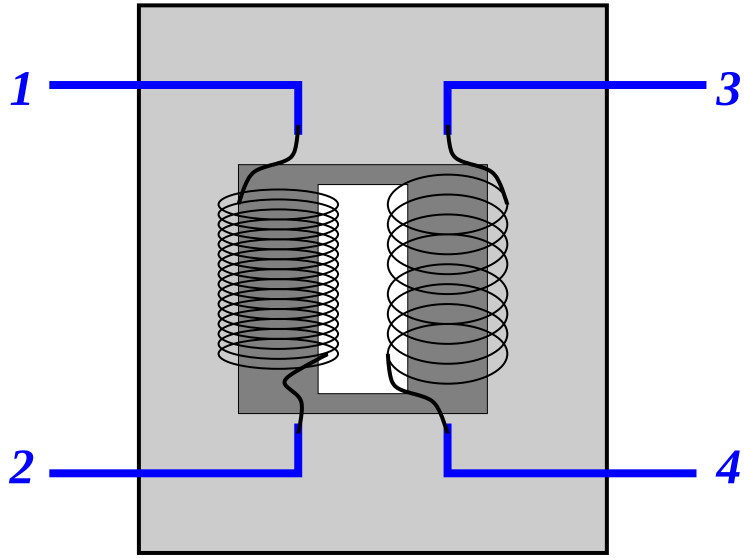
Examples

Examples of 1-port 2-terminal devices:

**resistors, capacitors, inductors, memristors,
any 2-terminal circuit composed of these.**



Examples

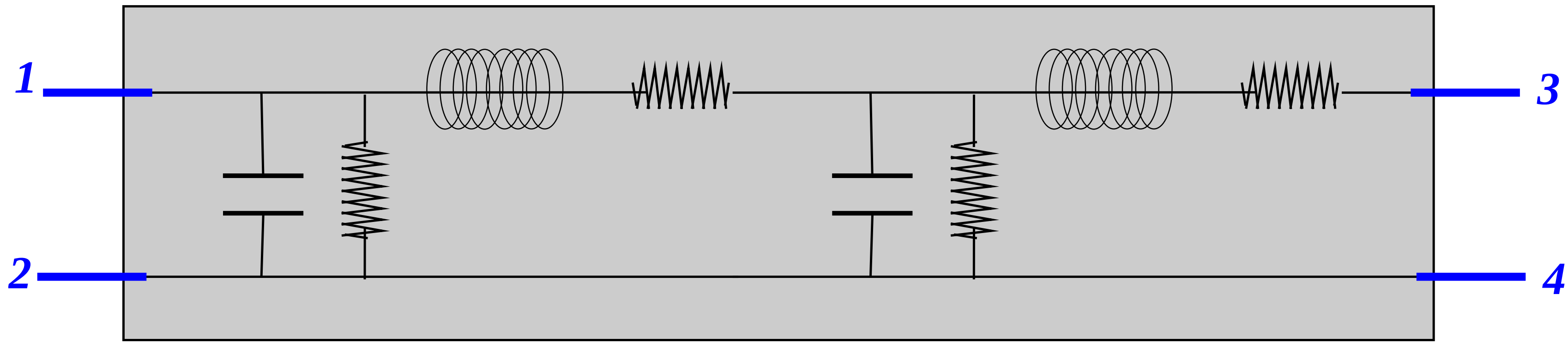


$$V_1 - V_2 = n\left(V_3 - V_4\right), -nI_1 = I_3 \qquad I_1 + I_2 = 0, I_3 + I_4 = 0$$

Terminals $\{1,2\}$ **and** $\{3,4\}$ **(and** $\{1,2,3,4\}$) **form ports.**

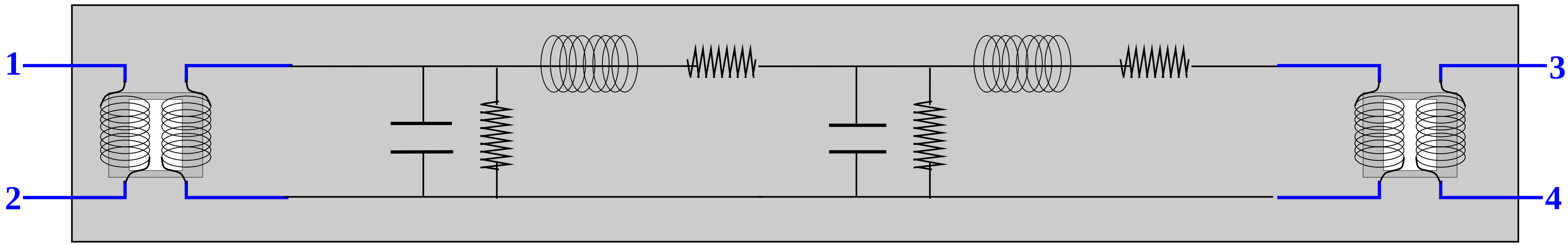
For trafo, energy in at port $\{1,2\}$ **= energy out at port** $\{3,4\}$.

Examples

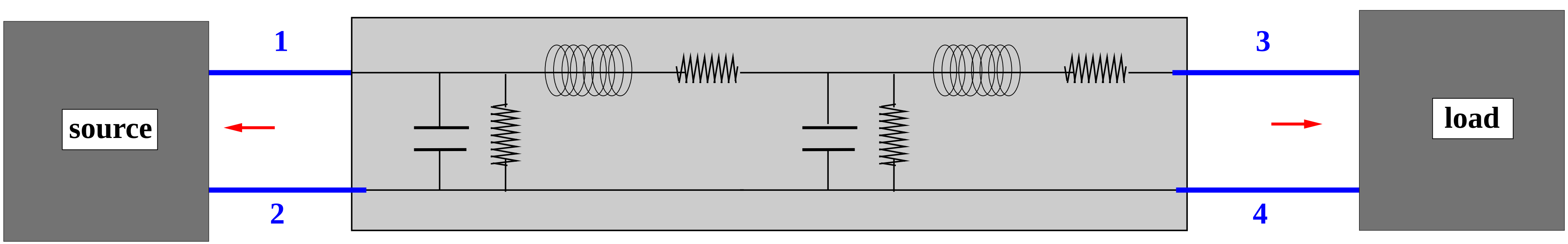


Terminals $\{1, 2, 3, 4\}$ **form a port. But** $\{1, 2\}$ **and** $\{3, 4\}$ **do not.**

Examples



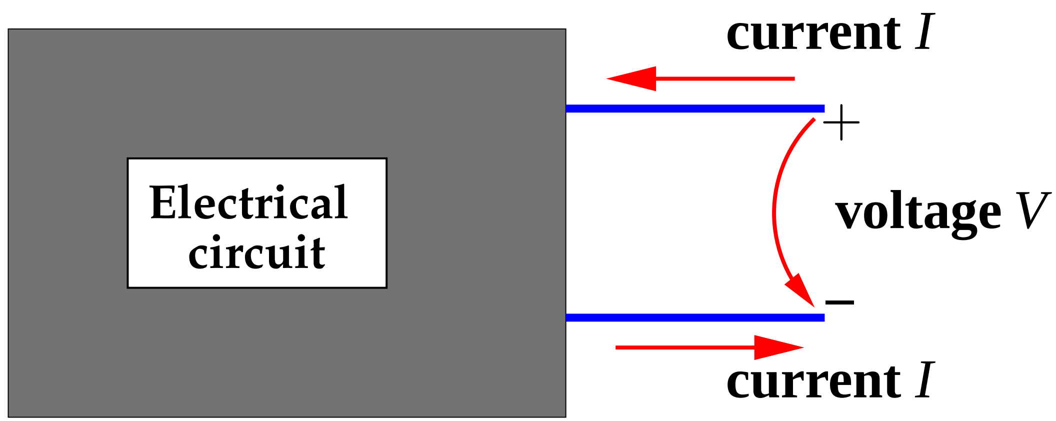
Terminals $\{1, 2\}$ and $\{3, 4\}$ form a port.



Terminals $\{1, 2\}$ and $\{3, 4\}$ form 'internal' ports.

CLASSICAL CIRCUIT THEORY

1-ports



Power in $= VI,$ **energy in** $= \int VI\,dt.$

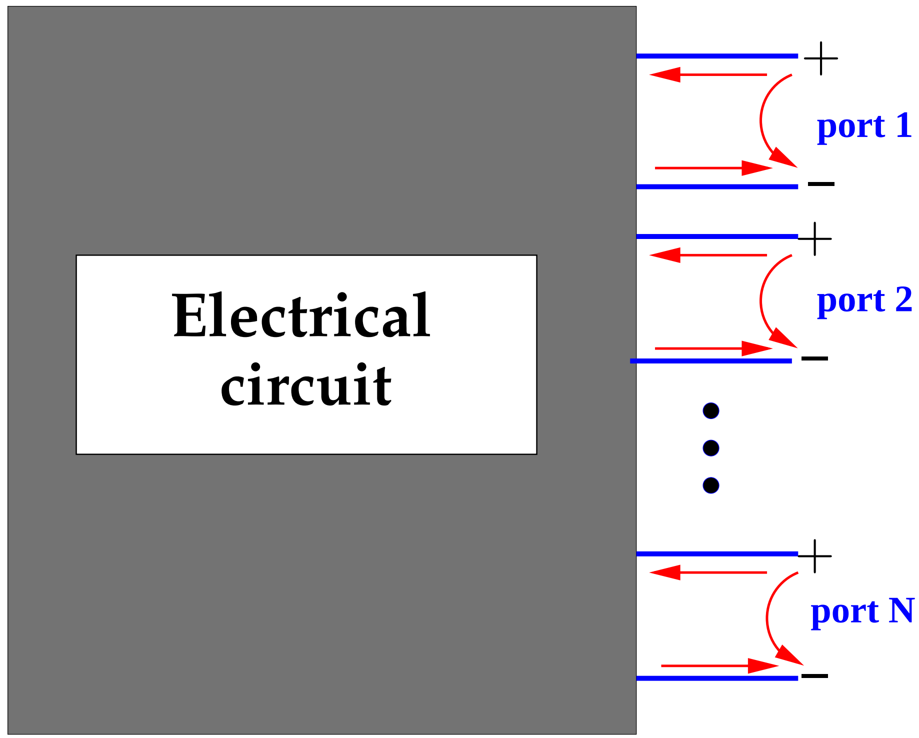
In particular, the energy does NOT flow along the terminals, but into the 'port'.

Power and energy are not a 'local' quantities.

They involve action at a distance.

N-ports

How is the situation for circuits with more terminals?



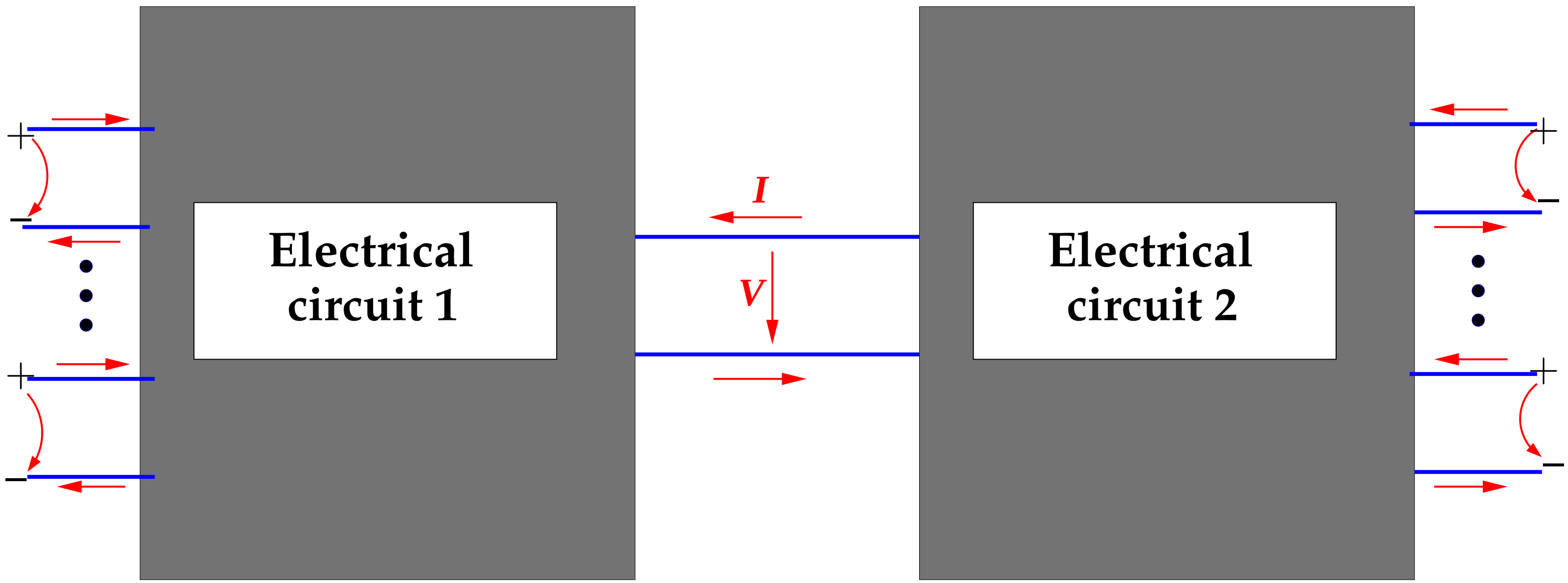
$\rightsquigarrow$ **N-port, N-port synthesis, resistive N-port problem,...**

Energy transmission port-by-port.

N-ports

$\leadsto$ **N-port, N-port synthesis, resistive N-port problem,...**

Energy transmission port-by-port.



If the connected terminals form a port, then

energy transmitted along the port $= \int V I \, dt$.

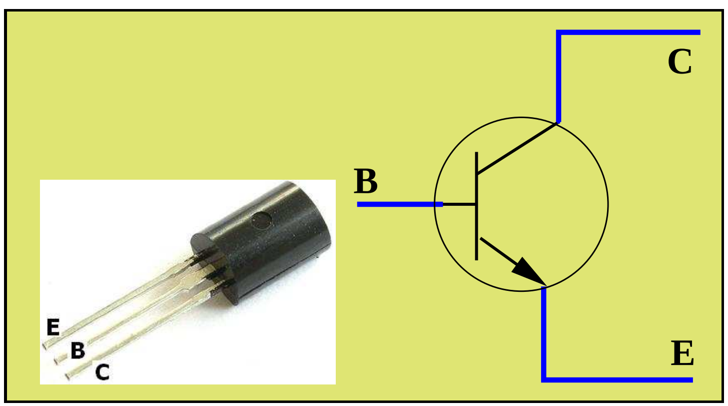
N-ports

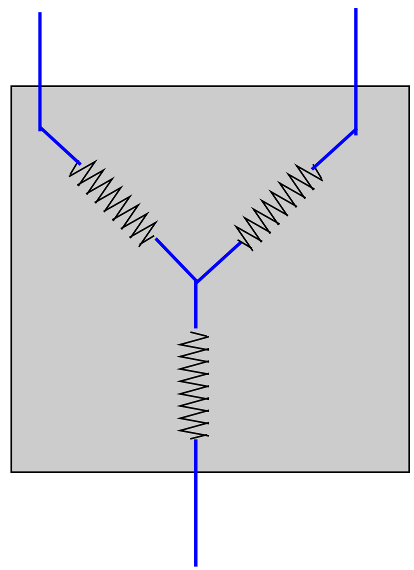
N-port theory does not deal well with the physics of circuits.

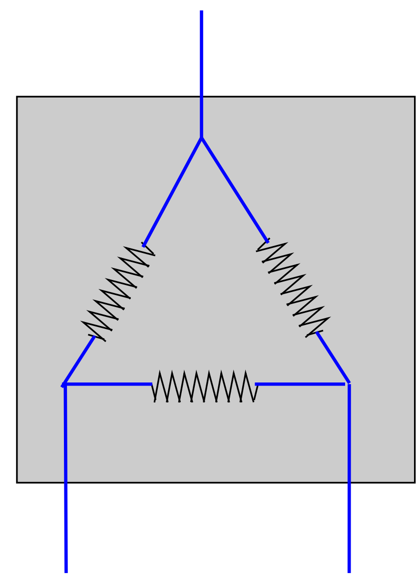
N-ports

N-port theory does not deal well with the physics of circuits.

For example, it does not fit 3-terminal devices.

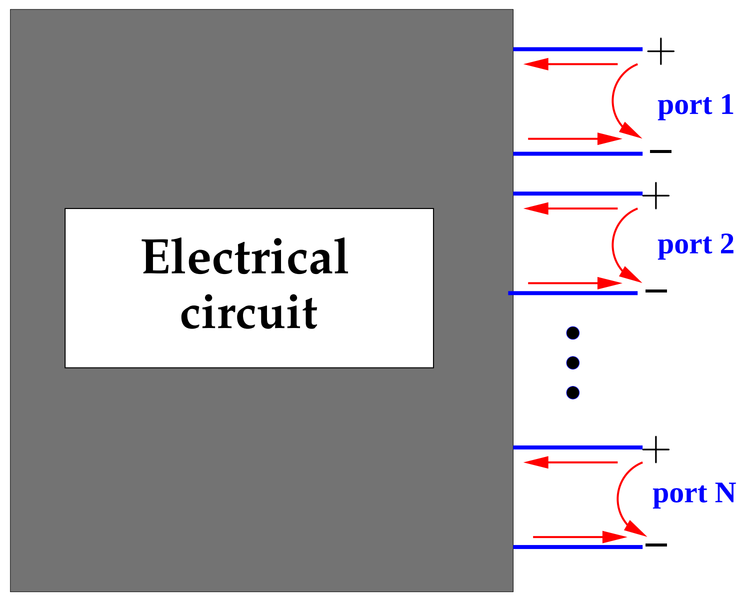






N-ports

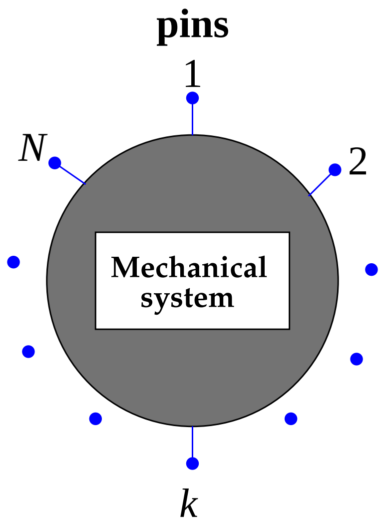
N-port theory **does not deal well with the physics** of circuits, and **input/output thinking** made the confusion complete.

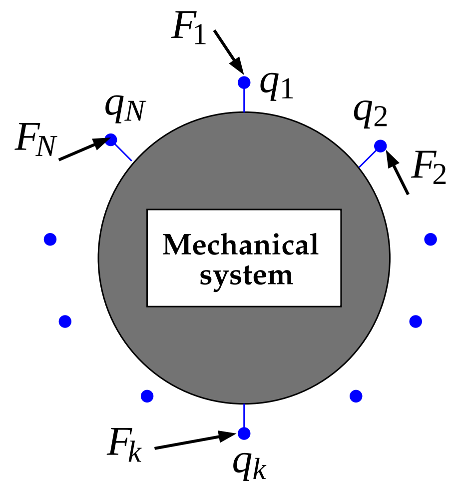


Weird physics, bizarre pedagogy.

MECHANICAL SYSTEMS

The behavior

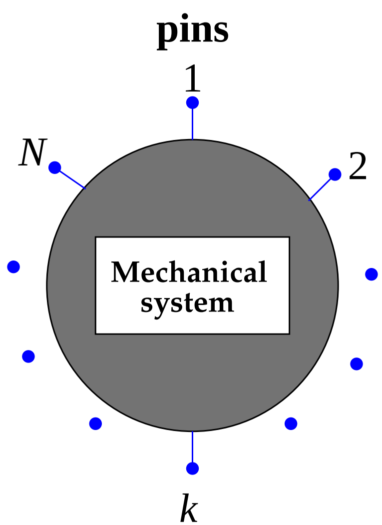


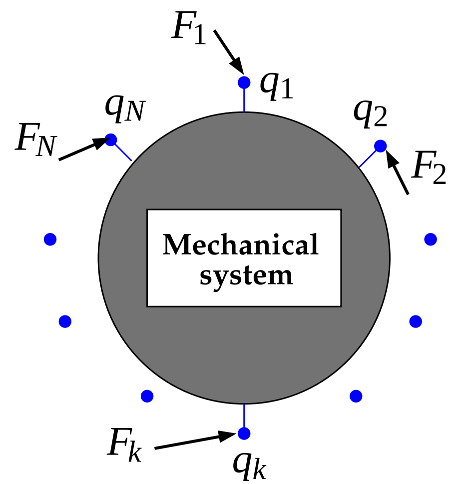


At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

The behavior





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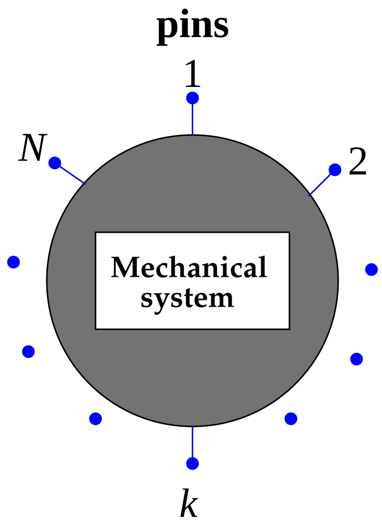
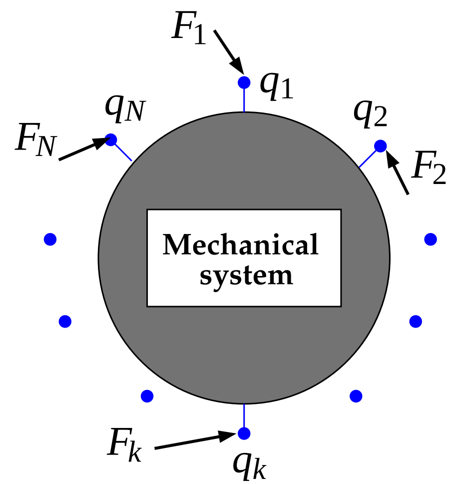
$\rightsquigarrow$ **position/force trajectories** $(q,F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

What are the analogues of KVL, KCL, interconnection?

The behavior

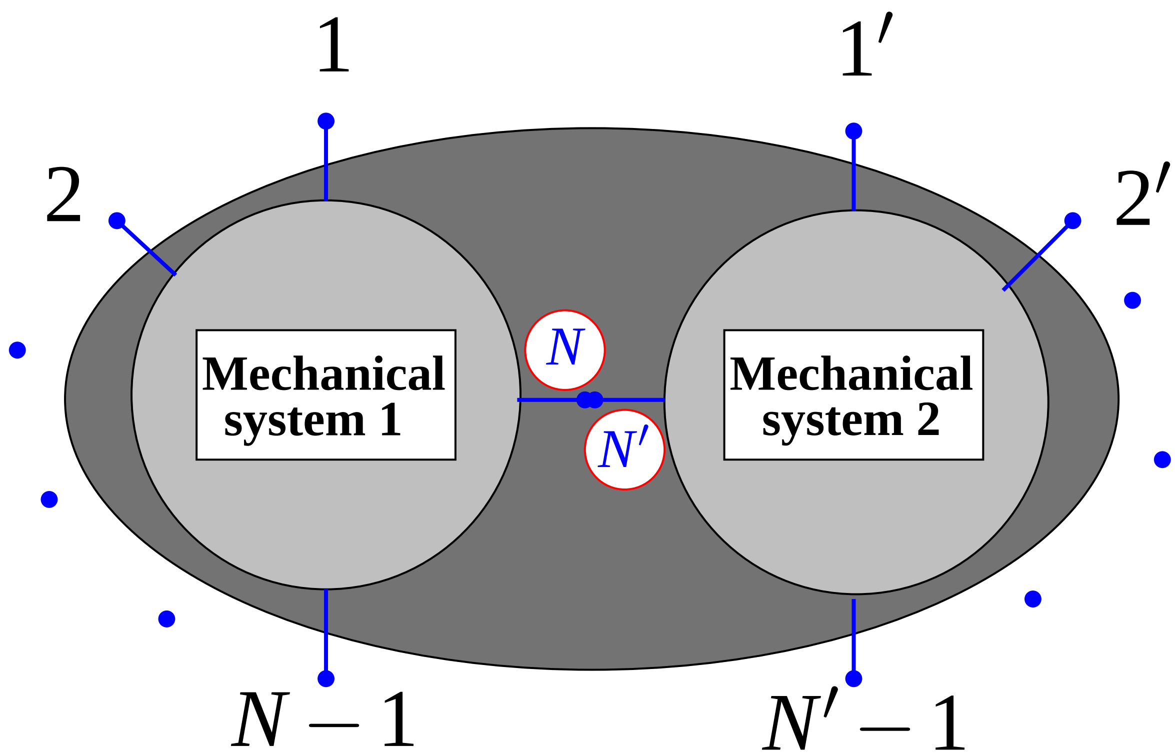




- **invariance under uniform motion** $:\Leftrightarrow (q_1, q_2, \ldots, q_N, F_1, F_2, \ldots, F_N) \in \mathscr{B}$ **and** $v : t \in \mathbb{R} \mapsto (a + bt) \in \mathbb{R}^\bullet$**, imply** $(q_1 + v,\ q_2 + v, \ldots, q_N + v, F_1, F_2, \ldots, F_N) \in \mathscr{B}$.

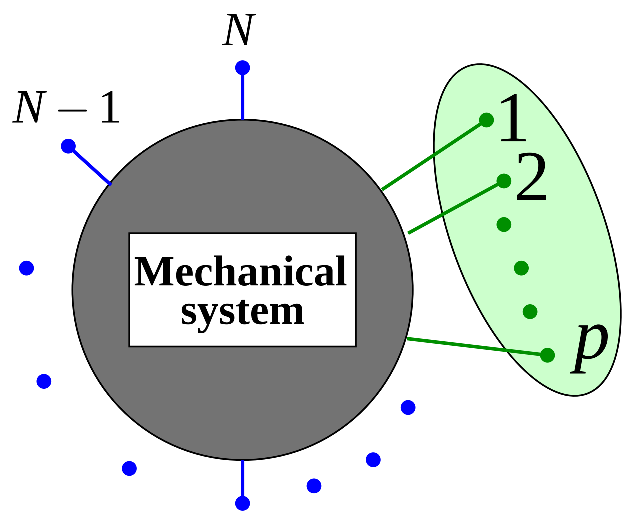
- **Kirchhoff's force law (KFL)** $:\Leftrightarrow (q_1, q_2, \ldots, q_N, F_1, F_2, \ldots, F_N) \in \mathscr{B}$ **implies** $F_1 + F_2 + \cdots + F_N = 0$.

Interconnection



$$q_N = q_{N'} \quad \textbf{and} \quad F_N + F_{N'} = 0.$$

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

and $v : t \in \mathbb{R} \mapsto (a + bt) \in \mathbb{R}^\bullet, a, b \in \mathbb{R}^\bullet$

$$\Rightarrow \quad (q_1 + v, \ldots, q_p + v, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B}$$

and $F_1 + F_2 + \cdots + F_p = 0.$

Power and energy

If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

$$\textbf{power in} \;=\; F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

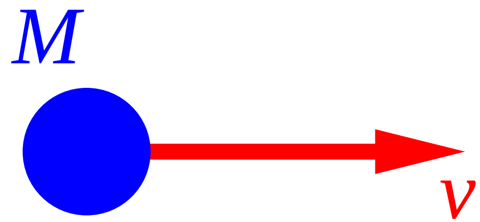
and

$$\textbf{energy in} \;=\; \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid
unless these terminals form a port !

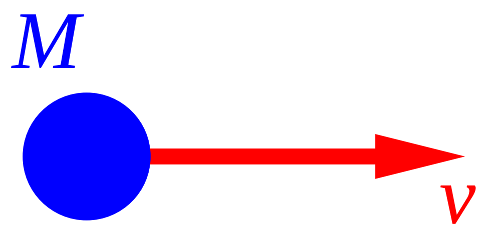
KINETIC ENERGY

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

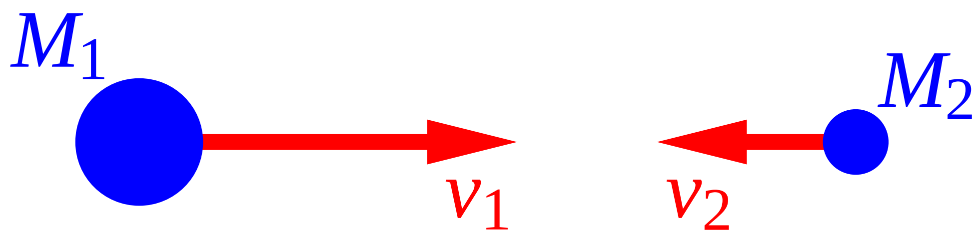
$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2}\, M \, ||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

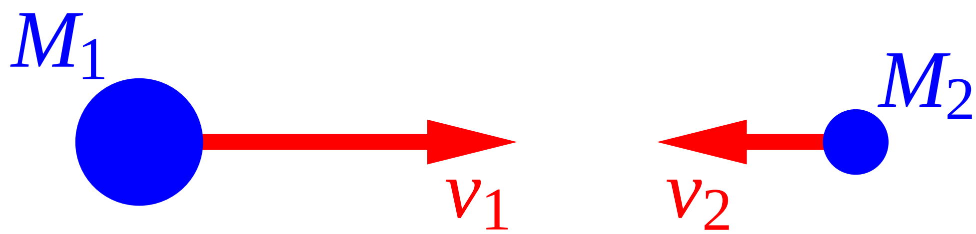
This formula is not invariant under uniform motion.

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

Kinetic energy and invariance under uniform motions



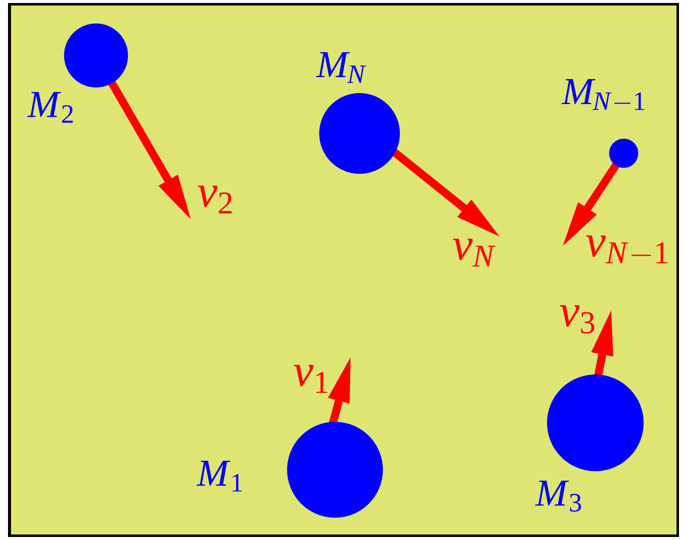
What is the kinetic energy?

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2}\ \frac{M_1\ M_2}{M_1 + M_2}\ ||v_1 - v_2||^2$$

Invariant under uniform motion.

Can be justified by mounting a damper or a spring between the masses.

Kinetic energy

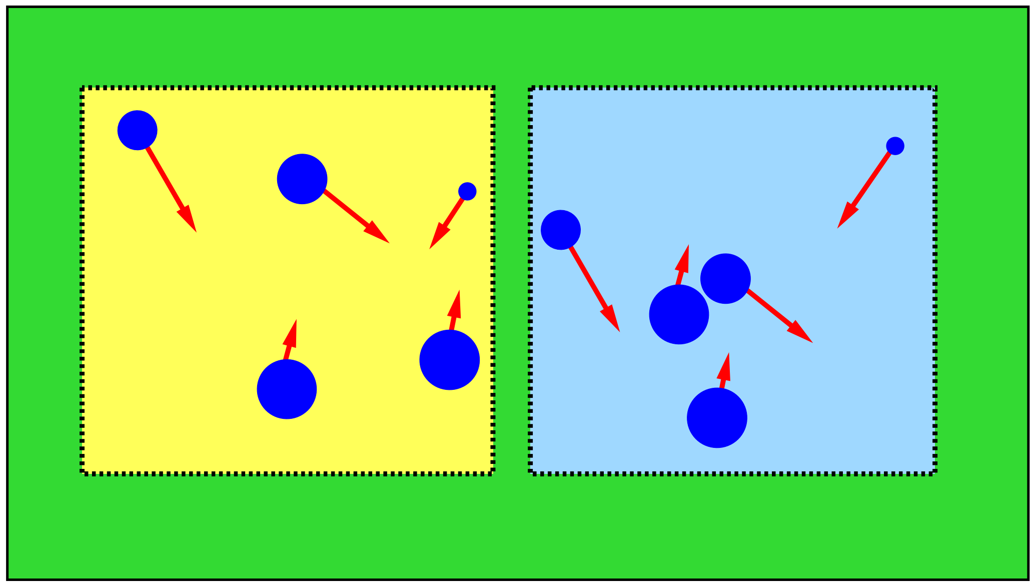


$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2 .$$

KFL $\Rightarrow$ $$\frac{d}{dt} \mathscr{E}_{\textbf{kinetic}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i .$$

Kinetic energy

Kinetic energy is not additive.



Total kinetic energy $\neq$ sum of the parts.

Kinetic energy

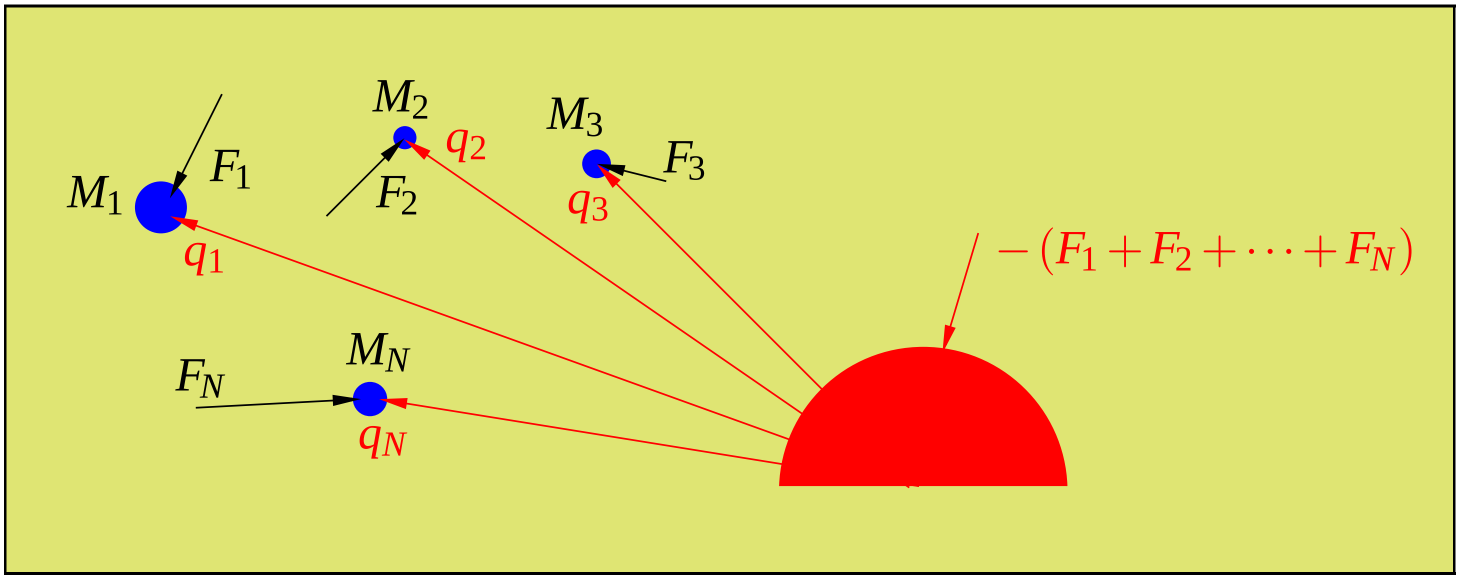
$$\mathscr{E}_{\textbf{kinetic}} = \boxed{\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.}$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{classical}} = \boxed{\frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.}$$

Kinetic energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -\left(F_1 + F_2 + \cdots + F_N\right)$,



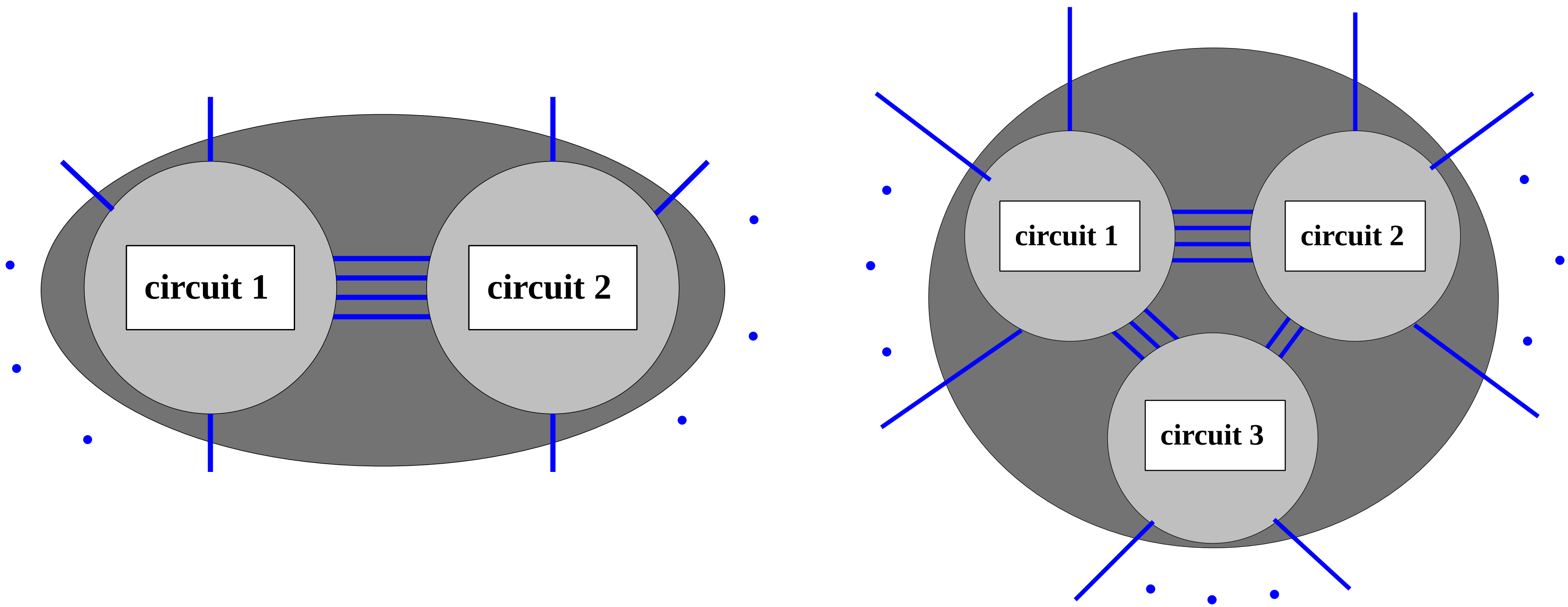
measure velocities w.r.t. this infinite mass, then

$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \qquad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i\, ||v_i||^2.$$

PORTS and TERMINALS

Energy transfer



One cannot speak about

"the energy transferred from circuit 1 to circuit 2"
or *"from the environment to circuit 1"*,

unless the relevant terminals form a port.

Conclusion

Terminals are for interconnection, ports are for energy transfer.

Interconnection is ‘local’,
power and energy transfer involve ‘action at a distance’.

Conclusion

Terminals are for interconnection, ports are for energy transfer.

Interconnection is ‘local’,
power and energy transfer involve ‘action at a distance’.

The basis of bond-graph (and related) modeling methodologies that

‘In physical systems, the interaction between subsystems is always related to an exchange of energy’

is flawed.

Conclusion

The physical basis for theoretical engineering settings, be they

- **information theory with its binary symmetric channel,**
- **additive stochastic noise with stochastic signals,**
- **ARMAX models in system identification,**
- **input/output partitions for system descriptions and related interconnection ideas,**
- **N-port circuit theory,**

is lacking, or weak.

Copies of the lecture frames will be available from/at

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http://www.esat.kuleuven.be/∼jwillems

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

 

Enjoy!