OPEN PROBLEMS IN THE AREA OF POLE PLACEMENT

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Abstract

In recent years there has been significant progress towards the solution of various versions of the pole placement problem. An important factor has been the behavioral theory which provided a simplified framework to tackle many of these problems. In this short article we provide an overview on the achieved results and we give a list of open research questions in the area of pole placement.

Keywords: Static pole placement, feedback stabilization, inverse eigenvalue problems, behavioral theory.

1 Introduction

The static and the dynamic output pole placement problem belong to the prominent design problems of modern control theory and we refer to the survey articles [6, 12, 22, 24] where also more references to the literature are provided. Various facets of the pole placement problem attracted many researchers over the years. It was been recognized right at the beginning of the problem that the output pole placement problem is nonlinear in nature and a simple solution based on techniques from linear algebra cannot be expected.

In recent years significant progress has been achieved. This progress is due in a major part to a better understanding of the system theoretic ingredients and its relation to algebraic geometry. Helpful in this regard is the behavioral approach which comes in its formulation closest to the algebraic geometric nature of the pole placement problem.

Classically, the static output feedback problem is formulated in the following way: Let A, B, C be real matrices of size $n \times n$, $n \times m$, and $p \times n$, respectively. These 3 matrices describe a linear system of the form

$$\frac{d}{dt}x = Ax + Bu,\; y = Cx. \tag{1}$$

Let $\phi \in \mathbb{R}[\xi]$ be a monic polynomial of degree n. Then the static pole placement problem asks for conditions which guarantee the existence of a $m \times p$ matrix K such that

$$\det(\xi I - A - BKC) = \phi(\xi).$$

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After elimination of the latent variable x, equation (1) describes a linear differential system. This elimination can be done as follows in the controllable case. If $D(\xi)^{-1}N(\xi)$ is a left coprime factorization of the transfer function $C(\xi I - A)^{-1}B$ then the manifest behavior is described by

$$\left(D\left(\frac{d}{dt}\right) \; -N\left(\frac{d}{dt}\right)\right) \left(\begin{array}{c} y \\ u \end{array} \right) = 0. \tag{2}$$

It is easy to verify that the design of a static compensator is equivalent to the construction of matrices K_1, K_2 such that

$$\det \left[\begin{array}{cc} D(\xi) & -N(\xi) \\ K_1 & K_2 \end{array} \right] = \phi(\xi) \in \mathbb{R}[\xi]. \tag{3}$$

In the next section we summarize some of the major known pole placement results. The essence of the problems and their solutions are most transparent in the behavioral language. For the connection between the behavioral point of view and the classical state space formulation we refer to [28, 31]. The following development of the theory follows closely the description given in [21].

2 A summary of known pole placement results

Recall from [28] that a *dynamical system* Σ is a triple $\Sigma = (T, W, \mathbb{B})$, where $T \subset \mathbb{R}$ is the time axis, W is the signal space and $\mathbb{B} \subset W^T$ is called the behavior. For simplicity we restrict attention sequel to dynamical systems Σ whose time axis $T = \mathbb{R}$, whose signal space $W = \mathbb{R}^{m+p}$ and whose behavior $\mathbb{B} \subset C^\infty(\mathbb{R}, \mathbb{R}^{m+p})$ has a so called "kernel representation", i.e. there exist a polynomial matrix $P(\xi)$ such that

$$\mathbb{B} = \{w(t) \in C^\infty(\mathbb{R}, \mathbb{R}^{m+p}) \mid P\left(\frac{d}{dt}\right) w(t) = 0\}. \tag{4}$$

The class of behaviors having a kernel representation forms the class of observable behaviors and they are sometimes also called 'AR'-systems (compare with [27, 28]). Every such AR-system comes with two important invariants called the *rank* $r(\Sigma)$ and the *McMillan degree* $n(\Sigma)$, which are defined as follows: $r(\Sigma)$ is equal to the minimal number of rows needed for a polynomial matrix $P(\xi)$ which describes the behavior $\mathbb{B}$ in a representation of the form (4). We will call such a representation a (row) minimal representation.

If the polynomial matrix $P(\xi)$ is row minimal then we define the McMillan degree $n(\Sigma)$ as the maximal degree of the full size minors in one and therefore any minimal representation.

A system $\Sigma = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B})$ is called autonomous if the rank $r(\Sigma) = m+p$. If the $(m+p) \times (m+p)$ matrix $P(\xi)$ describes an autonomous behavior then the nonzero polynomial $\det P(\xi)$ is called the *characteristic polynomial* of Σ and it will be abbreviate with χ_Σ. χ_Σ is a projective invariant of the autonomous behavior, i.e. if

$$\det P(\xi) = a_0 + a_1 s + \cdots + a_n s^n$$

then $(a_0, \ldots, a_n)$ defines a unique one dimensional subspace, i.e. a point in the projective space $\mathbb{P}^n$. This point then only depends on the autonomous system Σ and does not depend on the particular representation. The roots of $\det P(\xi)$ are by definition the *poles* of Σ.

Feedback in the behavioral theory is defined in the following way: Assume that $\Sigma_1 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_1)$ and $\Sigma_2 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_2)$ are two AR-systems. Then the interconnected system $\Sigma_1 \wedge \Sigma_2$ is defined as:

$$\Sigma_1 \wedge \Sigma_2 := (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_1 \cap \mathbb{B}_2).$$

We say $\Sigma_1 \wedge \Sigma_2$ is a regular interconnection (see [31]) if the ranks "add up", i.e. if

$$r(\Sigma_1 \wedge \Sigma_2) = r(\Sigma_1) + r(\Sigma_2)$$

and we speak of a singular interconnection otherwise.

As it is immediate from the definition the interconnected system $\Sigma_1 \wedge \Sigma_2$ is defined through

$$\begin{pmatrix} P_1\left(\frac{d}{dt}\right) \\ P_2\left(\frac{d}{dt}\right) \end{pmatrix} w(t) = 0,$$

where P_1, P_2 are polynomial matrices representing Σ_1 respectively Σ_2. If P_1, P_2 are in addition row minimal representations then one verifies that $\Sigma_1 \wedge \Sigma_2$ is a regular interconnection if, and only if $\binom{P_1}{P_2}$ is row minimal.

The fundamental question, which is in fact a generalized pole placement problem and which implies many of the "traditional" pole placement questions, is now as follows:

Problem 1 Let m, p, n, q be fixed positive integers. Under what condition is it true that for a generic set of systems system $\Sigma_1 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_1)$ having rank $r(\Sigma_1) = p$ and McMillan degree $n(\Sigma_1) = n$ the following holds: For every polynomial $\phi \in \mathbb{R}[\xi]$ of degree $n+q$ there exists a system $\Sigma_2 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_2)$ having rank $r(\Sigma_2) = m$ and McMillan degree $n(\Sigma_2) = q$ such that $\Sigma_1 \wedge \Sigma_2$ forms a regular interconnection having characteristic polynomial $\chi_{\Sigma_1 \wedge \Sigma_2} = \phi$.

Problem 1 is the behavioral formulation of the dynamic pole placement problem. Closely related is the following formulation, which can be done for an arbitrary base field $\mathbb{F}$:

Problem 2 Let $\mathbb{F}$ be a field and let m, p, n, q be fixed positive integers. Let P_1 be a $p \times (m+p)$ matrix with entries in $\mathbb{F}[\xi]$ whose $p \times p$ minors have degree at most n. Let $\phi(\xi) \in \mathbb{F}[\xi]$ be a polynomial of degree $n+q$. Under what conditions does there exist a $m \times (m+p)$ matrix P_2 with entries in $\mathbb{F}[\xi]$ such that the $m \times m$ minors of P_2 have degree at most q and

$$\det \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \phi.$$

If Problem 2 has a positive answer for a 'generic set' of $p \times (m+p)$ matrices of degree n then we will say that the generic rank p system of McMillan degree n is arbitrary pole assignable in the class of feedback compensators of McMillan degree q. If $q = 0$ we will say that the generic rank p system of McMillan degree n is arbitrary pole assignable by static feedback compensators. Of course it is a major difficulty to make the notion of genericity precise in the formulation and we refer to [21] for details on this question.

2.1 Static pole placement results

The major results in the area of static pole placement are as follows:

Theorem 3 (Brockett and Byrnes [5]) *If the base field $\mathbb{F}$ is algebraically closed and if $mp \geq n$ then the generic rank p system of McMillan degree n is arbitrary pole assignable by static feedback compensators. Moreover if $mp = n$ then the number of non-equivalent feedback compensators assigning a particular closed loop characteristic polynomial is independent of the closed loop polynomial $\phi(\xi) \in \mathbb{F}[\xi]$ and is equal (when counted with multiplicity) to*

$$d(m,p) = \frac{1!2!\cdots(p-1)!(mp)!}{m!(m+1)!\cdots(m+p-1)!} \tag{5}$$

Since $mp \geq n$ is a necessary condition Theorem 3 gives the best possible bound when the base field $\mathbb{F}$ is algebraically closed.

The number $d(m,p)$ is the degree of the Grassmann variety, which was computed in the last century by Schubert [23]. Although Theorem 3 assumes that the base field $\mathbb{F}$ is algebraically closed it does also provide some results for real pole assignment:

Corollary 4 *If $\mathbb{F} = \mathbb{R}$, $mp = n$, and $d(m,p)$ is odd, then the generic rank p system of McMillan degree n is arbitrary pole assignable by static real feedback compensators.*

Berstein determined when $d(m,p)$ is odd.

Proposition 5 (Berstein [2]) *The number $d(m,p)$ is odd if and only if $\min(m,p) = 1$ or $\min(m,p) = 2$ and $\max(m,p) = 2^t - 1$, where t is a positive integer.*

When $d(m,p)$ is even, the best known sufficiency result over the reals is due to Wang:

Theorem 6 (Wang [25]) *If $\mathbb{F} = \mathbb{R}$ and $mp > n$, then the generic rank p system of McMillan degree n is arbitrary pole assignable by static feedback compensators.*

In the last 3 years several elementary proofs of Wang's theorem were given [11, 18, 19].

2.2 Dynamic pole placement results

As for the static pole placement problem there are for the dynamic problem some general necessary conditions. A simple dimension argument first carried out in [30] reveals that

$$q(m+p-1) + mp \geq n \tag{6}$$

is a necessary condition for the generic rank p system of McMillan degree n to be arbitrary pole assignable in the class of feedback compensators of McMillan degree q.

In [16, 17] the sufficiency result of Brockett and Byrnes Theorem 3 was extended:

Theorem 7 (Rosenthal-Ravi-Wang [16, 17]) *If the base field $\mathbb{F}$ is algebraically closed and if $q(m+p-1) + mp \geq n$ then the generic rank p system of McMillan degree n is arbitrary pole assignable in the class of feedback compensators of degree at most q. Moreover if there is equality in* (6) *then the number of non-equivalent feedback compensators assigning a particular closed loop characteristic polynomial is independent of the closed loop polynomial $\phi(\xi) \in \mathbb{F}[\xi]$ and is equal (when counted with multiplicity) to*

$$d(m,p,q) = (-1)^{q(m+1)}(mp + q(m+p))! \sum_{n_1+\cdots+n_m=q} \frac{\prod_{k<j}(j-k+(n_j-n_k)(m+p))}{\prod_{j=1}^{m}(p+j+n_j(m+p)-1)!} \tag{7}$$

Note that the number $d(m,p,0)$ is equal to $d(m,p)$ as introduced in (5). As in the case of the static pole placement problem condition (6) is also sufficient if $d(m,p,q)$ is odd. If $d(m,p,q)$ is even the strongest known sufficiency result over the reals is:

Theorem 8 (Rosenthal-Wang [21]) *Let $\mathbb{F} = \mathbb{R}$ and assume that*

$$q(m+p-1) + mp - \min(r_m(p-1), r_p(m-1)) > n \tag{8}$$

where $r_m = q - m[q/m]$ and $r_p = q - p[q/p]$ are the remainders of q divided by m and p, respectively. Then the generic rank p system of McMillan degree n is arbitrary pole assignable in the class of real feedback compensators of degree at most q.

3 A list of open problems

3.1 Necessary and sufficient conditions for generic pole assignment over the reals

When the Schubert number $d(m,p)$ is even there is a difference of one degree of freedom between the sufficiency condition of Wang ($mp > n$) and the general necessary condition of Wang ($mp \geq n$). Moreover the following lemma states that in general $mp \geq n$ in not a sufficient condition for generic pole placement with real static compensators:

Lemma 9 (Willems and Hesselink [30]) *If $\mathbb{F} = \mathbb{R}$ and if $m = p = 2$ and $n = 4$ then the static pole placement problem is not generically solvable over the reals $\mathbb{R}$.*

It has been conjectured by S.-W. Kim that $m = p = 2$, $n = 4$ is the only case where $mp = n$ is not a sufficient condition for generic pole assignment with static compensators over the reals. This conjecture was disproved by Rosenthal and Sottile in [20] by exhibiting a concrete counter example in the situation where $m = 2$ and $p = 4$. It has been conjectured:

Conjecture 10 (Rosenthal-Sottile [20]) *If $d(m,p)$ is even and $n = mp$, then the static pole placement problem is not generically solvable over the reals $\mathbb{R}$.*

This conjecture is open. For the general dynamic problem much less is known and the gap between the general necessary condition (6) and the best known sufficiency condition (8) is in general quite wide and it can be as large as $(m-1)(p-1)$. It is a challenging task to further narrow this gap. In addition it would be of interest to have exact algebraic criterions which would allow one to determine the minimal order of a real compensator needed, which achieved arbitrary pole assignability over the reals. For the static pole placement problem such algebraic conditions have been recently derived in [20].

3.2 Numerical algorithms for pole placement

The sufficiency conditions in Theorems 3,6,7 and 8 are mainly theoretical in nature and one has to say that there are no good numerical algorithms available in many situations where we know the existence of a feedback compensator. Theorem 6 and 8 were derived by a technique called linearization around a 'dependent compensator'. In principle this technique can be used to actually compute feedback compensators and we refer to [21, 26] for more details.

Geometrically dependent compensators form the *base locus* (see [17]) of the associated pole placement map and the numerical paradigm should probably be to construct solutions which are as far as possible away from the base locus.

In principle it is possible to tackle any pole placement problem directly through the defining polynomial equations and applying methods like e.g. Groebner basis computations. Such an approach was taken in [20] but it is clear that computations will be limited to small dimensions.

In summary we consider it a challenge for the numerical analysts to derive stable numerical algorithms to solve pole placement problems.

3.3 The invariant polynomial assignment problem

The characteristic polynomial forms in general not the only invariant of an autonomous behavior. A finer set of invariants are the invariant factors. Note that two autonomous behaviors are isomorphic if and only they have the same invariant factors. This follows readily from the fact that the Smith forms of the polynomial matrices describing the autonomous behaviors have to be the same. It is therefore reasonable to ask for necessary and sufficient conditions which guarantee the assignment of a set of invariant factors for a system $\Sigma_1 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_1)$ having rank $r(\Sigma_1) = p$. We refer to [31] for more details.

3.4 Feedback stabilization versus pole placement

It has been shown in [7, 14] that if a system is not generically pole assignable then there exist an open set of systems (open in the Euclidean topology) which cannot be stabilized. Based on those results one might think that the study of pole assignability does cover the question of stabilizability. This is only true in part. We believe that it would be worthwhile to have results which allows one to describe the set of systems of a fixed McMillan degree which cannot be stabilized by regular feedback interconnections of compensators of degree at most q.

3.5 The problem of simultaneous pole placement and simultaneous stabilization

Simultaneous pole placement tries to answer the following question: Given a set of plants $\Sigma_i = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B}_i)$, $i = 1, \ldots, r$ degree $n(\Sigma_i) = n_i$ and having constant rank $r(\Sigma_1) = p$. Let ϕ_i be a set of polynomials of degree $n_i + q$. We are seeking a feedback compensator $\Sigma = (\mathbb{R}, \mathbb{R}^{m+p}, \mathbb{B})$ of degree q and rank $r(\Sigma) = m$ such that all feedback interconnections $\Sigma_i \wedge \Sigma$ are regular and the characteristic polynomial $\chi_{\Sigma_i \wedge \Sigma} = \phi_i$.

In the work of Ghosh [8] some sufficiency results were derived. Those results are however far from being optimal. We would like to note that the question is an algebraic problem and this question is fully decidable. This is different from the simultaneous stabilization problem studied by Blondel [4, 3].

3.6 The problem of decentralized pole placement

Let $P(\xi)$ describe a linear AR-system of rank p and McMillan degree n. The decentralized pole placement problem asks for the construction of a set of compensators $Q_1(\xi), \ldots, Q_r(\xi)$ such that a desired closed loop characteristic polynomial of the form

$$\phi(\xi) = \det \left[\begin{array}{cccc} \multicolumn{4}{c}{P(\xi)} \\ \hline Q_1(s) & 0 & \ldots & 0 \\ 0 & Q_2(s) & \ldots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & Q_r(s) \end{array} \right] \tag{9}$$

can be achieved. In [15] necessary and sufficient conditions over the complex numbers were derived. There are however no strong results for real pole assignment known. For the control of e.g. power systems a theory for the stabilization of a plant through decentralized feedback compensators would be desirable.

3.7 General matrix extension problems

It has been explained in [22] that many pole placement problems can be expressed as matrix extension problems. It would be desirable to have strong necessary and strong sufficient conditions available which cover a wide range of matrix extension problems. A first general result in this direction has been derived in [9].

3.8 General interpolation problems

The dynamic pole placement problem as described in subsection 2.2. is part of a wide range of general interpolation problems. This class of interpolation problems includes problems like the left or right tangential interpolation problem and the partial realization problem and the interested reader can find details in [1]. It is an interesting fact that those interpolation problems have recently attracted many researchers in conformal quantum field theory and in geometry in general and the reader should consult [13]. It would be desirable to have strong necessary and strong sufficient results both over the reals and over the complex numbers. Good numerical algorithms would be desirable as well.

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