INFINITE-DIMENSIONAL SYSTEMS DESCRIBED BY FIRST ORDER PDE's

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ABSTRACT

This paper is concerned with the characterization of systems described by first order PDE's in terms of Markovian properties. It is shown that for 2D autonomous systems with infinite-dimensional behavior the existence of a description by means of first order PDE's is equivalent to strong-Markovianity.

1. INTRODUCTION

First order ODE's and PDE's are relevant not only due to simulation issues, but also due to the fact that they are often associated with state/Markov properties. In very broad terms, such properties mean that, given any partition of the evolution domain into a "past", a "present", and a "future" region, the values of the system trajectories on the "present" region summarize the system memory, in the sense that the future evolution only depends on those values, needing thus no extra information from the past. It is shown in [1] that for systems given by ODE's the existence of a first order description is equivalent to the Markov property. The situation is somewhat more complicated for systems described by PDE's. In fact, for systems evolving over multidimensional domains, two Markov properties, weak- and strong-Markovianity, can be considered. It has recently been shown in [2] that the existence of a first order description is sufficient, but not necessary for weak-Markovianity; however, for the case of systems with finite-dimensional behavior first order representability is equivalent to strong-Markovianity.

The aim of this paper is to move a step forward in the characterization of first order representability in terms of Markovianity and analyze what happens for systems with infinite-dimensional behavior. We consider the case of autonomous systems, i.e, systems without free variables, in which the infinite - dimensionality of the behavior is due to the existence of an infinite-dimensional set of initial conditions. In particular, we focus on the 2D case and prove that, in this case, similar to what happens for finite-dimensional behaviors, the existence of a description by means of first order PDE's is indeed equivalent to strong - Markovianity.

2. INFINITE-DIMENSIONAL SYSTEMS DESCRIBED BY PDE'S

This paper deals with multidimensional ($\mathtt{n}$D) behavioral systems that can be represented as the solution set of a system of linear PDE's with constant coefficients. Let $R \in \mathbb{R}^{\bullet\times\mathtt{w}}[s_1, \cdots, s_\mathtt{n}]$ (the set of real polynomial matrices in $\mathtt{n}$ indeterminates with $\mathtt{w}$ columns) and associate with this matrix the following system of PDE's

$$R(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_\mathtt{n}})w = 0. \qquad (1)$$

The behavior $\mathfrak{B}$ defined by this system of PDE's is simply its solution set over an appropriate domain. Here we consider as domain the set of all continuous functions $\mathcal{C}^0(\mathbb{R}^\mathtt{n}, \mathbb{R}^\mathtt{w})$. Hence

$$\mathfrak{B} = \{w \in \mathcal{C}^0(\mathbb{R}^\mathtt{n}, \mathbb{R}^\mathtt{w}) \mid (1) \text{ holds in the distributional sense}\}.$$

As $\mathfrak{B}$ is the kernel of a partial differential operator, we refer to it as a *kernel behavior*, and denote it as $\ker\left(R(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_\mathtt{n}})\right)$. The PDE (1) is called a *kernel*

* Research partially supported by the *Unidade de Investigação Matemática e Aplicações* (UIMA), University of Aveiro, Portugal, through the *Programa Operacional "Ciência e Tecnologia e Inovação"* (POCTI) of the *Fundação para a Ciência e Tecnologia* (FCT), co-financed by the European Union fund FEDER.

† Research supported by the Belgian Federal Government under the DWTC program Interuniversity Attraction Poles, Phase V, 2002–2006, Dynamical Systems and Control: Computation, Identification and Modelling, by the KUL Concerted Research Action (GOA) MEFISTO–666, and by several grants en projects from IWT-Flanders and the Flemish Fund for Scientific Research.

representation of $\mathfrak{B} = \ker\left(R(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_{\mathtt{n}}})\right)$.

Our aim is to focus on infinite-dimensional autonomous kernel behaviors, i.e., behaviors that have no free variables, in the sense that no component of the system variable w can be arbitrarily chosen in $\mathcal{C}^0(\mathbb{R}^{\mathtt{n}}, \mathbb{R})$, but are infinite-dimensional subspaces of $\mathcal{C}^0(\mathbb{R}^{\mathtt{n}}, \mathbb{R}^{\mathtt{w}})$ (due to the infinite dimension of their initial condition sets). For the sake of simplicity we shall restrict attention in this conference paper to the 2D univariate case, i.e., we take $\mathtt{n} = 2$ and $\mathtt{w} = 1$; this means that we consider behaviors with one variable evolving in $\mathbb{R}^2$. The general situation will later be reported elsewhere.

Thus, the kernel representations to be considered are associated with 2D polynomial columns

$$R(s_1, s_2) = \begin{bmatrix} r_1(s_1, s_2) \\ \vdots \\ r_{\mathtt{q}}(s_1, s_2) \end{bmatrix},$$

where the $r_i(s_1, s_2)$ are 2D polynomials. Factoring out the greatest common divisor $p(s_1, s_2)$ of these polynomials yields:

$$R(s_1, s_2) = F(s_1, s_2)p(s_1, s_2), \tag{2}$$

where,

$$F(s_1, s_2) = \begin{bmatrix} p_1(s_1, s_2) \\ \vdots \\ p_q(s_1, s_2) \end{bmatrix}$$

and

$$p_i(s_1, s_2)p(s_1, s_2) = r_i(s_1, s_2), \;\; i = 1, \ldots, \mathtt{q}.$$

Since the polynomials $p_i(s_1, s_2)$ have no common factors, they have at most a finite number of common zeros and hence the variety

$$\mathcal{V} := \{(\lambda_1, \lambda_2) \in \mathbb{C}^2 \mid p_i(\lambda_1, \lambda_2) = 0, i = 1, \ldots, \mathtt{q}\} \tag{3}$$

is finite, [3].

On the other hand, if $\mathfrak{B} = \ker\left(R(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\right)$ is infinite-dimensional, then the polynomial $p(s_1, s_2)$ cannot be a unit (i.e., a nonzero constant), otherwise $\mathfrak{B}$ would coincide with $\ker\left(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\right)$ which is finite dimensional. Since the case where $p(s_1, s_2)$ equals zero is trivial, we shall henceforth assume that this polynomial is nonconstant.

3. nD MARKOVIAN SYSTEMS

We consider two versions of Markovianity. The first, *weak - Markovianity*, is defined as follows. Let Π be the set of 3-way partitions (S_-, S_0, S_+) of $\mathbb{R}^{\mathtt{n}}$ such that S_- and S_+ are open and S_0 is closed; given a partition $\pi = (S_-, S_0, S_+) \in \Pi$ and a pair of trajectories (w_-, w_+) that coincide on S_0, define the *concatenation* of (w_-, w_+) along π as the trajectory $w_- \wedge_\pi w_+$ that coincides with w_- on $S_0 \cup S_-$ and with w_+ on $S_0 \cup S_+$.

Definition 1 *A multidimensional behavior $\mathfrak{B} \subseteq (\mathbb{R}^{\mathtt{w}})^{\mathbb{R}^{\mathtt{n}}}$ is said to be* weak-Markovian *if for any partition $\pi \in \Pi$ and any pair of trajectories $w_-, w_+ \in \mathfrak{B}$ such that $w_{-|S_0} = w_{+|S_0}$, the trajectory $w_- \wedge_\pi w_+$ is also an element of $\mathfrak{B}$.*

The second version, strong-Markovianity requires that the restriction of a behavior to linear subspaces of $\mathbb{R}^n$ also has concatenability properties.

Unlike what happens in the finite-dimensional case, the restriction of an infinite-dimensional kernel behavior $\mathfrak{B}$ to a subspace $\mathcal{S}$ of $\mathbb{R}^n$ is not always a kernel behavior. Therefore in the sequel we consider the following kernel behavior associated to the restriction $\mathfrak{B}|_{\mathcal{S}}$ of $\mathfrak{B}$ to $\mathcal{S}$.

Definition 2 *Given a kernel behavior $\mathfrak{B}$ defined over $\mathbb{R}^n$ and a linear subspace $\mathcal{S}$ of $\mathbb{R}^n$, define the behavior $\mathcal{K}(\mathfrak{B}|_{\mathcal{S}})$ as the smallest kernel behavior containing the restriction $\mathfrak{B}|_{\mathcal{S}}$ of $\mathfrak{B}$ to $\mathcal{S}$.*

Our definition of strong-Markovianity for a behavior $\mathfrak{B}$ requires that $\mathcal{K}(\mathfrak{B}|_{\mathcal{S}})$ is Markovian. More concretely, given a subspace $S \subseteq \mathbb{R}^{\mathtt{n}}$, let Π_S be the set of 3-way partitions (S_-, S_0, S_+) of S such that S_- and S_+ are open (in S) and S_0 is closed (in S).

Definition 3 *A multidimensional behavior $\mathfrak{B} \subseteq (\mathbb{R}^{\mathtt{w}})^{\mathbb{R}^{\mathtt{n}}}$ is said to be* strong-Markovian *if for any subspace S of $\mathbb{R}^{\mathtt{n}}$, any partition $\pi_S \in \Pi_S$, and any pair of trajectories $w_-, w_+ \in \mathcal{K}(\mathfrak{B}|_S)$ such that $w_{-|S_0} = w_{+|S_0}$, the trajectory $w_- \wedge_\pi w_+$ is an element of $\mathcal{K}(\mathfrak{B}|_S)$.*

Clearly, strong-Markovianity implies weak - Markovianity. Moreover, these two properties coincide for one-dimensional behaviors.

Let $\mathfrak{B}$ be an nD behavior defined by a first order PDE

$$(\sum_{i=1}^{\mathtt{n}} R_i \frac{\partial}{\partial x_i} + R_0)w = 0. \tag{4}$$

It is easy to see that this implies weak-Markovianity. However, as shown in [2] the reciprocal is not true. It is therefore natural to ask whether first order PDE's generate behaviors that are strong-Markovian and, reciprocally, whether strongly Markovian behaviors given by PDE's (1) admit first order representations (4). It is proven in [2] that for *finite dimensional* behaviors, strong-Markovianity and first order representability are indeed equivalent. In this paper we show that the same happens for autonomous systems with infinite-dimensional behaviors.

4. FIRST ORDER PDE'S AND MARKOVIANITY

As mentioned before, we shall focus on the univariate two-dimensional case.

Let $\mathfrak{B} \subset \mathcal{C}^0(\mathbb{R}^2, \mathbb{R})$ be the 2D behavior with kernel representation associated to the matrix R in (2). Given $\alpha \in \mathbb{R}$, define the following behaviors:

$$\mathfrak{B}_{2D}^{\alpha} := \{w \in \mathfrak{B} \mid \quad \forall t \in \mathbb{R} \ \exists c \in \mathbb{R} \ \forall x_2 \in \mathbb{R} \quad w(t - \alpha x_2, x_2) = c\} \tag{5}$$

and

$$\mathfrak{B}_{1D}^{\alpha} := \{v \in \mathcal{C}^0(\mathbb{R}, \mathbb{R}) \mid \quad v(t) = w(t - \alpha x_2, x_2), \quad t \in \mathbb{R}, w \in \mathfrak{B}_{2D}^{\alpha}\}. \tag{6}$$

Note that $\mathfrak{B}_{2D}^{\alpha}$ consists of all the trajectories in $\mathfrak{B}$ that are constant along all the lines $\mathcal{L}_t^{\alpha} := \{(x_1, x_2) \mid x_1 + \alpha x_2 = t\}$, $t \in \mathbb{R}$, while $\mathfrak{B}_{1D}^{\alpha}$ is the 1-D behavior obtained by following this constant value across these lines. It is not difficult to check that

$$\begin{aligned} \mathfrak{B}_{2D}^{\alpha} &= \mathfrak{B} \cap \ker(\frac{\partial}{\partial x_2} - \alpha \frac{\partial}{\partial x_1}) \\ &= \ker\big(R(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) \cap \ker(\frac{\partial}{\partial x_2} - \alpha \frac{\partial}{\partial x_1}) \\ &= \ker\big(R(\frac{\partial}{\partial x_1}, \alpha \frac{\partial}{\partial x_1})\big) \\ &= \ker\big(\pi_\alpha(\frac{\partial}{\partial x_1}) \tilde{p}_\alpha(\frac{\partial}{\partial x_1})\big), \end{aligned} \tag{7}$$

with

$$\pi_\alpha(s) := \gcd(p_1(s, \alpha s), \ldots, p_{\mathsf{q}}(s, \alpha s)) \tag{8}$$

and

$$\tilde{p}_\alpha(s) := p(s, \alpha s). \tag{9}$$

On the other hand $\mathfrak{B}_{1D}^{\alpha}$, that can alternatively be given by

$$\mathfrak{B}_{1D}^{\alpha} = \{v \in \mathbb{R}^{\mathbb{R}} \mid v(t) = w(t, 0), t \in \mathbb{R}, w \in \mathfrak{B}_{2D}^{\alpha}\},$$

is a 1D behavior whose trajectories correspond to the restriction of the trajectories in $\mathfrak{B}_{2D}^{\alpha}$ to the x_1-axis. Thus, due to (7), we have that:

$$\mathfrak{B}_{1D}^{\alpha} = \ker(\pi_\alpha(\frac{d}{dt}) \tilde{p}_\alpha(\frac{d}{dt})). \tag{10}$$

In order to show that strong-Markovianity implies first order representability, we start by proving that weak-Markovianity alone already implies that the polynomial $p(s_1, s_2)$ in (2) is first order.

Lemma 1 *Let* $\mathfrak{B} = \ker\big(R(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) \subset \mathcal{C}^0(\mathbb{R}^2, \mathbb{R})$ *be an infinite-dimensional 2D weak-Markovian kernel behavior and let* $\alpha \in \mathbb{R}$. *Then the behavior* $\mathfrak{B}_{1D}^{\alpha}$ *is a 1D Markovian behavior.*

Proof. In order to prove this result it suffices to show that every trajectory v of $\mathfrak{B}_{1D}^{\alpha}$ such that $v(0) = 0$ is concatenable with the zero trajectory, i.e., if $\Pi = ((-\infty, 0), \{0\}, (0, +\infty))$ then $v \wedge_\Pi 0 \in \mathfrak{B}_{1D}^{\alpha}$. Let then $v \in \mathfrak{B}_{1D}^{\alpha}$ be a trajectory such that $v(0) = 0$. Take $w \in \mathfrak{B}_{2D}^{\alpha}$ such that $v(t) = w(t, 0)$. Then, $w(-\alpha x_2, x_2) = w(0, 0) = v(0) = 0$, i.e, w is zero on the line $\mathcal{L}_0^{\alpha}$. By the weak-Markovianity of $\mathfrak{B}$, this implies that w is concatenable with the zero trajectory along the obvious partition $\Pi_0 = (\mathcal{S}_-, \mathcal{L}_0^{\alpha}, \mathcal{S}_+)$ of $\mathbb{R}^2$ determined by the line $\mathcal{L}_0^{\alpha}$. In other words, $w^* := w \wedge_{\Pi_0} 0 \in \mathfrak{B}$. But w^* is also a trajectory of $\mathfrak{B}_{2D}^{\alpha}$. Moreover, its corresponding trajectory in $\mathfrak{B}_{1D}^{\alpha}$, $v^*(t) := w^*(t, 0)$, coincides with $v \wedge_\Pi 0$. This shows that v is concatenable with the zero trajectory, as desired. ■

Corollary 1 *Let* $\mathfrak{B} = \ker\big(R(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) \subset \mathcal{C}^0(\mathbb{R}^2, \mathbb{R})$ *be an infinite-dimensional 2D weak-Markovian kernel behavior and* $p(s_1, s_2)$ *be the corresponding right factor in factorization (2). Then* $p(s_1, s_2)$ *is a 2D first order polynomial, i.e.,* $p(s_1, s_2) = a_1 s_1 + a_2 s_2 + a_0$, *for suitable coefficients* $a_0, a_1, a_2 \in \mathbb{R}$.

Proof. Assume that $\mathfrak{B}$ is weak-Markovian and let $\alpha \in \mathbb{R}$. Then, since 1D-Markovianity is equivalent to first order representability [1], by (10) and Lemma 1, the polynomial $\pi_\alpha(s)\tilde{p}_\alpha(s)$ must have degree not higher than 1. Note that, since the variety $\mathcal{V}$ defined in (3) is finite, the polynomials $p_i(s, \alpha s), i = 1, \ldots, \mathsf{q}$ are not all zero polynomials. This implies that $\pi_\alpha(s)$ is not the zero polynomial. Therefore the degree of $\tilde{p}_\alpha(s)$ cannot be higher than 1. Now, let $p(s_1, s_2) = \sum_{i,j} p_{ij} s_1^i s_2^j$, then

$$\tilde{p}_\alpha(s) = \sum_k (\sum_{i+j=k} p_{ij} \alpha^j) s^k,$$

and hence

$$\sum_{i+j=k} p_{ij} \alpha^j = 0, \quad \forall \alpha \in \mathbb{R}, \quad k \geq 2.$$

This implies that $p_{ij} = 0$, for $i + j \geq 2$ and $p(s_1, s_2) = a_1 s_1 + a_2 s_2 + a_0$, with $a_1 = p_{10}, a_2 = p_{01}, a_0 = p_{00}$. ■

Without loss of generality, we shall henceforth take $a_2 = 0$ (if this were not the case, then a linear change of variable in the (x_1, x_2)-plane could be made to yield this situation). Since $p(s_1, s_2)$ has previously been assumed to be nonconstant, we may also take $a_1 = 1$ without loss of generality. Thus

$$p(s_1, s_2) = s_1 + a_0$$

and

$$\mathfrak{B} = \ker\big(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})(\frac{\partial}{\partial x_1} + a_0)\big)$$

We next show that, in case $\mathfrak{B}$ is strong-Markovian, $\ker\big(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) = \{0\}$.

Lemma 2 *Let* $\mathfrak{B} = \ker\big(R(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) \subset \mathcal{C}^0(\mathbb{R}^2, \mathbb{R})$ *be an infinite-dimensional 2D strong-Markovian behavior, and consider the corresponding 2D polynomial matrix* $F(s_1, s_2)$ *given by factorization (2). Then*

$$\ker\big(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) = \{0\}$$

.

Proof. Take an arbitrary $\beta \in \mathbb{R}$ and consider the subspace $\mathcal{S}_\beta = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_2 = \beta x_1\}$. If $\mathfrak{B}$ is strong-Markovian, then $\mathcal{K}(\mathfrak{B}|_{\mathcal{S}_\beta})$ is a 1D Markovian behavior given by

$$\begin{aligned}\mathcal{K}(\mathfrak{B}|_{\mathcal{S}_\beta}) &= \ker\big(F(\frac{d}{dt}, \beta\frac{d}{dt})\tilde{p}_\beta(\frac{d}{dt})\big) \\ &= \ker(\pi_\beta(\frac{d}{dt})\tilde{p}_\beta(\frac{d}{dt})),\end{aligned}$$

with π_β and $\tilde{p}_\beta$ defined as in (8) and (9).

But, taking into account the considerations made in the proof of Corollary 1, the polynomial $\pi_\beta(s)\,\tilde{p}_\beta(s)$ $= \pi_\beta(s)(s + a_0)$ must have degree not higher than 1. Therefore $\pi_\beta(s)$ (which is non null) must be a nonzero constant for all $\beta \in \mathbb{R}$, and

$$\begin{aligned}\mathcal{K}(\mathfrak{B}|_{\mathcal{S}_\beta}) &= \ker(\tilde{p}_\beta(\frac{d}{dt})) \\ &= \ker(\frac{d}{dt} + a_0) \\ &= span\{e^{-a_0 t}\}. \qquad (11)\end{aligned}$$

We now show that this implies that $\ker\big(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) = \{0\}$.

Indeed, suppose that this is not the case. Then (the complexification of) $\ker\big(F\big)$ contains a trajectory of the form $\hat{w}(x_1, x_2) = e^{\lambda_1 x_1 + \lambda_2 x_2}$ and all the trajectories w such that

$$(\frac{\partial}{\partial x_1} + a_0)w(x_1, x_2) = e^{\lambda_1 x_1 + \lambda_2 x_2} \qquad (12)$$

are in (the complexification of) $\mathfrak{B}$.

If $\lambda_1 \neq -a_0$, (12) has solutions of the form

$$w(x_1, x_2) = k(x_2)e^{-a_0 x_1} + \frac{1}{\lambda_1 + a_0}e^{\lambda_1 x_1 + \lambda_2 x_2}, \quad k(.) \in C^0(\mathbb{R}, \mathbb{R}).$$

Thus, (the complexification of) $\mathcal{K}(\mathfrak{B}|_{\mathcal{S}_\beta})$ contains all the trajectories v such that

$$v(t) = ke^{-a_0 t} + \frac{1}{\lambda_1 + a_0}e^{(\lambda_1 + \beta)t}, k \in \mathbb{R}.$$

In particular, taking $\beta = 0$, we conclude that

$$\{v_0 \mid v_0(t) = ke^{-a_0 t} + \frac{1}{\lambda_1 + a_0}e^{\lambda_1 t}, k \in \mathbb{R}\}$$

is a subset of (the complexification of) $\mathcal{K}(\mathfrak{B}|_{\mathcal{S}_0})$, which (taking into account that $\lambda_1 \neq -a_0$) contradicts (11).

If $\lambda_1 = -a_0$, one can easily verify that

$$\{v_0 \mid v_0(t) = ke^{-a_0 t} + te^{-a_0 t}, k \in \mathbb{R}\}$$

is a subset of $\mathcal{K}(\mathfrak{B}|_{\mathcal{S}_0})$, which also contradicts (11). ■

Note that, in case $\ker\big(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) = \{0\}$, we have $\ker\big(R(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) = \ker\big(F(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})p(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2})\big) = \ker(p(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}))$. Thus, Corollary 1 and Lemma 2 clearly imply that every strong-Markovian infinite - dimensional kernel behavior $\mathfrak{B} \subset C^0(\mathbb{R}^2, \mathbb{R})$ can be described by a first order PDE, i.e., $\mathfrak{B} = \ker(a_1\frac{\partial}{\partial x_1} + a_2\frac{\partial}{\partial x_2} + a_0)$. Conversely, it is not difficult to prove that $\mathfrak{B} = \ker(a_1\frac{\partial}{\partial x_1} + a_2\frac{\partial}{\partial x_2} + a_0)$ is strong-Markovian. This yields our main result.

Theorem 1 *Let* $\mathfrak{B} \subset \mathcal{C}^0(\mathbb{R}^2, \mathbb{R})$ *be an infinite - dimensional 2D kernel behavior. Then the following are equivalent:*

1. $\mathfrak{B}$ *is strong-Markovian*
2. $\mathfrak{B}$ *is described by one first order PDE, i.e.,* $\mathfrak{B} = \ker(a_1\frac{\partial}{\partial x_1} + a_2\frac{\partial}{\partial x_2} + a_0)$, *for suitable real coefficients* a_0, a_1, a_2.

5. CONCLUSION

This paper reports on some results how to characterize infinite-dimensional systems described by first order PDE's in terms of Markovian properties. For the particular case of univariate infinite-dimensional 2D systems, it was proven that representability by one first order PDE is equivalent to strong-Markovianity. This result points in the same direction as the results already obtained in [2] for the finite-dimensional case, suggesting that this straight connection between Markovianity and first order PDE's might also exist in more general cases.

6. REFERENCES

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