Control as Interconnection*

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Abstract

This paper puts forward the idea of system interconnection as the central idea of control. It contrasts this with intelligent control, which refers to the usual measurement-to-control-action feedback type of control. As illustrations, stabilization and linear-quadratic control are treated from this vantage point.

1 Introduction.

The purpose of this essay is to question the universal appropriateness of the usual *signal flow graph*, input/output structure which is invariably taken as the starting point in control. We will argue that it is much more reasonable and pragmatic to view instead *interconnection* as the basic idea in control.

This paper is written in honor of George Zames at the occasion of his sixtieth birthday. I first met George in the mid-sixties when I was a beginning graduate student at MIT. My doctoral dissertation, which appeared in 1968, later expanded into the monograph [1], dealt with input/output stability. It was greatly influenced by Zames' seminal papers [2] and built on the ideas of $\mathcal{L}_2$-stability, the small loop gain theorem, and the positive operator theorem which he had laid out in [2]. One of the things which George's work taught me was to appreciate the importance of clear and elegant problem formulations (as $\mathcal{L}_2$-stability) and of general principles (as the small loop gain theorem). The present paper is written in this spirit.

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The usual approach to thinking about control design takes the feedback loop shown in figure 1 as its starting point. The control inputs drive the actuators, while the sensors produce the measured outputs. In this view, the aim of control theory is to design a feedback processor, i.e., a device that processes the observed outputs in order to compute the control inputs. Very powerful control design principles have emerged from this paradigm, combining identification and parameter estimation, observers and state reconstructors, motion planning enhanced with set point servo control and gain scheduling, etc.

Figure 1: Intelligent control

We will refer to the control scheme of figure 1 as an *intelligent controller.* Before elaborating, we would like to make a short comment on the use of the word *intelligent.* We use it because the controller of figure 1 involves *observation* (through the sensors), *analysis* (for example in the form of state and parameter estimation), *decision making* for example, in the form of an (adaptive) control algorithm), and *action* (through the actuators). Of course, this scheme could involve a very low level of intelligence (for example in a room thermostat). Without wanting to engage in the AI debate, we feel that for the purposes of this paper the use of the term *intelligent* is an appropriate one for clarifying the issues which will be discussed. In fact we use *intelligent* control in contrast to *passive* control.

2 Passive control.

Many, if not most, control devices used in engineering practice do not function as intelligent controllers. In many such devices it is unnatural to view the controller action as feedback, and to think of it in terms of a signal flow graph.

Examples of such situations abound: mechanical dampers to attenuate vibrations, commercial devices for temperature, flow and pressure control, passive elements introduced in electrical signal processing devices in order to improve frequency transfer characteristics, etc. When one analyzes such devices it is often impossible to regard one variable as being measured and used in order to decide what value another variable should take on. When a resistor is added in a circuit in order to improve the response characteristics, it makes no sense to view the current as input and the voltage as output, or vice-versa. To think of a damper, for example a damper in a car, as a device that *measures* the position (or velocity) of the body and the chassis, and *decides* from there what force to exert on the body of the car, is an anthropomorphic caricature (and an unnecessary one at that). It makes just a much (or better just as little) sense to turn the situation around and to view a damper as a device that measures the force exerted on the piston and decides how fast to make the piston move. In other examples, say a simple expansion value with a spring for pressure control, it would perhaps be possible to *think* of the pressure as the measured input and the extension of the spring as the control output. However, it is obvious that this signal flow is merely in our minds (and could hence be useful for enhancing our understanding of the situation), not in nature.

Our point of view will be the following. When a controller is attached to a plant there are some variables which interface between the controller and the plant. Before the controller is attached, these variables have to obey only the laws imposed by the plant. After the controller is put in place, they have to obey *both* the laws of the plant and of the controller. Through this intervention, the dynamic behavior of the plant variables is adjusted. This adjustment effects not only the interfacing variables, but, through these, also the other plant variables. This is important when the to-be-controlled variables are not available for interconnection. By properly choosing the controller one can achieve in this way desired dynamic characteristics for the interconnected system.

The plant and the controller may or may not have an input/output structure when viewed from the interfacing variables. When this input/output structure is present, then the intelligent control paradigm is suitable. When this input/output structure is not present, then it is not. The aim of this paper is to put forward a framework for control design which does not take the signal flow input/output structure as its starting point, but treats it as a special case.

In order to avoid misunderstandings, we would again like to emphasize that there are many situations where the intelligent control paradigm is eminently suitable. It is a deep and attractive paradigm which is basically a must whenever logic devices are involved in the controller. Thus it will undoubtedly gain in importance as logic devices become cheaper and more reliable and more intelligence can be incorporated in controllers. Nevertheless, it remains puzzling why control theory textbooks, those with a practical as well as those with a theoretical outlook, have, ever since the subject was formalized, chosen to work in an input/output framework. By regarding control this way, our subject can be viewed as a part of signal processing, instead of making it a part of integrated system design, of properly designing subsystems.

3 Control as interconnection.

The behavioral framework [3,4,5] provides a suitable setting for our purposes. One of the important features of this approach is that it does not take the usual input/output structure as its starting point. Instead, it lays out a framework for discussing dynamical systems in which all external system variables are treated on equal footing. However, it recognizes *ab initio* the importance in modelling of internal variables, auxiliary variables which unavoidably need to be introduced in the modelling process and which are different from the variables whose behavior the model aims at. These internal variables are called *latent variables*, while the variables which are being modelled are called *manifest* variables.

In control applications, it is natural to distinguish between those variables which are available for control and those which are not. Using again the above language we will call the variables available for control, *manifest* variables, and the remaining plant variables, *latent* variables. We can think of the manifest variables as *control variables*, and of the latent variables as *to-be–controlled* variables. This leads to the setting for control shown in figure 2.

Figure 2: Control as interconnection

However, for the sake of exposition we will first assume that *all* the plant variables are available to the controller. This leads to the situation shown in figure 3. This will now be formalized.
Let $\Sigma_p = (T, W, \mathfrak{B}_p)$ be the *plant*, viewed as a dynamical system (in the sense explained in [5]). Thus $T \subseteq \mathbb{R}$ denotes the time-axis. For the purposes of this

Figure 3: Control as interconnection

paper, in which we will consider for simplicity continuous-time systems, think $T = \mathbb{R}$. W denotes the space of plant variables. Think $W = \mathbb{R}^q . \mathfrak{B}_p$ is a subset of W^T, i.e., a subset of all maps from T to W. The (model of the) plant imposes certain laws on the plant variables. This results in the fact that some time function $w : T \to W$ (namely those in $\mathfrak{B}_p$) are declared possible by the plant model, while the others (those not in $\mathfrak{B}_p$) are declared impossible. $\mathfrak{B}_p$ is called the *behavior* of Σ_p.

The *controller* $\Sigma_c = (T, W, \mathfrak{B}_c)$ is, as Σ_p, a dynamical system. It imposes new laws on the variables: the functions $w : T \to W$ are now required to lie in $\mathfrak{B}_c$.

The *controlled* system Σ is the interconnection of Σ_p and Σ_c. This is denoted as $\Sigma = \Sigma_p \wedge \Sigma_c$. The *interconnection* is defined as

$$\Sigma = \Sigma_p \wedge \Sigma_c := (T, W, \mathfrak{B}_p \cap \mathfrak{B}_c) \tag{1}$$

In other words, in the interconnected system, the realizable trajectories $w : T \to W$ will have to obey both the laws imposed by the plant and by the controller. The control design problem is thus:
Given the plant Σ_p and given a family of admissible controllers $\mathfrak{C}$, find a $\Sigma_c \in \mathfrak{C}$ such that $\Sigma_p \wedge \Sigma_c$ has desirable properties.

4 Stabilization.

We will now explain, as an example, how the stabilization question for linear time-invariant systems can be formulated in this setting. Let $\mathcal{L}^q$ denote the family of linear differential systems with q variables. Thus each element of $\mathcal{L}^q$ is described through a polynomial matrix $R \in \mathbb{R}^{\bullet \times q}[\xi]$ by the differential equation

$$R(\frac{d}{dt})w = 0 \tag{2}$$

(2) yields the dynamical system $\Sigma = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B}) \in \mathcal{L}^q$, where $\mathfrak{B}$ consists of all $w : \mathbb{R} \to \mathbb{R}^q$ satisfying (2). The precise meaning of what it signifies that w satisfies a system of differential equations as (2), is not important for the purposes of this paper.

Call $\Sigma = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B}) \in \mathcal{L}^q$ *controllable* [5] if for all $w_1, w_2 \in \mathfrak{B}$ there exists $t' > 0$ and $w \in \mathfrak{B}$ such that $w(t) = w_1(t)$ for $t < 0$ and $w(t + t') = w_2(t)$ for $t \geq 0$. Call it *stabilizable* if for all $w \in \mathfrak{B}$ there exists $w' \in \mathfrak{B}$ such that $w(t) = w'(t)$ for $t < 0$ and such that $\lim_{t\to\infty} w'(t) = 0$. It can be shown that (2) is controllable [5] if and only if $rank(R(\lambda)) = rank(R)$ for all $\lambda \in \mathbb{C}$ and stabilizable if and only if this holds for all $\lambda \in \mathbb{C}^+ := \{\lambda \in \mathbb{C} | Re(\lambda) \geq 0\}$.

Next, call Σ *autonomous* if $w_1, w_2 \in \mathfrak{B}$ and $w_1(t) = w_2(t)$ for $t < 0$ imply $w_1 = w_2$. It can be shown that (2) is autonomous if and only if $rank(R) = q$. Finally, define Σ to be *stable* if $w \in \mathfrak{B}$ implies $\lim_{t\to\infty} w(t) = 0$. It is easy to see that stability requires autonomy. In describing systems $\Sigma \in \mathcal{L}^q$ by differential equations (2) (many R's will generate the same $\mathfrak{B}$!) we can always assume that R has q rows. If the number of rows in the original R is less than q, then this is trivial: simply add zero rows to R. If it is more than q then this fact requires an (easy) proof. Let $\Sigma \in \mathcal{L}^q$ and let it be described by (2), with $R \in \mathbb{R}^{q\times q}[\xi]$. Now define χ_Σ, the *characteristic polynomial* of Σ, by $\chi_\Sigma := det(R)$. Thus Σ is autonomous if and only if $\chi_\Sigma \neq 0$, and stable if and only if χ_Σ is Hurwitz.

Now consider a plant $\Sigma_p \in \mathcal{L}^q$. Assume that it is described by

$$P(\frac{d}{dt})w = 0 \tag{3}$$

Take as controllers elements of $\mathcal{L}^q$. A typical element is thus described by

$$C(\frac{d}{dt})w = 0 \tag{4}$$

Now restrict the class of admissible controllers to those for which

$$rank(\begin{bmatrix} P \\ C \end{bmatrix}) = rank(P) + rank(C) = q \tag{5}$$

Thus in this case $\mathfrak{C} = \{\Sigma_c \in \mathcal{L}^q | \Sigma_c$ can be described by (4), and (5) will be satisfied$\}$
We will return to condition (5) later. For the moment, treat it as a technical requirement. As far as stabilization and pole placement is concerned, the following results can be obtained:

1. *Let $\Sigma_p \in \mathcal{L}^q$. Then there exists $\Sigma_c \in \mathfrak{C}$ such that $\Sigma_p \wedge \Sigma_c$ is stable if and only if Σ_p is stabilizable.*

2. *Let $\Sigma_p \in \mathcal{L}^q$. Then the following conditions are equivalent:*

   (i) Σ_p is controllable
   (ii) For each monic polynomial $r \in \mathbb{R}[\xi]$, there exists a $\Sigma_c \in \mathfrak{C}$ such that $\chi_{\Sigma_p \wedge \Sigma_c} = r$

5 Implementation of controllers.

In intelligent control, it is usually taken for granted that every controller which processes the sensor measurements and delivers the actuator inputs is admissible. From an applications point of view, it is somewhat surprising that this very broad feasibility and implementability of intelligent controllers has not been questioned more often. In many practical circumstances it is difficult to see why and how such sophisticated devices should be used. Computer control is important, but it is not the whole picture.

In addition, there is usually some reference to causality which is sometimes formulated as implying the absence of a differentiating action. This last condition is then justified by referring to noise amplification which differentiating controllers may cause.

There are, however, many control devices which, if we insist in viewing them as input/output processors, will act as differentiators, but which cause no trouble at all. Take as an example a controller consisting of a mass/spring/damper combination which is attached to a given mass and serves to hold this given mass in a particular equilibrium, while achieving a gentle transient response. A traditional door closing mechanism is an example of a device which functions exactly in this way. Our point of view is that it is much better in such examples not to insist on an input/output interpretation, but simply take the interconnection point of view.

Nevertheless, the implementation issue is an important one. In our approach, it can simply not be avoided and should be incorporated in the specifications of $\mathfrak{C}$. In particular, the following question occurs: *if $\Sigma_p \in \mathcal{L}^q$ and we want to control it by means of a controller $\Sigma_c \in \mathcal{L}^q$, how would we achieve this?*

For example:

(i) Assume that the control terminals correspond to electrical terminals or terminals of a mechanical system, can the controller be realized using passive components?

(ii) Can the controller be implemented by means of an input/output device? In other words, can we choose some of the control variables and make them act as inputs to the controller, while the other variables act as outputs?

(iii) Is (ii) possible while making the transfer function of the controller proper? In this case the controller could in principle be implemented as an intelligent controller using logic devices.

(i) is a research question, for which we have obtained some partial results. We will not report them here. We can give complete answers to (ii) and (iii). However, before doing this, we need to introduce some integer invariants of $\mathcal{L}^q$. Define three integer valued maps on $\mathcal{L}^q$ as follows:

$$\begin{aligned} m: &\quad \mathcal{L}^q \rightarrow \{0,1,2,\ldots,q\} \\ p: &\quad \mathcal{L}^q \rightarrow \{0,1,2,\ldots,q\} \\ n: &\quad \mathcal{L}^q \rightarrow \{0,1,2,\ldots\} = \mathbb{Z}_+ \end{aligned}$$

Take $\Sigma \in \mathcal{L}^q$, let it be represented by (2), and assume (without loss of generality - see [5]) that R has full row rank. Define

$$\begin{aligned} m(\Sigma): &= q - rowdimension(R) \\ p(\Sigma): &= rowdimension(R) \\ n(\Sigma): &= McMillandegree(R) \end{aligned}$$

Recall that the *McMillan degree* of a full row rank polynomial matrix is the largest degree of its maximal size minors. In terms of minimal input/state/output representations, $m(\Sigma), p(\Sigma)$ and $n(\Sigma)$ can be given a very concrete interpretation. They correspond respectively to the number of input variables, the number of output variables, and the number of state variables of Σ [5].

Let $\Sigma_p \in \mathcal{L}^q$ and $\Sigma_c \in \mathcal{L}^q$, and assume that $\Sigma_p \wedge \Sigma_c$ is autonomous. Assume that

$$p(\Sigma_p) + p(\Sigma_c) = p(\Sigma_p \wedge \Sigma_c) = q \tag{6}$$

(note that condition (5) is precisely equivalent to this). Then it can be shown that the control variables can be partitioned in such a way that Σ_p and Σ_c have a complementary input/output structure, and with the transfer function in Σ_p proper, but that of Σ_c in general not proper. If we want the transfer function of Σ_c to be proper and the feedback system to be well posed, then we need also

$$n(\Sigma_p) + n(\Sigma_c) = n(\Sigma_p \wedge \Sigma_c) \tag{7}$$

For a precise statement see [6]. Generalizations to the case that $\Sigma_p \wedge \Sigma_c$ is not autonomous are also possible.

6 LQ-control.

Let $\Sigma \in \mathcal{L}^q = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B})$ and assume for simplicity that it is controllable. Let (2) be a representation of it. Assume, to avoid smoothness difficulties, that $\mathfrak{B}$ consists of the C^∞ solutions of (2). As in [5] we will call this a *kernel representation* of Σ. Consider also the two-variable polynomial matrix $L \in \mathbb{R}^{q\times q}[\zeta, \eta]$ and assume that it is symmetric, i.e., $L(\zeta, \eta) = L^T(\eta, \zeta)$. L induces the quadratic differential form $Q_L : C^\infty(\mathbb{R}, \mathbb{R}^q) \to C^\infty(\mathbb{R}, \mathbb{R})$ defined by

$$Q_L(w) := \sum_{k,\ell} (\frac{d^k w}{dt^k})^T L_{k\ell} (\frac{d^\ell w}{dt^\ell}) \tag{8}$$

where

$$L(\zeta, \eta) =: \sum_{k,\ell} L_{k\ell} \zeta^k \eta^\ell \tag{9}$$

Consider now the following optimization problem: *Determine* $\mathfrak{B}^* \subseteq \mathfrak{B}$ *such that* $w^* \in \mathfrak{B}^*$ *implies*

(i) (*stability*) $\lim_{t\to\infty} w^*(t) = 0$

(ii) *(optimality) for all* $\Delta \in \mathfrak{B}$ *of compact support, there should hold:*

$$\int_{-\infty}^{+\infty} (Q_L(w^* + \Delta) - Q_L(w^*))dt \geq 0 \tag{10}$$

View (10) as a question of determining the optimal trajectories of the system (2) with the cost functional $\int Q_L(w)dt$. Note that this formulation of the *LQ*-problem departs radically from the classical formulation: we do not start from a state model, there are no specified initial conditions, no inputs, the cost functional may contain higher order derivatives.

The optimal behavior $\mathfrak{B}^*$ can be characterized as follows. It is non-empty (equivalently, $0 \in \mathfrak{B}^*$) if and only if there exists a polynomial matrix $X \in \mathbb{R}^{q\times \bullet}[\xi]$ such that

$$L(-i\omega, i\omega) + X^T(-i\omega)R(i\omega) + R^T(-i\omega)X(i\omega) \geq 0 \tag{11}$$

for all $\omega \in \mathbb{R}$. Assume that this is the case with the left hand side of (11) of rank $m(\Sigma)$ (regard this as a technical condition, similar to observability in the classical formulation). Then $\mathfrak{B}^*$ can be computed as follows: find $X \in \mathbb{R}^{q\times\bullet}[\xi]$ and $C \in \mathbb{R}^{m(\Sigma)\times q}$ such that

$$L(-\xi, \xi) + X^T(-\xi)R(\xi) + R^T(-\xi)X(\xi) = C^T(-\xi)C(\xi) \tag{12}$$

and such that

$$det(\begin{bmatrix} R \\ C \end{bmatrix}) \text{is Hurwitz} \tag{13}$$

Proofs and further details will be given in [7]. This way of obtaining $\mathfrak{B}^*$ is reminiscent of the spectral factorization approach to LQ control as explained for example in [8, section 26]. Equation (12) is a complete generalization of the Riccati equation.

Note that our way of thinking about optimal control *comes up with* $\mathfrak{B}^*$*, the optimal behavior, not with the optimal controller* ! The question thus remains how to implement the closed loop system $\Sigma^* = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B}^*)$. We will take this up in the next section.

7 Implementation of controlled behavior.

Let $\Sigma_p = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B}_p) \in \mathcal{L}^q$ be a plant. A system $\Sigma = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B}) \in \mathcal{L}^q$ is said to be a *subsystem* of Σ_p if $\mathfrak{B} \leq \mathfrak{B}_p$. A controller $\Sigma_c = (\mathbb{R}, \mathbb{R}^q, \mathfrak{B}_c)$ is said to *implement* Σ if

$$\Sigma = \Sigma_p \wedge \Sigma_c \tag{14}$$

This implementation problem is very akin to what, particularly in the Russian literature, is called a *synthesis*. Note that if we do not restrict Σ_c, then it is trivial to solve this problem: take $\Sigma_c = \Sigma$. The problem becomes interesting when Σ_c is further constrained, for example to be passive, or such that in addition to (14), (6), or (6) and (7) hold.

The one case where we have rather complete results is specifying when (14) is implementable with a controller satisfying (6). The result says that if $\Sigma \in \mathcal{L}^q$ is *any* subsystem of a controllable system $\Sigma_p \in \mathcal{L}^q$, then there exists a $\Sigma_c \in \mathcal{L}^q$ such that (14) and (6) hold. Using the results mentioned in section 5, this implies that Σ can be implemented using a controller with an input/output structure which is complementary (in the sense that the input/output structure of the plant and the controller go in opposite directions) to that of the input/output structure of the plant but with a transfer function which may not be proper. Thus any subsystem of a controllable system is implementable using an improper controller of feedback type.

Let us now return to the optimal LQ controller of section 6. The algorithm given there yields the optimal behavior $\mathfrak{B}^*$, i.e., the family of all optimal trajectories. The question is how to implement $\mathfrak{B}^*$ by means of a controller. Let $\Sigma^* :=$ $(\mathbb{R}, \mathbb{R}^q, \mathfrak{B}^*)$. Obviously $\Sigma^* \in \mathcal{L}^q$. Then every time someone proposes a controller Σ_c, we can check whether $\Sigma^* = \Sigma_p \wedge \Sigma_c$. Note that this departs from the usual situation in LQ control, since, while our $\mathfrak{B}^*$ is unique, there will be many $\Sigma_c \in \mathcal{L}^q$ such that

$$\Sigma_p \wedge \Sigma_c = \Sigma^* \tag{15}$$

In particular it can be shown that since Σ_p is controllable, there exists a Σ_c such (15) and (6) hold. In addition, it can be shown that if the 2-variable polynomial matrix L is of degree 0 (i.e., if it is a constant), then Σ_c can be taken to be a memoryless state controller.

We close this section by making the remark that the non-uniqueness of the controller Σ_c which achieves (15) has important practical consequences, since it shows that optimal control can be achieved by means of controllers which in the traditional point of view would not be admissible controllers.

8 Control of latent variables.

We will now return to the situation of figure 2, in which not all the plant variables are available to the controller. Assume again that the plant is described by a constant coefficient linear differential equation. This yields

$$R(\frac{d}{dt})w = M(\frac{d}{dt})\ell \tag{16}$$

as the plant model, with $w = col(w_1, \ldots, w_q)$ the manifest variables (the variables available to the controller) and $\ell = col(\ell_1, \ldots, \ell_d)$ the latent variables (the to-be-controlled variables). The question which we will discuss is what controlled behavior can be achieved by a controller

$$C(\frac{d}{dt})w = 0 \tag{17}$$

which acts on the manifest variables only.

We will call (16) *observable* [5] if ℓ can be deduced from w, i.e., if whenever (w, ℓ_1) and (w, ℓ_2) satisfy (16), then $\ell_1 = \ell_2$ must hold. It can be shown [5] that (16) is observable if and only if the complex matrix $M(\lambda)$ has full column rank for all $\lambda \in \mathbb{C}$. In this case (16) can equivalently, in the sense that (16) and (18) have the same solutions (w, ℓ), be described by a system of differential equations of the form

$$\ell = M'(\frac{d}{dt})w \tag{18a}$$

$$R'(\frac{d}{dt})w = 0 \tag{18b}$$

for suitable polynomial matrices R', M'.

Now, assume that a controller is designed for (16) which disregards the fact that only the w's are available for interconnection:

$$C_1(\frac{d}{dt})w + C_2(\frac{d}{dt})\ell = 0 \tag{19}$$

Then, using (18a), it easily follows that the alternative controller

$$(C_1 + C_2 M')(\frac{d}{dt})w = 0 \tag{20}$$

will achieve the same controlled behavior. In other words, the solutions of (16, 19) will be the same as those of (16, 20). It follows that *for an observable system* any controlled behavior which can be achieved by a controller using the (w, ℓ)'s as control variables, can also be achieved by a controller using only the w's as control variables. We remark, however, that the observability requirement is more severe than it might appear at first sight. For example, the usual situation of additive noise in the measurements already obstructs observability.

9 Epilogue.

In this paper we have put forward a theory of control in which system *interconnection* is the central idea. This in contrast with what we have called *intelligent* control, where we can view a controller as a signal processor, processing the measurements in order to compute the control action. This last type of controller is actually a special type of interconnection. In fact, the issue of what interconnections can be implemented by means of an intelligent controller comes up naturally and was briefly discussed in the present paper.

Ever since control theory was established as a scientific discipline, it has chosen to formalize its questions in an input/output setting. This can be observed in older and in more recent texts alike [8, 9,10,11]. Also mathematical system theory, which can be seen as an outgrowth of control theory, has invariably adopted the input/output framework. This may be seen, for example, from the attempts at axiomatization in [1,12].

One thing is clear: as a basic structure for modelling dynamical systems, the input/output framework is unsuitable. In an off-the-shelf modelling package, modules (standard elements) and interconnections (standard ways of interconnecting standard elements) are the key components: modelling proceeds by *tearing* (examining the interconnections) and *zooming* (examining the subsystems) in a hierarchical fashion. Physical components (resistors, capacitors, transformers, masses, spring, dampers, etc, etc.) will be specified by giving their parameter values. The specification of the system architecture will tell how the components are inserted in the system. As such, it is unnecessary, awkward, and illogical to specify the components in input/output form. It may, but need not be the case that the overall system needs to be specified in input/output form, for example because of the presence of logic devices in the system. This input/output structure will have its effect on the components and may translate into an input/output structure on a particular component. However in what input/output structure this particular component will function will depend on the architecture. It is because of such considerations that modelling packages based on simple interconnection ideas (as SPICE) are bound to be much more usable that those based on input/output thinking (as MATLAB's SIMULINK).

True, many of the issues underlying the behavioral framework have been touched on before. The limitations of input/output thinking is very clearly addressed in books on circuit theory [13,14]. Also Zames' original papers [2] on $\mathcal{L}_2$-stability work with input/output relations, rather than with maps (as in [1]). This input/output *relation* point of view was even more strongly emphasized by Safonov [15], where, however, the for-that-time-radical-step of dropping inputs and outputs altogether was not taken. The need to work with both external and internal variables is one of the key ingredients in the state space description of dynamical systems. The feeling that the state is but a limited implementation of this need for involving internal variables in models is what led Rosenbrock to introduce (always in an input/output setting) the *partial state* [16]. We may view this as a precursor of our latent variables. Also descriptor system formulations of state models (implicit systems, singular systems) point to discomfort with the usual input/state/output approach. Indeed, anyone who examines the suitability of input/output structures for models obtained from first principles will discover that it is unreasonable to take $\frac{dx}{dt} = f(x,u); y = f(x,u)$ as the starting point for dynamics.

The fact that in optimal control, it is sometimes convenient to first define the optimal behavior and the proceed to find the optimal controller is implicit in Brockett's approach [8, section 26].

Finally, the fact that it is the solution set of equations and not the equations themselves that are important in modelling underlies the system representation questions, from co-prime factorizations to Hankel matrices to the state space isomorphism theorem.

Whenever an axiomatic framework, as the behavioral setting, is put forward, whenever its effectiveness is argued and compared to an existing framework, as the input/output setting, it is unavoidable that there will be countless links with older work. It is unavoidable that many researchers will recognize their own discomforts. The merit of the behavioral framework has been in bringing the appropriate framework to the foreground explicitly. Mathematics is mainly discovery of existing structures, and inventing the language to discuss them in.

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