Tensor-Based Signal Processing

Lieven De Lathauwer

BioTensors Closing Workshop

WP 1–4

WP1: Prior knowledge incorporation in tensor decomposition

WP2: Updating tensor decompositions

WP3: Coupling tensor decompositions

WP4: Software platform for tensor based biomedical blind source separation

Some "other" team contributions

- Otto Debals: tensorization, large-scale NMF ($\rightarrow$ SeLMA WP3A)

$$\mathbf{M} = \mathbf{A} \cdot \mathbf{S}^T \qquad \mathbf{a}_r \text{ and/or } \mathbf{s}_r \text{ NN poly}$$

- Xiao-Feng Gong: double coupled tensor decompositions and blind source separation/independent vector analysis ($\rightarrow$ visit July 1–15, 2019)

$$\mathbf{x}^{(m)}(t) = \mathbf{A}^{(m)} \cdot \mathbf{s}^{(m)}(t) \quad \Rightarrow \quad \mathcal{T}^{(m,n)} = \left[\!\left[\mathbf{A}^{(m)}, \mathbf{A}^{(n)*}, \mathbf{C}^{(m,n)} \right]\!\right]$$

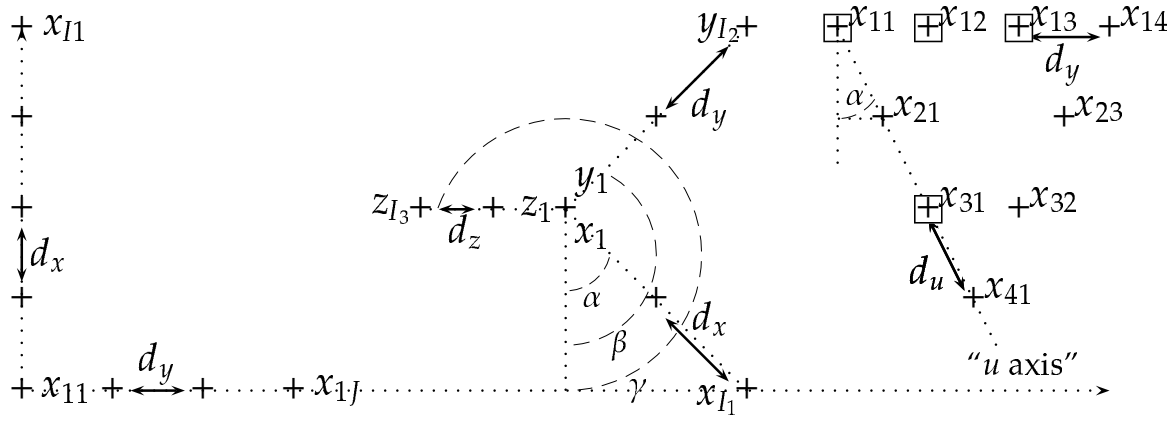
- Chuan Chen: factor analysis

$$\begin{array}{rcl} \mathbf{x} & = & \mathbf{A} \cdot \mathbf{s} + \mathbf{n} \qquad \mathbf{A} \in \mathbb{C}^{I \times R} \\ E\{\mathbf{n}\mathbf{n}^T\} = \sigma_N^2 \mathbf{I} & \longrightarrow & E\{\mathbf{n}\mathbf{n}^T\} = \mathrm{diag}(\sigma_{n_1}^2, \sigma_{n_2}^2, \ldots) \\ R \leqslant I - 1 & & R < I - 1 \\ \text{matrix EVD} & & \text{MLSVD-like extension} \end{array}$$

- Frederik Van Eeghem: convolutive extensions of ICA, tensor similarity

- Mikael Sørensen: array processing, coupled decompositions, MHR, nonuniform arrays and multirate sampling, fiber sampled CPD

  Coupled CPD and structured arrays:



L-shape Y-shape Triangular

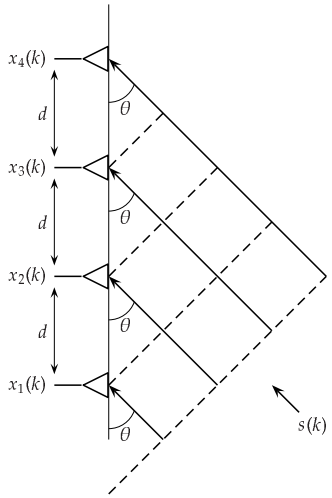
$$\mathbf{Y}^{(1)} = \mathbf{M}^{(1)} \cdot \mathbf{S}^T \qquad \ldots \qquad \mathbf{Y}^{(N)} = \mathbf{M}^{(N)} \cdot \mathbf{S}^T$$

$$\mathcal{Y}^{(1)} = \sum_r \mathbf{a}_r^{(1)} \otimes \mathbf{b}_r^{(1)} \otimes \mathbf{s}_r \qquad \mathcal{Y}^{(N)} = \sum_r \mathbf{a}_r^{(N)} \otimes \mathbf{b}_r^{(N)} \otimes \mathbf{s}_r$$

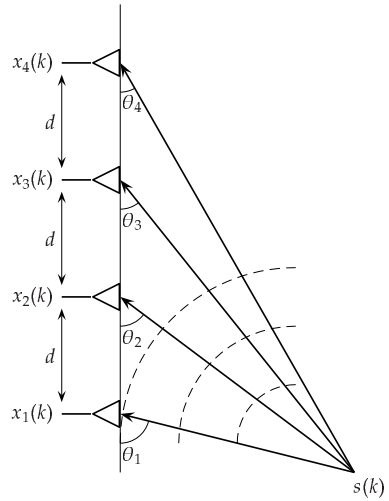
Uniqueness, reduction to GEVD (cf. ESPRIT), optimization (init)

Sources in near field:

Far field:



Near field:



Array in near field $\sim$ superposition of smaller arrays in far field

CCPD for smaller arrays, source signals in common

(spline-type approach)

- Eric Evert: CPD existence, uniqueness despite noise

$$\mathcal{T}^{(exact)} = [\![\mathbf{A}, \mathbf{B}, \mathbf{C}]\!] \qquad \text{rank}(\mathbf{A}) = \text{rank}(\mathbf{B}) = \text{rank}(\mathcal{T}) = R \qquad \forall r : \|\mathbf{c}_r\| = 1$$

$$\|\mathcal{T}^{(exact)} - \mathcal{T}^{(obs)}\|_F < \frac{1}{2}\sigma_{min}(\mathbf{A})\sigma_{min}(\mathbf{B}) \min_{r \neq s}(|\sin(\mathbf{c}_r, \mathbf{c}_s)|)$$

- Alwin Stegeman: sets of polynomial equations

$$\left\{ \begin{array}{rcl} y & = & a_1 x + b_1 \approx 0 \\ & \vdots & \\ y & = & a_n x + b_n \approx 0 \end{array} \right. \longrightarrow \left\{ \begin{array}{rcl} y & = & p_1(x) \approx 0 \\ & \vdots & \\ y & = & p_n(x) \approx 0 \end{array} \right.$$

roots $p(x) =$ eigenvalues companion matrix
$\rightarrow$ RQI, SVD-based computation

- Patrick Kürschner: sets of multivariate polynomial equations

$$\left\{ \begin{array}{rcl} p_1(x_1, \ldots, x_m) & \approx & 0 \\ & \vdots & \\ p_n(x_1, \ldots, x_m) & \approx & 0 \end{array} \right.$$

$\rightarrow$ CPD/BTD-based computation

Workshops and education

- TDA 2016, Leuven
- ICIAM 2019, Valencia, Spain, July 15–19: 6-part minisymposium (24 talks)(with A. Uschmajew, K. Usevich)
- Eusipco 2019, A Coruña, Spain, Sept. 2–6: double session (with SVH)

- Winter School on Tensor Decompositions and Blind Signal Separation, Jan. 2016, Leuven
- Graduate School Tensor Decompositions and (Large-Scale) Applications, Jan. 2017, Leuven (with N. Sidiropoulos, SOCN)
- EURASIP Summer school on Tensor-Based Signal Processing, Aug. 2018 (with SVH)

- Survey paper IEEE Signal Processing Magazine, March 2015 (with A. Cichocki, D. Mandic)(SPS best paper award)
- Invited overview paper IEEE TSP, July 2017 (with N. Sidiropoulos)(32 p. + 12 p. suppl.)(most popular)

Other projects

- FWO project Block Term Decompositions (with M. Van Barel)(ended)
- FWO project Tensor-Based Similarity (with JS)(ended)
- ERC-SG Interdiffusion (promoter: Nele Moelans)
- C1 project n-D (with BDM)
- EOS SeLMA (LDL, BDM, PP, Van Barel; Markovsky, Ishteva (VUB); Gillis (UMons); Absil, Glineur (UCL))

Research

General theme: Tensor tools for mathematical engineering:

- algebraic foundations
- numerical algorithms
- signal processing/data analysis concepts
- specific applications