INTERCONNECTED SYSTEMS

Jan C. Willems

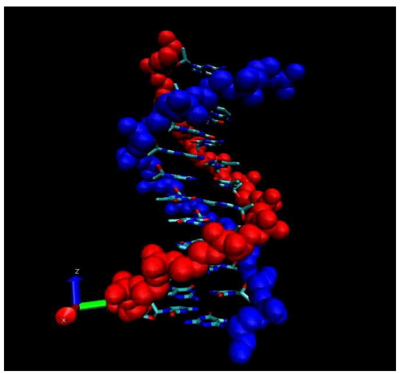
Colloquium, SISTA, KULeuven **November 18, 2008**

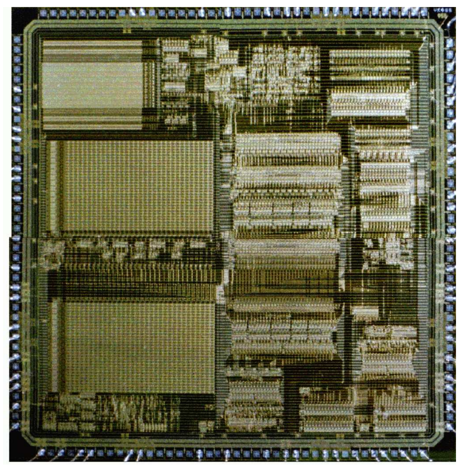
Outline

- Open, connected, and modular
- Classical dynamical systems
- Input/output systems
- Modeling by tearing, zooming, and linking
- Signal flow graphs
- Bond graphs
- Circuit diagrams

Systems


ZYGOTE





Features

- **open**
- **interconnected**
- **modular**
- **dynamic**

Features

- **open**
- **interconnected**
- **modular**
- **dynamic**

Aim:

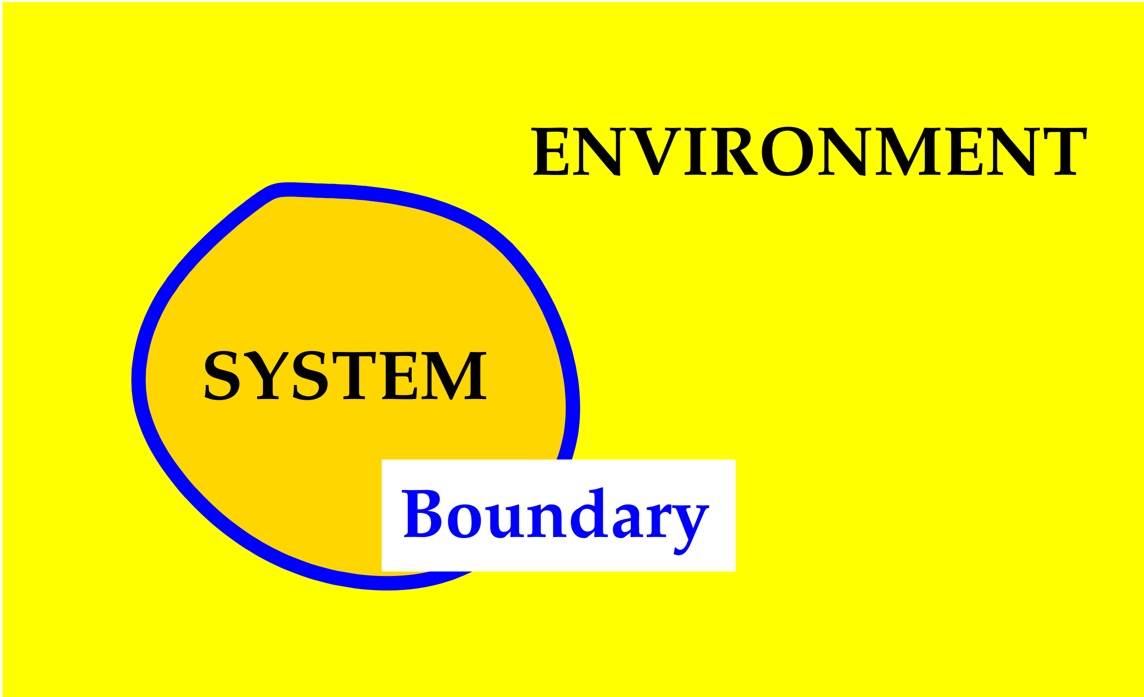
develop a suitable mathematical language

aimed at computer-assisted modeling.

Modeling $\Leftrightarrow$ ***Describing reality accurately***

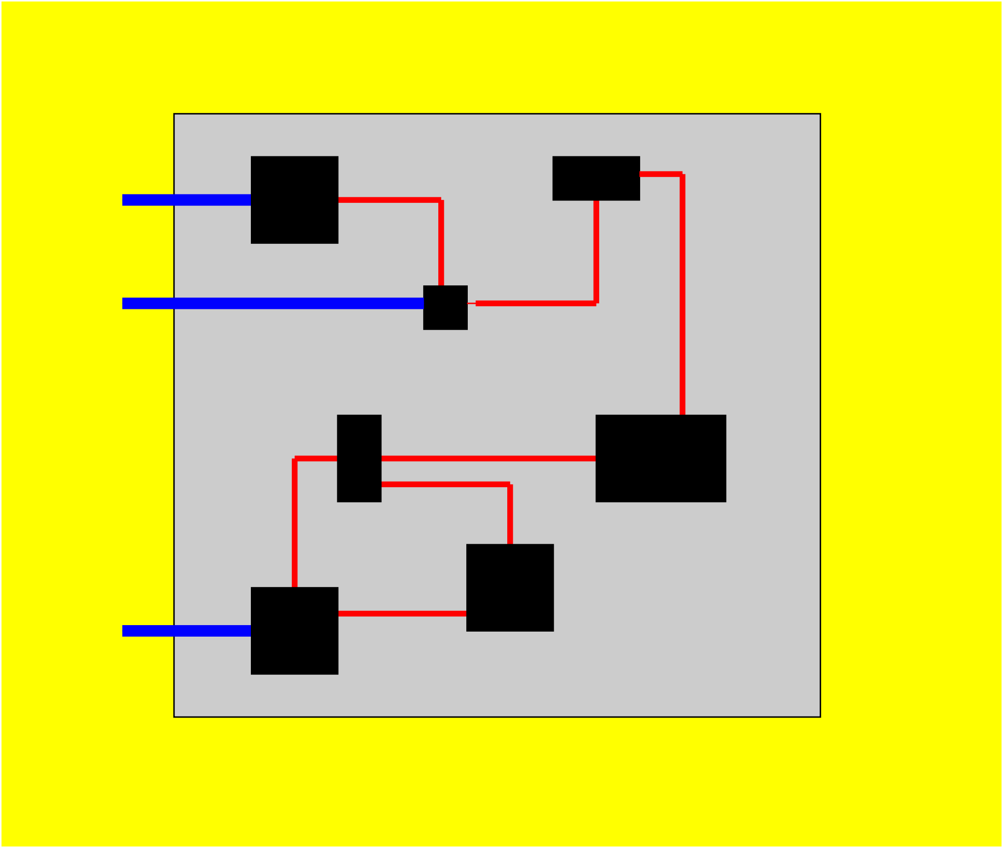
Open, connected, modular

Open



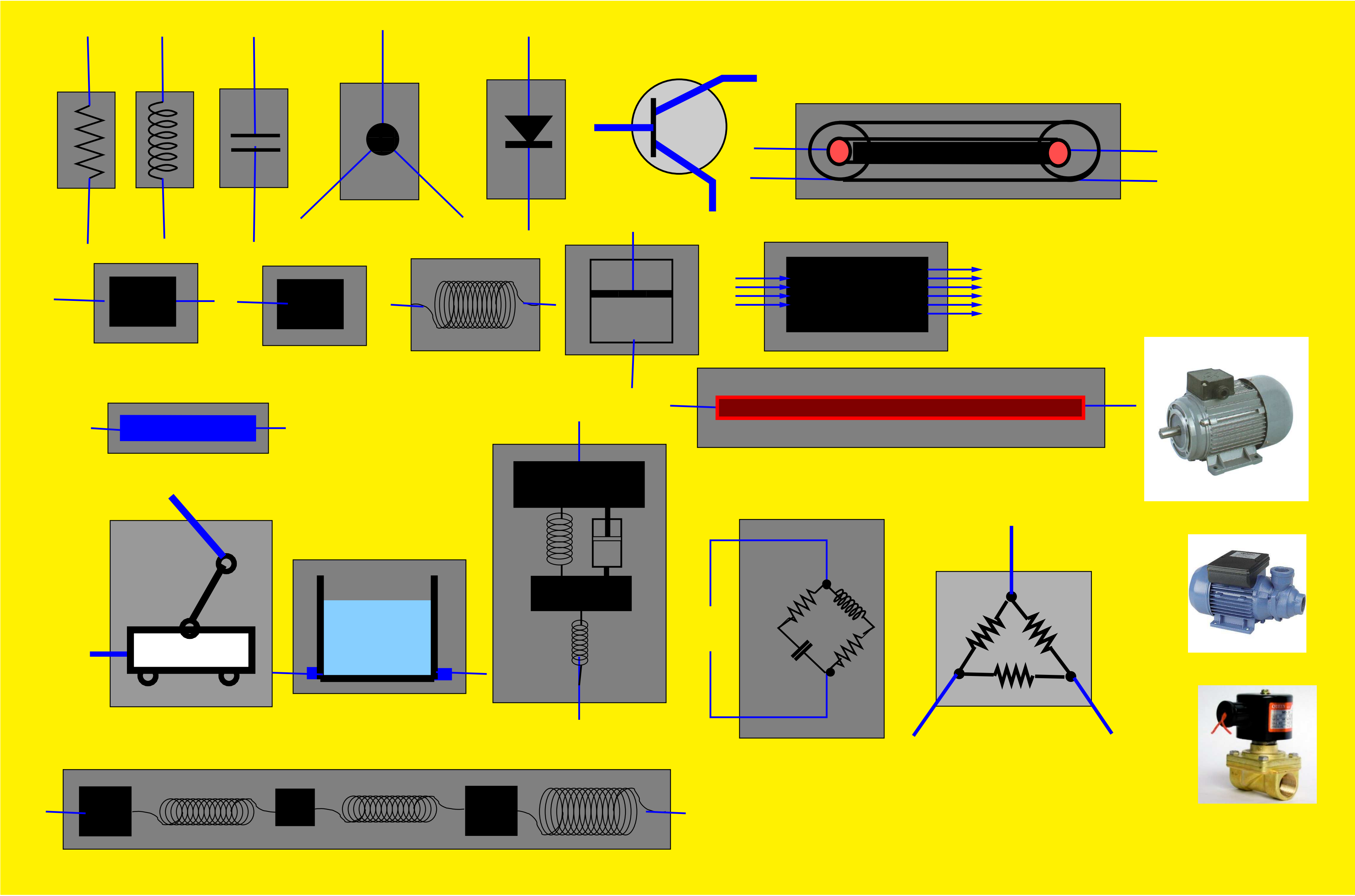
Systems interact with their environment

Connected



Systems consist of an architecture of interconnected subsystems

Modular



Systems are modular: composed of ‘building blocks’

The development of the notion

of a dynamical system

Closed dynamical systems

Closed dynamical systems

K.1, K.2, & K.3

$$\rightsquigarrow \quad \frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|\frac{d}{dt}w(t)|^2} = 0$$

$$\rightsquigarrow \textbf{ with } x = (w, \tfrac{d}{dt}w) \quad \rightsquigarrow \quad \frac{d}{dt}x = f(x)$$

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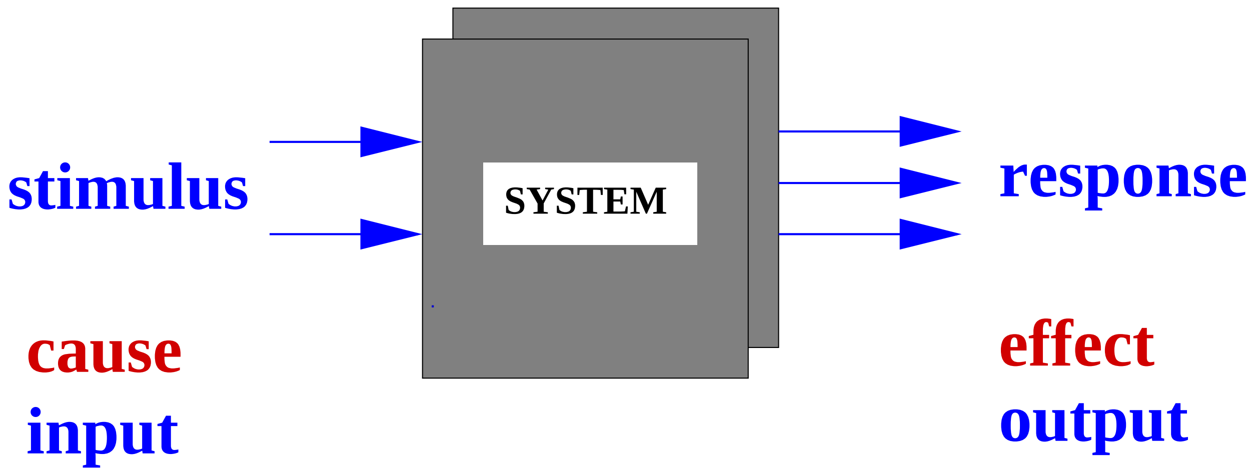
$\rightsquigarrow$ **flows as paradigm of dynamics** $\rightsquigarrow$ **closed systems**

Motion determined by initial conditions:
a popular (but inadequate) paradigm for modeling dynamics.

Very frequently in mathematics and physics (chaos theory, synchronization, classical mechanics, QM, ...)

Inputs and outputs

Input/output systems



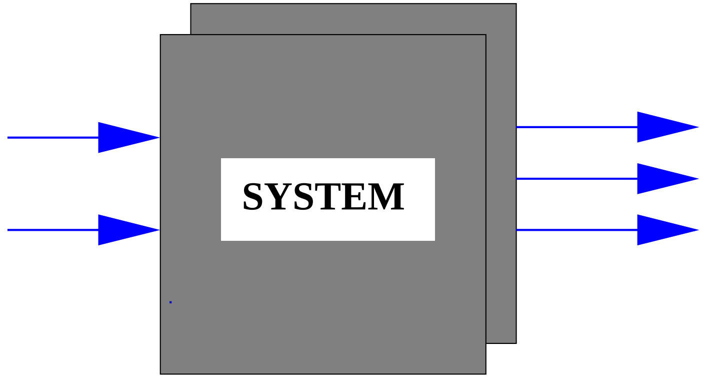
Transfer functions, impedances, convolutions, Volterra series, ...

Input/output systems

stimulus

cause

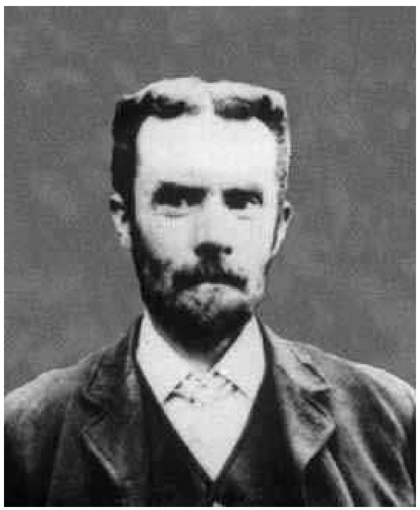
input



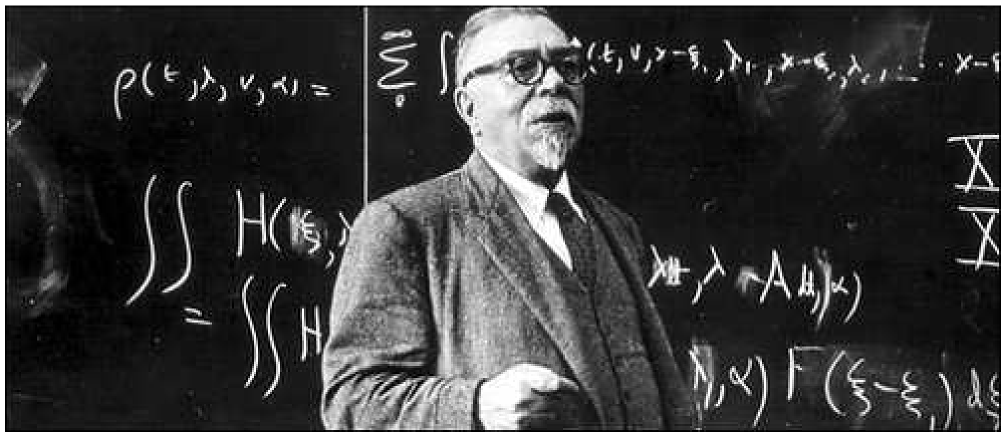
response

effect

output



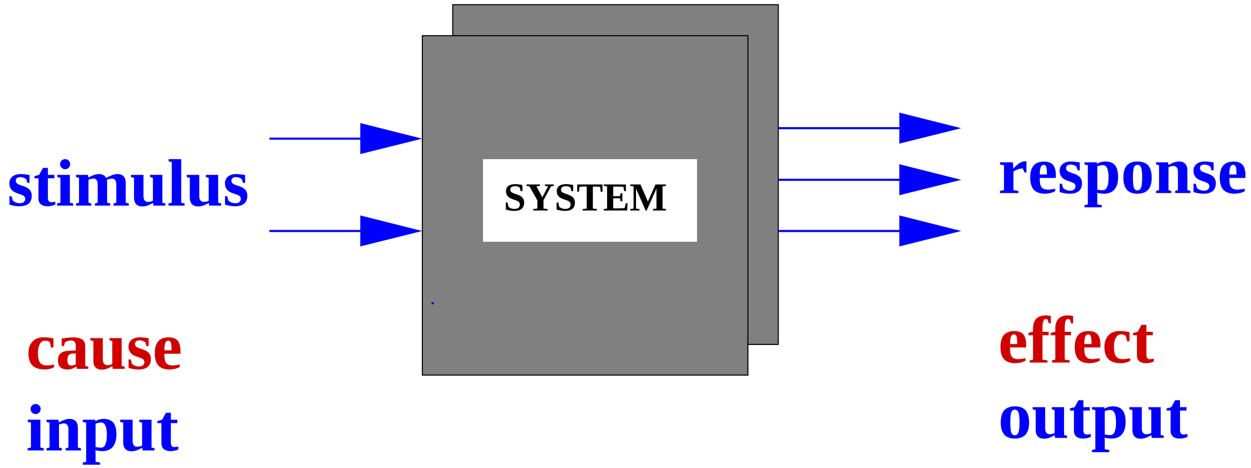
Oliver Heaviside (1850-1925)



Norbert Wiener (1894-1964)

and the many electrical circuit theorists

Mathematical description



$$y(t) = \int_{0 \text{ or } -\infty}^{t} H(t-t')u(t')\, dt'$$

$$y(t) = H_0(t) + \int_{-\infty}^{t} H_1(t-t')u(t')\, dt' + \int_{-\infty}^{t}\int_{-\infty}^{t'} H_2(t-t', t'-t'')u(t')u(t'')\, dt'dt'' + \cdots$$

Far from the physics. Fails to deal with ‘initial conditions’.

Awkward for nonlinear models, ...

Input/state/output systems

Around 1960: a paradigm shift to

$$\frac{d}{dt}x = f(x,u),\ y = g(x,u)$$

Rudolf Kalman (1930-)

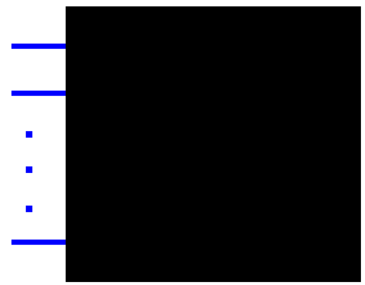
- **open**
- **ready to be interconnected**
  outputs of one system $\mapsto$ inputs of another
- **deals with initial conditions**
- **incorporates nonlinearities, time-variation**
- **models many physical phenomena**
- $\cdots$

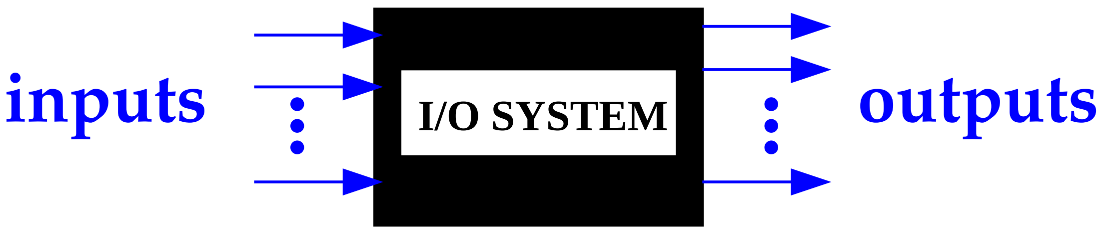
This framework turned out to be very effective and useful!

Theme

Theme of this lecture

We are accustomed to view an open dynamical system as an input/output structure (with or without the state)





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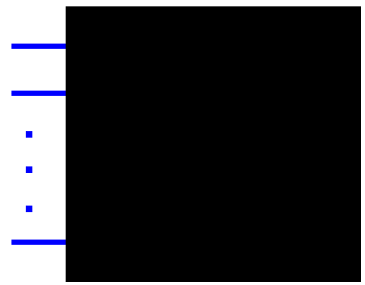
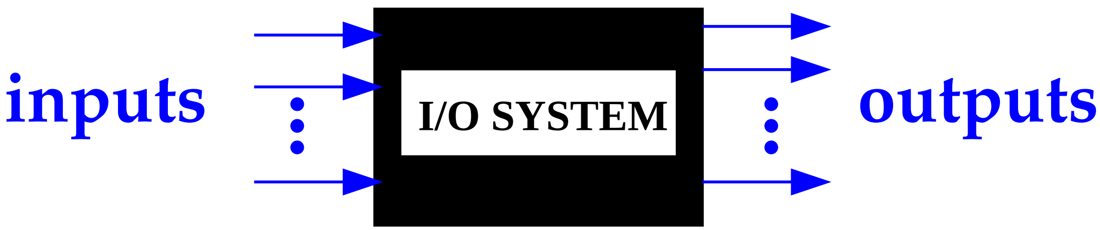
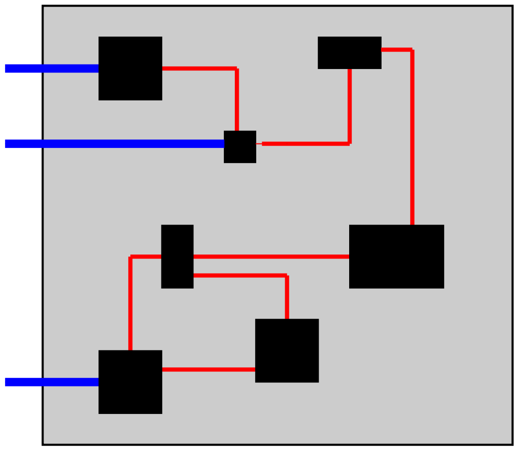


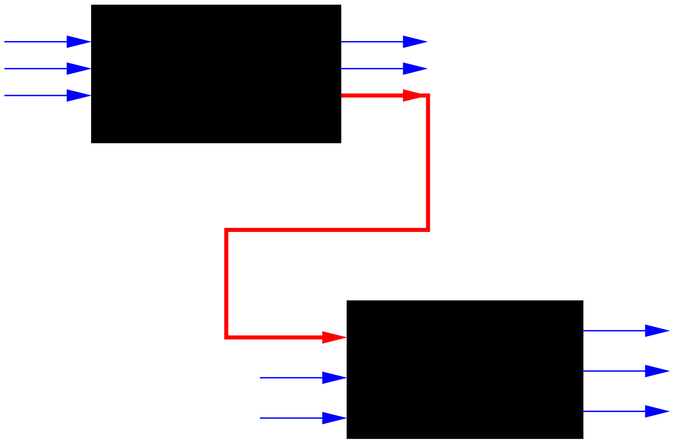


Is this an appropriate abstraction of models of physical systems?

Theme of this lecture

And we are also accustomed to view interconnection as output-to-input assignment





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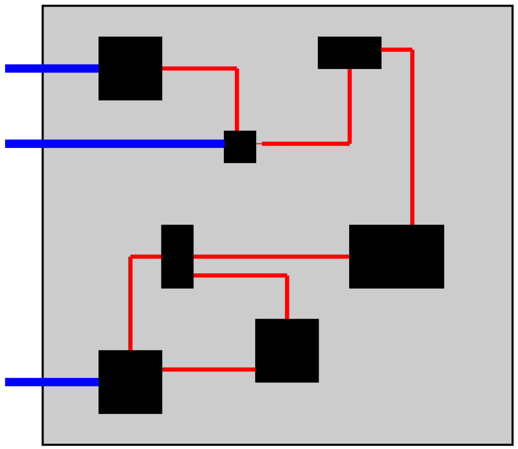
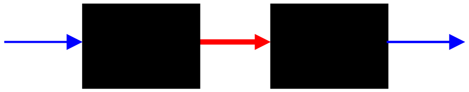
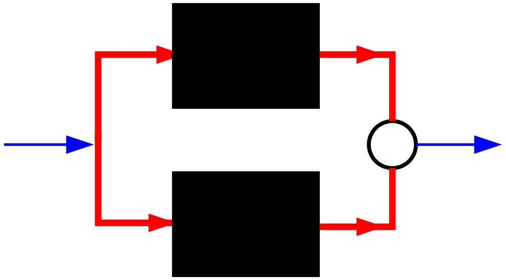


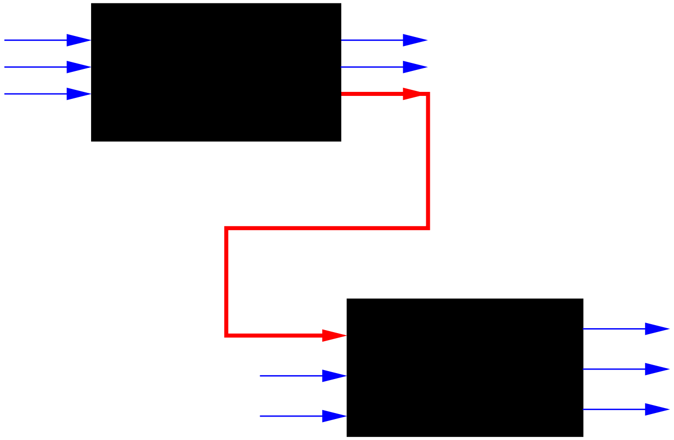
Feedback



Series



Parallel



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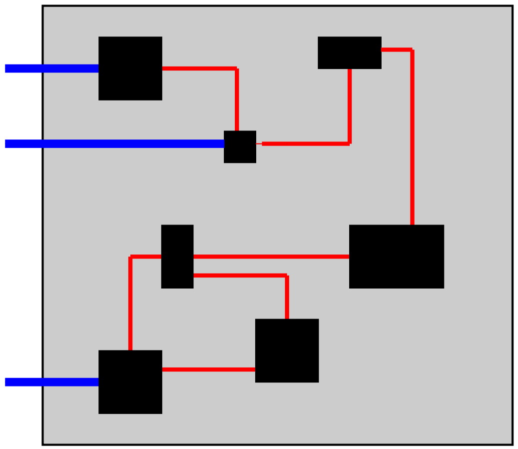

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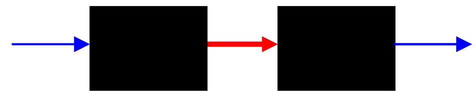

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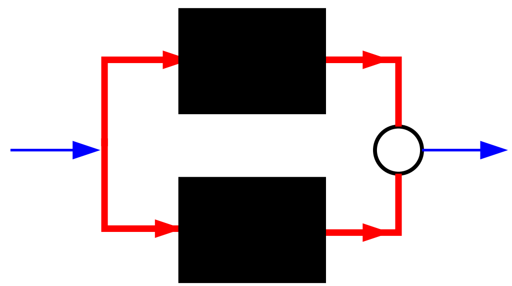

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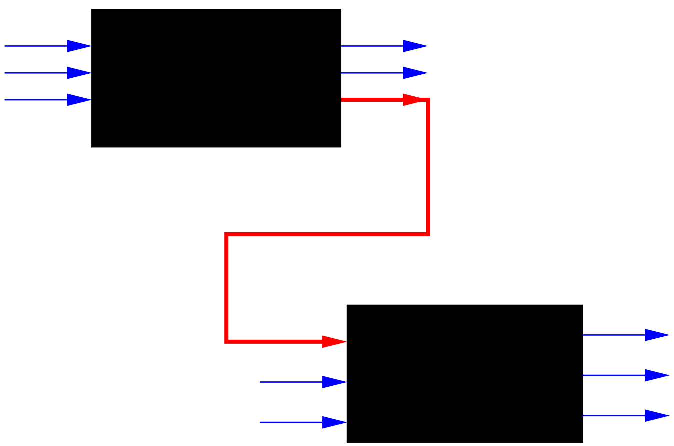
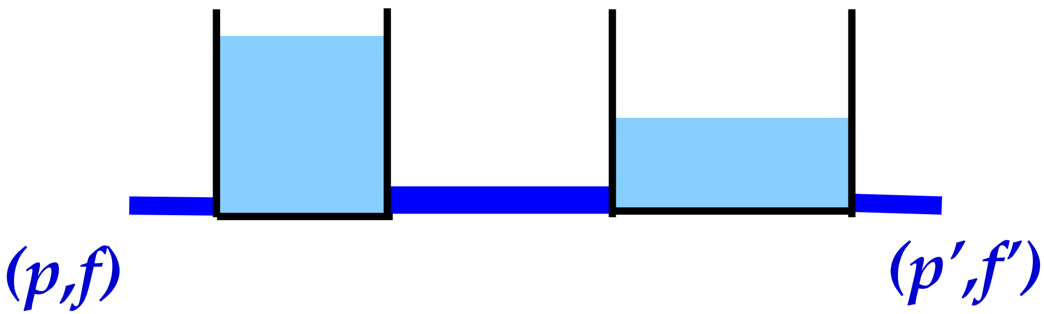


Is this an appropriate abstraction of interconnection of physical systems?

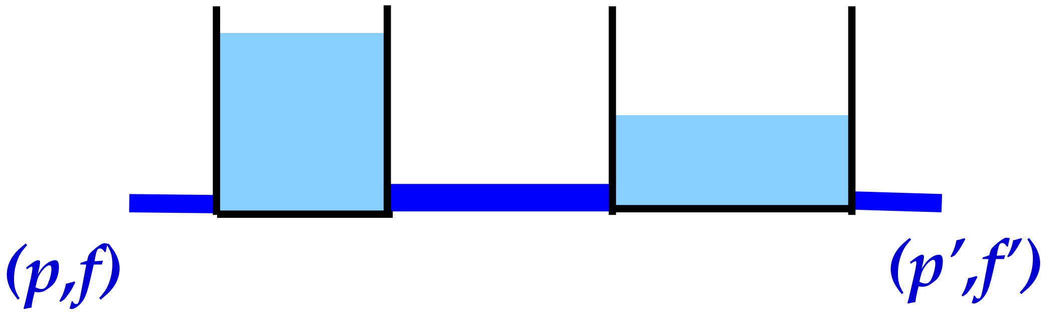
An example

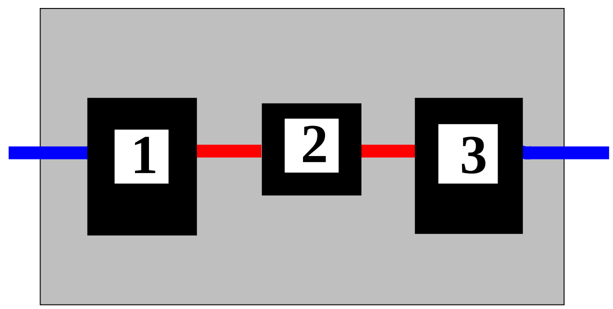
Tearing

Tearing



Tearing





Subsystems 1 and 3 (tanks):

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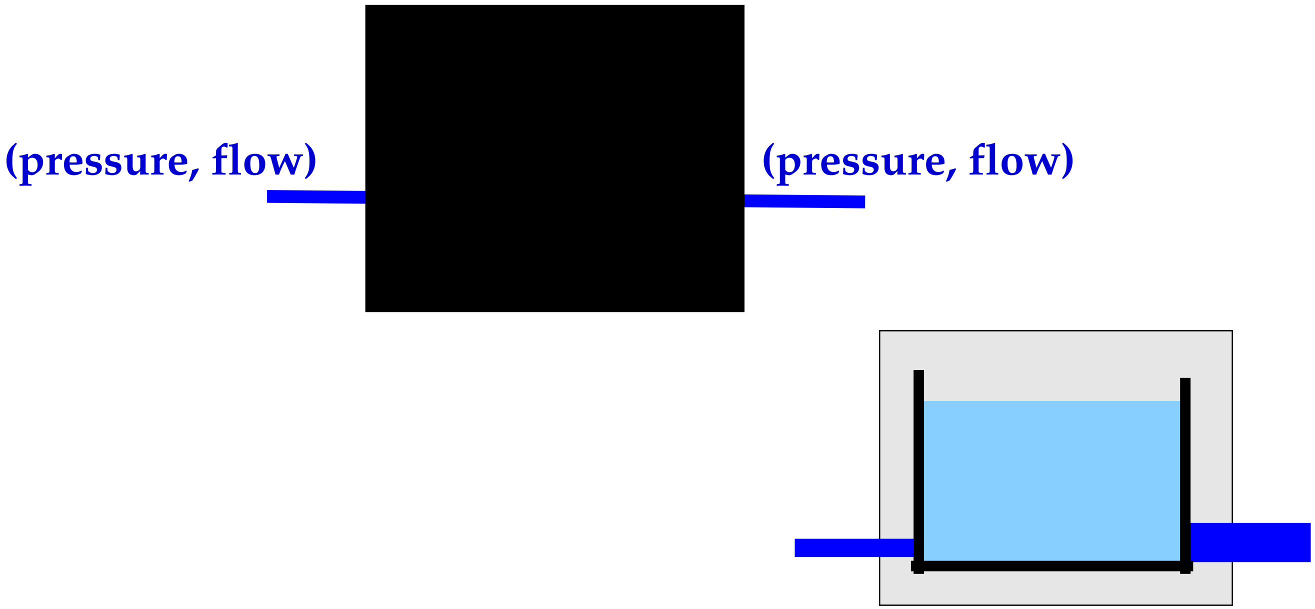

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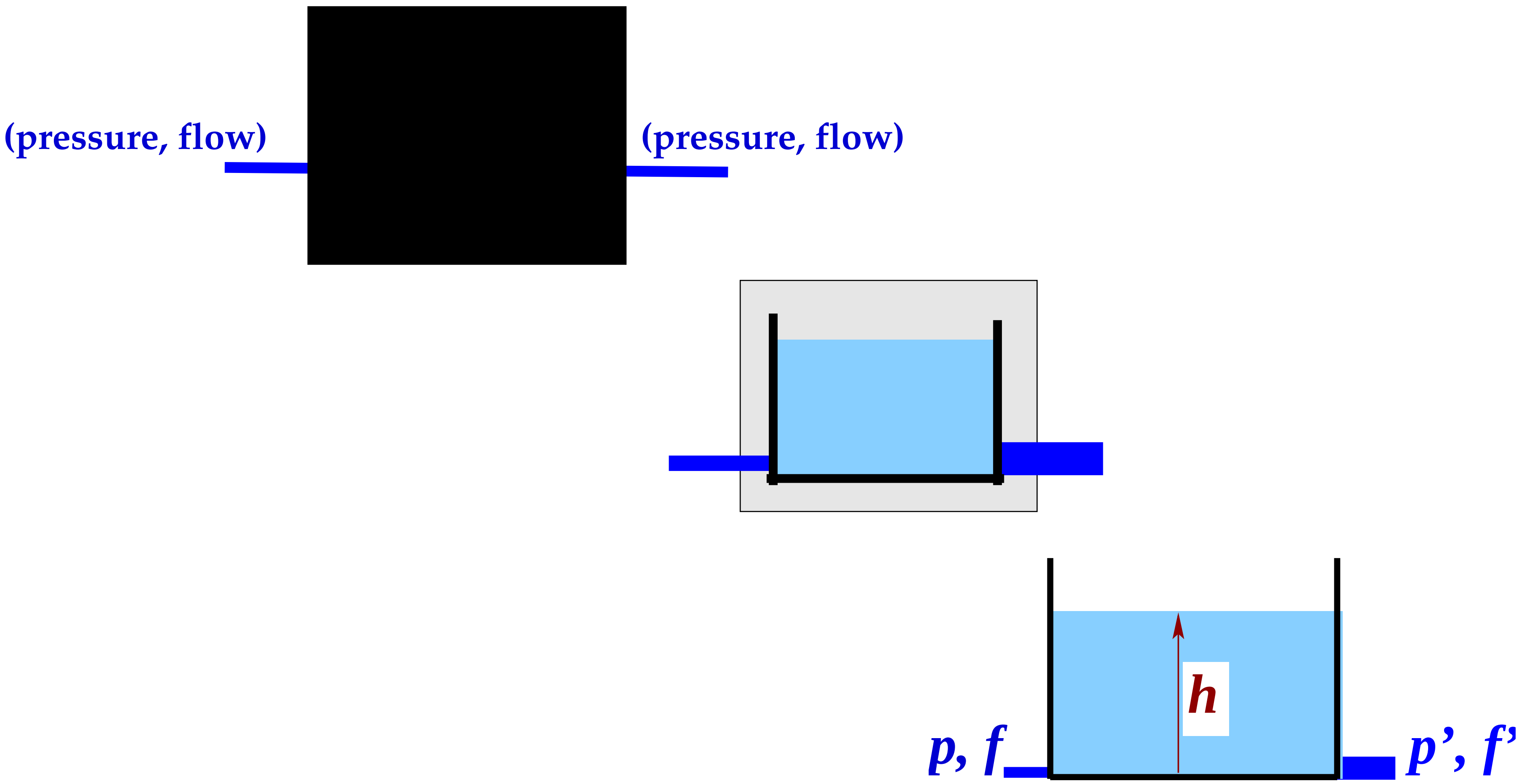
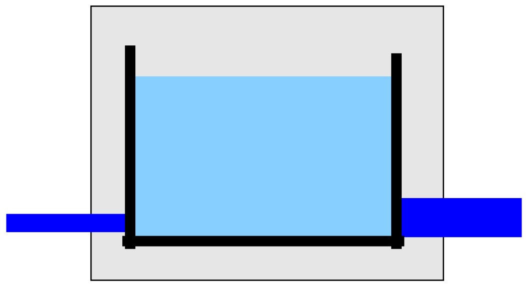
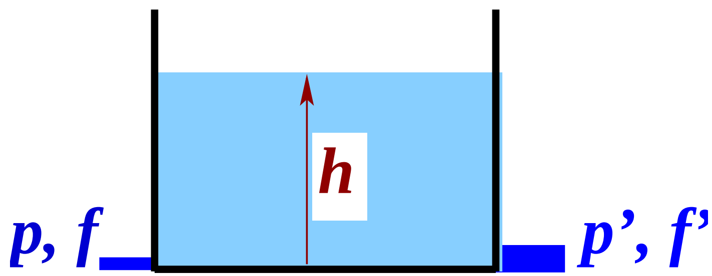


Zooming

Subsystems 1 and 3 (tanks):



$$
\begin{aligned}
A\frac{d}{dt}h &= f + f', \\
Bf &= \begin{cases} \sqrt{|p - p_0 - \rho h|} & \textbf{if } p - p_0 \geq \rho h, \\ -\sqrt{|p - p_0 - \rho h|} & \textbf{if } p - p_0 \leq \rho h, \end{cases} \\
Cf' &= \begin{cases} \sqrt{|p' - p_0 - \rho h|} & \textbf{if } p' - p_0 \geq \rho h, \\ -\sqrt{|p' - p_0 - \rho h|} & \textbf{if } p' - p_0 \leq \rho h, \end{cases}
\end{aligned}
$$



Zooming

Subsystem 2 (pipe):

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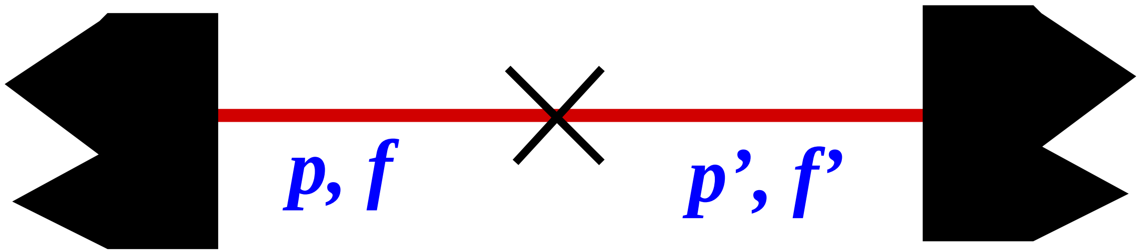
$$f = -f', \quad p - p' = \alpha f$$

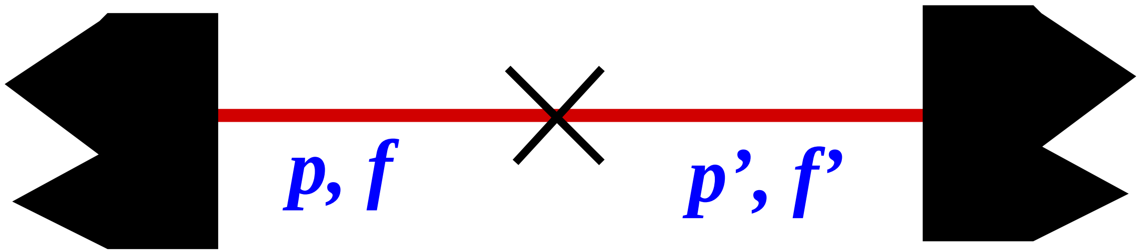
Linking

Interconnection laws:



Linking

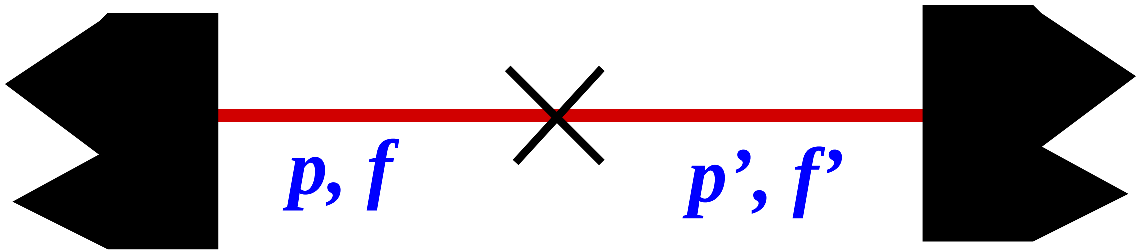
Interconnection laws:



$$p = p', \qquad f + f' = 0$$

Linking

Interconnection laws:



$$p = p', \qquad f + f' = 0$$

Leads to the complete model:

$$A_1 \frac{d}{dt} h_1 = f_1 + f_1',$$

$$B_1 f_1 = \begin{cases} \sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \geq \rho h_1, \\ -\sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \leq \rho h_1, \end{cases} \qquad \textbf{(blackbox 1)}$$

$$C_1 f_1' = \begin{cases} \sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \geq \rho h_1, \\ -\sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \leq \rho h_1, \end{cases}$$

$$f_2 = -f_2', \quad p_2 - p_2' = \alpha f_2, \qquad \textbf{(blackbox 2)}$$

$$A_3 \frac{d}{dt} h_3 = f_3 + f_3',$$

$$C f_3 = \begin{cases} \sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \geq \rho h_3, \\ -\sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \leq \rho h_3, \end{cases} \qquad \textbf{(blackbox 3)}$$

$$C_3 f_3' = \begin{cases} \sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \geq \rho h_3, \\ -\sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \leq \rho h_3, \end{cases}$$

$$p_1' = p_2, \quad f_1' + f_2 = 0, \; p_2' = p_3, \quad f_2' + f_3 = 0. \qquad \textbf{(interconnection)}$$

$$p_{\text{left}} = p_1, \; f_{\text{left}} = f_1, \; p_{\text{right}} = p_3', \; f_{\text{right}} = f_3'. \qquad \textbf{(manifest variable assignment)}$$

NB

This tableau of equations is the endpoint of a straightforward first-principles-modeling procedure.

- **Unclear (and, in fact, irrelevant) input/output structure for the terminal variables,**
  both in the overall system and in the subsystems
- **Many variables, indivisibly, at the same terminal**
- **Interconnection = variable sharing**
- **No signal flows, no output-to-input assignment**

Behavioral systems

Behavioral approach

A dynamical system

$:\Leftrightarrow$ **a family of time trajectories,** ***'the behavior'***

Interconnection $\Leftrightarrow$ ***'variable sharing'***

Control $\Leftrightarrow$ ***interconnection***

Modeling of interconnected physical systems is the strongest case for 'behaviors'. We deal mainly with this aspect today.

Terminals

We consider systems that interact with their environment through terminals

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There are many
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civil engineering, pneumatic, ... connections
that can be viewed this way, exactly, literally.

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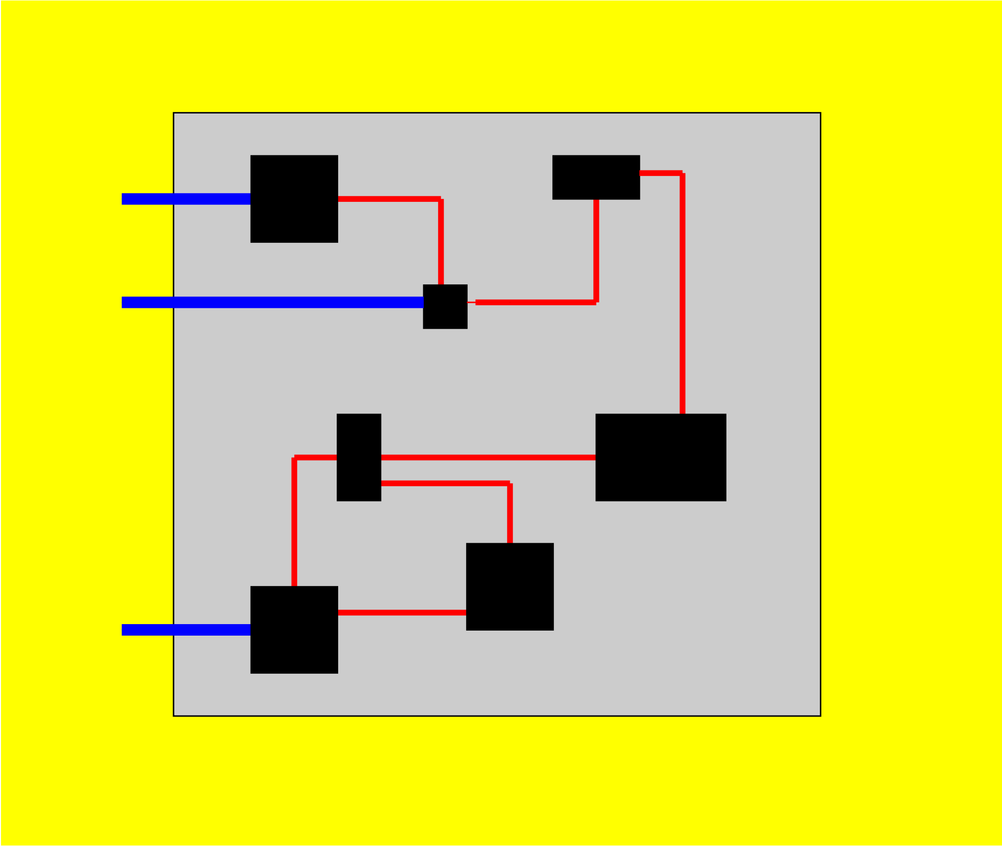
civil engineering, pneumatic, ... connections
that can be viewed this way, exactly, literally.

The clearest example is an electrical connection.
A terminal = a single wire, and interconnection = soldering of wires.

Interconnection architecture

Objective

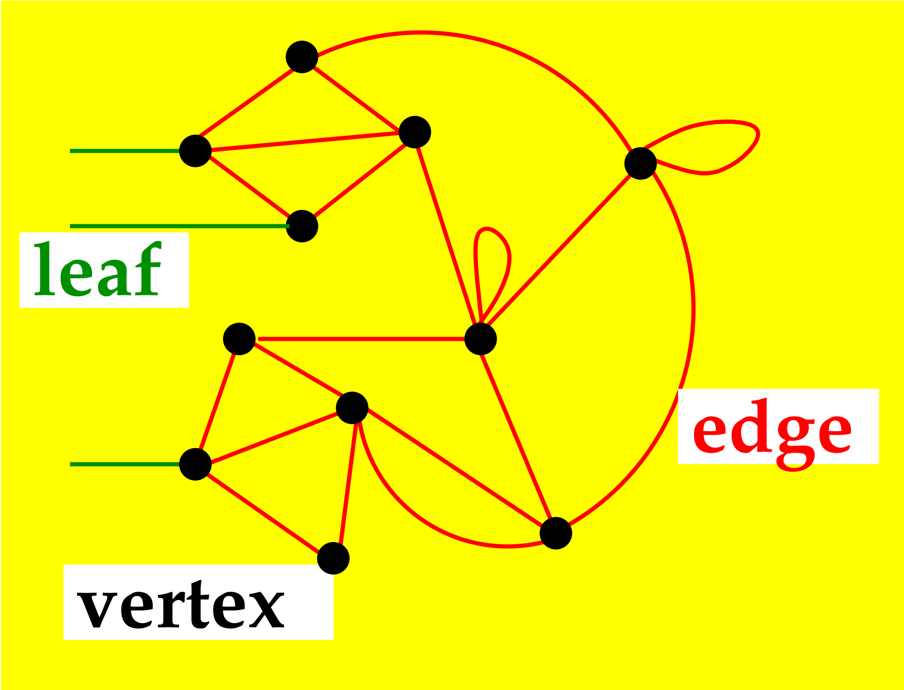


Formalize mathematically interconnection of systems.

Graph with leaves

Architecture:

graph with leaves



vertices	$\rightsquigarrow$	**systems with terminals**
edges	$\rightsquigarrow$	**connected terminals**
leaves	$\rightsquigarrow$	**interaction with environment**
terminals	$\rightsquigarrow$	**system variables**

Behavioral equations

1. **Module equations** for each vertex.

   Relation among the variables on the terminals.

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

3. **Manifest variable assignment**

   Specifies the variables of interest.

Behavioral equations

1. **Module equations** for each vertex.

   Relation among the variables on the terminals.

   Behavioral equations stored as (parametrized) modules in a data-base.

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

   Interconnection laws stored in a data-base.
   Laws depend on terminal type:
   electrical / mechanical / hydraulic / etc.

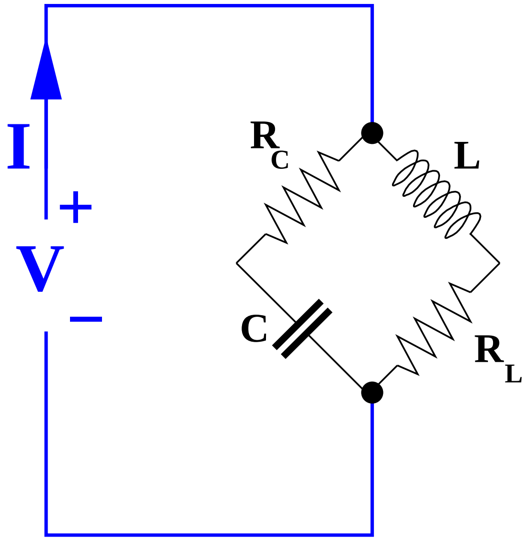
3. **Manifest variable assignment**

   Specifies the variables of interest.

An example

RLC circuit

Model the port behavior of



by tearing, zooming, and linking.

RLC circuit

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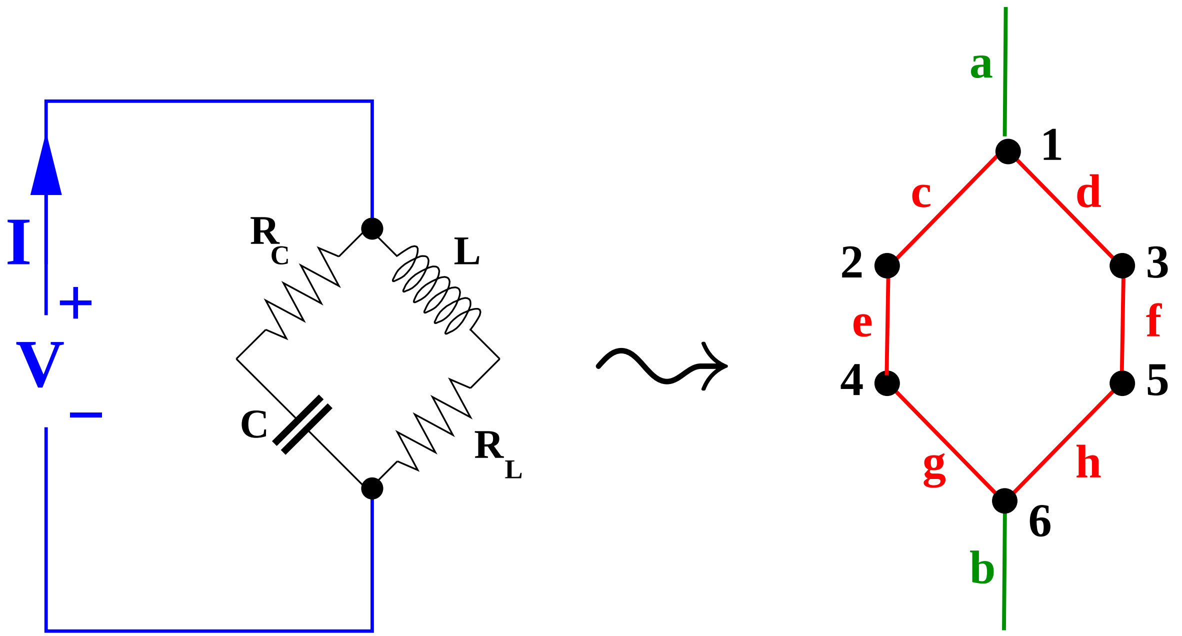
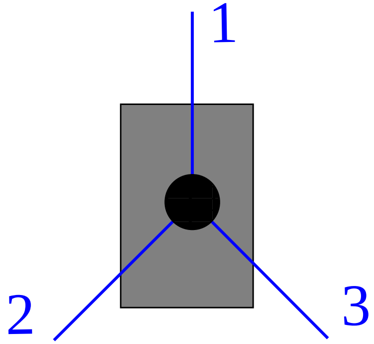


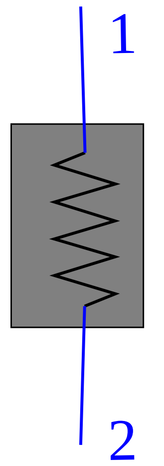
by tearing, zooming, and linking.

In each vertex there is a module ⇝ module equations
each terminal involves 2 variables (potential, current)
in each edge there is an electrical interconnection ⇝
interconnection equations

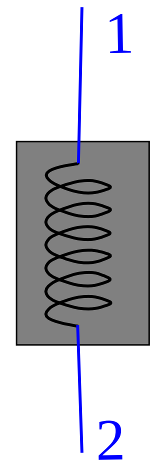
Modules



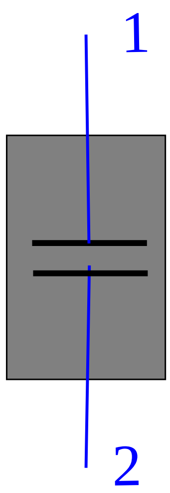
connector1



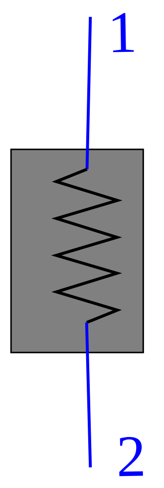
resistor1



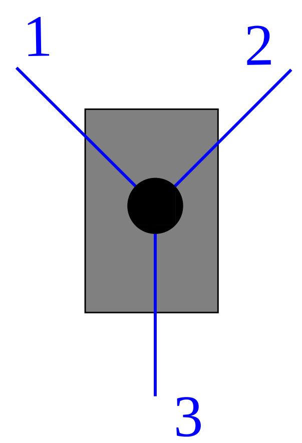
inductor



capacitor

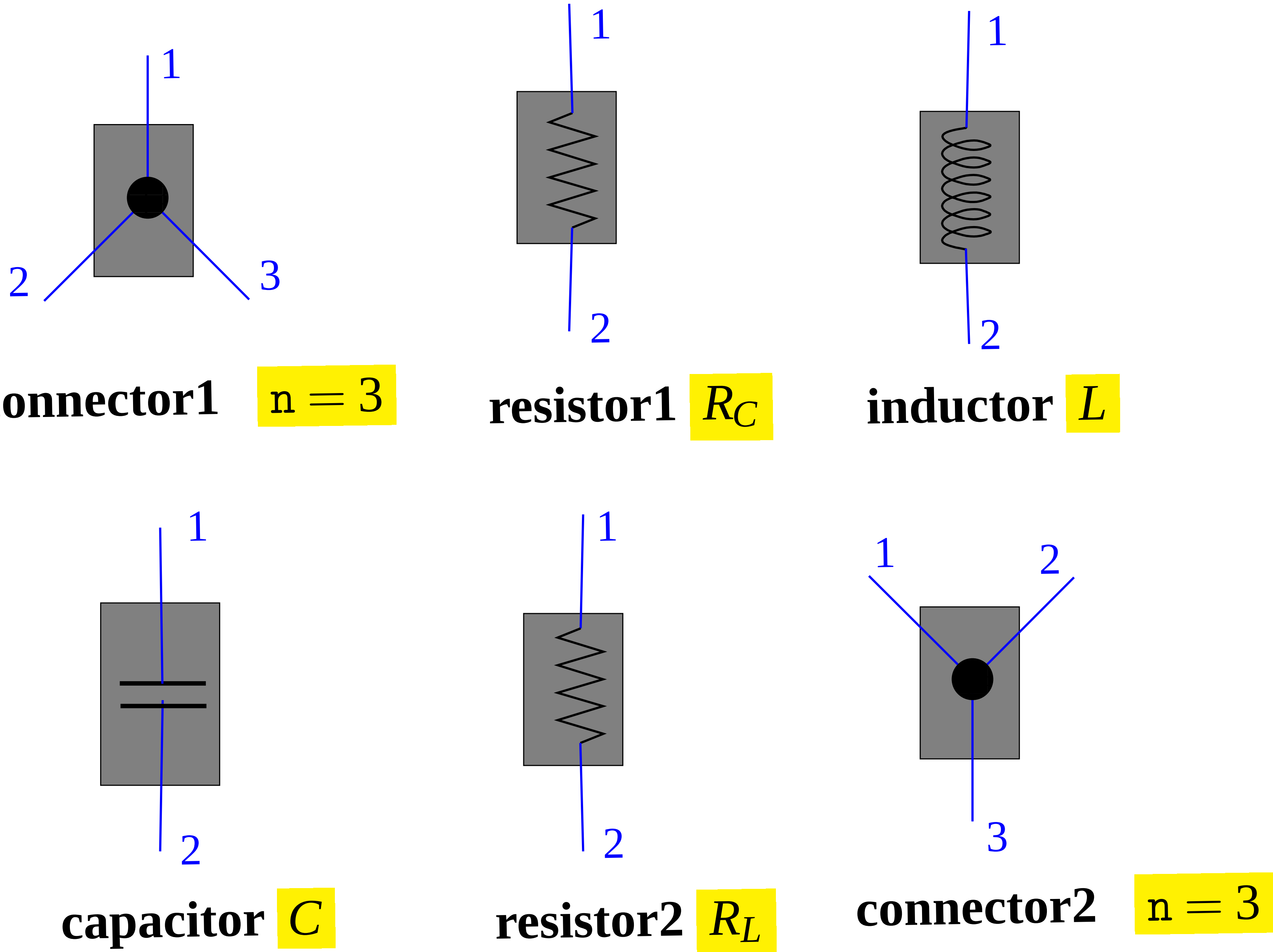


resistor2



connector2

Modules


1
2
3
connector1
n = 3
1
2
resistor1
R_C
1
2
inductor
L
1
2
capacitor
C
1
2
resistor2
R_L
1
2
3
connector2
n = 3

Vertices ⇒ module equations

$$\textbf{vertex 1}: \quad V_{\text{connector1,1}} = V_{\text{connector1,2}} = V_{\text{connector1,3}}$$

$$I_{\text{connector1,1}} + I_{\text{connector1,2}} + I_{\text{connector1,3}} = 0$$

$$\textbf{vertex 2}: \quad V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1}, \; I_{R_C,1} + I_{R_C,2} = 0$$

$$\textbf{vertex 3}: \quad L\frac{d}{dt}I_{L,1} = V_{L,1} - V_{L,2}, \; I_{L,1} + I_{L,2} = 0$$

$$\textbf{vertex 4}: \quad C\frac{d}{dt}\left(V_{C,1} - V_{C,2}\right) = I_{C,1}, \; I_{C,1} + I_{C,2} = 0$$

$$\textbf{vertex 5}: \quad V_{R_L,1} - V_{R_L,2} = R_L I_{R_L,1}$$

$$I_{R_L,1} + I_{R_L,2} = 0$$

$$\textbf{vertex 6}: \quad V_{\text{connector2,1}} = V_{\text{connector2,2}} = V_{\text{connector2,3}}$$

$$I_{\text{connector2,1}} + I_{\text{connector2,2}} + I_{\text{connector2,3}} = 0$$

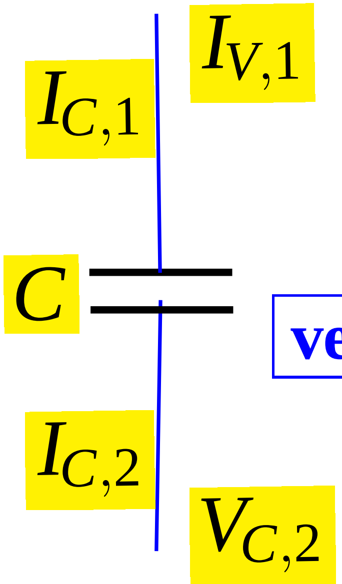
Vertices ⇒ module equations

$$V_{\text{connector1},1} = V_{\text{connector1},2} = V_{\text{connector1},3}$$

$$I_{\text{connector1},1} + I_{\text{connector1},2} + I_{\text{connector1},3} = 0$$

$$V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1},\ I_{R_C,1} + I_{R_C,2} = 0$$

$$L\frac{d}{dt}I_{L,1} = V_{L,1} - V_{L,2},\ I_{L,1} + I_{L,2} = 0$$



vertex 4:

$$C\frac{d}{dt}\left(V_{C,1} - V_{C,2}\right) = I_{C,1},\ I_{C,1} + I_{C,2} = 0$$

$$V_{R_L,1} - V_{R_L,2} = R_L I_{R_L,1}$$

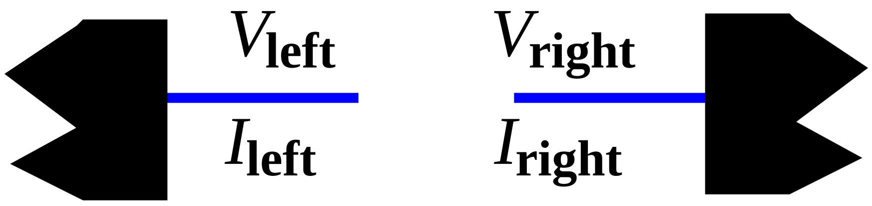
$$I_{R_L,1} + I_{R_L,2} = 0$$

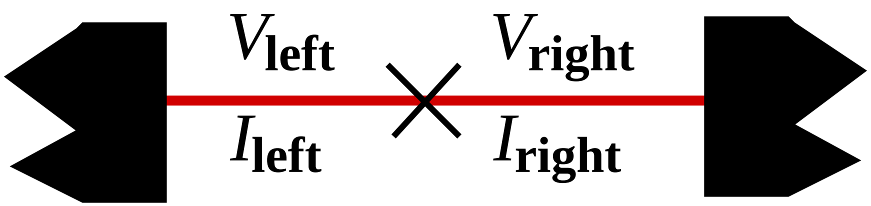
$$V_{\text{connector2},1} = V_{\text{connector2},2} = V_{\text{connector2},3}$$

$$I_{\text{connector2},1} + I_{\text{connector2},2} + I_{\text{connector2},3} = 0$$

Interconnection

All interconnections are of electrical type





Interconnection equations:

potential left $=$ **potential right** $\leadsto$ $V_{\text{left}} = V_{\text{right}}$

current left $+$ **current right** $= 0$ $\leadsto$ $I_{\text{left}} + I_{\text{right}} = 0$

Edges ⇒ interconnection equations

edge c :	$V_{R_{C,1}} = V_{\text{connector1,2}}$	$I_{R_{C,1}} + I_{\text{connector1,2}} = 0$
edge d :	$V_{L,1} = V_{\text{connector1,3}}$	$I_{L,1} + I_{\text{connector1,3}} = 0$
edge e :	$V_{R_{C,2}} = V_{C,1}$	$I_{R_{C,2}} + I_{C,1} = 0$
edge f :	$V_{L,2} = V_{R_{C,1}}$	$I_{L,2} + I_{R_{L,1}} = 0$
edge g :	$V_{C,2} = V_{\text{connector2,1}}$	$I_{C,2} + I_{\text{connector2,1}} = 0$
edge h :	$V_{R_{L,2}} = V_{\text{connector2,2}}$	$I_{R_{L,2}} + I_{\text{connector2,2}} = 0$

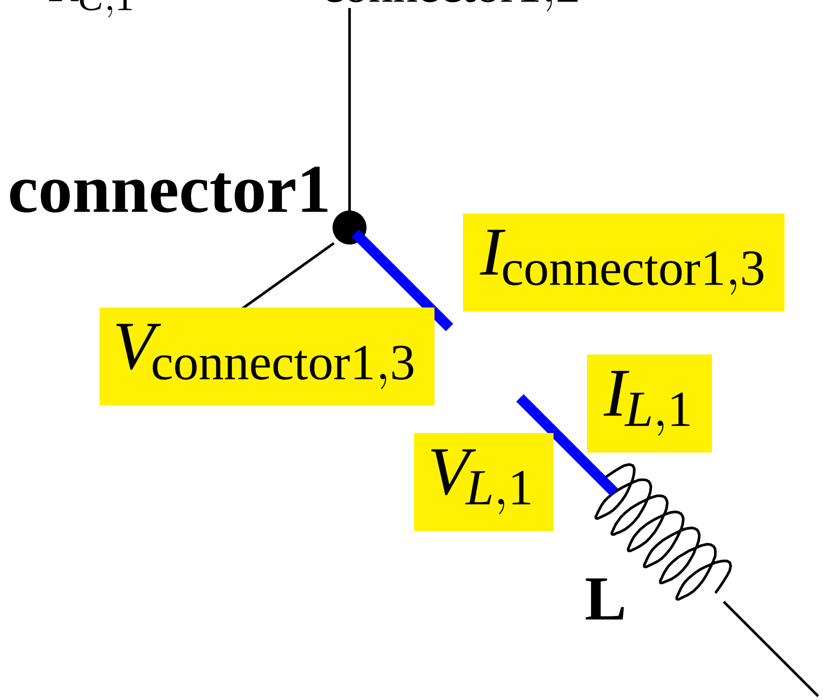
Interconnection equations

$$V_{R_{C,1}} = V_{\text{connector1,2}} \qquad I_{R_{C,1}} + I_{\text{connector1,2}} = 0$$

edge d:

$$V_{L,1} = V_{\text{connector1,3}}$$

$$I_{L,1} + I_{\text{connector1,3}} = 0$$



$$V_{R_{C,2}} = V_{C,1} \qquad I_{R_{C,2}} + I_{C,1} = 0$$

$$V_{L,2} = V_{R_{C,1}} \qquad I_{L,2} + I_{R_{L,1}} = 0$$

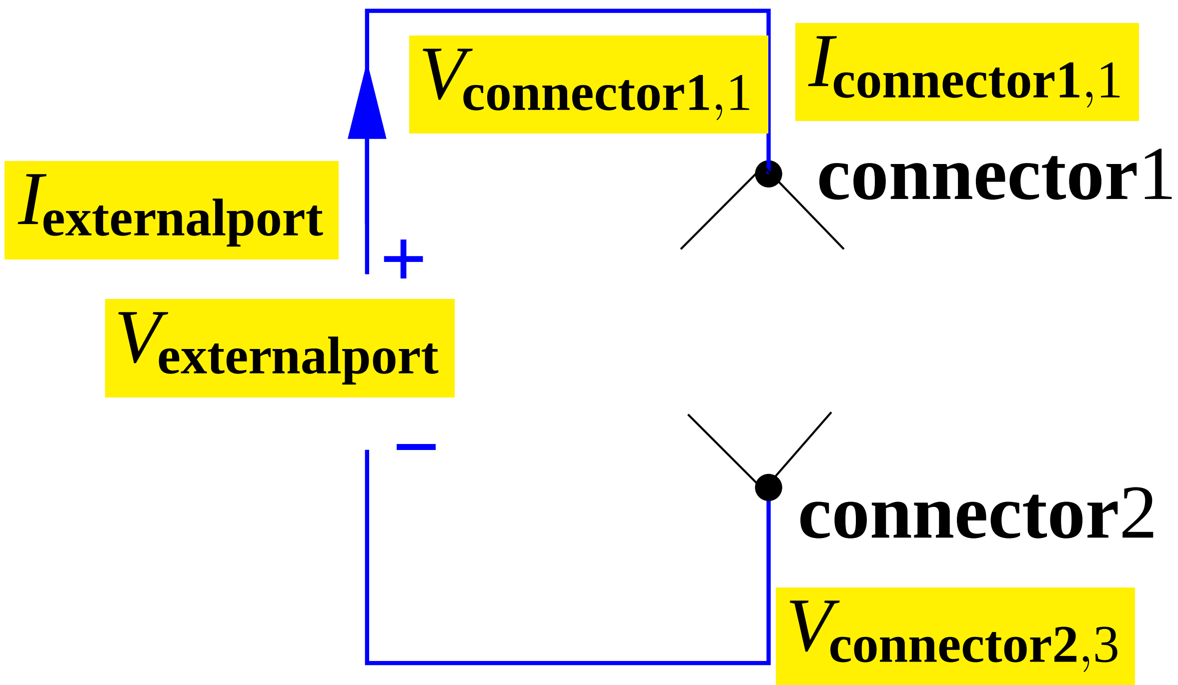
$$V_{C,2} = V_{\text{connector2,1}} \qquad I_{C,2} + I_{\text{connector2,1}} = 0$$

$$V_{R_{L,2}} = V_{\text{connector2,2}} \qquad I_{R_{L,2}} + I_{\text{connector2,2}} = 0$$

Manifest variable assignment

$$V_{\text{externalport}} = V_{\text{connector1},1} - V_{\text{connector2},3}$$

$$I_{\text{externalport}} = I_{\text{connector1},1}$$



Complete model

vertex 1 : $V_{\text{connector1},1} = V_{\text{connector1},2} = V_{\text{connector1},3}$

$I_{\text{connector1},1} + I_{\text{connector1},2} + I_{\text{connector1},3} = 0$

vertex 2 : $V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1},\ I_{R_C,1} + I_{R_C,2} = 0$

vertex 3 : $L \frac{d}{dt} I_{L,1} = V_{L,1} - V_{L,2},\ I_{L,1} + I_{L,2} = 0$

vertex 4 : $C \frac{d}{dt} \left(V_{C,1} - V_{C,2} \right) = I_{C,1},\ I_{C,1} + I_{C,2} = 0$

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edge h : $V_{R_L,2} = V_{\text{connector2},2}$

$I_{R_L,2} + I_{\text{connector2},2} = 0$

$$V_{\text{externalport}} = V_{\text{connector},1,1} - V_{\text{connector2},3}$$

$$I_{\text{externalport}} = I_{\text{connector1},1}$$

Port behavior

$$\mathscr{B} = \{ (V_{\text{externalport}}, I_{\text{externalport}}) : \mathbb{R} \to \mathbb{R}^2 \mid$$

$\exists$ latent variables trajectories

$$(V_{\text{connector}_1,1}, I_{\text{connector}_1,1}, \ldots, \ldots) : \mathbb{R} \to \mathbb{R}^{28}$$

such that

$$V_{\text{connector}_1,1} = V_{\text{connector}_1,2} = V_{\text{connector}_1,3},$$

$$\vdots$$

$$I_{\text{externalport}} = I_{\text{connector1,1}}$$

i.e., all 24 equations are satisfied$\}$

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$$\vdots$$

$$I_{\text{externalport}} = I_{\text{connector1,1}}$$

i.e., all 24 equations are satisfied$\}$

Can we simplify this expression for $\mathscr{B}$?

Port behavior

$\rightsquigarrow$ **the dynamical system with behavior $\mathscr{B}$ specified by:**

<u>Case 1</u>: $CR_C \neq \dfrac{L}{R_L}$

$$\boxed{\left(\frac{R_C}{R_L} + \left(1 + \frac{R_C}{R_L}\right) CR_C \frac{d}{dt} + CR_C \frac{L}{R_L} \frac{d^2}{dt^2}\right) V = \left(1 + \frac{L}{R_L} \frac{d}{dt}\right)\left(1 + CR_C \frac{d}{dt}\right) R_C I}$$

$\rightsquigarrow$ $\mathscr{B}$ **= all solutions** $(V, I) : \mathbb{R} \to \mathbb{R}^2$

Port behavior

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Case 2: $\quad CR_C = \frac{L}{R_L}$

$$\boxed{\left(\frac{R_C}{R_L} + CR_C \frac{d}{dt}\right) V = \left(1 + CR_C \frac{d}{dt}\right) R_C I}$$

$\rightsquigarrow$ $\mathscr{B}$ **= all solutions** $(V, I) : \mathbb{R} \to \mathbb{R}^2$

Port behavior

Thm: **In LTIDSs latent variables can be eliminated !**

$\rightsquigarrow$ **the dynamical system with behavior $\mathscr{B}$ specified by:**

Case 1: $CR_C \neq \frac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + \left(1+\frac{R_C}{R_L}\right)CR_C\frac{d}{dt} + CR_C\frac{L}{R_L}\frac{d^2}{dt^2}\right)V = \left(1+\frac{L}{R_L}\frac{d}{dt}\right)\left(1+CR_C\frac{d}{dt}\right)R_C I$$

Case 2: $CR_C = \frac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + CR_C\frac{d}{dt}\right)V = \left(1+CR_C\frac{d}{dt}\right)R_C I$$

$\rightsquigarrow$ $\mathscr{B}$ **= all solutions** $(V, I) : \mathbb{R} \rightarrow \mathbb{R}^2$

The elimination theorem

Elimination

Consider

$$R\left(\tfrac{d}{dt}\right)w = M\left(\tfrac{d}{dt}\right)\ell \qquad R, M \in \mathbb{R}[\xi]^{\bullet\times\bullet}$$

$$\mathscr{B} = \left\{ w \mid \exists \ell \textbf{ such that } R\left(\frac{d}{dt}\right)w = M\left(\frac{d}{dt}\right)\ell \right\}.$$

Ex.:

$$\frac{d}{dt}x = Ax + Bu, \;\; y = Cx + Du, \quad w = \begin{bmatrix} u \\ y \end{bmatrix}$$

$$\frac{d}{dt}Ex = Ax + Bu, \;\; y = Cx + Du, \quad w = \begin{bmatrix} u \\ y \end{bmatrix} \qquad \text{(DAE)}$$

etc.

Consider

$$R\left(\tfrac{d}{dt}\right)w = M\left(\tfrac{d}{dt}\right)\ell \qquad R, M \in \mathbb{R}[\xi]^{\bullet\times\bullet}$$

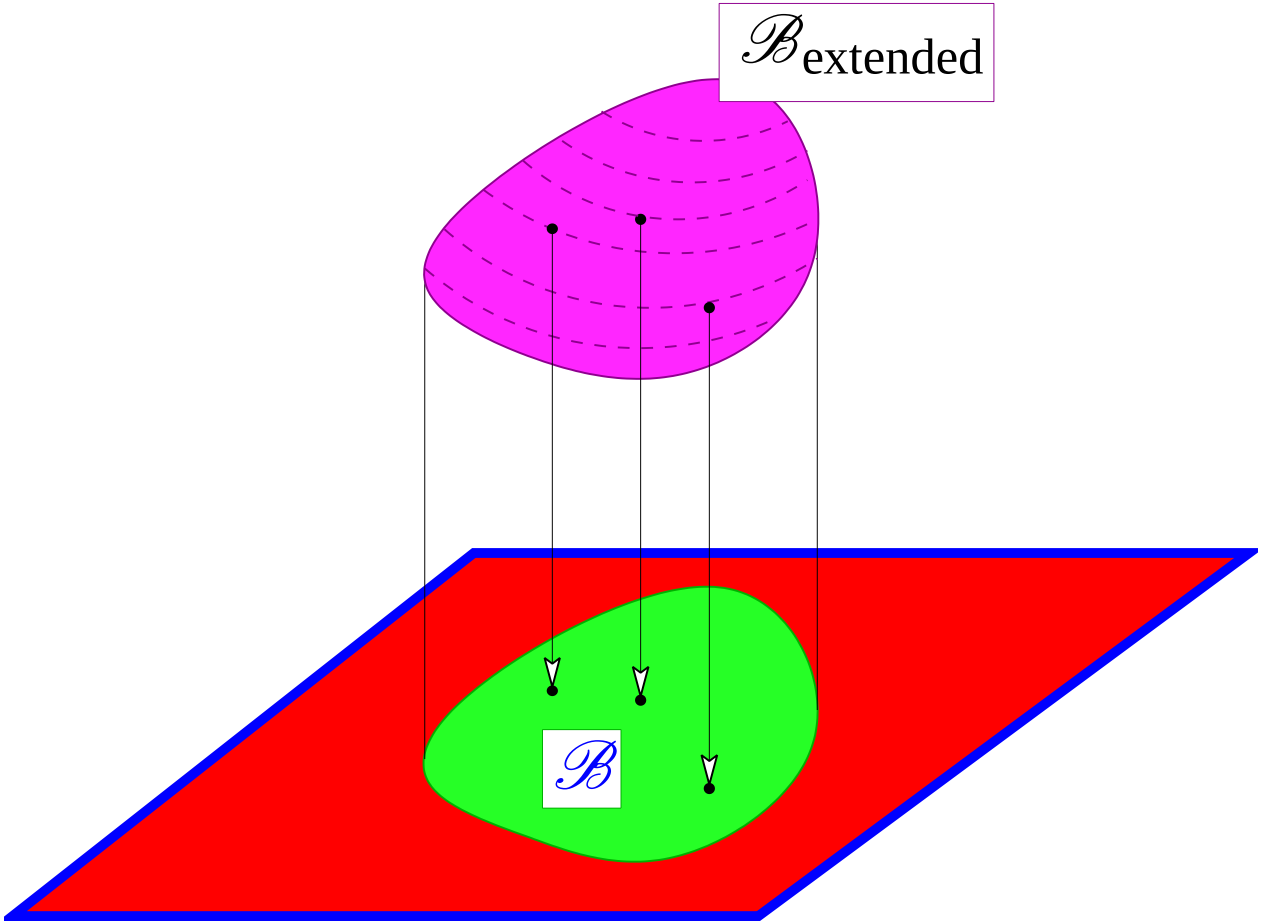
$$\mathscr{B} = \left\{ w \mid \exists \ell \text{ such that } R\left(\frac{d}{dt}\right)w = M\left(\frac{d}{dt}\right)\ell \right\}.$$

Theorem (Elimination theorem)

There is a polynomial matrix $R' \in \mathbb{R}[\xi]^{\bullet\times\mathtt{w}}$ **such that** $\mathscr{B}$ **is the solution set of**

$$R'\left(\frac{d}{dt}\right)w = 0.$$

Elimination



The projection of the set of solutions of a linear constant coefficient ODE is again the set of solutions of a linear constant coefficient ODE.

Other methodologies

Signal flow graphs

input/output thinking

There are many many examples where output-to-input connection is eminently natural:

input/output thinking

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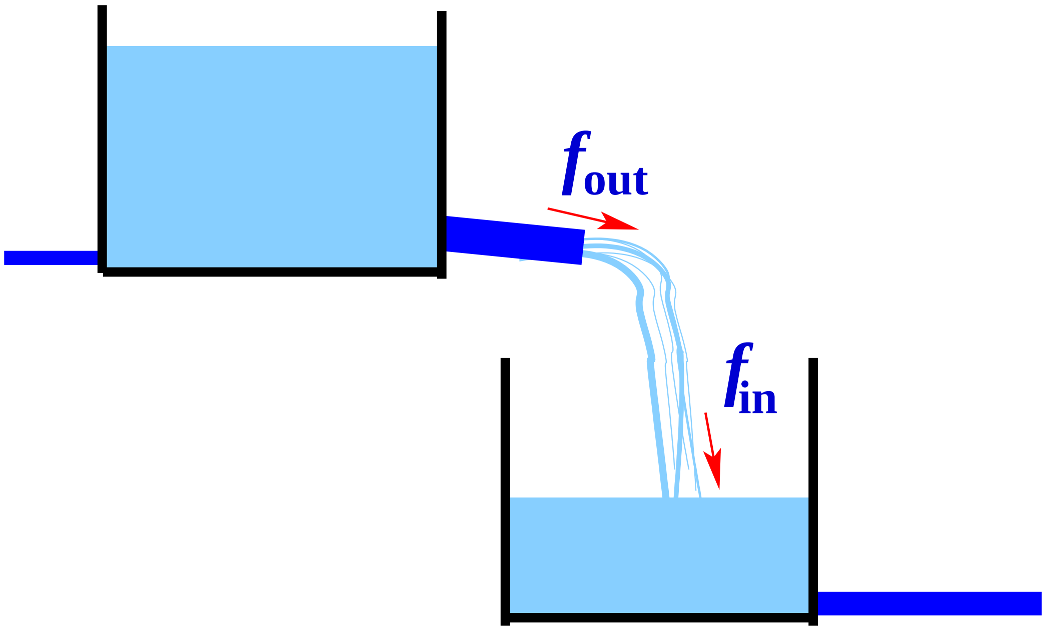

input/output partition

Assume that one of these variables acts as input, the other as output.

input/output partition

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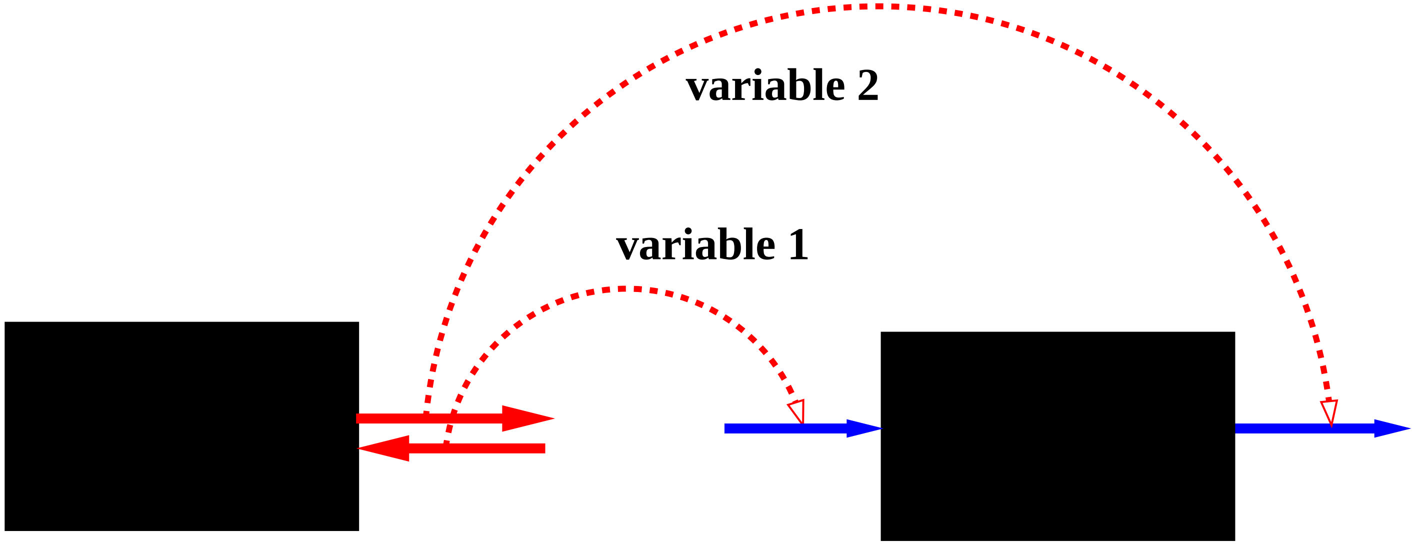
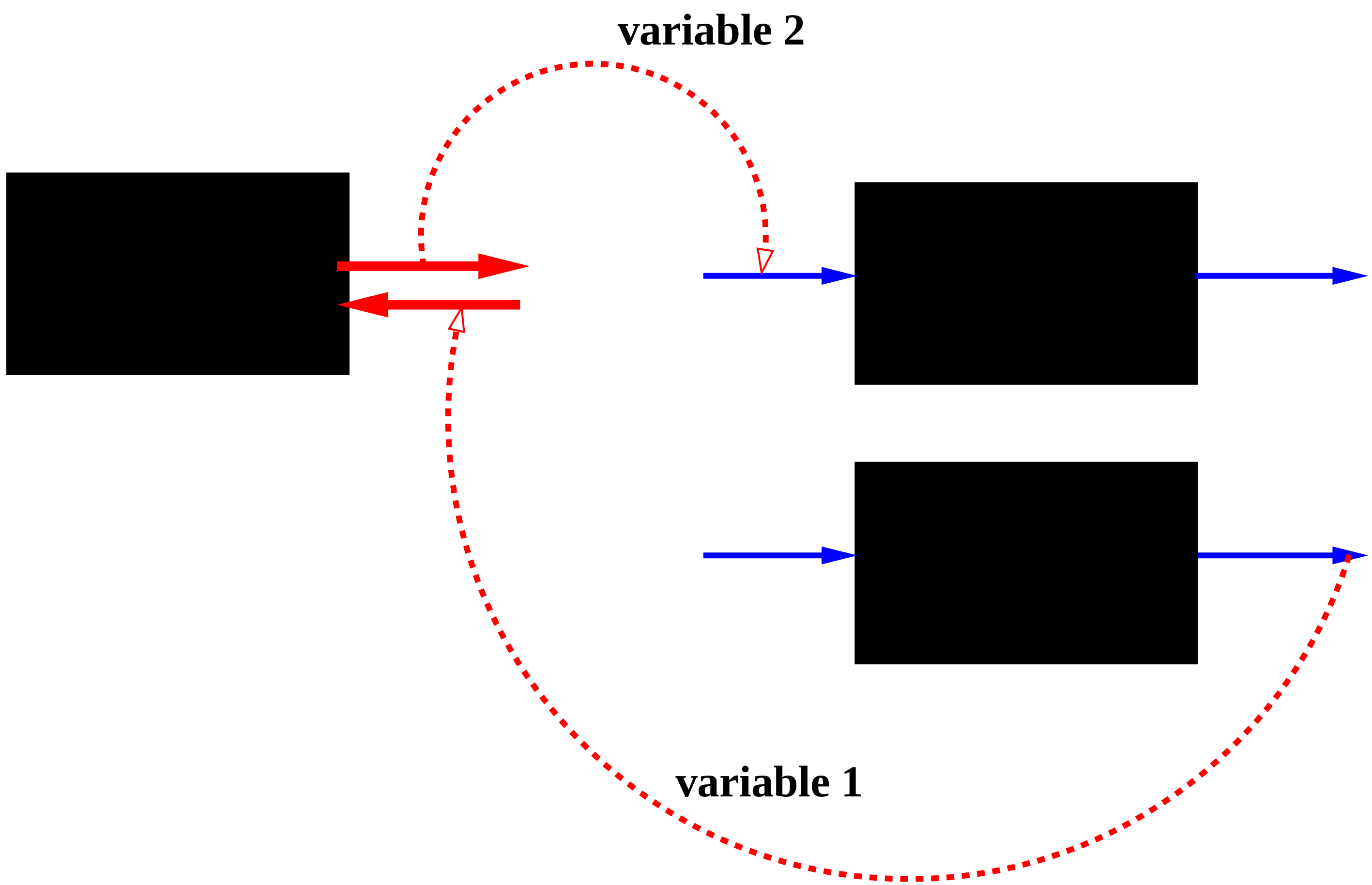
Block diagrams



- shows terminal variables separate
- suggests that inputs and outputs occur at different physical points

Pedagogically awkward, confusing, unreal.

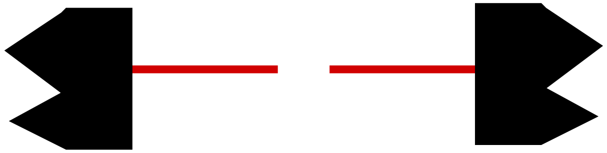
Block diagrams



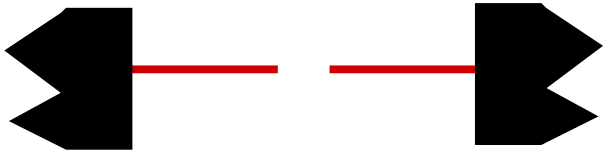
- allows impossible input-output connections

Does not respect the physics.

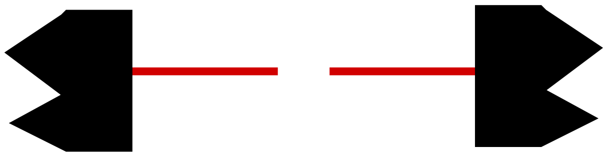
Signal flows and interconnections

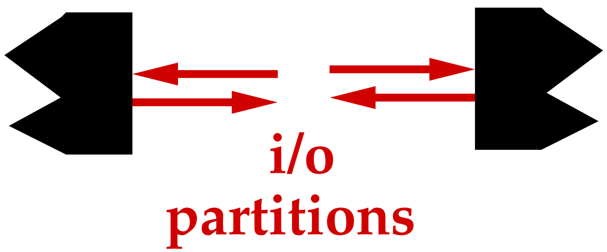


Signal flows and interconnections

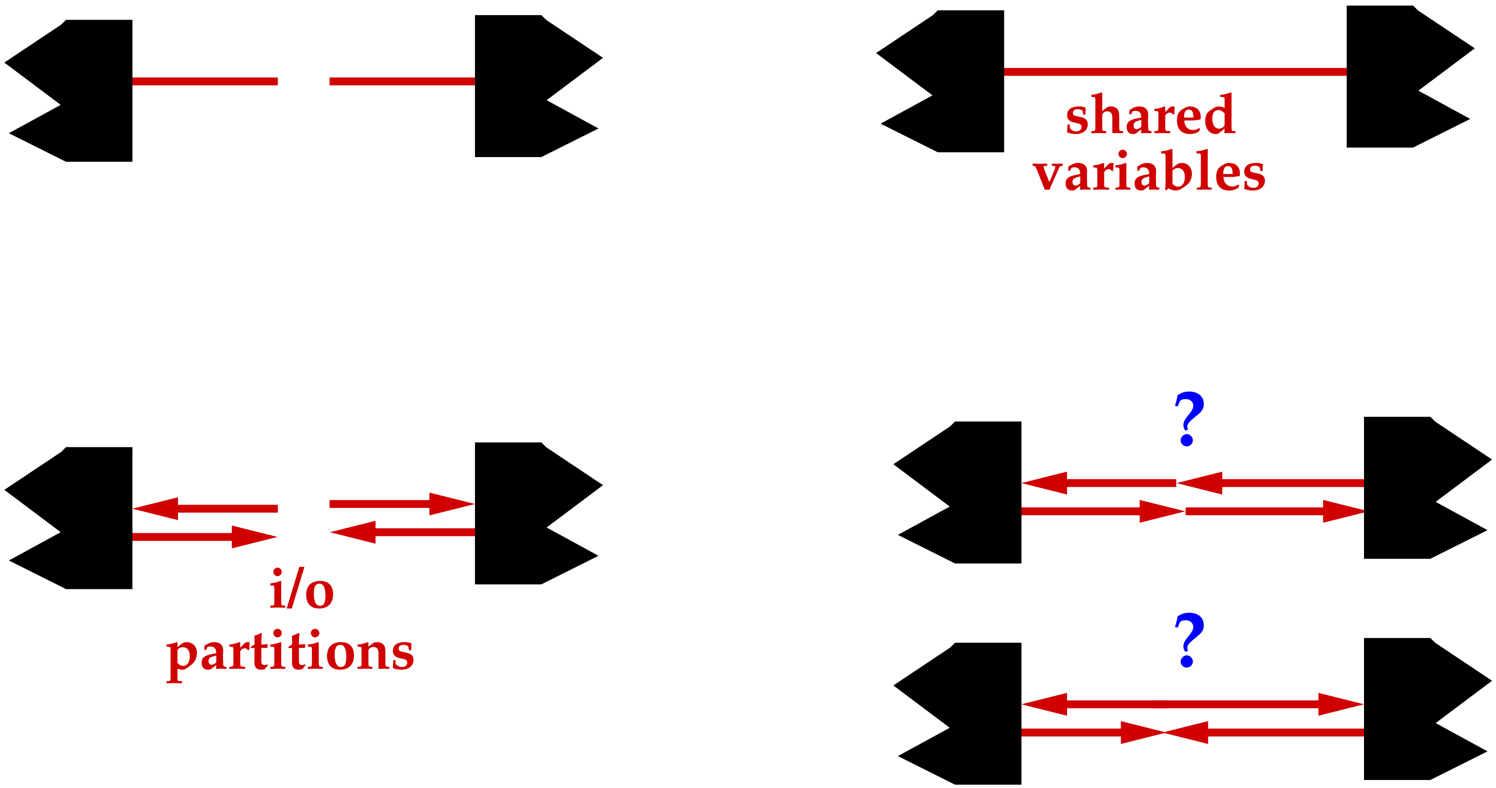


Signal flows and interconnections

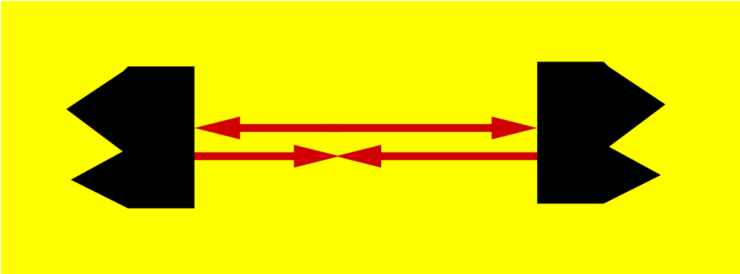


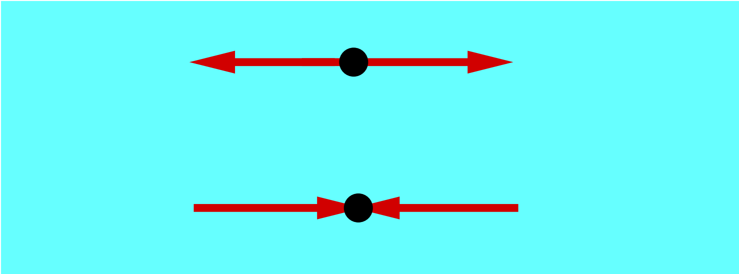


Signal flows and interconnections



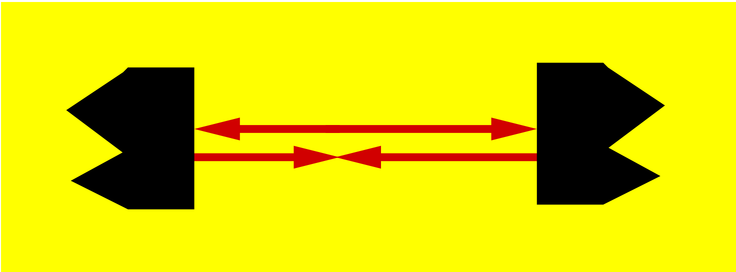
Signal flows and interconnections

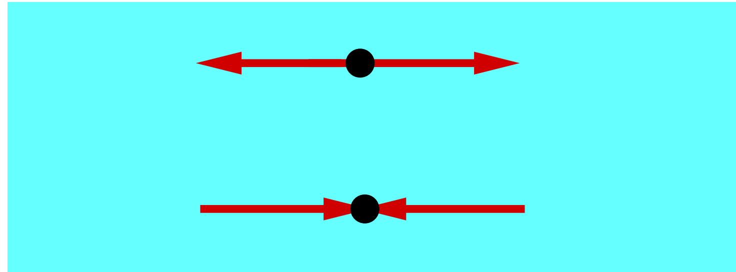




Forbidden?
Unlikely?

Signal flows and interconnections





Forbidden?
Unlikely?

For physical systems

input-to-input & output-to-output

assignment very prevalent:

force to force; pressure to pressure; heat flow to heat flow; temperature to temperature; mass flow to mass flow; ...

Physical systems are not signal processors

The input/output approach as the primary and universal view of open systems is a historical misconception.

The sooner it is abandoned as a starting point, the better.

The input/output approach as the primary and universal view of open systems is a historical misconception.

- **It fails in the most elementary examples.**
- **It does not deal adequately with interconnections.**
- **It breaks symmetries.**
- **It does not respect the physics.**
- **It is pedagogically ineffective.**

The sooner it is abandoned as a starting point, the better.

“Block diagrams unsuitable for serious physical modeling

- the control/physics barrier”

“Behavior based (declarative) modeling is a good alternative”

Karl Åström
(1934 -)

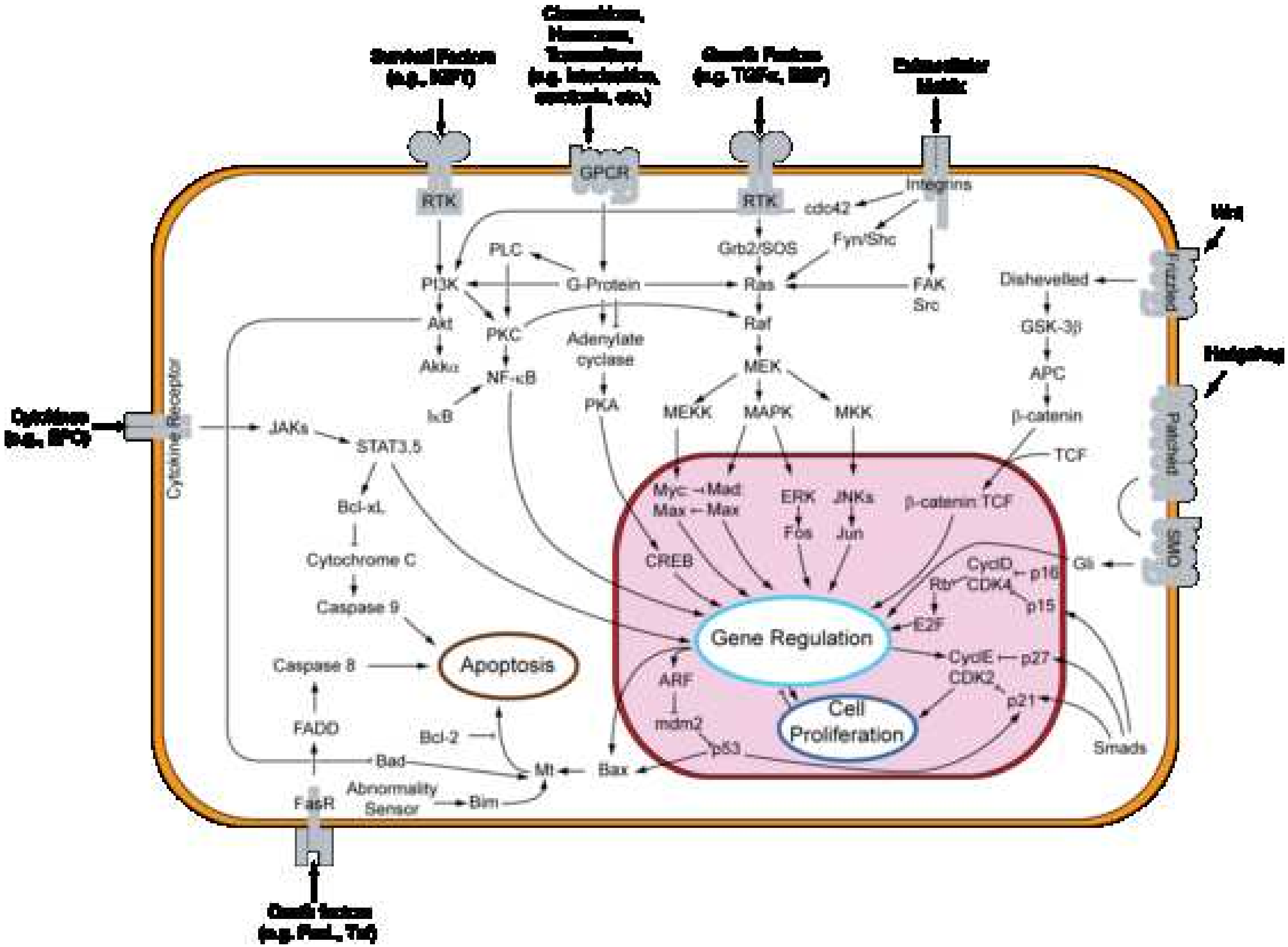
from K.J. Åström, ***Present Developments in Control Applications***

IFAC 50-th Anniversary Celebration
Heidelberg, September 12, 2006.

Notes & arrows

My dear young man, don't take it too hard.
Your work is ingenious. It's quality work.
But there are simply too many notes that's all ...

Notes & arrows

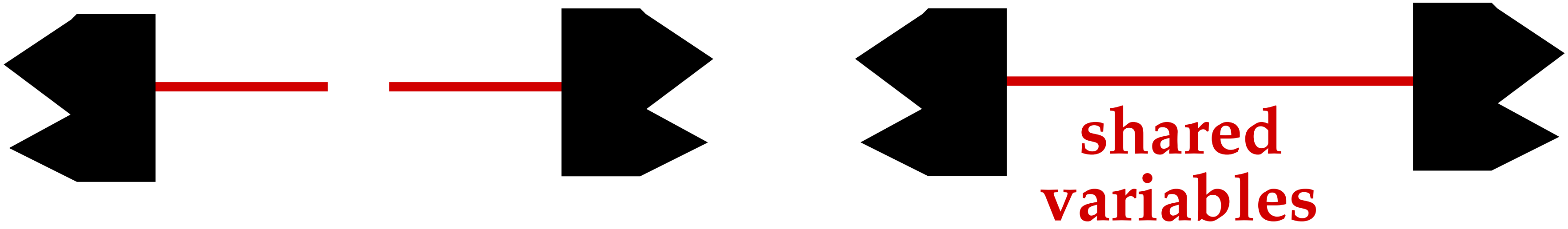


Ingenious. Quality work.

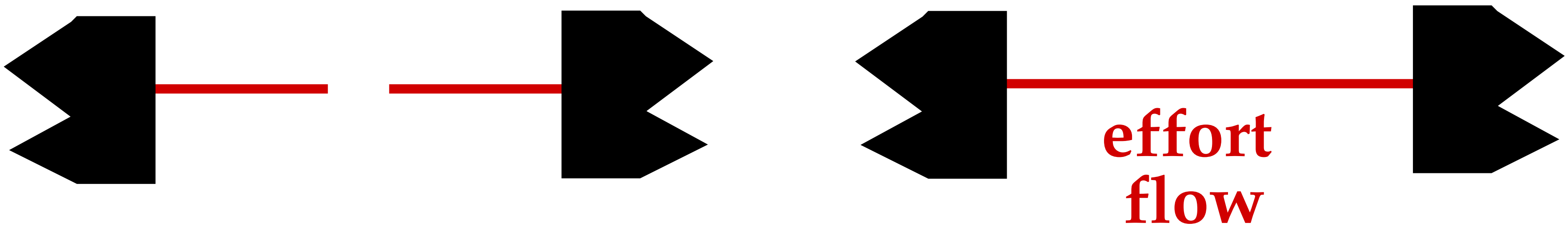
But there are simply too many arrows, that's all ...

Bond graphs

Bond graphs



Bond graphs



Interconnection variables consist of

an effort and a flow **effort × flow = power**

Interconnection ⇔

[efforts equal] & [flows sum to 0]

⇒ power equal

'Power is the universal currency of physical systems'

Bond graphs

Interconnection variables:

- **voltage & current**
- **force & velocity**
- **pressure & mass flow**
- **temperature & heat flow**

  temperature & $\dfrac{\textbf{heat flow}}{\textbf{temperature}}$
- **...**

Bond graphs

Interconnection variables:

- **voltage & current**
- **force & velocity**
- **pressure & mass flow**
- **temperature & heat flow**

  temperature & $\frac{\textbf{heat flow}}{\textbf{temperature}}$
- **...**

effort × flow = power?

Remarks

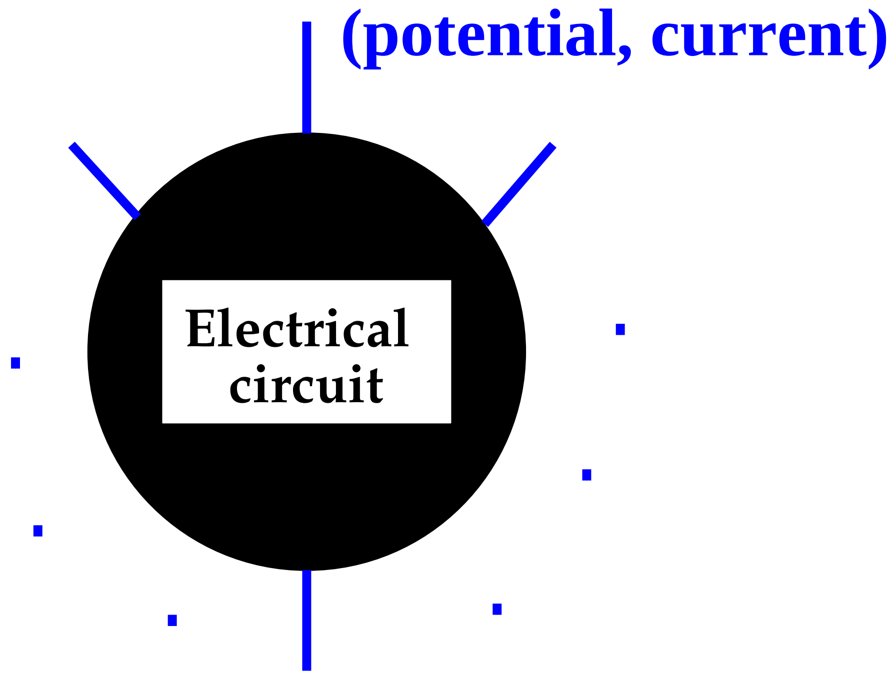
- Mechanical interconnections equate positions, not velocities.
- Not all interconnections involve equating energy transfer.
- Terminals are for interconnection, ports for energy transfer.

Remarks

- **Terminals for interconnection, ports for energy transfer**

This last point is illustrated for electrical interconnections.

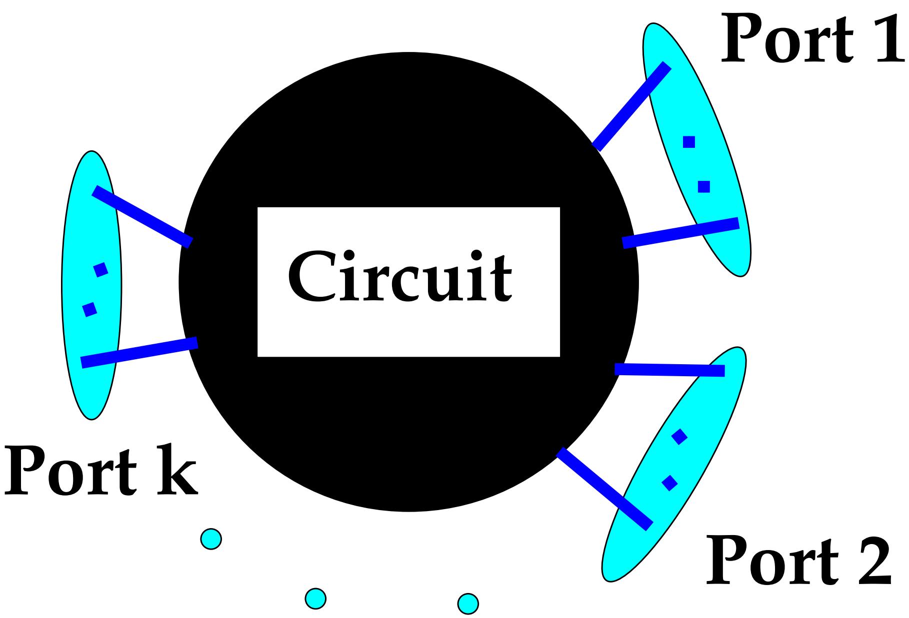
Terminals versus ports



Terminal variables and behavior:

$$(V_1, I_1, V_2, I_2, \ldots, V_{\mathtt{n}}, I_{\mathtt{n}}) \rightsquigarrow \textbf{ behavior } \mathscr{B} \subseteq \left(\mathbb{R}^{2\mathtt{n}}\right)^{\mathbb{R}}$$

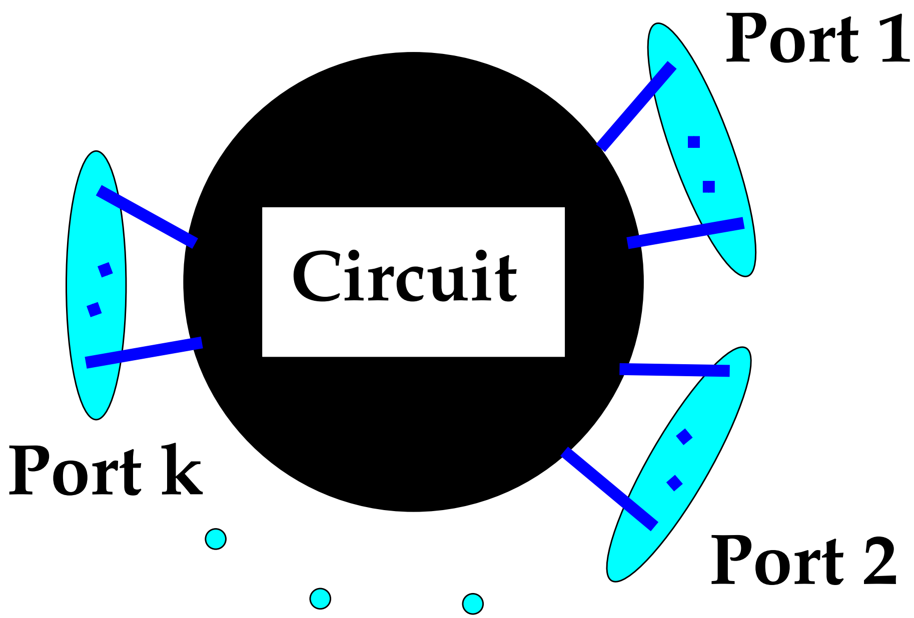
Terminals versus ports



Port :⇔

sum currents = 0
potentials + constant
⇒ potentials

Terminals versus ports



Port :⇔

sum currents = 0
potentials + constant
⇒ potentials

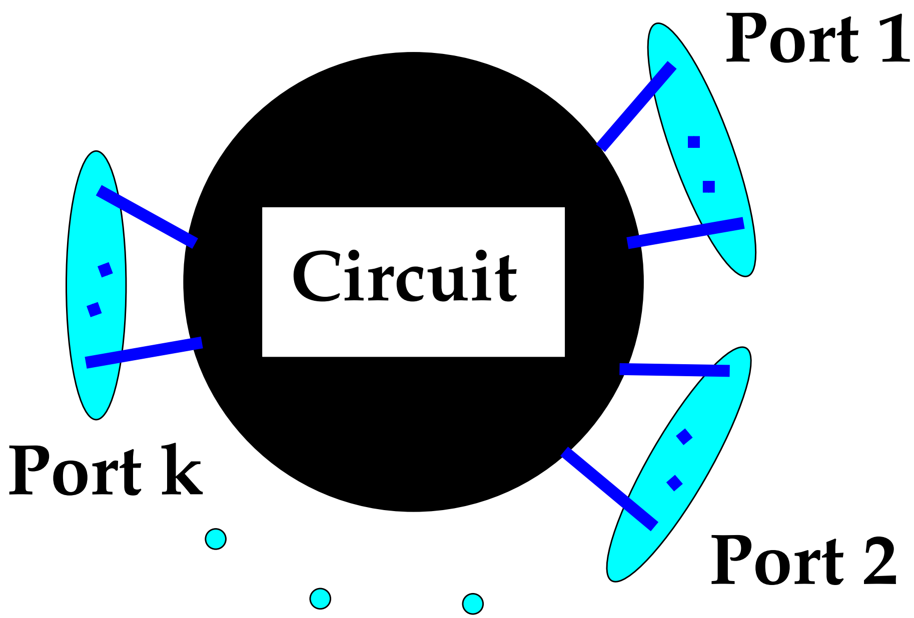
$$\left(\boxed{V_1, I_1 \ldots, V_{\mathrm{p}}, I_{\mathrm{p}}}, V_{\mathrm{p}+1}, \ldots, I_{\mathrm{n}}\right) \in \mathscr{B}, \alpha : \mathbb{R} \rightarrow \mathbb{R}$$

$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_{\mathrm{p}} + \alpha, I_{\mathrm{p}}}, V_{\mathrm{p}+1}, \ldots, I_{\mathrm{n}}\right) \in \mathscr{B}$$

$$\boxed{I_1 + \cdots + I_{\mathrm{p}}} = 0$$

Terminals versus ports



Port :⇔

sum currents = 0
potentials + constant
⇒ potentials

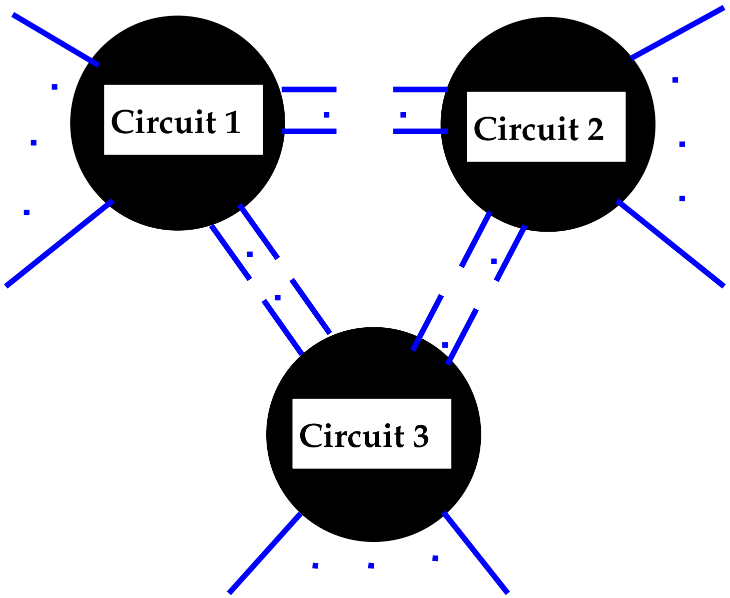
The behavioral equations contain the variables $V_1, V_2 \ldots, V_{\mathrm{p}}$ **only as the differences**

$$V_{\mathrm{i}} - V_{\mathrm{j}} \qquad \textbf{for } \mathrm{i}, \mathrm{j} = 1, \ldots \mathrm{p}$$

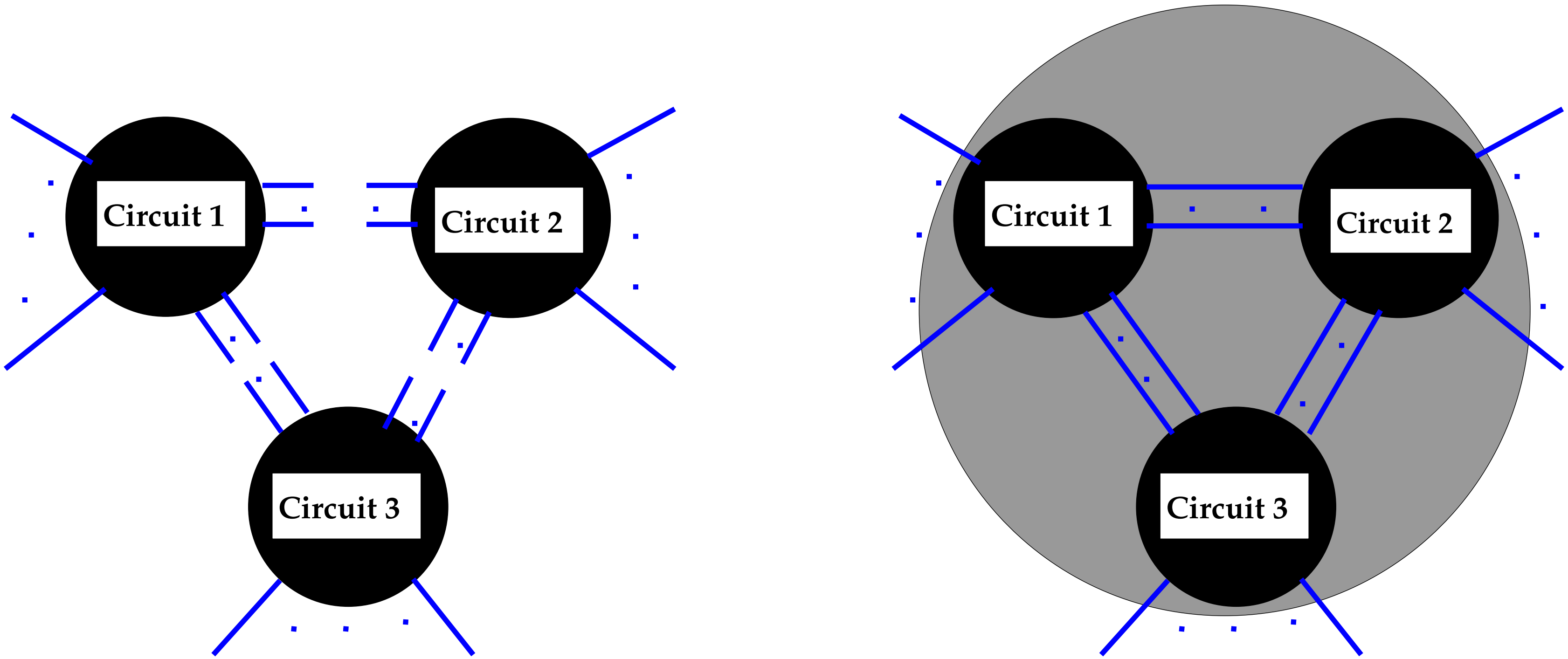
and contain the equation

$$I_1 + I_2 + \cdots + I_{\mathrm{p}} = 0$$

Terminals versus ports



Terminals versus ports



Interconnection through terminals, energy transfer through ports. **One cannot speak about**

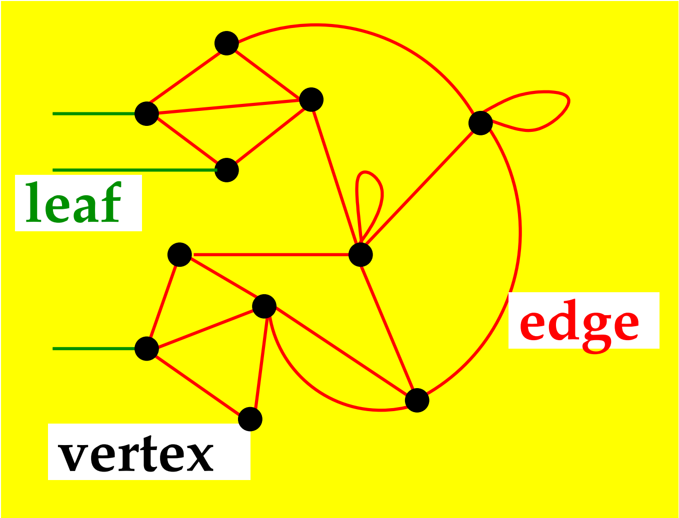
"the energy transferred from circuit 1 to circuit 2"

unless their interconnected terminals form a port.

Hierarchy

New modules from old ones

Tearing, zooming, linking is hierarchical:



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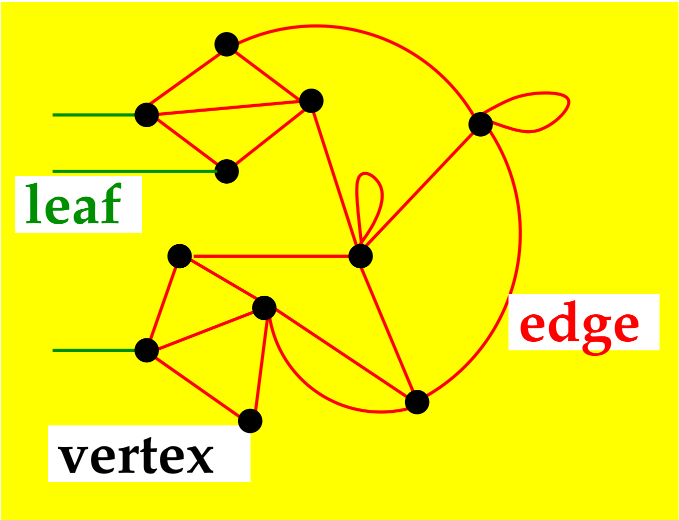
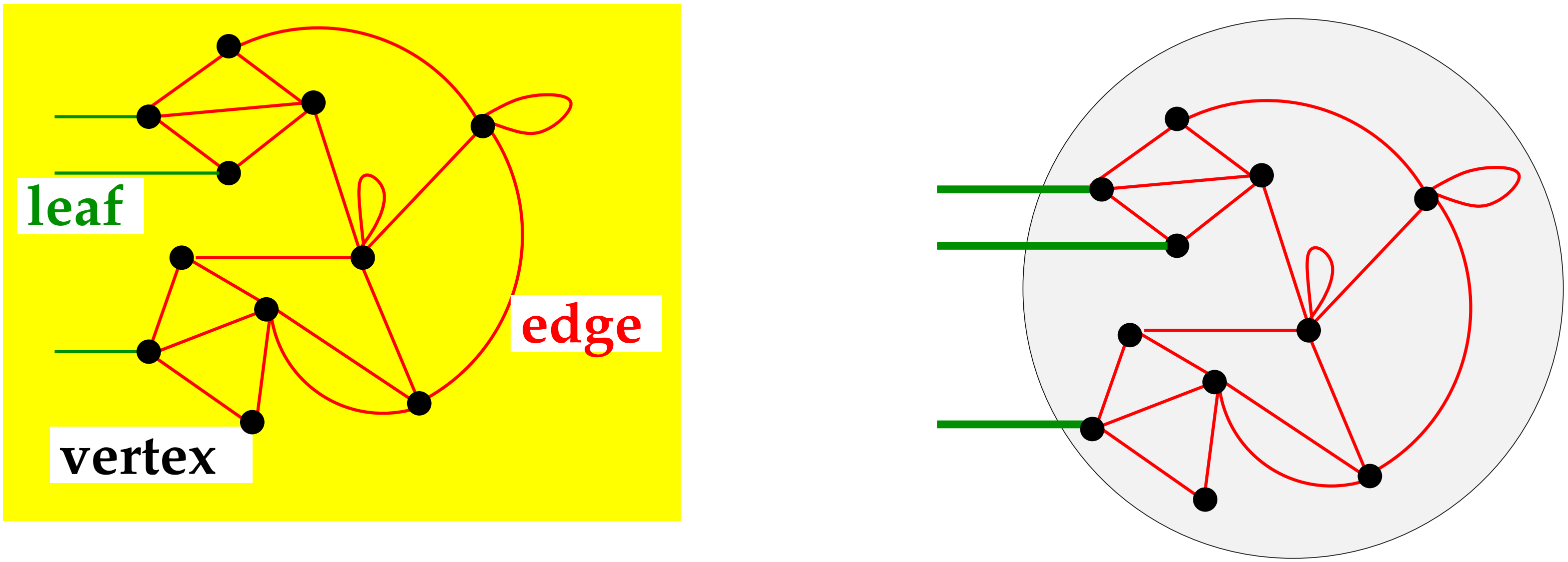


Embed modules in vertices, obtain behavioral equations for the interconnected system, eliminate the latent variables,

New modules from old ones

Tearing, zooming, linking is hierarchical:



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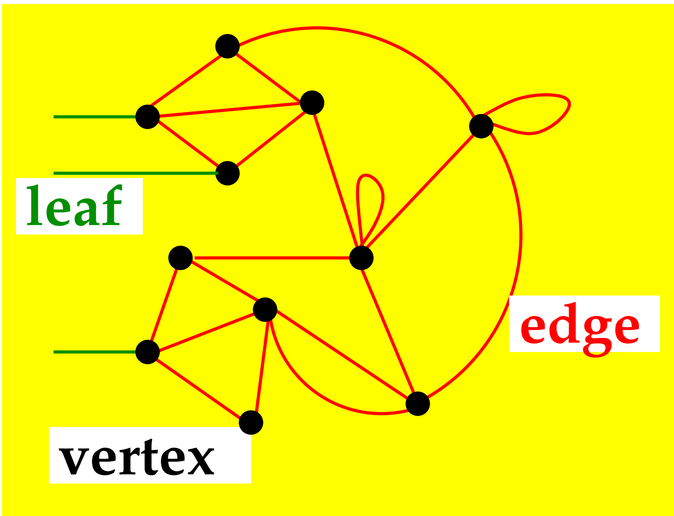
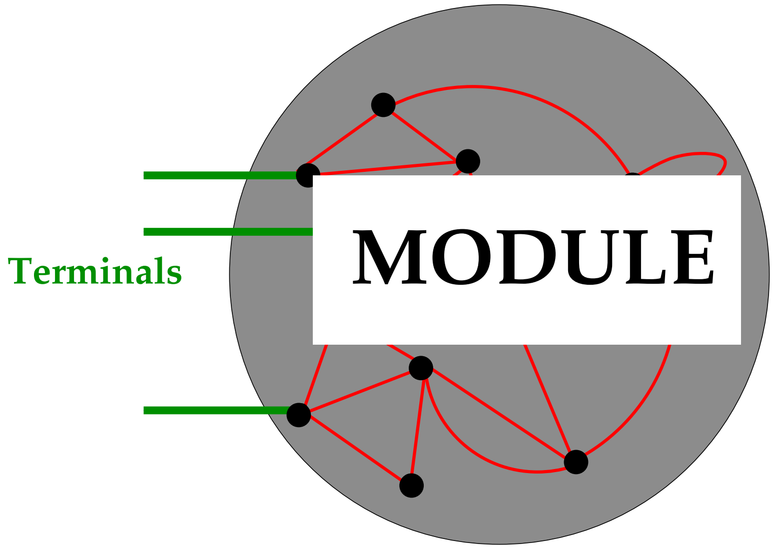
New modules from old ones

Tearing, zooming, linking is hierarchical:

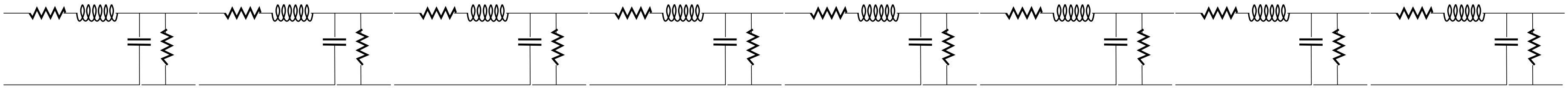




Embed modules in vertices, obtain behavioral equations for the interconnected system, eliminate the latent variables, and view the interconnected system as a module with terminals in a new interconnection architecture.

Example

Model the behavior of the external terminal voltages and currents of the following circuit:



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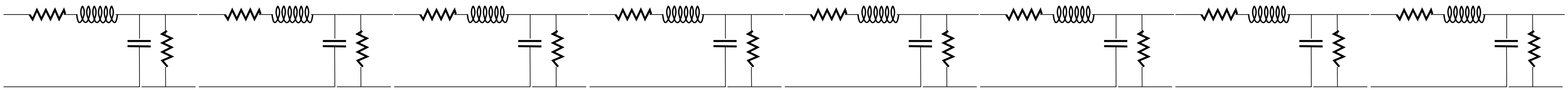
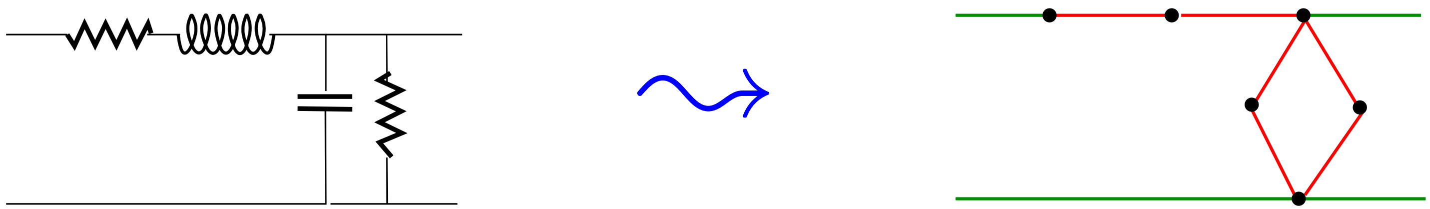


One section:



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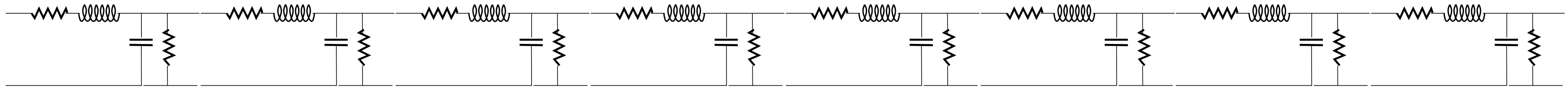

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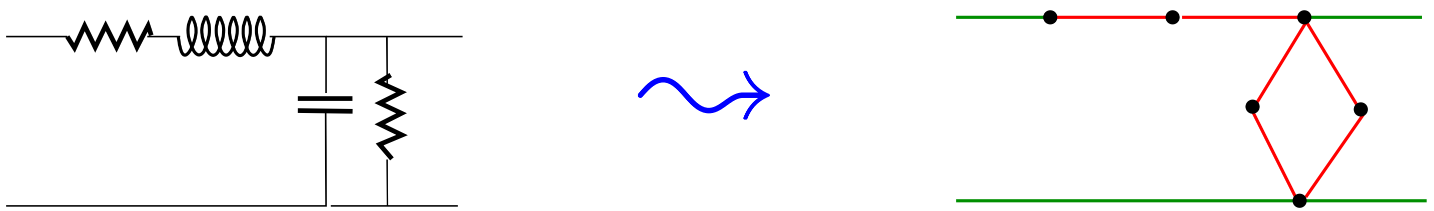
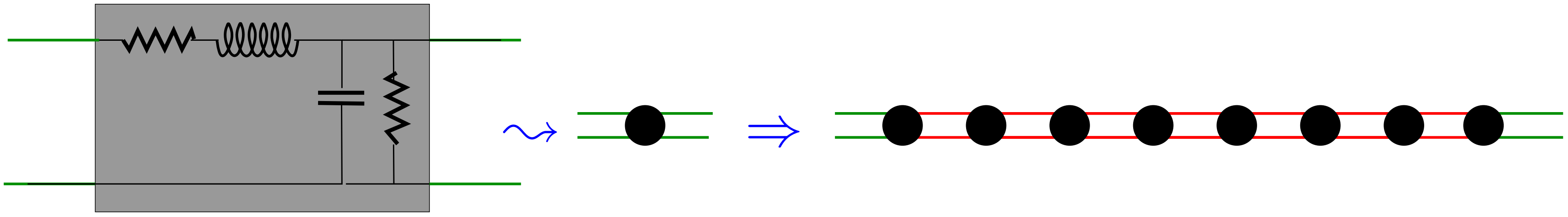


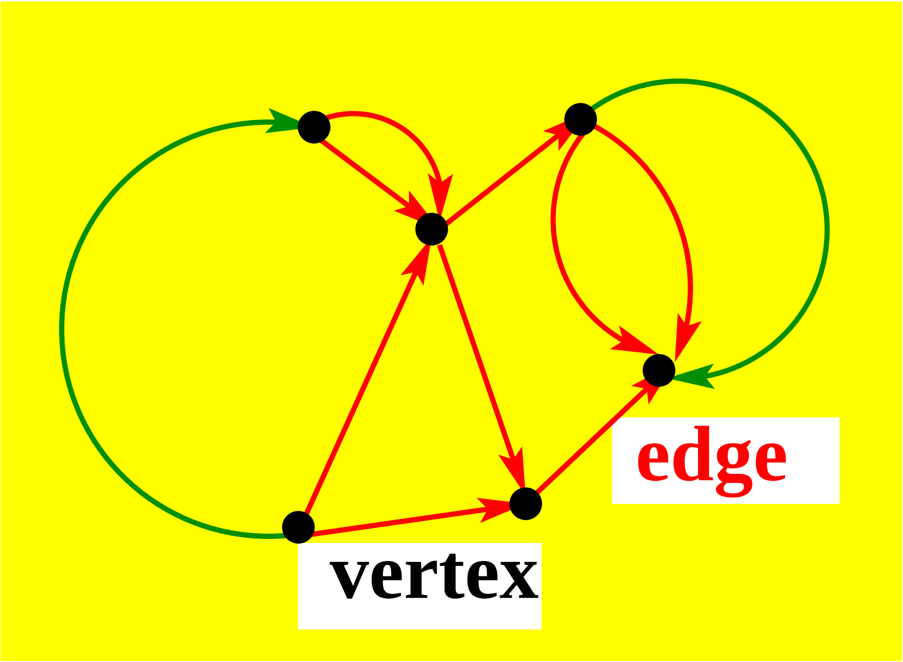
Hierarchical combination:



Circuit diagrams

Circuits and graphs

Classical circuit theory evolves around a digraph with 2-terminal elements or external ports in the edges and connections in the vertices.



Circuits and graphs

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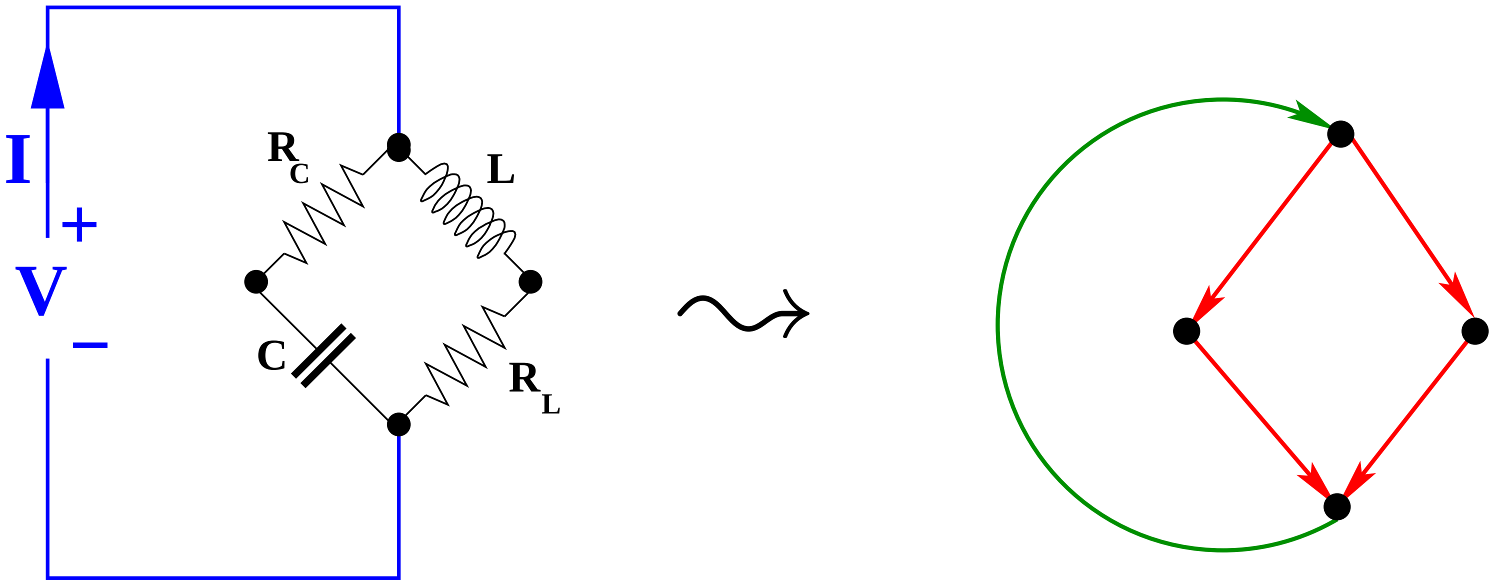

Circuits and graphs

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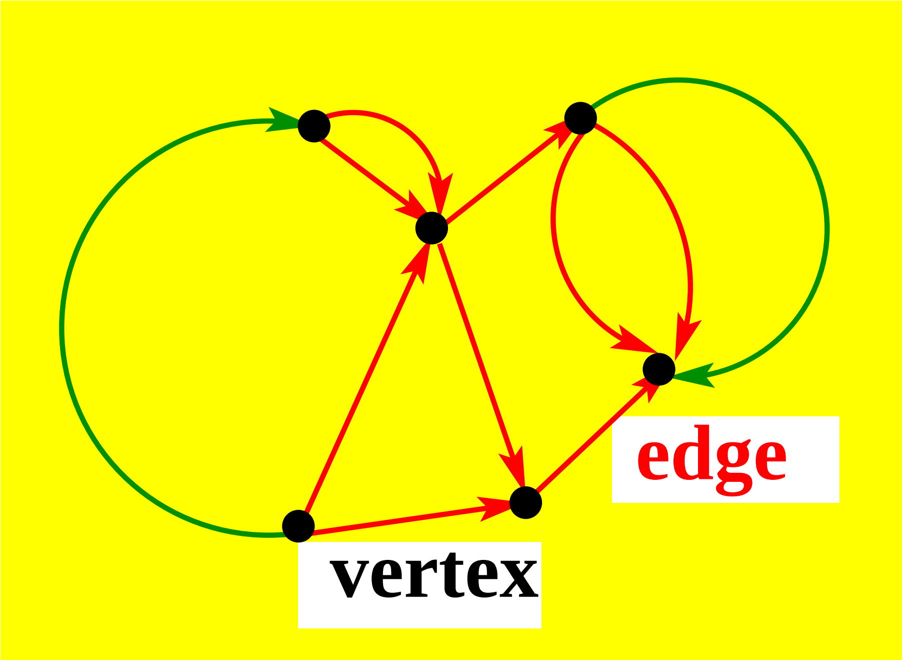


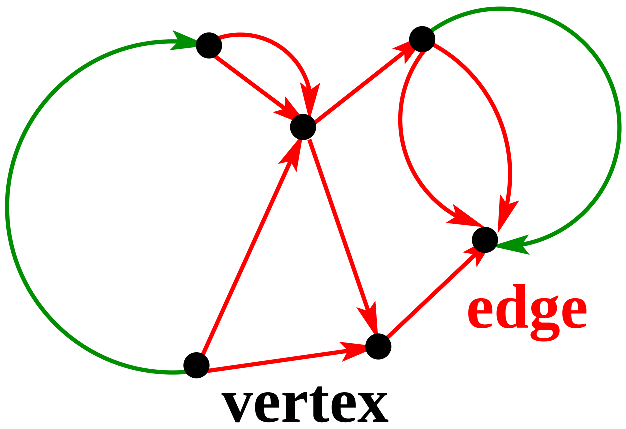
Associate a voltage drop and a current with each edge, and embed an element (say, R, L, or C) in each ‘internal’ edge.

Limitations

This methodology is limited:

- **It can only deal with 2-terminal elements and 2-terminal external ports.**
- **It is purely port oriented. It does not articulate that terminals, not ports make the interconnections.**
- **It is not hierarchical**
  An already-modeled-circuit cannot be reused as a subsystem in a larger circuit diagram.

Embedding a circuit in a graph



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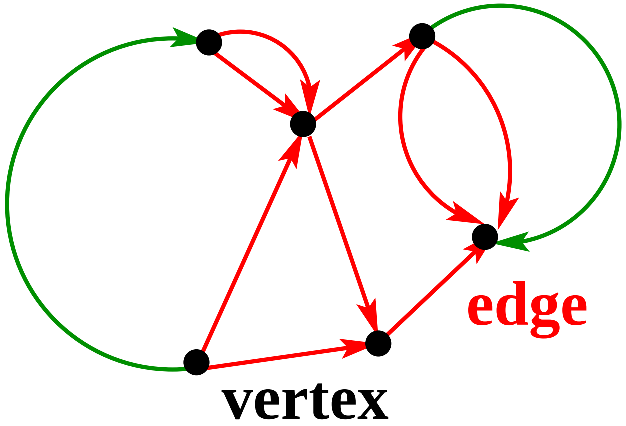
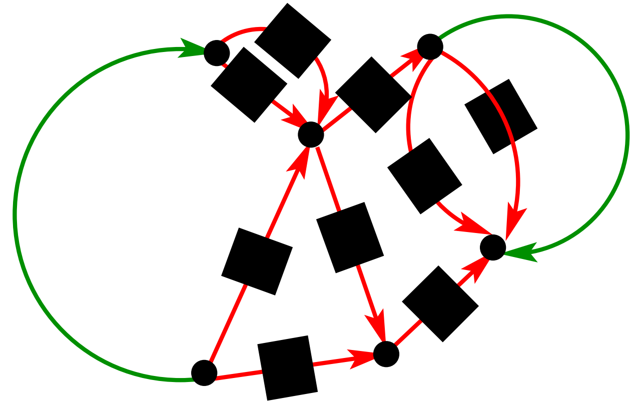
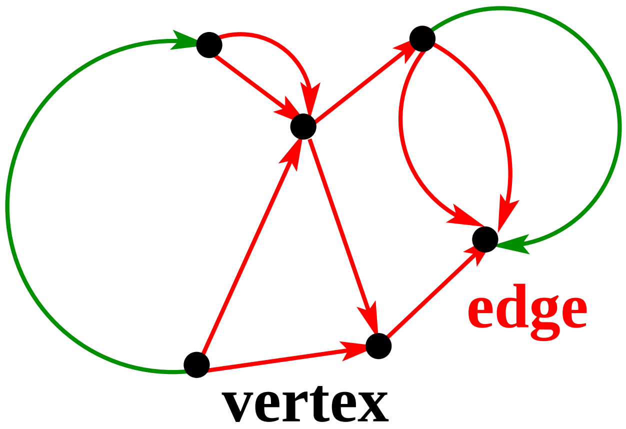
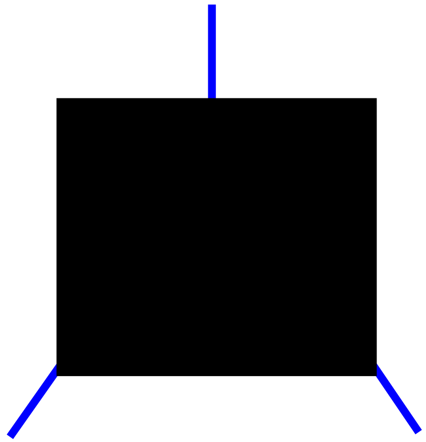




Perfect for 2-terminal one-ports

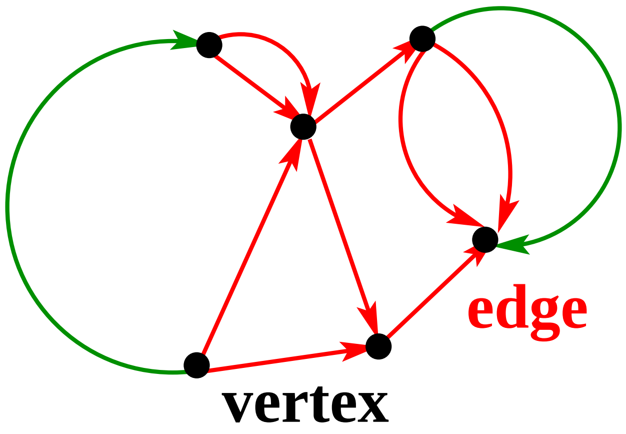
Embedding a circuit in a graph

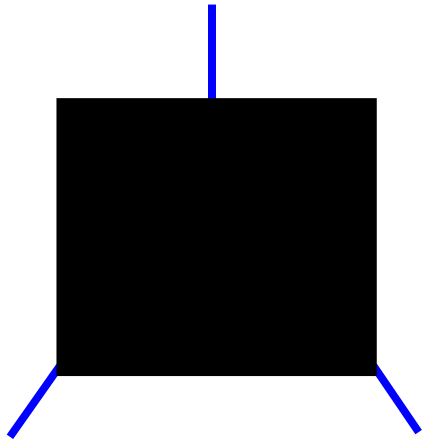




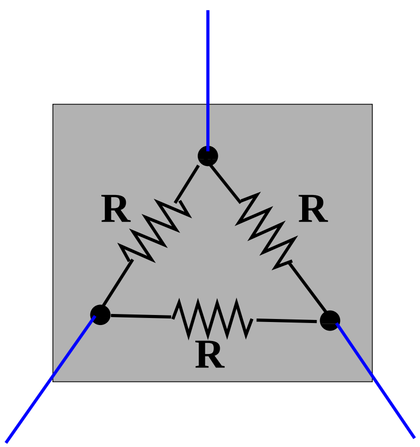
There is no way to embed a 3-terminal circuit in a circuit graph,

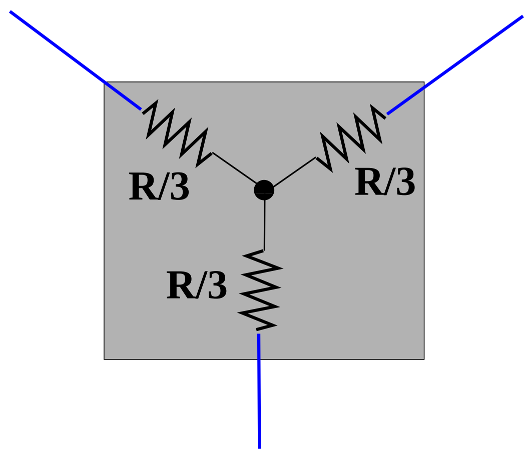
Embedding a circuit in a graph



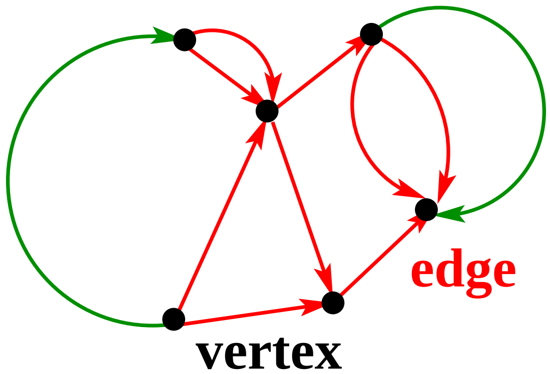


There is no way to embed a 3-terminal circuit in a circuit graph, unless we tear the blackbox into its components



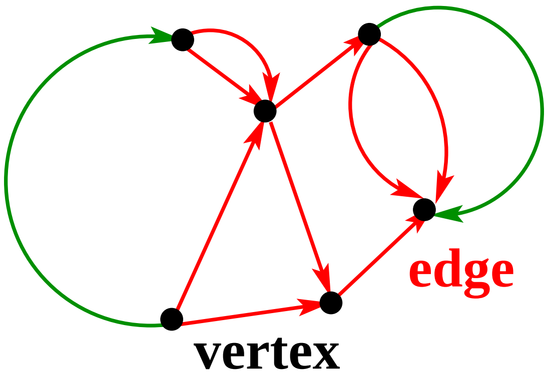


Embedding a circuit in a graph



If we imbed a 4-terminal circuit into a circuit graph, it has to be a 2-port.

Embedding a circuit in a graph



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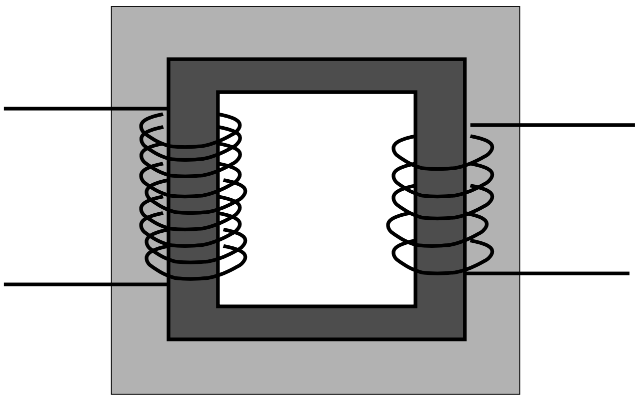
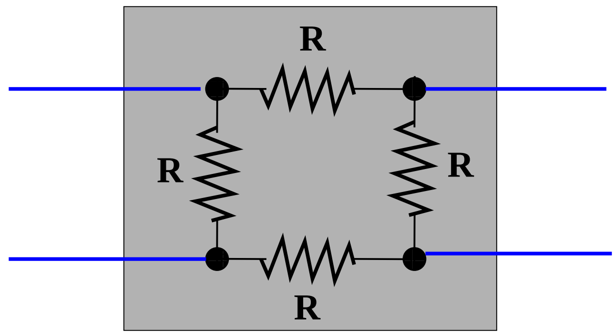


embeddable

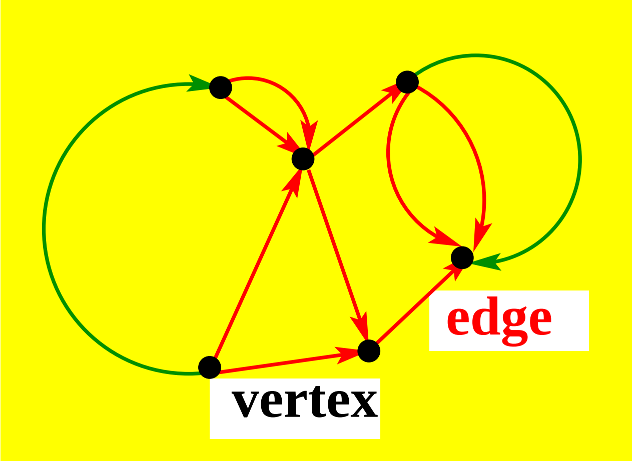


not embeddable

Vertices and edges

In circuit graphs,

subsystems are in the edges , connections are in the vertices



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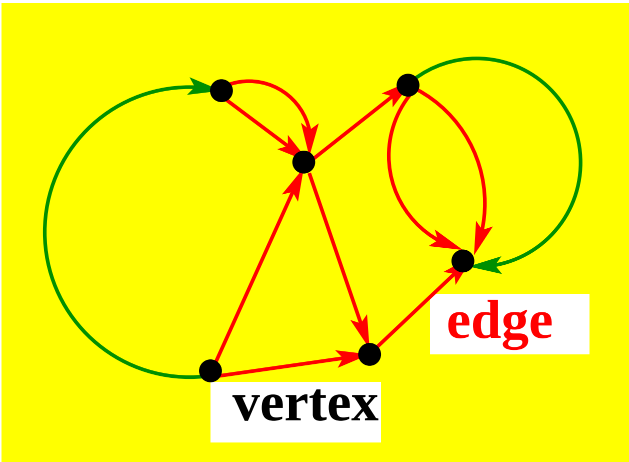

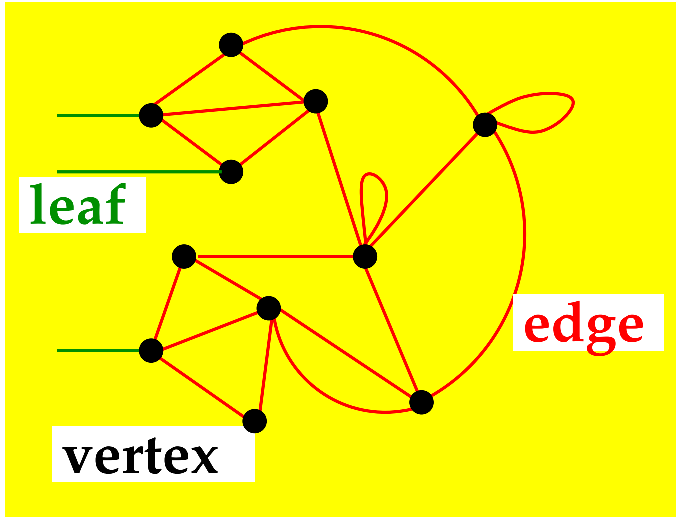

Contrast with tearing, zooming, linking:

subsystems are in the vertices , connections are in the edges

Summary

Main points

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- Hierarchical procedure
- Importance of latent variables and the elimination theorem
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- Control is interconnection, sensor output to actuator input feedback important special case
- Need generalization to distributed terminals, etc.

Thoughts to take home

1. **A dynamical system = a family of trajectories.**
2. **Interconnection = variable sharing**
3. **Control = interconnection**

Want to read about it? See

The behavioral approach to open and interconnected systems,

***Control Systems Magazine*, Volume 27, pages 46-99, 2007.**

The lecture frames are available from/at

`http://www.esat.kuleuven.be/~jwillems`

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