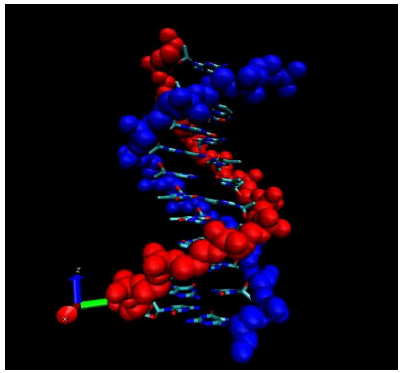
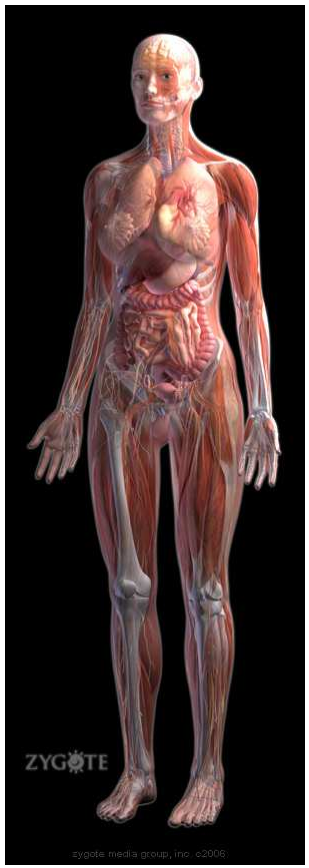
INTERCONNECTED SYSTEMS

Jan Willems, K.U. Leuven, Flanders, Belgium

Seminar, Kyoto University **July 22, 2008**

Systems

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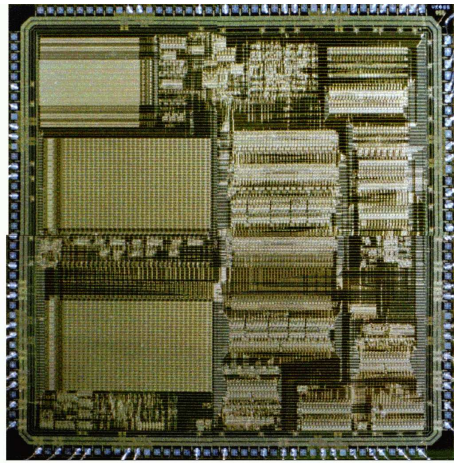


Features

- **open**
- **interconnected**
- **modular**
- **dynamic**

Features

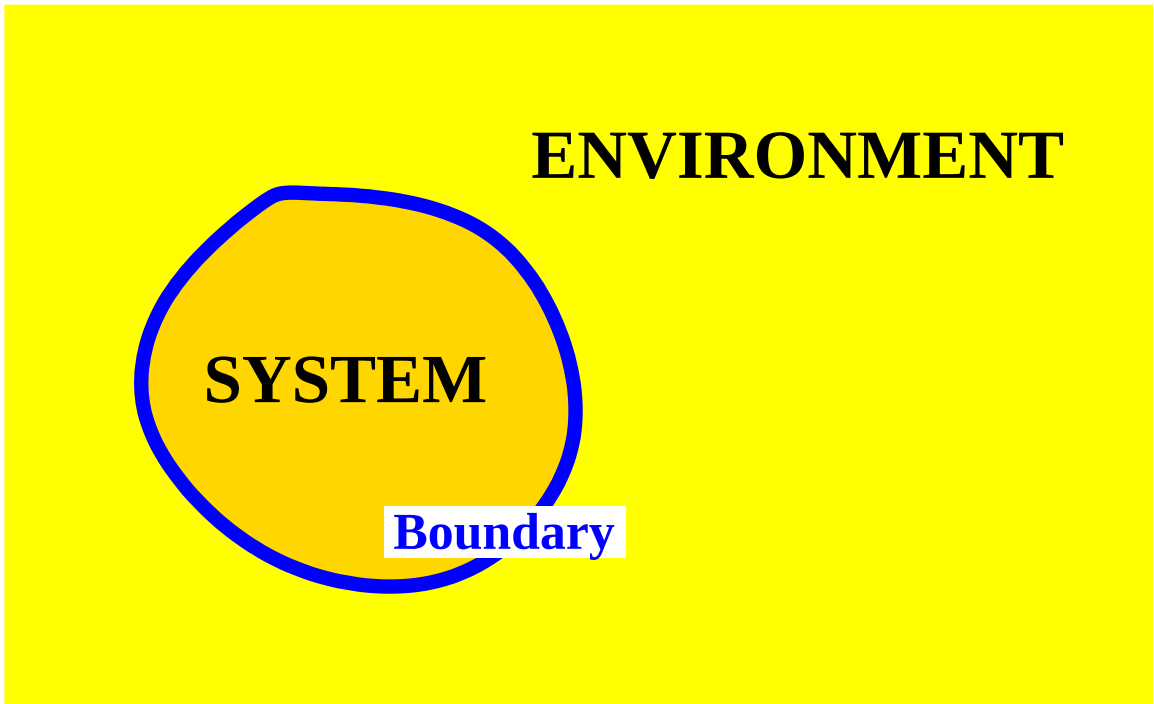
- **open**
- **interconnected**
- **modular**
- **dynamic**

Theme of this seminar:

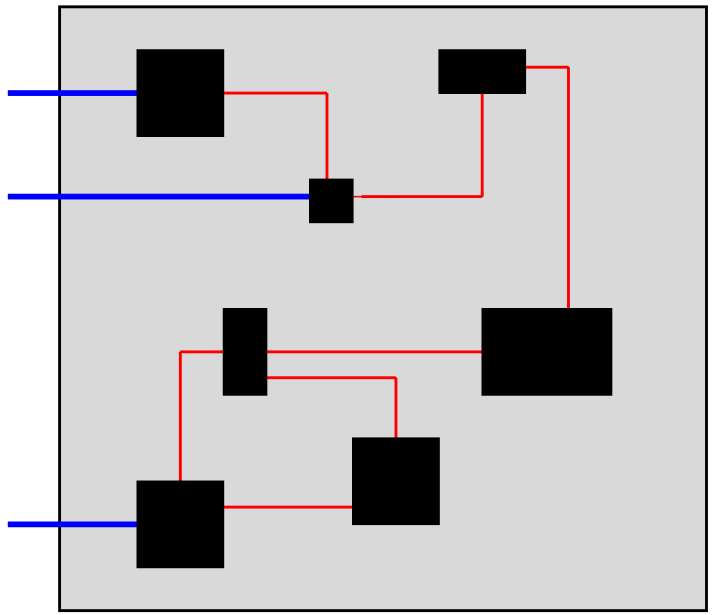
develop a suitable mathematical framework

Open and Connected

Open



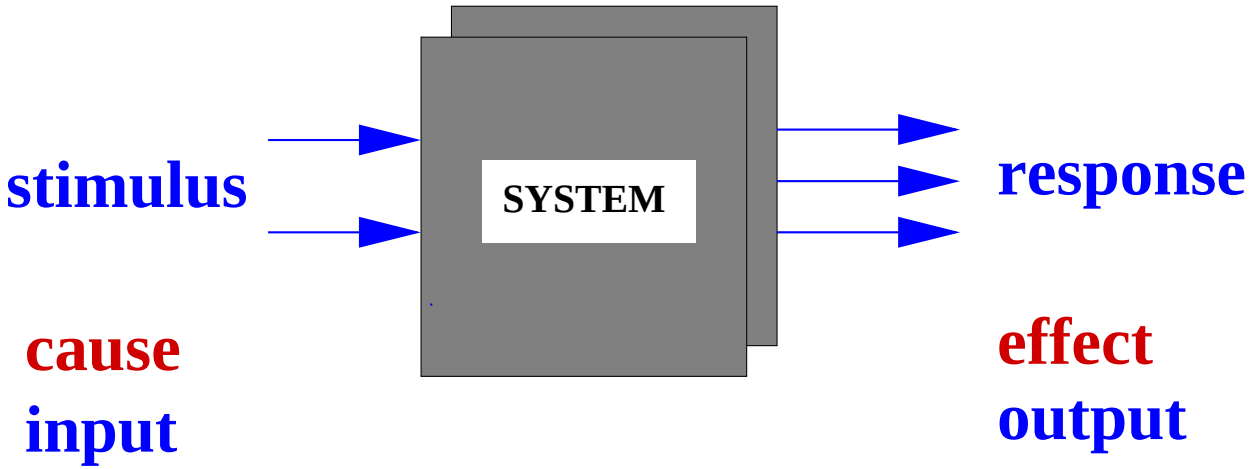
Connected

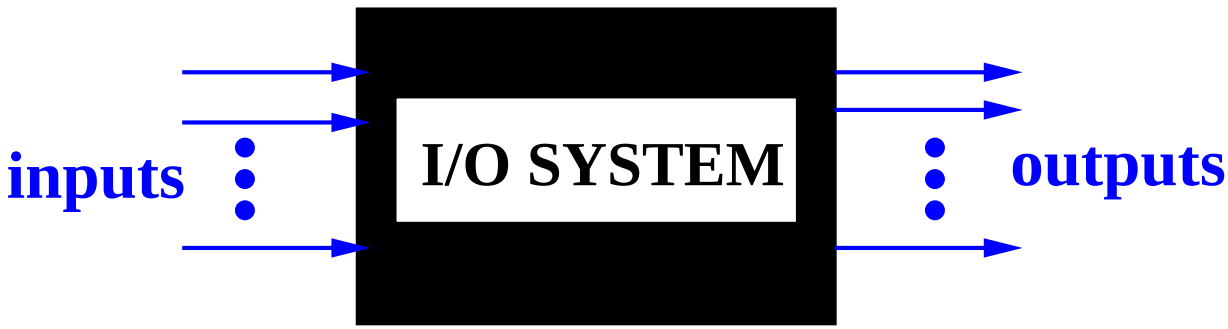


Architecture with subsystems

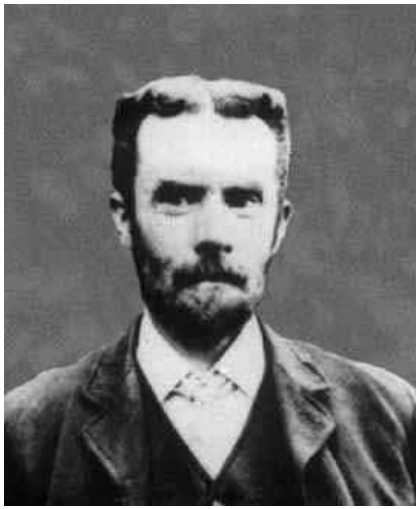
Inputs and outputs

Input/output systems

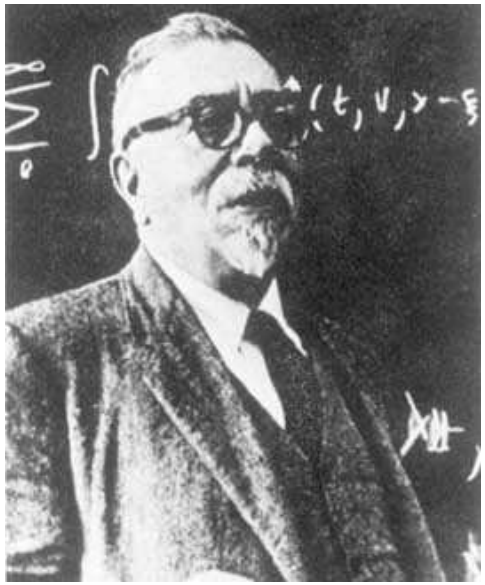




The originators



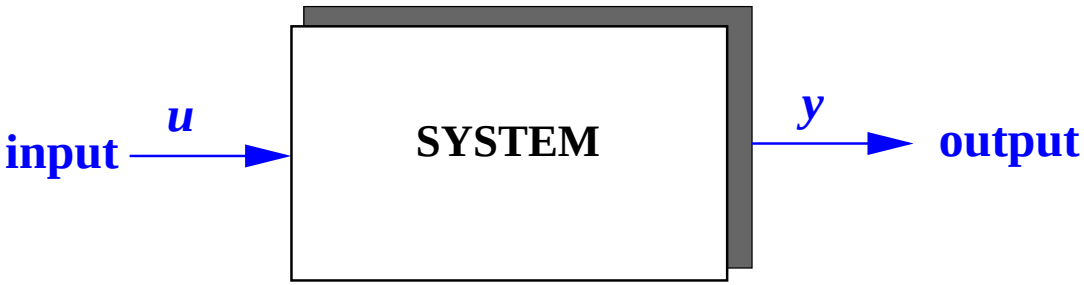
Oliver Heaviside (1850-1925)



Norbert Wiener (1894-1964)

and the many electrical circuit theorists ...

Mathematical description

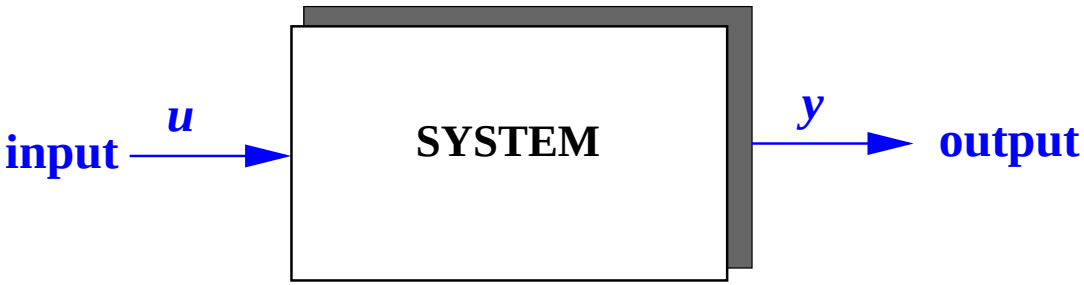


u: input, y: output, p **and** q **polynomials**

$G(s) = \frac{q(s)}{p(s)}$ **transfer functions, impedances, admittances.**

PID rules. Bode, Nyquist, Nichols. Lead-lag. Root-locus.

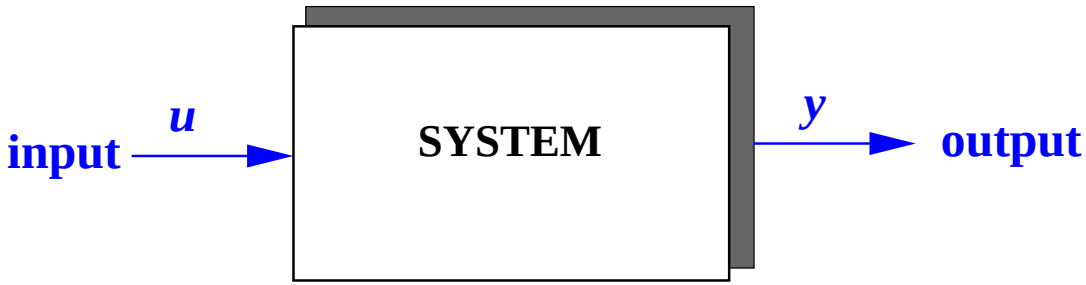
Mathematical description



$$y(t) = \int_{0 \text{ or } -\infty}^{t} H(t-t')u(t')\,dt'$$

$$y(t) = H_0(t) + \int_{-\infty}^{t} H_1(t-t')u(t')\,dt' + \int_{-\infty}^{t}\int_{-\infty}^{t'} H_2(t-t',t'-t'')u(t')u(t'')\,dt'dt'' + \cdots$$

Mathematical description



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Awkward nonlinear

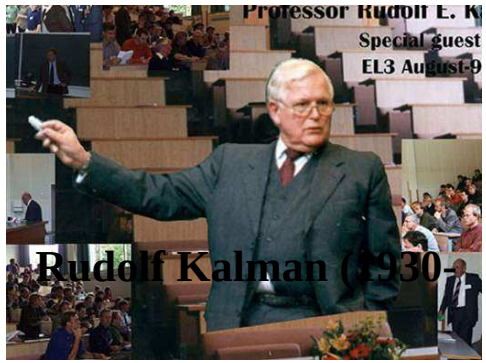
Far from the physics

Fail to deal with ‘initial conditions’.

Input/state/output systems

Around 1960: a paradigm shift to

$$\frac{d}{dt}x = f(x,u),\ y = g(x,u)$$



Rudolf Kalman (1930–)

Input/state/output systems

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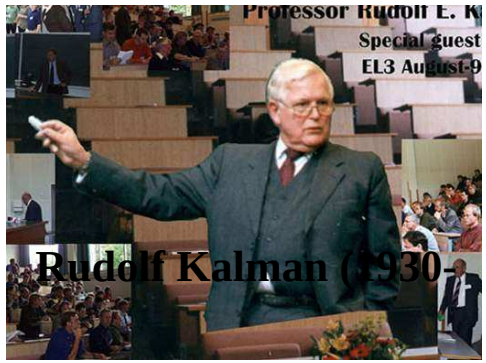


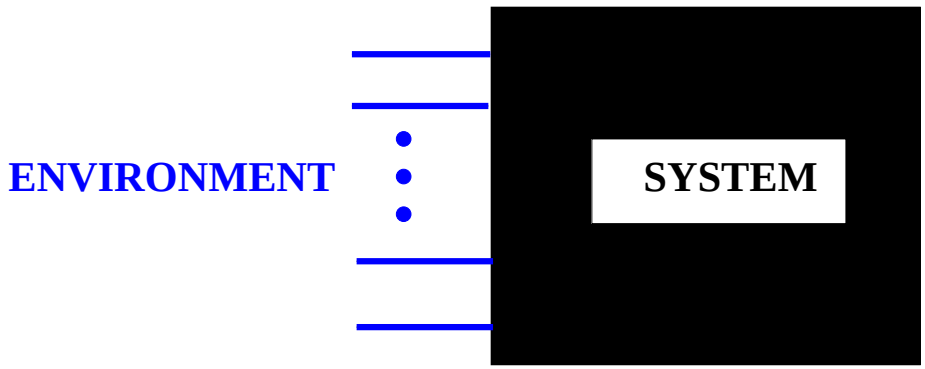
Rudolf Kalman (1930–)

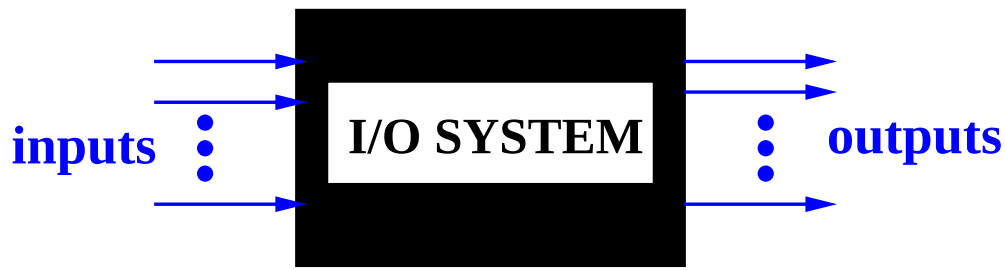
1. **open**
   ready to be interconnected
   outputs of one system $\mapsto$ inputs of another
2. **deals with initial conditions**
3. **incorporates nonlinearities, time-variation**
4. **models many physical phenomena**
5. $\ldots$

Theme

Theme of this lecture

We are accustomed to view an open dynamical system as an input/output structure

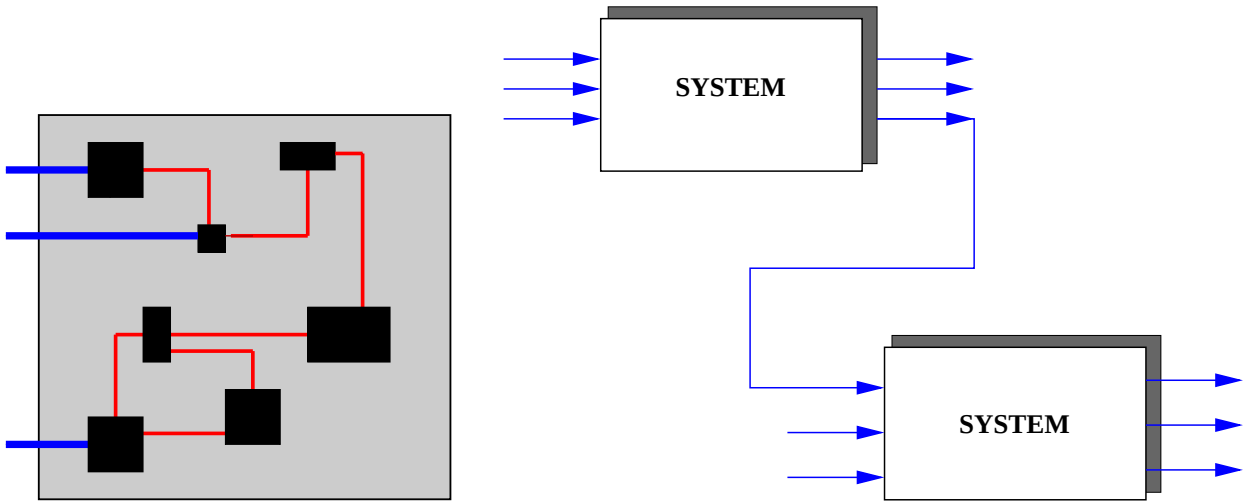




Is this appropriate for modeling physical systems?

Theme of this lecture

& interconnection as output-to-input assignment.



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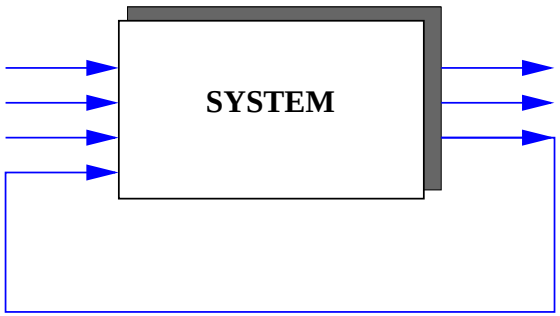
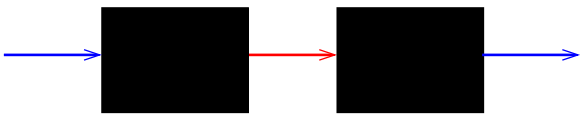
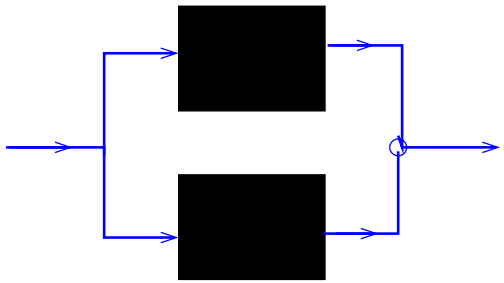




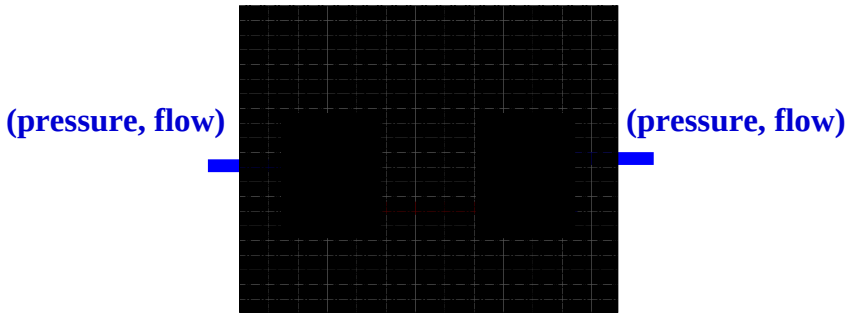


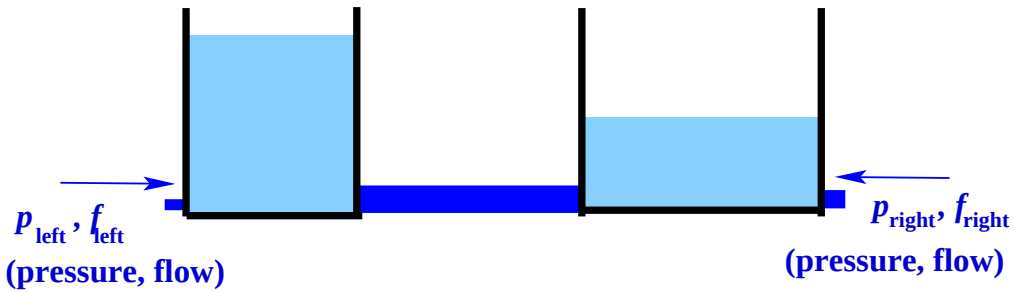
Feedback **Series** **Parallel**

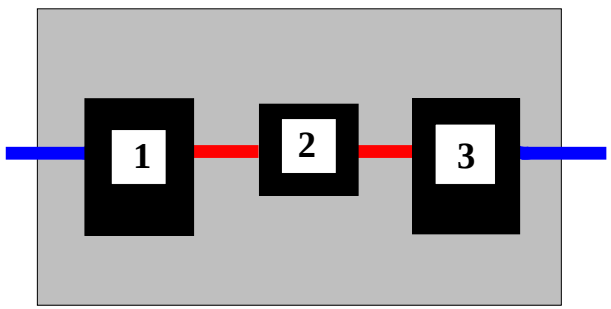
Is this appropriate for modeling physical systems?

Interconnection in physical systems

Example

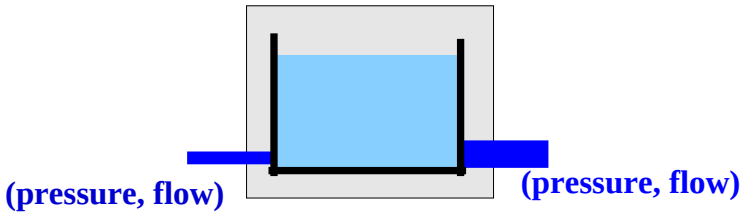

(pressure, flow)
(pressure, flow)

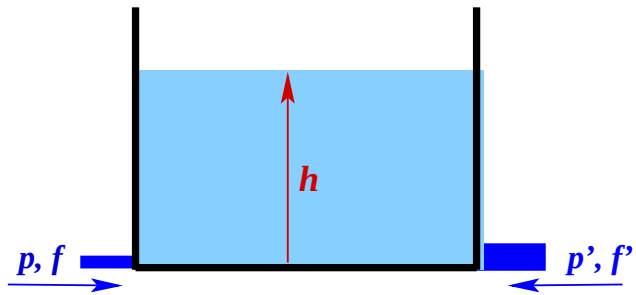

p_{left}, f_{left}
(pressure, flow)
p_{right}, f_{right}
(pressure, flow)


1
2
3

Example

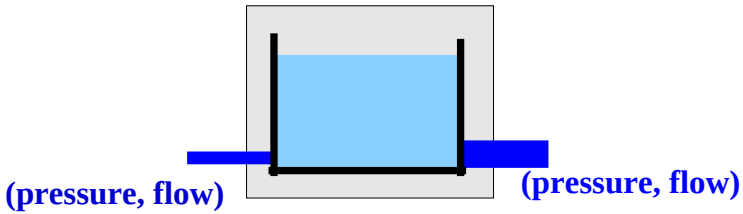
Subsystems 1 and 3:

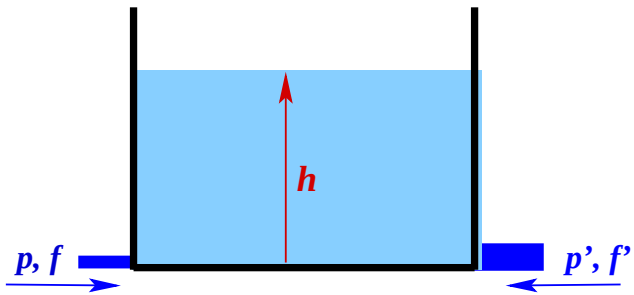




Example

Subsystems 1 and 3:

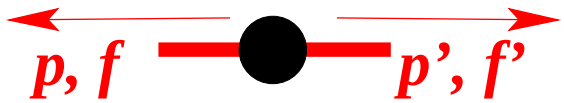




Subsystem 2:

Example

Interconnection laws:



$$p = p', \qquad f + f' = 0.$$

$$A_1 \tfrac{d}{dt} h_1 = f_1 + f_1',$$

$$B_1 f_1 = \begin{cases} \sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \geq \rho h_1, \\ -\sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \leq \rho h_1, \end{cases} \tag{1}$$

$$C f_1' = \begin{cases} \sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \geq \rho h_1, \\ -\sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \leq \rho h_1, \end{cases}$$

$$f_2 = -f_2', \quad p_2 - p_2' = \alpha f_2, \tag{2}$$

$$A_3 \tfrac{d}{dt} h_3 = f_3 + f_3',$$

$$C f_3 = \begin{cases} \sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \geq \rho h_3, \\ -\sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \leq \rho h_3, \end{cases} \tag{3}$$

$$C_3 f_3' = \begin{cases} \sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \geq \rho h_3, \\ -\sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \leq \rho h_3, \end{cases}$$

$$p_1' = p_2, \;\; f_1' + f_2 = 0, \; p_2' = p_3, \;\; f_2' + f_3 = 0. \tag{4}$$

$$p_{\text{left}} = p_1, \; f_{\text{left}} = f_1, \; p_{\text{right}} = p_3', \; f_{\text{right}} = f_3'. \tag{5}$$

- **Unclear input/output structure for terminal variables**
- **Many variables, indivisibly, at the same terminal**
- **Interconnection = variable sharing**
- **No signal flows, no output-to-input assignment**

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"Block diagrams unsuitable for serious physical modeling

- the control/physics barrier"

"Behavior based (declarative) modeling is a good alternative"

from K.J. Åström, *Present Developments in Control Applications*

IFAC 50-th Anniversary Celebration
Heidelberg, September 12, 2006.

Behavioral systems

A dynamical system

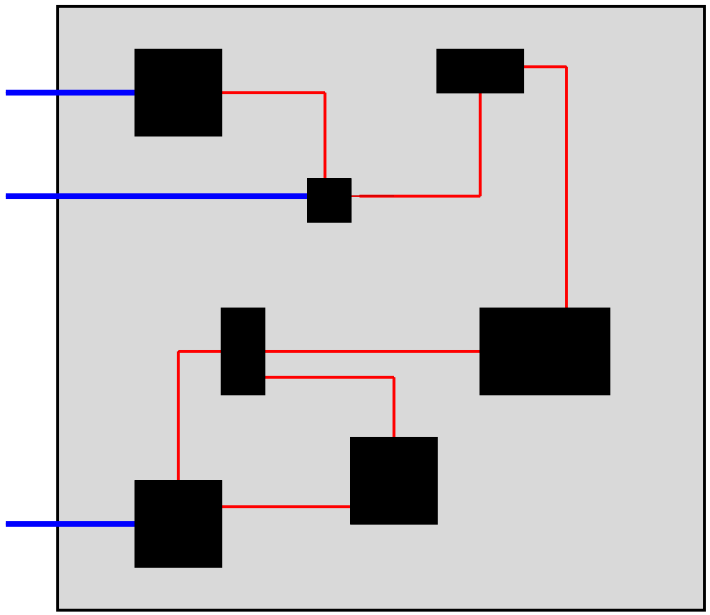
: $\Leftrightarrow$ a family of time functions, *'the behavior'*

Interconnection : $\Leftrightarrow$ ***'variable sharing'.***

Control : $\Leftrightarrow$ ***interconnection.***

Modeling of interconnected physical systems is the strongest case for 'behaviors'.

Objective



To develop a mathematical framework for dealing with interconnection of (open , dynamical) systems.

Objective

To develop a mathematical framework for dealing with interconnection of (open , dynamical) systems.

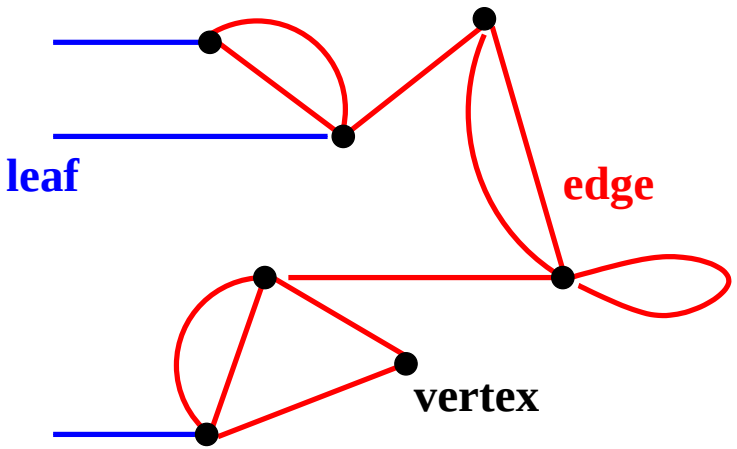
Competing philosophies:

- **input/output signal flow graphs**
- **circuit diagrams (loops, nodes)**
- **bond graphs (across, through, power)**
- **object-oriented modeling (SPICE, Modelica, ...)**
- **...**

ormalization of interconnection architectu

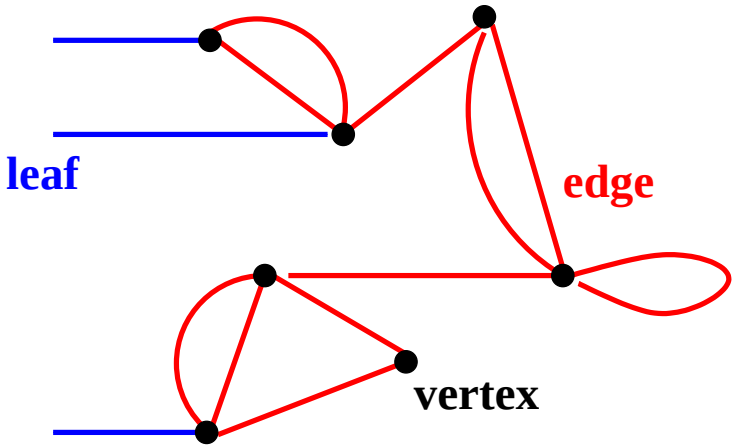
Architecture & module embedding

Architecture

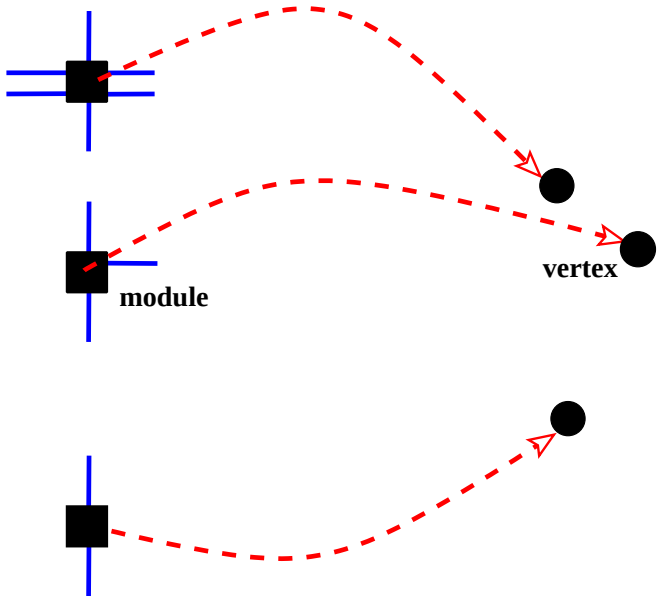


Architecture & module embedding

Architecture

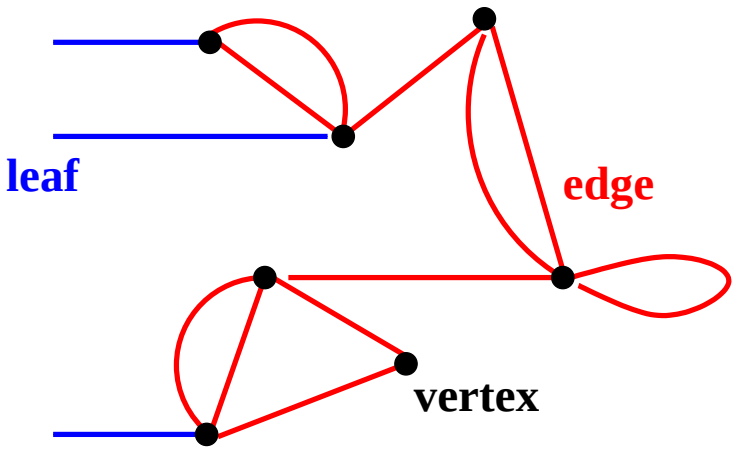


Modules (systems) in the vertices



Architecture & module embedding

Architecture



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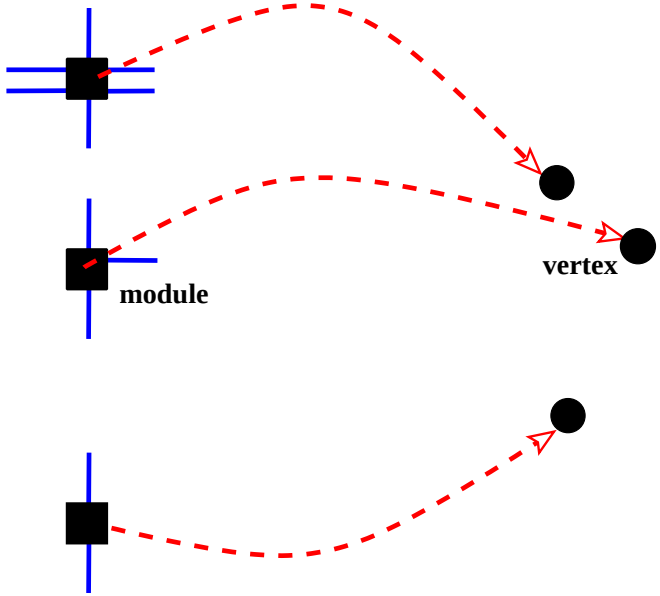
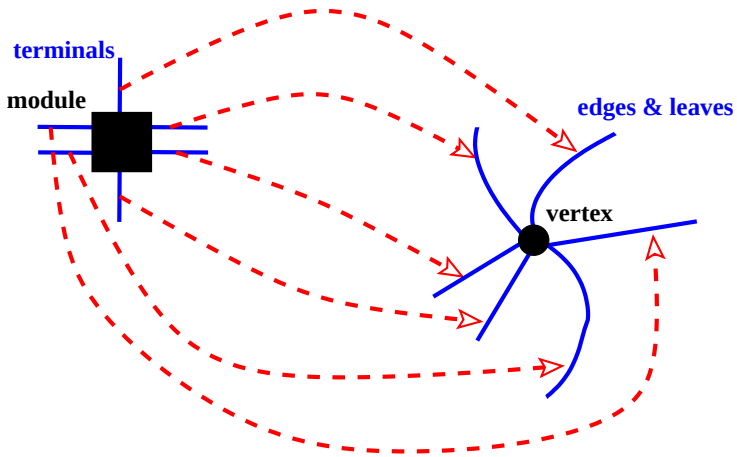


Terminals in the edges



Interconnection architecture

A *graph with leaves* defined as $\mathscr{G} = (\mathbb{V}, \mathbb{E}, \mathbb{L}, \mathscr{A})$

- **$\mathbb{V}$ the set of *vertices*,**
- **$\mathbb{E}$ the set of *edges*,**
- **$\mathbb{L}$ the set of *leaves*,**
- **$\mathscr{A}$ the *adjacency map*.**

$\mathscr{A}$ associates

- **with each edge $e \in \mathbb{E}$ an unordered pair**

$$\mathscr{A}(e) = [v_1, v_2] \quad v_1, v_2 \in \mathbb{V},$$

- **with each leaf $\ell \in \mathbb{L}$ an element $\mathscr{A}(\ell) = v \in \mathbb{V}$.**

Module embedding

The *module embedding* associates

a module with each vertex,

a $1 \leftrightarrow 1$ assignment between the

edges and leaves adjacent to the vertex and

the terminals of the module.

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a module with each vertex,

a $1 \leftrightarrow 1$ assignment between the

edges and leaves adjacent to the vertex and

the terminals of the module.

Vertices specify the subsystems,

edges how terminals of subsystems are connected,

leaves how the interconnected system interacts with the environment.

Module embedding

Vertices $\rightsquigarrow$ Subsystems

Edges $\rightsquigarrow$ Interconnections

Manifest variables

The *manifest variable assignment* is a map that assigns the manifest variables as a function of the terminal (or, more general, the module) variables.

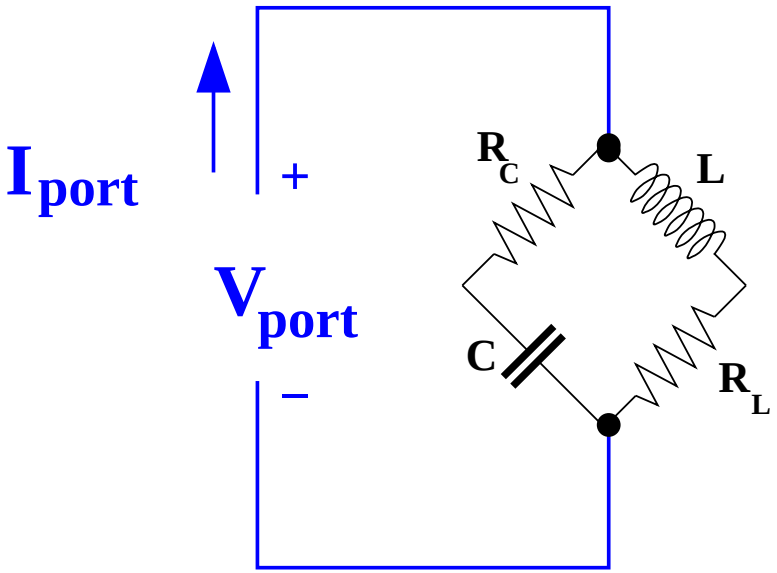
The terminal variables are henceforth considered as latent variables.

Behavioral equations

1. **Module equations** for each vertex.
   Relation among the variables on the terminals of the subsystems.
2. **Interconnection equations** for each edge.
   Equating the variables on the terminals associated with the same edge.
3. **Manifest variable assignment**
   Specifies the variables of interest.

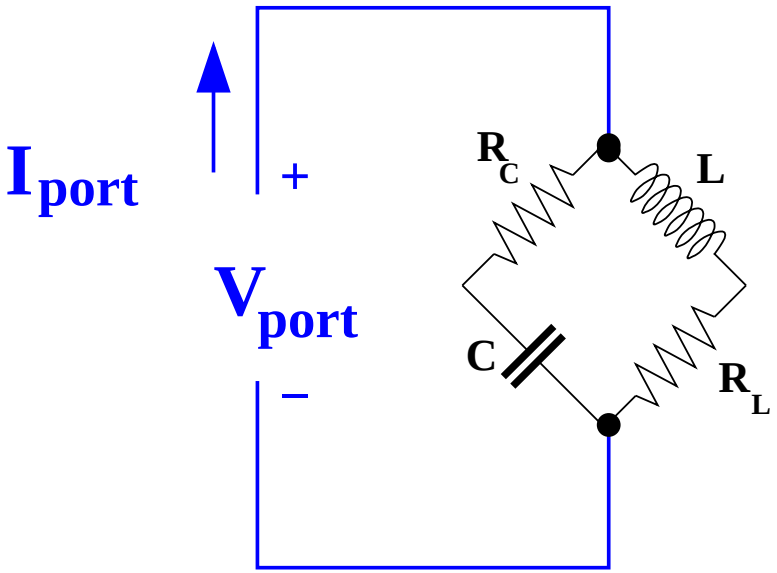
A very classical example

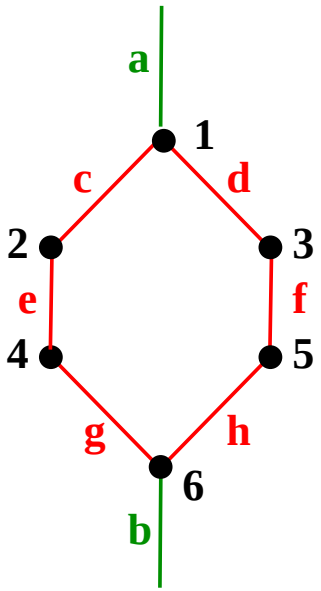
RLC circuit



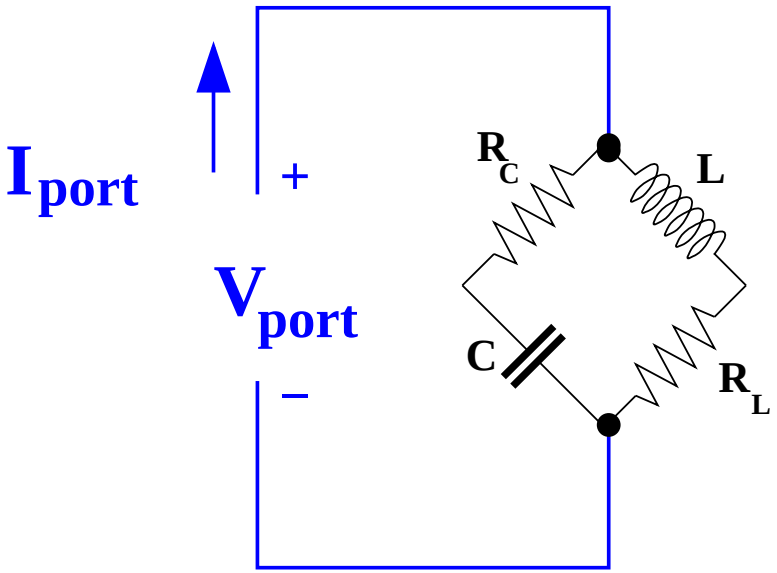
¡¡¡ **Model the port behavior !!!**

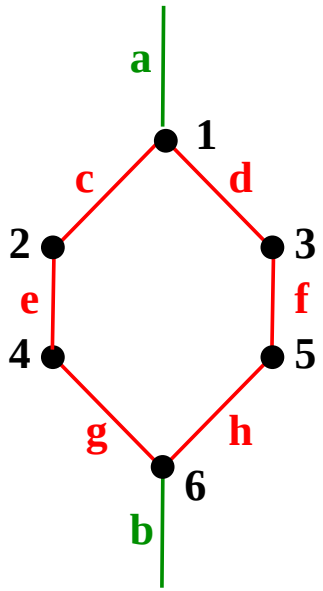
RLC circuit





RLC circuit





$$R_C \mapsto 2, R_L \mapsto 5, C \mapsto 4, L \mapsto 3, \text{connector}_1 \mapsto 1, \text{connector}_2 \mapsto 6,$$

$$1_{R_C} \mapsto c, 2_{R_C} \mapsto e, 1_{R_L} \mapsto f, 2_{R_L} \mapsto h, 1_C \mapsto e, 2_C \mapsto g, 1_L \mapsto d, 2_L \mapsto f,$$

$$1_{connector_1} \mapsto a, 2_{connector_1} \mapsto c, 3_{connector_1} \mapsto d,$$

$$1_{connector_2} \mapsto b, 2_{connector_2} \mapsto g, 3_{connector_2} \mapsto h.$$

Module equations

vertex 1

$$V_{1_{\text{connector}_1}} = V_{2_{\text{connector}_1}} = V_{3_{\text{connector}_1}},$$

$$I_{1_{\text{connector}_1}} + I_{2_{\text{connector}_1}} + I_{3_{\text{connector}_1}} = 0;$$

vertex 2

$$V_{1_{R_C}} - V_{2_{R_C}} = R_C I_{1_{R_C}},\ I_{1_{R_C}} + I_{2_{R_C}} = 0;$$

vertex 3

$$L\frac{d}{dt} I_{I_L} = V_{1_L} - V_{2_L},\ I_{1_L} + I_{2_L} = 0;$$

vertex 4

$$C\frac{d}{dt}\left(V_{1_C} - V_{2_C}\right) = I_{1_C},\ I_{1_C} + I_{2_C} = 0;$$

vertex 5

$$V_{1_{R_L}} - V_{2_{R_L}} = R_L I_{1_{R_L}},\ I_{1_{R_L}} + I_{2_{R_L}} = 0;$$

vertex 6

$$V_{1_{\text{connector}_2}} = V_{2_{\text{connector}_2}} = V_{3_{\text{connector}_2}},$$

$$I_{1_{\text{connector}_2}} + I_{2_{\text{connector}_2}} + I_{3_{\text{connector}_2}} = 0.$$

Interconnection equations

edge c $V_{1_{R_C}} = V_{2_{\text{connector}_1}},\ I_{1_{R_C}} + I_{2_{\text{connector}_1}} = 0;$

edge d $V_{1_L} = V_{3_{\text{connector}_1}},\ I_{1_L} + I_{3_{\text{connector}_1}} = 0;$

edge e $V_{2_{R_C}} = V_{1_C},\ I_{2_{R_C}} + I_{1_C} = 0;$

edge f $V_{2_L} = V_{1_{R_C}},\ I_{2_L} + I_{1_{R_C}} = 0;$

edge g $V_{2_C} = V_{1_{\text{connector}_2}},\ I_{2_C} + I_{1_{\text{connector}_2}} = 0;$

edge h $V_{2_{R_L}} = V_{2_{\text{connector}_2}},\ I_{2_{R_L}} + I_{2_{\text{connector}_2}} = 0.$

Manifest variable assignment

$$V_{\text{external port}} = V1_{\text{connector}_1} - V3_{\text{connector}_2}$$

$$I_{\text{external port}} = I1_{\text{connector}_1}$$

Manifest variable assignment

$$V_{\text{external port}} = V1_{\text{connector}_1} - V3_{\text{connector}_2}$$

$$I_{\text{external port}} = I1_{\text{connector}_1}$$

The module equations
+ the interconnection constraints
+ the manifest variable assignment
form the complete model for the behavior of

$$(V_{\text{external port}}, I_{\text{external port}})$$

Prevalence of latent variables $\rightsquigarrow$ elimination theory.

Manifest variable assignment

$$V_{\text{external port}} = V1_{\text{connector}_1} - V3_{\text{connector}_2}$$

$$I_{\text{external port}} = I1_{\text{connector}_1}$$

Behavior = all

$$(V_{\text{external port}}, I_{\text{external port}}) : \quad \mathbb{R} \rightarrow \mathbb{R}^2$$

$$\exists \quad \ldots, V1_{R_C}, \ldots, I3_{\text{connector}_2} \; \mathbb{R} \rightarrow \mathbb{R}^{\cdots} \qquad \textbf{such that ...}$$

Manifest behavior

$\rightsquigarrow$ **the dynamical system** $\Sigma = \left(\mathbb{R}, \mathbb{R}^2, \mathscr{B}\right)$ **with behavior** $\mathscr{B}$ **specified by:**

Case 1: $CR_C \neq \dfrac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + \left(1 + \frac{R_C}{R_L}\right) CR_C \frac{d}{dt} + CR_C \frac{L}{R_L} \frac{d^2}{dt^2}\right) V = \left(1 + CR_C \frac{d}{dt}\right) \left(1 + \frac{L}{R_L} \frac{d}{dt}\right) R_C I$$

Case 2: $CR_C = \dfrac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + CR_C \frac{d}{dt}\right) V = \left(1 + CR_C\right) \frac{d}{dt} R_C I$$

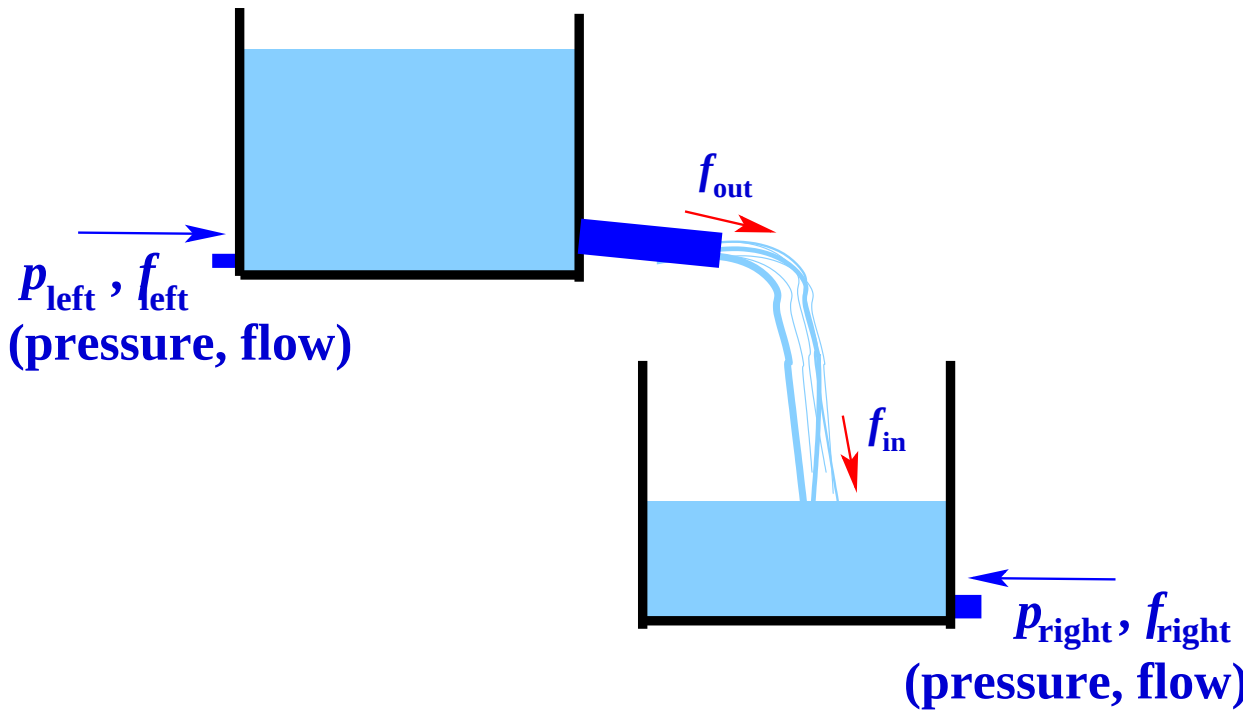
$\rightsquigarrow$ **behavior** $\mathscr{B}$ **=** **all solutions** $(V, I) : \mathbb{R} \to \mathbb{R}^2$

Other methodologies

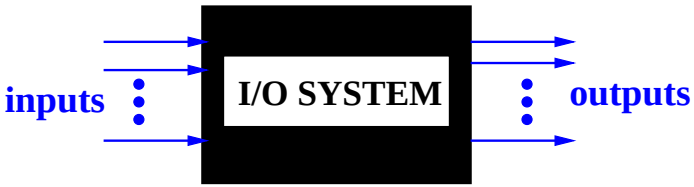
Signal flow graphs

input/output thinking

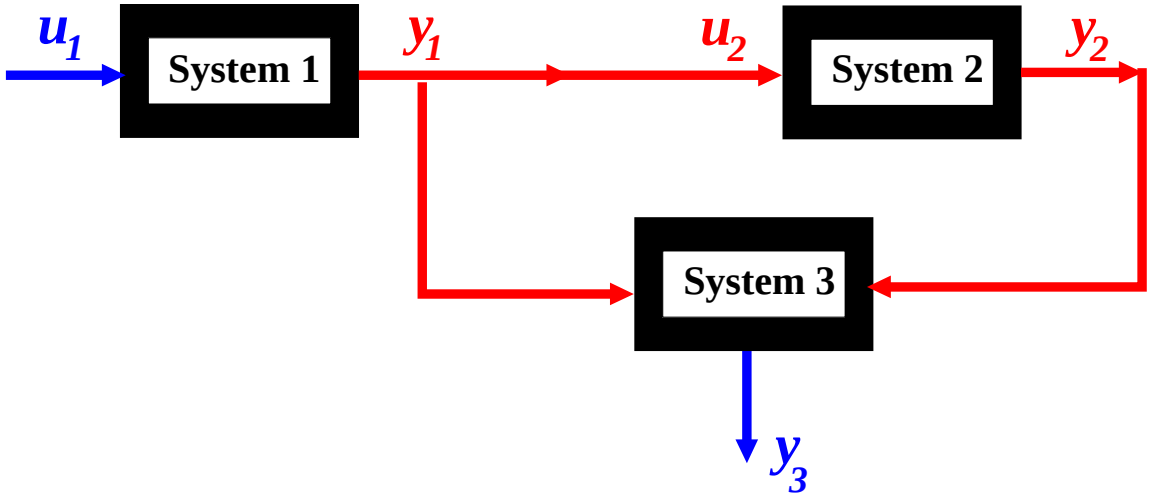
There are many many examples where output-to-input connection is eminently natural:



input/output thinking



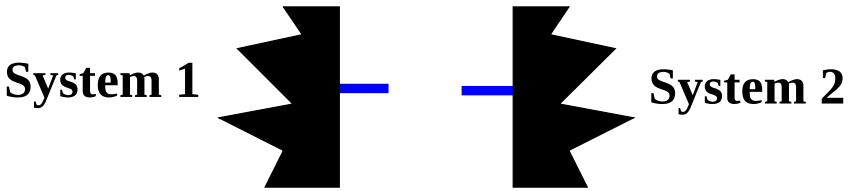
(a)



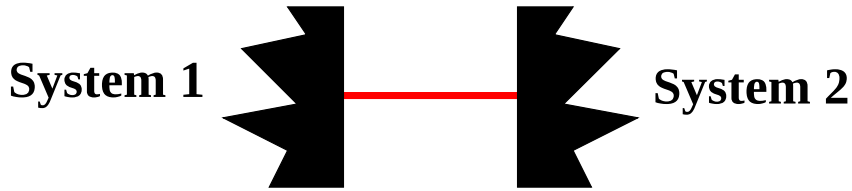
(b)

- shows terminal variables separate
- suggests that inputs and outputs occur at different points
- allows impossible input-output connections

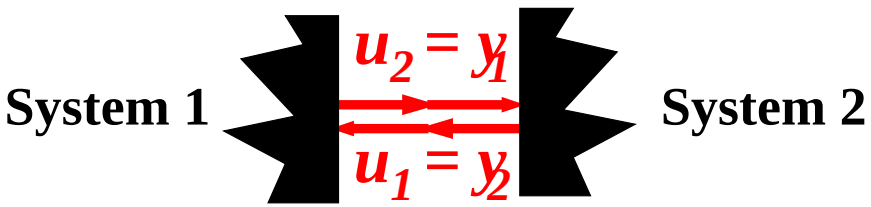
input/output thinking



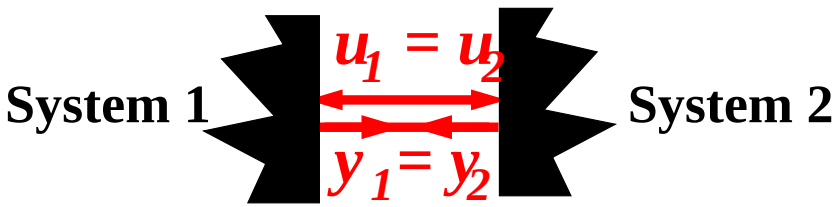
(a)



(b)



(c)



(d)

For physical systems

input-to-input & output-to-output

assignment very prevalent.

Physical systems are not signal processors.

Bond graphs

Bond graphs

Interconnection variables:

a flow and an effort **product = power**

- **current & voltage**
- **velocity & force**
- **mass flow & pressure**
- **heat flow& temperature**
- $\frac{\text{heat flow}}{\text{temperature}}$ **& temperature**
- **...**

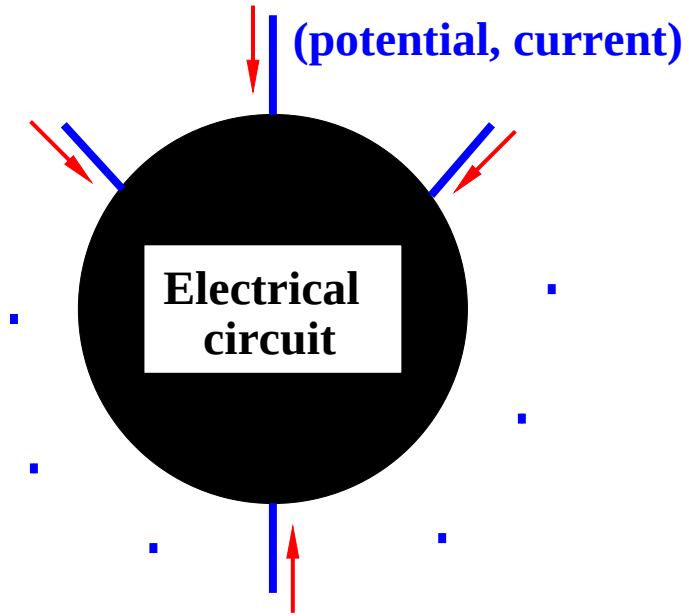
Bond graphs

Interconnection variables:

a flow and an effort **product = power**

1. **Mechanical interconnections equate positions, not velocities**
2. **Not all interconnections involve equating energy transfer**
3. **Terminals are for interconnection, ports are for energy transfer**

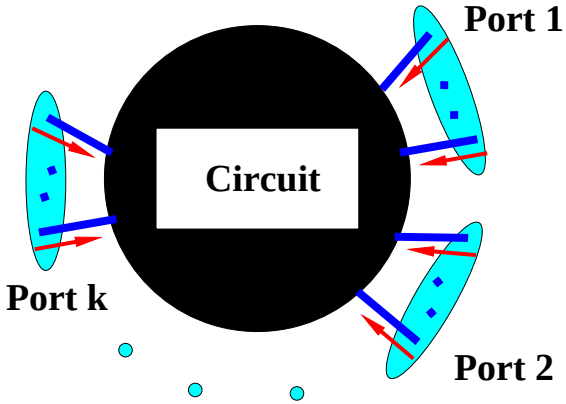
Terminals versus ports



Terminal variables and behavior:

$$(V_1, I_1, V_2, I_2, \ldots, V_{\mathtt{n}}, I_{\mathtt{n}}) \rightsquigarrow \textbf{ behavior } \mathscr{B} \subseteq \left(\mathbb{R}^{2\mathtt{n}}\right)^{\mathbb{R}}$$

Terminals versus ports



Port $:\Leftrightarrow$ **sum currents = 0**

potentials + constant $\Rightarrow$ potentials

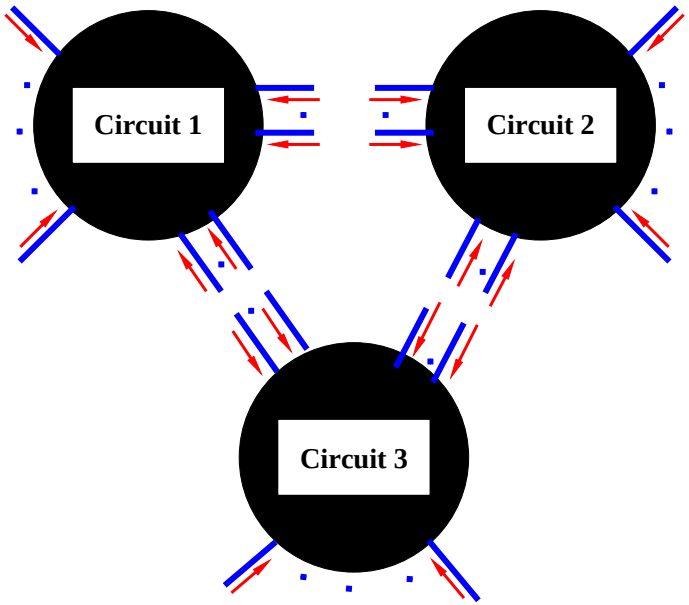
$$\left(\boxed{V_1, I_1 \ldots, V_{\mathrm{p}}, I_{\mathrm{p}}}, V_{\mathrm{p}+1}, \ldots, I_{\mathrm{n}}\right) \in \mathscr{B}, \alpha : \mathbb{R} \longrightarrow \mathbb{R}$$

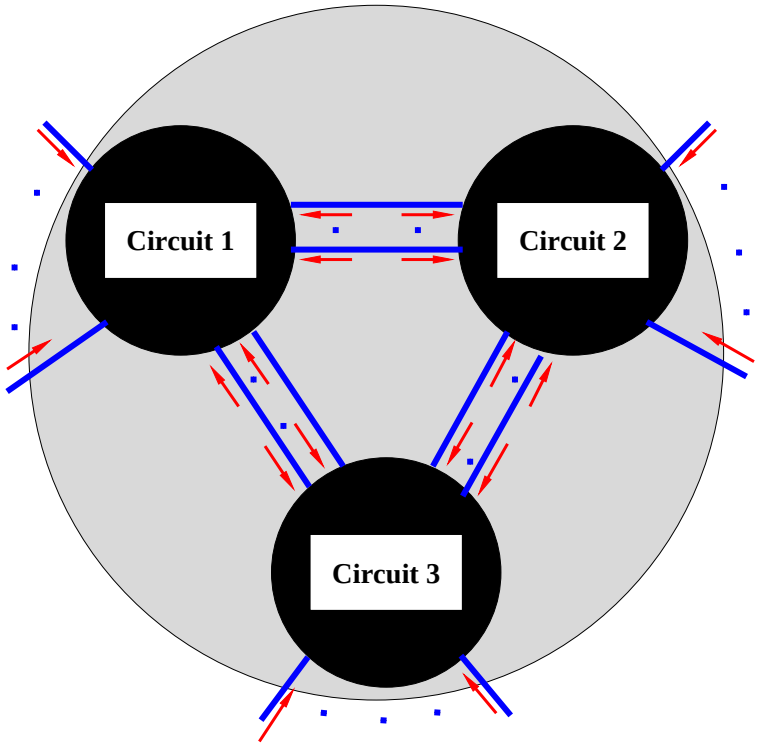
$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_{\mathrm{p}} + \alpha, I_{\mathrm{p}}}, V_{\mathrm{p}+1}, \ldots, I_{\mathrm{n}}\right) \in \mathscr{B}$$

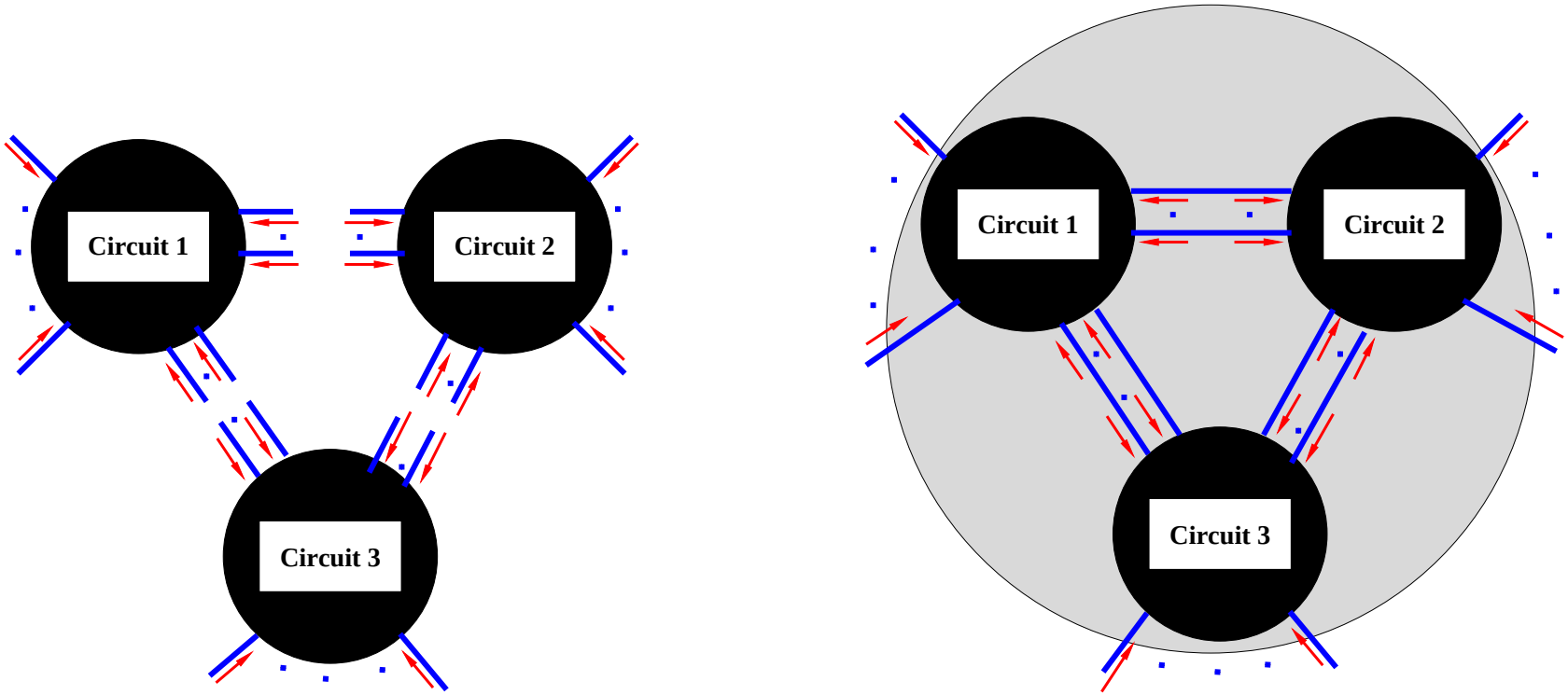
$$\boxed{I_1 + \cdots + I_{\mathrm{p}}} = 0$$

Terminals versus ports





Terminals versus ports

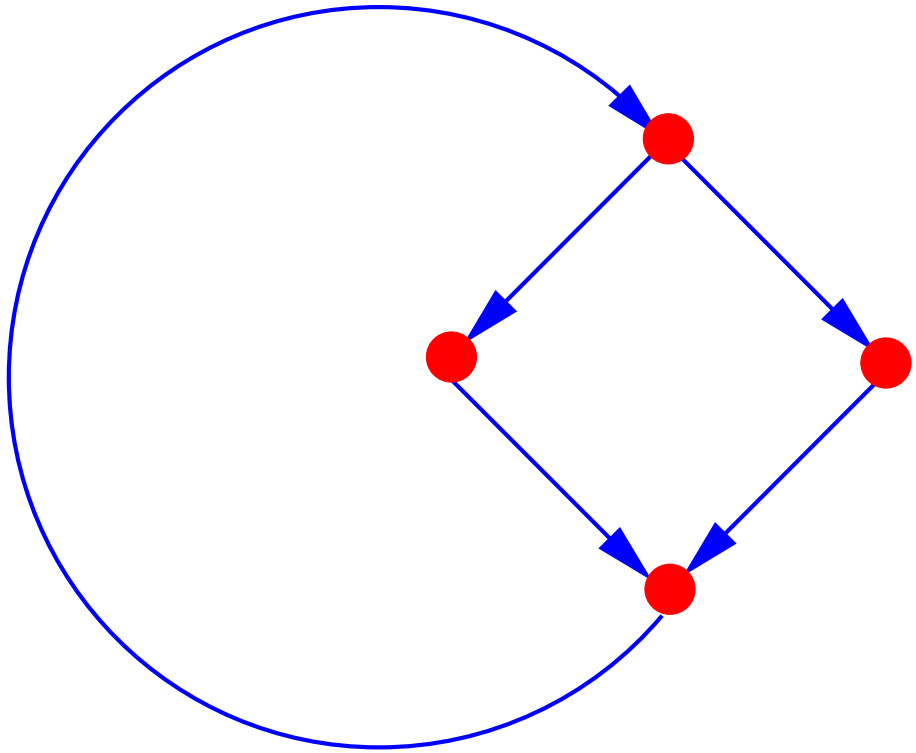


Interconnection via terminals, energy transfer via ports; one cannot talk about

"the energy transferred from circuit 1 to circuit 2"

Circuit diagrams

Circuit diagrams



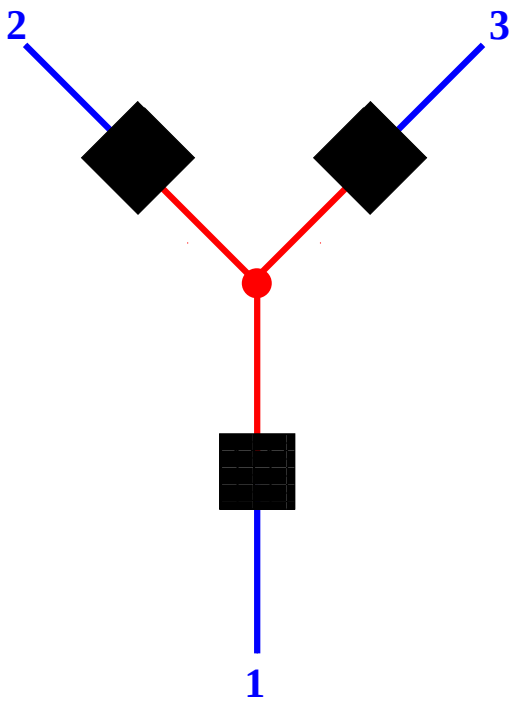
Circuit diagrams with

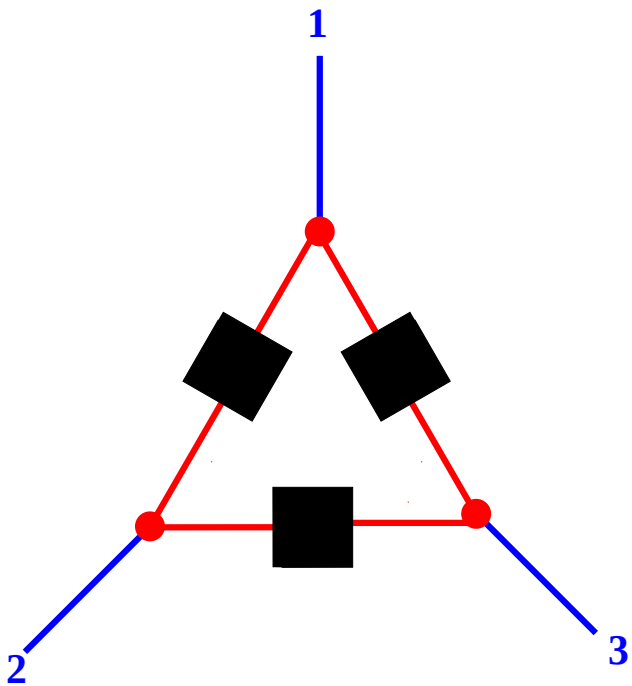
nodes & branches & KVL & KCL

are only effective with 2-terminal 1-ports.

Circuit diagrams

Not closed under composition





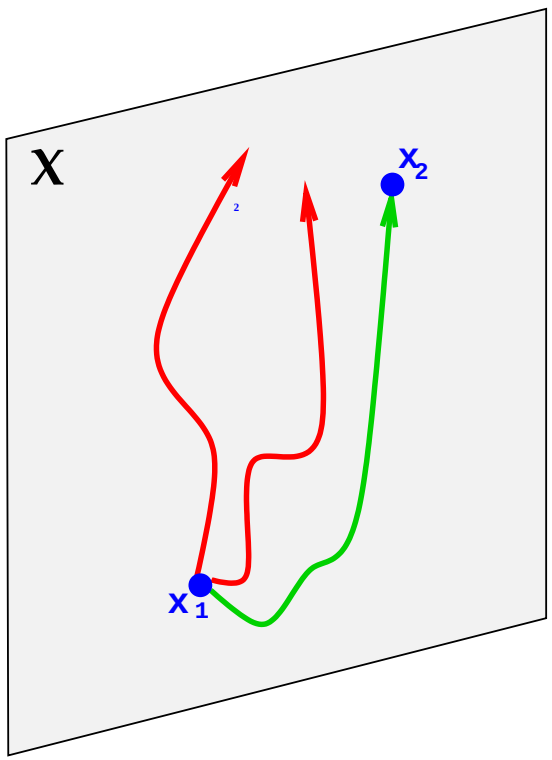
Various facets of control

Path planning

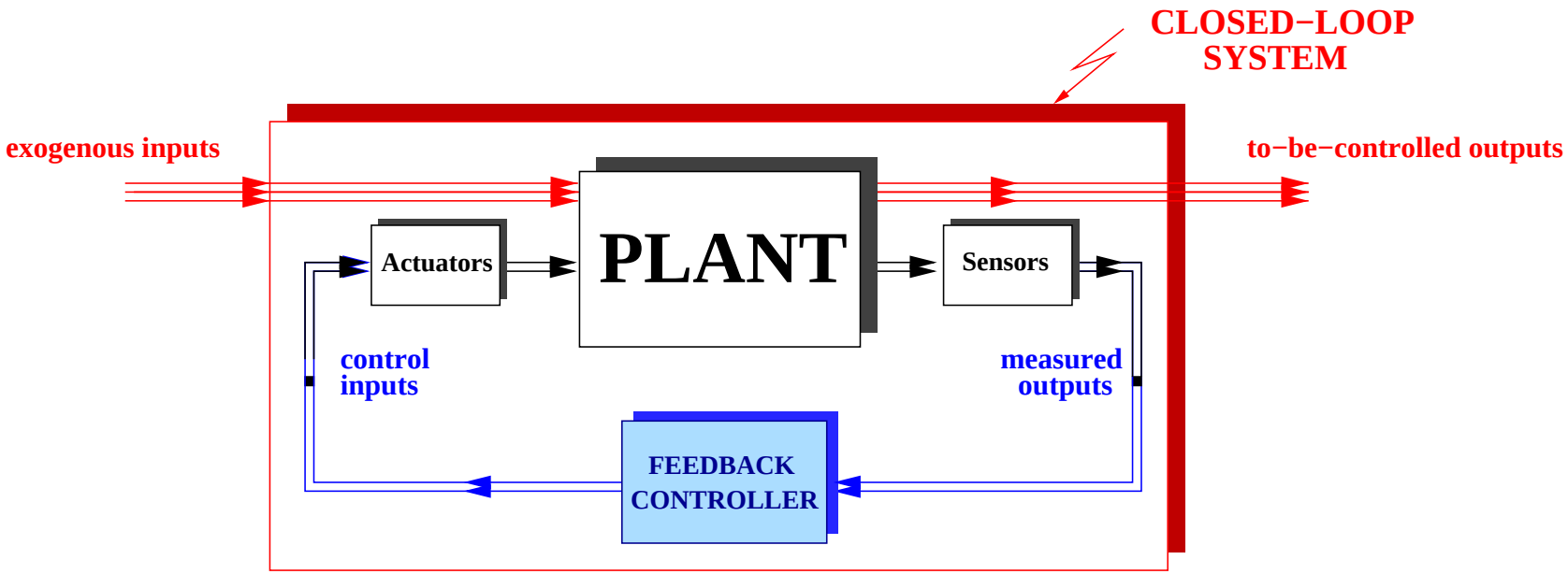
$$\frac{d}{dt}x = f(x,u)$$

Choose time-function $u(\cdot) : [0,T] \to \mathbb{U}$ so as to achieve (optimal) state transfer.

'open loop control'



Decision making



Choose map from sensor outputs to actuator inputs so as to achieve good (optimal) performance.

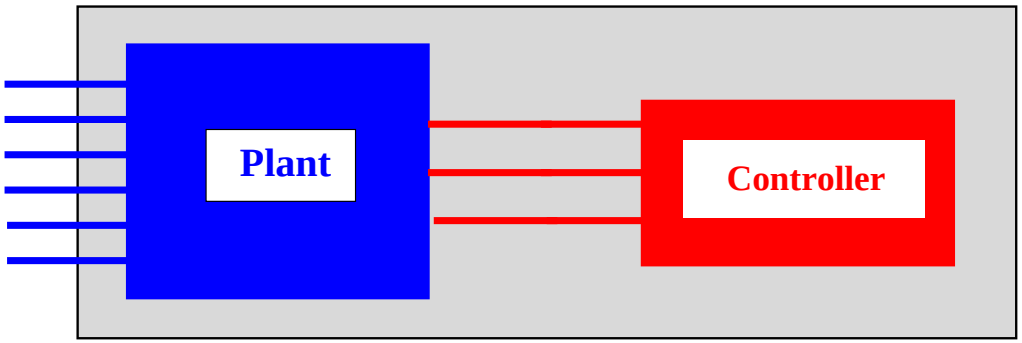
‘feedback control’

‘closed loop control’

‘intelligent control’

Embedded control

Embedded systems

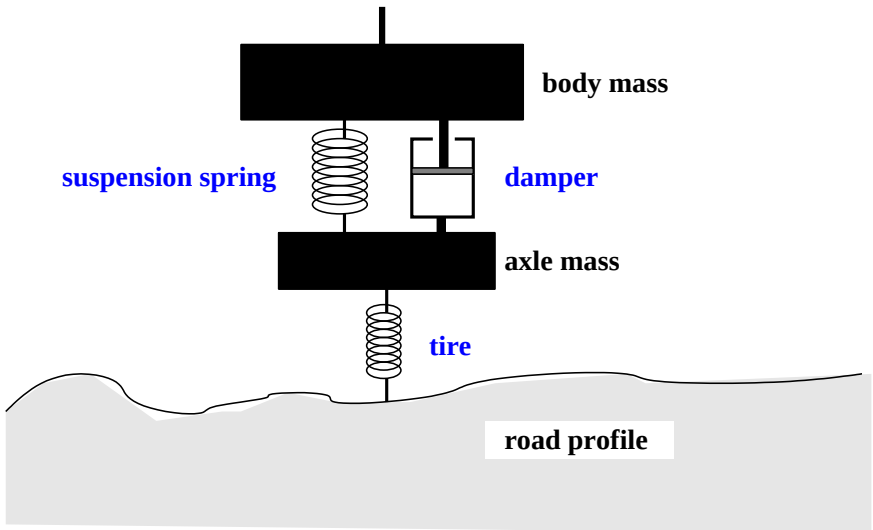


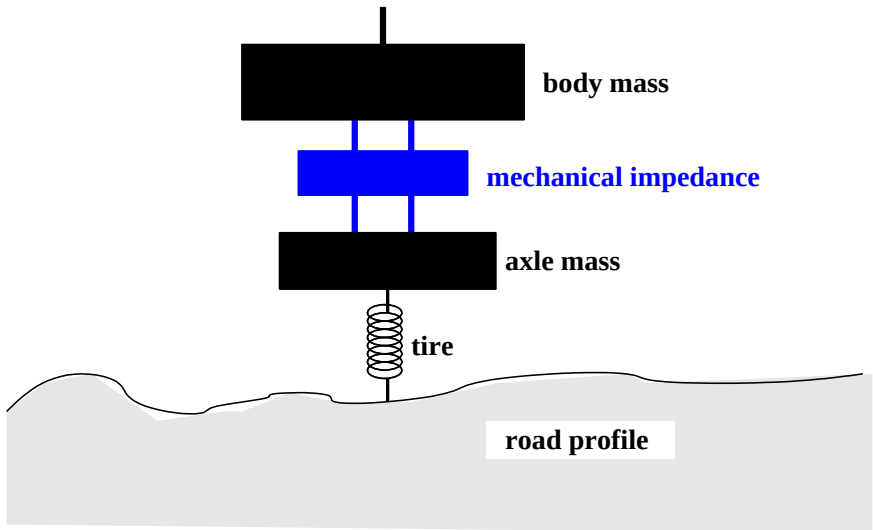
Choose controller so as to achieve good (optimal) performance of the interconnected system

‘control as interconnection’

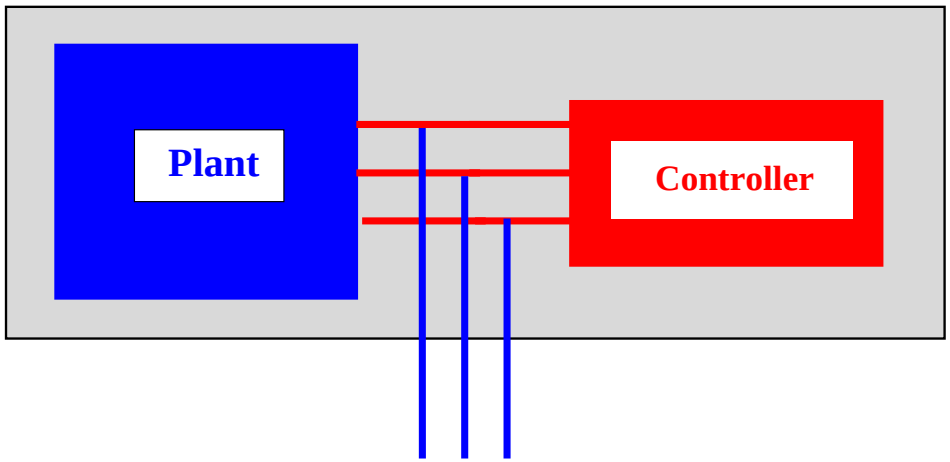
‘integrated system design’

Example


body mass
suspension spring
damper
axle mass
tire
road profile


body mass
mechanical impedance
axle mass
tire
road profile

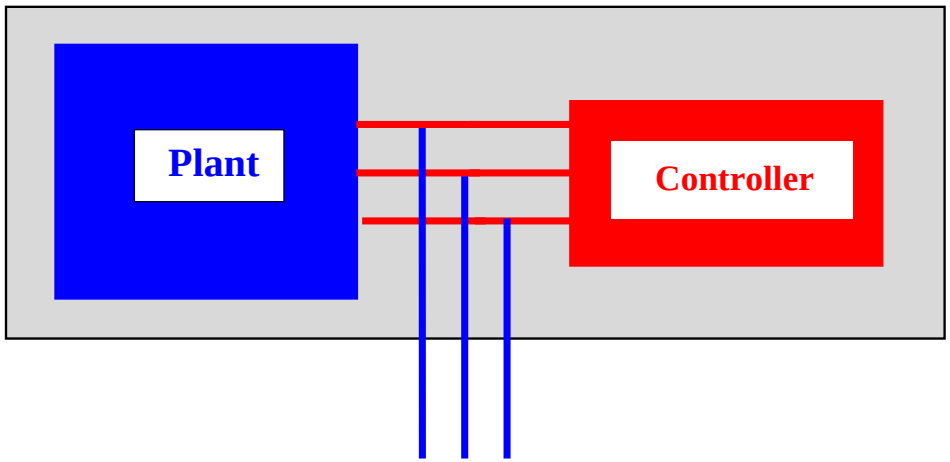
Control as interconnection



Plant behavior $\mathscr{P}$, controller behavior $\mathscr{K}$,

controlled behavior $\mathscr{P} \cap \mathscr{K}$.

Control as interconnection



Plant behavior $\mathscr{P}$, controller behavior $\mathscr{K}$,

controlled behavior $\mathscr{P} \cap \mathscr{K}$.

$$\frac{d^2y}{dt^2} + (1+a)y = 0$$

Robustness

Robust stability, but $||\frac{1}{s} - \frac{1}{s+a}|| = \infty||$**.**

Robustness

Viewing plant as a behavior, rather than i/o map $\rightsquigarrow$

Robustness: Given $\mathscr{P}$, stabilized by $\mathscr{K}$, how close to $\mathscr{P}$ needs $\mathscr{P}'$ be to be also stabilized by $\mathscr{K}$?

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$$\text{`gap'}\,(\mathscr{P},\mathscr{P}') < \text{`margin'}\,(\mathscr{P},\mathscr{K})$$

Robustness

‘gap’ $(\mathscr{P}, \mathscr{P}') <$ ‘margin’ $(\mathscr{P}, \mathscr{K})$

Overview

Conclusions

- **Interconnection = variable (terminal) sharing**
- **Modeling by physical systems proceeds by tearing, zooming, and linking**
- **Hierarchical procedure**

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- **The behavioral approach & its view of system interconnection are a pedagogical 'must'**

Behavioral systems

- **Gets the physics right**

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- Advantages in SYSID, etc.
- Far easier pedagogically
- ...

Details & copies of frames are available from/at

Jan.Willems@esat.kuleuven.be

http://www.esat.kuleuven.be/∼jwillems

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

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