



# **CONTROLLABILITY & OBSERVABILITY**

**in a**

**NEW PERSPECTIVE**

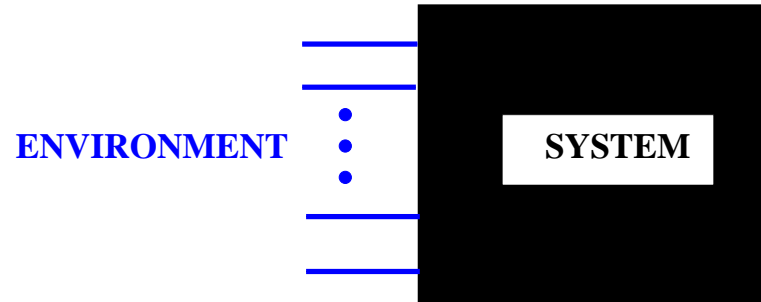
**Jan C. Willems  
K.U. Leuven, Belgium**

**Seminar ANU, Canberra**

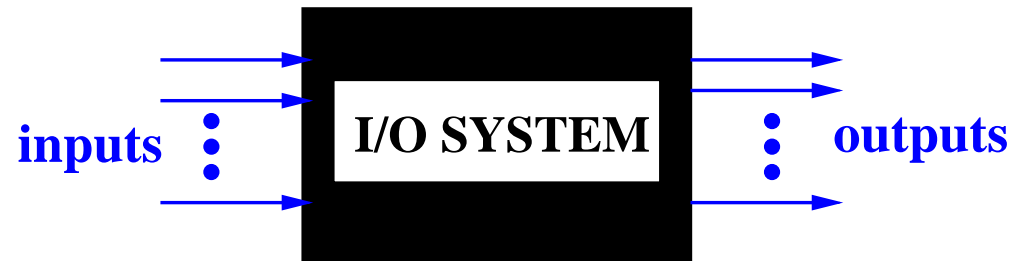
**February 22, 2007**

# Motivation

# Open and Connected



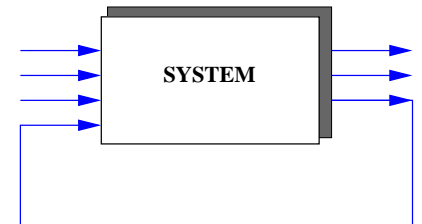
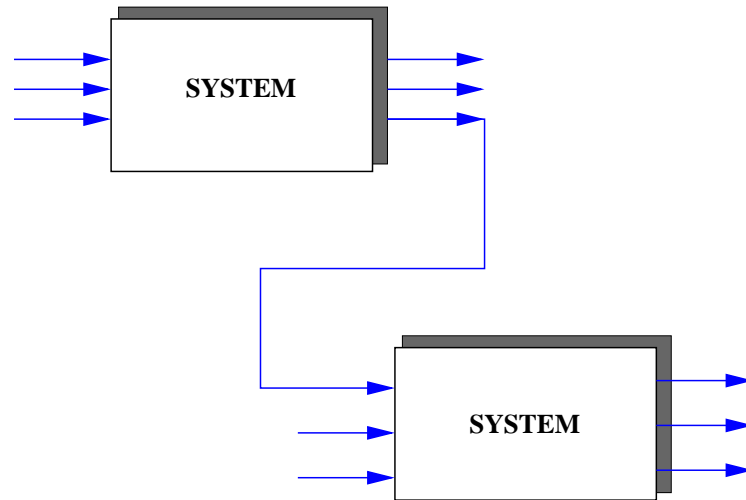
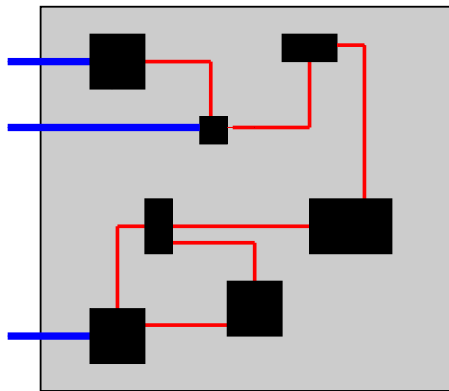
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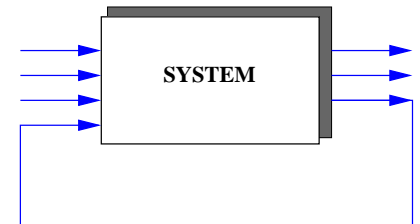
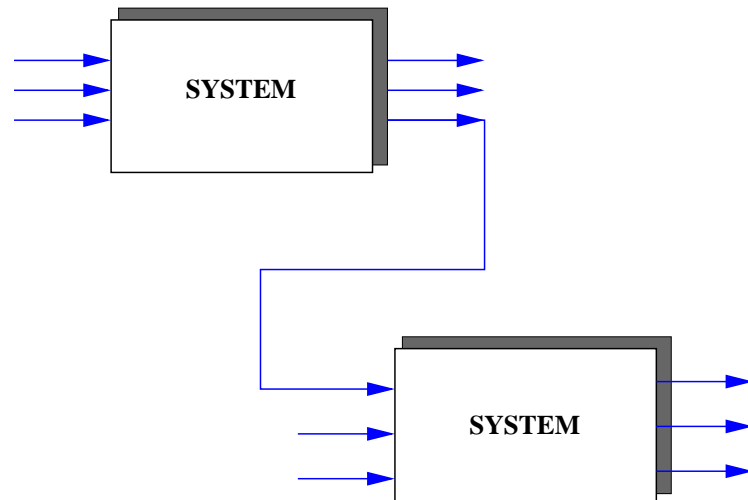
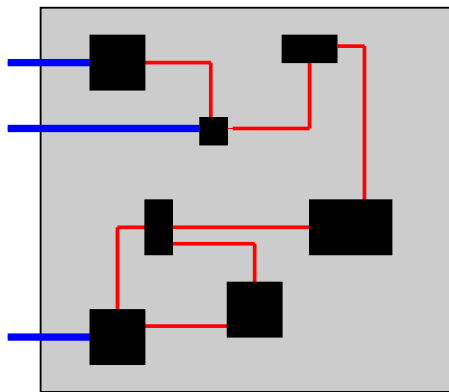
and an interconnection as a **output-to-input assignment**.



# Open and Connected

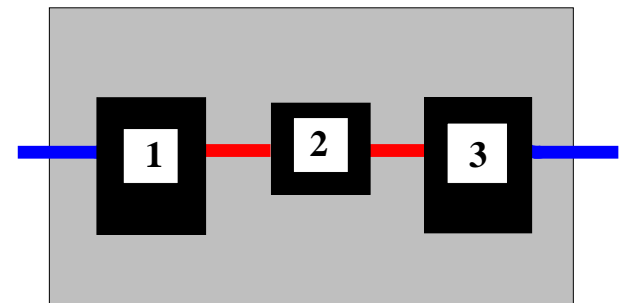
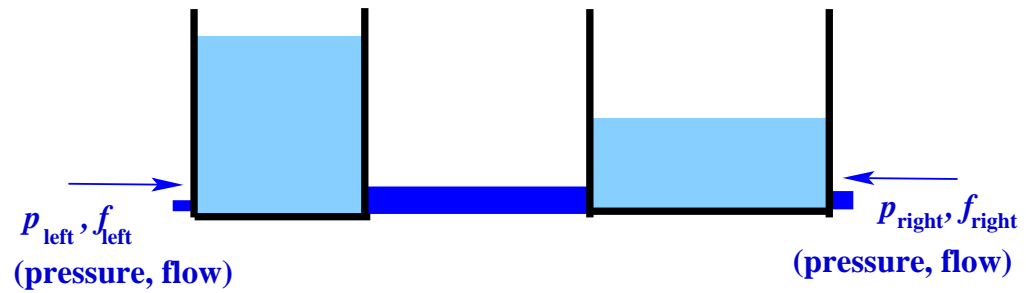
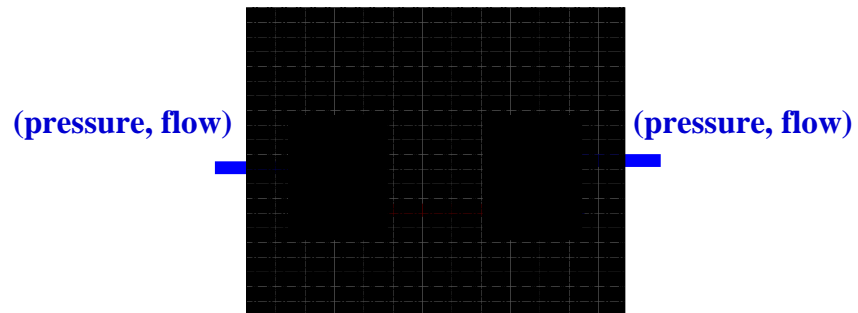
In system theory, we are accustomed to view a dynamical system as an **input/output map**

and an interconnection as a **output-to-input assignment**.



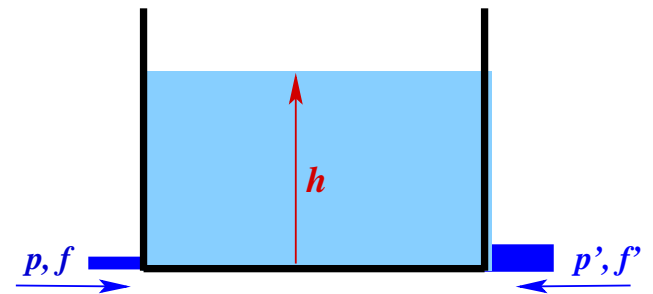
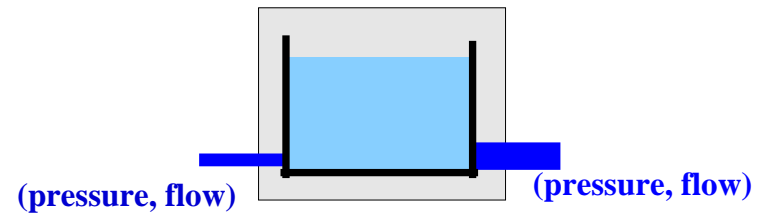
Is this appropriate for modeling **physical** systems?  
If not, how should we proceed instead?

# Example



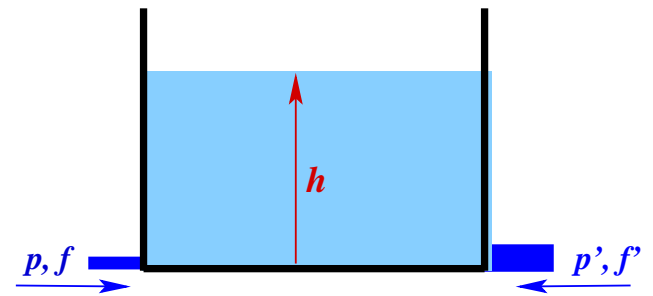
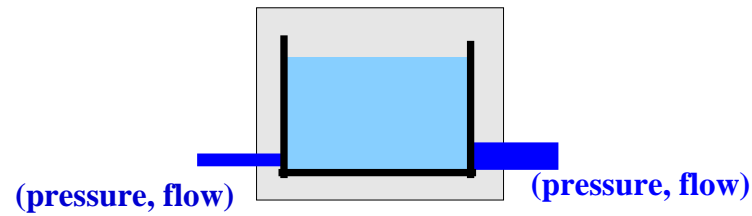
# Example

## Subsystems 1 and 3:



# Example

## Subsystems 1 and 3:



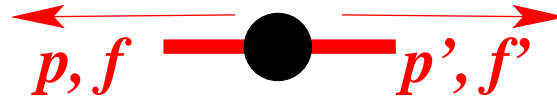
## Subsystem 2:





## Example

Interconnection laws:



$$p = p', \quad f + f' = 0.$$

$$\begin{aligned}
A_1 \frac{d}{dt} h_1 &= f_1 + f'_1, \\
B_1 f_1 &= \begin{cases} \sqrt{|p_1 - p_0 - \rho h_1|} & \text{if } p_1 - p_0 \geq \rho h_1, \\ -\sqrt{|p_1 - p_0 - \rho h_1|} & \text{if } p_1 - p_0 \leq \rho h_1, \end{cases} \\
C f'_1 &= \begin{cases} \sqrt{|p'_1 - p_0 - \rho h_1|} & \text{if } p'_1 - p_0 \geq \rho h_1, \\ -\sqrt{|p'_1 - p_0 - \rho h_1|} & \text{if } p'_1 - p_0 \leq \rho h_1, \end{cases}
\end{aligned} \tag{1}$$

$$f_2 = -f'_2, \quad p_2 - p'_2 = \alpha f_2, \tag{2}$$

$$\begin{aligned}
A_3 \frac{d}{dt} h_3 &= f_3 + f'_3, \\
C f_3 &= \begin{cases} \sqrt{|p_3 - p_0 - \rho h_3|} & \text{if } p_3 - p_0 \geq \rho h_3, \\ -\sqrt{|p_3 - p_0 - \rho h_3|} & \text{if } p_3 - p_0 \leq \rho h_3, \end{cases} \\
C_3 f'_3 &= \begin{cases} \sqrt{|p'_3 - p_0 - \rho h_3|} & \text{if } p'_3 - p_0 \geq \rho h_3, \\ -\sqrt{|p'_3 - p_0 - \rho h_3|} & \text{if } p'_3 - p_0 \leq \rho h_3, \end{cases}
\end{aligned} \tag{3}$$

$$p'_1 = p_2, \quad f'_1 + f_2 = 0, \quad p'_2 = p_3, \quad f'_2 + f_3 = 0. \tag{4}$$

$$p_{\text{left}} = p_1, \quad f_{\text{left}} = f_1, \quad p_{\text{right}} = p'_3, \quad f_{\text{right}} = f'_3. \tag{5}$$

## Conclusion

- **Unclear input/output structure for terminal variables**
- **Many variables, indivisibly, at the same terminal**
- **Interconnection = variable sharing**
- **...**

## Conclusion

- Unclear input/output structure for terminal variables
- Many variables, indivisibly, at the same terminal
- Interconnection = variable sharing
- ...

*“Block diagrams unsuitable for serious physical modeling*

*- the control/physics barrier”*

*“Behavior based (declarative) modeling is a good alternative”*



from K.J. Åström

*Present Developments in Control Applications*



**IFAC 50-th Anniversary Celebration  
Heidelberg, September 12, 2006.**

## Remedy

### A dynamical system

$:\Leftrightarrow$  a family of time functions, *'the behavior'*.

**Interconnection**  $:\Leftrightarrow$  *'variable sharing'*.

Even though modeling of interconnected physical systems may be the strongest case for 'behaviors', I will not deal with this today.

# Concepts

## Models

A dynamical system  $:\Leftrightarrow (\mathbb{T}, \mathbb{W}, \mathfrak{B})$

$\mathbb{T} \subseteq \mathbb{R}$  ‘time set’

$\mathbb{W}$  ‘signal space’

$\mathfrak{B} \subseteq \mathbb{W}^{\mathbb{T}}$  the ‘behavior’

a family of trajectories  $\mathbb{T} \rightarrow \mathbb{W}$

henceforth, today,  $\mathbb{T} = \mathbb{R}, \mathbb{W} = \mathbb{R}^w$ .

Hence today  $\mathfrak{B}$  is a family of vector-valued continuous-time trajectories

$w : \mathbb{R} \rightarrow \mathbb{R}^w \in \mathfrak{B}$  means “ $w$  is compatible with the model”

$w : \mathbb{R} \rightarrow \mathbb{R}^w \notin \mathfrak{B}$  means “the models forbids  $w$ ”

## Models

The dynamical system  $(\mathbb{R}, \mathbb{R}^w, \mathfrak{B}) \rightsquigarrow \mathfrak{B}$

**linear**  $:\Leftrightarrow w_1, w_2 \in \mathfrak{B}, \alpha \in \mathbb{R}, \text{ imply } \alpha w_1 + w_2 \in \mathfrak{B}$

**time-invariant**  $:\Leftrightarrow w \in \mathfrak{B}, \sigma \text{ any shift, imply } \sigma w \in \mathfrak{B}$

**differential**  $:\Leftrightarrow$  ‘described’ by an ODE. ‘LTIDS’

$$R_0 w + R_1 \frac{d}{dt} w + \cdots + R_n \frac{d^n}{dt^n} w = 0.$$

$\rightsquigarrow$   $R \left( \frac{d}{dt} \right) w = 0$   $R$  typically ‘wide’  **LTIDS**

$R = R_0 + R_1 \xi + \cdots + R_n \xi^n$  **polynomial matrix.**

**Defines**  $\mathfrak{B} = \text{kernel} \left( R \left( \frac{d}{dt} \right) \right)$  ‘*kernel representation*’ of  $\mathfrak{B}$



## Models

For example,

$$P \left( \frac{d}{dt} \right) y = Q \left( \frac{d}{dt} \right) u, \quad w = \begin{bmatrix} u \\ y \end{bmatrix}, \quad P, Q \text{ polynomial matrices}$$

$$\frac{d}{dt}x = Ax + Bu, \quad y = Cx + Du, \quad w = \begin{bmatrix} u \\ y \\ x \end{bmatrix} \quad \text{or} \quad w = \begin{bmatrix} u \\ y \end{bmatrix}$$

$$y = G \left( \frac{d}{dt} \right) u, \quad w = \begin{bmatrix} u \\ y \end{bmatrix}, \quad P, Q \text{ matrices of rational f'ns}$$

$$\text{DAE's} \quad F \frac{d}{dt}x + Gx + Hw = 0$$

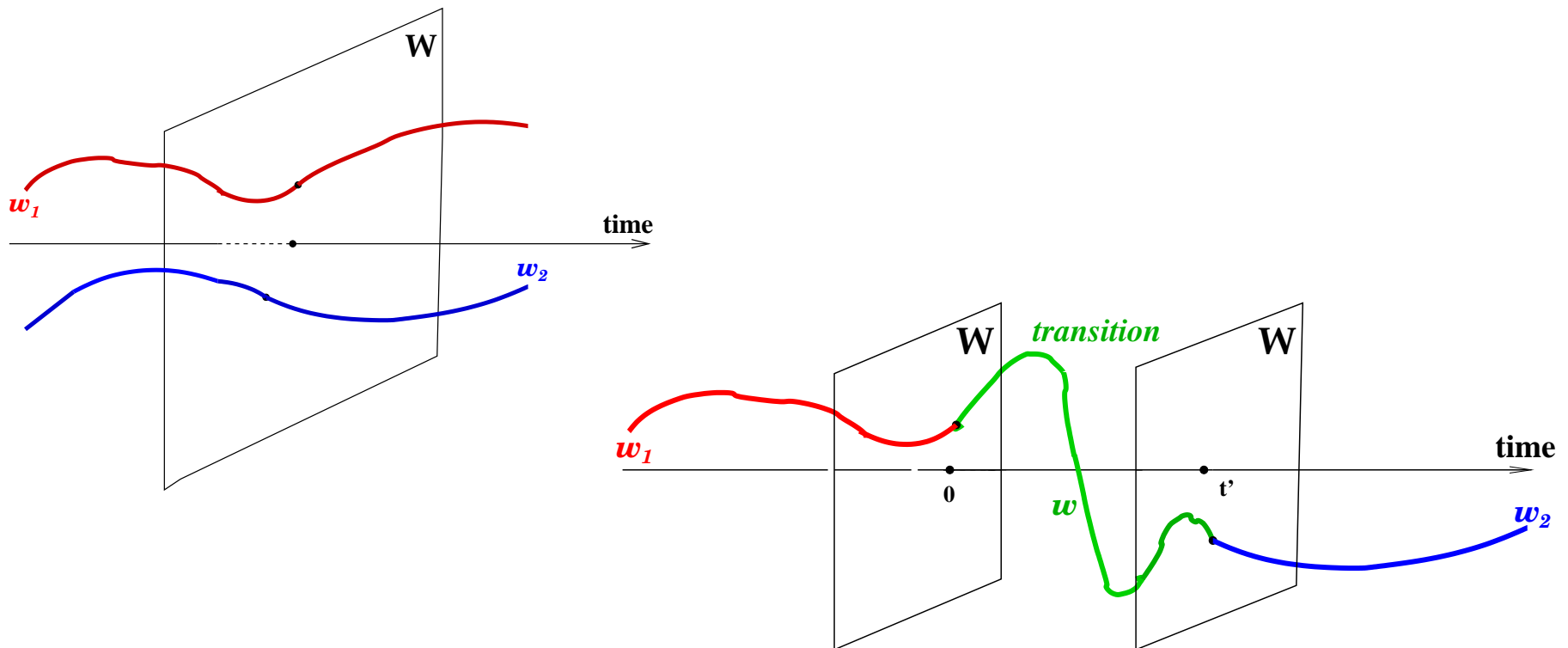
etc.

# Controllability

The time-invariant system  $(\mathbb{R}, \mathbb{R}^w, \mathfrak{B})$   $\rightsquigarrow$   $\mathfrak{B} \subseteq (\mathbb{R}^w)^{\mathbb{R}}$

**controllable**  $:\Leftrightarrow$

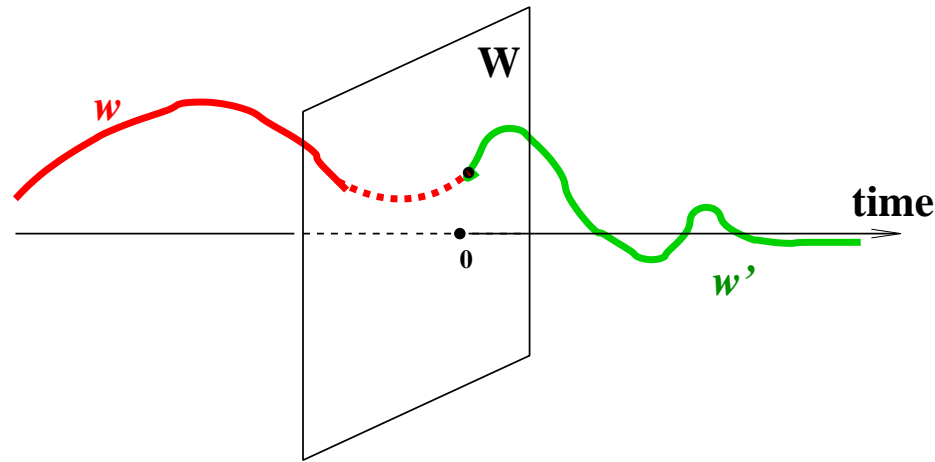
for all  $w_1, w_2 \in \mathfrak{B}$ , exists  $w \in \mathfrak{B}$  and  $T \geq 0$  such that



# Controllability

The time-invariant system  $(\mathbb{R}, \mathbb{R}^w, \mathfrak{B}) \rightsquigarrow \mathfrak{B} \subseteq (\mathbb{R}^w)^{\mathbb{R}}$

**stabilizable**  $:\Leftrightarrow$  for all  $w \in \mathfrak{B}$ , exists  $w' \in \mathfrak{B}$  such that



**stable**  $:\Leftrightarrow w \in \mathfrak{B}$  implies  $w(t) \rightarrow 0$  for  $t \rightarrow \infty$

**autonomous**  $:\Leftrightarrow$

$w_1, w_2 \in \mathfrak{B}, w_1(t) = w_2(t)$  for  $t < 0$  implies  $w_1 = w_2$

# Controllability

The time-invariant system  $(\mathbb{R}, \mathbb{R}^w, \mathfrak{B})$   $\leadsto \mathfrak{B} \subseteq (\mathbb{R}^w)^{\mathbb{R}}$

$$R \left( \frac{d}{dt} \right) w = 0$$

defines a **controllable system** iff

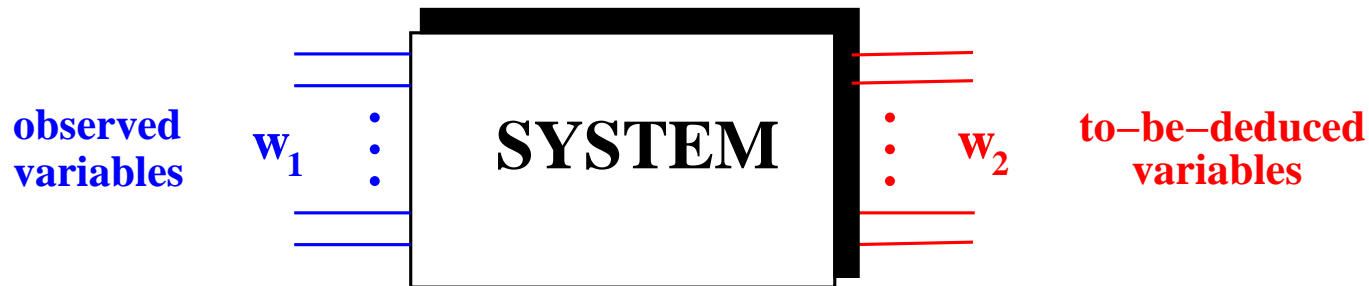
$R(\lambda)$  has the same rank for all  $\lambda \in \mathbb{C}$ .

a **stabilizable system** iff

$R(\lambda)$  has the same rank for all  $\lambda \in \bar{\mathbb{C}}_+$ .

# Observability

Consider the dynamical system  $(\mathbb{R}, \mathbb{R}^{w_1 \times w_2}, \mathfrak{B})$



$w_2$  observable from  $w_1$   $:\Leftrightarrow$

$$(w_1, w_2), (w_1, w'_2) \in \mathfrak{B} \Rightarrow w_2 = w'_2$$

$w_2$  detectable from  $w_1$   $:\Leftrightarrow$

$$(w_1, w_2), (w_1, w'_2) \in \mathfrak{B} \Rightarrow w_2(t) - w'_2(t) \rightarrow 0 \text{ for } t \rightarrow \infty$$

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There exists a map  $F : w_1 \mapsto w_2$  such that

$(w_1, w_2) \in \mathfrak{B} \Rightarrow w_2 = F(w_1)$  recovers  $w_2$  (asymptotically)

There are tests for

$$R_1 \left( \frac{d}{dt} \right) w_1 = R_2 \left( \frac{d}{dt} \right) w_2$$

# **LTIDS: Basic results**

# LTIDS

Recall

$$R \left( \frac{d}{dt} \right) w = 0$$

$R$  a polynomial matrix  $R \in \mathbb{R} [\xi]^{\bullet \times w} \rightsquigarrow \mathcal{L}^w, \mathcal{L}^\bullet$

Fact 1:  $\mathcal{L}^\bullet$  closed under addition, intersection, & projection



**Fact 1:**  $\mathcal{L}^\bullet$  closed under addition, intersection, & **projection**

**Consider**

$$R_1 \left( \frac{d}{dt} \right) w_1 + R_2 \left( \frac{d}{dt} \right) w_2 = 0 \rightsquigarrow \text{behavior } \mathfrak{B}$$

**Define**

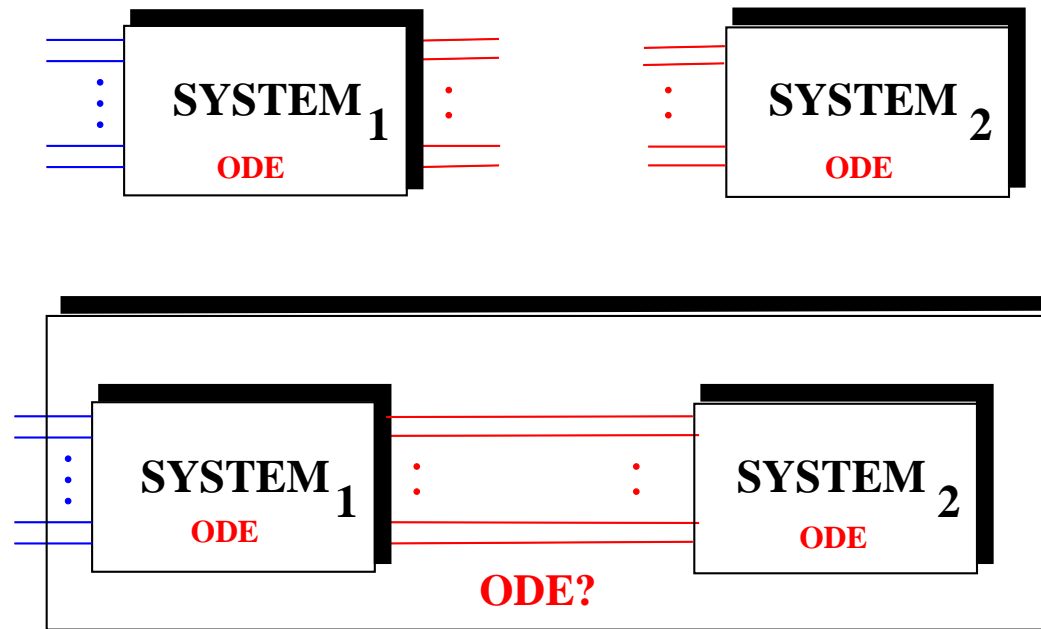
$$\mathfrak{B}_1 := \{w_1 \mid \exists w_2 \text{ such that } (w_1, w_2) \in \mathfrak{B}\}$$

**Elimination thm**  $\exists R$  such that  $\mathfrak{B}_1 = \text{kernel} \left( R \left( \frac{d}{dt} \right) \right)$ !

**E.g.**  $\frac{d}{dt}x = Ax + Bu, y = Cx + Du \Rightarrow P\left(\frac{d}{dt}\right)y = Q\left(\frac{d}{dt}\right)u$   
**linear DAE's always allow elimination of nuisance variables**

# LTIDS

Fact 1:  $\mathcal{L}^\bullet$  closed under addition, intersection, & projection



In LTIDS described by ODE if systems 1 and 2 are.  
In nonlinear case, very unlikely described by ODE, even if systems 1 and 2 are!

**Why are ODE's so common?**

# LTIDS

**Fact 1:**  $\mathcal{L}^\bullet$  closed under addition, intersection, & projection

**Fact 2:** Consequences of  $\mathfrak{B} \in \mathcal{L}^w$ :  $\mathbb{R}[\xi]$ -submodule of  $\mathbb{R}[\xi]^w$

$n \in \mathbb{R}[\xi]^w$  is a consequence of  $\mathfrak{B} : \Leftrightarrow n^\top \left( \frac{d}{dt} \right) \mathfrak{B} = 0$ .

E.g. Observability of

$$R_1 \left( \frac{d}{dt} \right) w_1 = R_2 \left( \frac{d}{dt} \right) w_2$$

equivalent to existence of consequences

$$w_2 = F \left( \frac{d}{dt} \right) w_1$$

# LTIDS

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E.g. detectability of

$$R_1 \left( \frac{d}{dt} \right) w_1 = R_2 \left( \frac{d}{dt} \right) w_2$$

equivalent to existence of consequences

$$H \left( \frac{d}{dt} \right) w_1 = F \left( \frac{d}{dt} \right) w_2, \quad H \text{ Hurwitz}$$

# LTIDS

Fact 1:  $\mathcal{L}^\bullet$  closed under addition, intersection, & projection

Fact 2: Consequences of  $\mathfrak{B} \in \mathcal{L}^w$ :  $\mathbb{R}[\xi]$ -submodule of  $\mathbb{R}[\xi]^w$

Fact 3: Controllability of  $\mathfrak{B} \in \mathcal{L}^w \Leftrightarrow \exists$  *image repr'ion*

Consider  $w = M \left( \frac{d}{dt} \right) \ell$

i.e.,  $w$ -behavior  $\mathfrak{B} = \text{image} \left( M \left( \frac{d}{dt} \right) \right)$ .

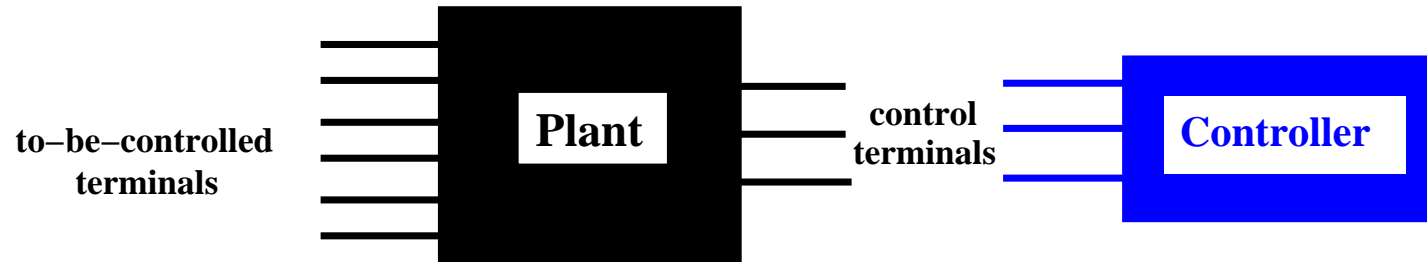
Elimination thm  $\Rightarrow \mathfrak{B} = \text{kernel} \left( R \left( \frac{d}{dt} \right) \right)$ , for some  $R$ .

So, all images are kernels, but what kernels are images?

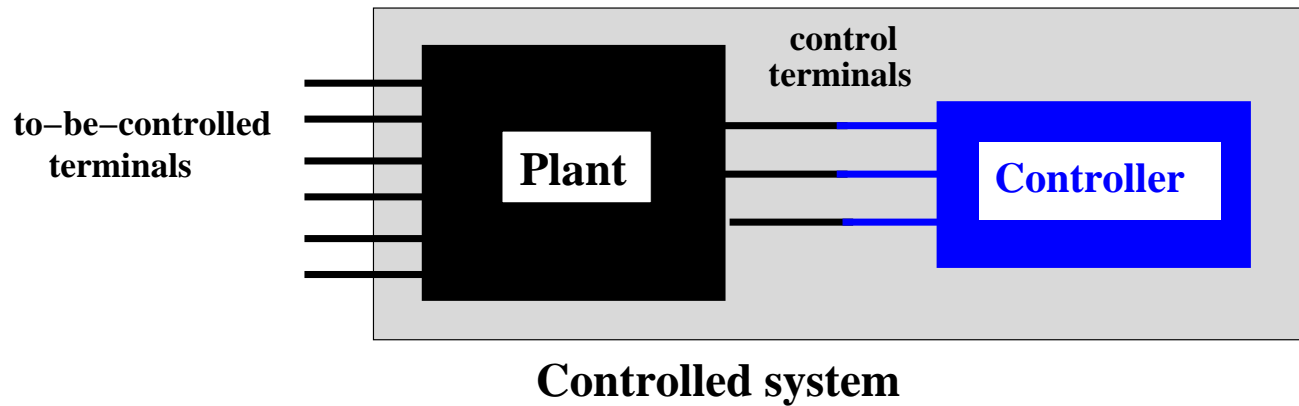
$\Leftrightarrow \mathfrak{B}$  is controllable

**Control**

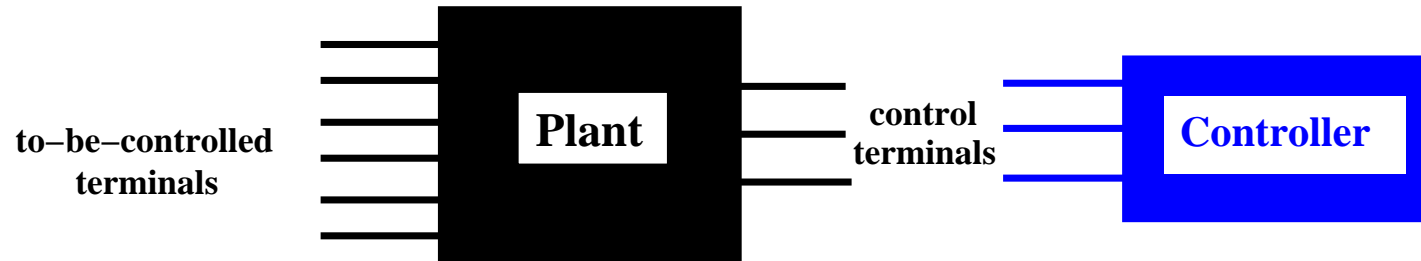
# Control as Interconnection



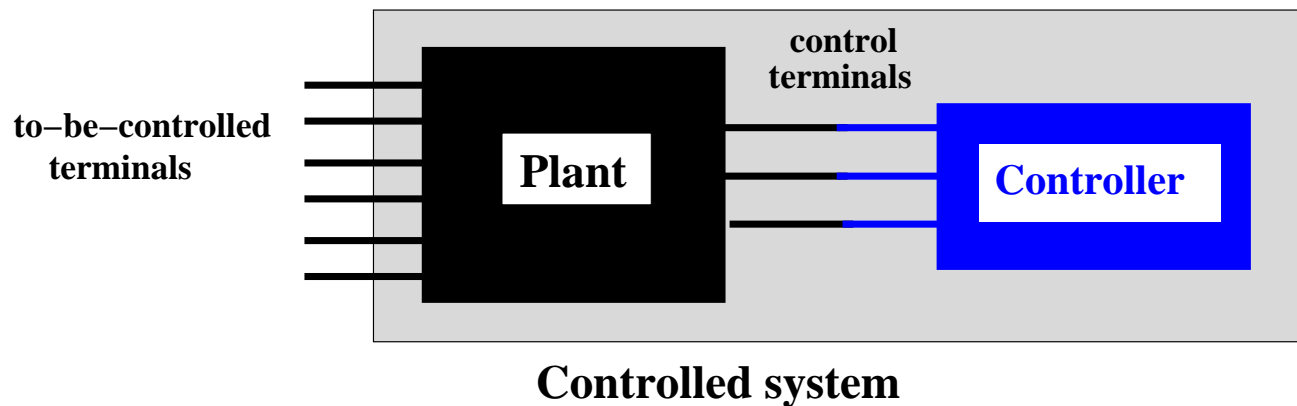
**Interconnect via control terminals:**



# Control as Interconnection



**Interconnect via control terminals:**

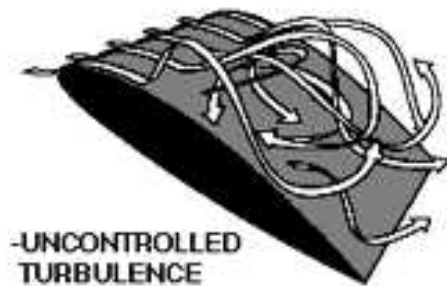


- Are all interconnections ‘reasonable’?
- Which controlled behaviors can be achieved?
- Parametrize all stabilizing controllers

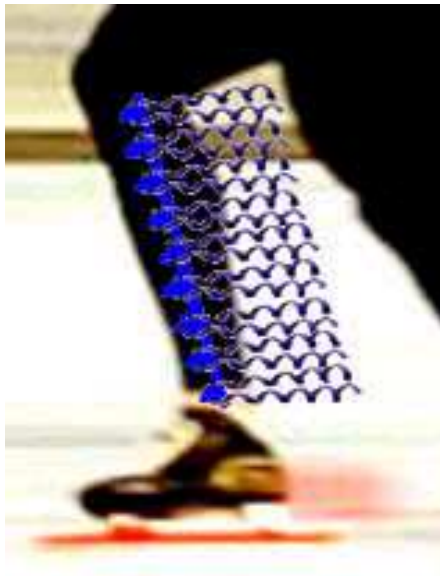
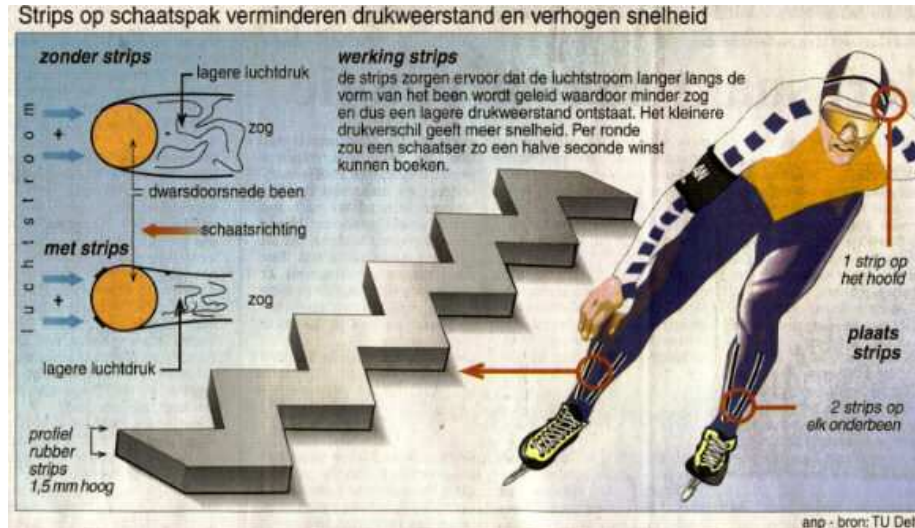


# Many controllers are not sensor-to-actuator

## Controlling turbulence:

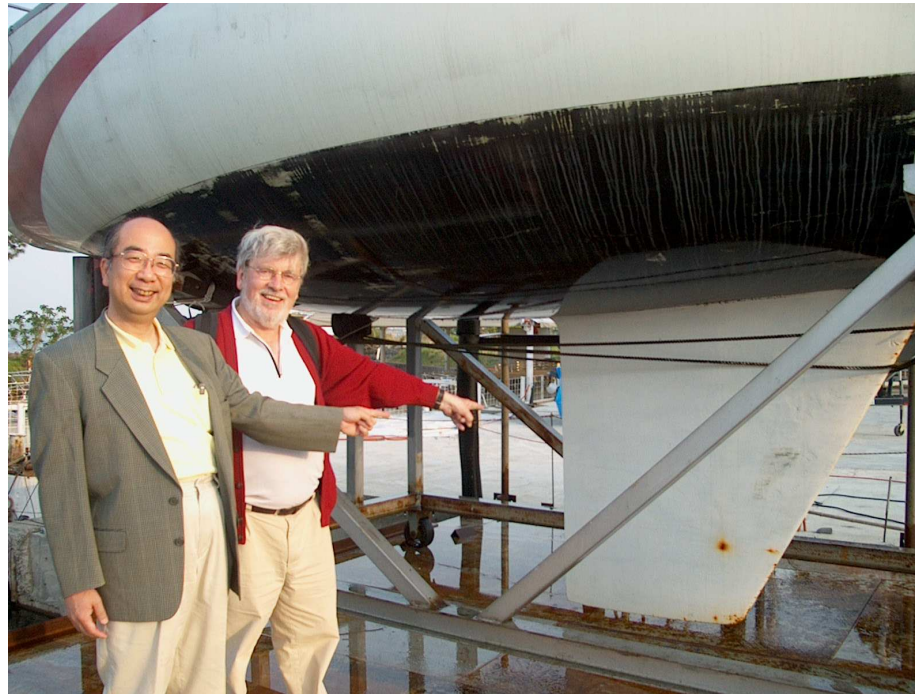


# Many controllers are not sensor-to-actuator



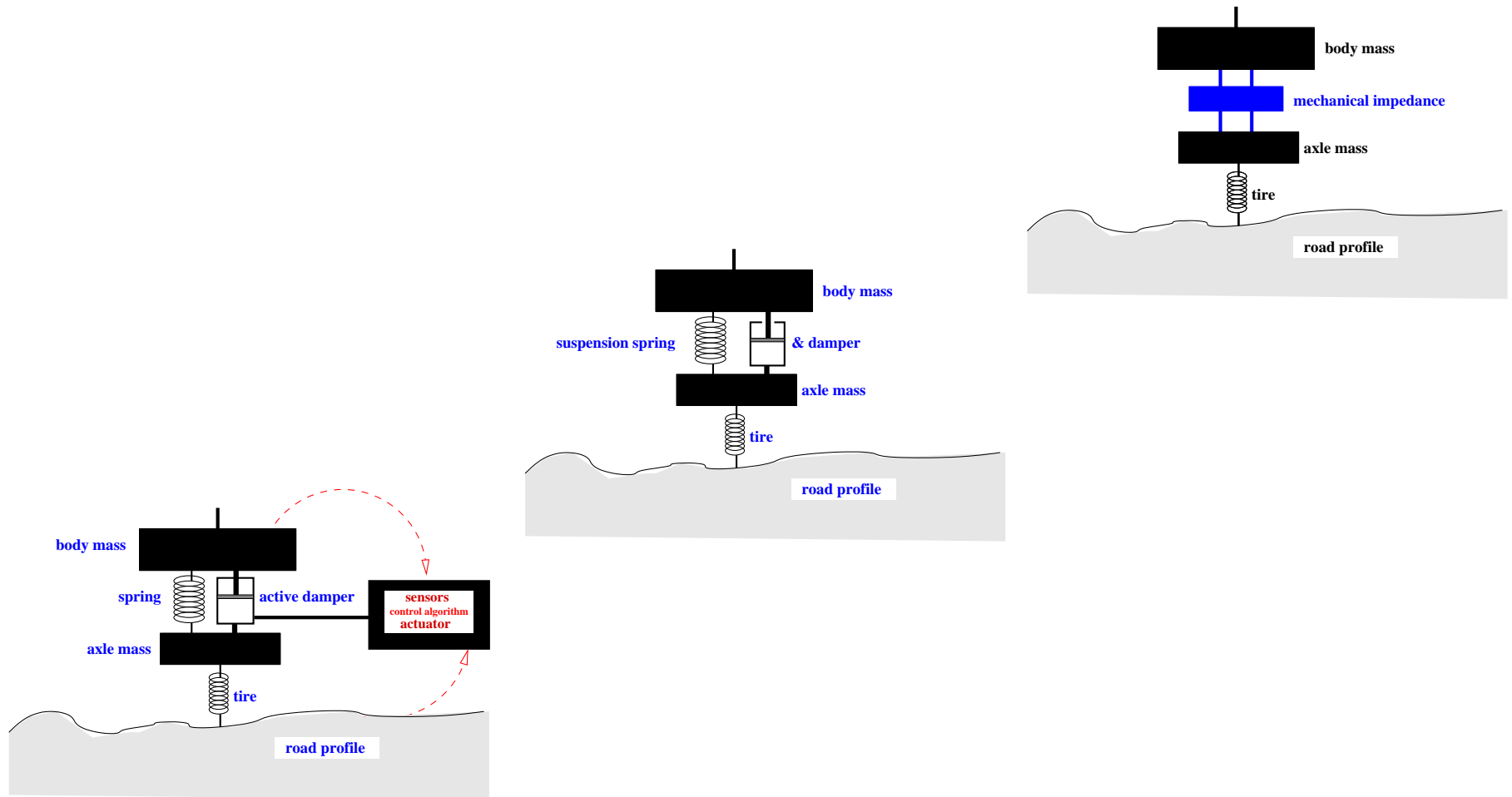
# Many controllers are not sensor-to-actuator

## Stabilization:

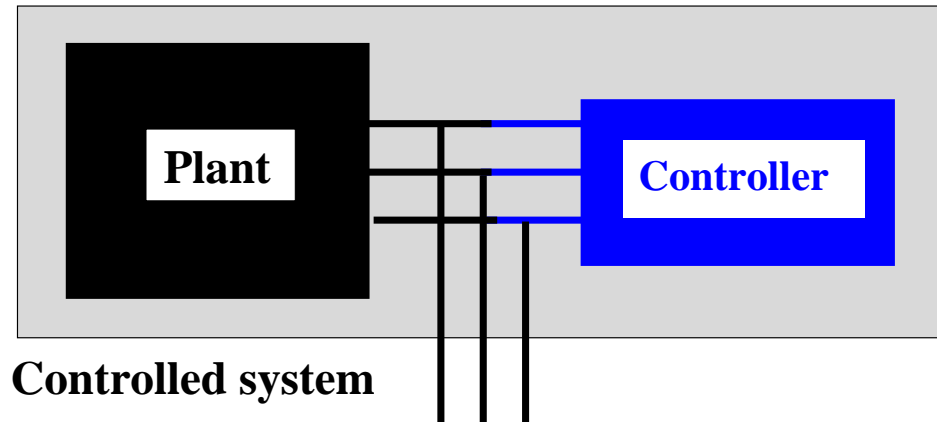


# Many controllers are not sensor-to-actuator

## Disturbance attenuation:



## Full Control



Let  $\mathfrak{B}$  be the plant behavior,  $\mathfrak{C}$  the controller behavior,

Then the controlled behavior  $\mathfrak{K} = \mathfrak{B} \cap \mathfrak{C} \subseteq \mathfrak{B}$

**Control means finding a subbehavior of the plant behavior**

Henceforth,  $\mathfrak{B} \in \mathcal{L}^w$ ,  $\mathfrak{C} \in \mathcal{L}^w \Rightarrow \mathfrak{K} = \mathfrak{B} \cap \mathfrak{C} \in \mathcal{L}^w$

## How to generate subbehaviors?

Plant & controller in kernel repr'ion.  $R$  is 'wide'

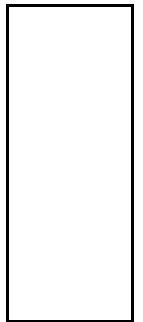


$$\boxed{R} \left( \frac{d}{dt} \right) w = 0 \quad \Rightarrow \quad \begin{bmatrix} R \\ C \end{bmatrix} \left( \frac{d}{dt} \right) w = 0$$

Plant in kernel & controller in image representation

$$\boxed{R} \left( \frac{d}{dt} \right) w = 0 \quad \Rightarrow \quad RC \left( \frac{d}{dt} \right) \ell = 0$$

Plant & controller in image representation.  $M$  is 'tall'



$$w = \boxed{M} \left( \frac{d}{dt} \right) \ell \quad \Rightarrow \quad \begin{bmatrix} M & C \end{bmatrix} \left( \frac{d}{dt} \right) \ell' = 0$$

'Squaring'  $\sim$  creating autonomous behavior

$\Rightarrow$  pole placement, stabilization, ...

## Regularity

2 notions of ‘well behaved’ controllers:

‘regular’ and ‘superregular’.

$\mathcal{C}$  is a **regular controller** for  $\mathfrak{B} : \Leftrightarrow$

$$p(\mathcal{K}) = p(\mathfrak{B}) + p(\mathcal{C})$$

$p :=$  number of eq’ns, of output variables.

$\mathcal{C}$  is a **superregular controller** for  $\mathfrak{B} : \Leftrightarrow$ , in addition,

$$n(\mathcal{K}) = n(\mathfrak{B}) + n(\mathcal{C})$$

$n :=$  number of state variables, ‘McMillan degree’.

## Regularity

$\mathcal{C}$  is a **regular controller** for  $\mathcal{B} : \Leftrightarrow$

$$p(\mathcal{K}) = p(\mathcal{B}) + p(\mathcal{C})$$

$(\pm)$  allows proper and improper controller transfer functions. The states need to be ‘prepared’ before interconnection.

$\mathcal{C}$  is a **superregular controller** for  $\mathcal{B} : \Leftrightarrow$ , in addition,

$$n(\mathcal{K}) = n(\mathcal{B}) + n(\mathcal{C})$$

$(\pm)$  allows only proper transfer functions in the controller. It is equivalent to feedback control.

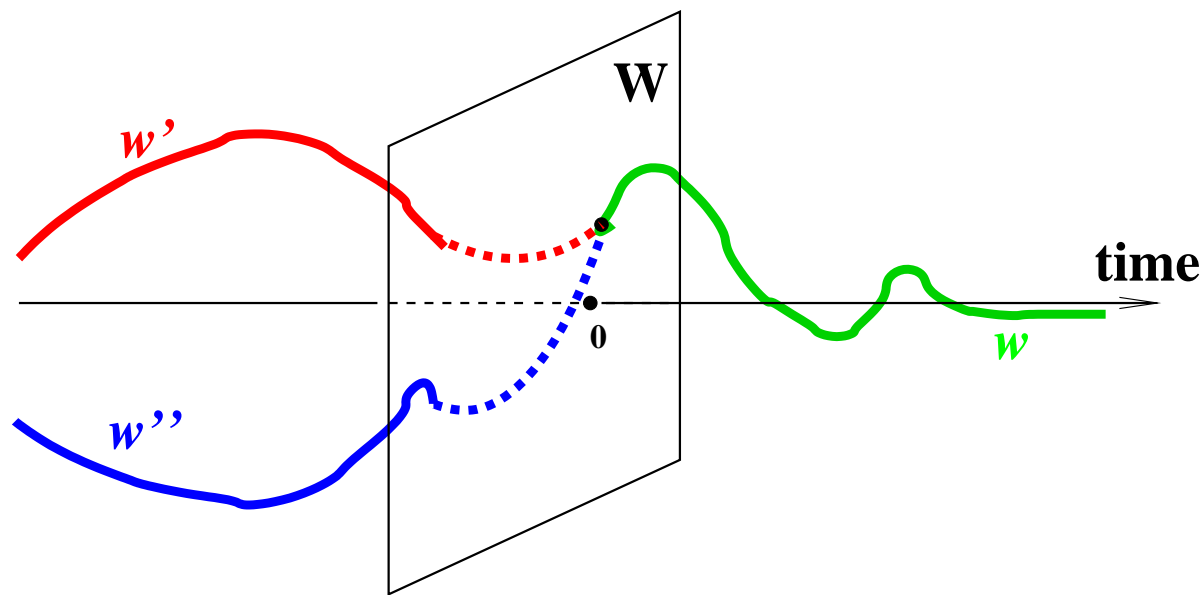


# Regularity

Superregularity also means:

‘the controller can take effect at any time’

$\forall w' \in \mathcal{B}, w'' \in \mathcal{C}, \exists w \in \mathcal{B} \cap \mathcal{C}$  such that



On regular controllers: Madhu Belur & Harry Trentelman, IEEE AC, 2002

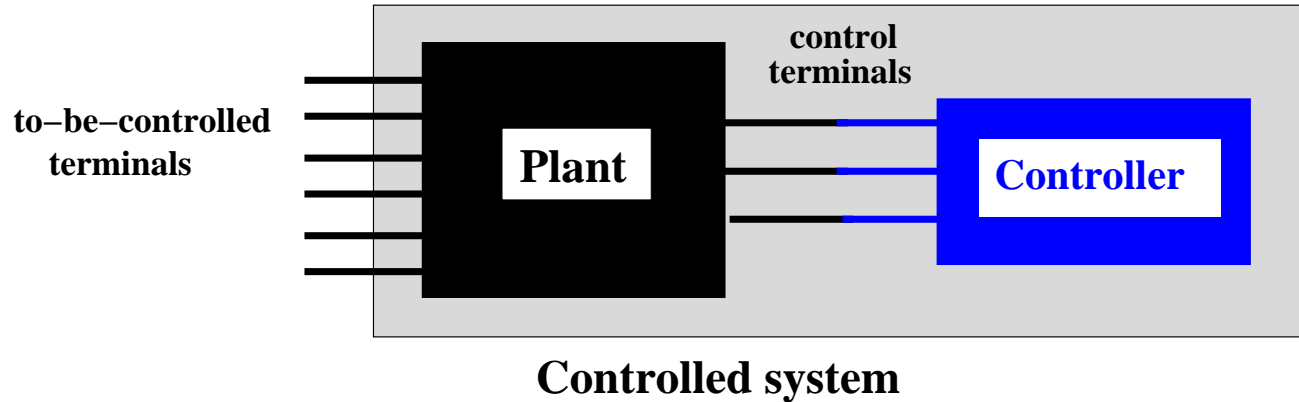
## Implementability

Assume that the plant  $\mathfrak{B} \in \mathcal{L}^w$  is controllable, then  
**any**  $\mathfrak{K} \subseteq \mathfrak{B}$  is implementable by a regular controller, i.e.

$$\forall \mathfrak{K} \in \mathcal{L}^w, \exists \mathfrak{C} \in \mathcal{L}^w \text{ such that } \mathfrak{K} = \mathfrak{B} \cap \mathfrak{C}$$

In order to be implementable by a superregular controller, we need  $n(\mathfrak{K})$  to be sufficiently high.

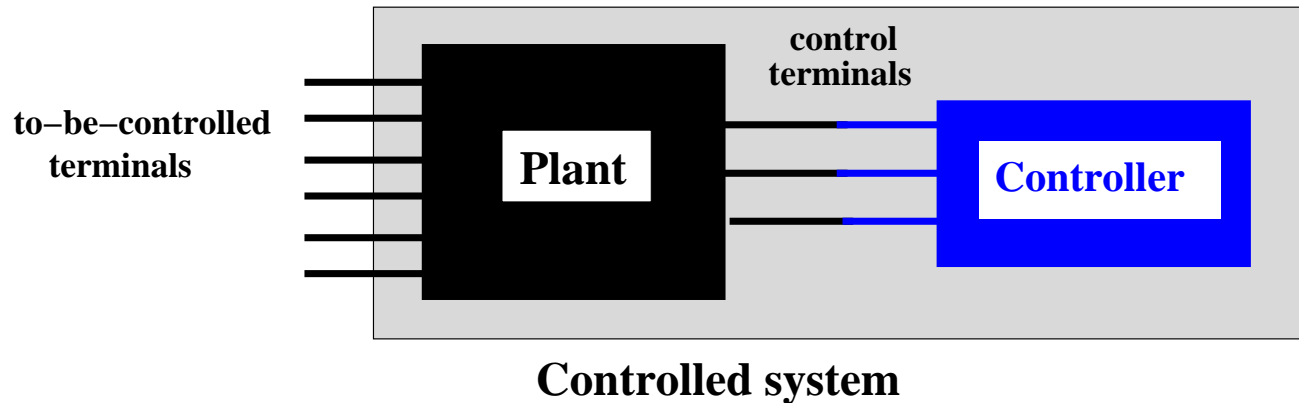
# Implementability



$w$  to-be-controlled variables,  $c$  control variables.

Assume behavior of plant, before control,  $\in \mathcal{L}^{w+c}$ .

# Implementability



Let  $\mathfrak{P} \in \mathcal{L}^w$  be the **plant behavior**, the behavior of to-be-controlled variables before the controller is applied.

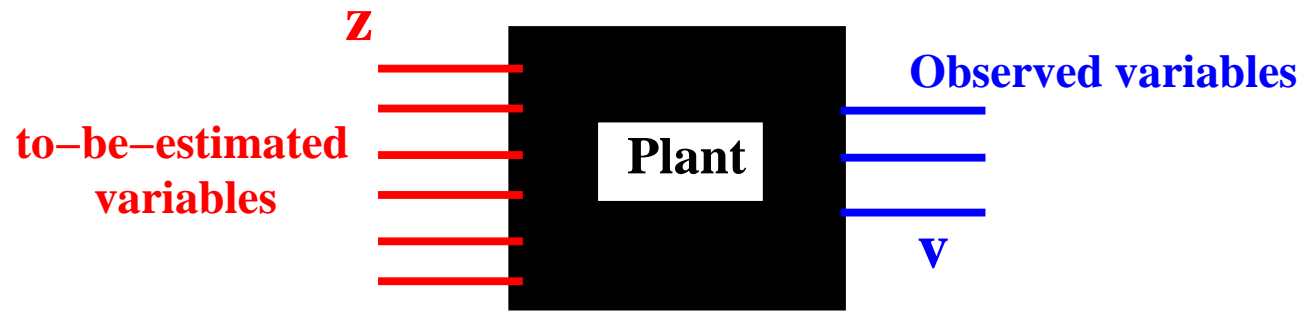
Let  $\mathfrak{N} \in \mathcal{L}^w$  be the **hidden behavior**, the behavior of to-be-controlled variables compatible with  $w = 0$ .

Assume  $\mathfrak{P}$  controllable.  $\mathfrak{K}$  is regularly implementable iff

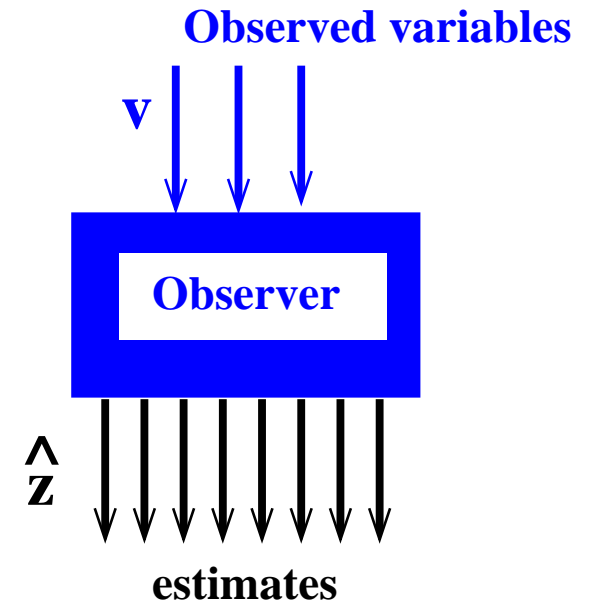
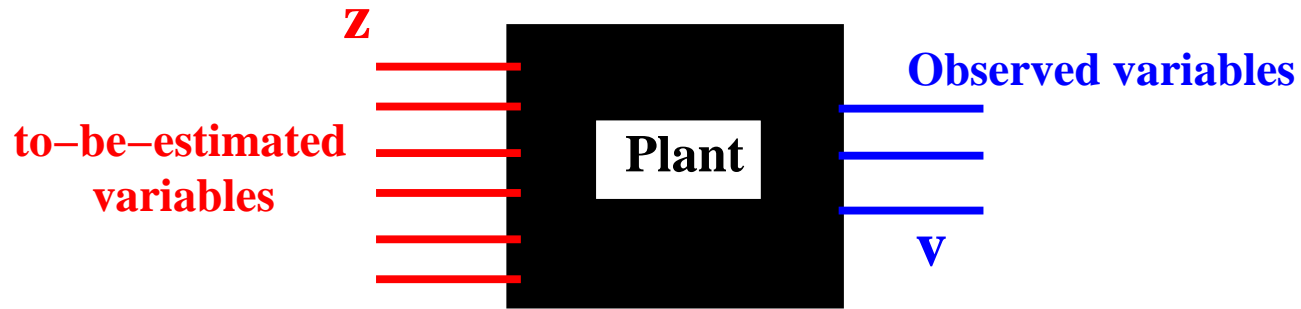
$$\mathfrak{N} \subseteq \mathfrak{K} \subseteq \mathfrak{P}$$

# **Observers: Joint work with Jochen**

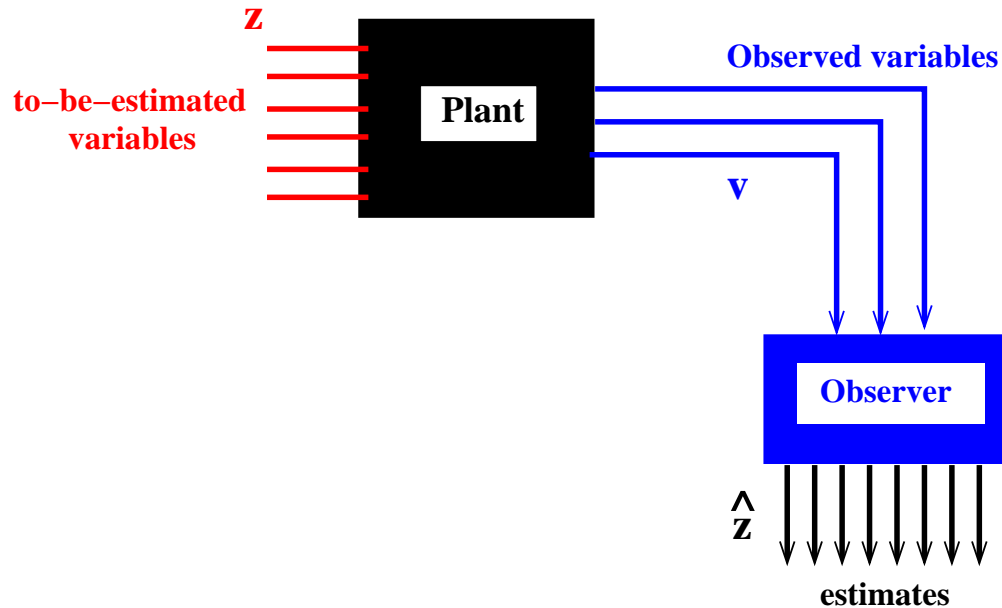
# Observer Architecture



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**Plant var.:**  $(v, z)$ :  $v$  observed,  $z$  to-be-estimated var.

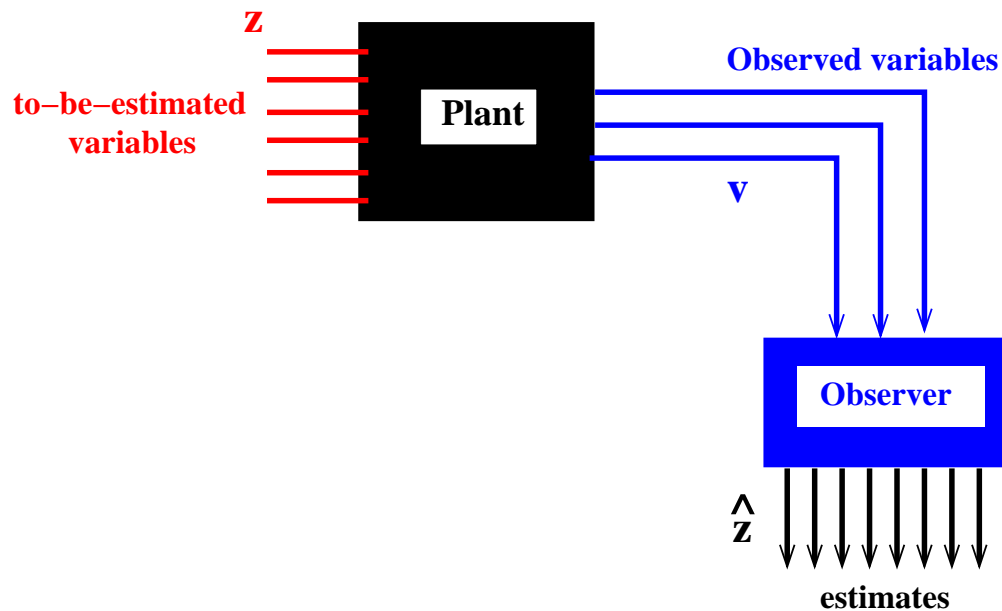
**Observer variables:**  $(v, \hat{z})$ :  $v$  observed,  $\hat{z}$  estimates

**Interconnected system variables:**  $v, z, \hat{z}$ .

**Estimation error:**

$$e = z - \hat{z}$$





Plant behavior:  $\mathfrak{B}$ , Observer behavior:  $\hat{\mathfrak{B}}$ , Error behavior:  $\mathfrak{E}$

Call  $\hat{\mathfrak{B}}$  a **replicator** of  $\mathfrak{B}$  if for all  $(y, z) \in \mathfrak{B}$ , there exists

$(y, \hat{z}) \in \hat{\mathfrak{B}}$  such that  $z = \hat{z}$ , i.e.  **$\mathfrak{B} \subseteq \hat{\mathfrak{B}}$**

**tracking** if the error behavior  $\mathfrak{E}$  is **autonomous**.

Thm: Assume plant  $\mathfrak{B}$  controllable,  $y$  'free' in observer  $\hat{\mathfrak{B}}$ .

**$\hat{\mathfrak{B}}$  is tracking iff it is a replicator**

**Observers means finding a supbehavior of the plant behavior**

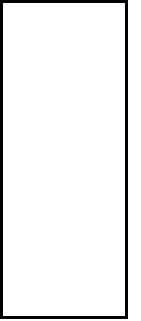
## How to generate **supbehaviors?**

**Plant in kernel representation.**

*R* is 'tall'

**Plant:**

$$R \left( \frac{d}{dt} \right) z = H \left( \frac{d}{dt} \right) v$$



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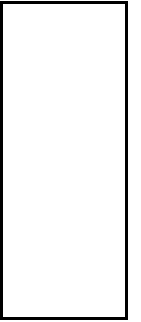
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**Error dynamics:**  $e = z - \hat{z}$  'eliminate'  $v, z, \hat{z} \Rightarrow$

$$F \left( \frac{d}{dt} \right) R \left( \frac{d}{dt} \right) e = 0$$

So, squaring up  $R$  to  $FR$

$\Rightarrow$  error autonomous, desired input structure.

**Pole placement, stabilization, ...**

## Example

**Plant equations in ‘observability’ canonical form:**

$$V \left( \frac{d}{dt} \right) v = 0, \quad z = Z \left( \frac{d}{dt} \right) v$$

**This canonical form exists iff  $z$  observable from  $v$  in the plant.**

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**Observer:**

$$\mathbf{P} \left( \frac{d}{dt} \right) \hat{\mathbf{z}} = \mathbf{P} \left( \frac{d}{dt} \right) \mathbf{Z} \left( \frac{d}{dt} \right) \mathbf{v} + \mathbf{S} \left( \frac{d}{dt} \right) \mathbf{V} \left( \frac{d}{dt} \right) \mathbf{v}$$

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$$P \left( \frac{d}{dt} \right) \hat{z} = P \left( \frac{d}{dt} \right) Z \left( \frac{d}{dt} \right) v + S \left( \frac{d}{dt} \right) V \left( \frac{d}{dt} \right) v$$

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Error dynamics:

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Choose  $P$  for stability,  $S$  for high frequency roll-off, etc.



# Conclusion

## The barrier

*“Block diagrams unsuitable for serious physical modeling*

*- the control/physics barrier”*

*“Behavior based (declarative) modeling is a good alternative”*



from K.J. Åström

*Present Developments in Control Applications*



**IFAC 50-th Anniversary Celebration  
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**Block diagrams are indeed unsuitable for serious physical modeling. Block diagrams also exclude many controllers!**

**Behaviors respect the physics, easier, more general concepts, block diagrams are a very important special case, ...**

**Details & copies of the lecture frames are available from/at**

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**Thank you**

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