SYSTEM INTERCONNECTION

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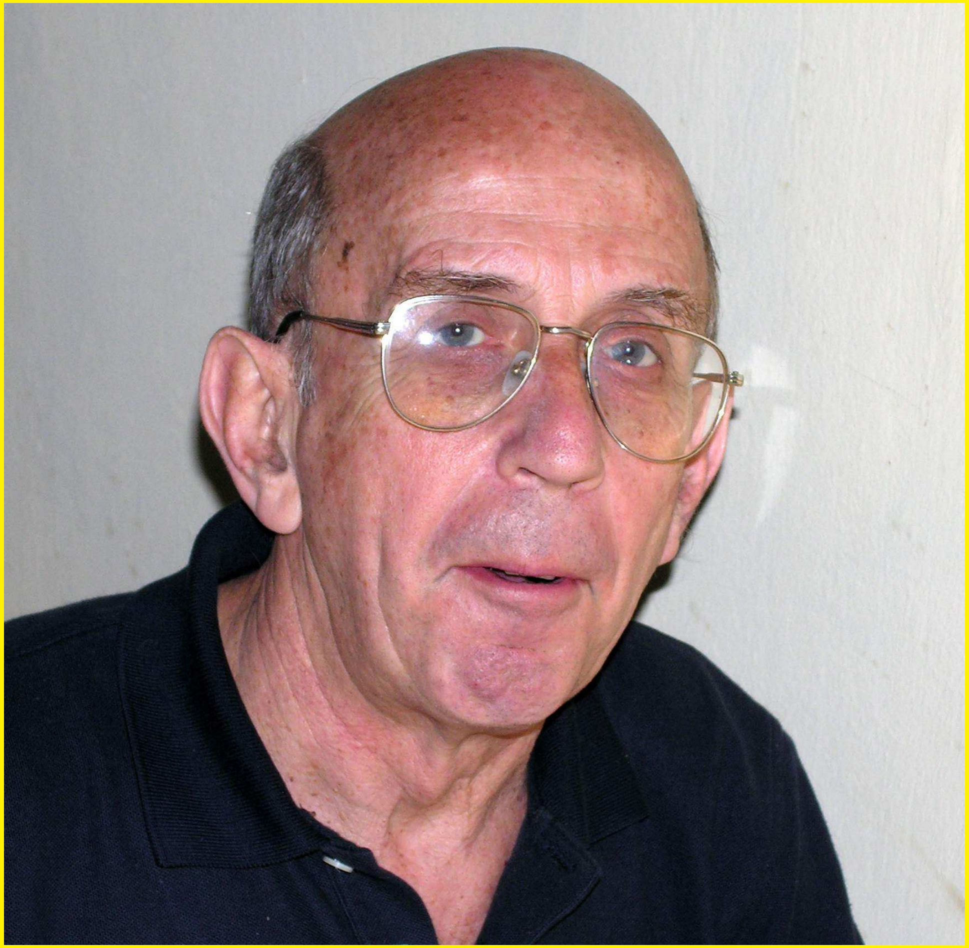
IsidoriFest **London, May 16, 2008**

On the occasion of Alberto’s 65-th



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SPECIAL ISSUE ON SYSTEM THEORY AND PHYSICS
Edited by R. W. Brockett

Papers *page*

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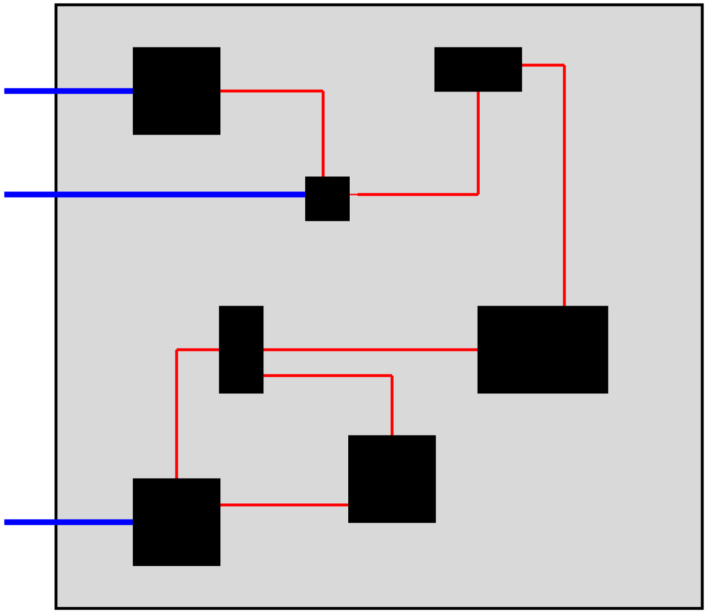
Objective



To develop a mathematical framework for dealing with interconnection of (open , dynamical) systems.

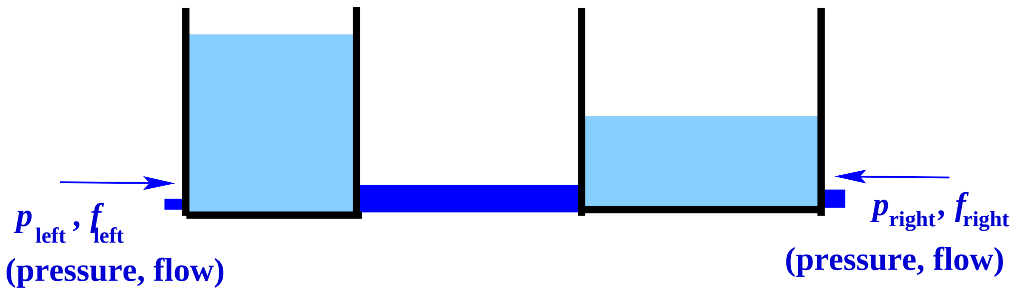
Objective

To develop a mathematical framework for dealing with interconnection of (open , dynamical) systems.

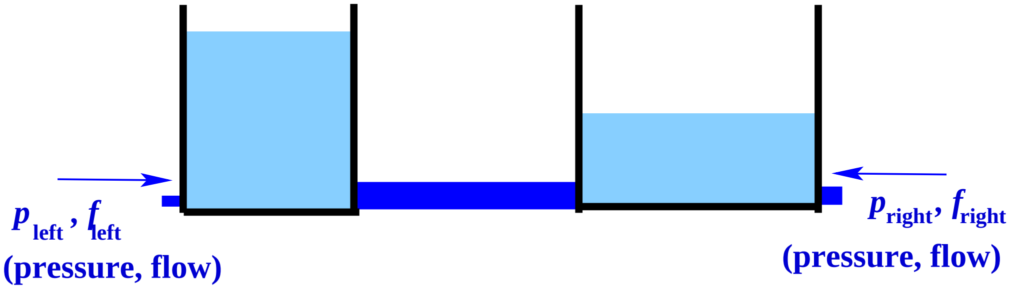
Competing philosophies:

- **input/output signal flow graphs**
- **circuit diagrams (loops, nodes)**
- **bond graphs (across, through, power)**
- **object-oriented modeling (SPICE, Modelica, ...)**
- **...**

A simple example



p left , f left
(pressure, flow)
p right, f right
(pressure, flow)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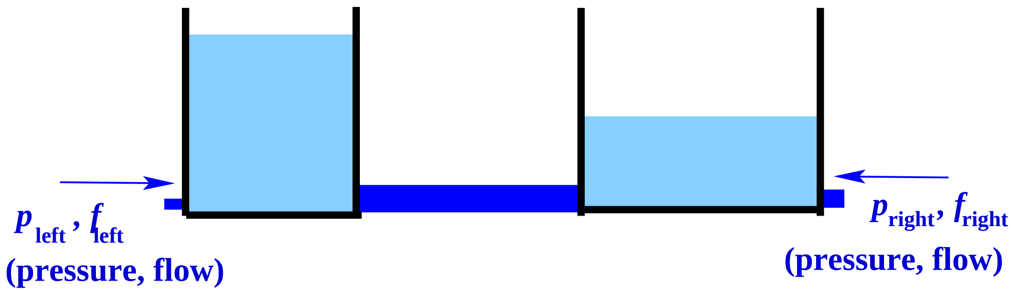
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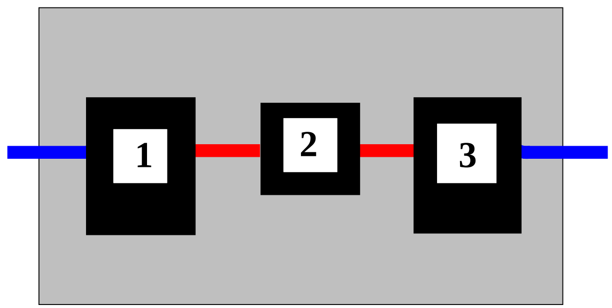
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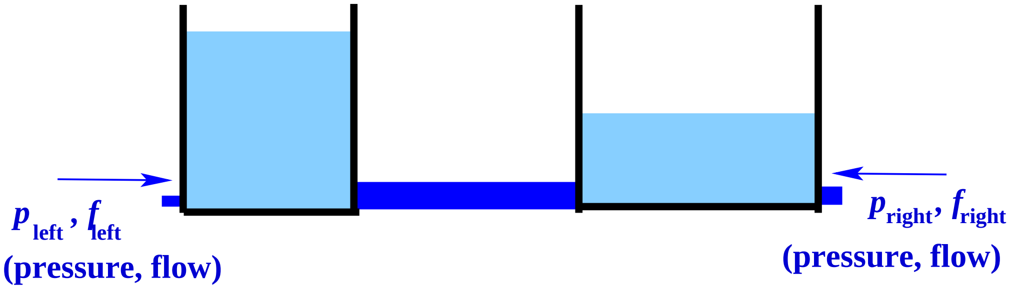
2
3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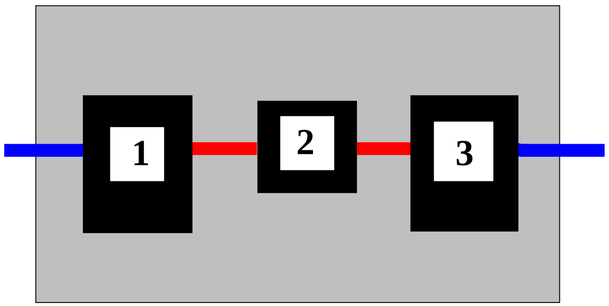
(pressure, flow)
(pressure, flow)

black box 1 & 3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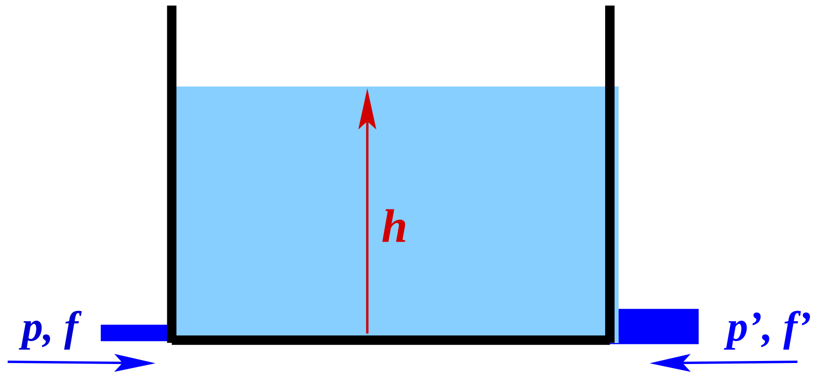
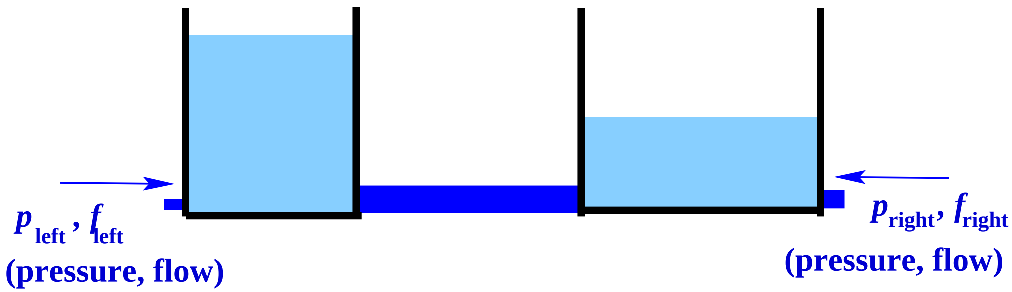
3


h
p, f
p', f'



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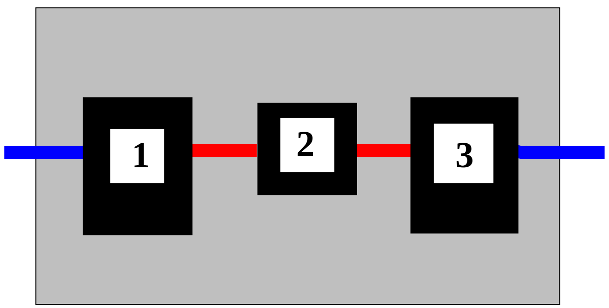
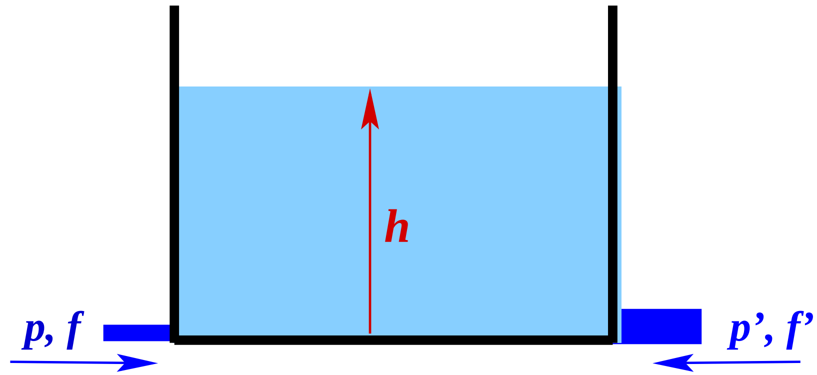
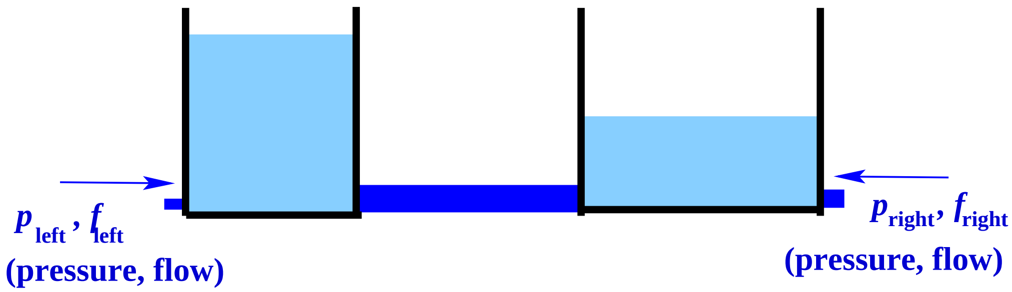
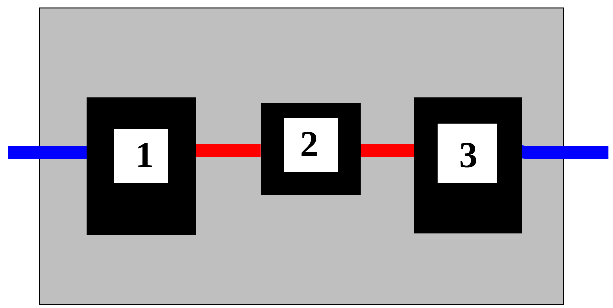


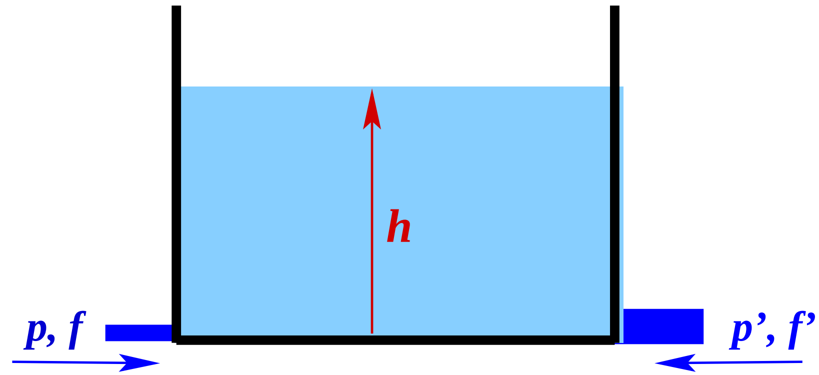


black box 2



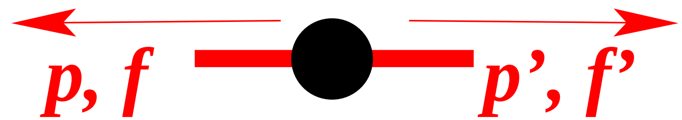


black box 1 & 3



black box 2

interconnections



Black box 1

$$A_1 \frac{d}{dt} h_1 = f_1 + f_1'$$

$$B_1 f_1 = \begin{cases} \sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \geq \rho h_1 \\ -\sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \leq \rho h_1 \end{cases}$$

$$C f_1' = \begin{cases} \sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \geq \rho h_1 \\ -\sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \leq \rho h_1 \end{cases}$$

Black box 2

$$f_2 = -f_2', \quad p_2 - p_2' = \alpha f_2$$

Black box 3

$$A_3 \frac{d}{dt} h_3 = f_3 + f_3'$$

$$C f_3 = \begin{cases} \sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \geq \rho h_3 \\ -\sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \leq \rho h_3 \end{cases}$$

$$C_3 f_3' = \begin{cases} \sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \geq \rho h_3 \\ -\sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \leq \rho h_3 \end{cases}$$

Interconnection laws

$$p_1' = p_2, \quad f_1' + f_2 = 0, \quad p_2' = p_3, \quad f_2' + f_3 = 0$$

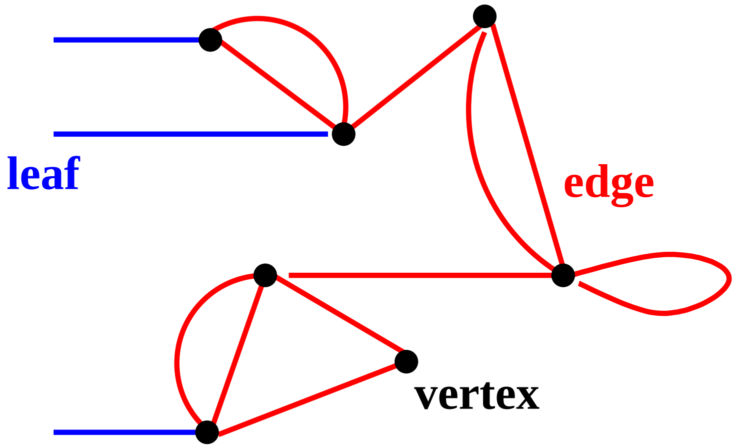
Variables of interest

$$p_{\text{left}} = p_1, \quad f_{\text{left}} = f_1, \quad p_{\text{right}} = p_3', \quad f_{\text{right}} = f_3'$$

Formalization

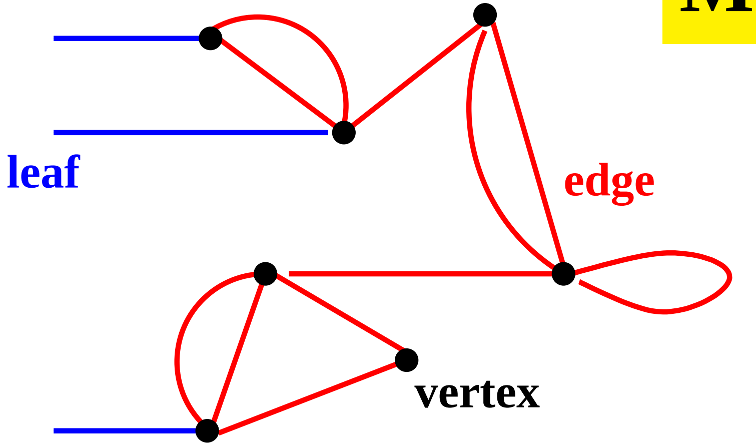
Architecture & module embedding

Architecture

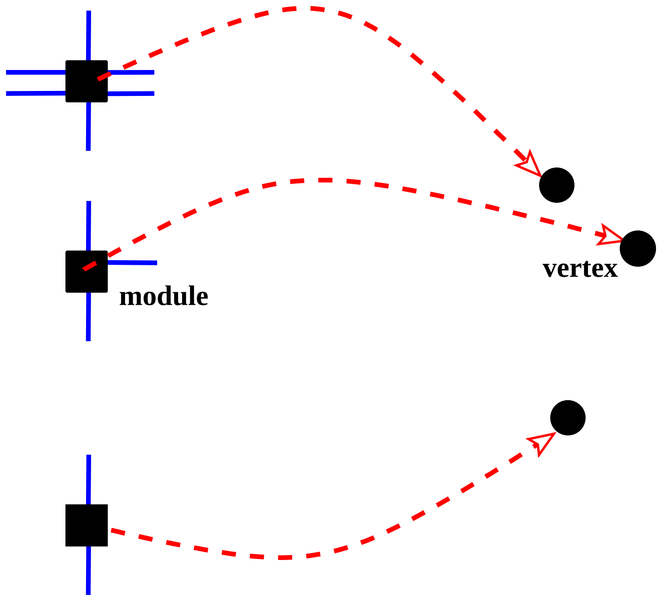


Architecture & module embedding

Architecture

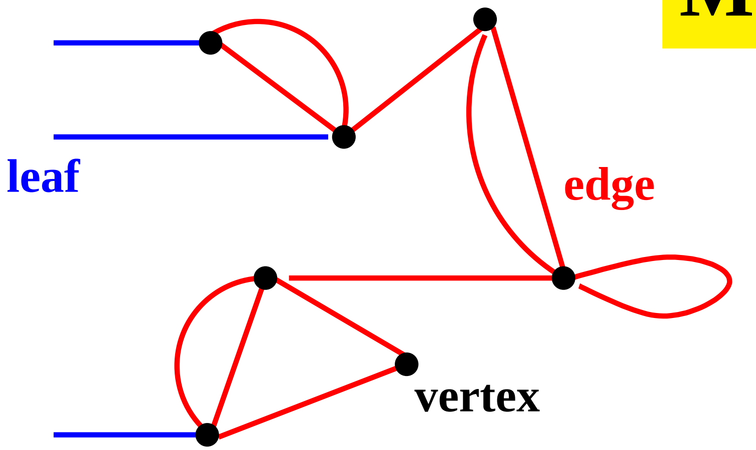


Modules (systems) in the vertices

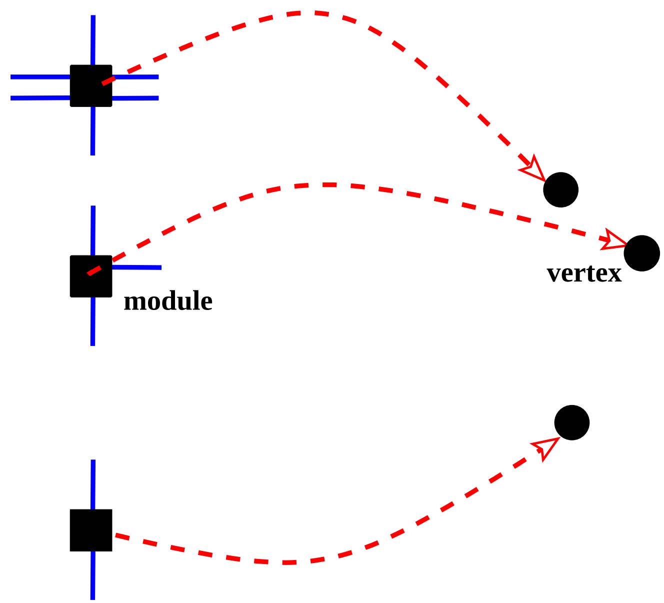


Architecture & module embedding

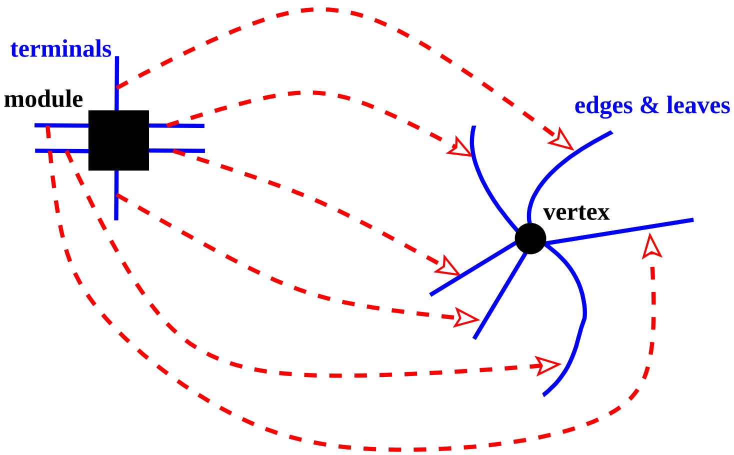
Architecture



Modules (systems) in the vertices



Terminals in the edges



Interconnection architecture

A ***graph with leaves*** **defined as** $\mathscr{G} = (\mathbb{V}, \mathbb{E}, \mathbb{L}, \mathscr{A})$

$\mathbb{V}$ **the set of *vertices*,**

$\mathbb{E}$ **the set of *edges*,**

$\mathbb{L}$ **the set of *leaves*,**

$\mathscr{A}$ **the *adjacency map*.**

$\mathscr{A}$ **associates**

with each edge $e \in \mathbb{E}$ **an unordered pair**

$$\mathscr{A}(e) = [v_1, v_2] \quad v_1, v_2 \in \mathbb{V},$$

with each leaf $\ell \in \mathbb{L}$ **an element** $\mathscr{A}(\ell) = v \in \mathbb{V}$**.**

Module embedding

The *module embedding* associates

a module with each vertex,

a $1 \leftrightarrow 1$ assignment between the

edges and leaves adjacent to the vertex and

the terminals of the module.

Module embedding

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a module with each vertex,

a $1 \leftrightarrow 1$ assignment between the

edges and leaves adjacent to the vertex and

the terminals of the module.

Vertices specify the subsystems,

edges how terminals of subsystems are connected,

leaves how the interconnected system interacts with the environment.

Module embedding

Vertices $\rightsquigarrow$ Subsystems

Edges $\rightsquigarrow$ Interconnections

Manifest variables

The *manifest variable assignment* is a map that assigns the manifest variables as a function of the terminal (or, more general, the module) variables.

The terminal variables are henceforth considered as latent variables.

Behavioral equations

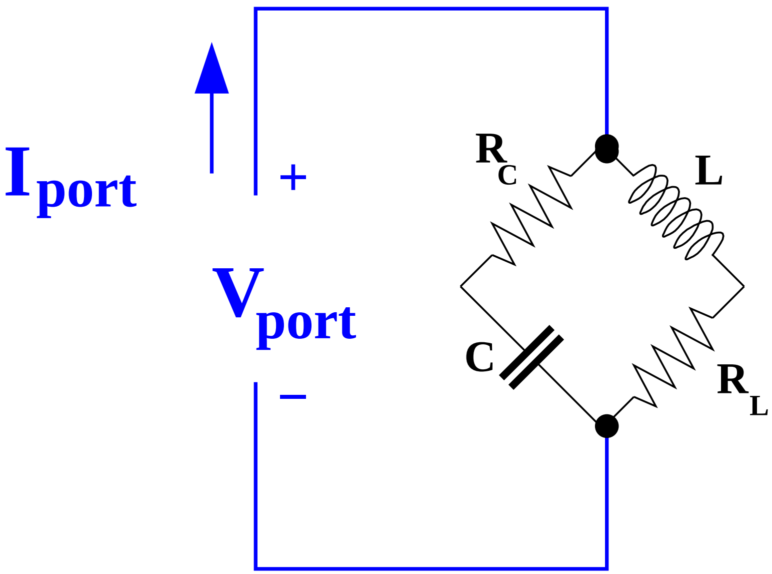
1. **Module equations** for each vertex.
   Relation among the variables on the terminals of the subsystems.

2. **Interconnection equations** for each edge.
   Equating the variables on the terminals associated with the same edge.

3. **Manifest variable assignment**
   Specifies the variables of interest.

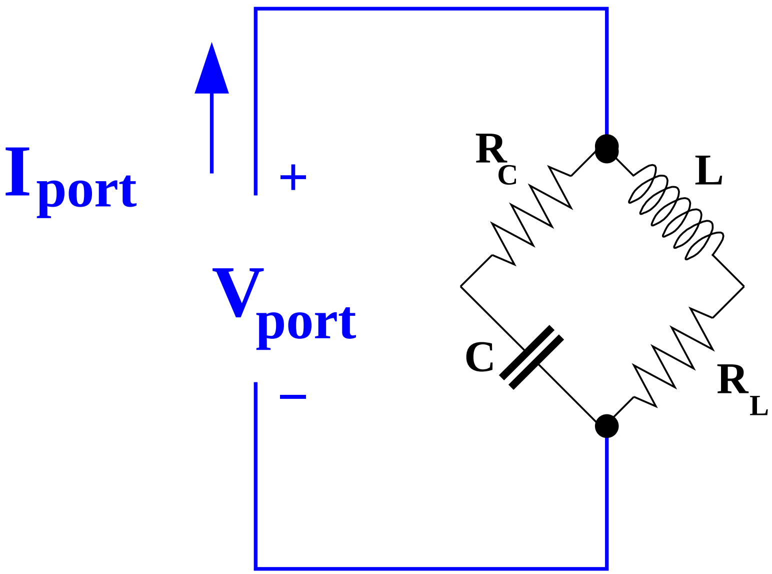
A very classical example

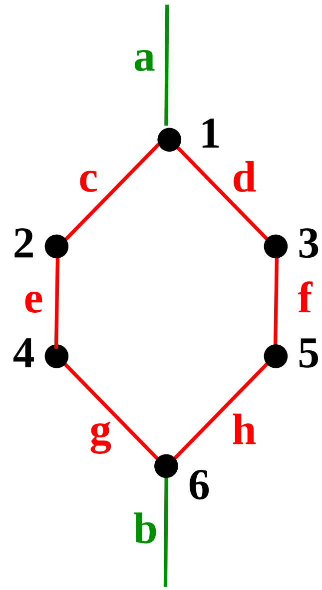
RLC circuit



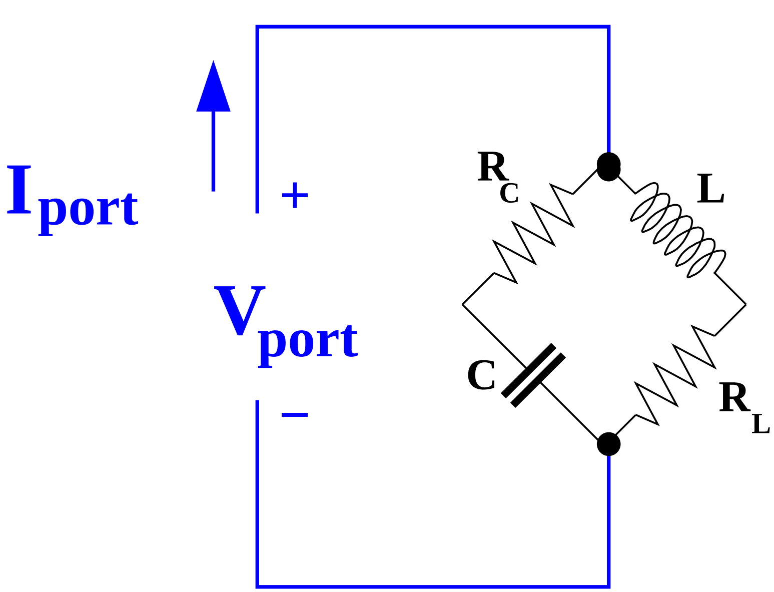
¡¡¡ Model the port behavior !!!

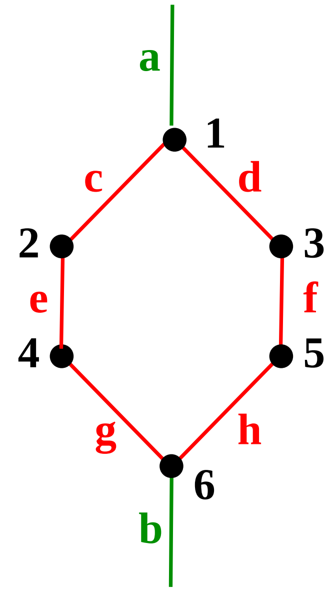
RLC circuit





RLC circuit





$$R_C \mapsto 2, R_L \mapsto 5, C \mapsto 4, L \mapsto 3, \text{connector}_1 \mapsto 1, \text{connector}_2 \mapsto 6,$$

$$1_{R_C} \mapsto c, 2_{R_C} \mapsto e, 1_{R_L} \mapsto f, 2_{R_L} \mapsto h, 1_C \mapsto e, 2_C \mapsto g, 1_L \mapsto d, 2_L \mapsto f,$$

$$1_{connector_1} \mapsto a, 2_{connector_1} \mapsto c, 3_{connector_1} \mapsto d,$$

$$1_{connector_2} \mapsto b, 2_{connector_2} \mapsto g, 3_{connector_2} \mapsto h.$$

Module equations

vertex 1

$$V_{1_{\text{connector}_1}} = V_{2_{\text{connector}_1}} = V_{3_{\text{connector}_1}},$$
$$I_{1_{\text{connector}_1}} + I_{2_{\text{connector}_1}} + I_{3_{\text{connector}_1}} = 0;$$

vertex 2

$$V_{1_{R_C}} - V_{2_{R_C}} = R_C I_{1_{R_C}},\ I_{1_{R_C}} + I_{2_{R_C}} = 0;$$

vertex 3

$$L\frac{d}{dt}I_{I_L} = V_{1_L} - V_{2_L},\ I_{1_L} + I_{2_L} = 0;$$

vertex 4

$$C\frac{d}{dt}\left(V_{1_C} - V_{2_C}\right) = I_{1_C},\ I_{1_C} + I_{2_C} = 0;$$

vertex 5

$$V_{1_{R_L}} - V_{2_{R_L}} = R_L I_{1_{R_L}},\ I_{1_{R_L}} + I_{2_{R_L}} = 0;$$

vertex 6

$$V_{1_{\text{connector}_2}} = V_{2_{\text{connector}_2}} = V_{3_{\text{connector}_2}},$$
$$I_{1_{\text{connector}_2}} + I_{2_{\text{connector}_2}} + I_{3_{\text{connector}_2}} = 0.$$

Interconnection equations

edge c $V_{1_{R_C}} = V_{2_{\text{connector}_1}},\ I_{1_{R_C}} + I_{2_{\text{connector}_1}} = 0;$

edge d $V_{1_L} = V_{3_{\text{connector}_1}},\ I_{1_L} + I_{3_{\text{connector}_1}} = 0;$

edge e $V_{2_{R_C}} = V_{1_C},\ I_{2_{R_C}} + I_{1_C} = 0;$

edge f $V_{2_L} = V_{1_{R_C}},\ I_{2_L} + I_{1_{R_C}} = 0;$

edge g $V_{2_C} = V_{1_{\text{connector}_2}},\ I_{2_C} + I_{1_{\text{connector}_2}} = 0;$

edge h $V_{2_{R_L}} = V_{2_{\text{connector}_2}},\ I_{2_{R_L}} + I_{2_{\text{connector}_2}} = 0.$

Manifest variable assignment

$$V_{\text{external port}} = V_{1_{\text{connector}_1}} - V_{3_{\text{connector}_2}}$$

$$I_{\text{external port}} = I_{1_{\text{connector}_1}}$$

Manifest variable assignment

$$V_{\text{external port}} = V_{1_{\text{connector}_1}} - V_{3_{\text{connector}_2}}$$

$$I_{\text{external port}} = I_{1_{\text{connector}_1}}$$

The module equations

+ the interconnection constraints

+ the manifest variable assignment

form the complete model for the behavior of

$$(V_{\text{external port}}, I_{\text{external port}})$$

Prevalence of latent variables $\rightsquigarrow$ elimination theory.

Manifest variable assignment

$$V_{\text{external port}} = V_{1_{\text{connector}_1}} - V_{3_{\text{connector}_2}}$$

$$I_{\text{external port}} = I_{1_{\text{connector}_1}}$$

Behavior = all

$$\left(V_{\text{external port}}, I_{\text{external port}}\right) : \quad \mathbb{R} \longrightarrow \mathbb{R}^2$$

$\exists \quad \ldots, V_{1_{R_C}}, \ldots, I_{3_{\text{connector}_2}} \mathbb{R} \longrightarrow \mathbb{R}^{\cdots}$ **such that ...**

Manifest behavior

$\rightsquigarrow$ **the dynamical system** $\Sigma = \left(\mathbb{R}, \mathbb{R}^2, \mathscr{B}\right)$ **with behavior** $\mathscr{B}$ **specified by:**

Case 1: $\quad CR_C \neq \dfrac{L}{R_L}$

$$\boxed{\left(\frac{R_C}{R_L} + \left(1 + \frac{R_C}{R_L}\right) CR_C \frac{d}{dt} + CR_C \frac{L}{R_L} \frac{d^2}{dt^2}\right) V = \left(1 + CR_C \frac{d}{dt}\right) \left(1 + \frac{L}{R_L} \frac{d}{dt}\right) R_C I}$$

Case 2: $\quad CR_C = \dfrac{L}{R_L}$

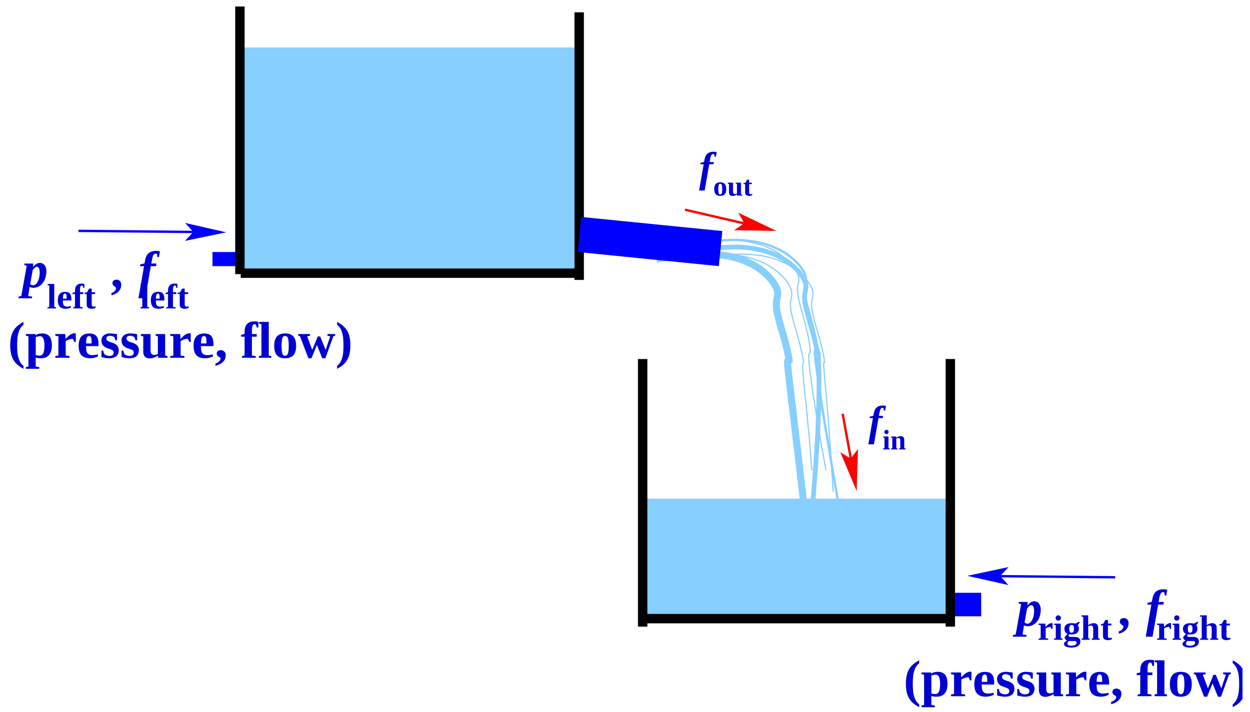
$$\boxed{\left(\frac{R_C}{R_L} + CR_C \frac{d}{dt}\right) V = \left(1 + CR_C\right) \frac{d}{dt} R_C I}$$

$\rightsquigarrow$ **behavior** $\mathscr{B}$ **= all solutions** $(V, I) : \mathbb{R} \rightarrow \mathbb{R}^2$

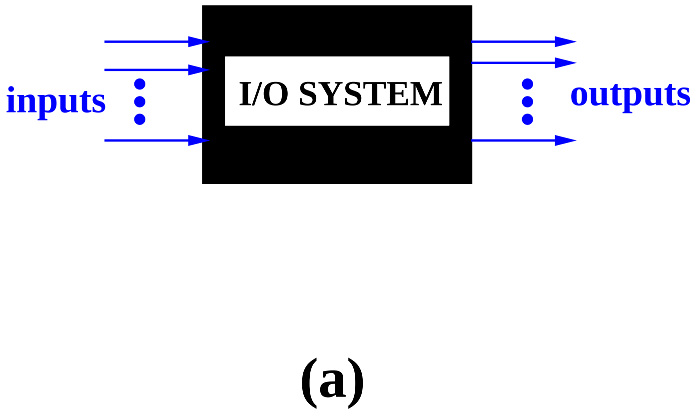
Other methodologies

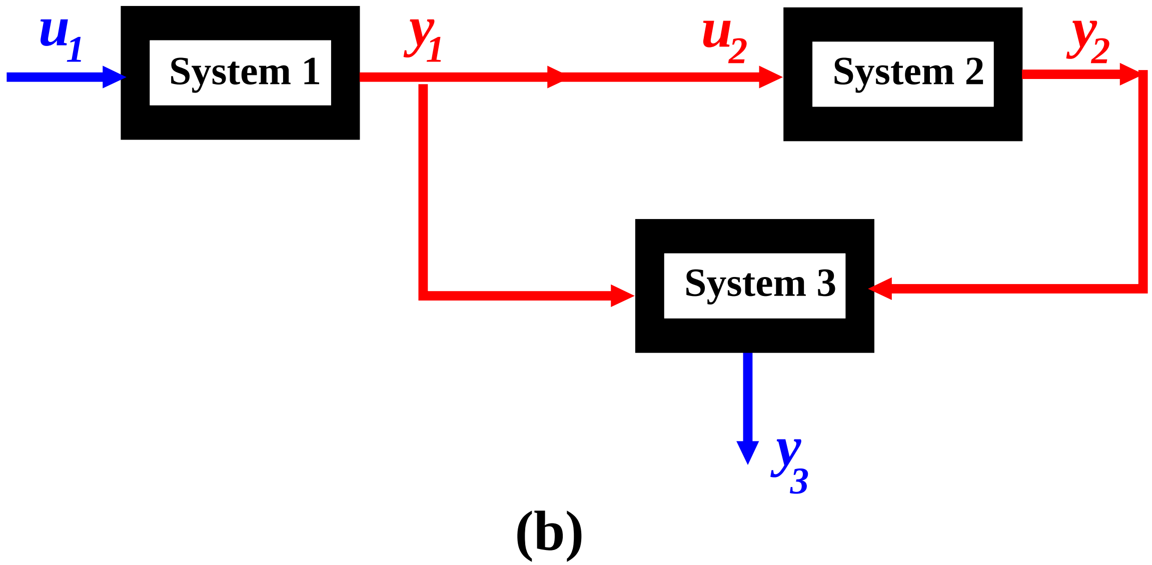
Signal flow graphs

There are many many examples where output-to-input connection is eminently natural:



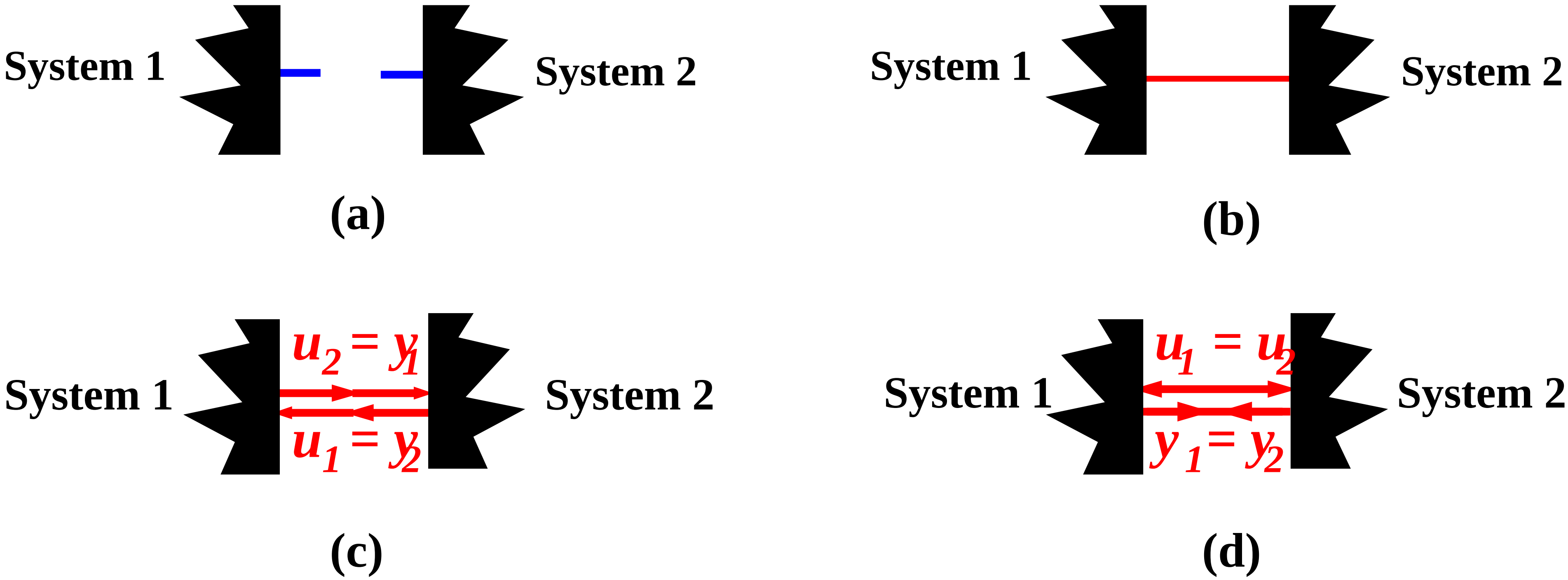
input/output thinking





- **shows terminal variables separate**
- **suggests that inputs and outputs occur at different points**
- **allows impossible input-output connections**

input/output thinking



For physical systems

input-to-input & output-to-output

assignment very prevalent.

Physical systems are not signal processors.

input/output thinking

"Block diagrams unsuitable for serious physical modeling

- the control/physics barrier"

"Behavior based (declarative) modeling is a good alternative"

from K.J. Åström, ***Present Developments in Control Applications***

IFAC 50-th Anniversary Celebration

Heidelberg, September 12, 2006.

Bond graphs

Interconnection variables:

a flow and an effort **product = power**

- **current & voltage**
- **velocity & force**
- **mass flow & pressure**
- **heat flow& temperature**
  $\frac{\text{heat flow}}{\text{temperature}}$ **& temperature**
- ...

Bond graphs

Interconnection variables:

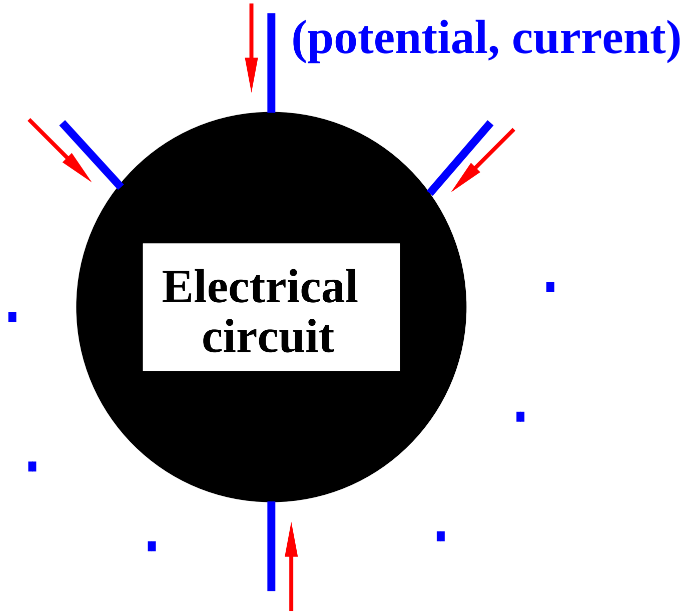
a flow and an effort **product = power**

1. **Mechanical interconnections equate positions, not velocities**

2. **Not all interconnections involve equating energy transfer**

3. **Terminals are for interconnection, ports are for energy transfer**

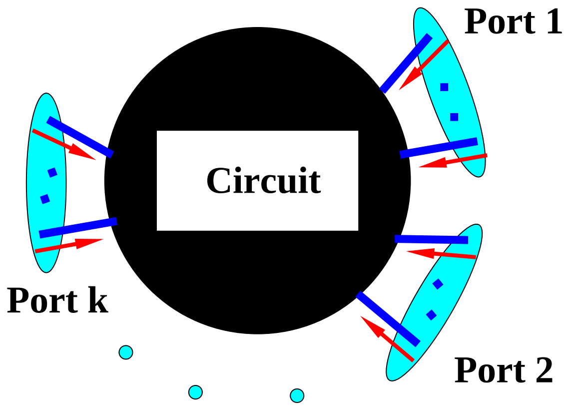
Terminals versus ports



Terminal variables and behavior:

$$(V_1, I_1, V_2, I_2, \ldots, V_n, I_n) \rightsquigarrow \textbf{ behavior } \mathscr{B} \subseteq \left(\mathbb{R}^{2n}\right)^{\mathbb{R}}$$

Terminals versus ports



Port :⇔ **sum currents = 0**

potentials + constant ⇒ potentials

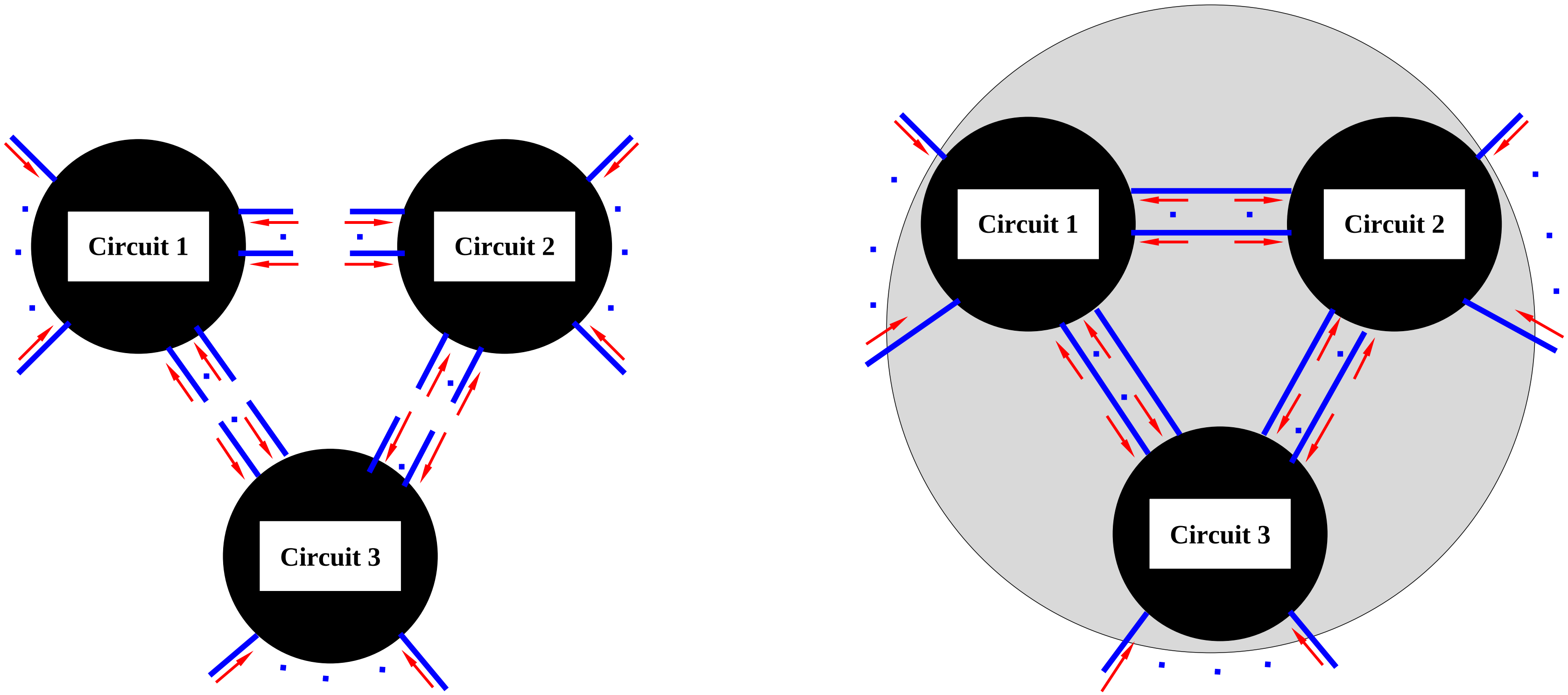
$$\left(\boxed{V_1, I_1 \ldots, V_{\mathrm{p}}, I_{\mathrm{p}}}, V_{\mathrm{p}+1}, \ldots, I_{\mathrm{n}}\right) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$$

$$\Downarrow$$

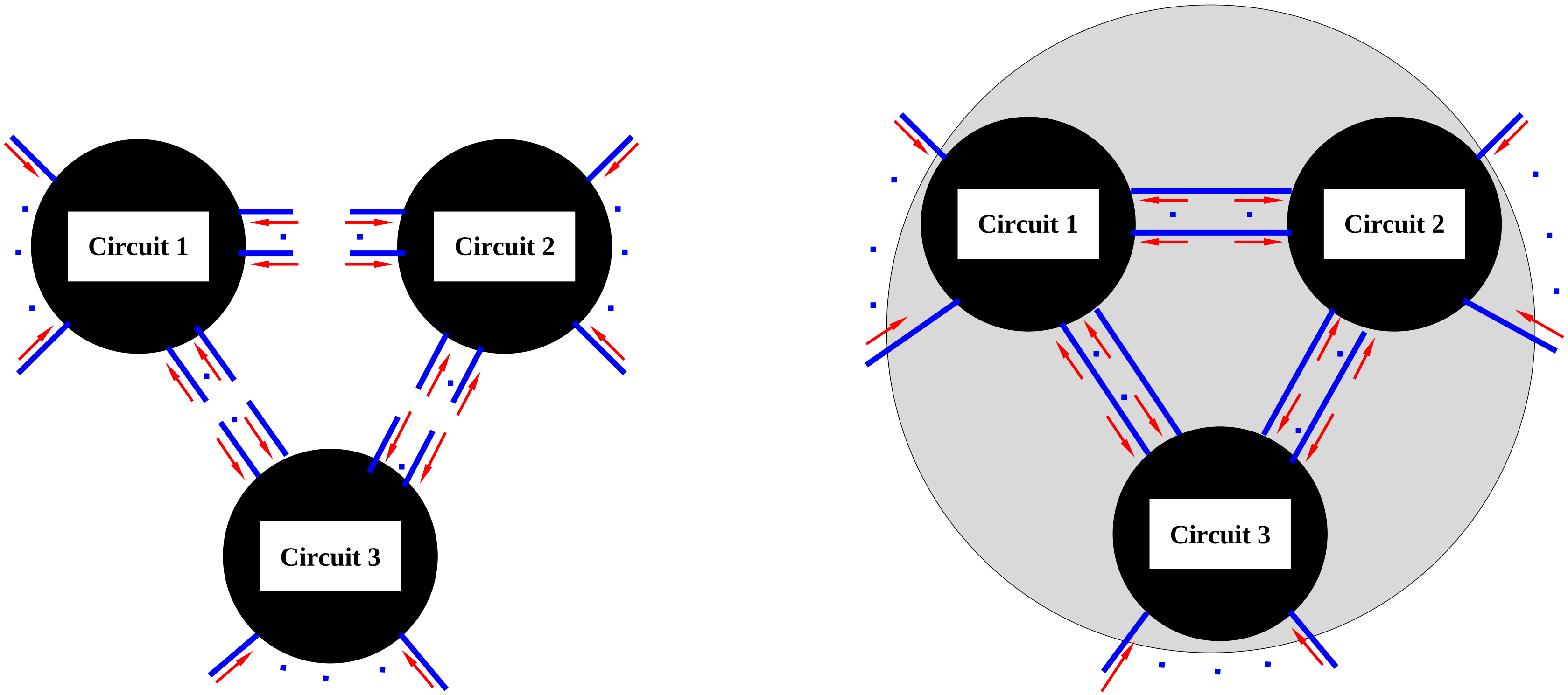
$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_{\mathrm{p}} + \alpha, I_{\mathrm{p}}}, V_{\mathrm{p}+1}, \ldots, I_{\mathrm{n}}\right) \in \mathscr{B}$$

$$\boxed{I_1 + \cdots + I_{\mathrm{p}}} = 0$$

Terminals versus ports



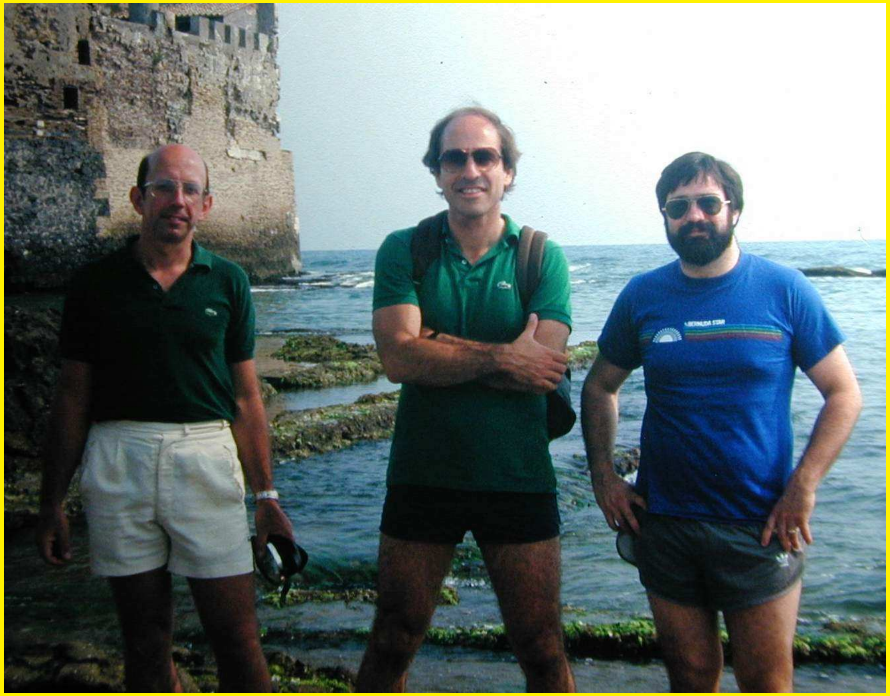
Terminals versus ports



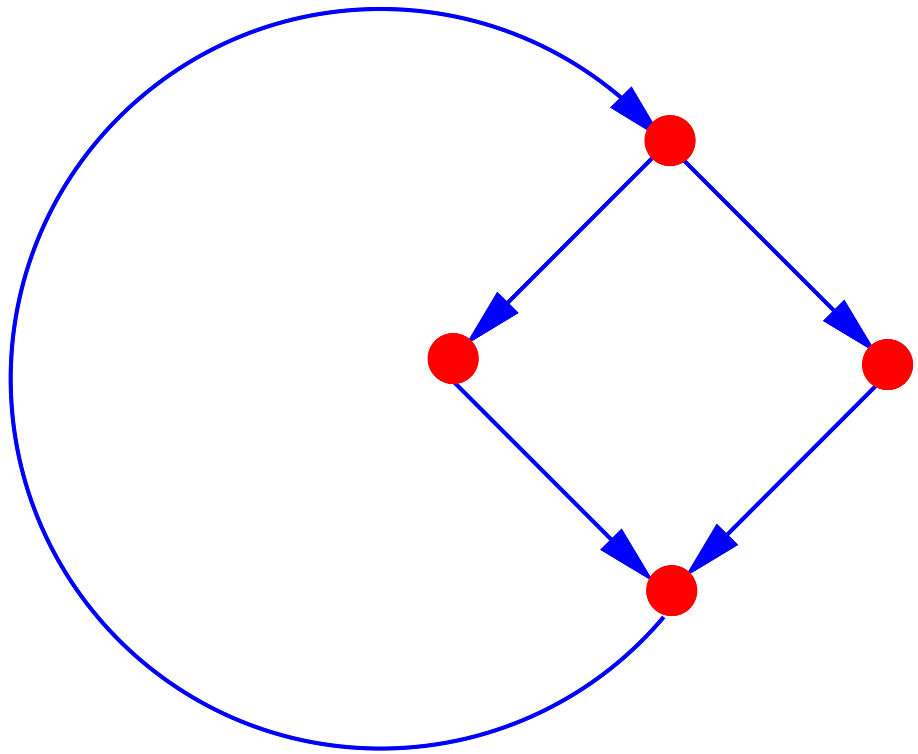
Interconnection via terminals, energy transfer via ports; one cannot talk about

“the energy transferred from circuit 1 to circuit 2”

Circuit diagrams



Circuit diagrams

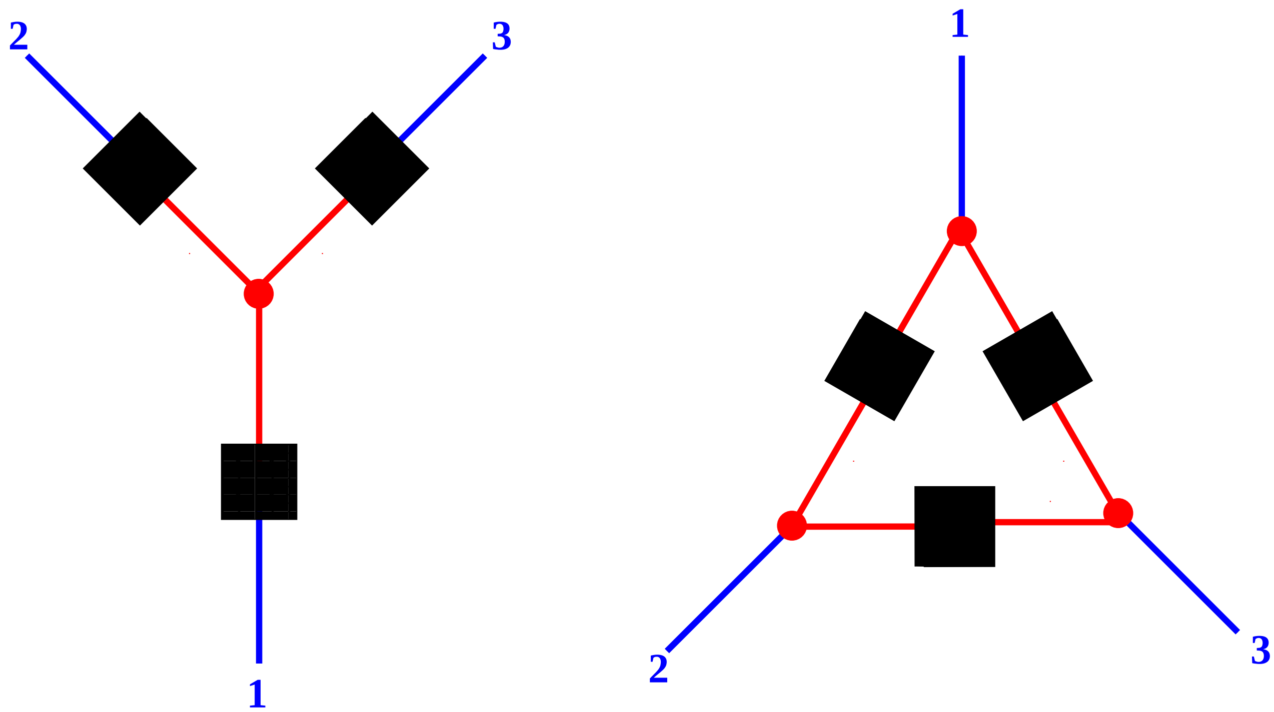


Circuit diagrams with

nodes & branches & KVL & KCL

are only effective with 2-terminal 1-ports.

Not closed under composition



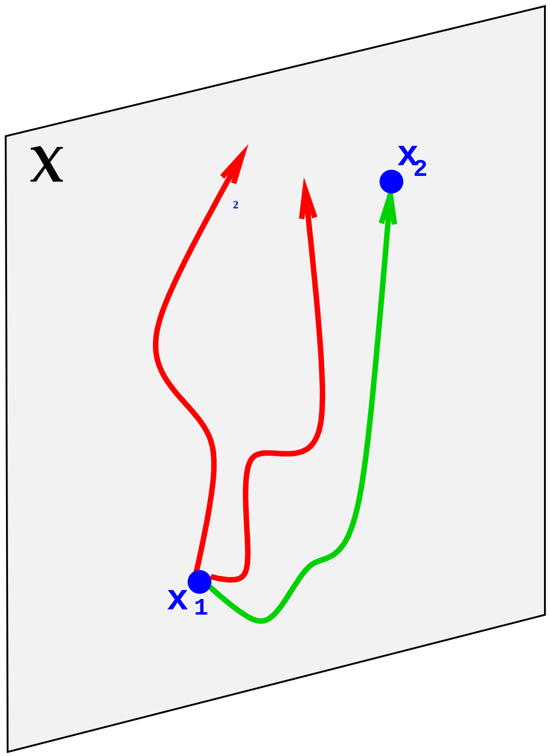
Various facets of control

Path planning

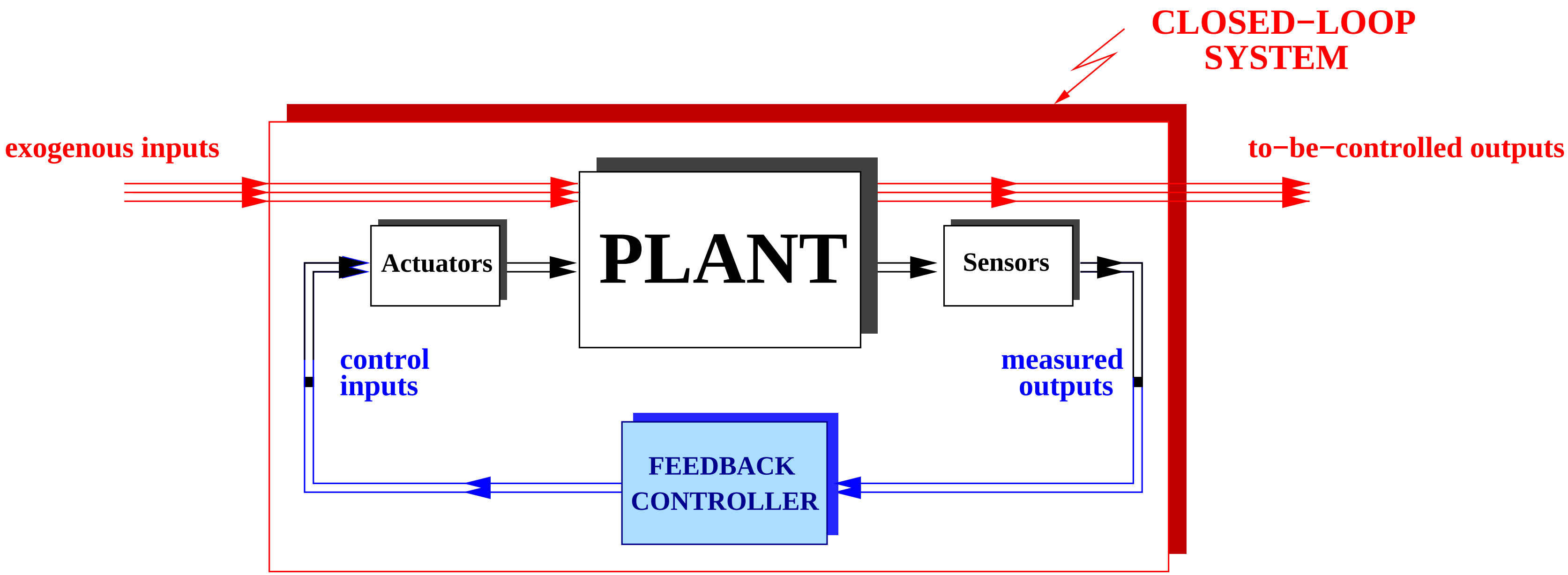
$$\frac{d}{dt}x = f(x,u)$$

Choose time-function $u(\cdot) : [0,T] \longrightarrow \mathbb{U}$ so as to achieve (optimal) state transfer.

'open loop control'



Decision making



Choose map from sensor outputs to actuator inputs so as to achieve good (optimal) performance.

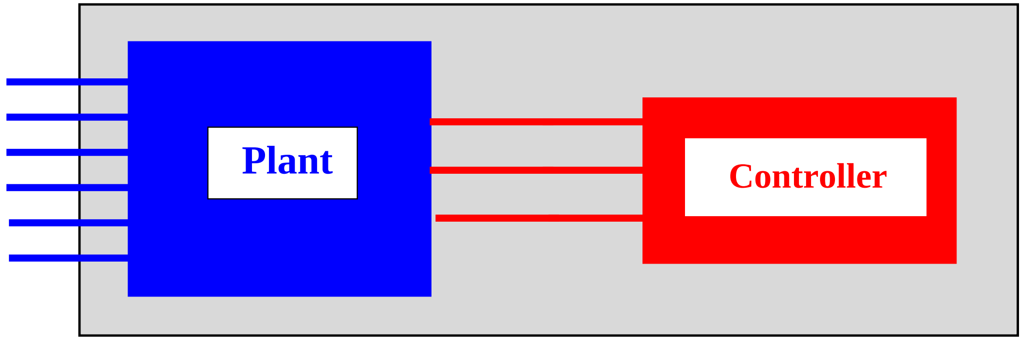
'feedback control'

'closed loop control'

'intelligent control'

Embedded control

Embedded systems

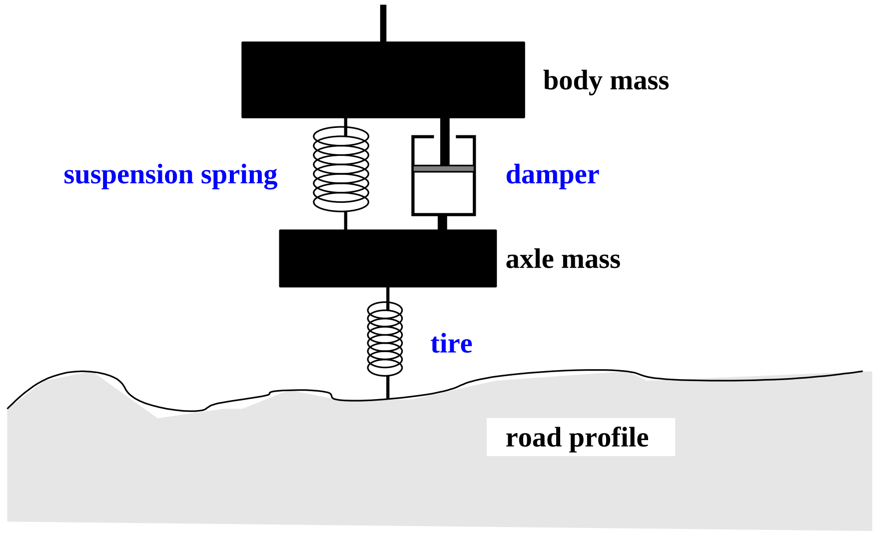


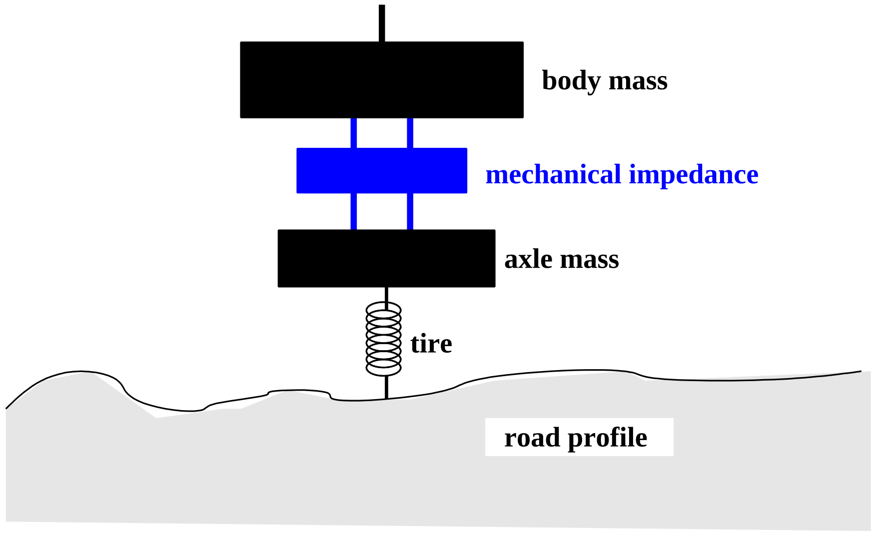
Choose controller so as to achieve good (optimal) performance of the interconnected system

'control as interconnection'

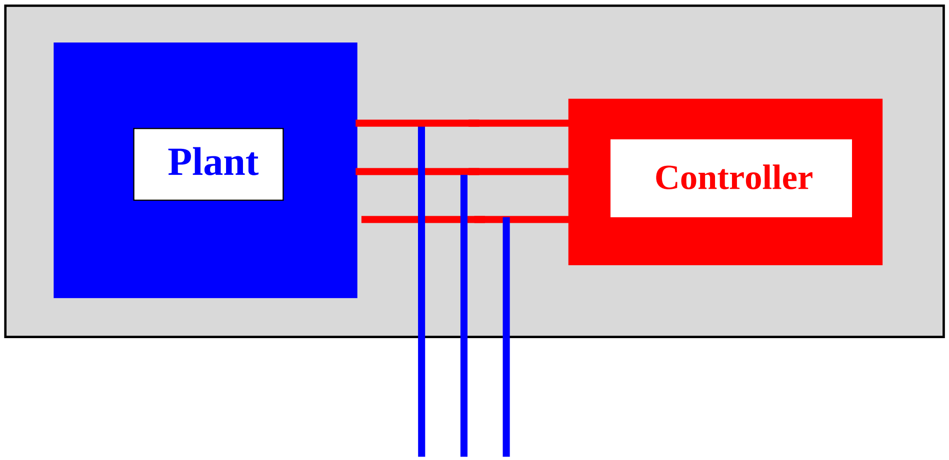
'integrated system design'

Example


body mass
suspension spring
damper
axle mass
tire
road profile


body mass
mechanical impedance
axle mass
tire
road profile

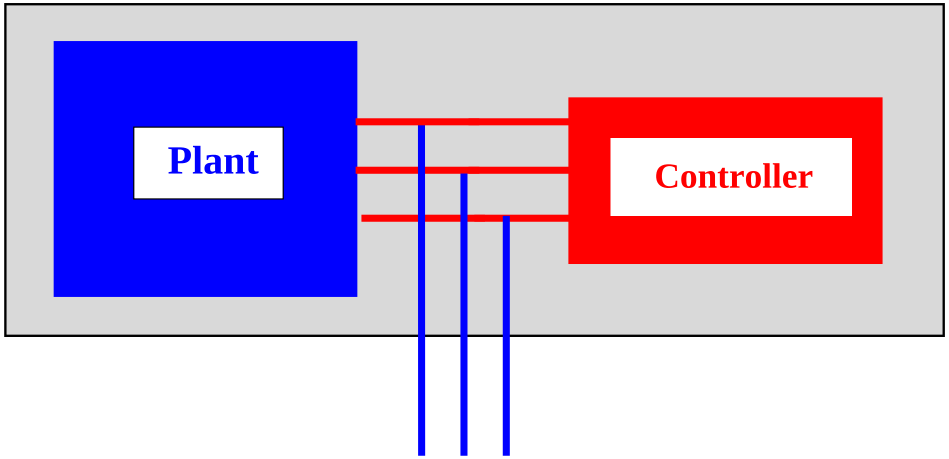
Control as interconnection



Plant behavior $\mathscr{P}$, controller behavior $\mathscr{K}$,

controlled behavior $\mathscr{P} \cap \mathscr{K}$.

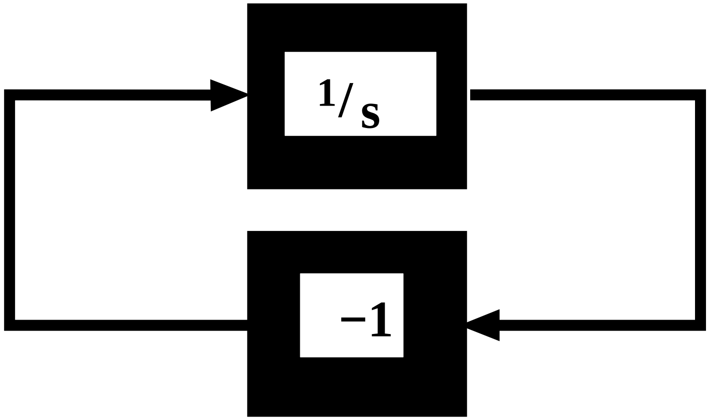
Control as interconnection



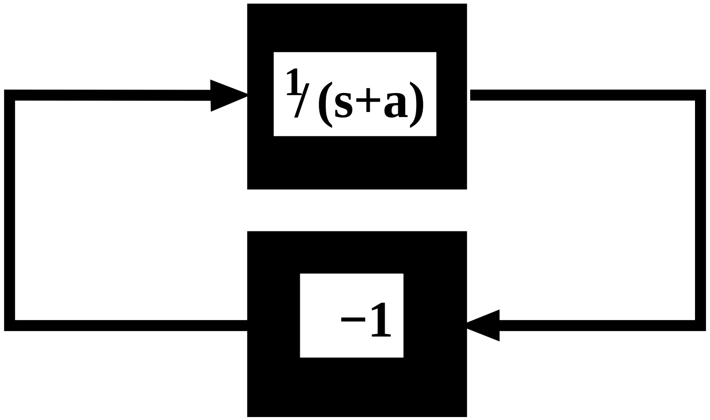
Plant behavior $\mathscr{P}$, controller behavior $\mathscr{K}$,

controlled behavior $\mathscr{P} \cap \mathscr{K}$.

Robustness



$$\frac{d}{dt}y + y = 0$$



$$\frac{d}{dt}y + (1+a)y = 0$$

Robust stability, but $||\frac{1}{s} - \frac{1}{s+a}|| = \infty||$**.**

Robustness

Viewing plant as a behavior, rather than i/o map $\rightsquigarrow$

Robustness: Given $\mathscr{P}$, stabilized by $\mathscr{K}$, how close to $\mathscr{P}$ needs $\mathscr{P}'$ be to be also stabilized by $\mathscr{K}$?

Robustness

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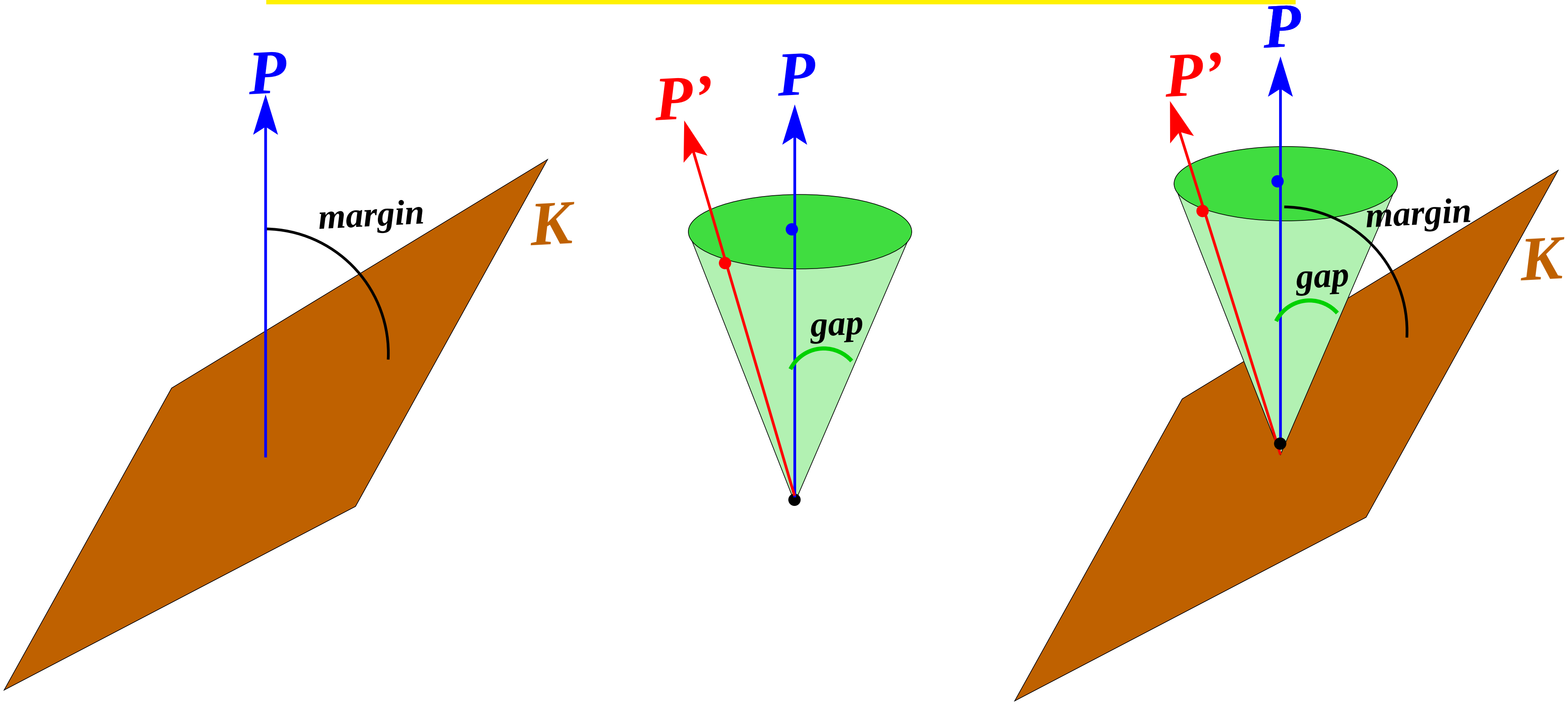
$$\textbf{‘gap’}\,(\mathscr{P},\mathscr{P}') < \textbf{‘margin’}\,(\mathscr{P},\mathscr{K})$$

Robustness

$$\text{'gap'}\,(\mathscr{P}, \mathscr{P}') < \text{'margin'}\,(\mathscr{P}, \mathscr{K})$$



Exactly approach used in robust control,

$\mathscr{L}_2-$, $\mathscr{H}_2-$ **'gap', 'ν-gap', ...**

Overview

Conclusions

- **Interconnection = variable (terminal) sharing**
- **Modeling by physical systems proceeds by tearing, zooming, and linking**
- **Hierarchical procedure**

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Conclusions

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- **Hierarchical procedure**
- **Importance of latent variables and the elimination theorem**
- **Limitations of input/output thinking**
- **The behavioral approach & its view of system interconnection are a pedagogical 'must'**

Reference: Jan C. Willems

The behavioral approach to open and interconnected systems

***Control Systems Magazine*, volume 27, pages 46 – 99, 2007**

Details & copies of the lecture frames are available from/at

`Jan.Willems@esat.kuleuven.be`

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Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Happy Birthday, Alberto !!!