INTERCONNECTED SYSTEMS

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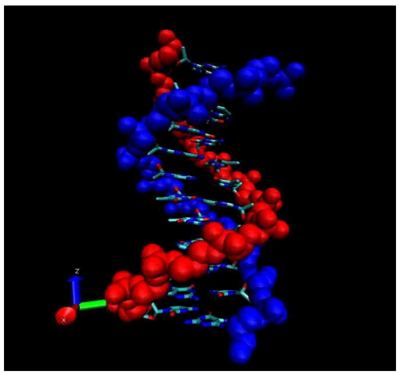
Mini-symposium, TU Delft **October 15, 2008**

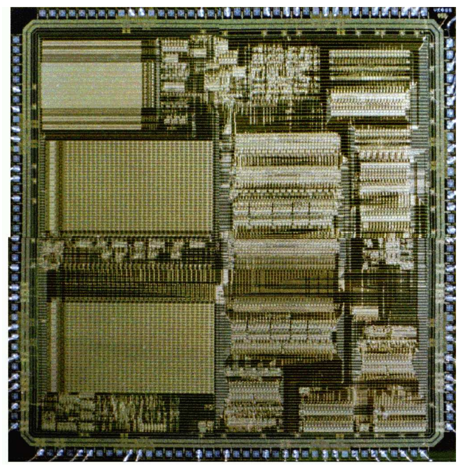
Outline

- Open, connected, and modular
- [Classical dynamical systems]
- [Input/output systems]
- Modeling by tearing, zooming, and linking
- [On canceling poles and zeros]
- [DAEs]
- Signal flow graphs
- Bond graphs
- Circuit diagrams
- [Control as interconnection]

Systems




ZYGOTE



Features

- open
- interconnected
- modular
- dynamic

Features

- **open**
- **interconnected**
- **modular**
- **dynamic**

Aim:

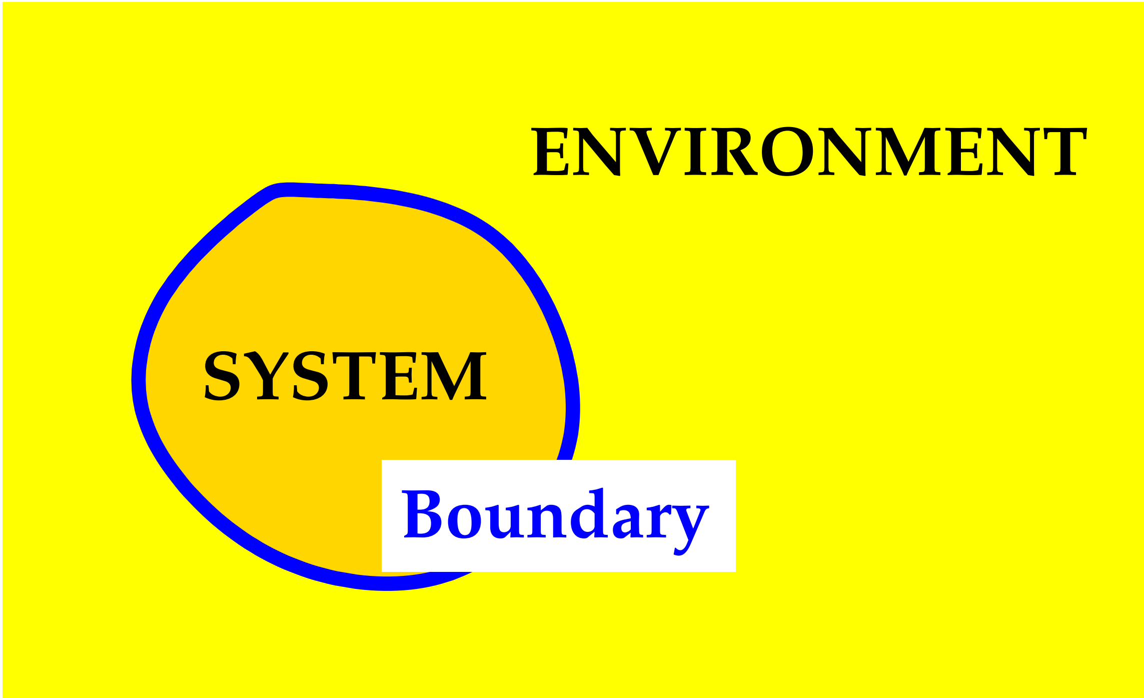
develop a suitable mathematical language

aimed at computer-assisted modeling.

Modeling $\Leftrightarrow$ ***Describing reality accurately***

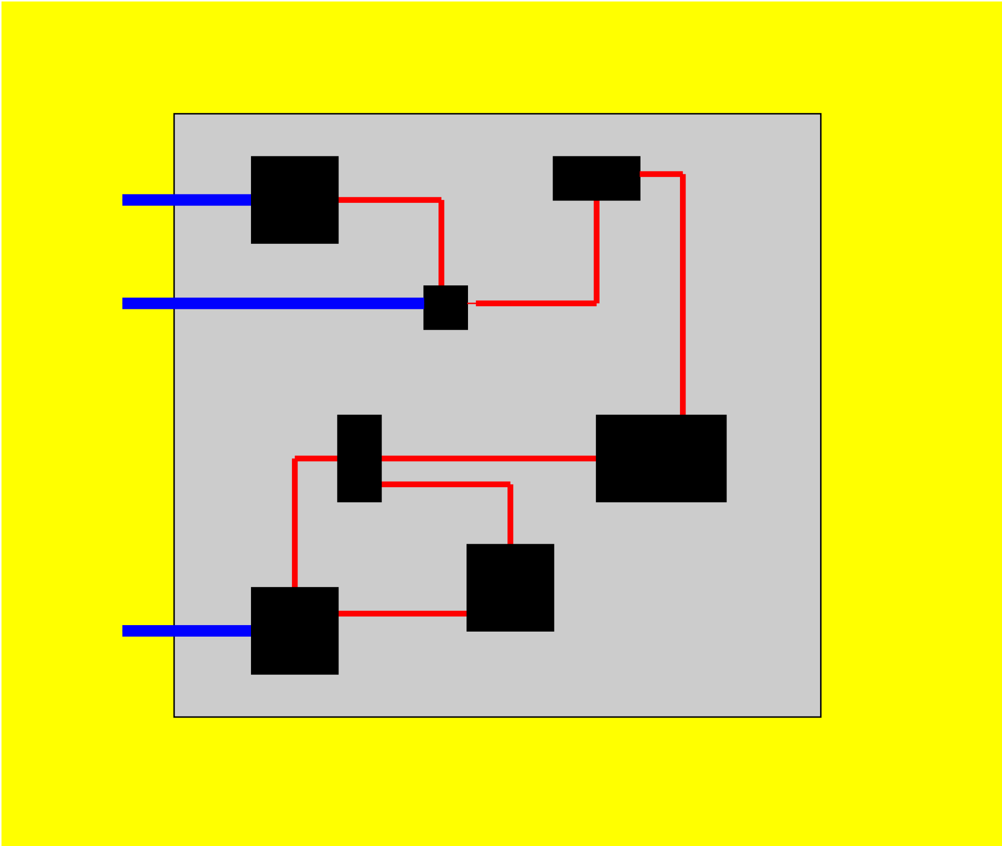
Open, connected, modular

Open



Systems interact with their environment

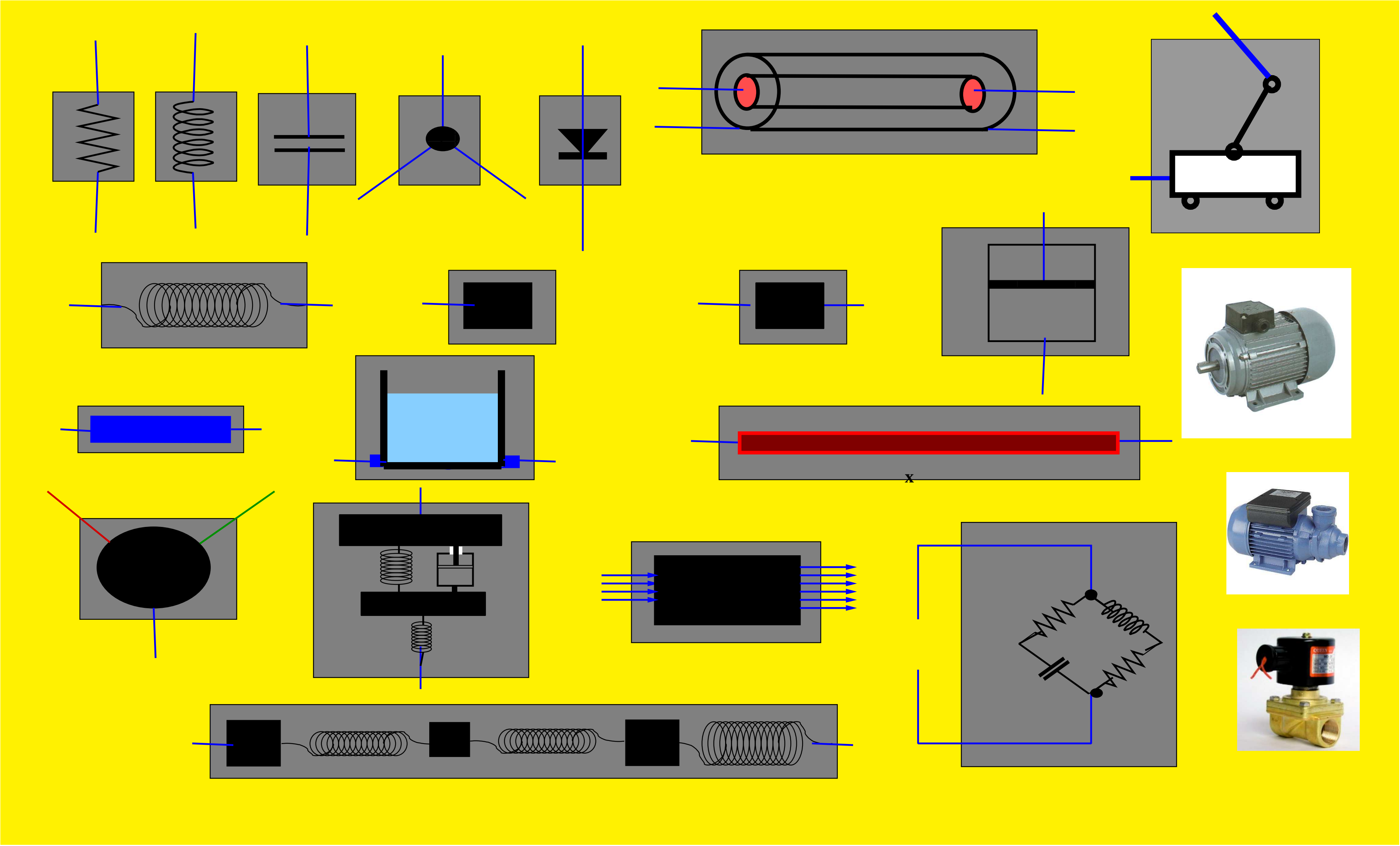
Connected



Systems consist of an architecture of interconnected subsystems

Modular

Systems are modular: composed of ‘building blocks’



The development of the notion of a dynamical system

Theme

First things first

1. **Get the physics right**
2. **The rest is mathematics**

R.E. Kalman

Opening lecture
IFAC World Congress

Prague, July 4, 2005

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Prima la fisica, poi la matematica

The missing link

- **Get the physics right**
- ***Translate the physics into mathematics***
- **The rest is mathematics**

What are the ‘right’ concepts?

What is the ‘natural’ generalization?

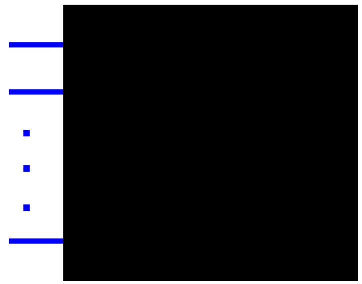
What are the ‘relevant’ questions?

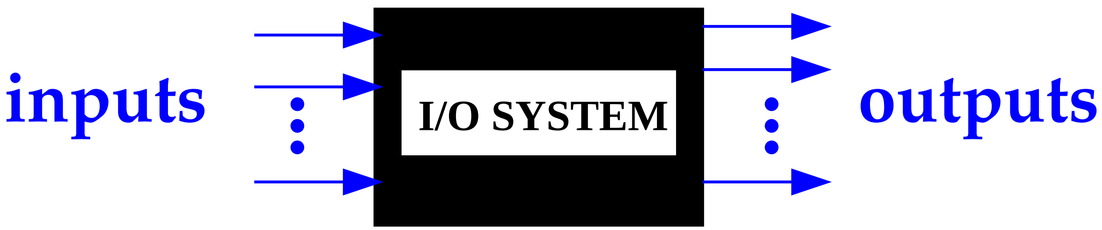
Closed dynamical systems

Inputs and outputs

Theme of this lecture

We are accustomed to view an open dynamical system as an input/output structure (with or without the state)





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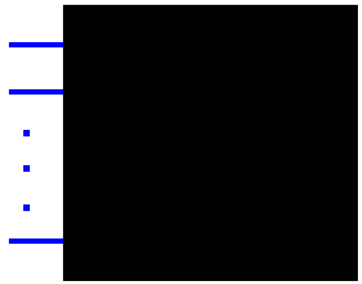
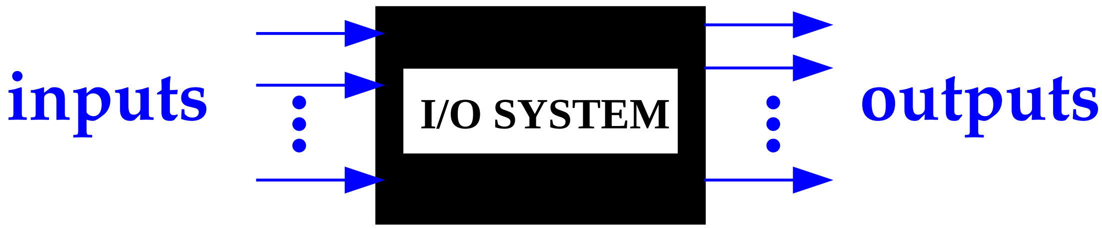
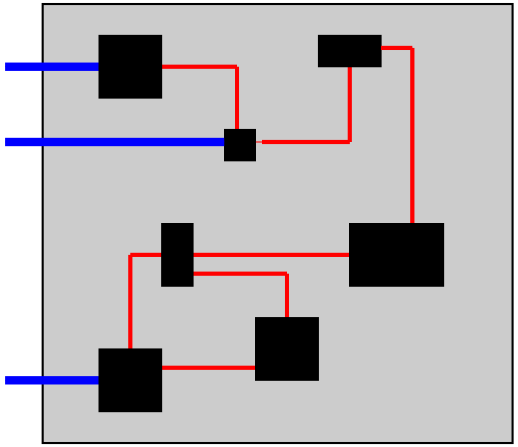




Is this an appropriate abstraction of models of physical systems?

Theme of this lecture

and interconnection as output-to-input assignment



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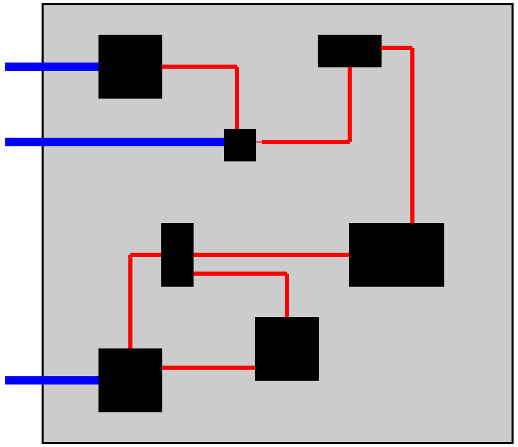

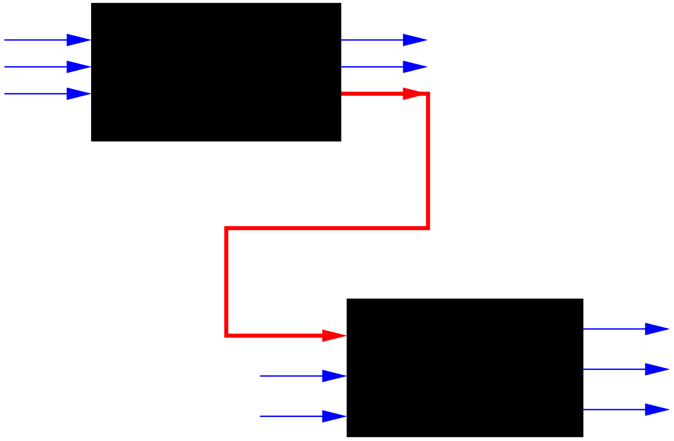

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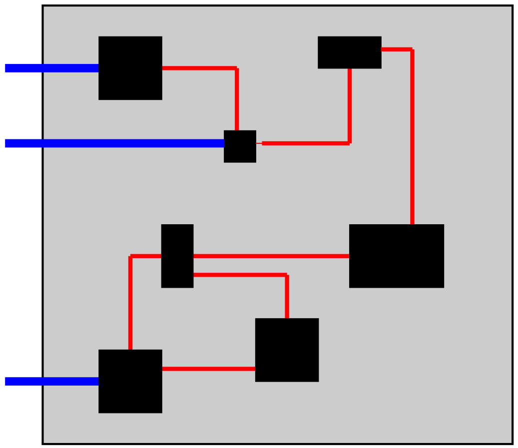
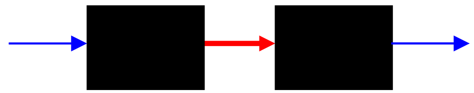
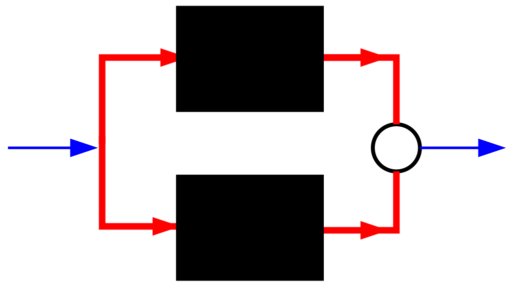


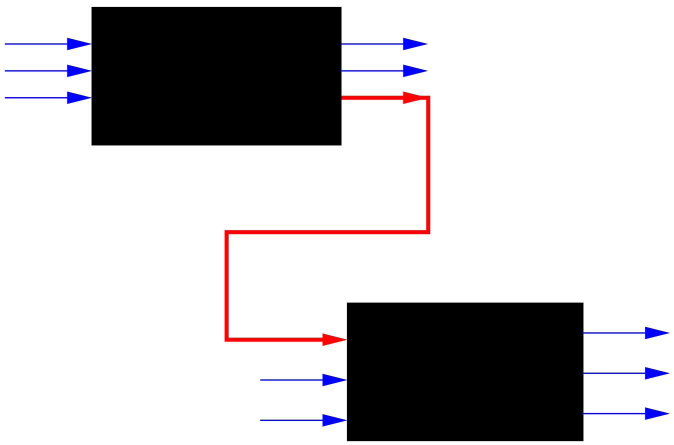
Feedback



Series



Parallel



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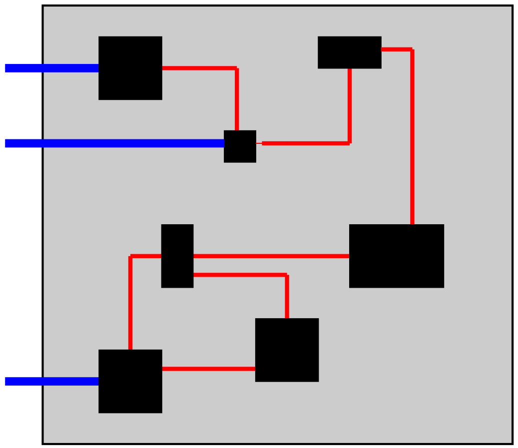

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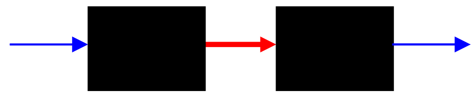

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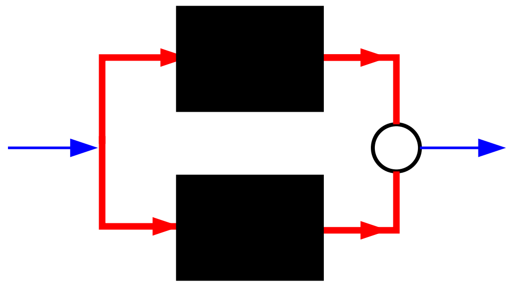

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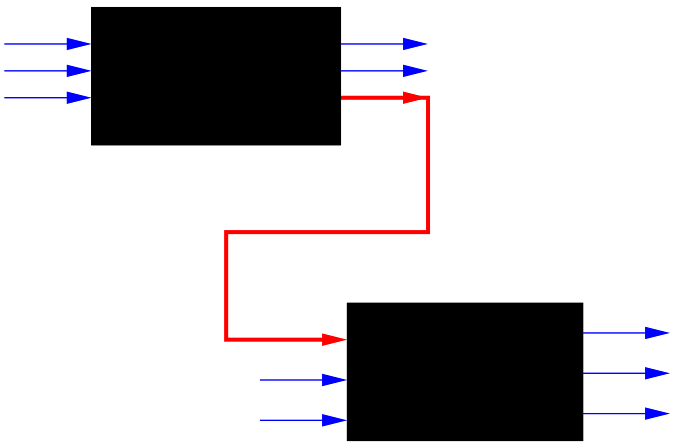
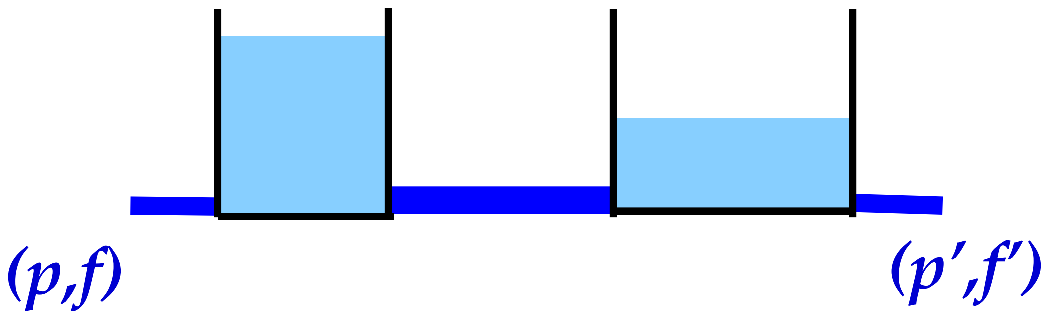


Is this an appropriate abstraction of interconnection of physical systems?

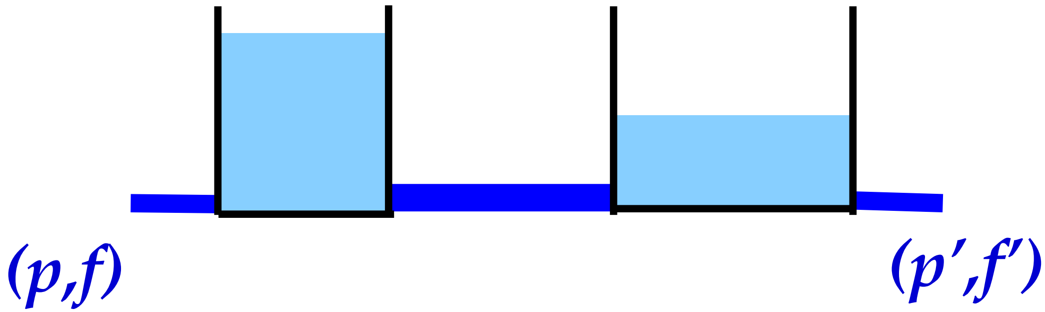
An example

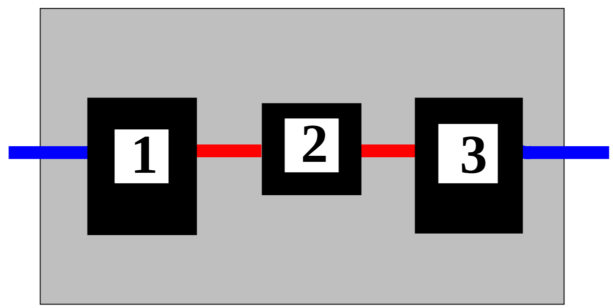
Tearing

Tearing



Tearing





Zooming

Subsystems 1 and 3 (tanks):

Zooming

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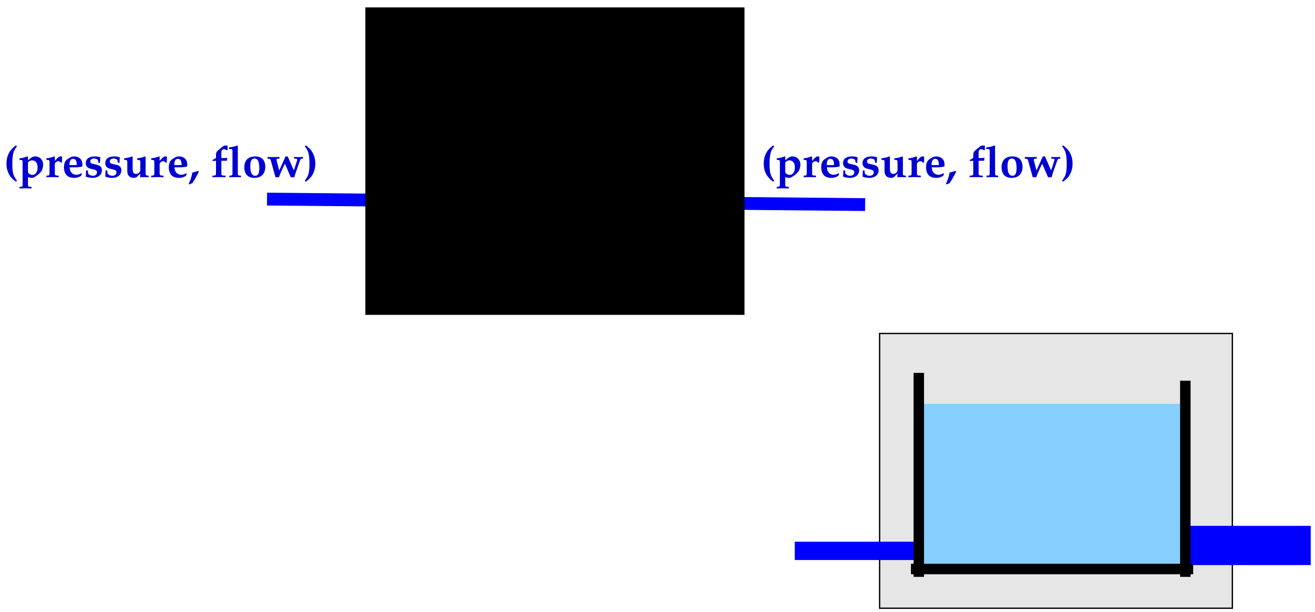

Zooming

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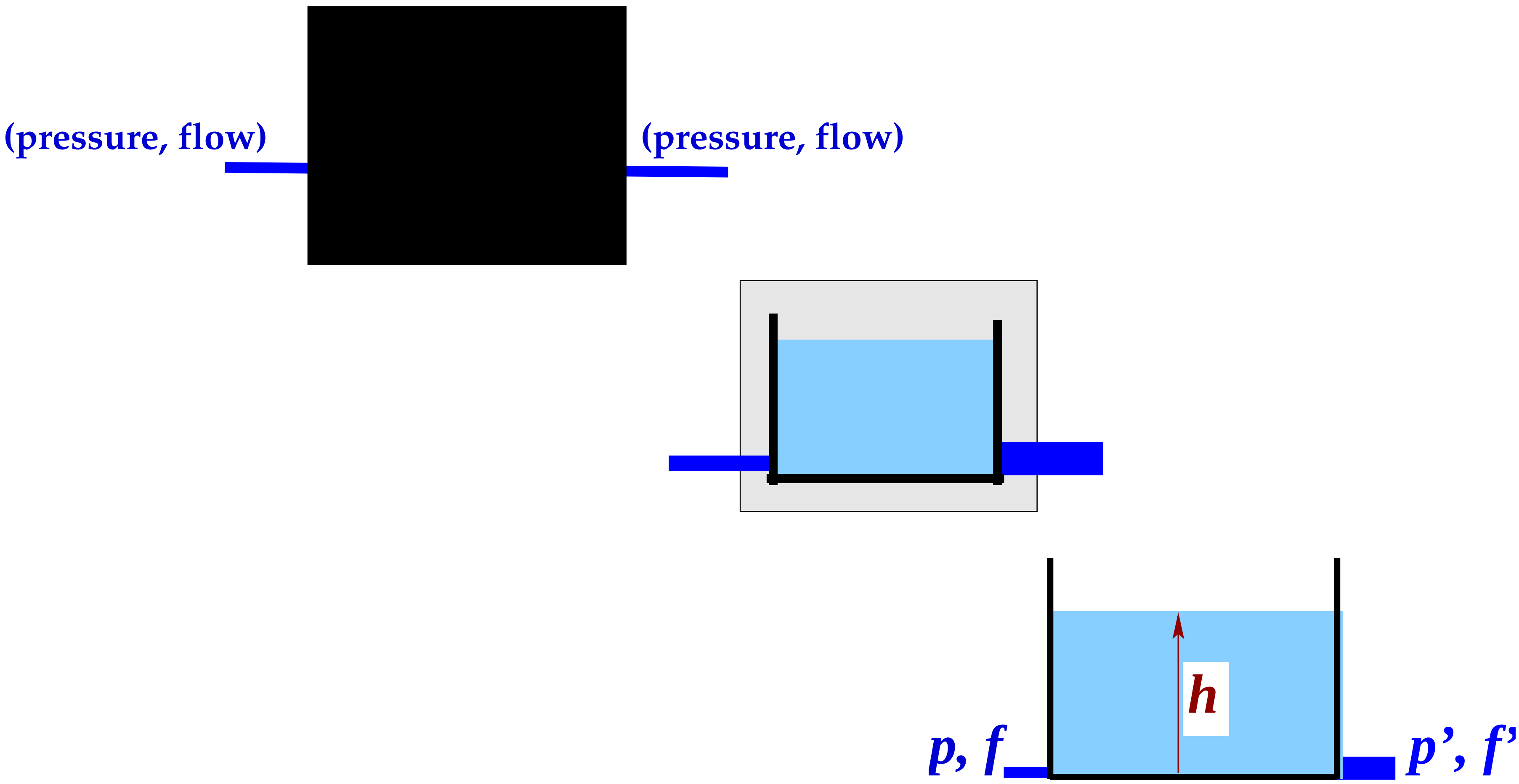

Zooming

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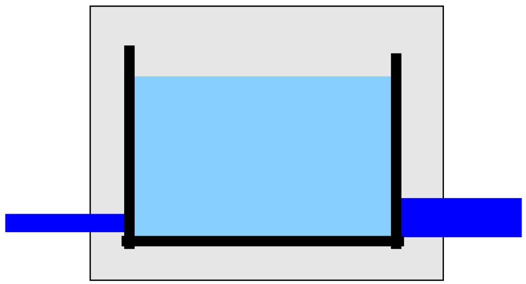
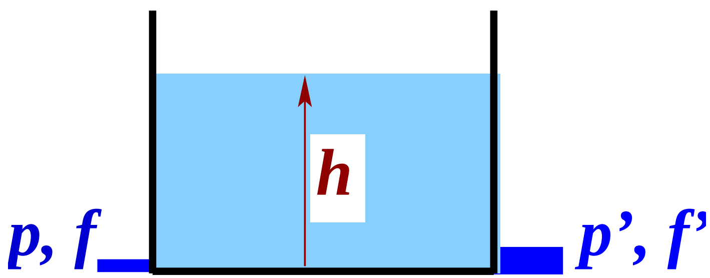


$$
\begin{aligned}
A\frac{d}{dt}h &= f + f', \\
Bf &= \begin{cases} \sqrt{|p - p_0 - \rho h|} & \textbf{if } p - p_0 \geq \rho h, \\ -\sqrt{|p - p_0 - \rho h|} & \textbf{if } p - p_0 \leq \rho h, \end{cases} \\
Cf' &= \begin{cases} \sqrt{|p' - p_0 - \rho h|} & \textbf{if } p' - p_0 \geq \rho h, \\ -\sqrt{|p' - p_0 - \rho h|} & \textbf{if } p' - p_0 \leq \rho h, \end{cases}
\end{aligned}
$$



Zooming

Subsystem 2 (pipe):

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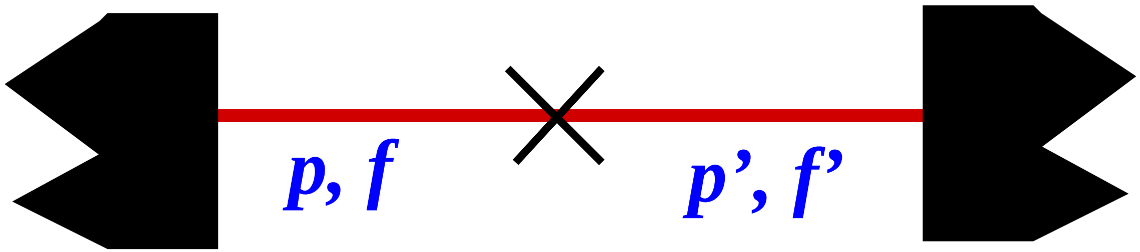
$$f = -f', \quad p - p' = \alpha f$$

Linking

Interconnection laws:



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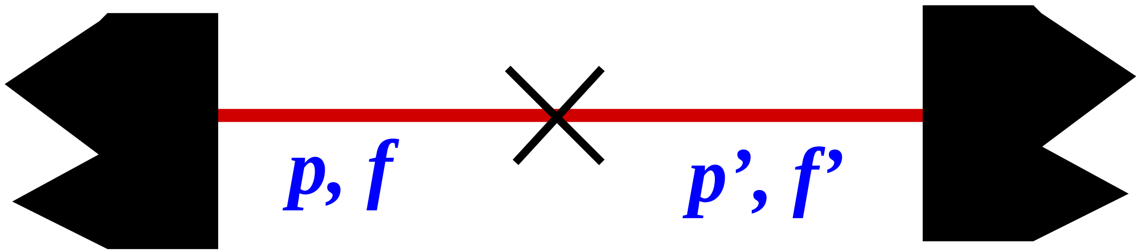
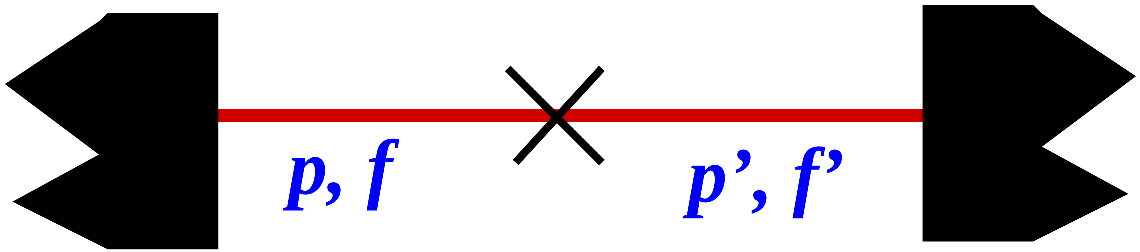


$$p = p', \qquad f + f' = 0$$

Linking

Interconnection laws:



$$p = p', \qquad f + f' = 0$$

Leads to the complete model:

$$A_1 \frac{d}{dt} h_1 = f_1 + f_1',$$

$$B_1 f_1 = \begin{cases} \sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \geq \rho h_1, \\ -\sqrt{|p_1 - p_0 - \rho h_1|} & \textbf{if } p_1 - p_0 \leq \rho h_1, \end{cases}$$ **(blackbox 1)**

$$C_1 f_1' = \begin{cases} \sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \geq \rho h_1, \\ -\sqrt{|p_1' - p_0 - \rho h_1|} & \textbf{if } p_1' - p_0 \leq \rho h_1, \end{cases}$$

$$f_2 = -f_2', \quad p_2 - p_2' = \alpha f_2,$$ **(blackbox 2)**

$$A_3 \frac{d}{dt} h_3 = f_3 + f_3',$$

$$C f_3 = \begin{cases} \sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \geq \rho h_3, \\ -\sqrt{|p_3 - p_0 - \rho h_3|} & \textbf{if } p_3 - p_0 \leq \rho h_3, \end{cases}$$ **(blackbox 3)**

$$C_3 f_3' = \begin{cases} \sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \geq \rho h_3, \\ -\sqrt{|p_3' - p_0 - \rho h_3|} & \textbf{if } p_3' - p_0 \leq \rho h_3, \end{cases}$$

$$p_1' = p_2, \quad f_1' + f_2 = 0, \; p_2' = p_3, \quad f_2' + f_3 = 0.$$ **(interconnection)**

$$p_{\text{left}} = p_1, \; f_{\text{left}} = f_1, \; p_{\text{right}} = p_3', \; f_{\text{right}} = f_3'.$$ **(manifest variable assignment)**

NB

This tableau of equations is the endpoint of a straightforward first-principles-modeling procedure.

- **Unclear (and, in fact, irrelevant) input/output structure for the terminal variables,**
  both in the overall system and in the subsystems
- **Many variables, indivisibly, at the same terminal**
- **Interconnection = variable sharing**
- **No signal flows, no output-to-input assignment**

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These remarks pertain to every physical interconnection.
And, ultimately, every interconnection is physical.

Behavioral systems

Behavioral approach

A dynamical system

:⇔ **a family of time trajectories, *'the behavior'***

Interconnection ⇔ ***'variable sharing'***

Control ⇔ ***interconnection***

Modeling of interconnected physical systems is the strongest case for 'behaviors'.

Terminals

We consider systems that interact with their environment through terminals

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There are many
electrical, mechanical, hydraulic, thermal,
civil engineering, pneumatic, ... connections
that can be viewed this way,
***exactly*, *literally*.**

For mechanical systems, think of interconnections as screwing, gluing, welding, ...
Hinges, hooks, etc. ought to be thought of as devices (modules).

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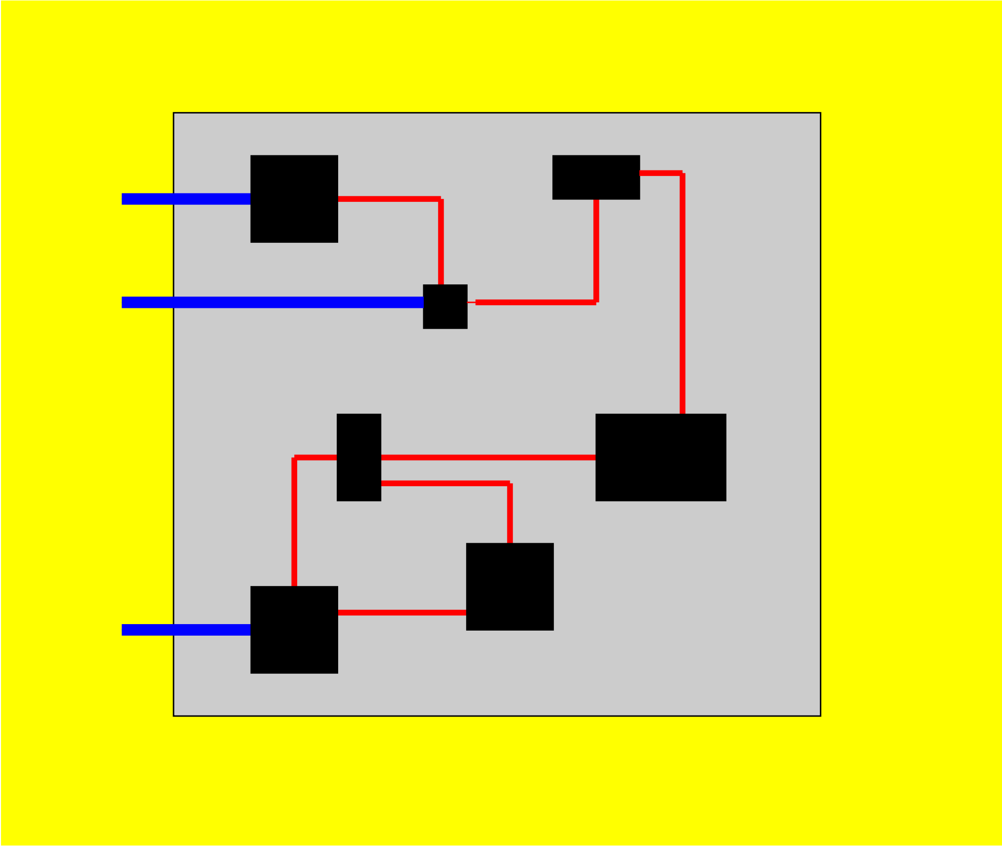
The clearest example is an electrical connection.
A terminal = a single wire.

Interconnection architecture

Objective

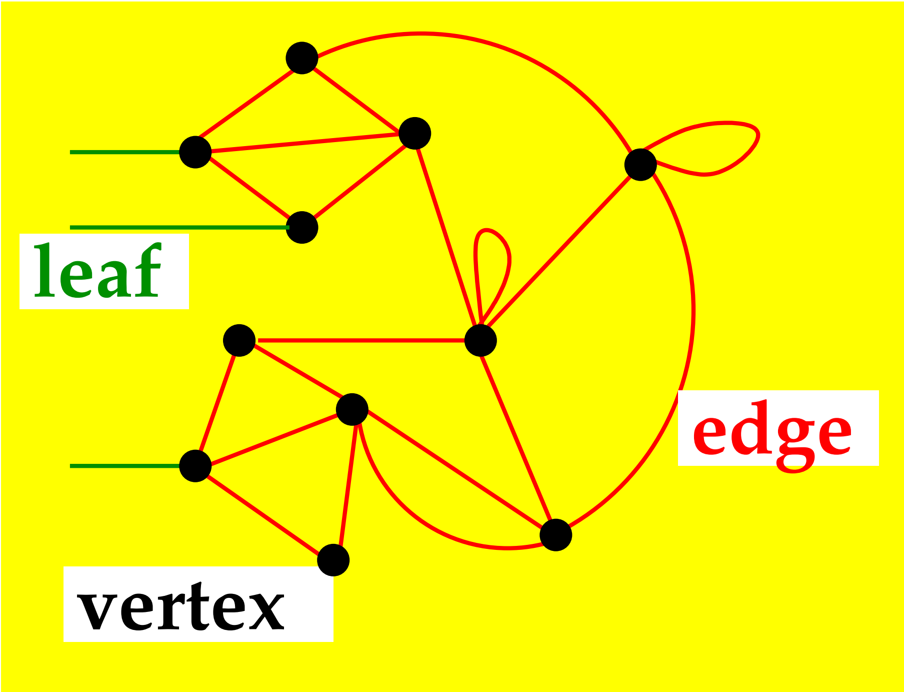


Formalize mathematically interconnection of systems.

Graph with leaves

Architecture:

graph with leaves



vertices	$\rightsquigarrow$	**systems with terminals**
edges	$\rightsquigarrow$	**connected terminals**
leaves	$\rightsquigarrow$	**interaction with environment**
terminals	$\rightsquigarrow$	**system variables**

Behavioral equations

1. **Module equations** for each vertex.
   Relation among the variables on the terminals.

2. **Interconnection equations** for each edge.
   Equating the variables on the terminals associated with the same edge.

3. **Manifest variable assignment**
   Specifies the variables of interest.

Behavioral equations

1. **Module equations** for each vertex.

   Relation among the variables on the terminals.

   Behavioral equations stored as (parametrized) modules in a data-base.

2. **Interconnection equations** for each edge.

   Equating the variables on the terminals associated with the same edge.

   Interconnection laws stored in a data-base.
   Laws depend on terminal type:
   electrical / mechanical / hydraulic / etc.

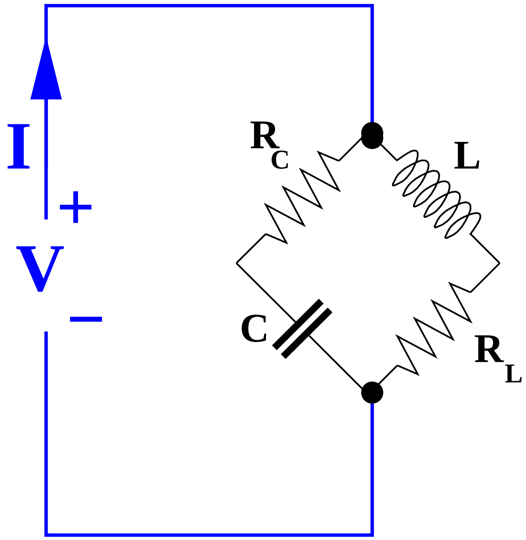
3. **Manifest variable assignment**

   Specifies the variables of interest.

An example

RLC circuit

Model the port behavior of



by tearing, zooming, and linking.

RLC circuit

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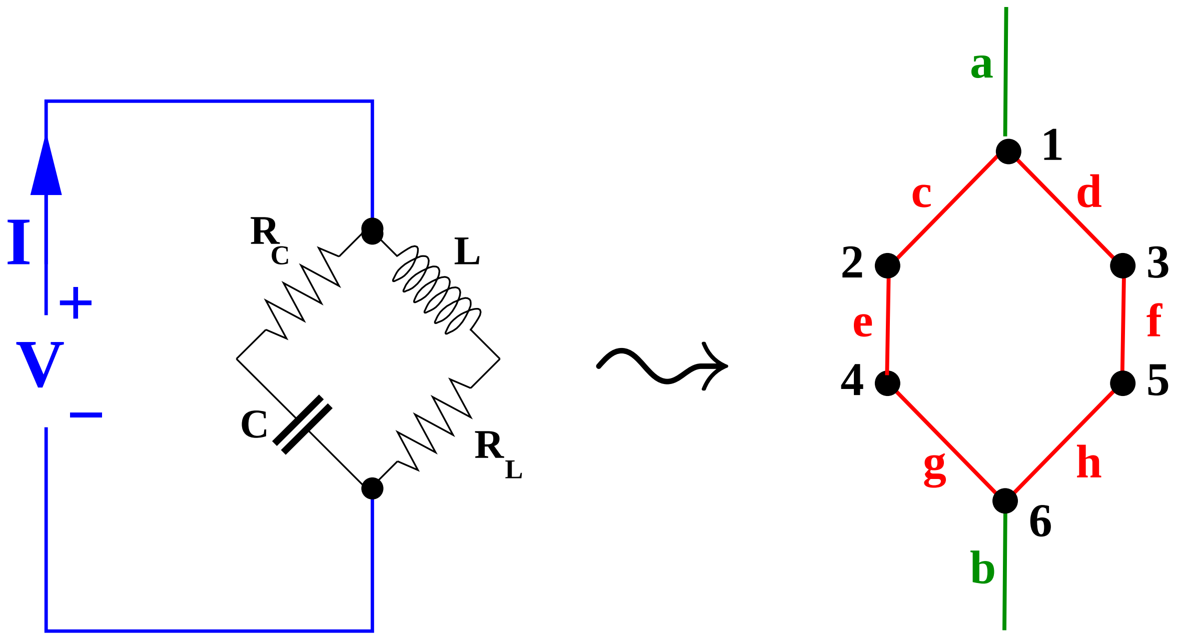
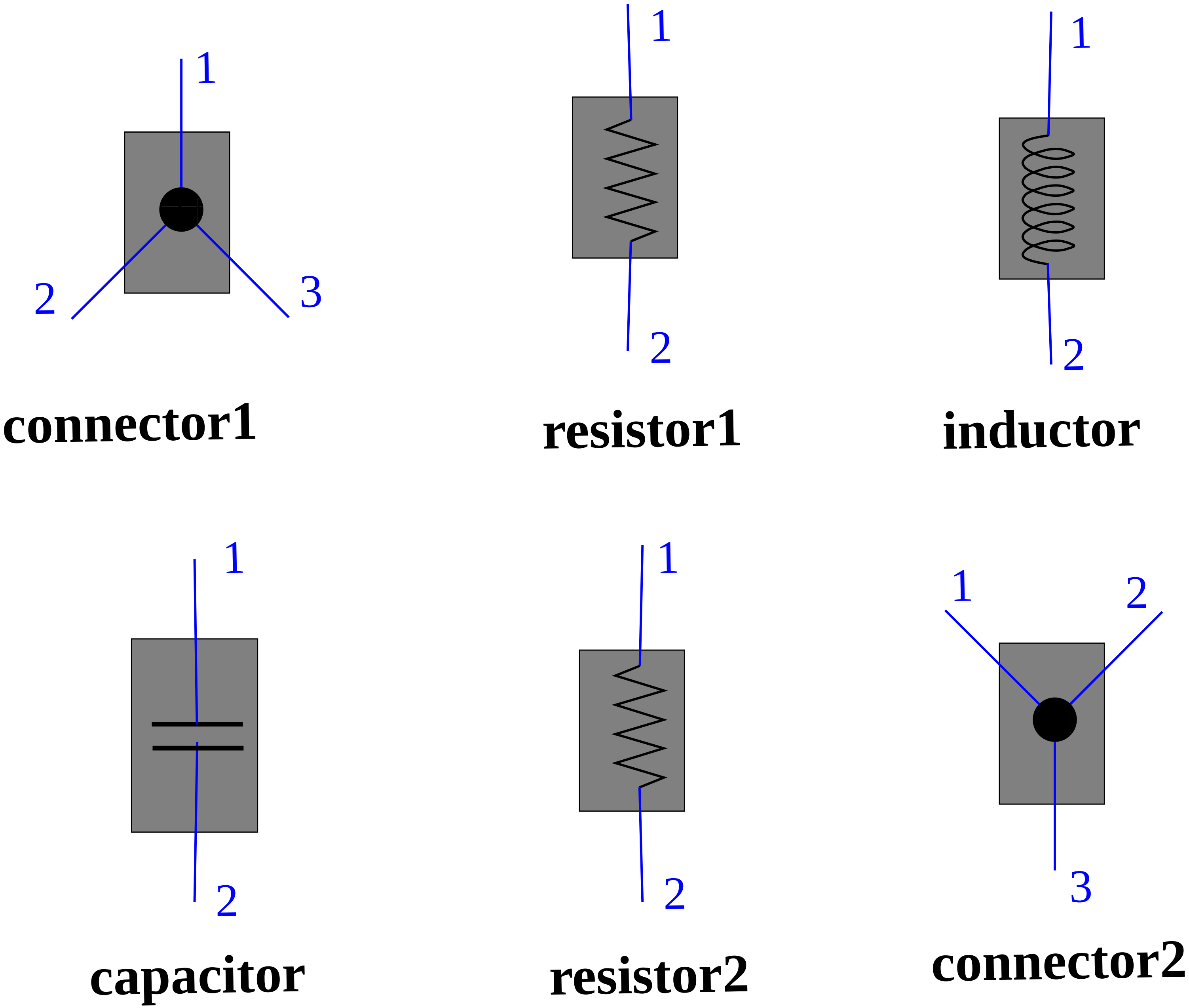


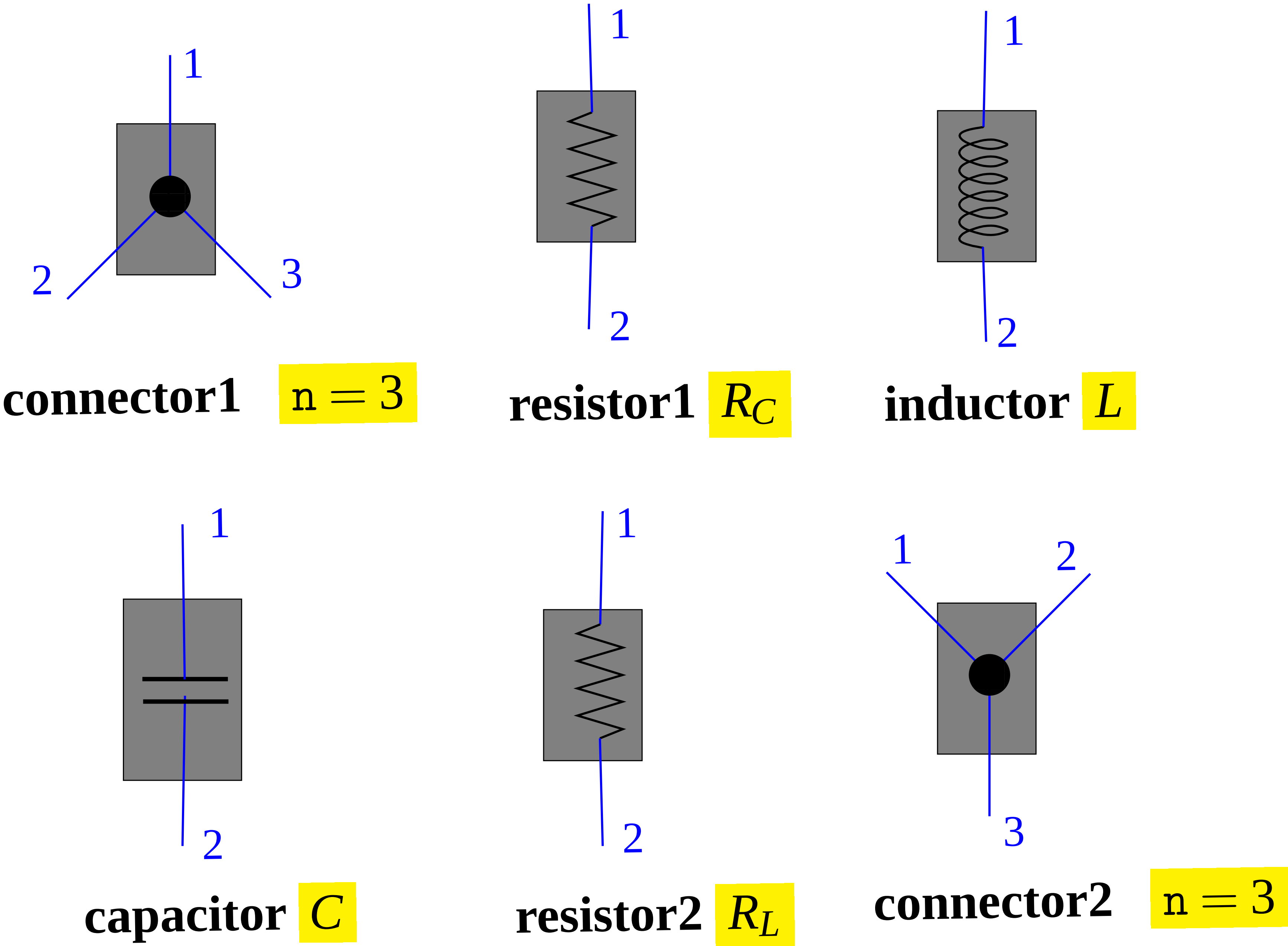
by tearing, zooming, and linking.

In each vertex there is a module ⇝ module equations
each terminal involves 2 variables (potential, current)
in each edge there is an electrical interconnection ⇝
interconnection equations

Modules


1
2
3
connector1
1
2
resistor1
1
2
inductor
1
2
capacitor
1
2
resistor2
1
2
3
connector2

Modules


connector1
n = 3
resistor1
R_C
inductor
L
capacitor
C
resistor2
R_L
connector2
n = 3

Module equations

vertex 1 : $V_{\text{connector1},1} = V_{\text{connector1},2} = V_{\text{connector1},3}$

$I_{\text{connector1},1} + I_{\text{connector1},2} + I_{\text{connector1},3} = 0$

vertex 2 : $V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1},\; I_{R_C,1} + I_{R_C,2} = 0$

vertex 3 : $L\frac{d}{dt}I_{L,1} = V_{L,1} - V_{L,2},\; I_{L,1} + I_{L,2} = 0$

vertex 4 : $C\frac{d}{dt}\left(V_{C,1} - V_{C,2}\right) = I_{C,1},\; I_{C,1} + I_{C,2} = 0$

vertex 5 : $V_{R_L,1} - V_{R_L,2} = R_L I_{R_L,1}$

$I_{R_L,1} + I_{R_L,2} = 0$

vertex 6 : $V_{\text{connector2},1} = V_{\text{connector2},2} = V_{\text{connector2},3}$

$I_{\text{connector2},1} + I_{\text{connector2},2} + I_{\text{connector2},3} = 0$

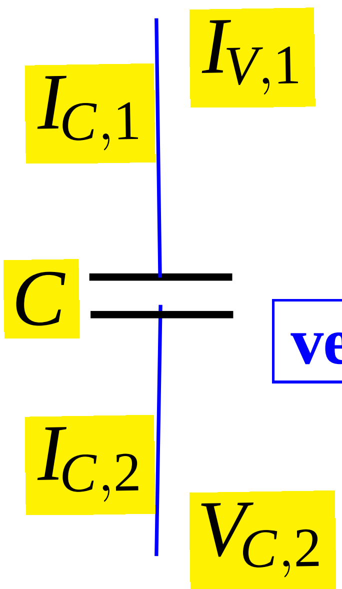
Module equations

$$V_{\text{connector1},1} = V_{\text{connector1},2} = V_{\text{connector1},3}$$

$$I_{\text{connector1},1} + I_{\text{connector1},2} + I_{\text{connector1},3} = 0$$

$$V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1},\ I_{R_C,1} + I_{R_C,2} = 0$$

$$L\frac{d}{dt} I_{L,1} = V_{L,1} - V_{L,2},\ I_{L,1} + I_{L,2} = 0$$



vertex 4:

$$C\frac{d}{dt}\left(V_{C,1} - V_{C,2}\right) = I_{C,1},\ I_{C,1} + I_{C,2} = 0$$

$$V_{R_L,1} - V_{R_L,2} = R_L I_{R_L,1}$$

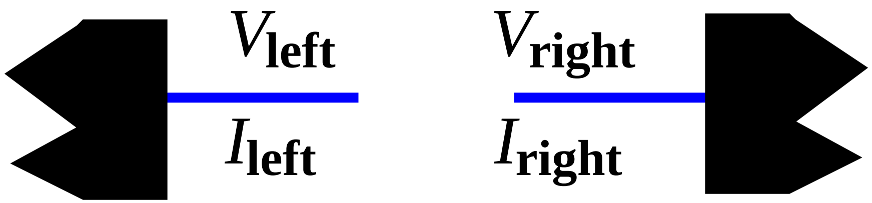
$$I_{R_L,1} + I_{R_L,2} = 0$$

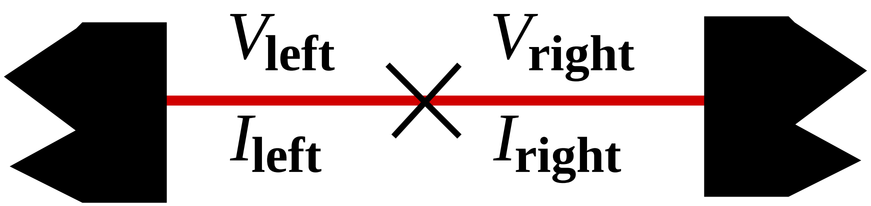
$$V_{\text{connector2},1} = V_{\text{connector2},2} = V_{\text{connector2},3}$$

$$I_{\text{connector2},1} + I_{\text{connector2},2} + I_{\text{connector2},3} = 0$$

Interconnection

All interconnections are of electrical type





Interconnection equations:

potential left $=$ **potential right** $\leadsto$ $V_{\mathbf{left}} = V_{\mathbf{right}}$

current left $+$ **current right** $= 0$ $\leadsto$ $I_{\mathbf{left}} + I_{\mathbf{right}} = 0$

Interconnection equations

edge c : $V_{R_{C,1}} = V_{\text{connector}1,2}$ $\quad$ $I_{R_{C,1}} + I_{\text{connector}1,2} = 0$

edge d : $V_{L,1} = V_{\text{connector}1,3}$ $\quad$ $I_{L,1} + I_{\text{connector}1,3} = 0$

edge e : $V_{R_{C,2}} = V_{C,1}$ $\quad$ $I_{R_{C,2}} + I_{C,1} = 0$

edge f : $V_{L,2} = V_{R_{C,1}}$ $\quad$ $I_{L,2} + I_{R_{L,1}} = 0$

edge g : $V_{C,2} = V_{\text{connector}2,1}$ $\quad$ $I_{C,2} + I_{\text{connector}2,1} = 0$

edge h : $V_{R_{L,2}} = V_{\text{connector}2,2}$ $\quad$ $I_{R_{L,2}} + I_{\text{connector}2,2} = 0$

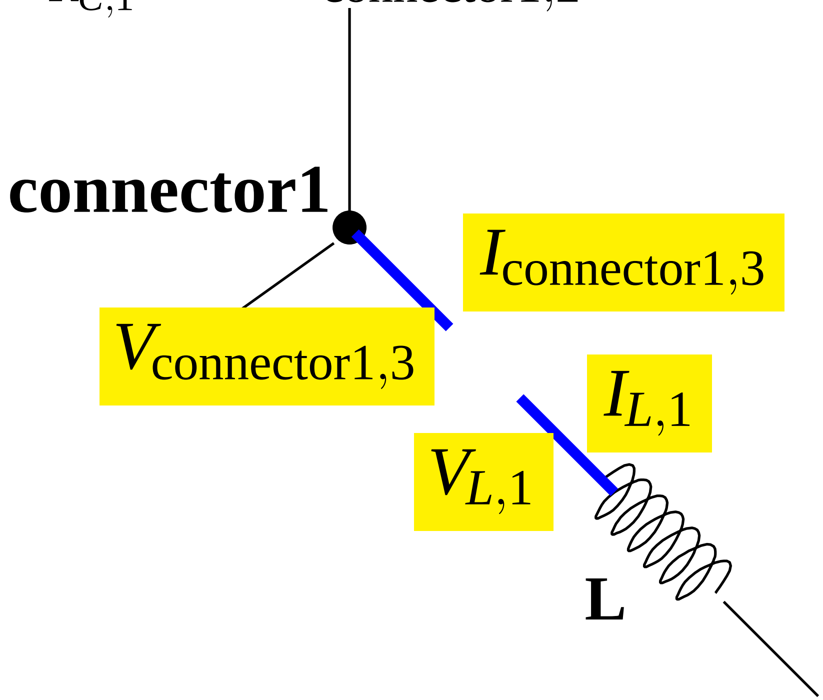
Interconnection equations

$$V_{R_{C,1}} = V_{\text{connector1,2}} \qquad I_{R_{C,1}} + I_{\text{connector1,2}} = 0$$

edge d:

$$V_{L,1} = V_{\text{connector1,3}}$$

$$I_{L,1} + I_{\text{connector1,3}} = 0$$



$$V_{R_{C,2}} = V_{C,1} \qquad I_{R_{C,2}} + I_{C,1} = 0$$

$$V_{L,2} = V_{R_{C,1}} \qquad I_{L,2} + I_{R_{L,1}} = 0$$

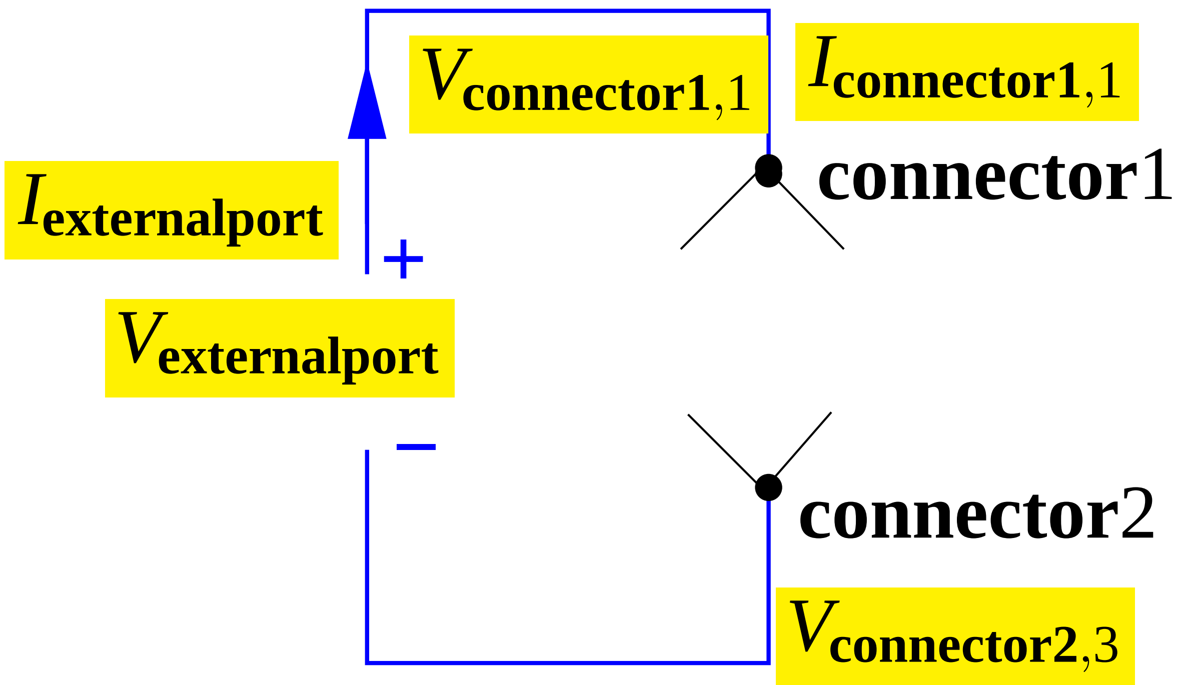
$$V_{C,2} = V_{\text{connector2,1}} \qquad I_{C,2} + I_{\text{connector2,1}} = 0$$

$$V_{R_{L,2}} = V_{\text{connector2,2}} \qquad I_{R_{L,2}} + I_{\text{connector2,2}} = 0$$

Manifest variable assignment

$$V_{\text{externalport}} = V_{\text{connector1},1} - V_{\text{connector2},3}$$

$$I_{\text{externalport}} = I_{\text{connector1},1}$$



Complete model

vertex 1 : $V_{\text{connector1,1}} = V_{\text{connector1,2}} = V_{\text{connector1,3}}$

$I_{\text{connector1,1}} + I_{\text{connector1,2}} + I_{\text{connector1,3}} = 0$

vertex 2 : $V_{R_C,1} - V_{R_C,2} = R_C I_{R_C,1},\ I_{R_C,1} + I_{R_C,2} = 0$

vertex 3 : $L\frac{d}{dt} I_{L,1} = V_{L,1} - V_{L,2},\ I_{L,1} + I_{L,2} = 0$

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$I_{R_L,1} + I_{R_L,2} = 0$

vertex 6 : $V_{\text{connector2,1}} = V_{\text{connector2,2}} = V_{\text{connector}_2\text{,3}}$

$I_{\text{connector2,1}} + I_{\text{connector2,2}} + I_{\text{connector2,3}} = 0$

edge c : $V_{R_{C,1}} = V_{\text{connector1,2}}$

$I_{R_{C,1}} + I_{\text{connector1,2}} = 0$

edge d : $V_{L_1} = V_{\text{connector1,3}}$

$I_{L_1} + I_{\text{connector1,3}} = 0$

edge e : $V_{R_{C,2}} = V_{C_1}$

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edge f : $V_{L_2} = V_{R_{C,1}}$

$I_{L_2} + I_{R_{L,1}} = 0$

edge g : $V_{C_2} = V_{\text{connector2,1}}$

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$$V_{\text{externalport}} = V_{\text{connector,1,1}} - V_{\text{connector2,3}}$$

$$I_{\text{externalport}} = I_{\text{connector1,1}}$$

Port behavior

$$\mathscr{B} = \{ (V_{\text{externalport}}, I_{\text{externalport}}) : \mathbb{R} \to \mathbb{R}^2 \mid$$

$\exists$ **latent variables trajectories**

$$(V_{\text{connector}_1,1}, I_{\text{connector}_1,1}, \ldots, \ldots) : \mathbb{R} \to \mathbb{R}^{28}$$

such that

$$V_{\text{connector}_1,1} = V_{\text{connector}_1,2} = V_{\text{connector}_1,3}, \ldots,$$

all 24 equations are satisfied$\}$

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all 24 equations are satisfied$\}$

Can we simplify this expression for $\mathscr{B}$?

Port behavior

$\rightsquigarrow$ **the dynamical system with behavior $\mathscr{B}$ specified by:**

Case 1: $CR_C \neq \dfrac{L}{R_L}$

$$\boxed{\left(\frac{R_C}{R_L}+\left(1+\frac{R_C}{R_L}\right)CR_C\frac{d}{dt}+CR_C\frac{L}{R_L}\frac{d^2}{dt^2}\right)V=\left(1+\frac{L}{R_L}\frac{d}{dt}\right)\left(1+CR_C\frac{d}{dt}\right)R_C I}$$

$\rightsquigarrow$ $\mathscr{B}$ **= all solutions** $(V,I):\mathbb{R}\rightarrow\mathbb{R}^2$

Port behavior

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Case 2: $CR_C = \frac{L}{R_L}$

$$\left(\frac{R_C}{R_L} + CR_C \frac{d}{dt}\right) V = \left(1 + CR_C \frac{d}{dt}\right) R_C I$$

$\rightsquigarrow$ $\mathscr{B}$ **= all solutions** $(V, I) : \mathbb{R} \to \mathbb{R}^2$

Port behavior

Thm: **In LTIDSs latent variables can be eliminated !**

$\rightsquigarrow$ **the dynamical system with behavior $\mathscr{B}$ specified by:**

Case 1: $CR_C \neq \dfrac{L}{R_L}$

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$\rightsquigarrow$ $\mathscr{B}$ **= all solutions** $(V,I) : \mathbb{R} \rightarrow \mathbb{R}^2$

The complete model is a linear constant coefficient DAE

vertex 1 : $V_{\text{connector1,1}} = V_{\text{connector1,2}} = V_{\text{connector1,3}}$

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$$\boxed{I_{\text{externalport}}} = I_{\text{connector1,1}}$$

Canceling poles and zeros

Other methodologies

Differential-algebraic equations (DAEs)

Signal flow graphs

input/output thinking

There are many many examples where output-to-input connection is eminently natural:

input/output thinking

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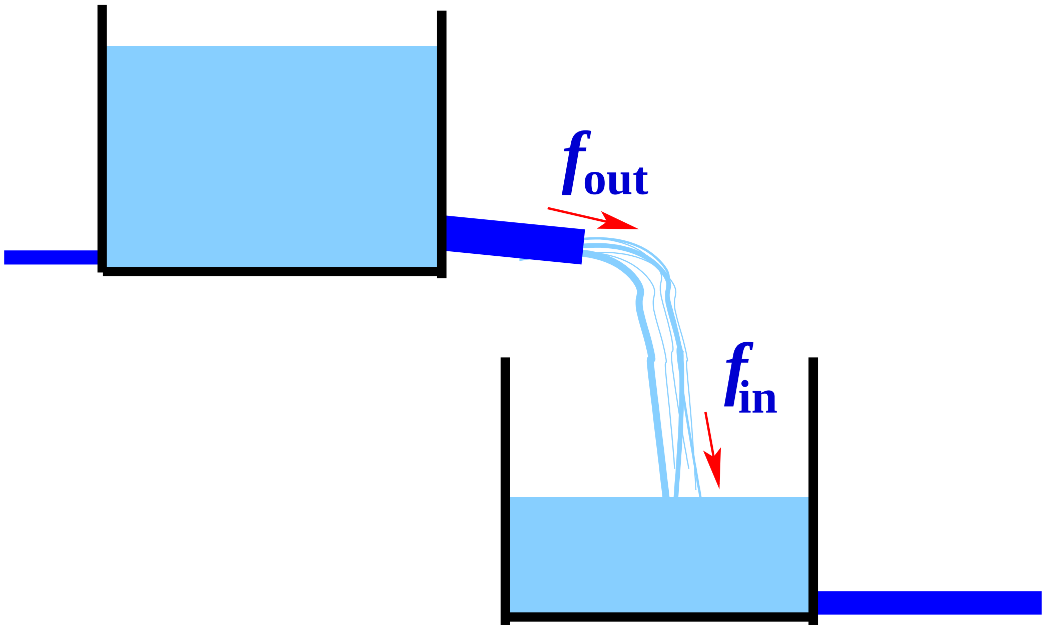
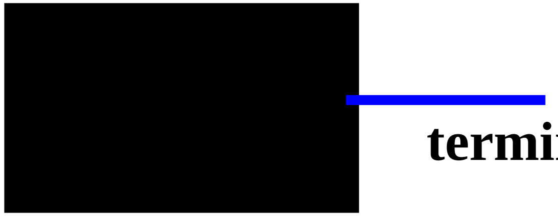


input/output partition



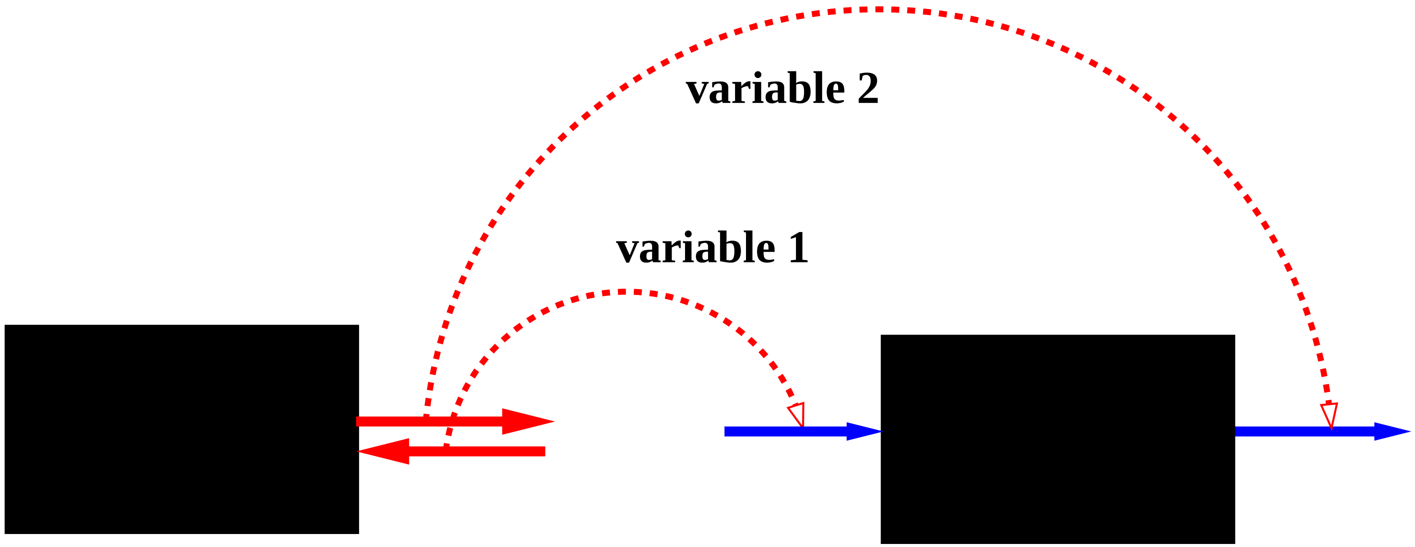
terminal with 2 physical variables

Assume that one of these variables acts as input, the other as output.

input/output partition

Assume that one of these variables acts as input, the other as output.

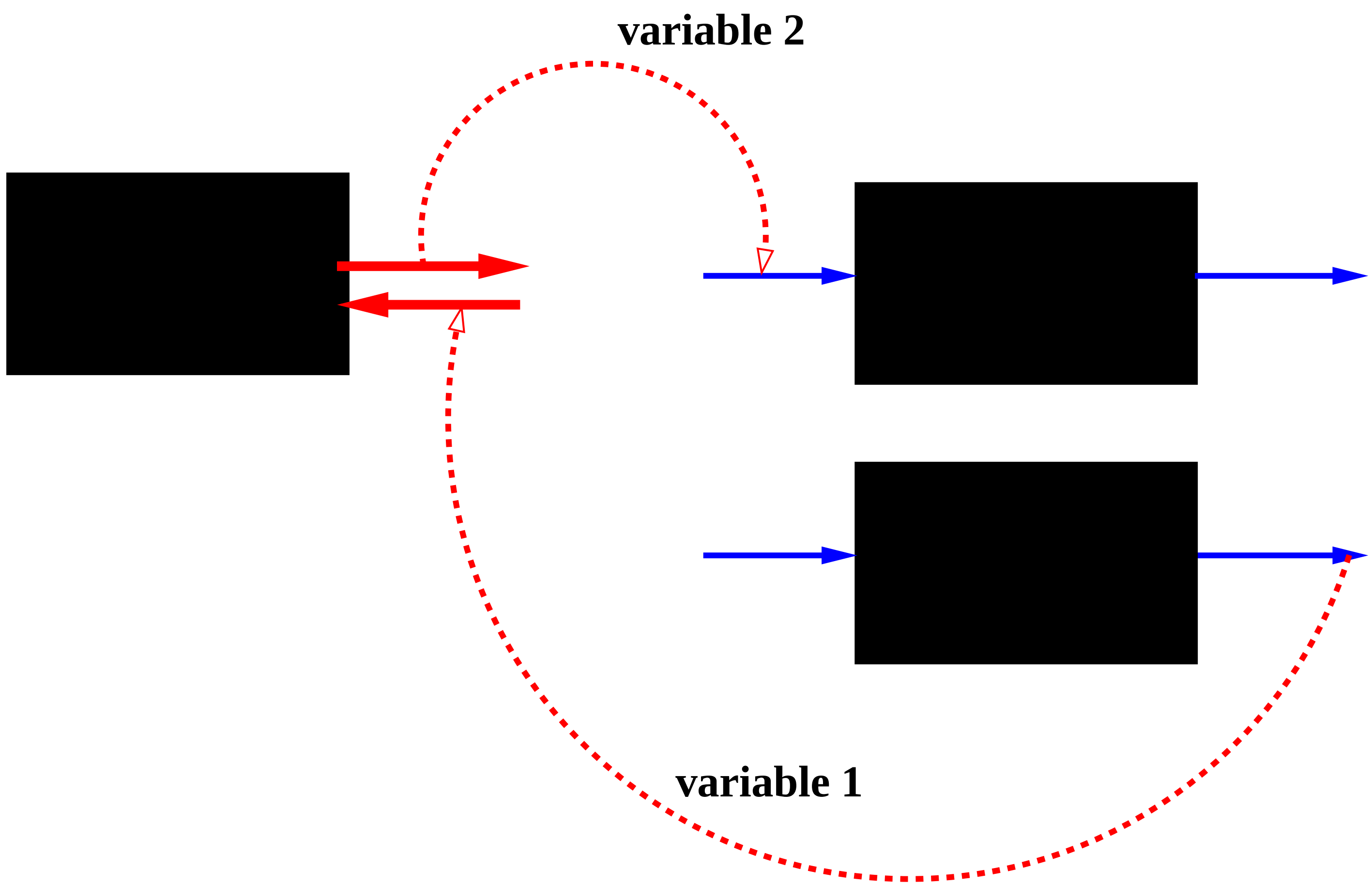
Block diagram



- shows terminal variables separate
- suggests that inputs and outputs occur at different physical points

Pedagogically awkward, confusing.

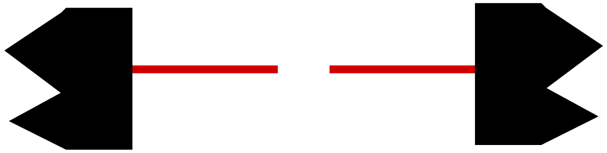
input/output thinking



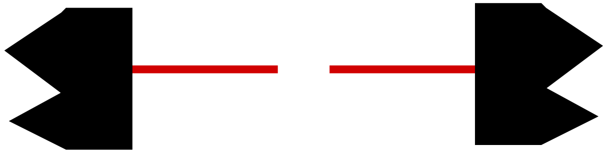
- allows impossible input-output connections

Does not respect the physics.

input/output thinking



input/output thinking



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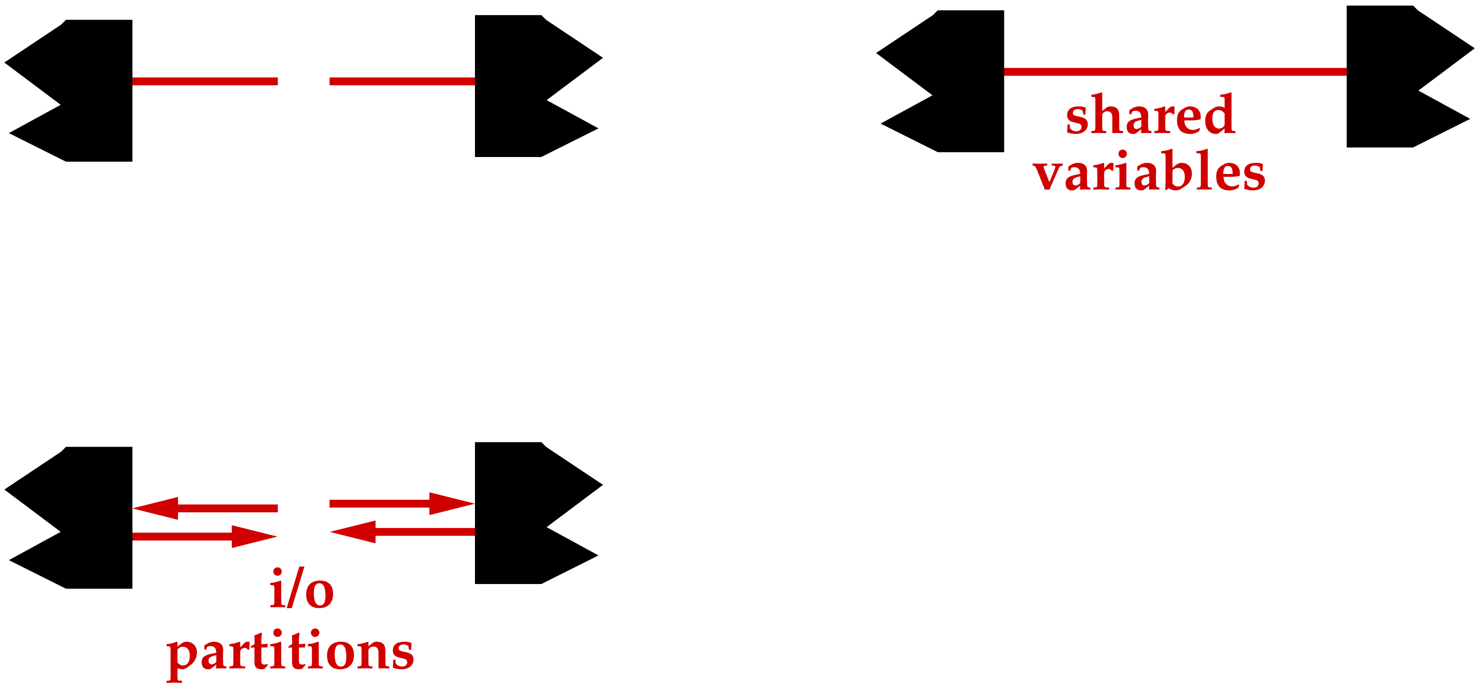

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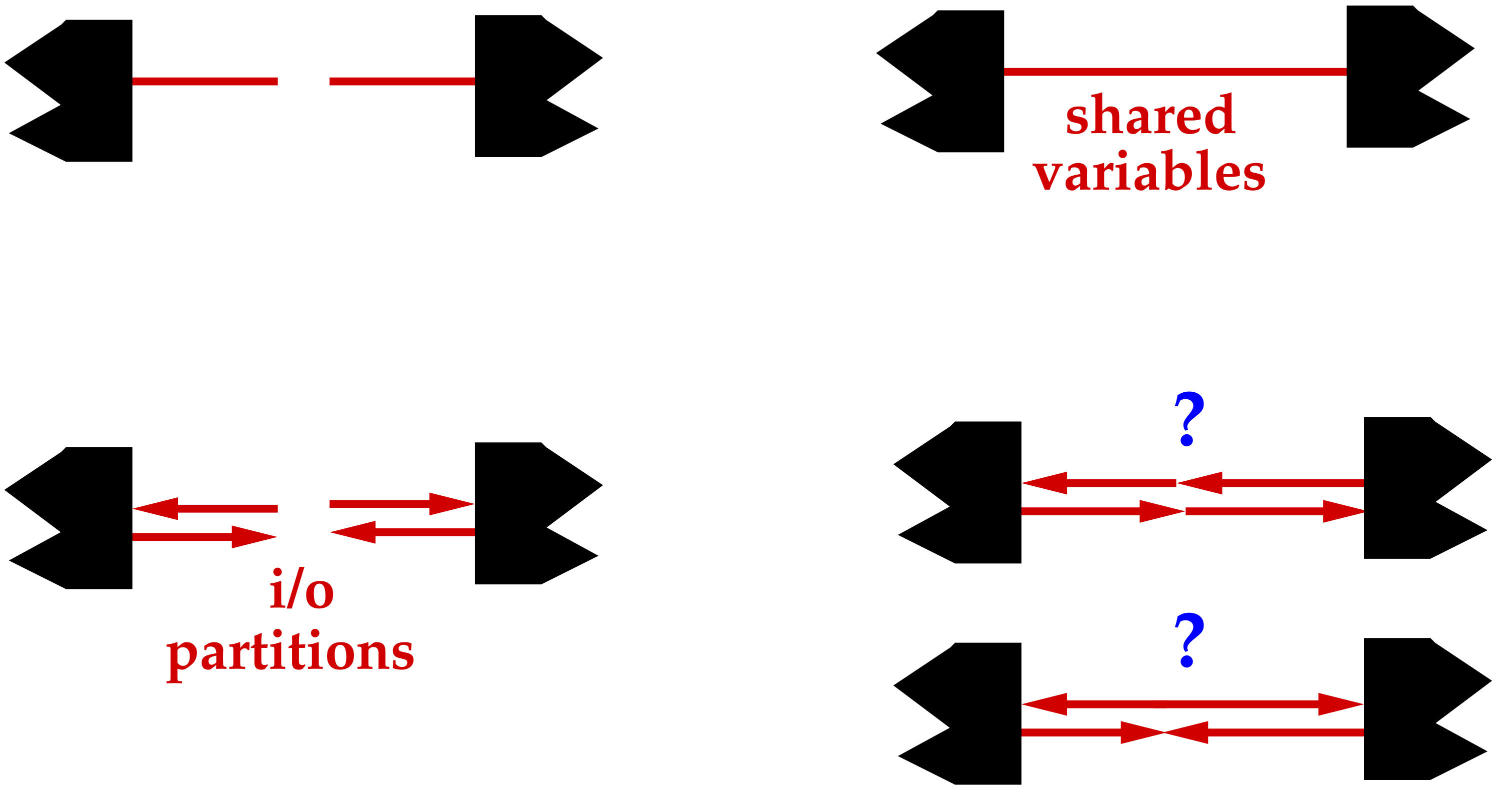

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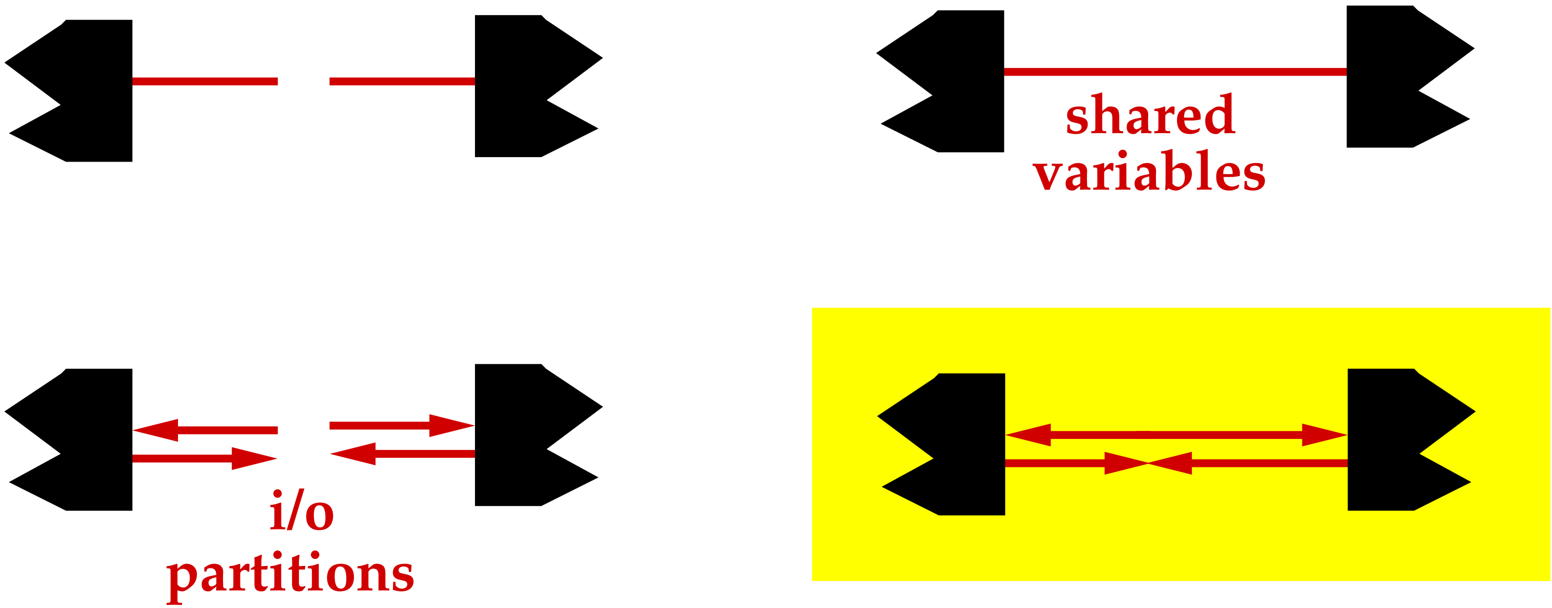


For physical systems

input-to-input & output-to-output

assignment very prevalent:

force to force; pressure to pressure; heat flow to heat flow; temperature to temperature; mass flow to mass flow; ...

Physical systems are not signal processors

The input/output approach as the primary and universal view of open systems is a historical misconception.

The sooner it is abandoned as a starting point, the better.

The input/output approach as the primary and universal view of open systems is a historical misconception.

- **It fails in the most elementary examples.**
- **It does not deal adequately with interconnections.**
- **It breaks symmetries.**

- **It does not respect the physics.**
- **It is pedagogically ineffective.**

The sooner it is abandoned as a starting point, the better.

“Block diagrams unsuitable for serious physical modeling

- the control/physics barrier”

“Behavior based (declarative) modeling is a good alternative”

Karl Åström

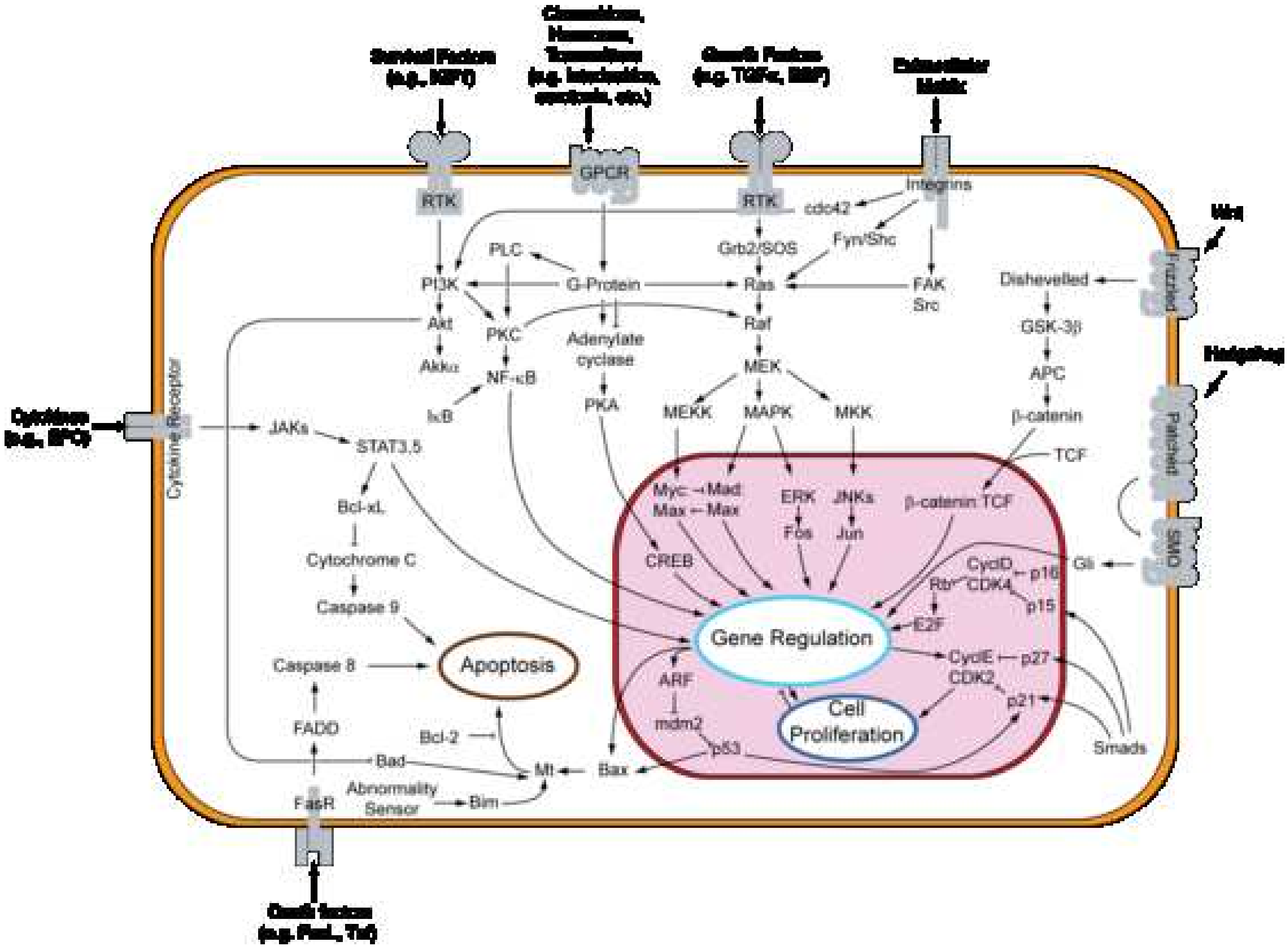
from K.J. Åström, *Present Developments in Control Applications*

IFAC 50-th Anniversary Celebration
Heidelberg, September 12, 2006.

Notes & arrows

My dear young man, don’t take it too hard.
Your work is ingenious. It’s quality work.
But there are simply too many notes that’s all ...

Notes & arrows



Ingenious. Quality work.

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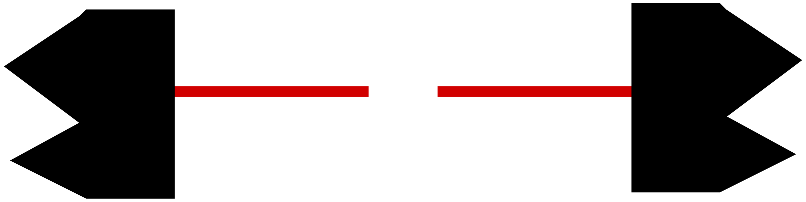
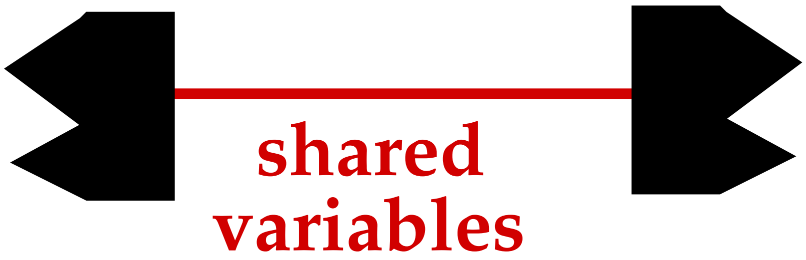
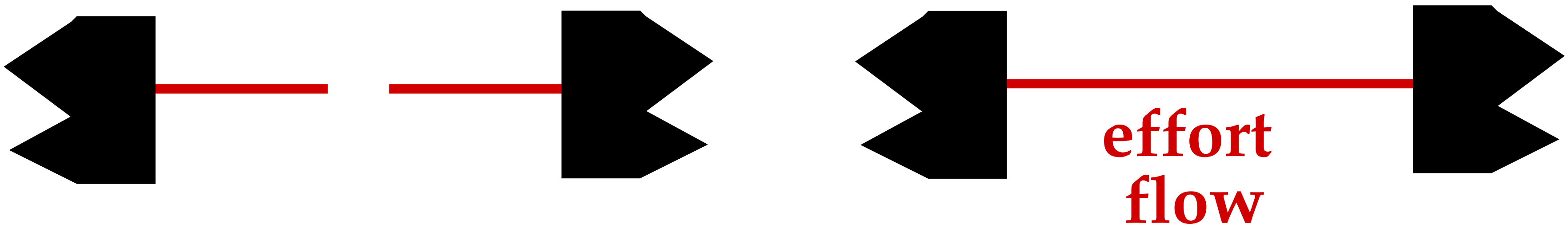
Bond graphs

Bond graphs





Bond graphs



Interconnection variables consist of

an effort and a flow **effort $\times$ flow = power**

Interconnection $\Leftrightarrow$

[efforts equal] & [flows sum to 0]

$\Rightarrow$ **power equal**

'Power is the universal currency of physical systems'

Bond graphs

Interconnection variables:

- voltage & current
- force & velocity
- pressure & mass flow
- temperature & heat flow

  temperature & $\dfrac{\text{heat flow}}{\text{temperature}}$
- ...

Remarks

- Mechanical interconnections equate positions, not velocities.
- Not all interconnections involve equating energy transfer.
- Terminals are for interconnection, ports for energy transfer.

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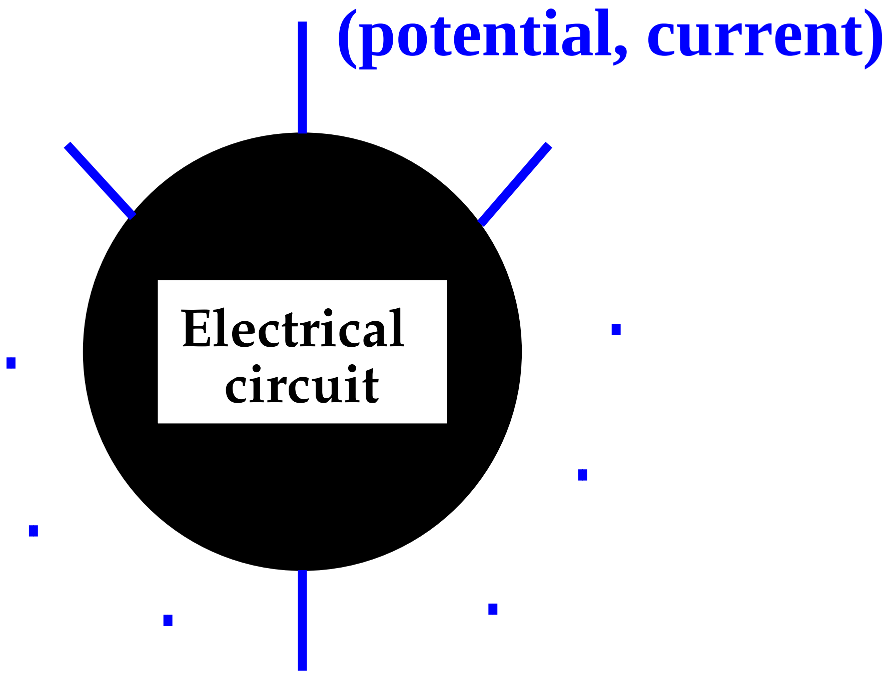
- Mechanical interconnections equate positions, not velocities.
- Not all interconnections involve equating energy transfer.
- Terminals for interconnection, ports for energy transfer

This last point is illustrated for electrical interconnections.

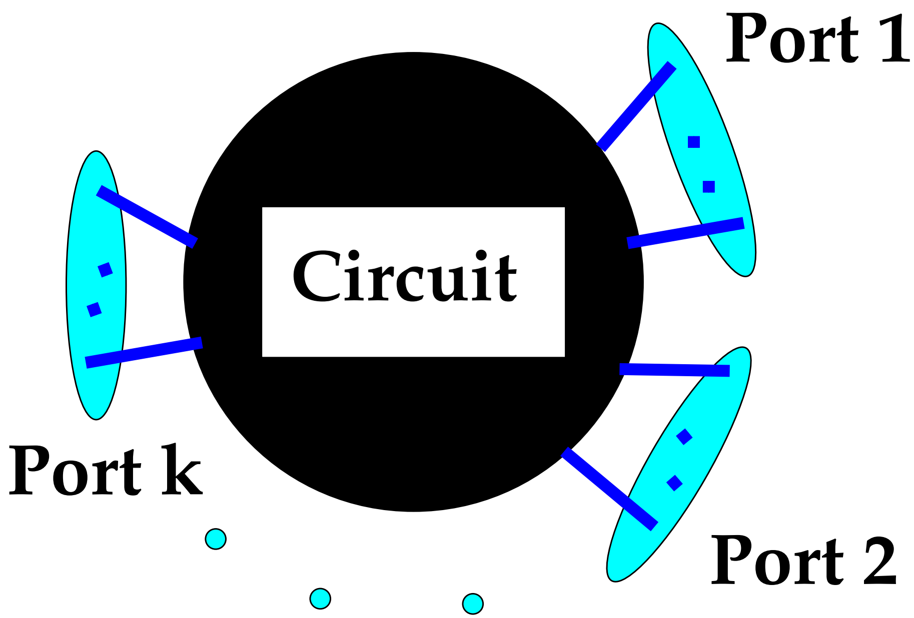
Terminals versus ports



Terminal variables and behavior:

$$(V_1, I_1, V_2, I_2, \ldots, V_{\mathtt{n}}, I_{\mathtt{n}}) \rightsquigarrow \textbf{ behavior } \mathscr{B} \subseteq \left(\mathbb{R}^{2\mathtt{n}}\right)^{\mathbb{R}}$$

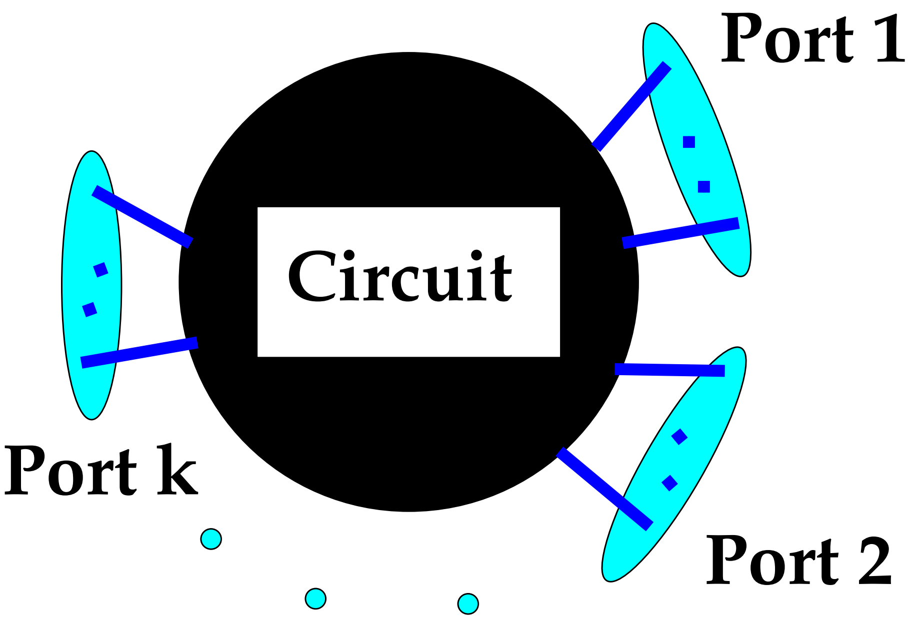
Terminals versus ports



Port :⇔

sum currents = 0
potentials + constant
⇒ potentials

Terminals versus ports



Port :⇔

sum currents = 0
potentials + constant
⇒ potentials

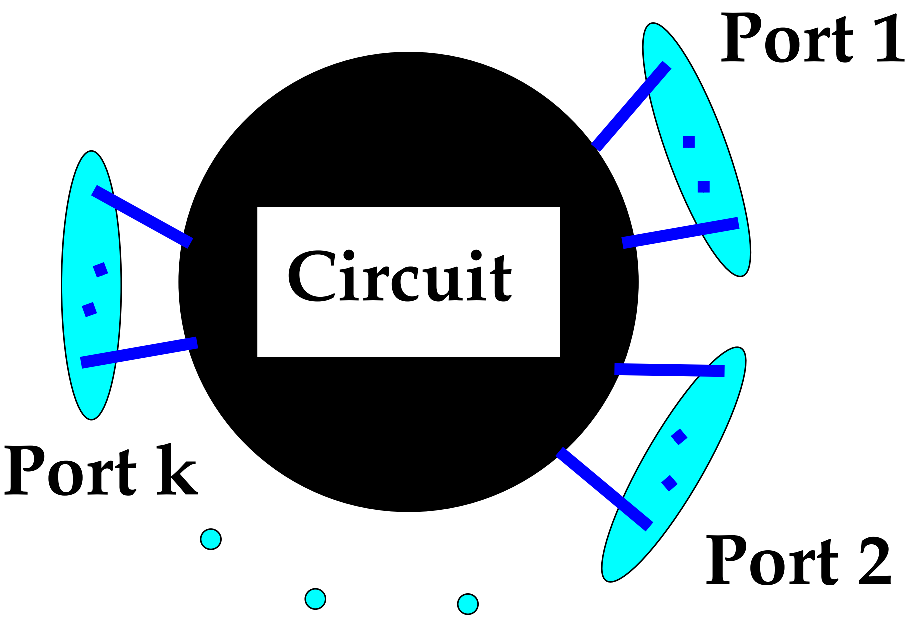
$$\left(\boxed{V_1, I_1 \ldots, V_p, I_p}, V_{p+1}, \ldots, I_n\right) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$$

$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_p + \alpha, I_p}, V_{p+1}, \ldots, I_n\right) \in \mathscr{B}$$

$$\boxed{I_1 + \cdots + I_p} = 0$$

Terminals versus ports



Port :$\Leftrightarrow$

sum currents = 0
potentials + constant
$\Rightarrow$ **potentials**

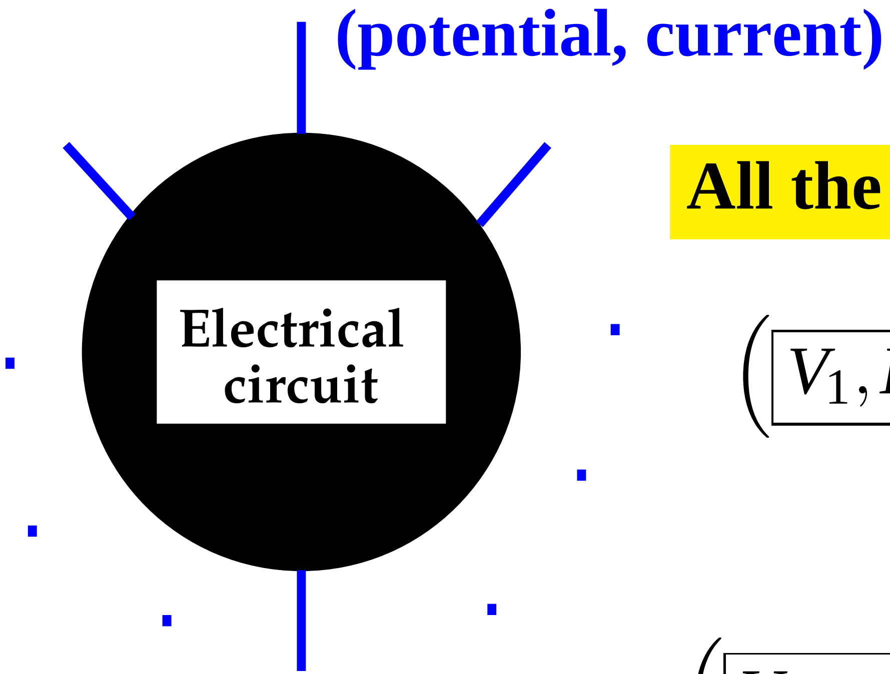
The behavioral equations contain the variables $V_1, V_2 \ldots, V_p$ **only as the differences**

$$V_i - V_j \qquad \textbf{for } i,j = 1, \ldots p$$

and contain the equation

$$I_1 + I_2 + \cdots + I_p = 0$$

Terminals versus ports



All the terminals together form a port

$$\left(\boxed{V_1, I_1 \ldots, V_{\mathtt{n}}, I_{\mathtt{n}}}\right) \in \mathscr{B}, \alpha : \mathbb{R} \to \mathbb{R}$$

$$\Downarrow$$

$$\left(\boxed{V_1 + \alpha, I_1, \ldots, V_{\mathtt{n}} + \alpha, I_{\mathtt{n}}}\right) \in \mathscr{B}$$

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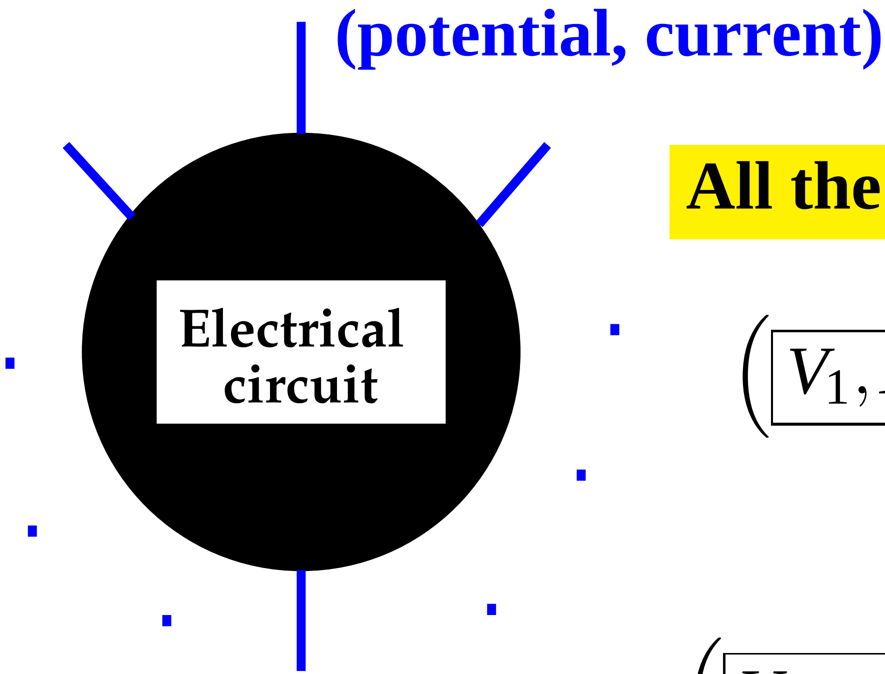
Terminals versus ports



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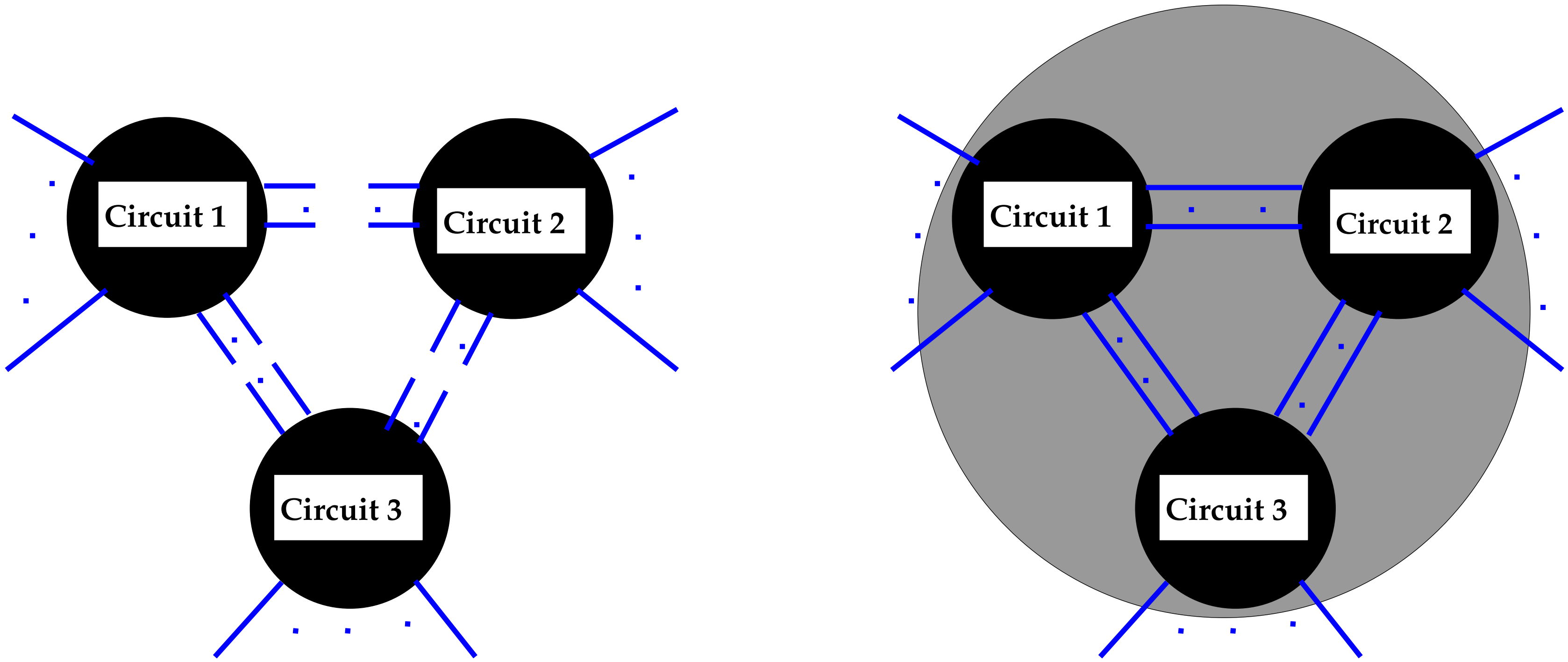
$$\boxed{I_1 + \cdots + I_n} = 0$$

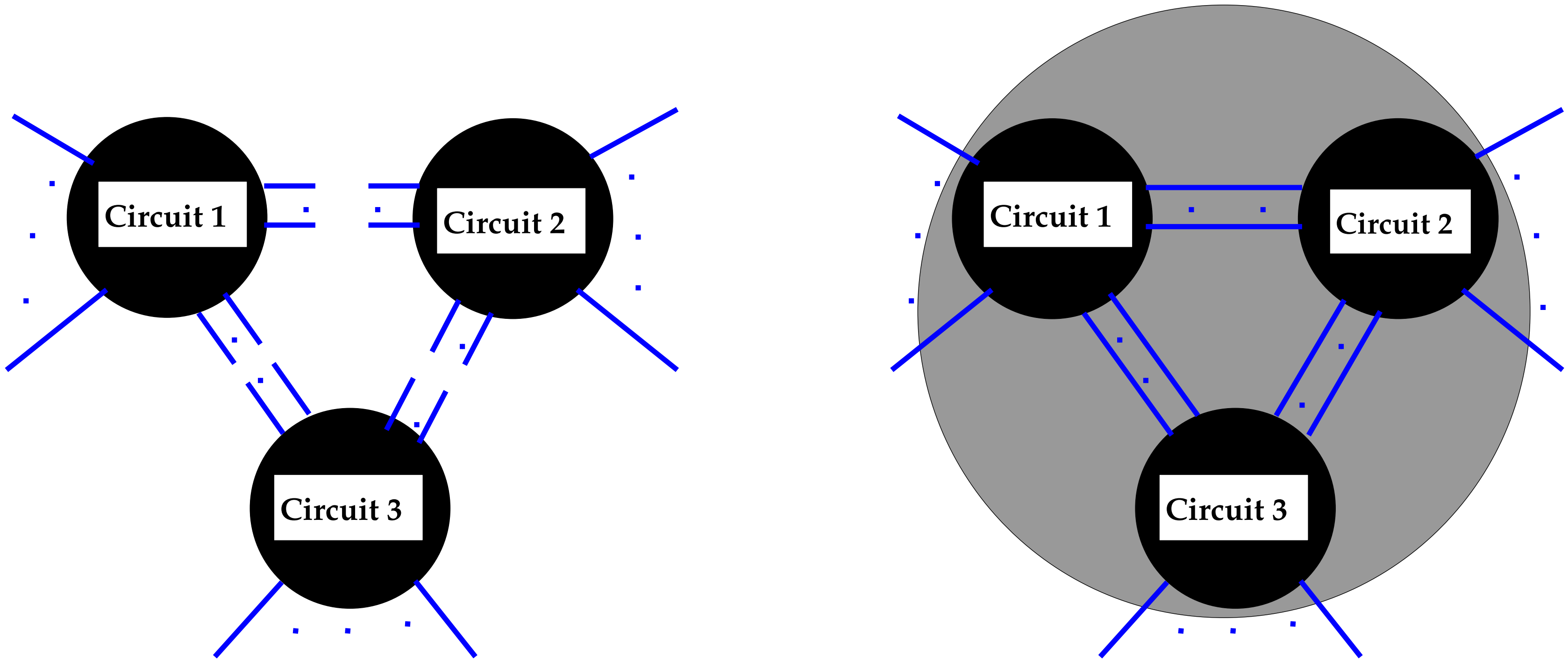
Viewed as 'laws' governing electrical circuits, these may be thought of as the Kirchhoff laws, KVL & KCL ,

This property is closed under interconnection.

Terminals versus ports



Terminals versus ports



Interconnection via terminals, energy transfer via ports.
One cannot speak about

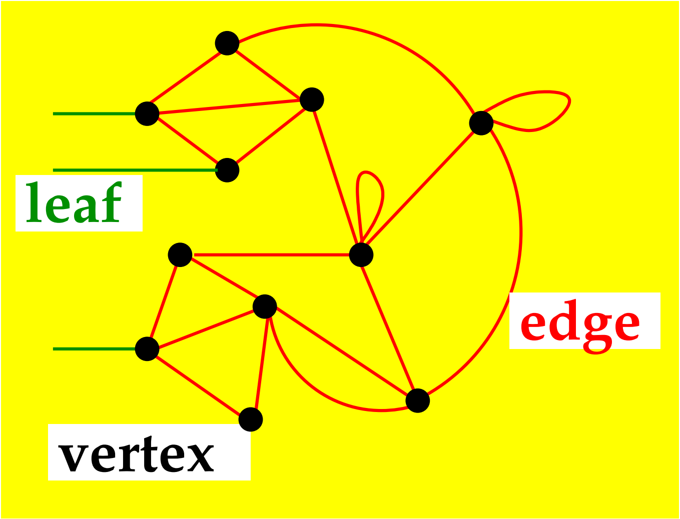
“the energy transferred from circuit 1 to circuit 2”

unless their interconnected terminals form a port.

Hierarchy

New modules from old ones

Tearing, zooming, linking is hierarchical:



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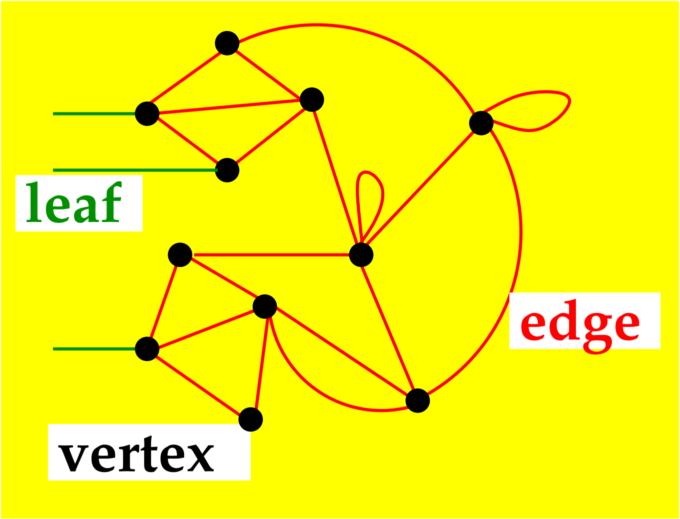

Embed modules in vertices, obtain behavioral equations for the interconnected system, eliminate the latent variables,

New modules from old ones

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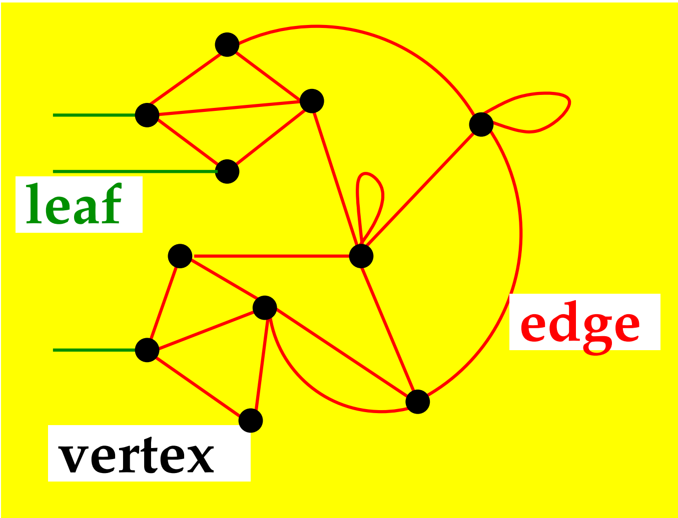
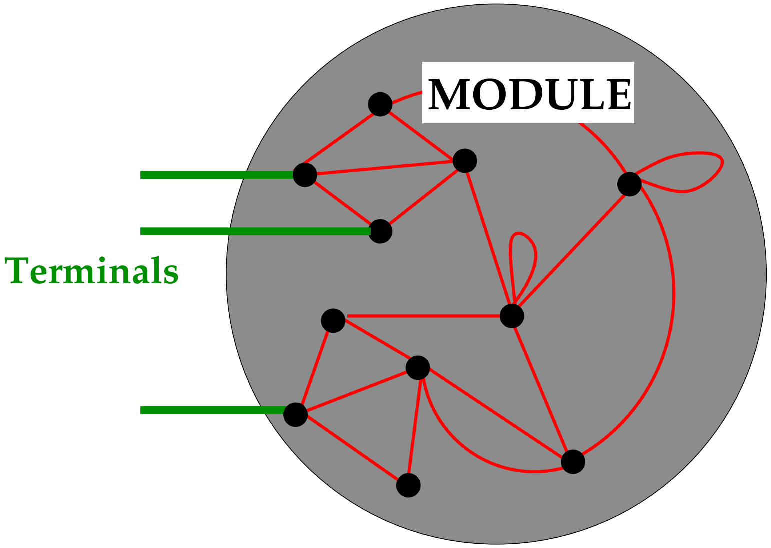
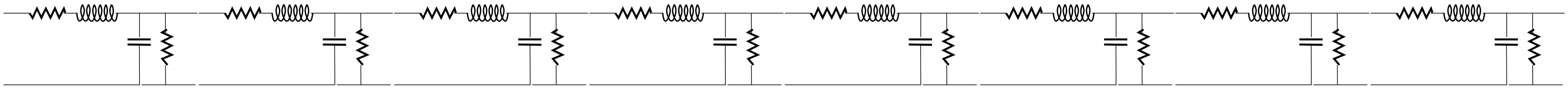




Embed modules in vertices, obtain behavioral equations for the interconnected system, eliminate the latent variables, and use interconnected system as a module with terminals in a new interconnection architecture.

Example

Model the behavior of the external terminal voltages and currents of the following circuit:



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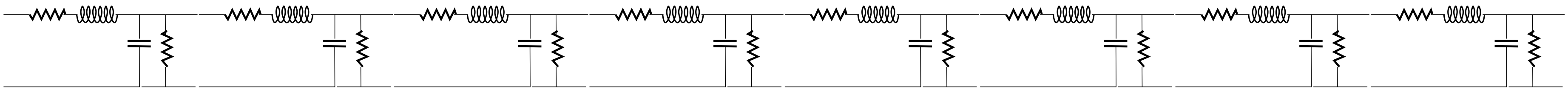
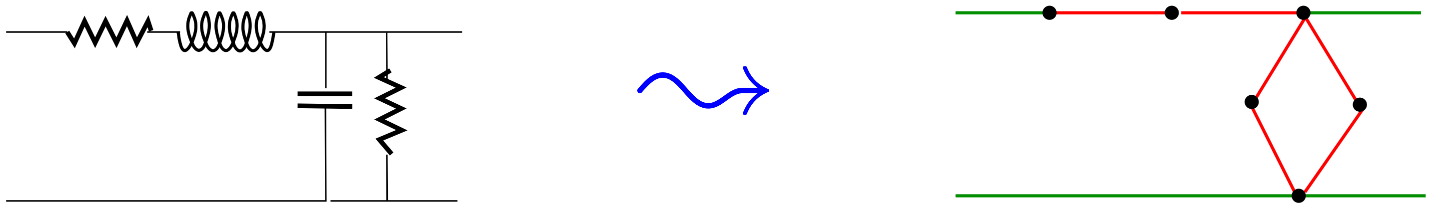


One section:



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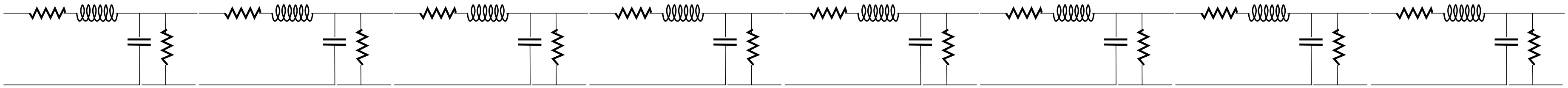

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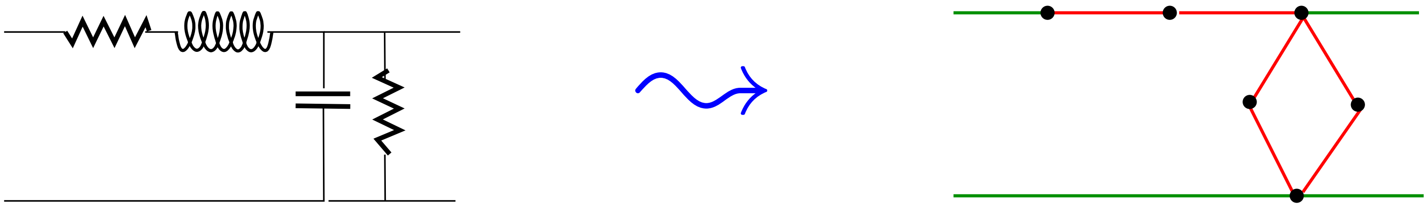
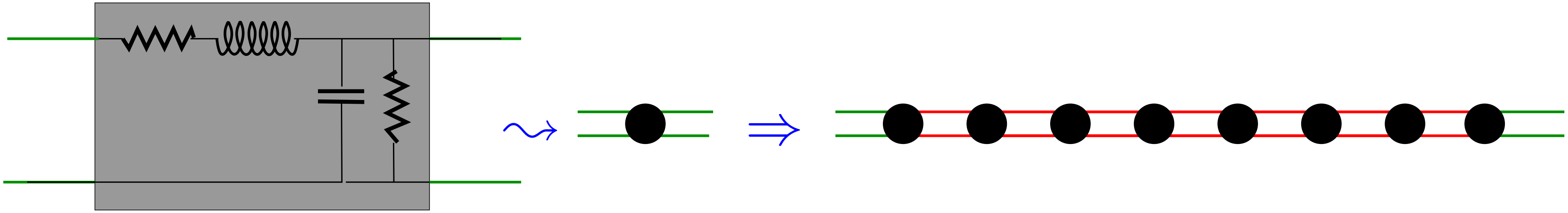


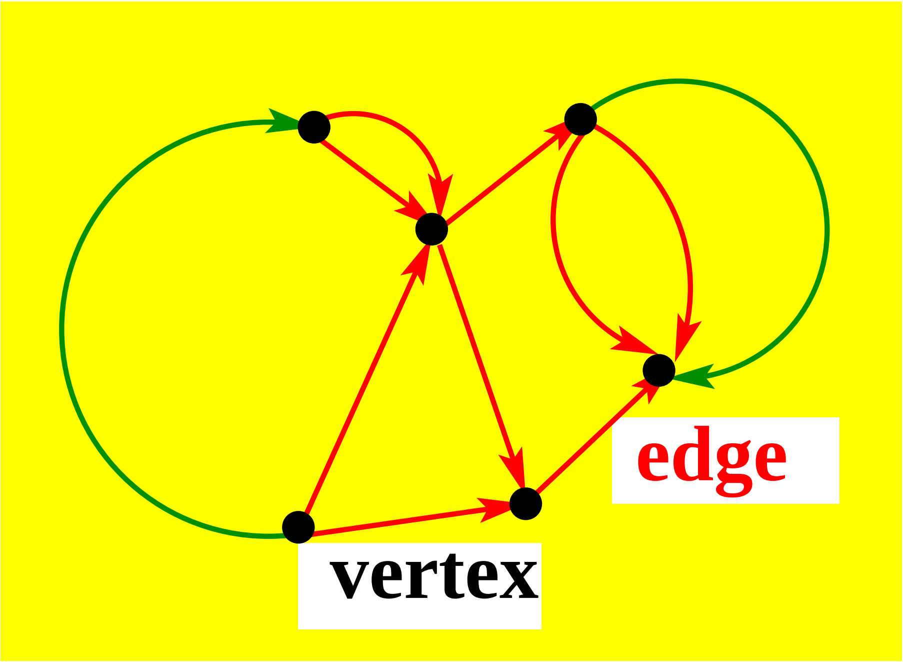
Hierarchical combination:



Circuit diagrams

Circuits and graphs

Classical circuit theory evolves around a digraph with 2-terminal elements or external ports in the edges and connections in the vertices.



Circuits and graphs

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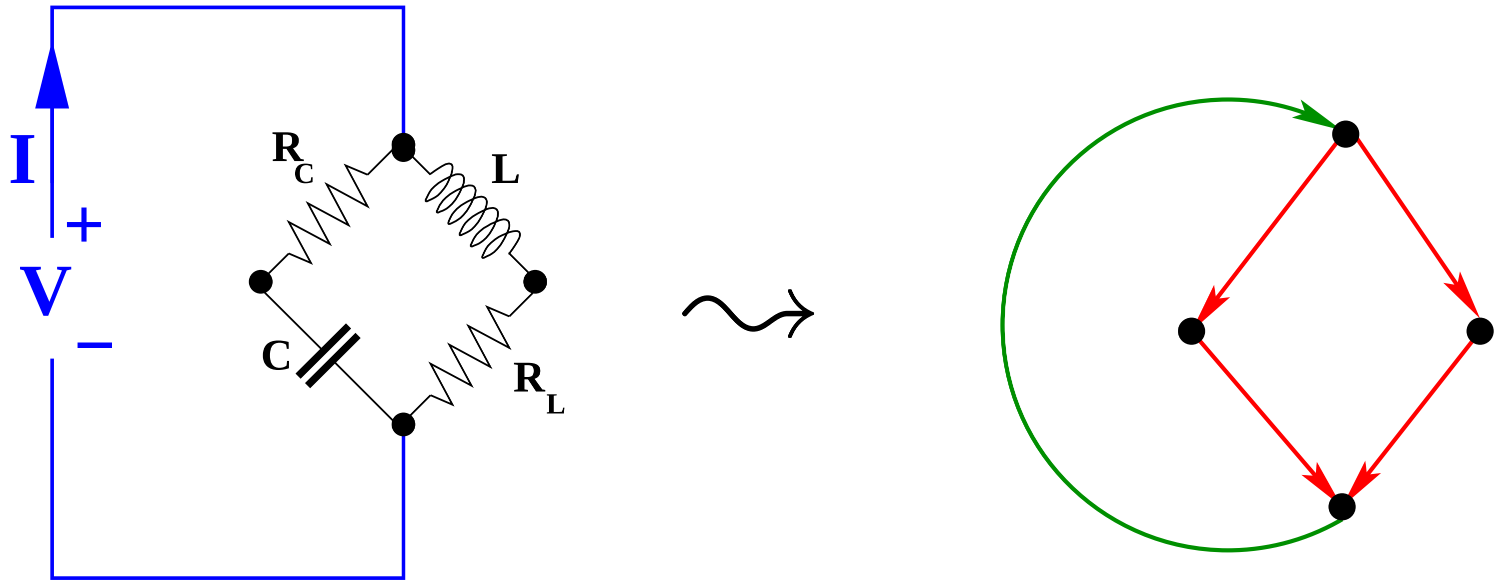
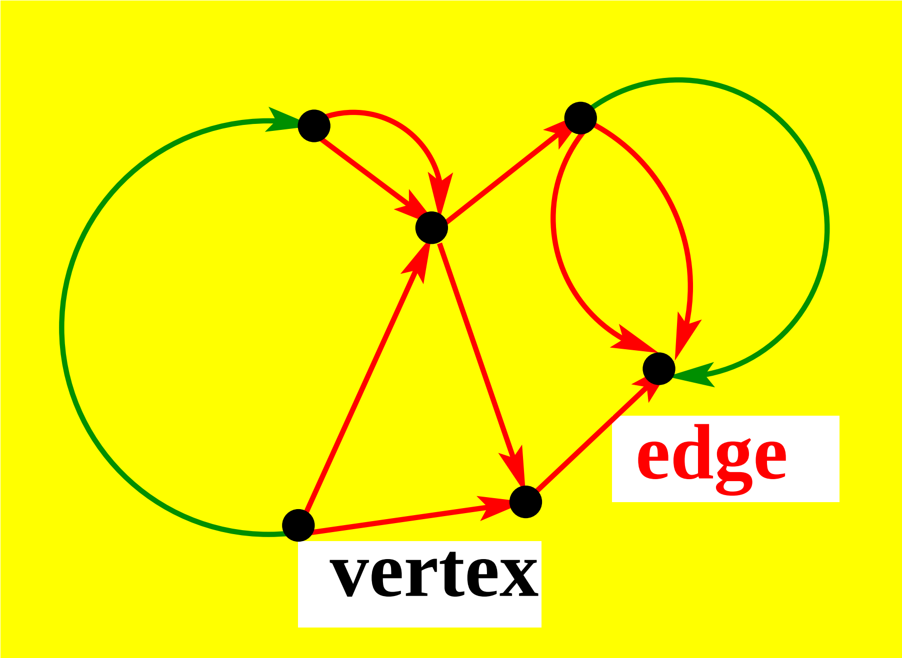


Circuits and graphs

Classical circuit theory evolves around a digraph with 2-terminal elements or external ports in the edges and connections in the vertices.



Associate a voltage drop and a current with each edge, and embed an element (say, R, L, or C) in each 'internal' edge.

KVL & KCL

Basic laws:

Kirchhoff's current law **for each vertex:**

$$\sum_{\text{edges adjacent to vertex}} \pm \textbf{ currents in edges } = 0$$

Kirchhoff's voltage law **for each cycle:**

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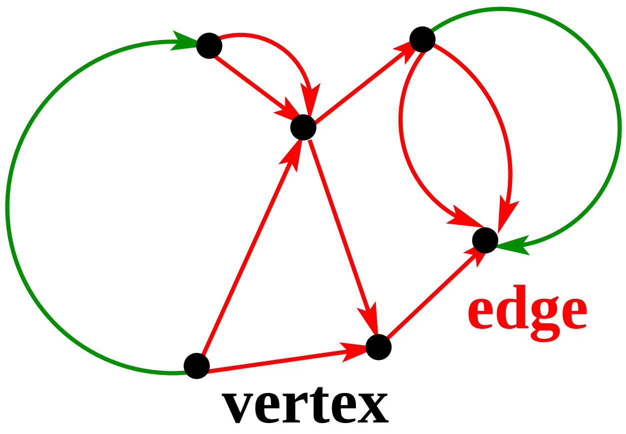
Combined with the constitutive laws of the elements in the 'internal' edges, this yields equations for the behavior (say, of the voltages and currents of the external ports).

Limitations

This methodology is very limited:

- **It can only deal with 2-terminal elements and 2-terminal external ports.**
- **It is purely port oriented. It does not articulate that terminals, not ports make the interconnections.**
- **It is not hierarchical**
  An already-modeled-circuit cannot be reused as a subsystem in a larger circuit diagram.

Embedding a circuit in a graph



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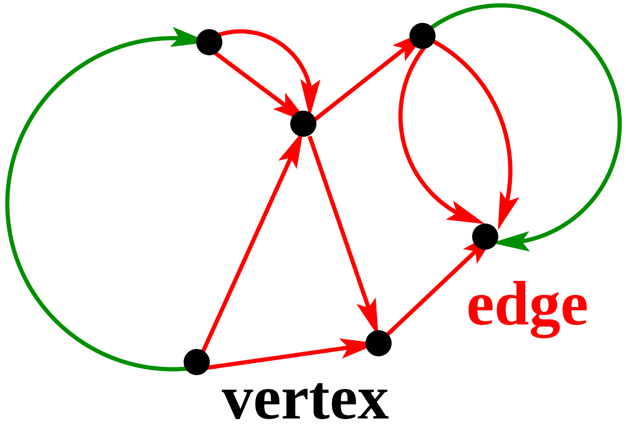

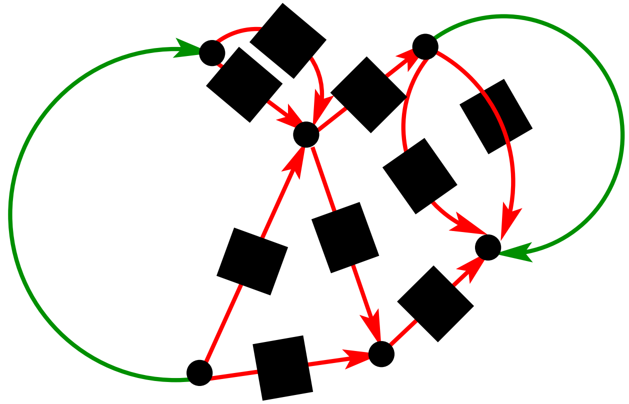

Perfect for 2-terminal one-ports

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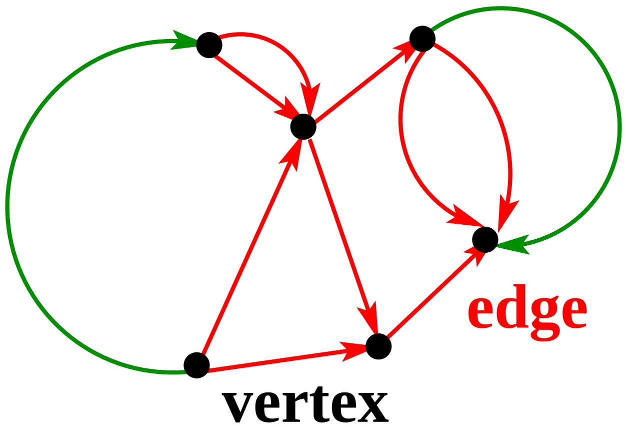

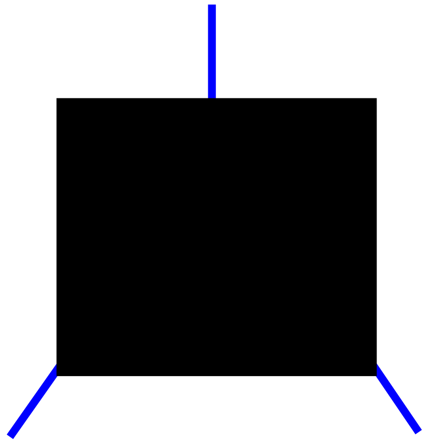

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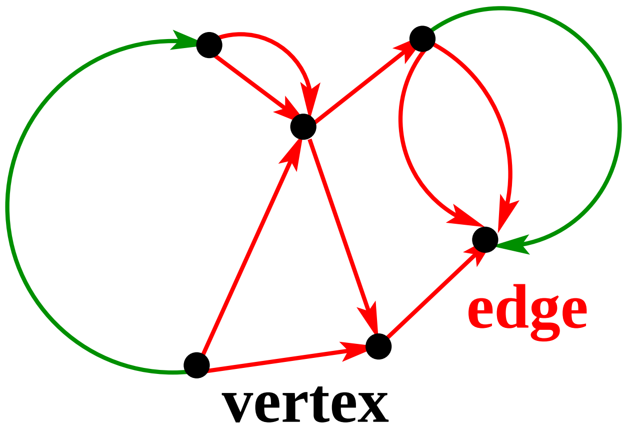
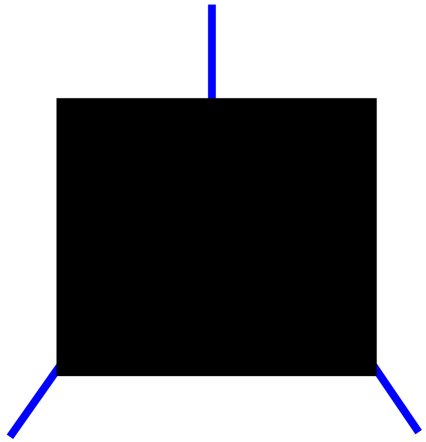
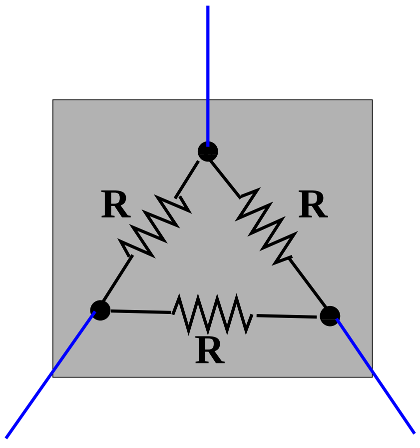
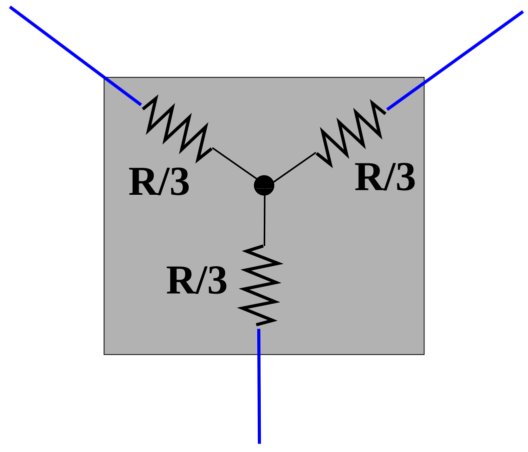
Embedding a circuit in a graph



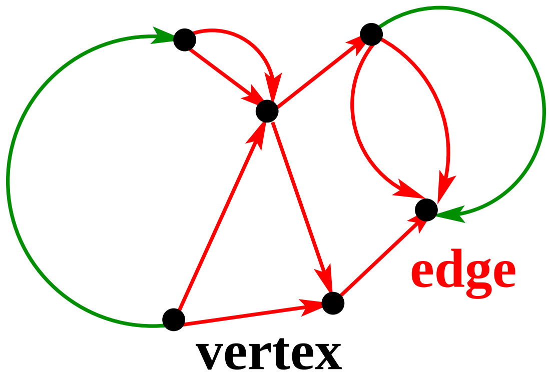


There is no way to embed a 3-terminal circuit in a circuit graph, unless we tear the blackbox into its components



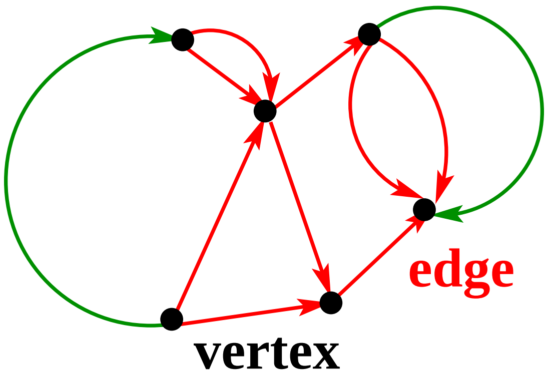


Embedding a circuit in a graph



If we imbed a 4-terminal circuit into a circuit graph, it has to be a 2-port.

Embedding a circuit in a graph



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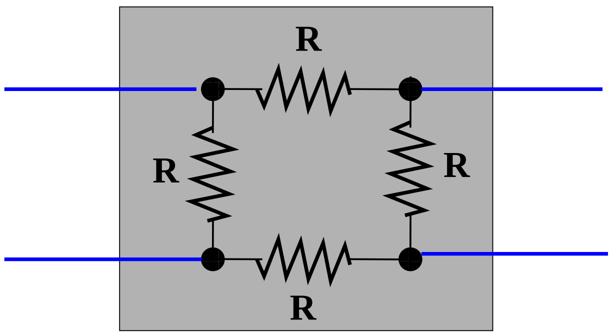
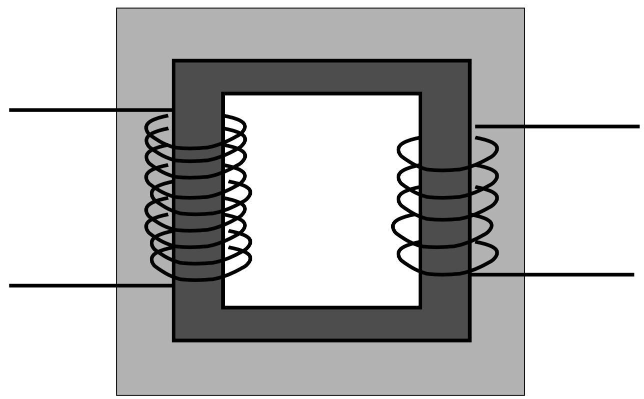


not embeddable

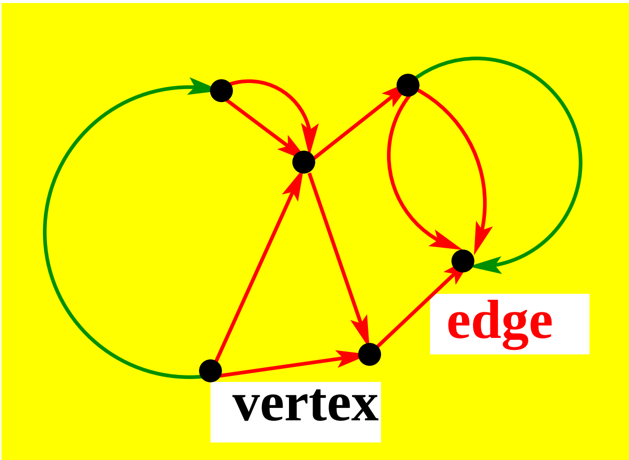


embeddable

Vertices and edges

In circuit graphs,

subsystems are in the edges , connections are in the vertices



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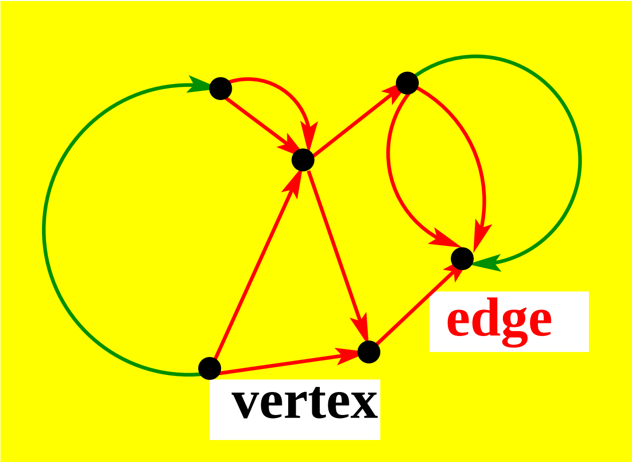

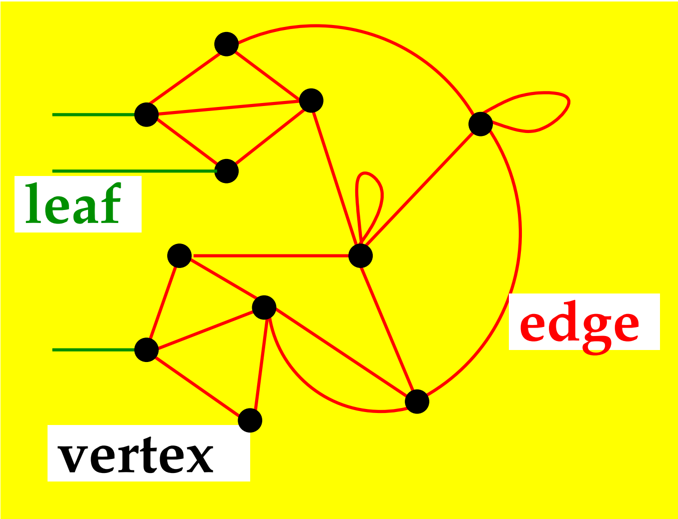

Contrast with tearing, zooming, linking:

subsystems are in the vertices , **connections are in the edges**

Various facets of control

Summary

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- **Control is interconnection, sensor output to actuator input feedback important special case**
- **Need generalization to distributed terminals, etc.**

Overview

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- Extends seamlessly to PDEs

1. **A dynamical system = a family of trajectories.**
2. **Interconnection = variable sharing**
3. **Control = interconnection**

Want to read about it? See

The behavioral approach to open and interconnected systems,

***Control Systems Magazine*, Volume 27, pages 46-99, 2007.**

The lecture frames are available from/at

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