Over SYSTEMEN,

THEORIEËN en THEOREMA'S

About SYSTEMS, THEORIES, and THEOREMS

Afscheidscollege, Groningen, 13 januari 2004

OPEN DYNAMICAL SYSTEMS

‘system’ := **the object which we are studying**

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‘dynamical’ :=

we are interested in its evolution over time

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‘system’ := **the object which we are studying**

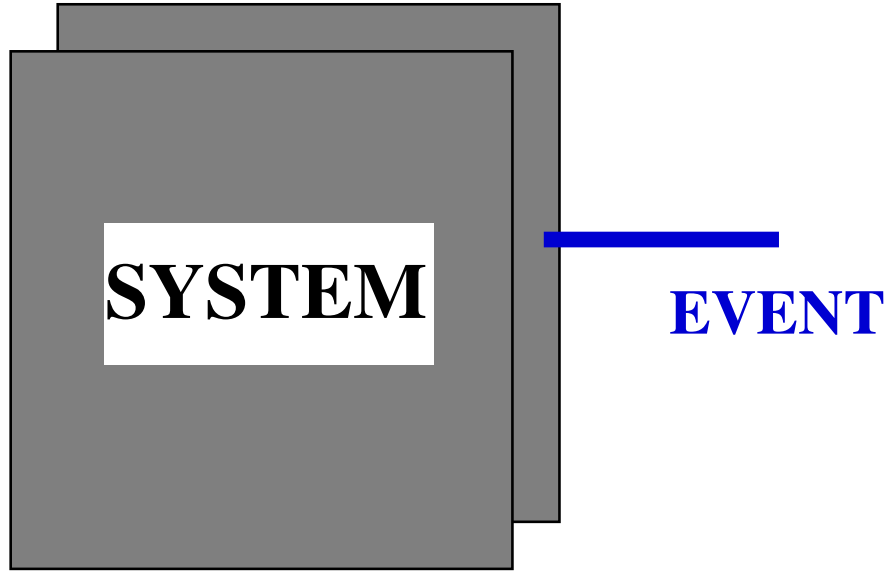
‘dynamical’ :=

we are interested in its evolution over time

‘open’ :=

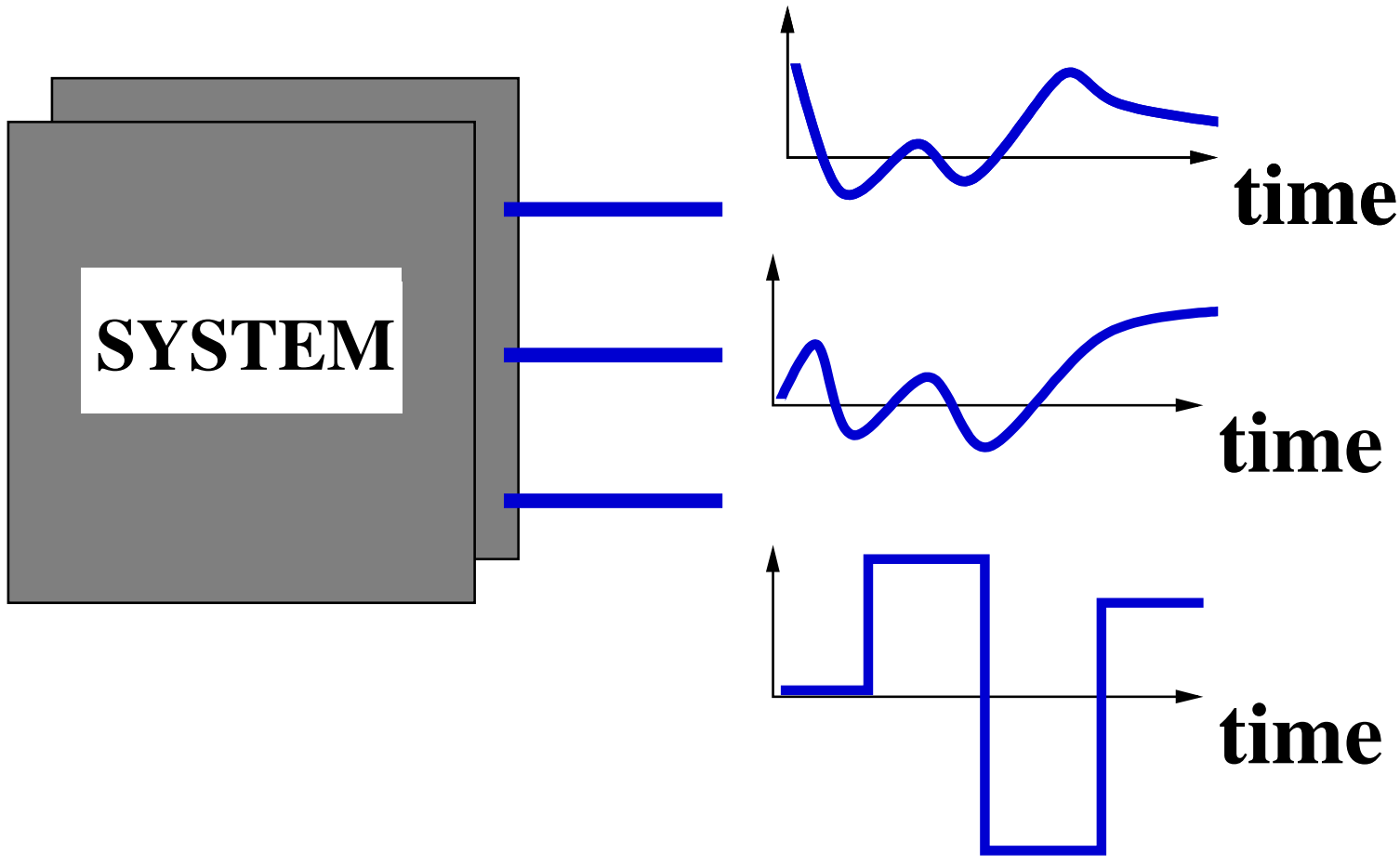
the system interacts with its environment

The BEHAVIOR



Which event sequences are possible?

The BEHAVIOR

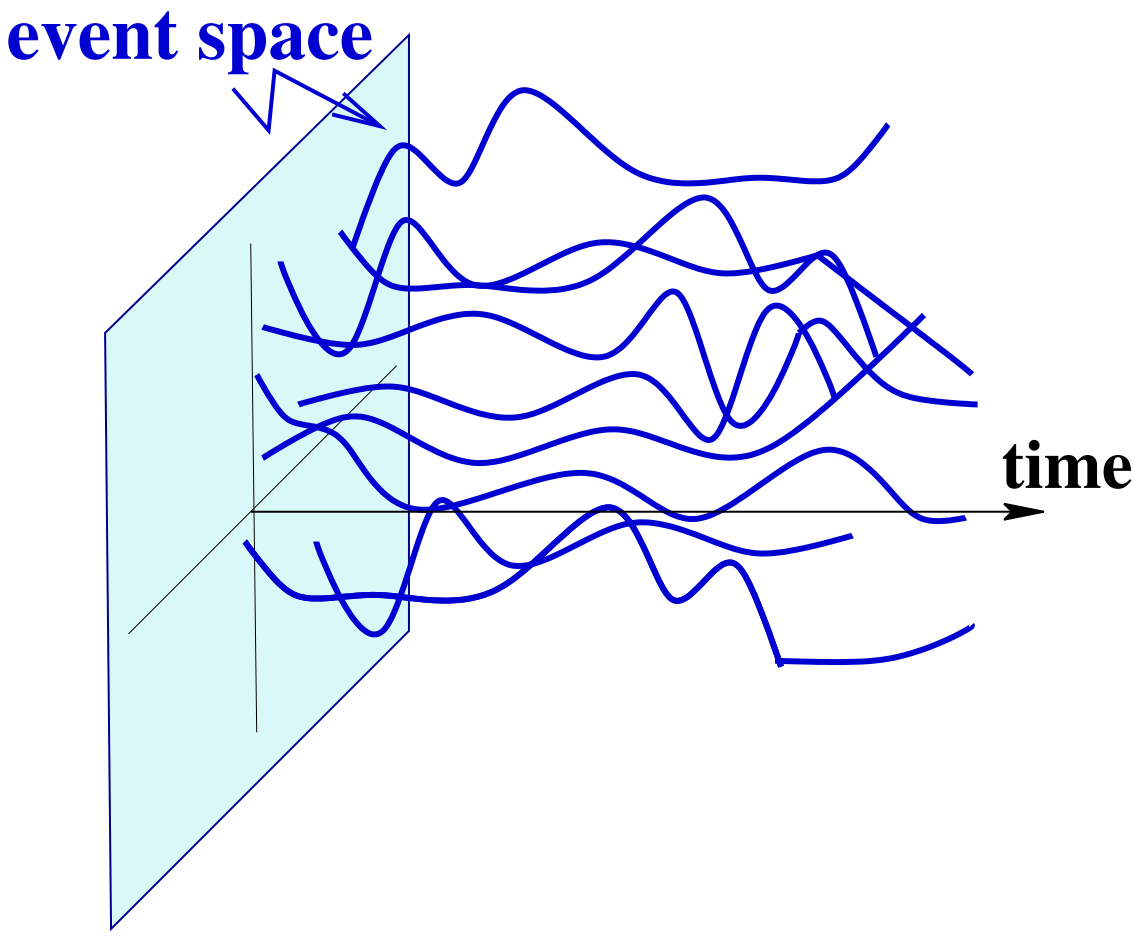


The BEHAVIOR

The behavior =

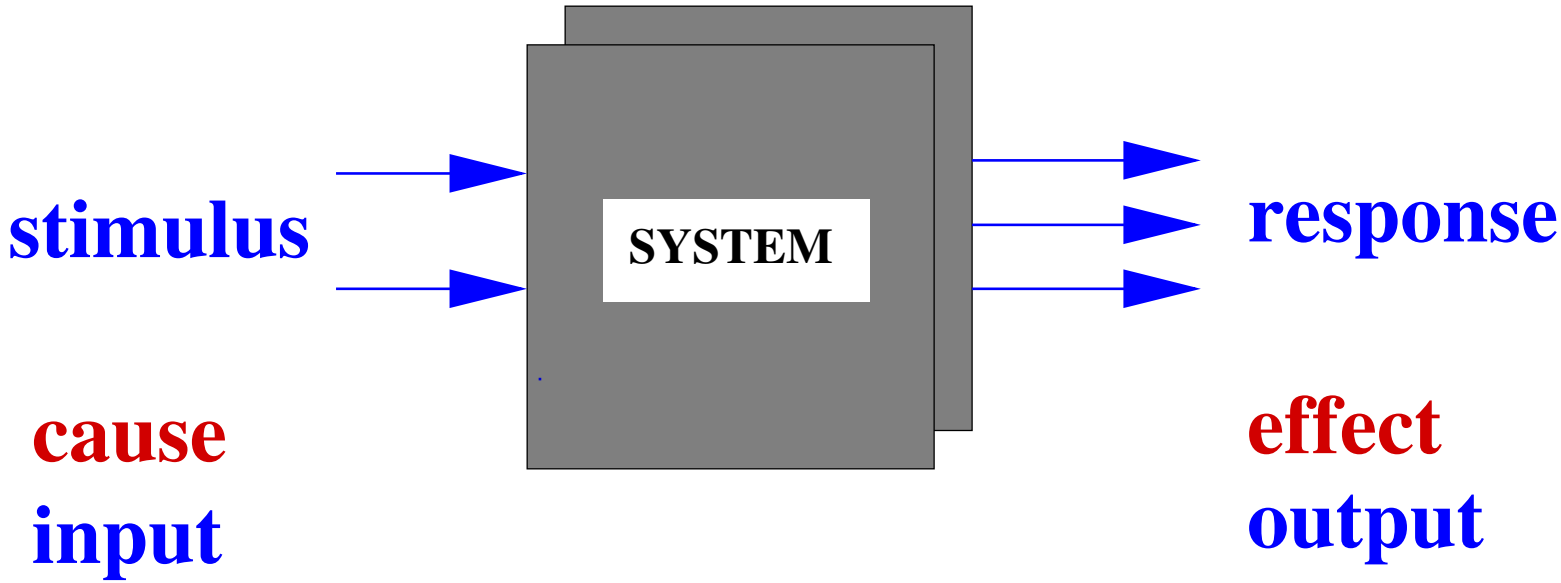
all trajectories of the system variables which, according to the mathematical model, are possible.

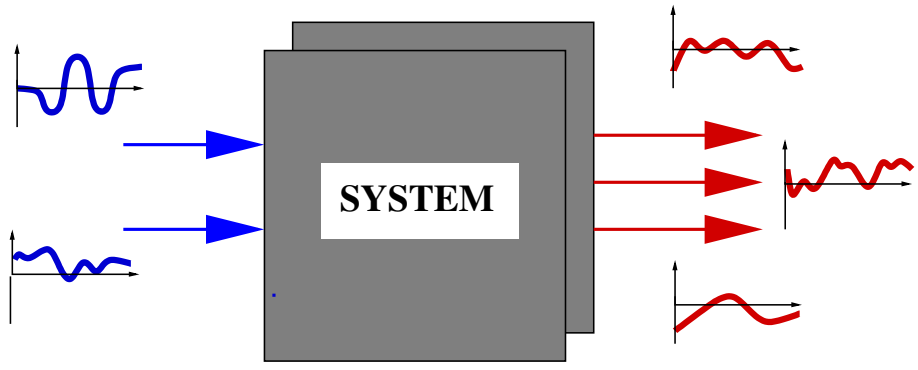
The BEHAVIOR



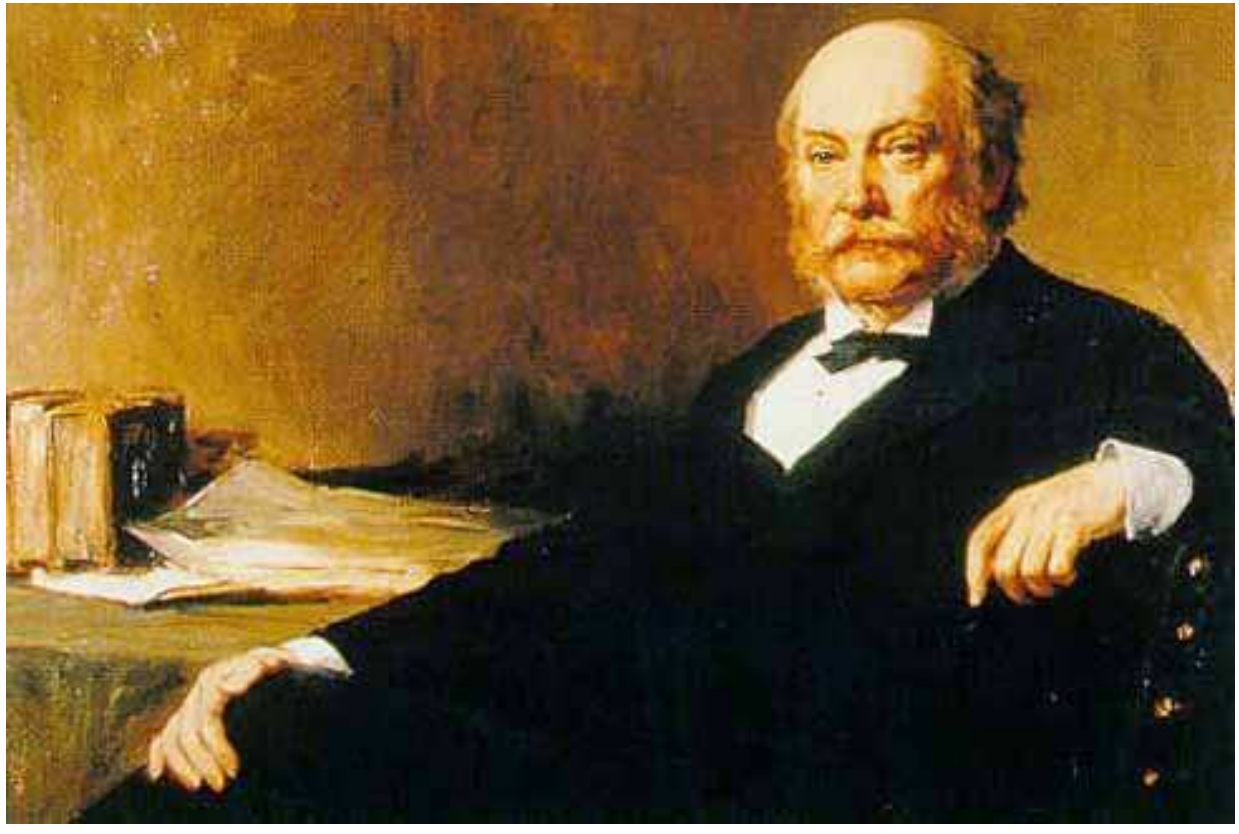
Totality of 'legal' trajectories =: the behavior

INPUT/OUTPUT SYSTEMS



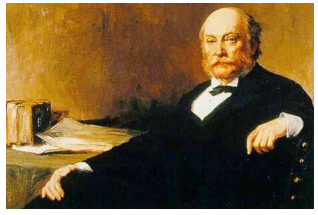


The HISTORY of I/O SYSTEMS



Lord Rayleigh (1842-1919)

The HISTORY of I/O SYSTEMS



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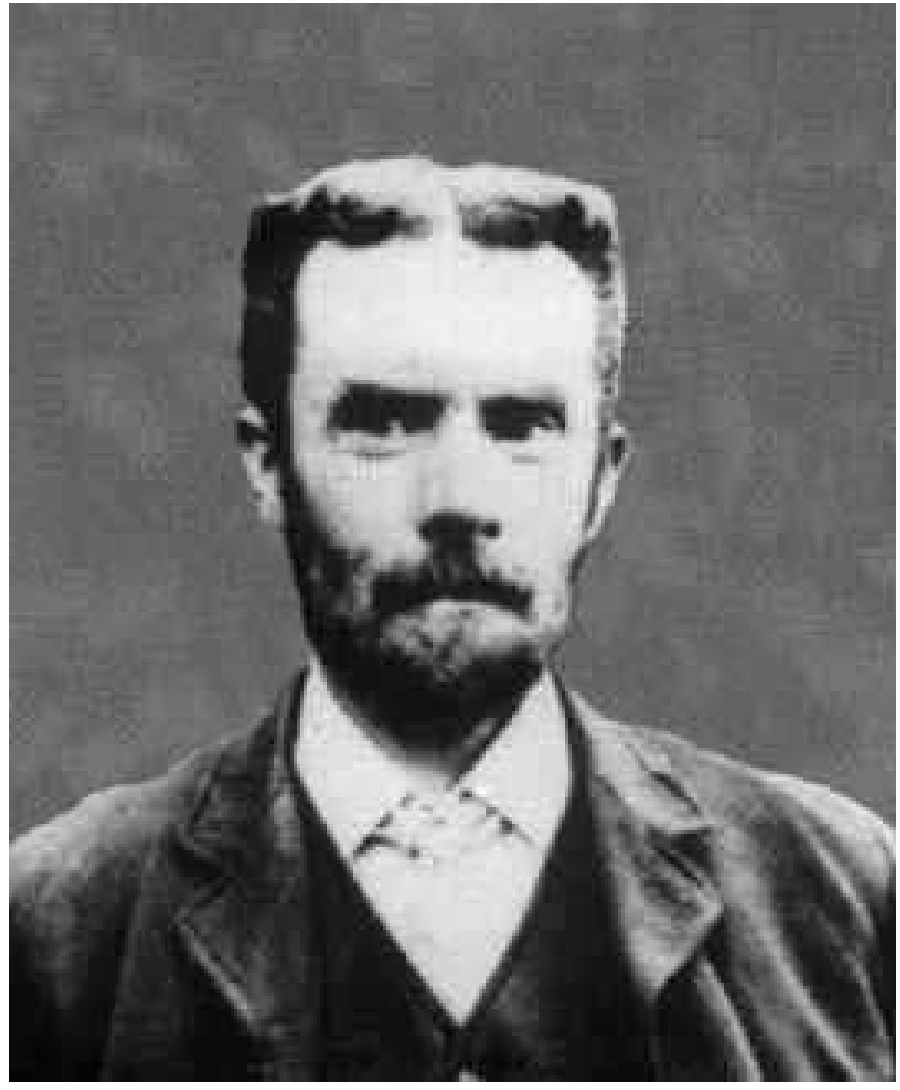

Oliver Heaviside (1850-1925)

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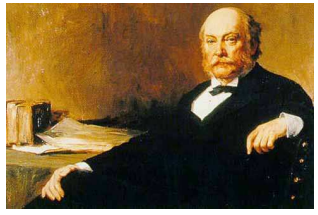

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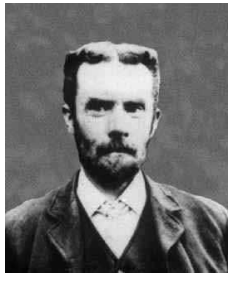

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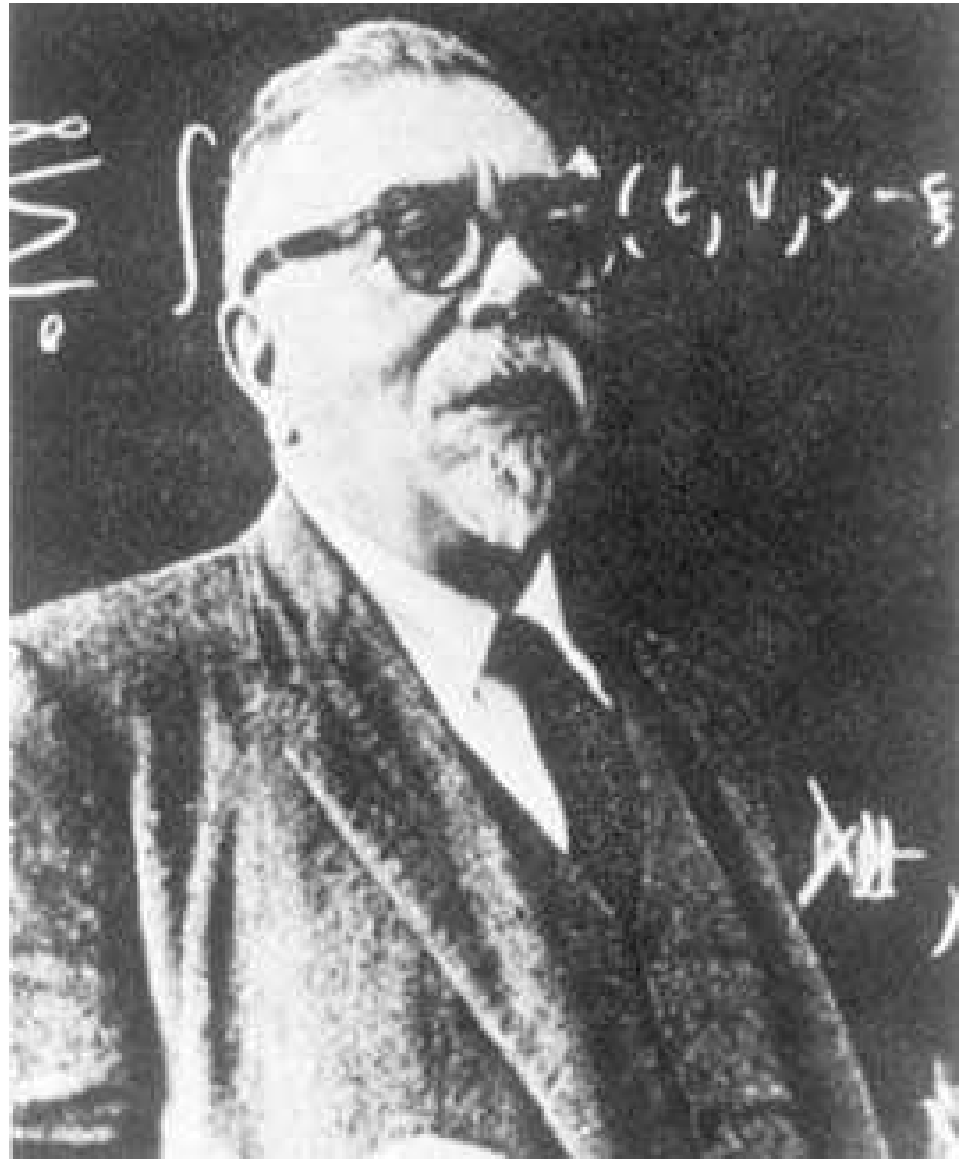

Norbert Wiener (1894-1964)

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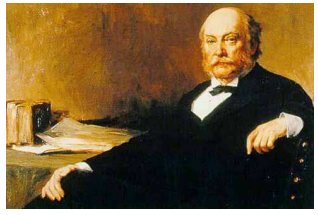

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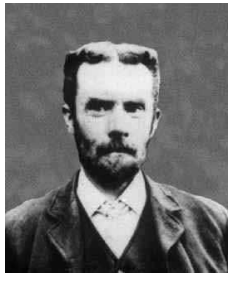

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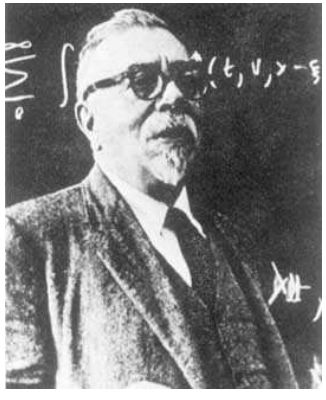


Norbert Wiener (1894-1964)

Rudolf Kalman (1930-)

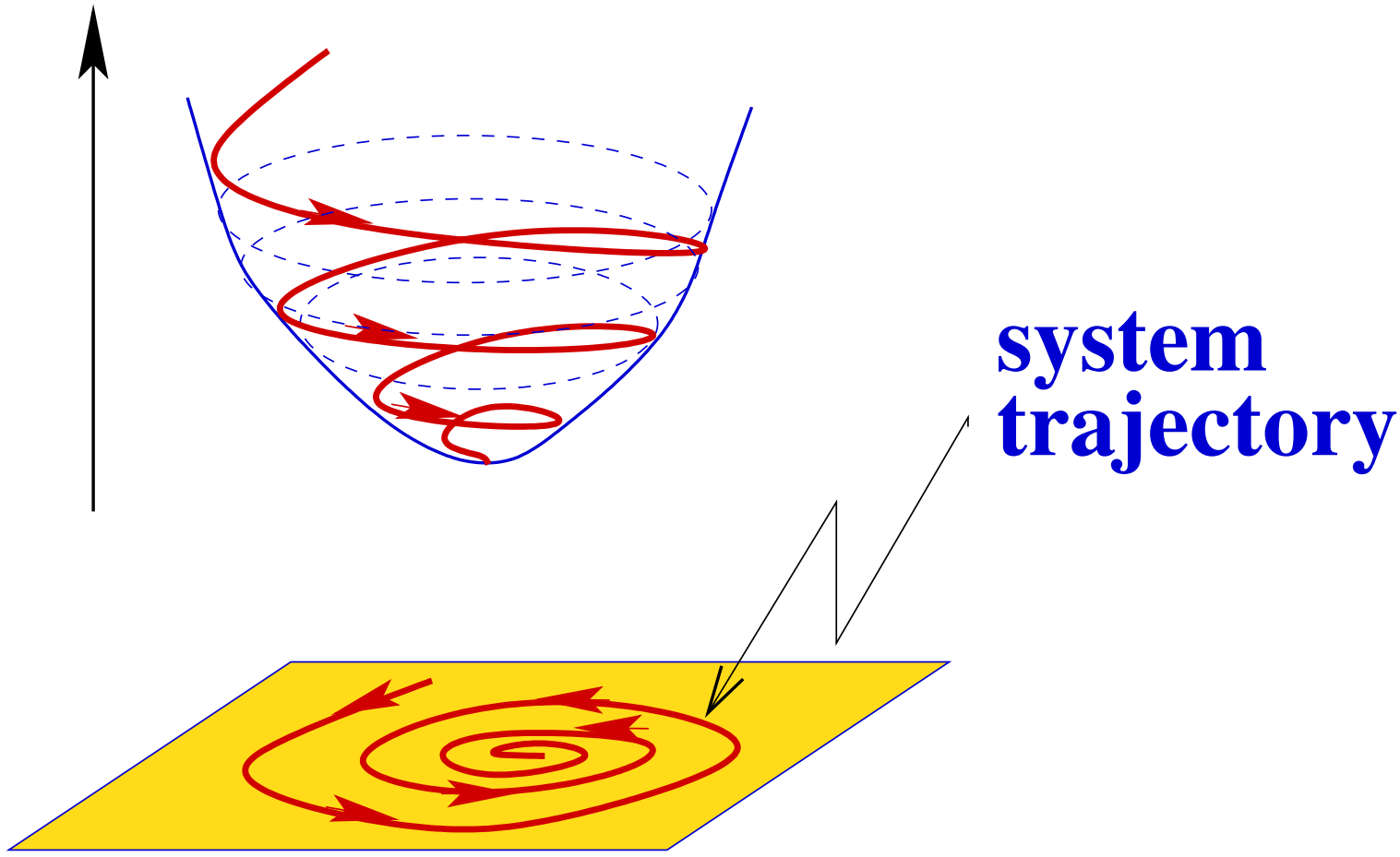
MIT, CAMBRIDGE

MIT, CAMBRIDGE

LYAPUNOV FUNCTIONS

Aleksandr Lyapunov (1857-1918)

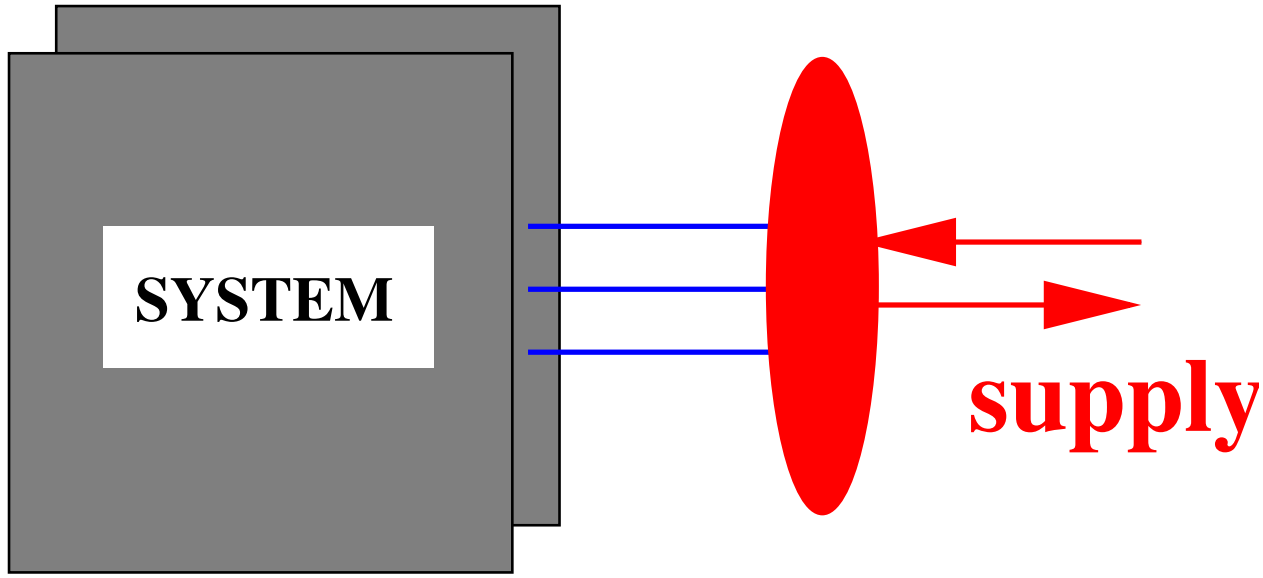
LYAPUNOV FUNCTIONS



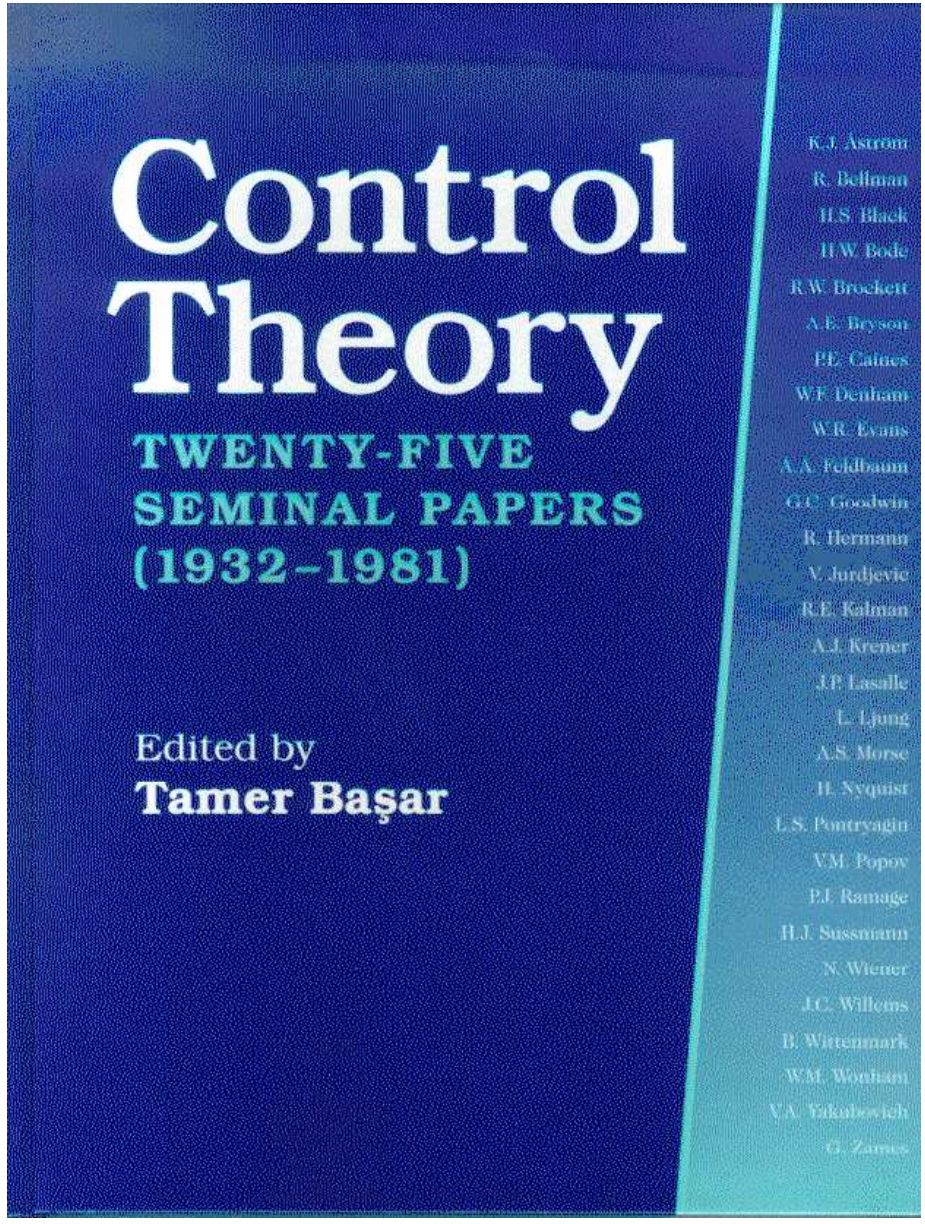
DISSIPATIVE SYSTEMS

A system is said to be dissipative :$\Leftrightarrow$

Initial storage + Supply $\leq$ Final storage



DISSIPATIVE SYSTEMS



To COUNT or to THINK?

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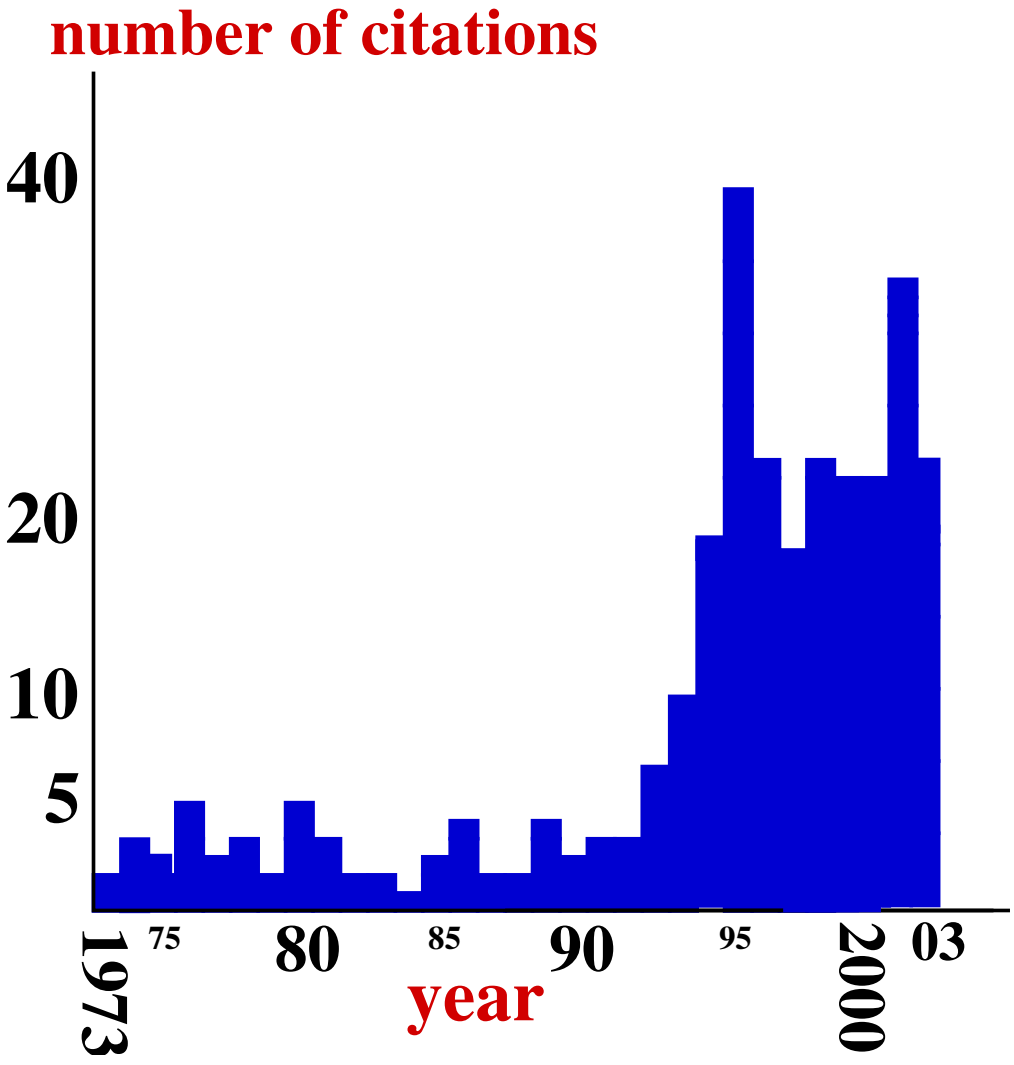
impact factor for year x

$$= \frac{\text{citations in year x to articles published in the journal in years x-1 and x-2}}{\text{number of articles published in the journal in years x-1 and x-2}}$$

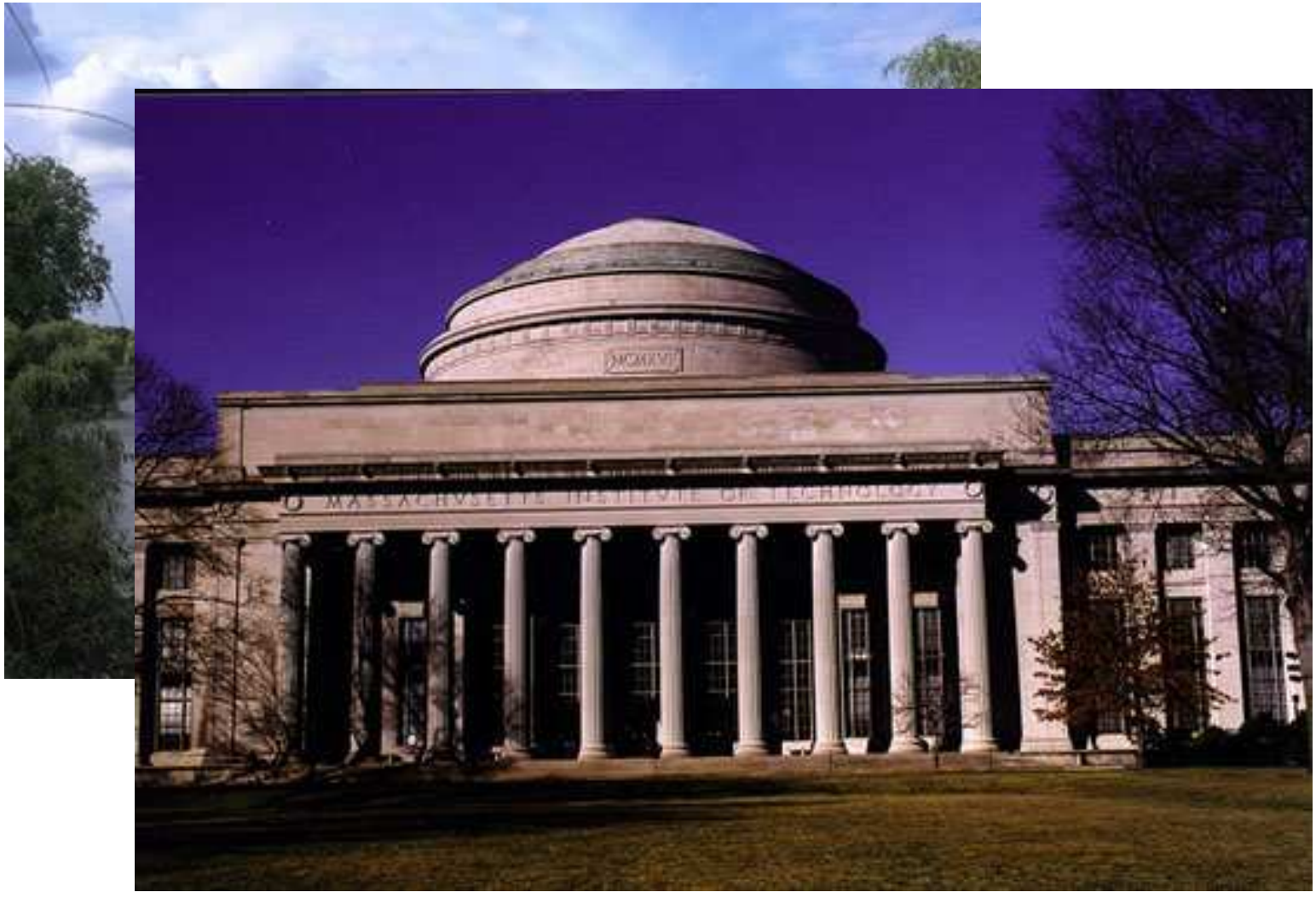
To COUNT or to THINK?



Years	Citations
1973-1982	36
1983-1992	31
1993-now	256

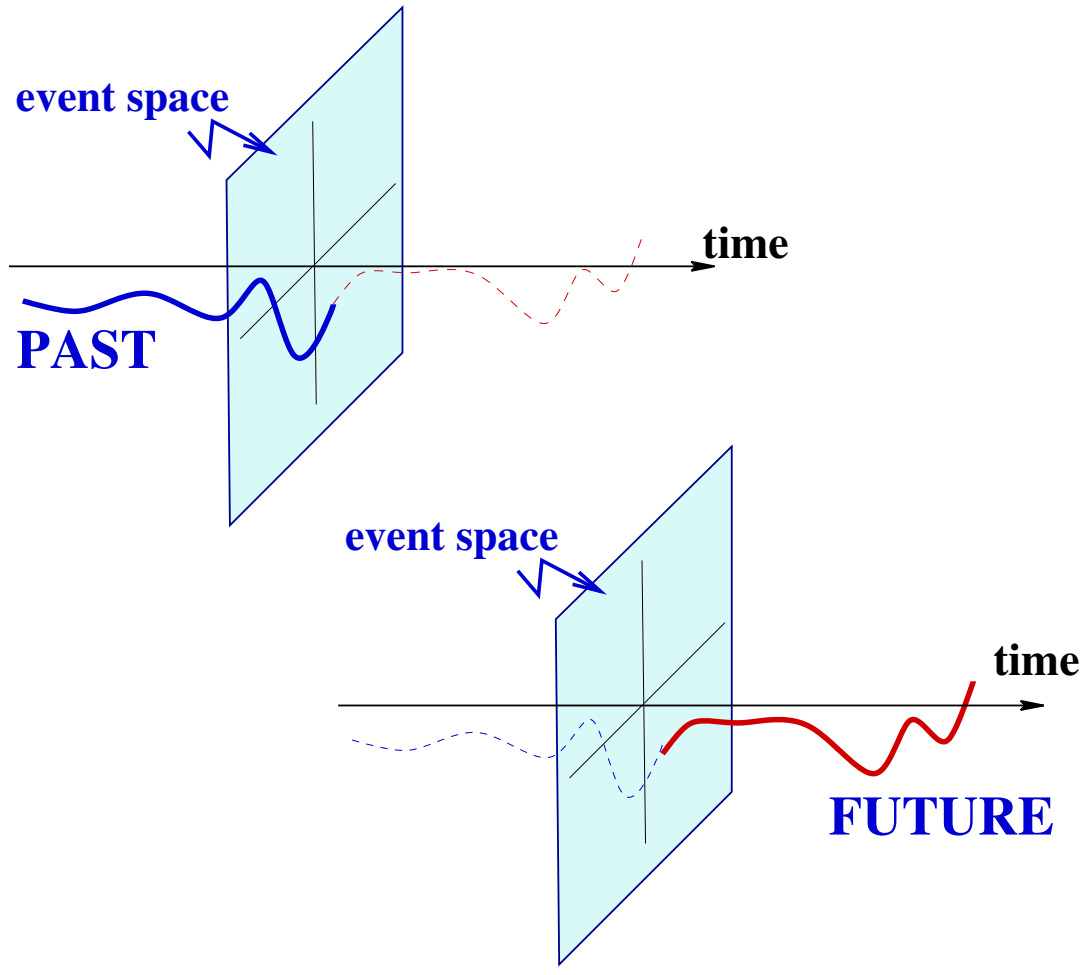
CAMBRIDGE, MIT, GRONINGEN

CAMBRIDGE, MIT, GRONINGEN



CAMBRIDGE, MIT, GRONINGEN

AUTONOMOUS SYSTEMS



Autonomous := past implies future

PLANETARY MOTION

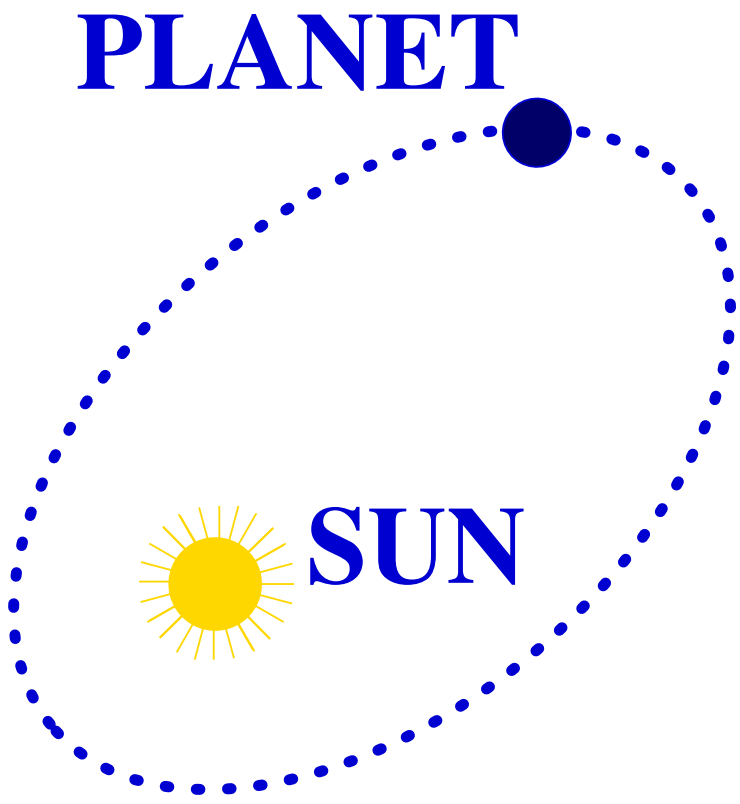
How can it move?

PLANETARY MOTION

Johannes Kepler (1571-1630)

PLANETARY MOTION

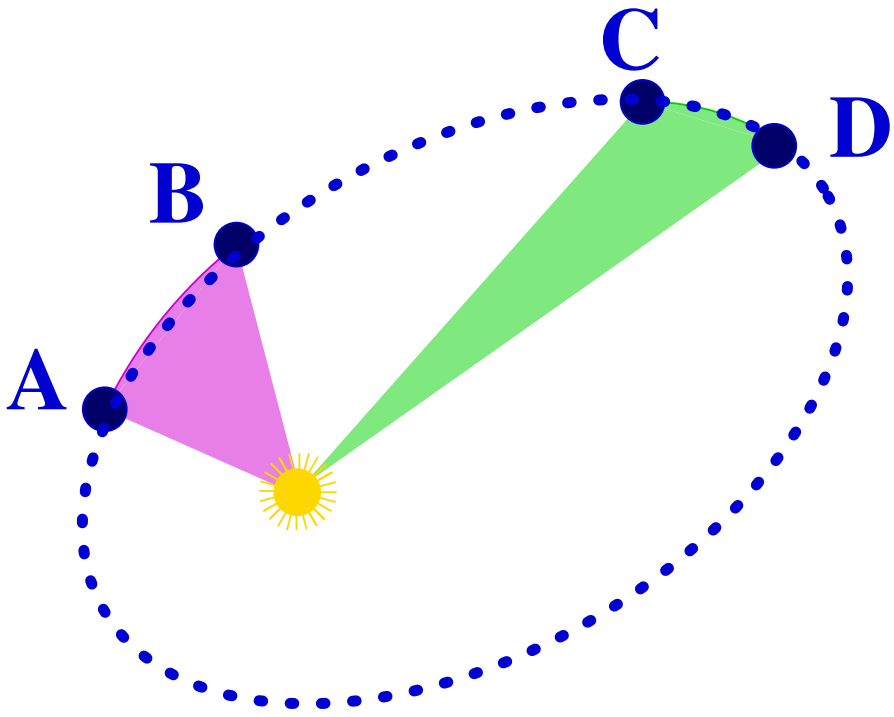
Kepler's first law



Ellipse, sun in focus

PLANETARY MOTION

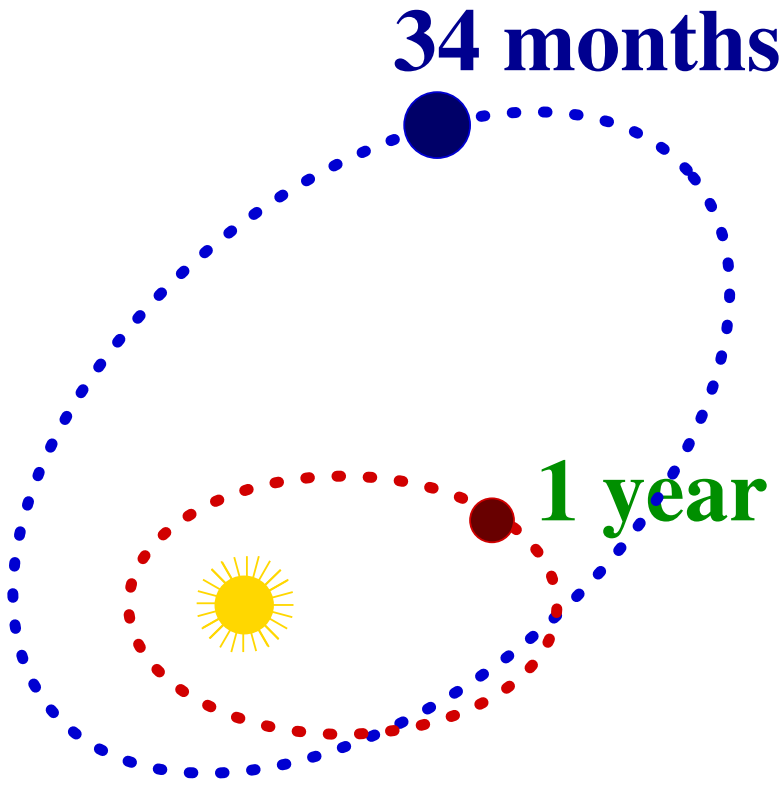
Kepler's second law



= areas in = times

PLANETARY MOTION

Kepler's third law

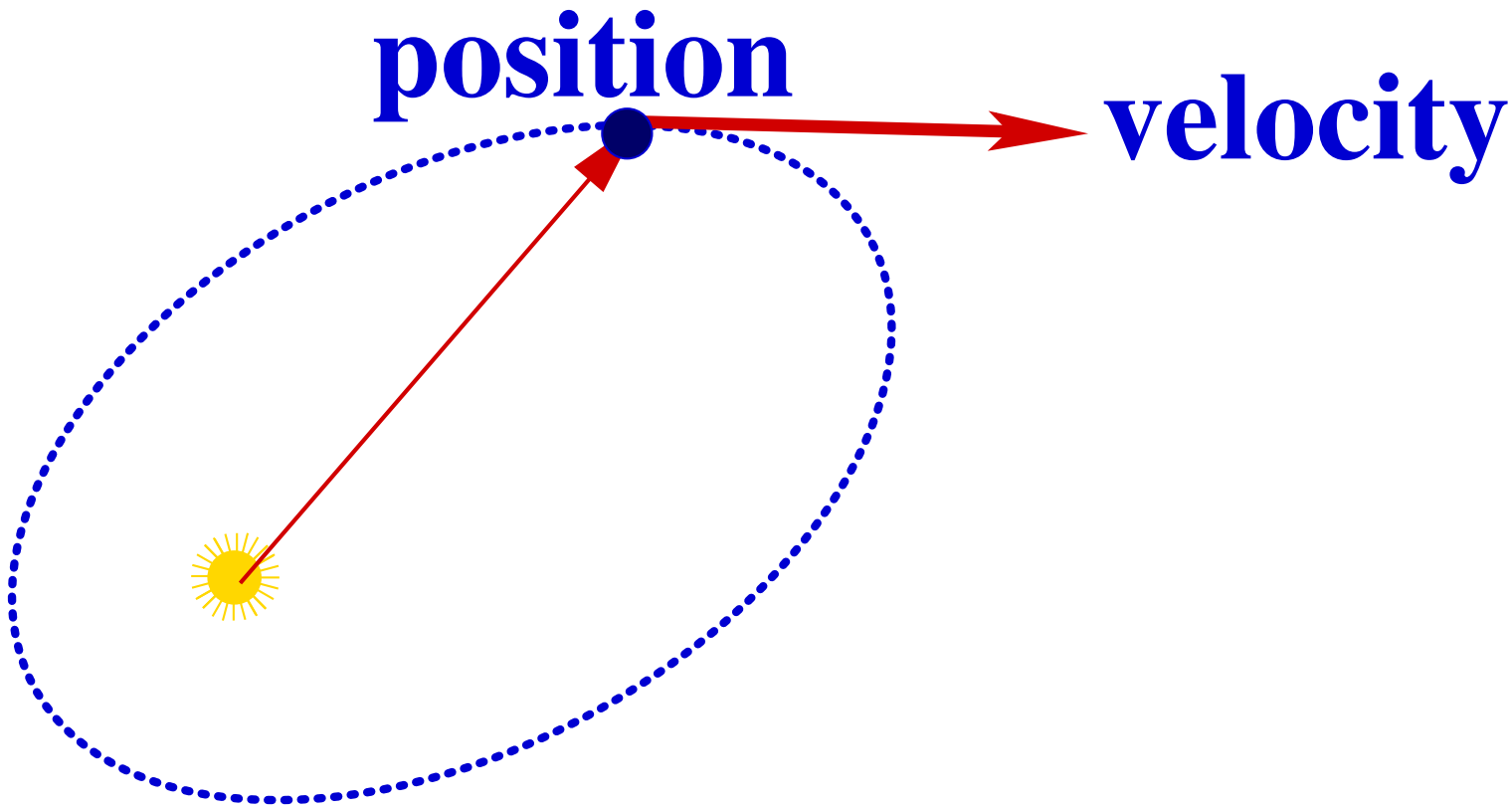


$$(\text{period})^2 = (\text{diameter})^3$$

Troja, Hiroshima, Srebrenica,

Maar de planeet zij draaide voort!

The STATE of the PLANET



The state = position and velocity

The EQUATION of the PLANET

acceleration = function of position and velocity

$$\frac{d^2}{dt^2}w(t) = F(w(t), \frac{d}{dt}w(t))$$

The EQUATION of the PLANET

acceleration = function of position and velocity

$$\frac{d^2}{dt^2}w(t) = F(w(t), \frac{d}{dt}w(t))$$

$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|\frac{d}{dt}w(t)|^2} = 0$$

Isaac Newton (1643-1727)

Isaac Newton (1643-1727)

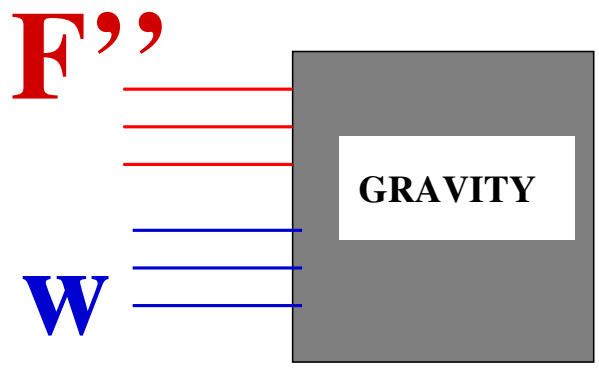
NEWTON's LAWS

Newton's 2nd law: force = mass * acceleration

$$F'(t) = m\frac{d^2}{dt^2}w(t)$$

NEWTON's LAWS

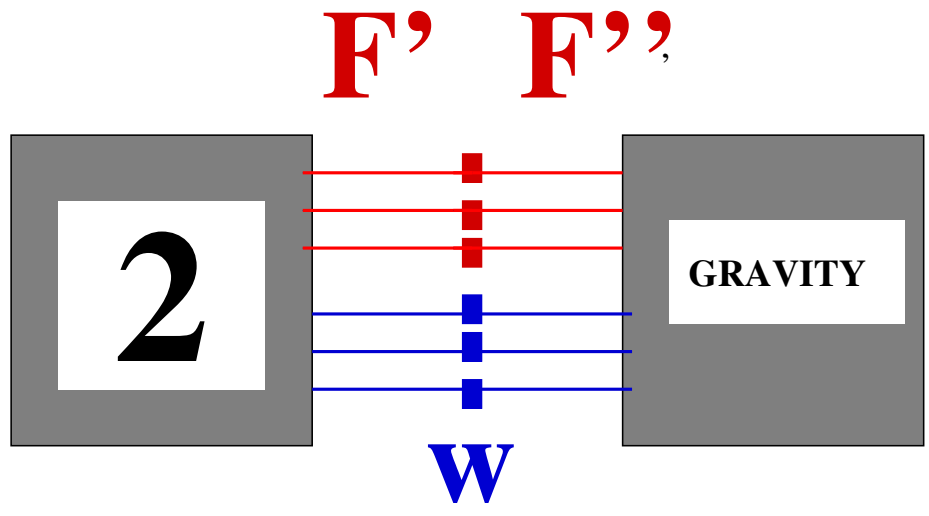
The law of gravitation: $\text{attraction} = \dfrac{\text{mass}}{(\text{distance})^2}$



$$F''(t) = m\frac{1_{w(t)}}{|\frac{d}{dt}w(t)|^2}$$

NEWTON's LAWS

Newton's third law: the sum of the forces = 0



$$F'(t) + F''(t) = 0$$

NEWTON's LAWS

$$F'(t) = m\frac{d^2}{dt^2}w(t)$$

$$F''(t) = m\frac{1_{w(t)}}{|\frac{d}{dt}w(t)|^2}$$

$$F'(t) + F''(t) = 0$$

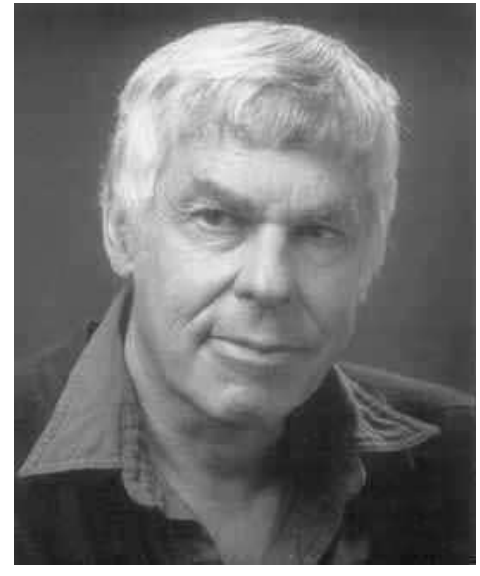
$$\Downarrow$$

$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|\frac{d}{dt}w(t)|^2} = 0$$

The HISTORY of CLOSED SYSTEMS

Henri Poincaré (1854-1912)

George Birkhoff (1884-1944)



Stephen Smale (1930-)

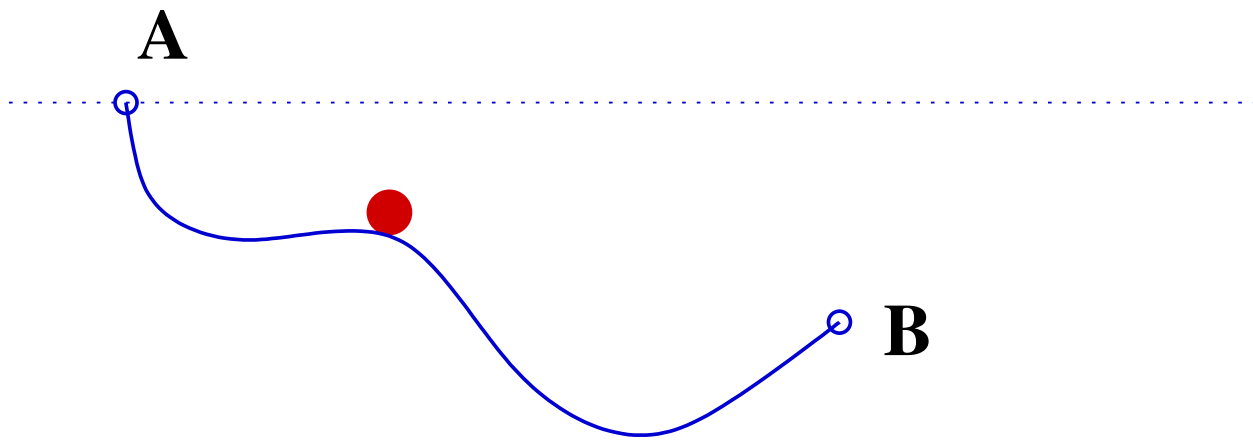
A system as a behavior

A fully general, consistent state construction

The BRACHISTOCHRONE

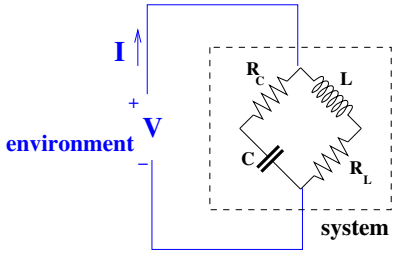
Johann Bernoulli (1667-1748)

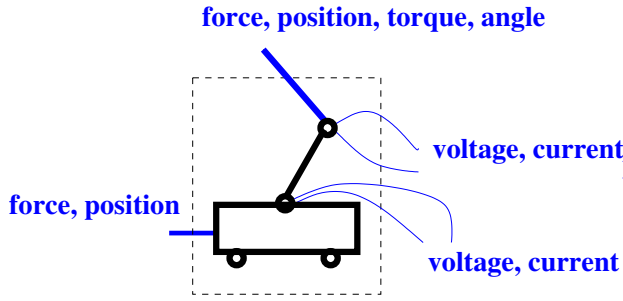
The BRACHISTOCHRONE

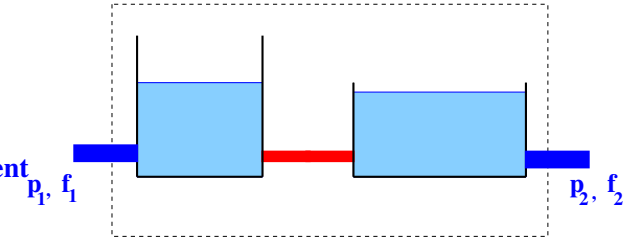


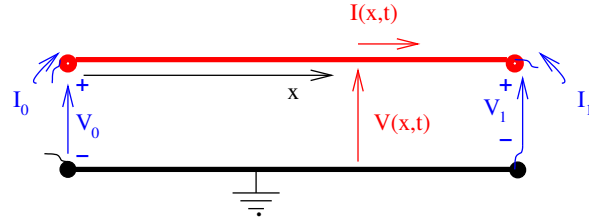
Let the ball roll as fast as possible from A to B !

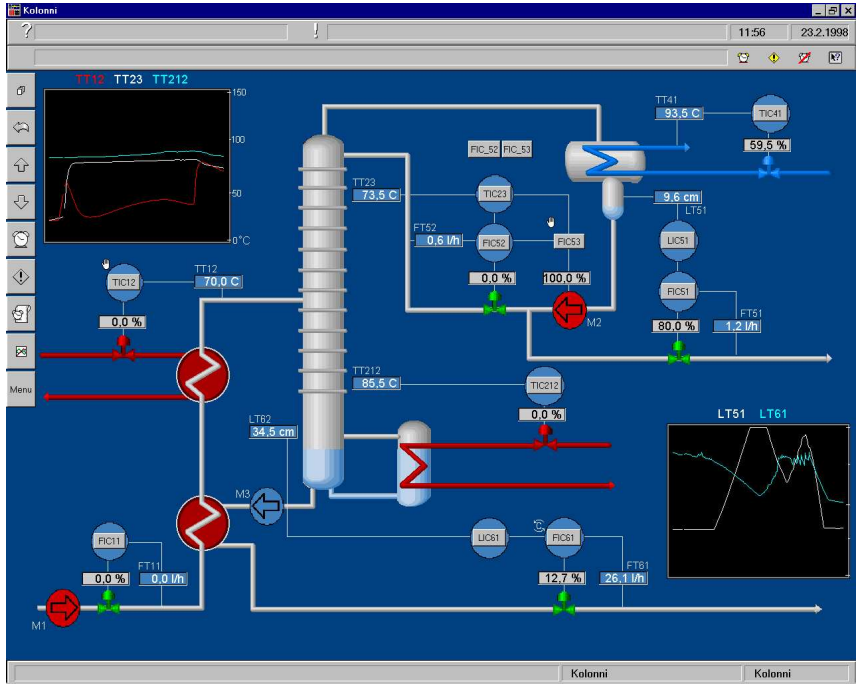
SYSTEMS

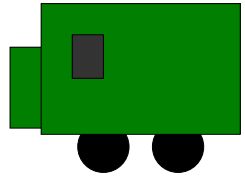

I
+
V
−
environment
R_C
L
C
R_L
system

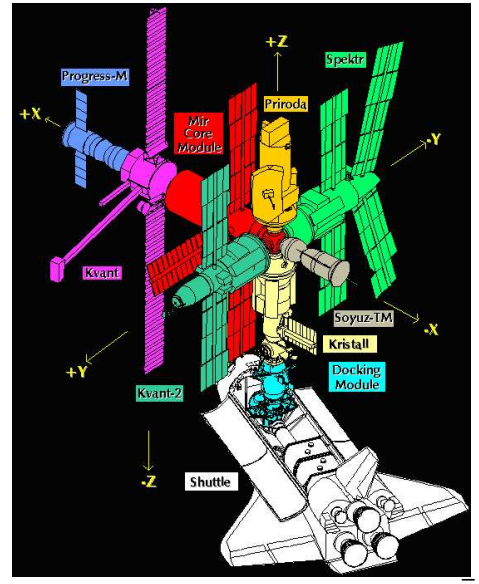

force, position, torque, angle
voltage, current
force, position
voltage, current


p1, f1
p2, f2

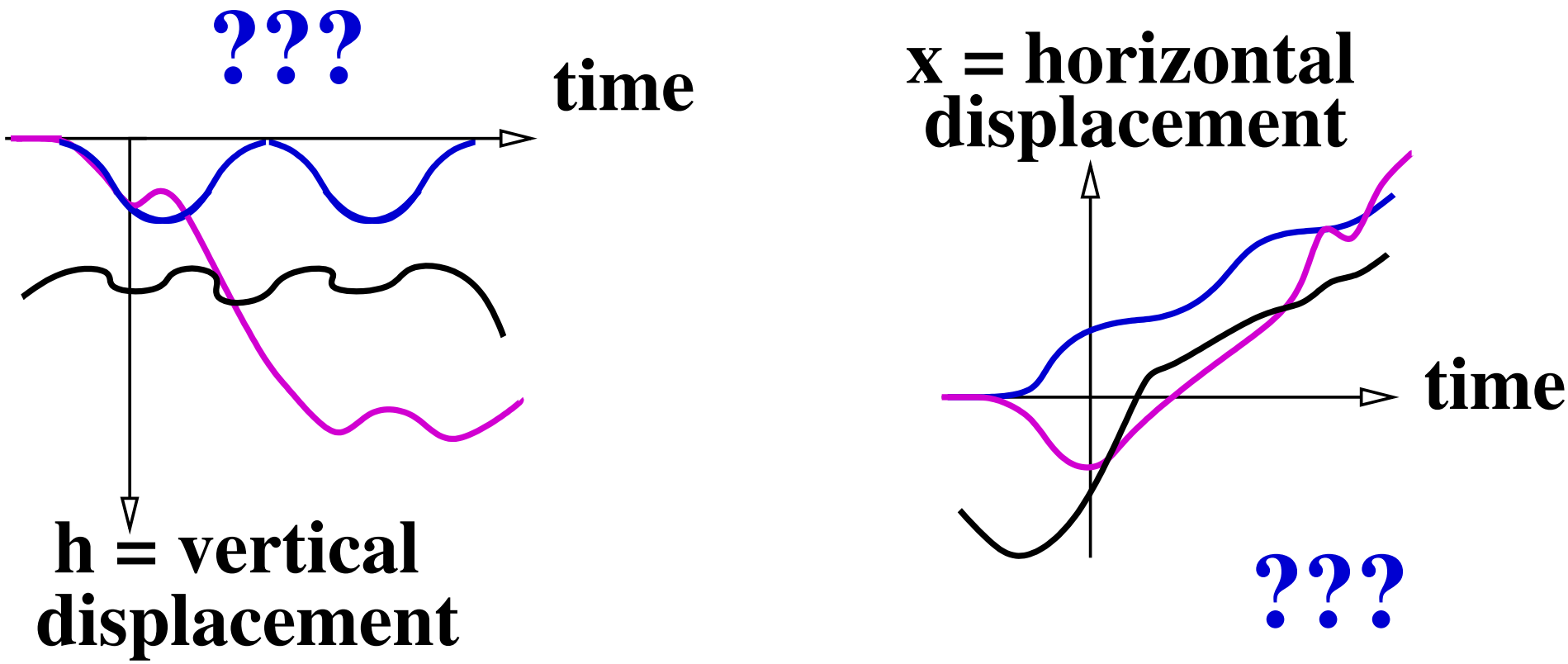

I(x,t)
I0
+
V0
−
x
V(x,t)
+
V1
−
I1


Kolonni
11:56
23.2.1998
TT12 TT23 TT212
TT41
93,5 C
TIC41
59,5 %
TT23
73,5 C
TIC23
9,6 cm
LT51
FT52
0,6 l/h
FIC52
FIC53
LIC51
0,0 %
100,0 %
TIC12
TT12
70,0 C
0,0 %
M2
FIC51
80,0 %
FT51
1,2 l/h
TT212
85,5 C
TIC212
0,0 %
LT62
34,5 cm
LT51 LT61
M3
FIC11
FT11
0,0 %
0,0 l/h
LIC61
FIC61
12,7 %
FT61
26,1 l/h
M1
Menu
Kolonni
Kolonni



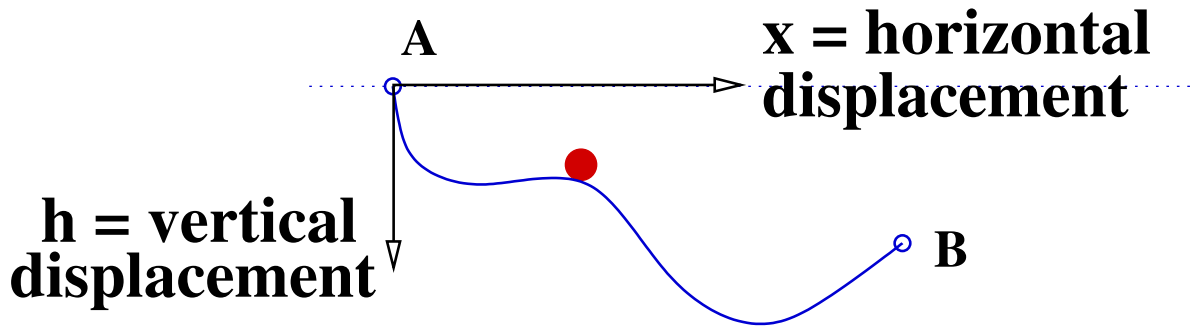

+Z
Spektr
Progress-M
+X
Priroda
Mir Core Module
-Y
Kvant
Soyuz-TM
-X
Kristall
+Y
Docking Module
Kvant-2
-Z
Shuttle

The BRACHISTOCHRONE



What is the behavior?

The BRACHISTOCHRONE



conservation of energy $\Leftrightarrow$

vertical displacement = (velocity)^2

$$h = (\frac{d}{dt}h)^2 + (\frac{d}{dt}x)^2$$

$\Rightarrow$ **Behavioral equation,** defines the behavior.

Prima la Musica,

poi le Parole.

Prima la Musica,

poi le Parole.

Prima la Física,

poi la Matematica.

RECAPITULATION

♠ **A dynamical system = a behavior**

RECAPITULATION

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♡ **From Kepler (closed system)**
to Newton's second law (open system)

RECAPITULATION

♠ **A dynamical system = a behavior**

♡ **From Kepler (closed system) to Newton's second law (open system)**

♢ **Dissipative systems**

RECAPITULATION

♠ A dynamical system = a behavior

♡ From Kepler (closed system)
to Newton's second law (open system)

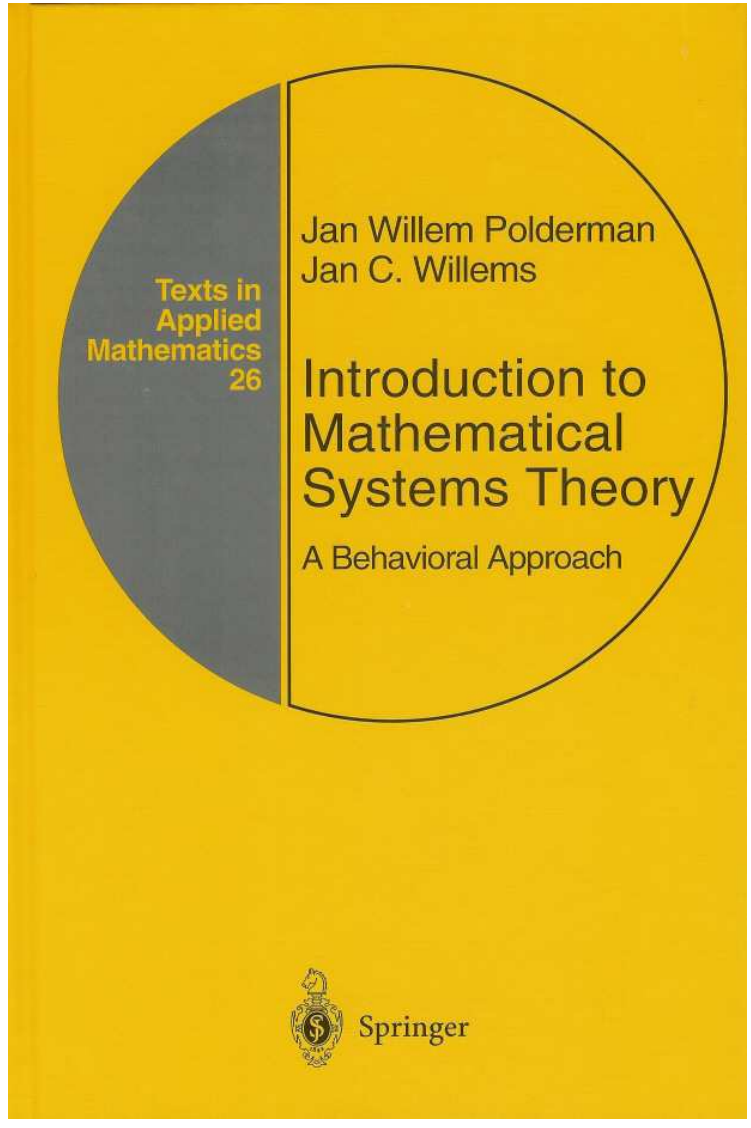
♢ Dissipative systems

♣ Almost invariant subspaces,
disturbance decoupling,
pole placement by static feedback,
instability by disc encirclement,
multipliers, . . .

Mais ou sont les neiges d'antan?

THANKS

THANKS


Texts in Applied Mathematics 26
Jan Willem Polderman
Jan C. Willems
Introduction to Mathematical Systems Theory
A Behavioral Approach
Springer

THANKS

Charles Desoer (1926-)

THANKS

Charles Desoer (1926-)

George Zames (1934-1997)

THANKS

Charles Desoer (1926-)

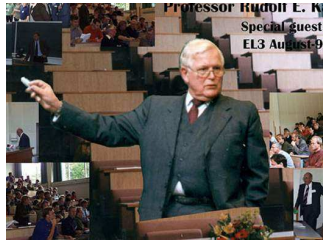
George Zames (1934-1997)

Rudolf Kalman (1930-)

THANKS

Charles Desoer (1926-)

George Zames (1934-1997)



Rudolf Kalman (1930-)

Roger Brockett (1938-)

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you

Thank you