CONTROL as INTERCONNECTION

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SISTA Seminar April 17, 2003

Program

- Part 1: Problem formulation
  - Jan Willems (SCD-SISTA, KU Leuven)
- Part 2: Controller implementability
  - Agung Julius (Twente University, NL)
- Part 3: Synthesis of dissipative systems
  - Madhu Belur (University of Groningen, NL)

The intelligent control paradigm

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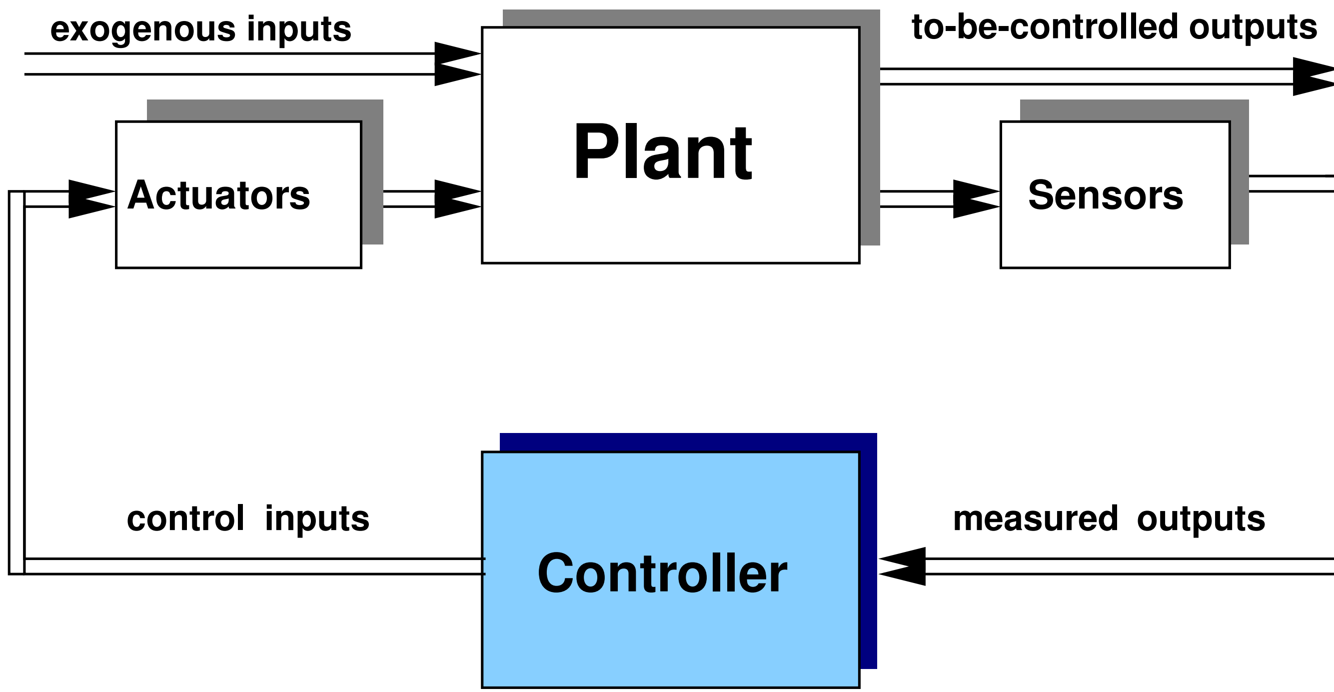

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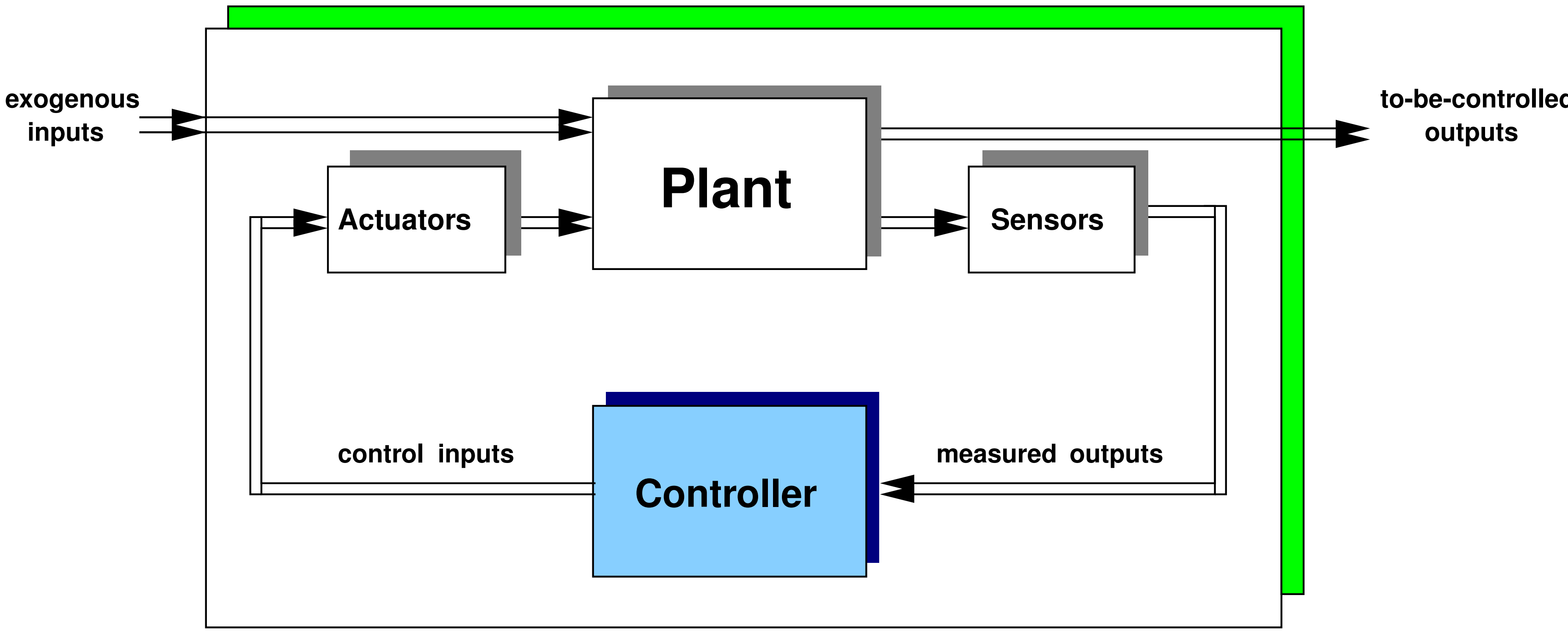

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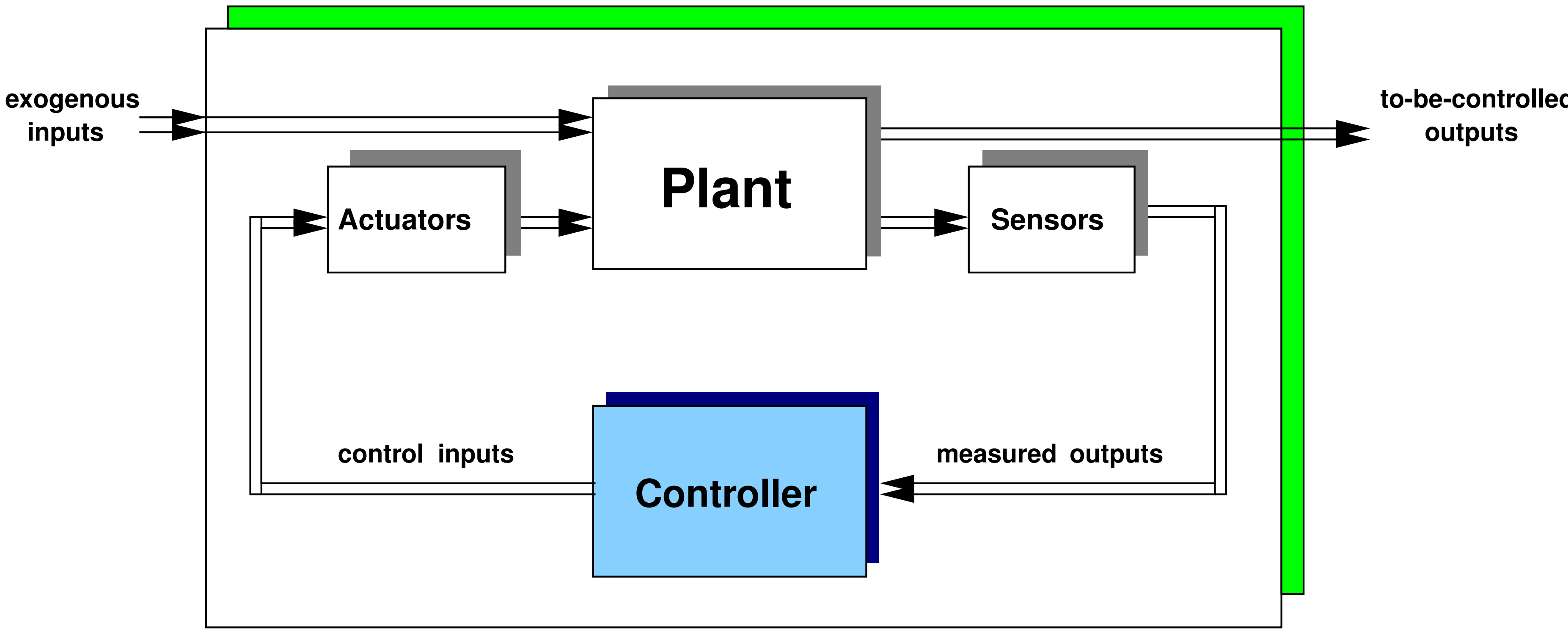

Plant, Controller, Cont’d system: i/o $\cong$ signal processors

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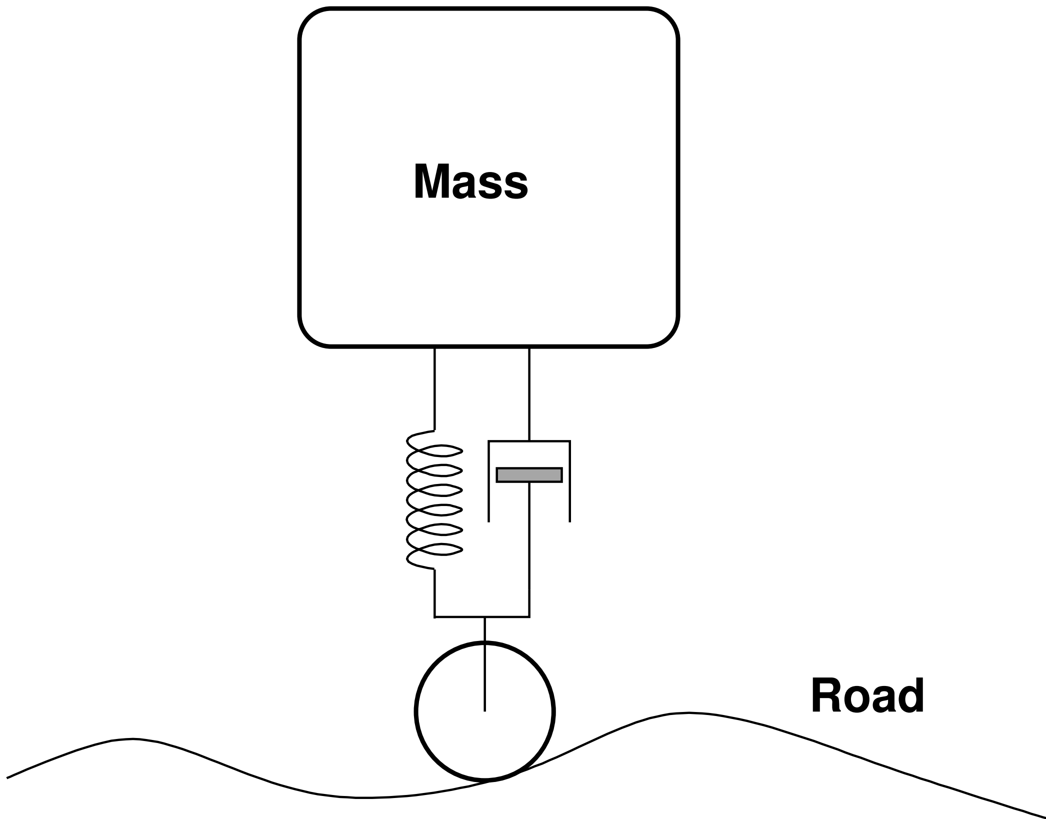
Automobile damper:



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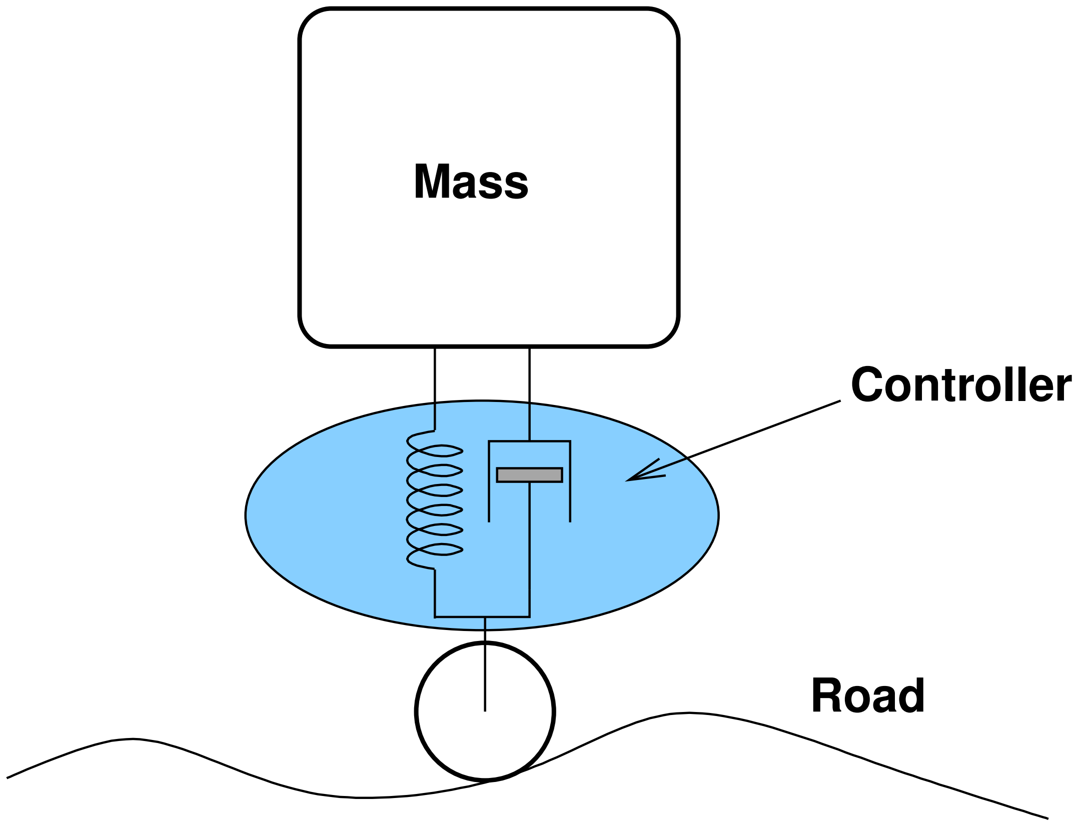

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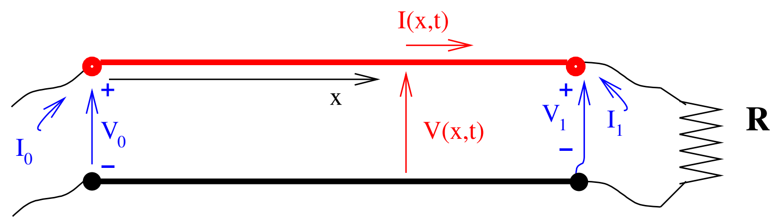
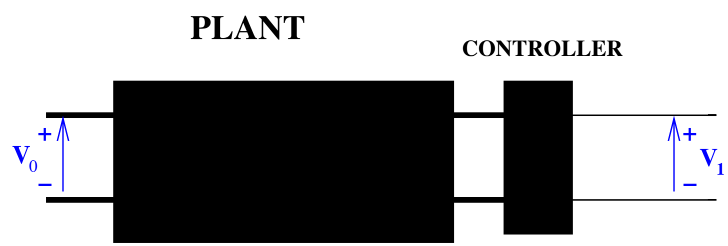
Terminated transmission line:





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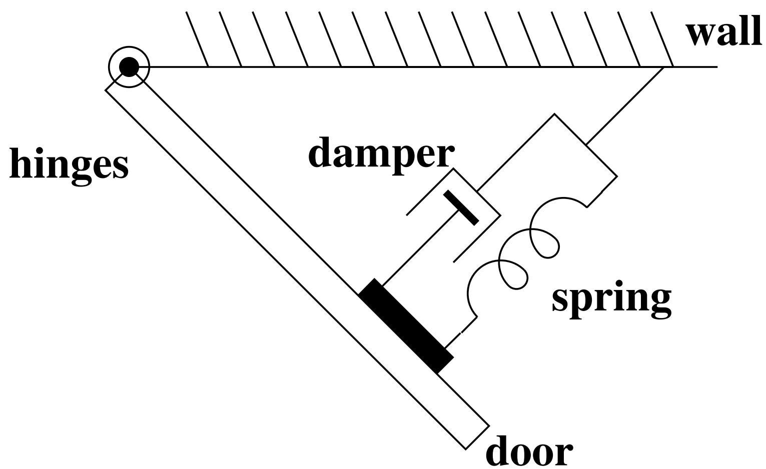
Door closing mechanism:



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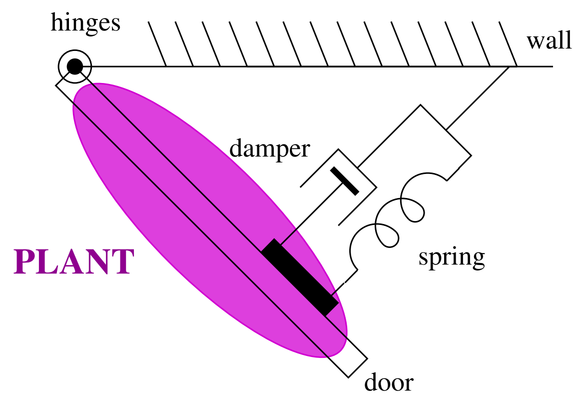

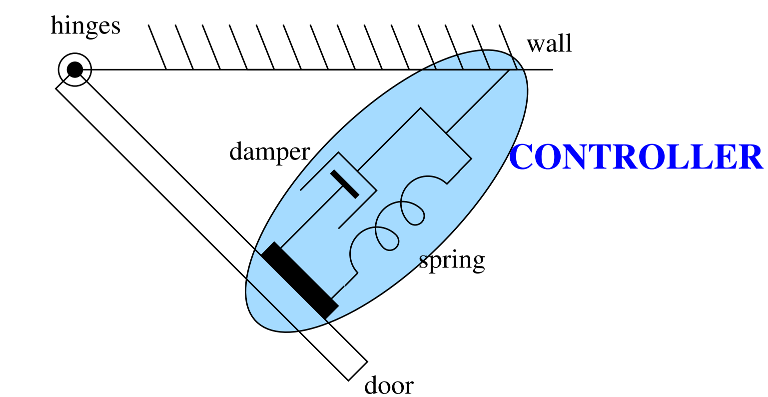

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Interconnected:

$$(M' + M'')\frac{d^2}{dt^2}\theta + D\frac{d}{dt}\theta + K\theta = F_e$$

Remarks:

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A broader framework is needed!

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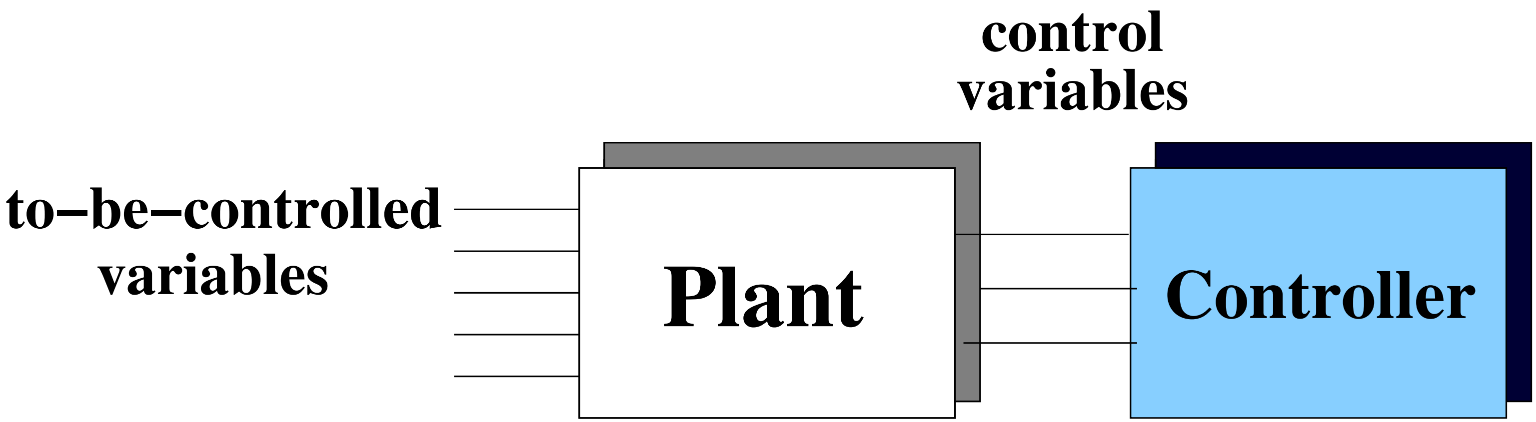

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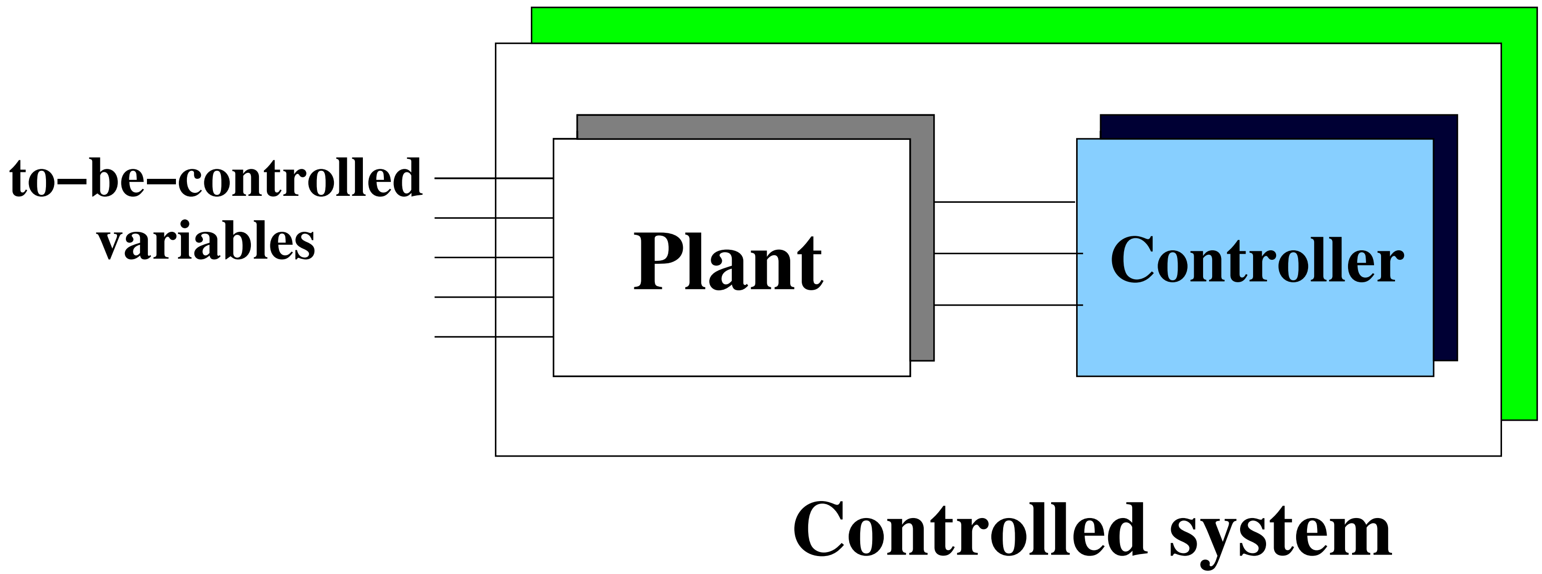

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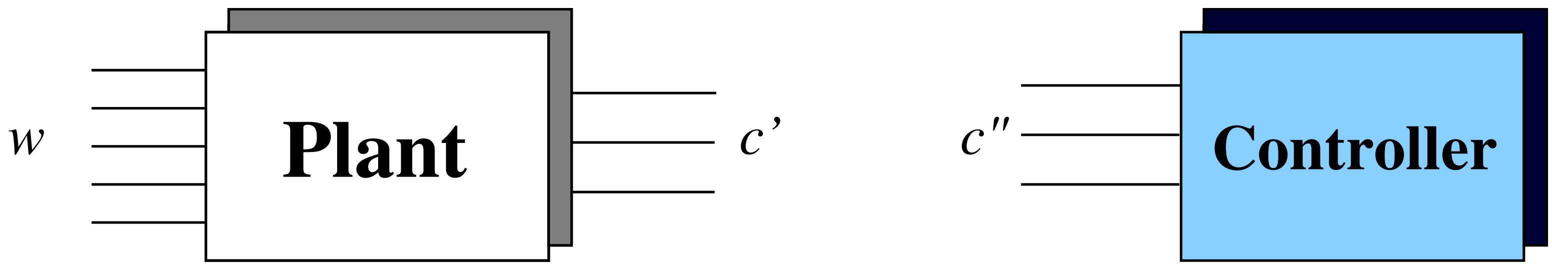

Interconnection law $\boxed{c' = c'' \quad =: c.}$ 'shared variables'

Restricts behavior of the $w's$ via the c's (=: **control**)

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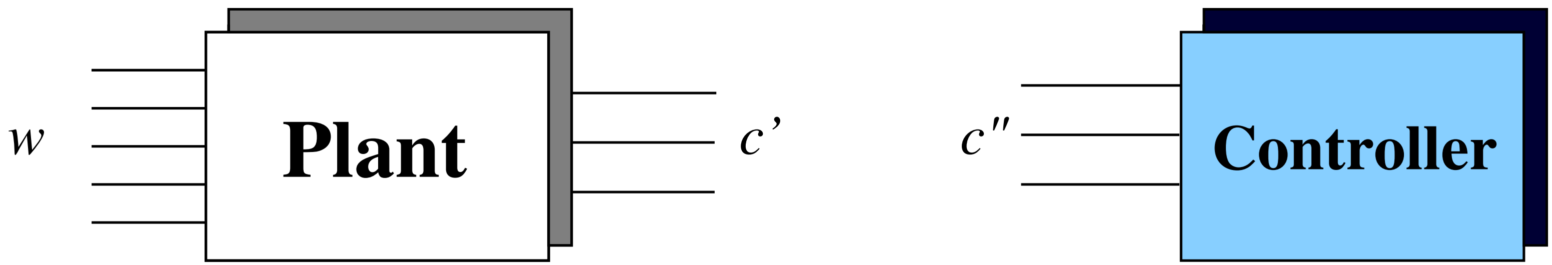

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CONTROL $\Leftrightarrow$ SUBSYSTEM DESIGN

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$\mathbb{T} \subseteq \mathbb{R} =$ 'time-set' today = $\mathbb{R}$

$\mathbb{W} =$ 'signal space' today = $\mathbb{R}^{\mathtt{w}}$

$\mathfrak{B}$ = the 'behavior' $\mathfrak{B} \subseteq \mathbb{W}^{\mathbb{T}}$

Dynamical system = $(\mathbb{T}, \mathbb{W}, \mathfrak{B})$

$\mathfrak{B}$ = the behavior = a family of trajectories,
maps: time-set $\rightarrow$ signal space

today: sol'ns of linear constant coefficient diff. eq'ns.

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$$\boxed{R_0 w + R_1 \frac{d}{dt} w + \cdots + R_{\mathrm{n}} \frac{d^{\mathrm{n}}}{dt^{\mathrm{n}}} w = 0,}$$

with $R_0, R_1, \cdots$ real matrices.

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with $R_0, R_1, \cdots$ real matrices. Shorthand notation

$$\boxed{R(\frac{d}{dt}) w = 0.}$$

R a real polynomial matrix

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$$\mathfrak{B} = \{w : \mathbb{R} \rightarrow \mathbb{R}^{\mathtt{w}} \mid R(\frac{d}{dt}) w = 0\}$$

Examples of systems $R(\frac{d}{dt})w = 0$

1.

$$p(\frac{d}{dt})y = q(\frac{d}{dt})u$$

$w = \begin{bmatrix} u \\ y \end{bmatrix}$, $R = [q \quad -p]$; SISO system with tf f'n $g = \frac{q}{p}$.

Examples of systems $R(\frac{d}{dt})w = 0$

2.

$$\frac{d}{dt}x = Ax + Bu \quad y = Cx + Cu$$

$$w = \begin{bmatrix} x \\ u \\ y \end{bmatrix}, R(\xi) = \begin{bmatrix} A - \xi I & B & 0 \\ C & D & -I \end{bmatrix}$$

Examples of systems $R(\frac{d}{dt})w = 0$

3.

$$E\frac{d}{dt}x = Ax + Bu$$

etc. etc.

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Notation: Σ or $\mathfrak{B} \in \mathfrak{L}^{\mathtt{w}}$

'Linear differential systems'

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'Linear differential systems'

$\mathfrak{L}^{\bullet}$ has very nice properties,
both mathematically and computationally!

Similar theory for difference eq'ns.

Elimination thm

Assume $\mathfrak{B} \in \mathfrak{L}^{\mathtt{w}_1+\mathtt{w}_2}$, described by

$$\boldsymbol{R_1(\frac{d}{dt})w_1 + R_2(\frac{d}{dt})w_2 = 0}$$

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$$\mathfrak{B}_1 = \{w_1 \mid \text{ there exists } w_2 \text{ such that } (w_1, w_2) \in \mathfrak{B}\}$$

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Theorem: YES!

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Theorem: YES!

$$\frac{d}{dt}x = Ax + Bu, y = Cx + Du \implies P(\frac{d}{dt})y = Q(\frac{d}{dt})u$$

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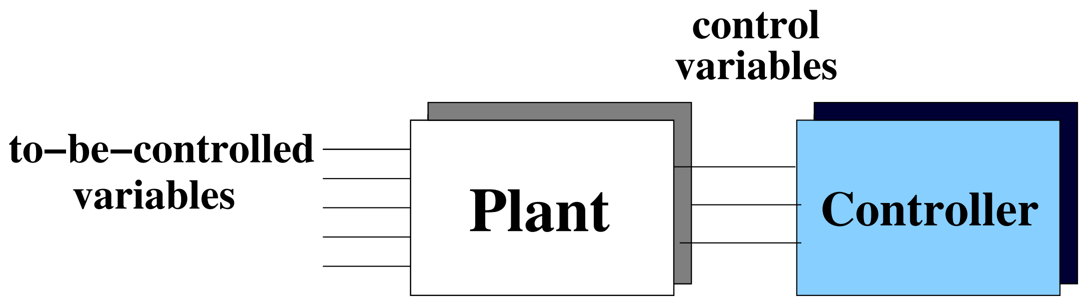

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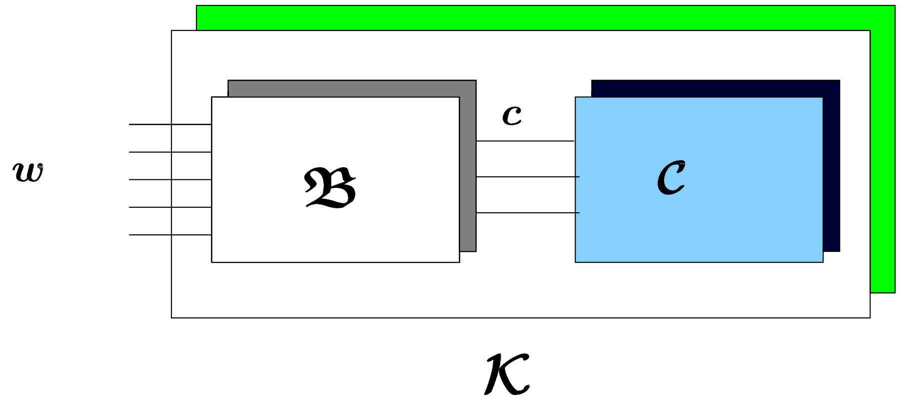

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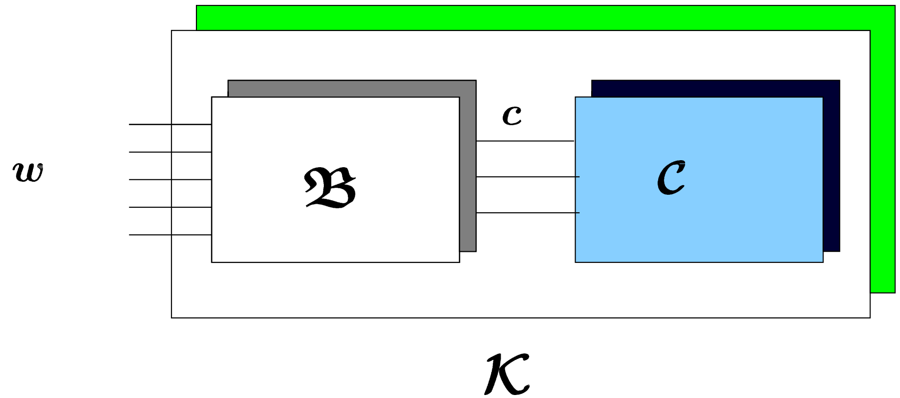

Elimination thm $\rightsquigarrow [\mathfrak{B} \in \mathfrak{L}^{\mathtt{w}+\mathtt{c}}, \mathcal{C} \in \mathfrak{L}^{\mathtt{c}} \Rightarrow \mathcal{K} \in \mathfrak{L}^{\mathtt{w}}]$

'$\mathcal{K}$ is implementable'; $\mathcal{C}$ 'implements' $\mathcal{K}$

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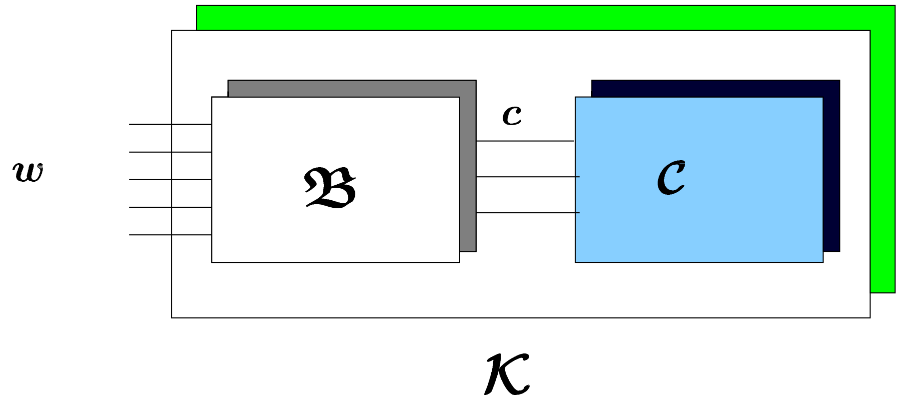

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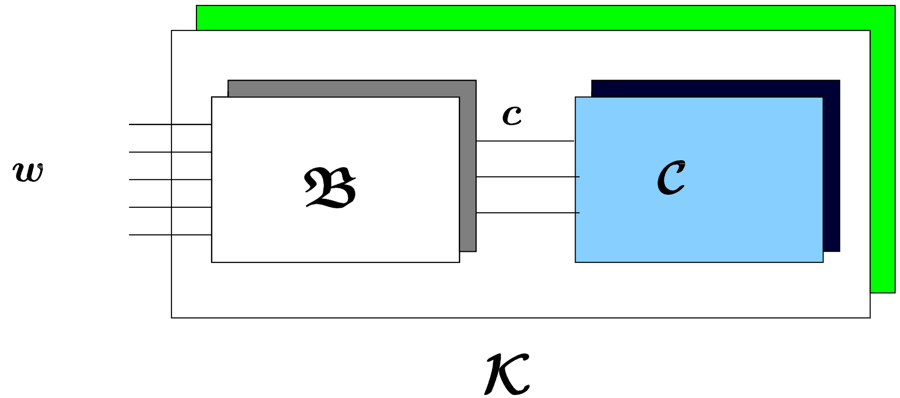

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Agung will tell you all about this and more.

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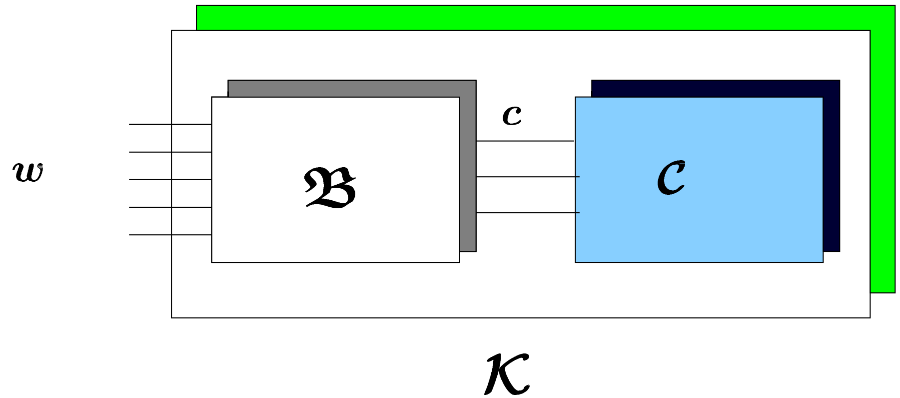

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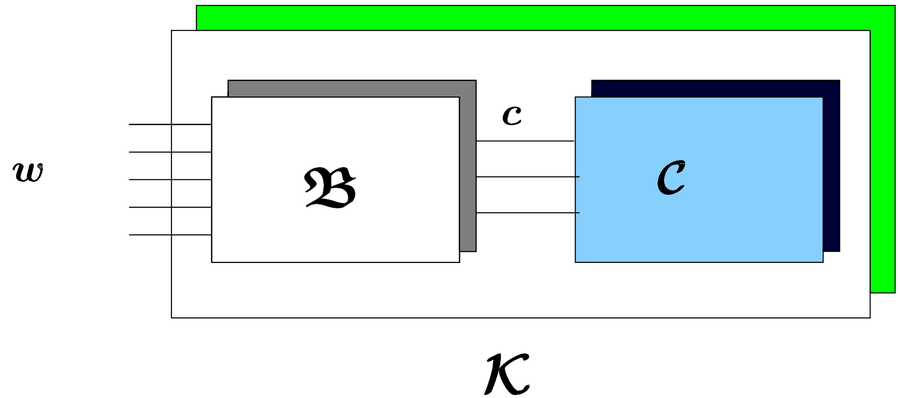

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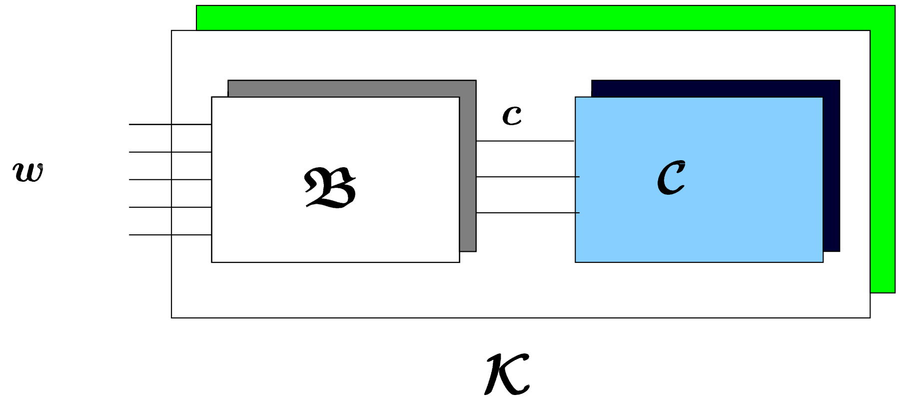

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3. Given $\mathfrak{B} \in \mathfrak{L}^{\mathtt{w}+\mathtt{c}}$, $\exists\, ?$ implementable $\mathcal{K} \in \mathfrak{L}^{\mathtt{w}}$ that is dissipative, implementable by a dissipative $\mathcal{C} \in \mathfrak{L}^{\mathtt{c}}$?

Controllability

One more def'n: Controllability of $\mathbf{\Sigma} = (\mathbb{T}, \mathbb{W}, \mathfrak{B})$

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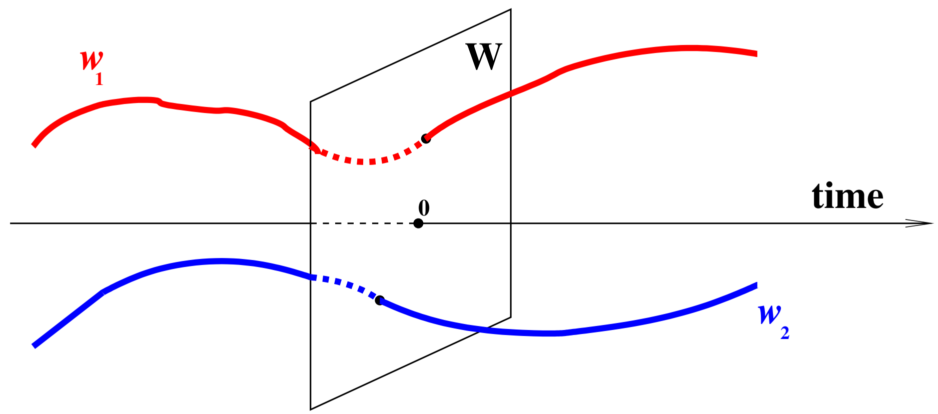

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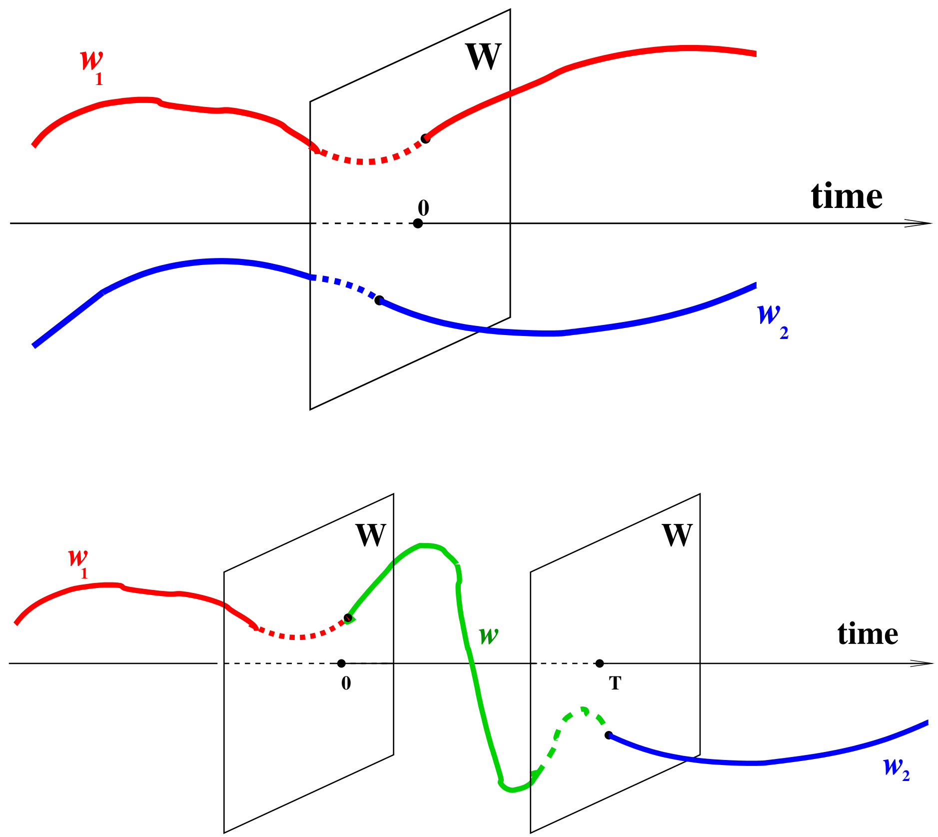

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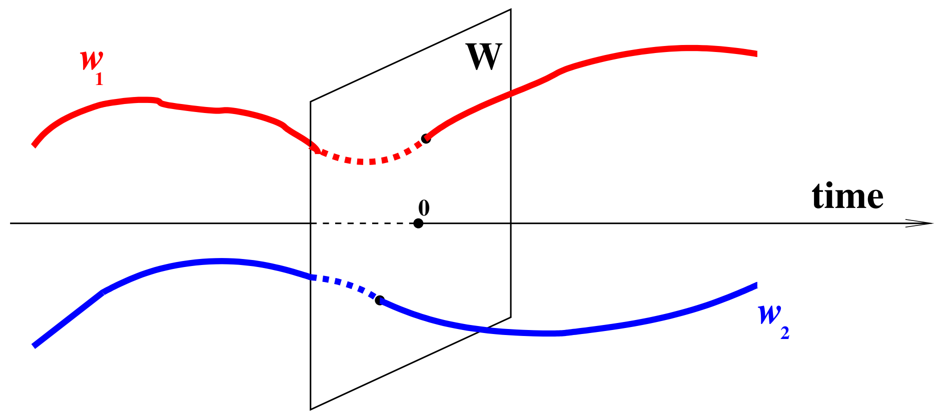

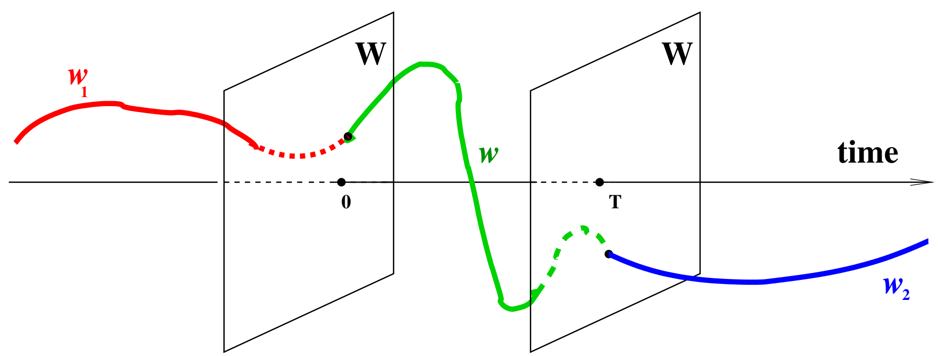

For $R(\frac{d}{dt})w = 0$ there are linear algebra conditions for controllability that act on $R_0, R_1, R_2, \ldots$.