STATE

and

FIRST ORDER REPRESENTATIONS

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CONJECTURE

A system with behavior described by
a linear constant coefficient PDE is
Markovian
if and only if the PDE is
first order.

$\mathtt{n}$-D linear differential systems

Set of independent variables $= \mathbb{R}^{\mathtt{n}}$; of dependent variables $= \mathbb{R}^{\mathtt{w}}$.

Let $R \in \mathbb{R}^{\bullet \times \mathtt{w}}[\xi_1, \cdots, \xi_{\mathtt{n}}]$, and consider the system of linear constant coefficient PDE's

$$R(\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_{\mathtt{n}}})w = 0 \quad (*)$$

This defines an $\mathtt{n}$-D system with behavior

$$\mathfrak{B} = \{w \in \mathfrak{D}'(\mathbb{R}^{\mathtt{n}}, \mathbb{R}^{\mathtt{w}}) \mid (*) \text{ holds } \}.$$

We refer to such behaviors as 'linear differential behaviors', and to $(*)$ as a 'kernel representation' of the behavior $\mathfrak{B}$.

Note: $\mathfrak{B}$ has many kernel representations.

Nice 3-way partitions

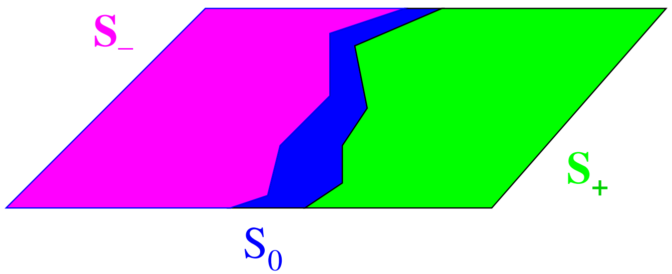
A 3-way partition (S_-, S_0, S_+) of $\mathbb{R}^n$

(all 3 non-empty, non-overlapping, union = $\mathbb{R}^n$)

is said to be a

nice 3-way partition of $\mathbb{R}^n$

if S_- and S_+ are open and S_0 is closed.



Think of S_- as the 'past', S_0 as the 'present', S_+ as the 'future'.

Markovian $\mathtt{n}$-D behaviors

A linear differential behavior $\mathfrak{B}$ is said to be

Markovian

if for all nice 3-way partitions (S_-, S_0, S_+) of $\mathbb{R}^{\mathtt{n}}$

and for all $w_1, w_2 \in \mathfrak{B} \cap \mathfrak{C}^\infty(\mathbb{R}^{\mathtt{n}}, \mathbb{R}^{\mathtt{w}})$

such that

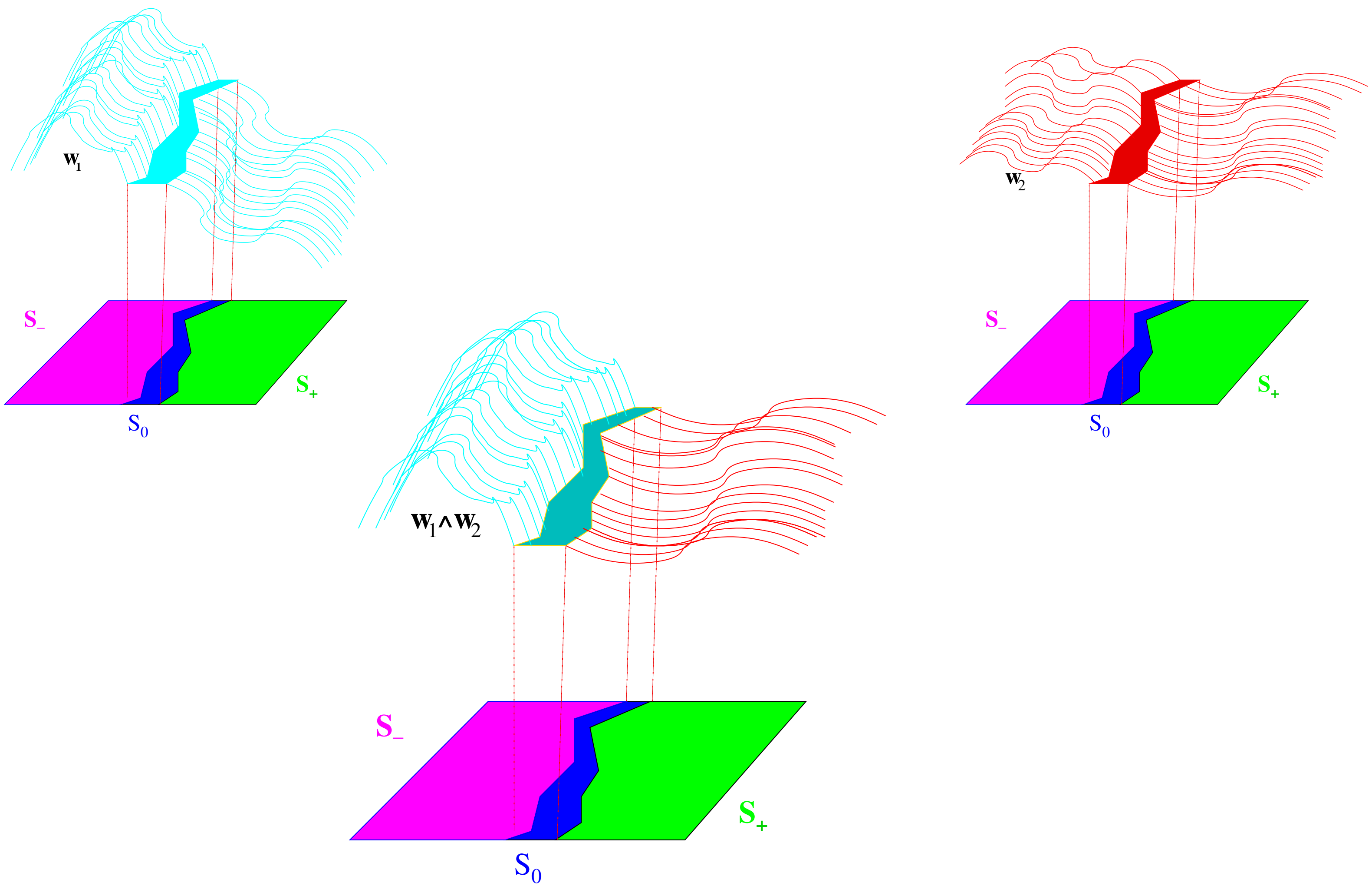
$$w_1(x) = w_2(x) \quad \text{for } x \in S_0,$$

there holds

$$w_1 \underset{S_0}{\wedge} w_2 \in \mathfrak{B}.$$

where $w_1 \underset{S_0}{\wedge} w_2$ **denotes the concatenation along** S_0**, i.e.**

$$(w_1 \underset{S_0}{\wedge} w_2)(x) := \begin{cases} w_1(x) & \textbf{for} \quad x \in S_- \\ w_1(x) = w_2(x) & \textbf{for} \quad x \in S_0 \\ w_2(x) & \textbf{for} \quad x \in S_+ \end{cases}$$



Markovian: $\quad w_1, w_2 \in \mathfrak{B} \quad \Rightarrow \quad w_1 \underset{S_0}{\wedge} w_2 \in \mathfrak{B}.$

Precise CONJECTURE

The linear differential behavior $\mathfrak{B}$ is Markovian

if and only if

it admits a kernel representation that is

first order.

I.e. a kernel representation of the form

$$R_0 w + R_1 \frac{\partial}{\partial x_1} w + R_2 \frac{\partial}{\partial x_2} w + \cdots + R_{\mathrm{n}} \frac{\partial}{\partial x_{\mathrm{n}}} w = 0$$

with $R_0, R_1, \ldots, R_{\mathrm{n}} \in \mathbb{R}^{\bullet \times \mathtt{w}}$ constant matrices

Notes: The **"if"-part** is easy.

The conjecture is true for $n = 1$, i.e. in the ODE case.

The conjecture is true if there is **only one** PDE.

Example of a Markovian system: Maxwell's equations.

Non-examples: Diffusion equation; Wave equation.

Situation is more involved if the set of independent variables is $\mathbb{Z}^n$ (cfr. Rocha)

Motivation: To understand **state and state construction** for n-D systems.

Manuscript & copies of the lecture frames are available from/at

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Thank you!