Observer Synthesis in the Behavioral Approach

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***Abstract*— This paper analyzes the observer design problem in the behavioral context. Observability and detectability notions are first introduced and fully characterized. Necessary and sufficient conditions for the existence of an observer, possibly an asymptotic or an exact one, are introduced, and a complete parameterization of all admissible observers is given. The problem of obtaining observers endowed with a (strictly) proper transfer matrix and the design of observer-based controllers are later addressed and solved. Finally, the above issues are particularized to the case of state-space systems, thus showing they naturally generalize well-known theorems of traditional system theory.**

***Index Terms*— Behaviors, latent variable models, observability/detectability, proper transfer matrices.**

I. Introduction

In the last decade, the behavioral point of view [11], [17], [18] has received an increasingly broader acceptance as an approach for modeling dynamic systems, and now it is generally viewed as a cogent framework for system analysis. One of the reasons for its success has to be looked for in the fact that it does not start with the input/output point of view for describing how a system interacts with its environment, but focuses on the set of system trajectories, the *behavior*, and hence on the mathematical model describing the relations among all system variables.

By assuming this point of view, important aspects of the classical system theory have been translated and solved, thus leading to interesting results, which are powerful generalizations of well-known theorems obtained within the input/output or state-space contexts. In particular, recently, the control problem has been posed in the behavioral setting [20], where it can be naturally viewed as a problem of systems interconnection. Although several issues have already been analyzed in some detail, the important question of estimating some system variables, not available for measurements, from others, which are measured, has not been treated yet.

The synthesis of an observer of the state for a (linear time-invariant) state-space system has been the object of considerable interest in classical system theory [1], [10]. The original theory of state observers was concerned with the problem of reconstructing (or estimating) the state from the corresponding inputs and outputs. This problem has been later generalized in various ways, and in the last years, a great deal of research has been aimed at state observers in the presence of unknown inputs (disturbances) [2]–[4], [12]–[16].

In this paper, we will be interested in the observer problem for linear, time-invariant (continuous-time) dynamic systems, which are described in behavioral terms by means of a set of differential equations. More precisely, we will consider a dynamic system $\Sigma = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B})$, whose trajectories $(\mathbf{w}_1, \mathbf{w}_2)$ satisfy some set of differential equations

$$R_2\left(\frac{d}{dt}\right)\mathbf{w}_2 = R_1\left(\frac{d}{dt}\right)\mathbf{w}_1$$

and we will assume $\mathbf{w}_1$ can be measured and $\mathbf{w}_2$ is unknown. The natural goal is that of designing an estimator of $\mathbf{w}_2$ based on the knowledge of $\mathbf{w}_1$, such that its estimation error goes to zero asymptotically.

To reach this goal, we shall first introduce, in Section II, the notions of observability and detectability of $\mathbf{w}_2$ from $\mathbf{w}_1$, and then provide (Section III) necessary and sufficient conditions for the existence of an (asymptotic) observer. This discussion then leads to a complete parameterization of all possible observers, in terms of the polynomial matrices involved in the system description.

Section IV deals with the problem of determining under what conditions it is possible to obtain (asymptotic) observers with a proper transfer matrix. Of course, the estimate $\hat{\mathbf{w}}_2$ produced by the observer can be used, together with the measured variable $\mathbf{w}_1$, to control the whole plant, thus obtaining, in Section V, an "observer-based controller" for the original plant.

Finally, in Section VI, the main definitions and results provided in this paper are particularized to the case of state-space models, thus showing they constitute natural extensions, to the behavioral setting, of the analogous definitions and results obtained in classic system theory.

In this paper, integers refering to dimensions of linear spaces or sizes of matrices are always denoted in typewriter fonts. For instance, $\mathbb{R}^{\mathtt{w}_i}$ denotes the linear space of real column vectors with $\mathtt{w}_i$ components, $\mathbb{R}^{\mathtt{r}\times\mathtt{w}_i}$ is the space of real $\mathtt{r}\times\mathtt{w}_i$ matrices, and $I_{\mathtt{w}_i}$ is the identity matrix of size $\mathtt{w}_i$. We also make the following convention: vectors $\mathbf{w}_i, \mathbf{d}$, and ℓ are elements of $\mathbb{R}^{\mathtt{w}_i}$, $\mathbb{R}^{\mathtt{d}}$, and $\mathbb{R}^{1}$, respectively. In keeping with the usual notation, when dealing with state-space systems, however, we will assume the state vector $\mathbf{x}$ is $\mathtt{n}$-dimensional, the output vector $\mathbf{y}$ is $\mathtt{p}$-dimensional, and the input vector $\mathbf{u}$ is $\mathtt{m}$-dimensional.

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II. Observability and Detectability

Consider a dynamic system $\Sigma = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B})$, with trajectories $(\mathbf{w}_1, \mathbf{w}_2)$, whose behavior $\mathfrak{B}$ is specified by the set of differential equations

$$R_2\left(\frac{d}{dt}\right)\mathbf{w}_2 = R_1\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{1}$$

with $R_1 \in \mathbb{R}[\xi]^{\mathtt{r}\times\mathtt{w}_1}$ and $R_2 \in \mathbb{R}[\xi]^{\mathtt{r}\times\mathtt{w}_2}$ polynomial matrices, and $\mathtt{w}_i := \dim \mathbf{w}_i, i = 1, 2$. In the sequel, we will assume the trajectories $(\mathbf{w}_1, \mathbf{w}_2)$ belong to $\mathfrak{L}^{\text{loc}}(\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2})$, the space of all measurable functions f from $\mathbb{R}$ to $\mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}$ for which the integral $\int_{t_1}^{t_2} \|f(t)\|\, dt$ is finite for all t_1 and t_2. Solutions $(\mathbf{w}_1, \mathbf{w}_2)$ that are not smooth are considered to be solutions in the distributional sense, with both the left- and right-hand sides of (1) considered in the sense of distributions. The interested reader is referred to [11] for details. The set of trajectories $(\mathbf{w}_1, \mathbf{w}_2)$ satisfying (1) will be denoted, for short, by $\ker([R_2(d/dt) \mid -R_1(d/dt)])$.

If we assume $\mathbf{w}_1$ can be exactly measured and $\mathbf{w}_2$ is completely unknown, it is natural to search for necessary and sufficient conditions for the existence of an estimator of $\mathbf{w}_2$ based on the knowledge of $\mathbf{w}_1$, whose estimation error goes to zero asymptotically. The first step toward this end is to introduce the notions of observability [18] and detectability.

Definition 2.1: Consider a dynamic system $\Sigma = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B})$, whose behavior $\mathfrak{B}$ is described as follows:

$$\mathfrak{B} := \left\{(\mathbf{w}_1, \mathbf{w}_2) \in (\mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2})^{\mathbb{R}} : R_2\left(\frac{d}{dt}\right)\mathbf{w}_2 = R_1\left(\frac{d}{dt}\right)\mathbf{w}_1\right\} \tag{2}$$

with $\mathtt{w}_i := \dim \mathbf{w}_i,\ i = 1, 2$. We say $\mathbf{w}_2$ is

- observable from $\mathbf{w}_1$, if $(\mathbf{w}_1, \mathbf{w}_2), (\mathbf{w}_1, \bar{\mathbf{w}}_2) \in \mathfrak{B}$ implies $\mathbf{w}_2 = \bar{\mathbf{w}}_2$;
- detectable from $\mathbf{w}_1$, if $(\mathbf{w}_1, \mathbf{w}_2), (\mathbf{w}_1, \bar{\mathbf{w}}_2) \in \mathfrak{B}$ implies

$$\mathbf{w}_2(t) - \bar{\mathbf{w}}_2(t) \underset{t\to+\infty}{\longrightarrow} 0.$$

Proposition 2.2: Consider the dynamic system $\Sigma = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B})$ described by (2), with $R_1 \in \mathbb{R}[\xi]^{\mathtt{r}\times\mathtt{w}_1}$ and $R_2 \in \mathbb{R}[\xi]^{\mathtt{r}\times\mathtt{w}_2}$. Then,

i) $\mathbf{w}_2$ is observable from $\mathbf{w}_1$ if and only if R_2 is a right prime matrix, or, equivalently, if and only if $R_2(\lambda)$ has full column rank for all $\lambda \in \mathbb{C}$;

ii) $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$ if and only if R_2 is of full column rank and the g.c.d. of its maximal-order minors is Hurwitz, or, equivalently, if and only if $R_2(\lambda)$ has full column rank for all $\lambda \in \mathbb{C}^+ := \{\lambda \in \mathbb{C} : \Re e(\lambda) \geq 0\}$.

Proof: 1) Assume R_2 is right prime, and let U be an $\mathtt{r} \times \mathtt{r}$ unimodular matrix such that

$$U(\xi)R_2(\xi) = \begin{bmatrix} I_{\mathtt{w}_2} \\ 0 \end{bmatrix}.$$

Once we conformably partition UR_1 as

$$\begin{bmatrix} N_1(\xi) \\ D_1(\xi) \end{bmatrix} = U(\xi)R_1(\xi)$$

we obtain the following equivalent description for the behavior $\mathfrak{B}$ of (2)

$$\mathfrak{B} = \left\{(\mathbf{w}_1, \mathbf{w}_2) \in (\mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2})^{\mathbb{R}} : D_1\left(\frac{d}{dt}\right)\mathbf{w}_1 = 0 \text{ and } \mathbf{w}_2 = N_1\left(\frac{d}{dt}\right)\mathbf{w}_1\right\} \tag{3}$$

which clearly proves $\mathbf{w}_2$ is observable from $\mathbf{w}_1$.

On the other hand, if R_2 were not right prime, its kernel would include some nonzero trajectory $\mathbf{v}$ [11]. So, $(\mathbf{w}_1, \mathbf{w}_2) \in \mathfrak{B}$ would also imply $(\mathbf{w}_1, \mathbf{w}_2 + \mathbf{v}) \in \mathfrak{B}$, thus ruling out observability.

2) The proof follows the same lines as the previous one. Indeed, $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$ if and only if $\ker(R_2(d/dt))$ is a stable autonomous behavior [11]; namely, it is the kernel of a full column rank matrix, and all its trajectories go to zero asymptotically. This amounts to saying R_2 is of full column rank with g.c.d. of its maximal-order minors, which is Hurwitz. □

III. Asymptotic Observers Design

Consider the dynamic system described by (2), with $\mathbf{w}_1$ as the measured variable and $\mathbf{w}_2$ as the to-be-estimated variable. The problem we will now address is that of introducing a sound definition of "observer." As a first requirement, an observer of $\mathbf{w}_2$ from $\mathbf{w}_1$ for system Σ should "accept" every sequence $\mathbf{w}_1$, which is part of a behavior trajectory $(\mathbf{w}_1, \mathbf{w}_2)$, and correspondingly produce some (in general, not unique) estimated trajectory $\hat{\mathbf{w}}_2$. This process amounts to saying an observer of Σ should not introduce additional constraints on the $\mathbf{w}_1$ components of the system trajectories. We refer to such a dynamic system as an "acceptor" of the signal $\mathbf{w}_1$ for Σ. As a further requirement, it is reasonable to assume the output of an observer is consistent when tracking $\mathbf{w}_2$, meaning when the trajectories $\hat{\mathbf{w}}_2$ and $\mathbf{w}_2$ coincide for a sufficiently long time, for instance in $(-\infty, 0]$, then they coincide all over the time. Therefore, an observer for Σ is a system that, corresponding to every $(\mathbf{w}_1, \mathbf{w}_2)$ in $\mathfrak{B}$, produces an estimate $\hat{\mathbf{w}}_2$ of the trajectory $\mathbf{w}_2$ and does not lose track of the correct trajectory once it has followed it over a sufficiently long time. Such an observer is said to be asymptotic if the estimate $\hat{\mathbf{w}}_2$ it provides represents a good asymptotic estimate of $\mathbf{w}_2$; namely, the sequence $\mathbf{w}_2(t) - \hat{\mathbf{w}}_2(t)$ goes to zero as t goes to $+\infty$. An asymptotic observer for Σ, which produces an estimate $\hat{\mathbf{w}}_2$ of $\mathbf{w}_2$, which coincides with $\mathbf{w}_2$ at each time instant, is an exact observer.

We may look for some intuition underlying these definitions. An acceptor is merely a system that can "treat," without any specific aim, the signal $\mathbf{w}_1$ produced by the plant. An observer is a system that also allows us to follow a given unobserved variable $\mathbf{w}_2$, provided the initial conditions of the observer have been set well, in accordance with the initial conditions of the plant.

With an asymptotic observer, no need exists to have the intial conditions set properly, but the price that we pay is that of achieving only an asymptotic tracking. An exact observer, finally, keeps track of the unobserved variables in an errorless way. The difficulty, however, is that this process requires, in general, differentiating the observations. So, in a sense, asymptotic observers, provided they can be designed within reasonable signal processing contraints (for example, no differentiations), appear to be the most reasonable observers to pursue.

From a theoretical point of view, the whole analysis can be carried on (as it will be in this section) with no explicit reference to implementability issues. For this reason, no restriction is here introduced on the class of dynamic systems we may want to observe. In Section IV, this aspect will be taken into account, by explicitly considering observers endowed with a (strictly) proper transfer matrix.

The notions now discussed are formalized in the following definitions.

Definition 3.1: Consider the dynamic system $\Sigma = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B})$, whose behavior $\mathfrak{B}$ is described by (2). The set of differential equations

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{w}}_2 = P\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{4}$$

with P and Q polynomial matrices of suitable dimensions, is said to describe the following:

- an acceptor of $\mathbf{w}_1$ for Σ, if for every $(\mathbf{w}_1, \mathbf{w}_2) \in \mathfrak{B}$, $\hat{\mathbf{w}}_2$ exists such that $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ satisfies (4);
- an observer of $\mathbf{w}_2$ from $\mathbf{w}_1$ for Σ, if whenever $(\mathbf{w}_1, \mathbf{w}_2)$ is in $\mathfrak{B}$, and $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ satisfies (4) with $\hat{\mathbf{w}}_2(t) = \mathbf{w}_2(t)$ for $t \in (-\infty, 0]$, $\hat{\mathbf{w}}_2(t) = \mathbf{w}_2(t)$ for all $t \in \mathbb{R}$;
- an asymptotic observer, if for every $(\mathbf{w}_1, \mathbf{w}_2)$ in $\mathfrak{B}$, and $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ satisfying (4), we have $\lim_{t\to+\infty} \mathbf{w}_2(t) - \hat{\mathbf{w}}_2(t) = 0$;
- an exact observer, if for every $(\mathbf{w}_1, \mathbf{w}_2)$ in $\mathfrak{B}$ and $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ satisfying (3.1), we have $\mathbf{w}_2 = \hat{\mathbf{w}}_2$.

In the sequel, as $\mathbf{w}_1$ will always represent the measured variable and $\mathbf{w}_2$ the unmeasurable variable, we will refer to the acceptors of $\mathbf{w}_1$ for Σ and to the observers of $\mathbf{w}_2$ from $\mathbf{w}_1$ for Σ simply as acceptors and observers for Σ.

Given an acceptor, described by (4), its behavior $\hat{\mathfrak{B}}$ is the set of all solutions $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ of the differential equation (4), and, by definition, it satisfies the following condition:

$$\begin{aligned}\mathcal{P}_1\mathfrak{B} &:= \{\mathbf{w}_1 : \exists(\mathbf{w}_1, \mathbf{w}_2) \in \mathfrak{B}\}\\ &\subseteq \{\mathbf{w}_1 : \exists(\mathbf{w}_1, \hat{\mathbf{w}}_2) \in \hat{\mathfrak{B}}\} =: \mathcal{P}_1\hat{\mathfrak{B}}.\end{aligned}$$

Among all of the trajectories of $\hat{\mathfrak{B}}$, however, we will be interested only in those produced corresponding to the trajectories of $\mathfrak{B}$, namely, in the set

$$\{(\mathbf{w}_1, \hat{\mathbf{w}}_2) \in \hat{\mathfrak{B}} : \mathbf{w}_1 \in \mathcal{P}_1\mathfrak{B}\}.$$

So, by assuming this point of view, it seems reasonable to regard as *equivalent* two acceptors, in particular, two observers, for the same system Σ, not if their behaviors $\hat{\mathfrak{B}}_1$ and $\hat{\mathfrak{B}}_2$ coincide, but if their behaviors satisfy the following condition:

$$\begin{aligned}&\{(\mathbf{w}_1, \hat{\mathbf{w}}_2) \in \hat{\mathfrak{B}}_1 : \mathbf{w}_1 \in \mathcal{P}_1\mathfrak{B}\}\\ &\quad = \{(\mathbf{w}_1, \hat{\mathbf{w}}_2) \in \hat{\mathfrak{B}}_2 : \mathbf{w}_1 \in \mathcal{P}_1\mathfrak{B}\}.\end{aligned}$$

For an observer described by (4), the difference variable $\mathbf{e} := \mathbf{w}_2 - \hat{\mathbf{w}}_2$ represents, of course, the *estimation error*. So, the previous definitions can be paraphrased by saying an observer for Σ is asymptotic (exact) if the set of its estimation error trajectories

$$\begin{aligned}\mathfrak{B}_e := \{\mathbf{e} = \mathbf{w}_2 - \hat{\mathbf{w}}_2 : \exists\ \mathbf{w}_1 \text{ s.t. } (\mathbf{w}_1, \mathbf{w}_2) \in \mathfrak{B},\\ (\mathbf{w}_1, \hat{\mathbf{w}}_2) \in \hat{\mathfrak{B}}\}\end{aligned}$$

is an autonomous and stable behavior (the zero autonomous behavior). Because autonomous behaviors can always be represented as kernels of nonsingular square matrices [11], some Hurwitz (unimodular) matrix $\Delta \in \mathbb{R}[\xi]^{\mathtt{w}_2\times\mathtt{w}_2}$ exists, i.e., a nonsingular matrix whose determinant is a Hurwitz polynomial (a nonzero constant term), such that $\mathfrak{B}_e = \ker\Delta$. The characteristic polynomial of the behavior $\mathfrak{B}_e$, namely, $\det\Delta$, will be called the *error-dynamics characteristic polynomial* (see, also, [20]).

Of course, an acceptor for Σ always exists: one can choose, for instance, Σ itself. So, the existence of an acceptor is not an issue. Necessary and sufficient conditions for the existence of (asymptotic or exact) observers, instead, are given in the following proposition.

Proposition 3.2: Consider a dynamic system $\Sigma = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B})$, whose behavior $\mathfrak{B}$ is described by (2).

i) A necessary and sufficient condition for the existence of an observer for Σ is that R_2 in (2) is a full column rank polynomial matrix.

ii) A necessary and sufficient condition for the existence of an asymptotic observer for Σ is that $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$.

iii) A necessary and sufficient condition for the existence of an exact observer for Σ is that $\mathbf{w}_2$ is observable from $\mathbf{w}_1$.

Proof: 1) Suppose an observer for Σ exists, described by (4). If R_2 would not be a full column rank matrix, some finite support sequence $\mathbf{v}$ would exist in its kernel, whose support is included in $(0, +\infty)$. So, corresponding to the behavior trajectory $(\mathbf{w}_1, \mathbf{w}_2) = (0, \mathbf{v}) \in \mathfrak{B}$, the observer would produce as a possible estimate of $\mathbf{w}_2 = \mathbf{v}$ the sequence $\hat{\mathbf{w}}_2 = 0$, which coincides with $\mathbf{v}$ in $(-\infty, 0]$, but not all over the time axis. This result contradicts the assumption that (4) is an observer.

In order to show the converse, assume R_2 is a full column rank polynomial matrix. Then, a unimodular matrix U exists such that

$$U(\xi)R_2(\xi) = \begin{bmatrix} D_2(\xi) \\ 0 \end{bmatrix}$$

with D_2 nonsingular. By conformably partitioning UR_1 as

$$\begin{bmatrix} N_1(\xi) \\ D_1(\xi) \end{bmatrix} := U(\xi)R_1(\xi)$$

we obtain for the behavior the following equivalent description:

$$D_2\left(\frac{d}{dt}\right)\mathbf{w}_2 = N_1\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{5}$$

$$0 = D_1\left(\frac{d}{dt}\right)\mathbf{w}_1. \tag{6}$$

It is immediate to verify $D_2(d/dt)\hat{\mathbf{w}}_2 = N_1(d/dt)\mathbf{w}_1$ represents an observer for Σ, with error-dynamics matrix $\Delta = D_2$.

2) Assume, first, an asymptotic observer for Σ exists. If $\mathbf{w}_2$ were not detectable from $\mathbf{w}_1$, two behavior sequences $(\mathbf{w}_1, \mathbf{w}_2)$ and $(\mathbf{w}_1, \bar{\mathbf{w}}_2)$ would exist such that $\mathbf{w}_2(t) - \bar{\mathbf{w}}_2(t)$ does not go to zero as t goes to $+\infty$. If $\hat{\mathbf{w}}_2$ is an estimate provided by the asymptotic observer corresponding to $\mathbf{w}_1$, it should be, at the same time

$$(\mathbf{w}_2(t) - \hat{\mathbf{w}}_2(t)) \underset{t\to+\infty}{\longrightarrow} 0 \quad \text{and}$$
$$(\bar{\mathbf{w}}_2(t) - \hat{\mathbf{w}}_2(t)) \underset{t\to+\infty}{\longrightarrow} 0$$

and, consequently, $\mathbf{w}_2(t) - \bar{\mathbf{w}}_2(t) = [\mathbf{w}_2(t) - \hat{\mathbf{w}}_2(t)] - [\bar{\mathbf{w}}_2(t) - \hat{\mathbf{w}}_2(t)]$ should asymptotically extinguish, a contradiction.

The proof of the converse follows the same lines of that in 1). Indeed, by assuming that $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$, or, equivalently, $R_2(\lambda)$ is of full column rank for every $\lambda \in \mathbb{C}^+$, we can obtain for the behavior the description (5), (6), with D_2 nonsingular Hurwitz. As a consequence, $D_2(d/dt)\hat{\mathbf{w}}_2 = N_1(d/dt)\mathbf{w}_1$ represents an asymptotic observer for Σ, with error-dynamics matrix $\Delta = D_2$.

3) Follows the same lines as the proof of 2). □

As our main interest in this paper is in asymptotic observers, from now on we will assume the behavior $\mathfrak{B}$ is described by (5) and (6), with D_2 nonsingular Hurwitz. Also, it entails no loss of generality assuming D_1 is of full row rank $\mathtt{d}_1$. In order to obtain a complete parameterization of the (asymptotic/exact) observers of Σ, we need the following technical lemma, in which it is shown that, given any acceptor for Σ (in particular, an observer), it is possible to obtain an equivalent one [i.e., producing the same set of trajectories $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ for every $\mathbf{w}_1$ in $\mathcal{P}_1\mathfrak{B}$] for which matrix Q is of full row rank.

Lemma 3.3: If $Q(d/dt)\hat{\mathbf{w}}_2 = P(d/dt)\mathbf{w}_1$ is an acceptor (in particular, an observer) for Σ, an equivalent acceptor (observer) $\bar{Q}(d/dt)\hat{\mathbf{w}}_2 = \bar{P}(d/dt)\mathbf{w}_1$ exists, with $\bar{Q}$ of full row rank.

Proof: Let U be a unimodular matrix that reduces Q to its (column) Hermite form $\left[\begin{smallmatrix}\bar{Q}\\ 0\end{smallmatrix}\right]$, with $\bar{Q}$ of full row rank. Then, we get

$$U(\xi)[Q(\xi) \quad -P(\xi)] = \begin{bmatrix} \bar{Q}(\xi) & -\bar{P}(\xi) \\ 0 & -V(\xi) \\ 0 & 0 \end{bmatrix}$$

and hence the acceptor can be equivalently described by the set of equations

$$\bar{Q}\left(\frac{d}{dt}\right)\hat{\mathbf{w}}_2 = \bar{P}\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{7}$$

$$0 = V\left(\frac{d}{dt}\right)\mathbf{w}_1. \tag{8}$$

By definition of acceptor, for every $\mathbf{w}_1 \in \ker(D_1(d/dt))$, (7) and (8) have to be fulfilled for some sequence $\hat{\mathbf{w}}_2$, and therefore $\ker(D_1(d/dt))$ must be included in $\ker(V(d/dt))$. So, the acceptor can be equivalently described by (7). □

Under the hypothesis that the matrix Q appearing in the observer equation is of full row rank, we can obtain deeper insights into the algebraic properties of the polynomial matrices P and Q involved in the observer description, and explicitly relate them to the matrices D_2, D_1, and N_1.

Theorem 3.4: Consider a plant Σ whose behavior $\mathfrak{B}$ is described by (5) and (6), with D_2 Hurwitz and D_1 of full row $\operatorname{rank} \mathtt{d}_1$. If P and Q are polynomial matrices, with Q of full row rank

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{w}}_2 = P\left(\frac{d}{dt}\right)\mathbf{w}_1$$

is an (asymptotic) observer for Σ if and only if a nonsingular (Hurwitz) matrix $Y \in \mathbb{R}[\xi]^{\mathtt{w}_2\times\mathtt{w}_2}$ and a polynomial matrix $X \in \mathbb{R}[\xi]^{\mathtt{w}_2\times\mathtt{d}_1}$ exist such that

$$[Q(\xi) \quad -P(\xi)] = [Y(\xi) \quad X(\xi)]\begin{bmatrix} D_2(\xi) & -N_1(\xi) \\ 0 & -D_1(\xi)\end{bmatrix}. \tag{9}$$

Moreover, the set $\mathfrak{B}_e$ of all possible error trajectories coincides with $\ker(Q(d/dt))$, which amounts to saying we can assume $\Delta = Q$.

Proof: Suppose, first, $Q(d/dt)\hat{\mathbf{w}}_2 = P(d/dt)\mathbf{w}_1$, with Q of full row rank, is an (asymptotic) observer for Σ, and hence $\mathfrak{B}_e$ is an autonomous (and stable) behavior. Corresponding to the behavior trajectory $(\mathbf{w}_1, \mathbf{w}_2) = (0, 0)$, the trajectory $(0, \hat{\mathbf{w}}_2)$, $\hat{\mathbf{w}}_2$ arbitrarily selected in $\ker(Q(d/dt))$, must be admissible for the observer, and hence $\mathbf{e} = 0 - \hat{\mathbf{w}}_2$ must be in $\mathfrak{B}_e$. This fact proves that $\ker(Q(d/dt))$ is included in $\mathfrak{B}_e$ and, hence, in turn, is autonomous (and stable). Because Q is full row rank, it has to be also of full column rank and, hence, nonsingular square. Moreover, if the observer is an asymptotic one, Q must be Hurwitz.

As $\mathbf{e} = 0$ is an admissible error trajectory, every trajectory $(\mathbf{w}_1, \mathbf{w}_2) \in \mathfrak{B}$ satisfies $Q(d/dt)\mathbf{w}_2 = P(d/dt)\mathbf{w}_1$. Therefore

$$\ker\left[Q\left(\frac{d}{dt}\right) \quad -P\left(\frac{d}{dt}\right)\right] \supseteq \ker\begin{bmatrix} D_2\left(\frac{d}{dt}\right) & -N_1\left(\frac{d}{dt}\right) \\ 0 & -D_1\left(\frac{d}{dt}\right)\end{bmatrix}$$

and thus polynomial matrices X and Y can be found such that (9) holds true. As $Q = YD_2$ is nonsingular (Hurwitz), Y has to be nonsingular (Hurwitz).

Assume, now, P and Q satisfy (9) for suitable polynomial matrices X and Y, with Y nonsingular (Hurwitz). Then, for every $(\mathbf{w}_1, \mathbf{w}_2) \in \mathfrak{B}$ and every estimate $\hat{\mathbf{w}}_2$, correspondingly determined from the observer equations, we get

$$\begin{aligned} Q\left(\frac{d}{dt}\right)&(\mathbf{w}_2 - \hat{\mathbf{w}}_2) \\ &= (YD_2)\left(\frac{d}{dt}\right)(\mathbf{w}_2 - \hat{\mathbf{w}}_2) \\ &= (YN_1)\left(\frac{d}{dt}\right)\mathbf{w}_1 - (YN_1 + XD_1)\left(\frac{d}{dt}\right)\mathbf{w}_1 = 0. \end{aligned}$$

This equation immediately proves $\mathfrak{B}_e$, as a subset of $\ker(Q(d/dt))$, is a (stable) autonomous behavior, and hence equation $Q(d/dt)\hat{\mathbf{w}}_2 = P(d/dt)\mathbf{w}_1$ describes an (asymptotic) observer. Moreover, as we have already proved $\ker(Q(d/dt))$ is also included in $\mathfrak{B}_e$, then $\mathfrak{B}_e$ coincides with $\ker(Q(d/dt))$. □

Remarks: For the problem analysis, it has been useful to adopt the behavior description (5), (6). If we assume, however, the behavior $\mathfrak{B}$ is described as in (2), with $\mathbf{w}_2$ detectable from $\mathbf{w}_1$, the asymptotic observers for Σ are those and those only described by (4) with Q and P polynomial matrices, Q nonsingular Hurwitz, satisfying

$$[Q(\xi) \quad -P(\xi)] = T(\xi)[R_2(\xi) \quad -R_1(\xi)] \qquad (10)$$

for some polynomial matrix T.

Also, by assuming the matrix Q appearing in the observer description is of full row rank, and hence nonsingular square, we have obtained a complete parameterization of all possible (asymptotic) observers. Loosening this constraint, indeed, would only produce a wider set of representations, not necessarily full row rank, for the same observers.

As a further result, we are now interested in analyzing what performances can be achieved from the asymptotic observers in terms of error dynamics. These performances can be evaluated by error-dynamics characteristic polynomials, as analyzed in the following corollary.

Corollary 3.5: Consider a dynamic system whose behavior $\mathfrak{B}$ is described by (5) and (6), with D_2 Hurwitz and D_1 of full row rank. Then,

i) for every (asymptotic) observer for Σ, the error-dynamics characteristic polynomial $\det\Delta$ is a (Hurwitz) polynomial satisfying the divisibility condition $\det D_2 \mid \det\Delta$ (i.e., $\det D_2$ divides $\det\Delta$);
ii) for every (Hurwitz) polynomial $\delta \in \mathbb{R}[\xi]$ with $\det D_2 \mid \delta$, an (asymptotic) observer exists whose error-dynamics characteristic polynomial coincides with δ.

Proof: Follows immediately from Theorem 3.4. □

IV. Proper Asymptotic Observers

Theorem 3.4 provides us with a useful parameterization of the observers for a system described by (5) and (6). Indeed, all observers for Σ can be described by means of the differential equation

$$(YD_2)\left(\frac{d}{dt}\right)\hat{\mathbf{w}}_2 = (YN_1 + XD_1)\left(\frac{d}{dt}\right)\mathbf{w}_1 \qquad (11)$$

with X and Y polynomial matrices of suitable dimensions, and the additional constraint that Y is Hurwitz if the observer is asymptotic.

This parameterization can be fruitfully exploited to investigate further relevant issues, in particular, that of determining the existence of (strictly) proper asymptotic observers, endowed with a (strictly) proper transfer matrix. Systems with proper or strictly proper transfer functions have desirable properties as signal processors, in the sense they smooth, rather than differentiate, signals. Thus (strictly), proper transfer functions can be expected to better noise immunity. Of course, this property becomes less important when we can infer more *a priori* smoothness for the observed signals.

From a mathematical point of view, we have to search for conditions guaranteeing that a matrix pair (Y, X) exists, with Y Hurwitz, such that

$$\hat{W}(\xi) := [Y(\xi)D_2(\xi)]^{-1}[Y(\xi)N_1(\xi) + X(\xi)D_1(\xi)]$$

is (strictly) proper rational. As shown in the following proposition, autonomous behaviors described as in (5) and (6) always admit strictly proper asymptotic observers.

Proposition 4.1: Let Σ be a dynamic system whose behavior $\mathfrak{B}$ is described by (5) and (6), with D_2 Hurwitz and D_1 having full row $\operatorname{rank} \mathtt{d}_1$. If $\mathfrak{B}$ is autonomous or, equivalently, D_1 is a nonsingular square matrix, a strictly proper asymptotic estimator of $\mathbf{w}_2$ from $\mathbf{w}_1$ exists.

Proof: Let $\mu_1, \mu_2, \cdots, \mu_{\mathtt{d}_1}$ denote the row degrees of D_1, and consider an Hurwitz matrix Y such that YD_2 is row reduced [5] with row degrees all greater to or equal to $\max_i \mu_i$.[1] By applying the matrix division algorithm, we can express YN_1 as

$$Y(\xi)N_1(\xi) = A(\xi)D_1(\xi) + R(\xi)$$

where A and R are polynomial matrices, and R satisfies

$$\deg i\text{th column of } R < \max_j \mu_j, \quad \forall\, j \in \{1, 2, \cdots, \mathtt{w}_1\}.$$

Therefore

$$\deg i\text{th row of } R < \deg i\text{th row of } YD_2, \\ \forall i \in \{1, 2, \cdots, \mathtt{w}_2\}$$

which implies $(YD_2)^{-1}[YN_1 - AD_1] = (YD_2)^{-1}R$ is a strictly proper observer for Σ. □

Remark: The above proposition not only proves autonomous behaviors admit strictly proper asymptotic estimators, but also shows how to construct one. Indeed, under the (unrestrictive) assumption that D_1 is row reduced with row degrees $\mu_1, \mu_2, \cdots, \mu_{\mathtt{d}_1}$, for every Hurwitz matrix Y such that YD_2 is row reduced with row degrees lower bounded by $\max_i \mu_i$, a polynomial matrix X can be found, such that $(YD_2)(d/dt)\hat{\mathbf{w}}_2 = (YN_1 + XD_1)(d/dt)\mathbf{w}_1$ represents a strictly proper asymptotic estimator, having $\Delta = YD_2$ as estimation error dynamics matrix.

The general problem, when $\mathfrak{B}$ is an arbitrary (not necessarily autonomous) behavior, is a little more involved. In order to solve it, we refer to the original behavior description and assume, without loss of generality, the behavior $\mathfrak{B}$ is described by the differential equation

$$R_2\left(\frac{d}{dt}\right)\mathbf{w}_2 = R_1\left(\frac{d}{dt}\right)\mathbf{w}_1 \qquad (12)$$

with $[R_2 \ \ -R_1] \in \mathbb{R}[\xi]^{\mathtt{r}\times(\mathtt{w}_2+\mathtt{w}_1)}$ a row reduced matrix [5] with row degrees $h_1, h_2, \cdots, h_{\mathtt{r}}$, $\mathtt{r} = \mathtt{w}_1 + \mathtt{d}_1$. Of course, $\mathbf{w}_2$ is assumed to be detectable from $\mathbf{w}_1$. The first step toward the solution is given by the following lemma, in which conditions for the existence of (strictly) proper observers for Σ, not necessarily asymptotic, are provided.

[1] A full row rank polynomial matrix M, with row degrees $\nu_1, \nu_2, \cdots,$ is said to be *row reduced* if the degree of at least one of its maximal-order minors coincides with $\sum_i \nu_i$.

Lemma 4.2: Consider a dynamic system Σ with behavior $\mathfrak{B}$ described by (12) and $\mathbf{w}_2$ detectable from $\mathbf{w}_1$. If $[R_{2hr}\ \ -R_{1hr}]$ denotes the leading row coefficient matrix [5] of $[R_2\ \ -R_1]$

i) a necessary and sufficient condition for the existence of an observer for Σ with proper transfer matrix $\hat{W}$ is that R_{2hr} has full column rank w_2;

ii) a necessary and sufficient condition for the existence of an observer for Σ with strictly proper transfer matrix $\hat{W}$ is that $S \in \mathbb{R}^{\mathrm{w}_2 \times (\mathrm{w}_2 + \mathrm{d}_1)}$ exists such that $S[R_{2hr}\ \ -R_{1hr}] = [I_{\mathrm{w}_2}\ \ 0]$.

Proof: i) Assume, first, an observer with proper transfer matrix $\hat{W}$ exists, and let $T = [t_{ij}]$ be a polynomial matrix such that the matrix pair (Q, P), obtained as

$$[Q(\xi) \mid -P(\xi)] := T(\xi)[R_2(\xi) \mid -R_1(\xi)]$$

provides a left matrix faction description (MFD) of $\hat{W}$, i.e., $\hat{W} = Q^{-1}P$. It entails no loss of generality, assuming $[Q \mid -P]$ is also row reduced, with row degrees $k_1, k_2, \cdots, k_{\mathrm{w}_2}$ and leading row coefficient matrix $[Q_{hr}\ \ -P_{hr}]$. By the properness assumption on $\hat{W}$, Q_{hr} is nonsingular. Moreover, as $[R_2\ -R_1]$ is row reduced, it follows that

$$k_i = \max_{j: t_{ij} \neq 0} \{\deg t_{ij} + \deg(j\text{th row of } [R_2\ \ -R_1])\}.$$

Let S be the real matrix whose (i, j)th entry coincides with the leading coefficient of t_{ij} when $\deg t_{ij} + \deg(j\text{th row of } [R_2\ \ -R_1]) = k_i$ and is zero otherwise. Clearly, the identity $Q_{hr} = SR_{2hr}$ holds true, thus proving R_{2hr} is of full column rank.

Conversely, suppose R_{2hr} has full column rank, and let $S = [s_{ij}]$ be one of its left inverses. Set $h := \max_i h_i$, and introduce the polynomial matrix $T(\xi) := [s_{ij}\xi^{h - h_j}]$. Then

$$[Q(\xi) \mid -P(\xi)] := T(\xi)[R_2(\xi) \mid -R_1(\xi)]$$

is a row-reduced matrix (with all row degrees equal to h), and the first $\mathrm{w}_2 \times \mathrm{w}_2$ submatrix of its leading row coefficient matrix coincides with I_{w_2}. Consequently, $\hat{W} = Q^{-1}P$ is proper rational.

ii) The proof follows the same lines as the previous one. $\square$

Theorem 4.3: Consider a dynamic system Σ with behavior $\mathfrak{B}$ described by (12) and $\mathbf{w}_2$ detectable from $\mathbf{w}_1$. If a (strictly) proper observer for Σ exists, a (strictly) proper asymptotic one exists.

Proof: Assume, as in the previous lemma, $[R_2\ -R_1] \in \mathbb{R}[\xi]^{\mathrm{r} \times (\mathrm{w}_2 + \mathrm{w}_1)}$ is row reduced with row degrees $h_1, h_2, \cdots, h_{\mathrm{r}}$ and leading row coefficient matrix $[R_{2hr}\ \ -R_{1hr}]$. If a proper observer for Σ exists, R_{2hr} has rank w_2 and, hence, a real matrix S exists, of suitable size, such that $SR_{2hr} = I_{\mathrm{w}_2}$. If the observer is strictly proper, in particular, S can be chosen in such a way that $SR_{1hr} = 0$ also holds true. Set, now,

$$[\hat{R}_2\ \ -\hat{R}_1] := \begin{bmatrix} \xi^{-h_1} & & & \\ & \xi^{-h_2} & & \\ & & \ddots & \\ & & & \xi^{-h_{\mathrm{r}}} \end{bmatrix} [R_2\ \ -R_1].$$

Clearly, $[\hat{R}_2\ \ -\hat{R}_1]$ is an element of $\mathbb{R}[\xi^{-1}]^{\mathrm{r} \times (\mathrm{w}_2 + \mathrm{w}_1)}$, and if we regard it as a polynomial in the negative powers of ξ, with coefficients in $\mathbb{R}^{\mathrm{r} \times (\mathrm{w}_2 + \mathrm{w}_1)}$, the coefficient matrix of the constant term coincides with $[R_{2hr}\ \ -R_{1hr}]$. Let $U \in \mathbb{R}[\xi^{-1}]^{\mathrm{r} \times \mathrm{r}}$ be a unimodular matrix (in $\mathbb{R}[\xi^{-1}]$) that reduces $\hat{R}_2$ to its Hermite form (still in $\mathbb{R}[\xi^{-1}]$):

$$U\hat{R}_2 = \begin{bmatrix} \hat{D}_2 \\ 0 \end{bmatrix}.$$

If we denote by U_0 the constant term of U, it entails no loss of generality, assuming $[I_{\mathrm{w}_2}\ \ 0]U_0$, coincides with S.

Clearly, the coefficient matrix of the constant term in $\hat{D}_2$, $\hat{D}_{20}$, coincides with the identity matrix. Moreover, by the detectability assumption, $\det \hat{D}_2 \in \mathbb{R}[\xi^{-1}]$ can be expressed as $d_2(\xi)/\xi^K$, for some positive integer K and some Hurwitz polynomial $d_2 \in \mathbb{R}[\xi]$ of degree K.

Corresponding to $\hat{T} := [I_{\mathrm{w}_2}\ \ 0]U \in \mathbb{R}[\xi^{-1}]^{\mathrm{w}_2 \times \mathrm{r}}$, the matrix pair $[\hat{Q} \mid -\hat{P}] := \hat{T}[\hat{R}_2 \mid -\hat{R}_1]$ provides a left MFD (over $\mathbb{R}[\xi^{-1}]$) of a (strictly) proper transfer matrix $\hat{W} := \hat{Q}^{-1}\hat{P}$, with all stable poles.Therefore, some nonsingular diagonal matrix $D \in \mathbb{R}[\xi]^{\mathrm{w}_2 \times \mathrm{w}_2}$ exists, with all monomial entries, such that the pair of polynomial matrices (Q, P), obtained as

$$[Q\ \ -P] := D\hat{T} \begin{bmatrix} \xi^{-h_1} & & & \\ & \xi^{-h_2} & & \\ & & \ddots & \\ & & & \xi^{-h_{\mathrm{r}}} \end{bmatrix}$$
$$[R_2\ \ -R_1] \in \mathbb{R}[\xi]^{\mathrm{w}_2 \times (\mathrm{w}_2 + \mathrm{w}_1)}$$

corresponds to a (strictly) proper asymptotic observer (4). $\square$

Remark: As a consequence of the above proposition and lemma, once we reduce the matrix $[R_2\ -R_1]$ involved in the behavior description to row-reduced form, the existence of a proper asymptotic observer is immediately checked by simply verifying R_{2hr} has full column rank. When so, by following the procedure described in the proof of Theorem 4.3, we can explicitly construct such an observer.

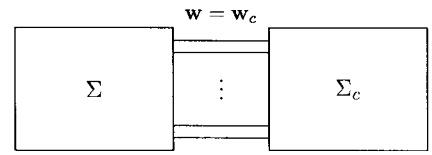

Fig. 1. Plant-controller connection.

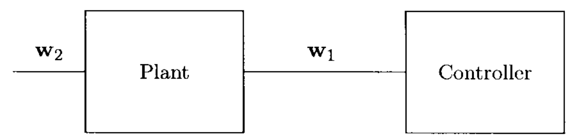

Fig. 2. $\mathbf{w}_1$-based controller.

V. THE CONTROL PROBLEM

The control problem that will be considered is that of designing a suitable device (*controller*), modeled as a dynamic system Σ_c, that can be applied to the plant Σ, thus producing a resulting system with desired properties. As recently emphasized in [20], the control problem is naturally stated as an interconnection problem, and the behavioral framework is

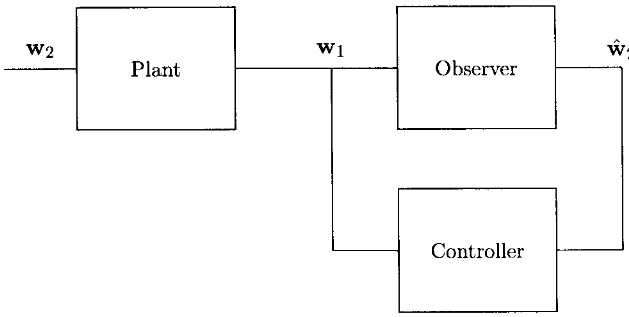

Fig. 3. Observer-based controller.

very convenient for this. So, we have to look for some system $\Sigma_c = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+\mathtt{w}_2}, \mathfrak{B}_c)$ that, once connected to Σ, as shown in Fig. 1, results in a *controlled system*

$$\Sigma \wedge \Sigma_c := (\mathbb{R}, \mathbb{R}^q, \mathfrak{B} \cap \mathfrak{B}_c) \tag{13}$$

with a desired behavior.

The control problem, under the assumption that all system variables are available for control, has been considered in [20]. In this paper, instead, we shall be concerned with the case in which not all variables are accessible for control purposes, namely with the situation in which the set of system variables $\mathbf{w}$ can be partitioned into two subvectors $\mathbf{w}^T = [\mathbf{w}_1^T \ \mathbf{w}_2^T]$, of which only $\mathbf{w}_1$ is available for control. Such a controller, which operates by restricting the set of admissible trajectories for the variable $\mathbf{w}_1$, will be called a $\mathbf{w}_1$*-based controller*.

In order to investigate what possibilities are offered by a controller of this kind, we start by assuming (without loss of generality) the plant behavior is described by the set of differential equations

$$D_2\left(\frac{d}{dt}\right)\mathbf{w}_2 = N_1\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{14}$$

$$0 = D_1\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{15}$$

with D_2 and D_1 both of full row rank, and consider a $\mathbf{w}_1$-based controller defined by the following representation:

$$0 = C_1\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{16}$$

where C_1 is a polynomial matrix (see Fig. 2).

As in [8] and [19], the problem we aim to address is that of designing a controller that makes the resulting controlled system autonomous and possibly stable (sometimes with a preassigned characteristic polynomial). This problem represents a reasonable extension of the classic regulation problem, and it allows us to focus immediately on the core of the problem, namely, on the autonomous part of the resulting connected system, for the controllable part plays no role in the stability analysis.

A $\mathbf{w}_1$-based controller that achieves these results is said to be *stabilizing*. The possibility of obtaining these properties is strictly related to the properties of Σ, as shown in the following theorem.

Theorem 5.1: Consider the dynamic system Σ whose behavior $\mathfrak{B}$ satisfies the differential equations (14) and (15). A necessary and sufficient condition for the existence of a $\mathbf{w}_1$-based controller that makes the resulting controlled system autonomous is that an observer for Σ exists; i.e., D_2 is nonsingular square. When so

i) if Σ is autonomous, namely, if D_1 is also nonsingular square, every $\mathbf{w}_1$-based controller makes the controlled system autonomous with its characteristic polynomial χ_{res} satisfying

$$\det D_2 \mid \chi_{\text{res}} \mid \det D_2 \det D_1 \tag{17}$$

and, conversely, for every polynomial χ_{res} satisfying (17), a $\mathbf{w}_1$-based controller exists such that the resulting autonomous system has characteristic polynomial χ_{res};

ii) if Σ is not autonomous $(\operatorname{rank} D_1 =: \mathtt{d}_1 < \mathtt{w}_1)$, for every $\mathbf{w}_1$-based controller that makes the controlled system autonomous, the characteristic polynomial χ_{res} satisfies

$$\det D_2 \mid \chi_{\text{res}} \tag{18}$$

and, conversely, for every polynomial χ_{res} satisfying (18), a $\mathbf{w}_1$-based controller exists that makes the resulting system autonomous with characteristic polynomial χ_{res}, consequently;

iii) a stabilizing $\mathbf{w}_1$-based controller exists if and only if $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$;

iv) a $\mathbf{w}_1$-based controller exists that makes the resulting connected system autonomous with an arbitrarily chosen characteristic polynomial if and only if $\mathbf{w}_2$ is observable from $\mathbf{w}_1$.

Proof: The behavior of the resulting controlled system is described by the following set of differential equations:

$$\begin{bmatrix} D_2 & -N_1 \\ 0 & -D_1 \\ 0 & -C_1 \end{bmatrix}\left(\frac{d}{dt}\right)\begin{bmatrix} \mathbf{w}_2 \\ \mathbf{w}_1 \end{bmatrix} = 0 \tag{19}$$

and hence some polynomial matrix C_1 exists such that the matrix describing the resulting behavior has full column rank if and only if D_2 is already of full column rank, and hence nonsingular square.

i) If Σ is autonomous, i.e., D_2 and D_1 are both nonsingular square, for every choice of C_1 the resulting system matrix is of full column rank, and hence the controlled system is still autonomous. Condition (17) follows immediately from the structure of the system matrix as given in (19). Conversely, assume χ_{res} is a polynomial satisfying (17), and express it as $\chi_{\text{res}} = \det D_2 \cdot p$, with $p \mid \det D_1$. By resorting to the Smith form of D_1, for instance, (see [5]), we can factorize D_1 as a product of polynomial matrices $D_1 = ST_p$, with $\det T_p = p$. Consequently, $C_1 = T_p$ is the desired $\mathbf{w}_1$-based controller.

ii) Assume, now, Σ is nonautonomous. Again, condition (18) follows immediately from the structure of the system matrix as given in (19). For the converse part, express, again, χ_{res} as $\chi_{\text{res}} = \det D_2 \cdot p$, and consider the Smith form of D_1

$$\Gamma(\xi) := [\tilde{\Gamma}(\xi) \quad 0] \in \mathbb{R}[\xi]^{\mathtt{d}_1 \times \mathtt{w}_1}.$$

Then, $D_1(\xi) = U(\xi)\Gamma(\xi)V(\xi)$ holds true, for suitable unimodular matrices U and V. Because D_1 is of full row rank, $\tilde{\Gamma}$ is nonsingular square, and it is easy to see that the g.c.d. of the maximal-order minors of

$$\begin{bmatrix} \tilde{\Gamma}(\xi) & 0 & 0 \\ I_{\mathtt{d}_1} & 0 & 0 \\ 0 & I_{\mathtt{w}_1-\mathtt{d}_1-1} & 0 \\ 0 & 0 & p \end{bmatrix}$$

is p. So, corresponding to the controller matrix

$$C_1(\xi) := \begin{bmatrix} I_{\mathtt{d}_1} & 0 & 0 \\ 0 & I_{\mathtt{w}_1-\mathtt{d}_1-1} & 0 \\ 0 & 0 & p \end{bmatrix} V(\xi)$$

we obtain an autonomous controlled system with characteristic polynomial χ_{res}. (iii) and (iv) follow immediately from (i) and (ii). □

As we have just seen, the possibility of achieving certain results by means of a $\mathbf{w}_1$-based controller depends, indeed, on how much information about the "missing variable," $\mathbf{w}_2$, can be deduced from $\mathbf{w}_1$. In particular, the possibility of stabilizing Σ by constraining only $\mathbf{w}_1$ depends on the fact that the information about $\mathbf{w}_2$ is "asymptotically correct," namely, that $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$. If this is the case, a reasonable approach to the problem solution could be that of exploiting an asymptotic estimate $\hat{\mathbf{w}}_2$ of $\mathbf{w}_2$, obtained from $\mathbf{w}_1$ by means of a suitable observer, and of designing a controller Σ_c that makes use of the pair $(\mathbf{w}_1, \hat{\mathbf{w}}_2)$ as if it was $(\mathbf{w}_1, \mathbf{w}_2)$. The advantage of such a control structure is that the controller outs in evidence the estimation aspect that is part of a dynamic controller. So, from a theoretical point of view, it is much more significant than a $\mathbf{w}_1$-based controller, in which this aspect is completely hidden. Moreover, this structure seems to be more suitable for addressing implementability issues, that, however, will not be explicitly taken into account here.

The situation just described represents the generalization of the analogous one for state-space models, and we will call a controller with this structure, connected to the original plant, as shown in Fig. 3, an *observer-based controller*. Our interest, as before, is in observer-based controllers that make the resulting connected system autonomous and stable and, hence, are stabilizing.

Once again, we assume the system behavior $\mathfrak{B}$ is described by (14) and (15), with D_2 Hurwitz and D_1 of full row rank, and introduce an asymptotic observer for Σ

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{w}}_2 = P\left(\frac{d}{dt}\right)\mathbf{w}_1 \tag{20}$$

with Q and P satifying condition (9) for some polynomial matrices X and Y, with Y Hurwitz. If we introduce a controller whose behavior $\mathfrak{B}_c$ is described by the following set of differential equations:

$$C_2\left(\frac{d}{dt}\right)\hat{\mathbf{w}}_2 = C_1\left(\frac{d}{dt}\right)\mathbf{w}_1, \tag{21}$$

the behavior of the whole connected system in Fig. 3, $\Sigma_{\text{res}} = (\mathbb{R}, \mathbb{R}^{\mathtt{w}_1+2\mathtt{w}_2}, \mathfrak{B}_{\text{res}})$, is described by

$$\begin{bmatrix} D_2 & 0 & -N_1 \\ 0 & 0 & -D_1 \\ 0 & Q & -P \\ 0 & C_2 & -C_1 \end{bmatrix}\left(\frac{d}{dt}\right)\begin{bmatrix} \mathbf{w}_2 \\ \hat{\mathbf{w}}_2 \\ \mathbf{w}_1 \end{bmatrix} = 0. \tag{22}$$

As $\mathfrak{B}_{\text{res}}$ can be expressed as the kernel of the polynomial matrix

$$M(\xi) := \begin{bmatrix} D_2(\xi) & 0 & -N_1(\xi) \\ 0 & 0 & -D_1(\xi) \\ 0 & Q(\xi) & -P(\xi) \\ 0 & C_2(\xi) & -C_1(\xi) \end{bmatrix}$$

to make it autonomous, we have to choose C_1 and C_2 so that M is of full column rank. If this is the case, $\mathfrak{B}_{\text{res}}$ is stable if and only if the g.c.d. of the maximal-order minors of M, the characteristic polynomial χ_{res} of $\mathfrak{B}_{\text{res}}$, is Hurwitz.

Theorem 5.2: Consider the controlled system described in Fig. 3, and set $\mathtt{d}_1 := \operatorname{rank} D_1$. If Σ is autonomous, namely, $\mathtt{d}_1$ coincides with $\mathtt{w}_1$

i) for every controller (21), the resulting controlled system Σ_{res} is autonomous with its characteristic polynomial χ_{res} satisfying

$$\det D_2 \mid \chi_{\text{res}} \mid \det D_2 \det Q \det D_1 \tag{23}$$

ii) for every polynomial χ_{res} that satisfies (23), a controller (21) exists such that the resulting autonomous system has characteristic polynomial χ_{res}.

If $\mathtt{d}_1 < \mathtt{w}_1$,

iii) for every controller (21) that makes the whole system Σ_{res} autonomous, the characteristic polynomial χ_{res} is a multiple of $\det D_2$;

iv) for every polynomial χ_{res} that is multiple of $\det D_2$, a controller exists (21) such that the resulting system is autonomous with characteristic polynomial χ_{res}.

Proof: i) and iii) follow immediately from the structure of M and from the fact that when $\mathfrak{B}$ is autonomous, the matrix

$$\begin{bmatrix} D_2(\xi) & 0 & -N_1(\xi) \\ 0 & 0 & -D_1(\xi) \\ 0 & Q(\xi) & -P(\xi) \end{bmatrix}$$

is already of full column rank.

ii) and iv) are proved along the same lines followed in the proof of Theorem 5.1, upon replacing matrix $-D_1$ with the full row rank matrix

$$T(\xi) := \begin{bmatrix} 0 & -D_1(\xi) \\ Q(\xi) & -P(\xi) \end{bmatrix}$$

and, hence, applying to T all previous reasonings, based on the Smith form, to explicitly construct the required controllers. □

Necessary and sufficient conditions for the existence of stabilizing observer-based controllers are immediately derived, as a straightforward consequence of the previous result.

Corollary 5.3: Given a plant Σ whose behavior $\mathfrak{B}$ is described by (14) and (15), with D_2 nonsingular square and D_1 of full row rank, a necessary and sufficient condition for the existence of a stabilizing observer-based controller is that $\mathbf{w}_2$ is detectable from $\mathbf{w}_1$; namely, D_2 is Hurwitz.

Remarks: It is worthwhile to note the gap existing between the results achieved (in terms of characteristic polynomials) by an observer-based controller and by a $\mathbf{w}_1$-based controller in case of detectability, is only apparent, and motivated by the fact that the characteristic polynomial we refer to in the case of an observer-based controller also takes into account the observer dynamics. If we considered just the behavior of the plant Σ, namely, the projection of $\mathfrak{B}_{\rm res}$ onto the variables $\mathbf{w}_1$ and $\mathbf{w}_2$, we would obtain the same results, as, indeed, observer-based controllers and $\mathbf{w}_1$-based controllers, under the detectability assumption, are completely equivalent.

Also, the development of observer-based controllers can of course be combined with the notions of singular and regular ($\mathbf{w}$-based) controllers extensively discussed in [20]. We prefer not to enter into these ramifications here, however.

VI. State-Space Models

To conclude, we aim to show the observer theory, here developed within the behavioral approach, is consistent with the classic one for state-space systems [9]. For sake of brevity, we will explicitly consider only the main definitions and results presented in Sections II–IV and skip the control problem, which is, nonetheless, of noteworthy interest.

Given an ($\mathtt{n}$-dimensional) state-space model, with $\mathtt{m}$ inputs and $\mathtt{p}$ outputs, i.e.,

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= F\mathbf{x}(t) + G\mathbf{u}(t), \\ \mathbf{y}(t) &= H\mathbf{x}(t) + J\mathbf{u}(t),\end{aligned} \qquad t \geq 0, \tag{24}$$

the set of its trajectories is equivalently described, in behavioral terms, as the set $\mathfrak{B}$ of all sequences $(\mathbf{x}, \mathbf{u}, \mathbf{y})$ satisfying

$$\begin{bmatrix} \frac{d}{dt}I_{\rm n} - F \\ H \end{bmatrix}\mathbf{x} = \begin{bmatrix} G & 0 \\ -J & I_{\rm p} \end{bmatrix}\begin{bmatrix} \mathbf{u} \\ \mathbf{y} \end{bmatrix}. \tag{25}$$

By Proposition 2.2, $\mathbf{x}$ is observable from $(\mathbf{u}, \mathbf{y})$ if and only if

$$\text{rank}\begin{bmatrix} \lambda I_{\rm n} - F \\ H \end{bmatrix} = \mathtt{n}, \quad \forall \lambda \in \mathbb{C},$$

and detectable from $(\mathbf{u}, \mathbf{y})$ if and only if

$$\text{rank}\begin{bmatrix} \lambda I_{\rm n} - F \\ H \end{bmatrix} = \mathtt{n}, \quad \forall \lambda \in \mathbb{C}^+.$$

These equations represent the well-know observability and detectability Popov–Belevitch–Hautus (PBH, for short) tests for state-space models [5].

Under the detectability assumption, an asymptotic state observer exists (see Proposition 3.2), based on the knowledge of the inputs and the outputs of the system. More precisely, all possible asymptotic state estimators can be described by the following set of differential equations:

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{x}} = P_u\left(\frac{d}{dt}\right)\mathbf{u} + P_y\left(\frac{d}{dt}\right)\mathbf{y} \tag{26}$$

with (Q, P_u, P_y) a triple of polynomial matrices satisfying the following constraints:

i) Q is nonsingular Hurwitz;

ii) $[Q(\xi) \quad -P_u(\xi) \quad -P_y(\xi)]$
$$= [Y(\xi) \quad X(\xi)]\begin{bmatrix} \xi I_{\rm n} - F & -G & 0 \\ H & J & -I_{\rm p} \end{bmatrix}$$

for suitable polynomial matrices Y and X.

Among these asymptotic observers, in particular, a Luenberger (full-order feedback) state observer exists. Indeed, if L is any $\mathtt{n} \times \mathtt{p}$ real matrix such that $F + LH$ is asymptotically stable, it is sufficient to assume $Y(\xi) = I_{\rm n}$ and $X(\xi) = -L$ in ii), thus obtaining the state observer

$$\left(\frac{d}{dt}I_{\rm n} - F - LH\right)\hat{\mathbf{x}} = G\mathbf{u} - L\mathbf{y} \tag{27}$$

which satisfies condition i), and whose estimation error dynamics matrix Δ coincides with $\xi I_{\rm n} - F - LH$.

Also, upon assuming detectability, we can look for (strictly) proper asymptotic state estimators, namely, observers described by

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{x}} = P_u\left(\frac{d}{dt}\right)\mathbf{u} + P_y\left(\frac{d}{dt}\right)\mathbf{y} \tag{28}$$

with

i) Q nonsingular Hurwitz;

ii) $[Q(\xi) \quad -P_u(\xi) \quad -P_y(\xi)]$
$$= [Y(\xi) \quad X(\xi)]\begin{bmatrix} \xi I_{\rm n} - F & -G & 0 \\ H & J & -I_{\rm p} \end{bmatrix},$$

for suitable polynomial matrices Y and X;

iii) $Q^{-1}[P_u \mid P_y]$ (strictly) proper rational.

The Luenberger observer is endowed with a strictly proper rational transfer matrix $\hat{W}(\xi) := (\xi I_{\rm n} - F - LH)^{-1}[G \ -L]$. So, the existence of a proper state observer is not an issue. It can be interesting, instead, to determine what matrices Q and, hence, Δ possibly describe the estimation error dynamics. The first step toward the solution is given by the following lemma, which proves that, to fulfill condition iii) above, it is sufficient $Q^{-1}P_y$ is proper rational.

Lemma 6.1: Consider an asymptotic state observer described as in (28), with matrices Q, P_u and P_y satisfying conditions i) and ii). Such an observer is (strictly) proper, i.e., fulfills iii), if and only if $Q^{-1}P_y$ is (strictly) proper.

Proof: Because matrix

$$\begin{bmatrix} \xi I_{\rm n} - F & -G & 0 \\ H & J & -I_{\rm p} \end{bmatrix}$$

is of full row rank, the pair (Y, X) appearing in ii) can be uniquely recovered from the observer matrices Q, P_u and P_y. Moreover, $X(\xi) = P_y(\xi)$ and $Y(\xi) = [Q(\xi) - P_y(\xi)H](\xi I_{\rm n} - F)^{-1}$. Consequently,

$$\begin{aligned} P_u(\xi) &= Y(\xi)G - X(\xi)J \\ &= Q(\xi)(\xi I_{\rm n} - F)^{-1}G - P_y(\xi)(H(\xi I_{\rm n} - F)^{-1}G + J) \end{aligned}$$

and therefore

$$\begin{aligned} Q^{-1}P_u = (\xi I_{\rm n} - F)^{-1}G - Q^{-1}(\xi)P_y(\xi) \\ \times (H(\xi I_{\rm n} - F)^{-1}G + J). \end{aligned}$$

So, $Q^{-1}P_u$ is (strictly) proper whenever $Q^{-1}P_y$ is.

The converse is obvious. □

As an immediate consequence of the above lemma, the search for proper asymptotic state observers is equivalent to the problem of determining proper asymptotic state observers for the autonomous system

$$\begin{bmatrix} \frac{d}{dt} I_{\mathrm{n}} - F \\ H \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ I_{\mathrm{p}} \end{bmatrix} \mathbf{y}. \tag{29}$$

This result is rather intuitive, because it expresses the fact that the forced state evolution could be easily removed without affecting the solution of the proper observer problem. Furthermore, it allows us to reduce ourselves to the special case of autonomous behaviors, previously analyzed.

We are now interested in getting some flavor of what can be the minimal complexity (in terms of realization dimension and, hence, of McMillan degree [5]) a proper (strictly proper) state observer can exhibit. The following proposition shows every row-reduced matrix Q, with row degrees lower bounded by $\max_i h_i - 1$ (by $\max_i h_i$), for which the diophantine equation $Q(\xi) = \bar{Y}(\xi)(\xi I_n - F) + \bar{X}(\xi)H$ is solvable, can appear as "denominator matrix" of some proper (strictly proper) state observer.

Proposition 6.2: Let $D_\ell^{-1} N_\ell$ be a left coprime MFD of $H(\xi I_{\mathrm{n}} - F)^{-1}$, with D_ℓ row reduced with row indexes $h_1, h_2, \ldots, h_{\mathrm{p}}$. For every polynomial pair $(\bar{Y}, \bar{X})$ such that $Q(\xi) := \bar{Y}(\xi)(\xi I_{\mathrm{n}} - F) + \bar{X}(\xi)H$ is row reduced with row degrees lower bounded by $\max_i h_i - 1$ $(\max_i h_i)$, a new pair (Y, X) exists such that $Y(\xi)(\xi I_{\mathrm{n}} - F) + X(\xi)H = Q(\xi)$ and

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{x}} = X\left(\frac{d}{dt}\right)\mathbf{y}$$

is a proper (strictly proper) state observer for (6.6), and hence

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{x}} = [YG - XJ]\left(\frac{d}{dt}\right)\mathbf{u} + X\left(\frac{d}{dt}\right)\mathbf{y}$$

is a proper (strictly proper) state observer for the state model (25).

Proof: It is sufficient to observe the set of possible solutions of the matrix equation

$$Q(\xi) = Y(\xi)(\xi I_{\mathrm{n}} - F) + X(\xi)H$$

can be expressed as

$$[Y(\xi) \mid X(\xi)] = [\bar{Y}(\xi) \mid \bar{X}(\xi)] + T(\xi)[-N_\ell(\xi) \mid D_\ell(\xi)], \quad T \in \mathbb{R}[\xi]^{\mathrm{n} \times \mathrm{p}}$$

and to apply the matrix division algorithm of Proposition 4.1 to the pair $(\bar{X}(\xi), D_\ell)$, thus getting $\bar{X}(\xi) = -T(\xi)D_\ell(\xi) + X(\xi)$, where X has all row degrees smaller than $\max_i h_i$. □

This result admits a rather interesting interpretation. As the row indexes $h_1, h_2, \ldots, h_{\mathrm{p}}$ are the well-known *observability indexes* of the pair (H, F) [5]–[7], the previous proposition states it is always possible to obtain a state observer (28) where Q, and hence the error dynamics matrix Δ (see Theorem 3.4), is row reduced with row degrees lower bounded by the maximum of the observability indexes. So, these indexes somehow provide a constraint on the minimal complexity the asymptotic state observers can possibly exhibit. This situation strictly reminds us of an analogous one for the classical output feedback compensator, in which, instead, the reachability indexes are involved [6], [7].

The characterization of strictly proper state observers is much simpler than the characterization of general proper state observers, as shown by the following proposition.

Proposition 6.3: Let

$$Q\left(\frac{d}{dt}\right)\hat{\mathbf{x}} = P_u\left(\frac{d}{dt}\right)\mathbf{u} + P_y\left(\frac{d}{dt}\right)\mathbf{y}$$

be an asymptotic state observer, whose matrices Q, P_u, and P_y satisfy conditions i) and ii) for some (uniquely determined) matrix pair (Y, X). Such an observer is strictly proper if and only if $Y^{-1}X$ is proper.

Proof: Assume the asymptotic observer is strictly proper, and let V be a unimodular matrix such that

$$\begin{aligned} &[V(\xi)Q(\xi) \quad -V(\xi)P_y(\xi)] \\ &\quad = [V(\xi)Y(\xi) \quad V(\xi)X(\xi)] \begin{bmatrix} \xi I_{\mathrm{n}} - F & 0 \\ H & -I_{\mathrm{p}} \end{bmatrix} \end{aligned}$$

is row reduced. If $[Q_{hr} \ -P_{yhr}]$ denotes the leading row coefficient matrix of $[V(\xi)Q(\xi) \ -V(\xi)P_y(\xi)]$, Q_{hr} is nonsingular, and $P_{yhr} = 0$. We, first, show

$$\deg i\text{th row of } (VY) \geq \deg i\text{th row of } (VX), \quad \forall i. \tag{30}$$

If not, some row, say, the jth, in VP_y would exist with degree greater to or equal to the degree of the corresponding row in VQ, thus contradicting the strict properness assumption on $(VQ)^{-1}(VP_y) = Q^{-1}P_y$. Then, however

$$\begin{aligned} &\deg i\text{th row of } (VQ) \\ &\quad \equiv \deg i\text{th row of } (VY) + 1, \\ &\quad > \deg i\text{th row of } (-VYF + XH), \quad \forall i. \end{aligned}$$

This fact implies, also, Q_{hr} coincides with the leading row coefficient matrix of VY, and hence VY is row reduced. So, as VY is row reduced and (30) holds true, $(VY)^{-1}(VX) = Y^{-1}X$ is proper.

Conversely, if $Y^{-1}X$ is proper, some unimodular matrix U exists that leads $[Y(\xi) \quad X(\xi)]$ to row-reduced form. If $[Y_{hr} \quad X_{hr}]$ denotes the leading row coefficient matrix of $[U(\xi)Y(\xi) \quad U(\xi)X(\xi)]$, Y_{hr} is nonsingular, and $[U(\xi)Q(\xi) \ -U(\xi)P_y(\xi)]$ is, in turn, row reduced, with leading row coefficient matrix $[Y_{hr} \ 0]$, thus proving $(UQ)^{-1}(UP_y) = Q^{-1}P_y$ is strictly proper. □

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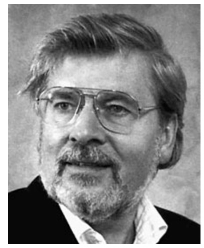

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