Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

1696: THE BIRTH OF OPTIMAL CONTROL

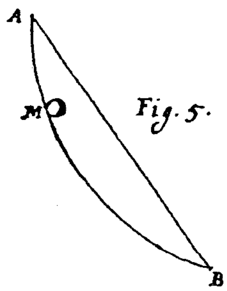
Jan C. Willems
Department of Mathematics
University of Groningen
P.O. Box 800
9700 AV Groningen, The Netherlands
email: J.C.Willems@math.rug.nl

The history of optimal control starts in 1696 in Groningen, a university town in the North of the Netherlands, with the story of the brachystochrone and Johann Bernoulli. He was Professor of Mathematics at the University of Groningen from 1695 to 1705. The purpose of this CDC talk is to explain the problem, sketch the solution, and tell a bit about the historical context in which it took place. This article is a short introduction to the talk.

In the June 1696 issue of *Acta Eruditorum*, Johann Bernoulli posed the following challenge to his contemporaries:

Invitation to all Mathematicians to solve a new problem.

> If in a vertical plane two points A and B are given, then it is required to specify the orbit AMB of the moveable point M, along which it, starting from A, and under the influence of its own weight, arrives at B in the shortest possible time. So that those who are keen of such matters will be tempted to solve this problem, is it good to know that it is not, as it may seem, purely speculative and without practical use. Rather it even appears, and this may be hard to believe, that it is very useful also for other branches of science than mechanics. In order to avoid a hasty conclusion, it should be remarked that the straight line is certainly the line of shortest distance between A and B, but it is not the one which is travelled in the shortest time. However, the curve AMB – which I shall divulge if by the end of this year nobody else has found it – is very well known among geometers.

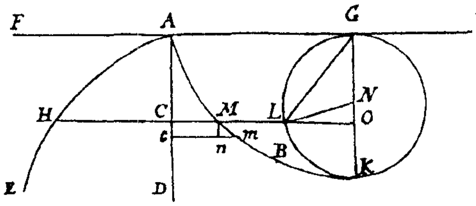


This publication, 300 years ago this year, marks the birth of optimal control. Optimization problems had been considered at least since the Greeks. One of the oldest is known as *Dido's problem* (inspired by the mythical story told by Vergilius in the *Aeneas* surrounding the foundation of Carthago): the problem of determining the shape of the figure of largest possible surface encircled by a curve of a given length. The solution to this problem was known to the Greeks: it is the circle – in their thinking, following Aristoteles, it *had to be* the circle, the perfect figure. Actually, it took until the 19–th century to prove this result in a way that meets our contemporary standards of rigor. Many other geometric optimization problems had been solved before. Newton had studied the shape of a body with minimal drag. Fermat and Huygens had interpreted the law of Snellius in optics as a minimization problem. However, Johann Bernoulli's brachystochrone seems to be the first problem which explicitely dealt with optimally controlling the path or the behavior of a dynamical system. As such, it seems appropriate to view the brachistochrone problem as the first problem of opti-

0-7803-3590-2/96 $5.00 © 1996 IEEE

mal control.

Johann Bernoulli called the fastest path the *brachystochrone* (from the Greek words *βράχιστος*: shortest, and *χρόνος*: time). As if time had stood still, it were also minimum time problems that propelled the development of optimal control in the early 1960's. Two outstanding features characterize the story of the solution of Johann Bernoulli's problem: the beauty of the solution, and the eminence of the personalities who took up Johann Bernoulli's challenge and solved the problem. The brachystochrone turns out to be a cycloid, the unique cycloid generated by a circle that rolls on the horizontal line passing through the point A, starts at A, and passes through B. It is easy to see that this defines the cycloid uniquely, provided, of course, that B is located not higher than A.



As is apparent from Johann Bernoulli's announcement, he was under the impression that the problem was new. However, Leibniz knew better: Galileo in his book on the *Two New Sciences* in 1638 formulated the brachystochrone problem and even suggested the solution: Galileo thought the brachystochrone was a circle through the points A and B, centered on the x–axis. Johann Bernoulli considered the fact that Galileo had been wrong on two counts, in thinking that the catenary was a parabola, and that the brachystochrone was a circle, as definitive evidence of the superiority of differential calculus (or the *Nova Methodus* as they called it).

Johann Bernoulli was extremely enthusiastic about the fact that the brachistochrone was a cycloid. This curve had been introduced by Galileo who gave it its name: *related to the circle.* Huygens had discovered that the the cycloid has a remarkable property: it is the only curve with the property that a body falling under its own weight is guided by this curve in such a way that it will oscillate with a period that is independent of the initial point where the body is released. Contrary to what Galileo thought, the circle has this property only approximately: the period of oscillation of a pendulum is a function of its amplitude. Huygens called this curve, the cycloid, therefore the *tautochrone* (from *τα ὐ τός*: equal, and *χρόνος*: time). Johann Bernoulli was amazed and somewhat puzzled, it seems, by the fact that the same curve had these two remarkable properties related to the time traveled on it by a body falling under its own weight: the cycloid is both the brachystochrone and the tautochrone. In fact, from this Hohann concluded the *Nature always acts in the simplest possible way.* That will be news to students of Quantum Mechanics.

Five mathematicians solved Johann Bernoulli's challenge, and not just any five! Johann himself; Leibniz, who called the problem *splendid* and sent his solution to Johann in a letter dated June 16, 1696; Johann's elder brother Jakob – his solution was published in the *Acta Eruditorum* of May 1697, in the same issue in which Johann's solution appeared; de l'Hôpital; and, finally, Newton, who presented his solution to the Royal Society on February 24, 1697. Newton published his solution (without proof) in the *Philosophical Transactions*, anonymously. However, Johann Bernoulli recognized the solution as Newton's, since: *ex ungue leonem* (you can tell the lion by its claws). Leibniz wrote also a short note in the issue of the *Acta Eruditorum* in which the solutions of the Bernoulli's appeared, remarking that he would not reproduce his own solution, since it was similar to theirs. He also noted who else, in his opinion, could solve the problem: *de l'Hôpital, Huygens, were he alive, Hudde, if he had not given up mathematics, and Newton, if he would take the trouble.* Hudde, incidentally, was a mathematician who had become mayor of Amsterdam. It is worth noting that Jakob Bernoulli's proof was quite different from Johann's. In appearance it seemed clumsier, and at first Johann made fun of it. However, Jakob's solution is more akin to the ideas of the calculus of variations, Hamilton-Jacobi theory, and dynamic programming, and is therefore also of great historical importance in the development of optimal control.

If we accept the brachystochrone problem as the birth of optimal control, the field made a spectacular start, involving Johann and Jakob Bernoulli, Wilhelm Gottfried Leibniz, the Marquis de l'Hôpital, Isaac Newton, and Galileo Galilei.

References.

[1] H.H. Goldstine, *A History of the Calculus of Variations from the 17th through the 19th Century*, Springer Verlag, 1980.

[2] J.A. van Maanen, *Een Complexe Grootheid, Leven en Werk van Johann Bernoulli 1667-1748*, Epsilon Uitgaven, Utrecht, 1995.