CONTROL as INTERCONNECTION

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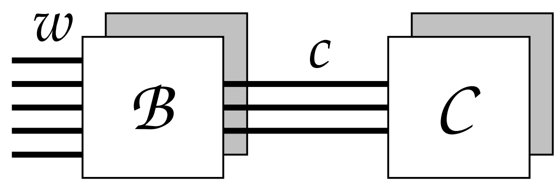
Control problems and interconnection

Problem: Find the controller $\mathcal{C}$ such that



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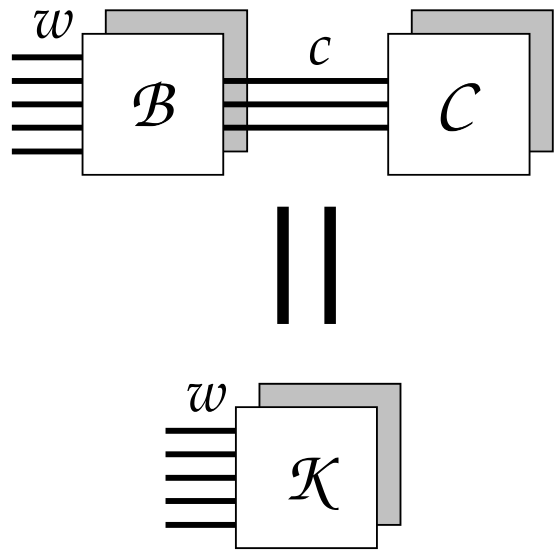
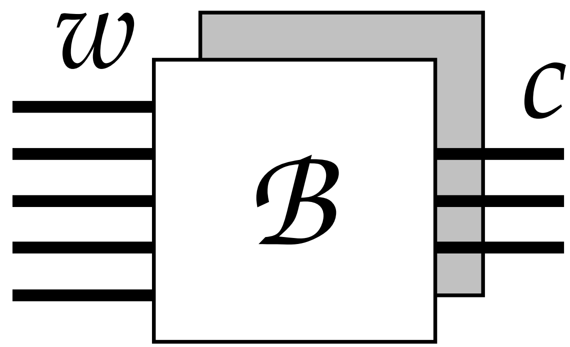


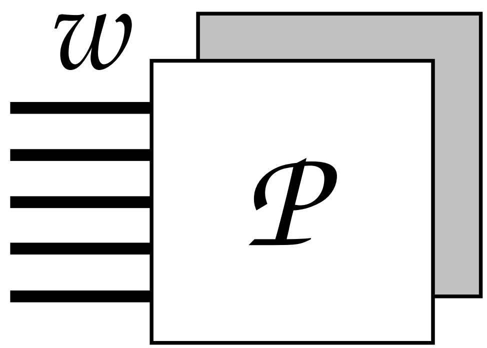
The full behavior $\mathfrak{B}$



- The behavior $\mathfrak{B}$ is given by all the solutions of $R\left(\frac{d}{dt}\right)w + M\left(\frac{d}{dt}\right)c = 0$.
- Both w and c appear explicitly in $\mathfrak{B}$

The behavior $\mathcal{P}$

- If we eliminate the variables c, from $\mathfrak{B}$ we get $\mathcal{P}$.



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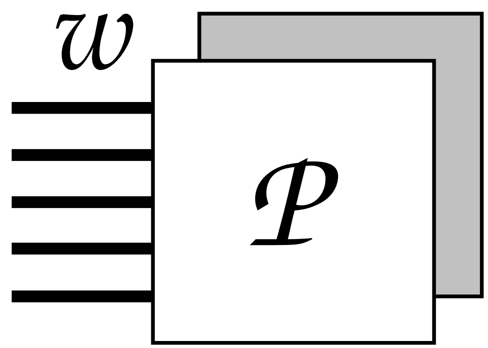

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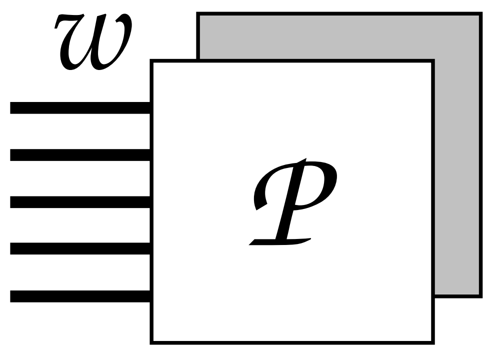

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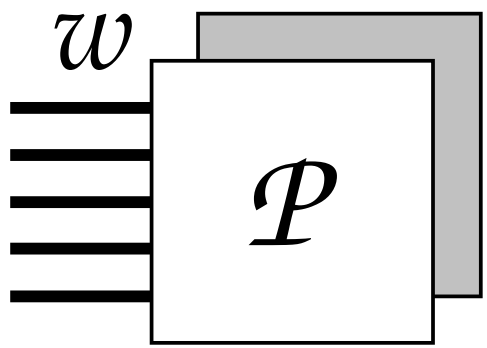

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- Hence, necessarily $\mathcal{K} \subset \mathcal{P}$

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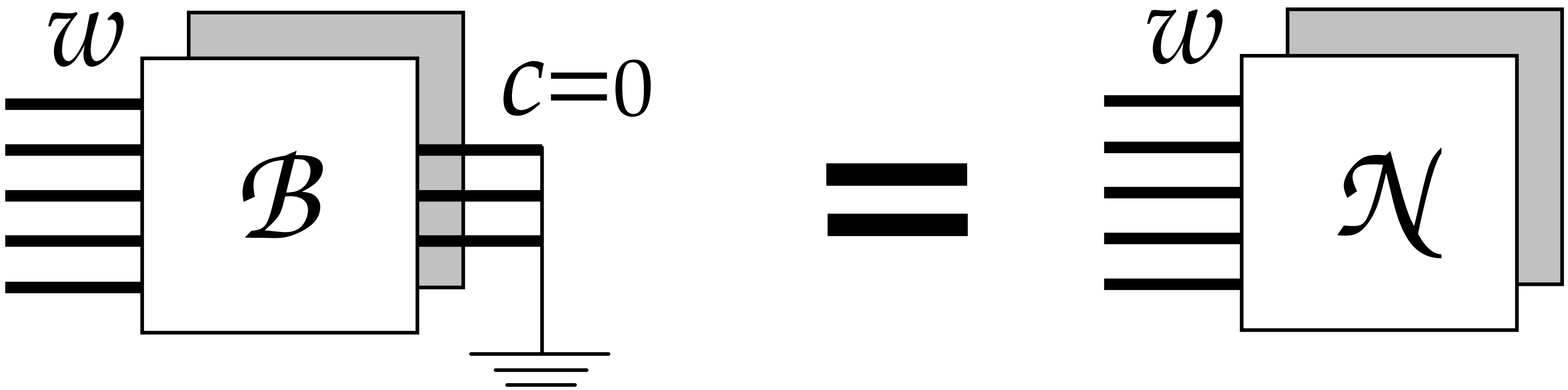

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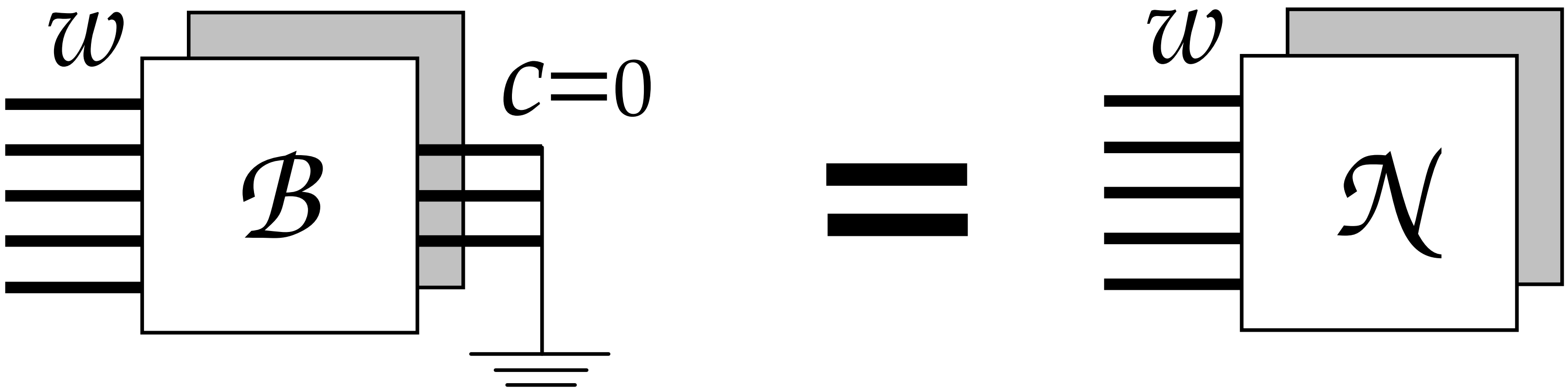

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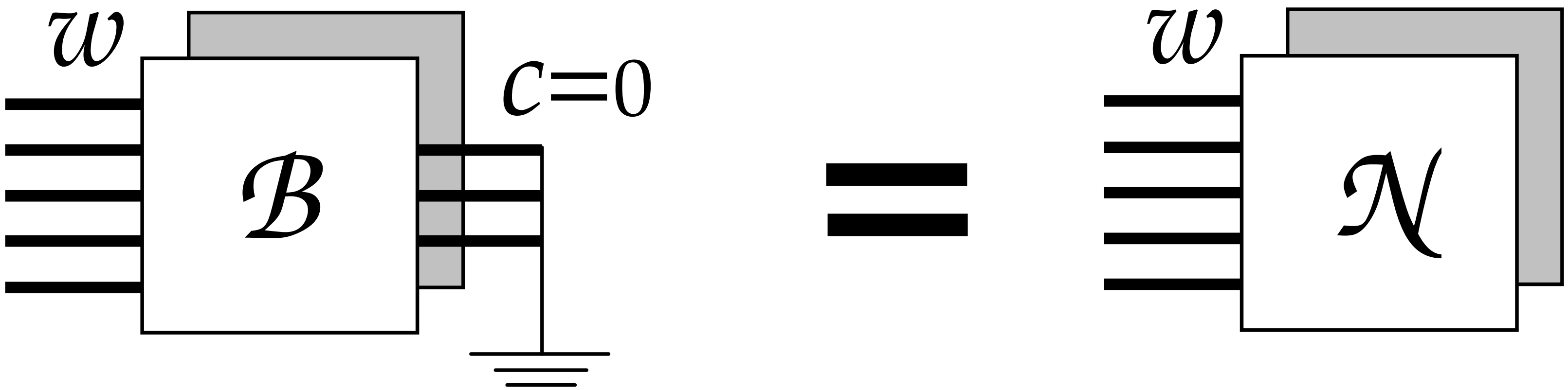

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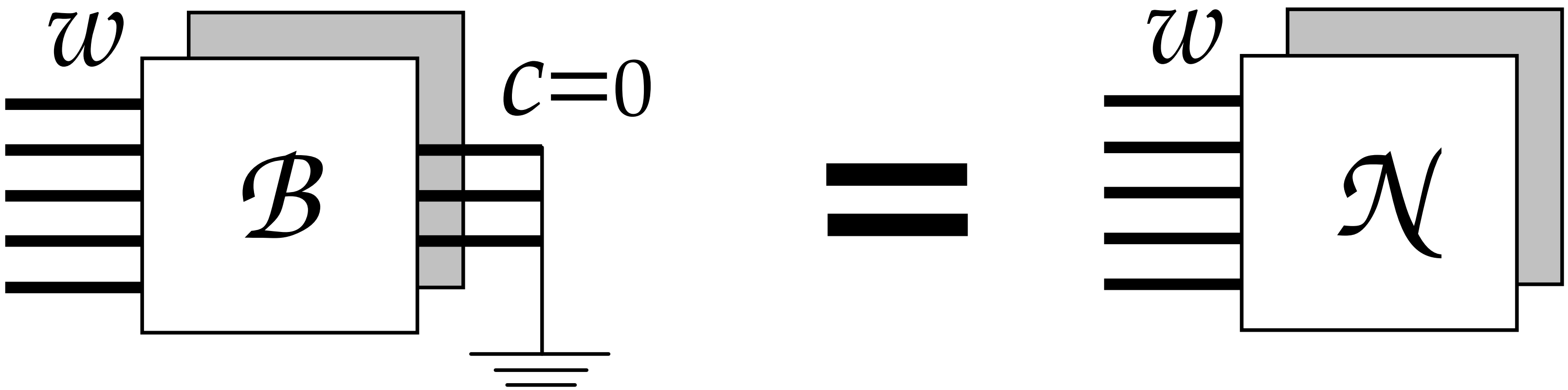

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Implementability of $\mathcal{K}$

Theorem: $\mathcal{K}$ is implementable, i.e. there exists a controller $\mathcal{C}$ such that $\mathfrak{B} \parallel_c \mathcal{C} = \mathcal{K}$ if and only if

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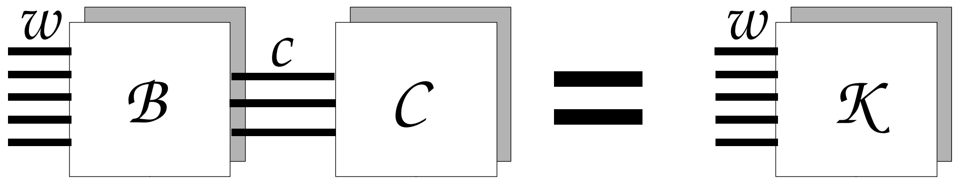
$(\Lleftarrow)$ This proof is constructive and not complicated, but will not be presented.

Control problem for general behaviors



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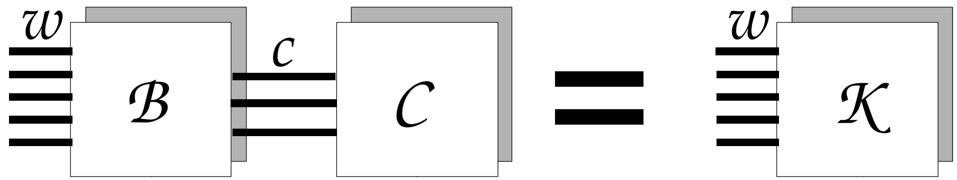

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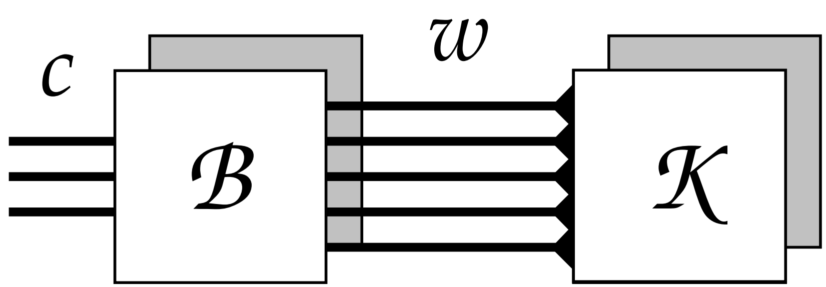
- Propose a canonical controller structure.



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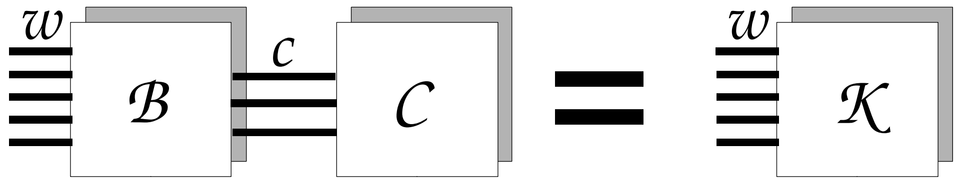

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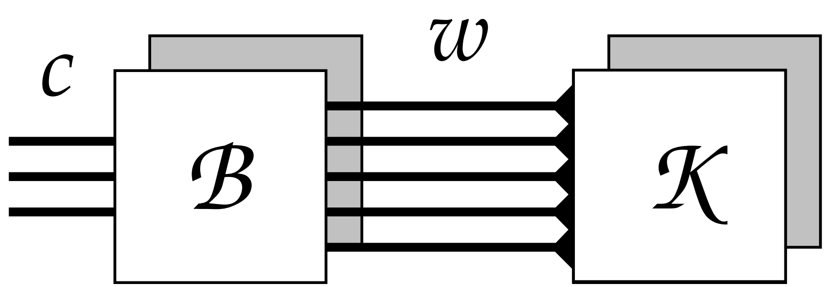
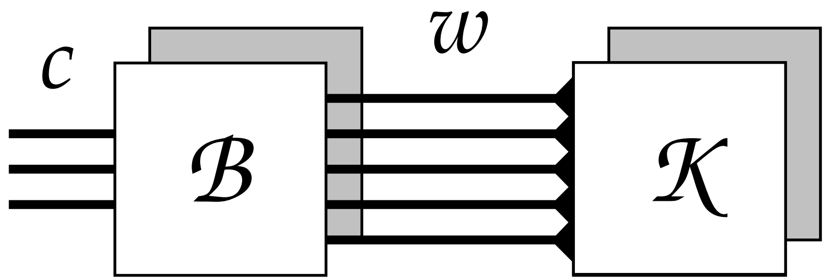


- $\mathcal{C}_{\text{can}} := \{c \,|\, \forall w, (w,c) \in \mathfrak{B} \Rightarrow w \in \mathcal{K}\}$.

About the canonical controller



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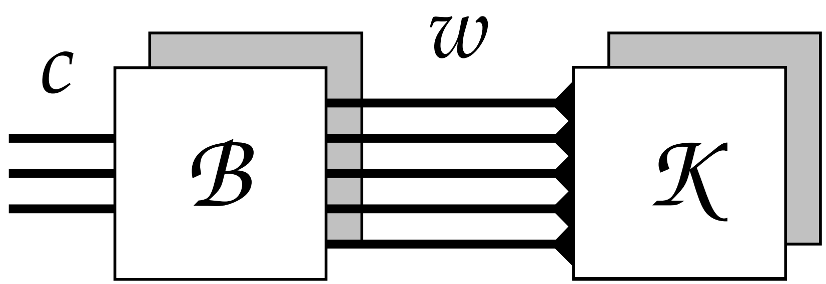

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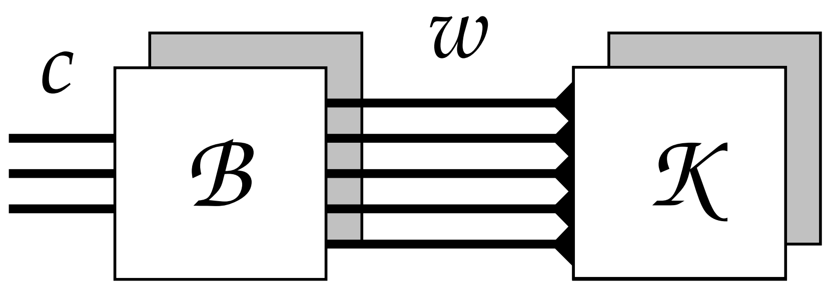

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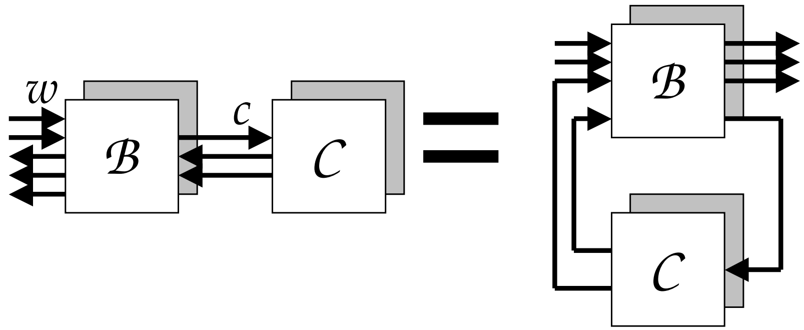
- $\mathcal{C}_{\text{can}}$ solves the control problem if and only if it is solvable at all.
- For linear systems case, computation of $\mathcal{C}_{\text{can}}$ can be done.

A second look on the control problem (1)

- Feedback control is a special case.



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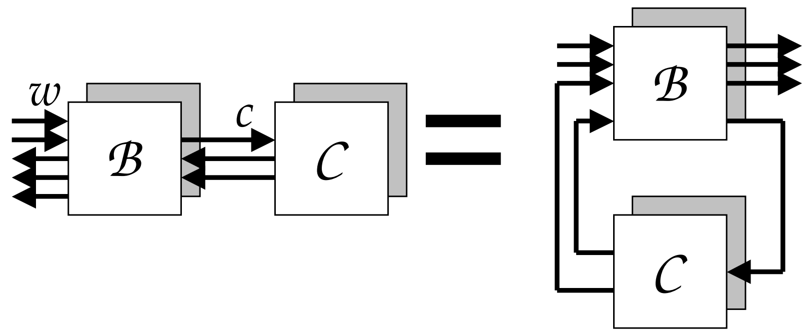

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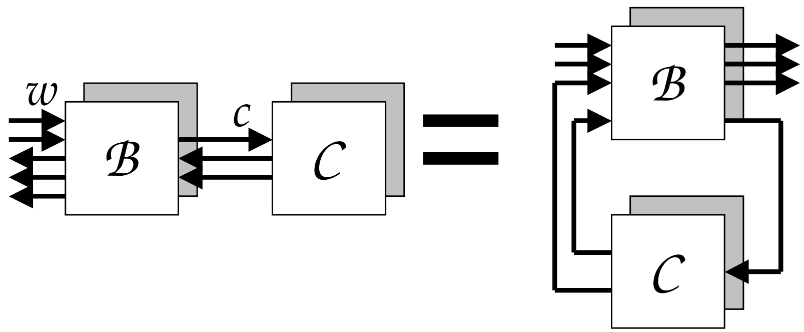

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- Accuracy is unlimited, all sub-behaviors are implementable.

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- Is the canonical controller compatible?

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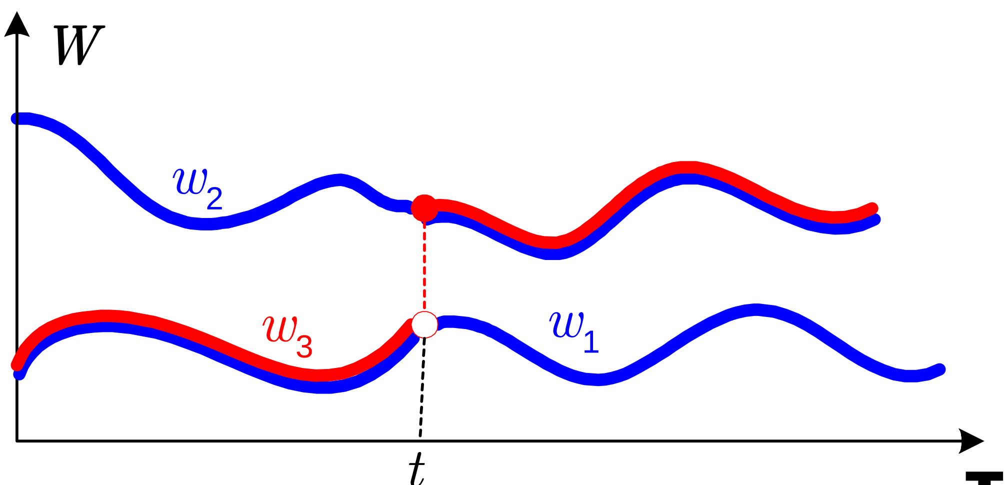

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- The interconnection $\mathfrak{B}_1 \parallel \mathfrak{B}_2$ is compatible at time t, if for any $w_i \in \mathfrak{B}_i$, $i = 1, 2$, there exist a $w \in \mathfrak{B}_1 \parallel \mathfrak{B}_2$, and $t \in \mathcal{T}$ such that $w_1 D_{\mathfrak{B}_1}(t) w$ and $w_2 D_{\mathfrak{B}_2}(t) w$.

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- For linear systems, uniform compatibility $\Leftrightarrow$ linear feedback with proper transfer functions.

Weak compatibility

- Let $w_1, w_2 \in \mathfrak{B}$ We say that w_1 is weakly directable to w_2 at time t (Notation : $w_1 D^*_{\mathfrak{B}}(t) w_2$) if there exists a trajectory $w_3 \in \mathfrak{B}$ and a $t' \leq t$ such that

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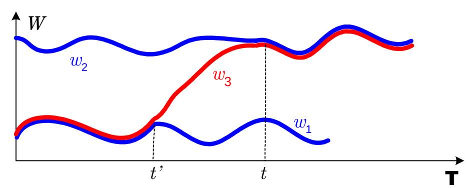

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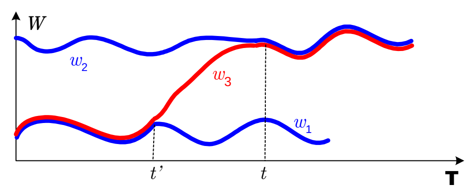

- The interconnection $\mathfrak{B}_1 \parallel \mathfrak{B}_2$ is weakly compatible at time t, if for any $w_i \in \mathfrak{B}_i$, $i = 1, 2$, there exist a $w \in \mathfrak{B}_1 \parallel \mathfrak{B}_2$, and $t \in \mathcal{T}$ such that $w_1 D^*_{\mathfrak{B}_1}(t) w$ and $w_2 D^*_{\mathfrak{B}_2}(t) w$.

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- $\mathcal{P}^{\text{ctr}}$ is uniquely defined and computable.
- Consequence: $\mathcal{N} = \{0\}$ and $\mathcal{P}$ controllable $\Rightarrow$ all $\mathcal{K} \subset \mathcal{P}$ is weakly comp. implementable.

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Theorem: Given any monic polynomial $r(\xi)$, it is possible to find a weakly comp. controller $\mathcal{C}$ such that $r(\xi)$ is the closed-loop characteristic polynomials if and only if $\mathcal{N} = \{0\}$ and $\mathcal{P}$ is controllable.

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- Weakly compatible implementability is characterized.
- Strongly compatible implementability is still an open problem.