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THE GEOMETRIC APPROACH TO CONTROL SYSTEM DESIGN: A TUTORIAL INTRODUCTION TO A FEW OF THE MAIN IDEAS.

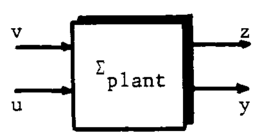
JAN C. WILLEMS

Mathematics Institute
P.O.Box 800, 9700 AV GRONINGEN
The Netherlands

In this talk we will discuss in a tutorial fashion some of the main applications of the geometric approach to control system design. We will briefly review the notions of controlles invariant, conditionally invariant, and controllability subspaces. Further, their 'almost' counterparts and the notion of (A,B,C)-pairs. We will show how these concepts yield the solution of the disturbance decoupling and of the non-interacting control problem.

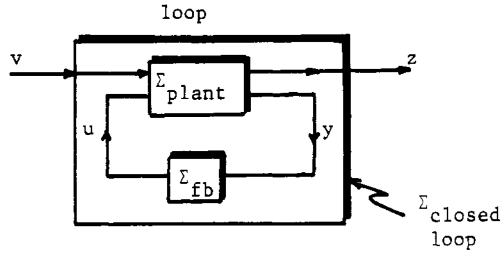
1. Control System Design

A typical control system Σ_{plant} has four types of external variables: exogenous inputs v, control inputs u, to-be-controlled outputs z, and observed outputs y. This structure is shown in the block diagram



A controller is a system $\Sigma_{feedback}$ having the observed output space as its input space and the control input space as its output space:

Feedback connection of Σ_{plant} with $\Sigma_{feedback}$ yields the closed loop system $\Sigma_{closed\ loop}$



having v as input and z as output.

A control system design problem consist in the question to design, for a given plant Σ_{plant} and a given design purpose, a feedback controller $\Sigma_{feedback}$, such that the closed loop system $\Sigma_{closed\ loop}$ meets the specified design requirement. For example, in the disturbance decoupling problem we assume that r is a disturbance and that in $\Sigma_{closed\ loop}$ z should be independent of r; in non-interacting control, we assume that

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} \text{ and } z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{bmatrix}$$

and we want that in $\Sigma_{closed\ loop}$, for each $j = 1,2,\ldots,k$, z_j is independent of v_i for $i \neq j$. We can also add stability or pole placement conditions to these design requirements.

2. Non-Interacting Control for Linear Systems

Consider the linear system

$$\Sigma_{plant}: \quad \dot{x} = Ax + Bu + \sum_{i=1}^{k} G_i v_i$$
$$y = Cx; \; z_i = H_i x$$

$$\Sigma_{feedback}: \quad \dot{w} = Kw + Ly \; ; \; u = Mw + Fy$$

yielding, in the obvious notation,

$$\Sigma_{closed\ loop}: \quad \dot{x}_e = A^e x^e + \sum_{i=1}^{k} G_i v_i \; ; \; z_i = H_i^e x^e$$

The purpose is to design, for a given Σ_{plant}, $\Sigma_{feedback}$, such that in $\Sigma_{closed\ loop}$ the transfer function is block diagonal.

We will show that the solution of this problem (with y = x) is a nice application of the notions of controlled invariant and of controllability subspaces.

3. Disturbance Decoupling

The disturbance decoupling problems for the linear case is specified by

$$\Sigma_{plant}: \quad \dot{x} = Ax + Bu + Gv \; ; \; y = Cx \; ; \; z = Hx$$

$$\Sigma_{feedback}: \quad \dot{w} = Kw + Ly \; ; \; u = Mw + Fy$$

$$\Sigma_{closed\ loop}: \quad \dot{x}^e = A^e x^e + G^e v \; ; \; z = H^e x^e$$

In disturbance decoupling we want that the transfer function of $\Sigma_{\text{closed loop}}$ is zero.

In almost disturbance decoupling we want that its transfer function has a norm which is less than any pressigned $\varepsilon > 0$ (of course, Σ_{fb} is allowed to depend on ε in this case). We shall review the solution to these problems. As we will see, solution of the disturbance decoupling problem is an illustration of Schumacher's (A,B,C)-pairs and the solution of the almost disturbance decoupling problem is an illustration of the notion of almost invariant subspaces.

We will also mention a few open problems in this area.