Virtual Control Lab: Inverted Pendulum

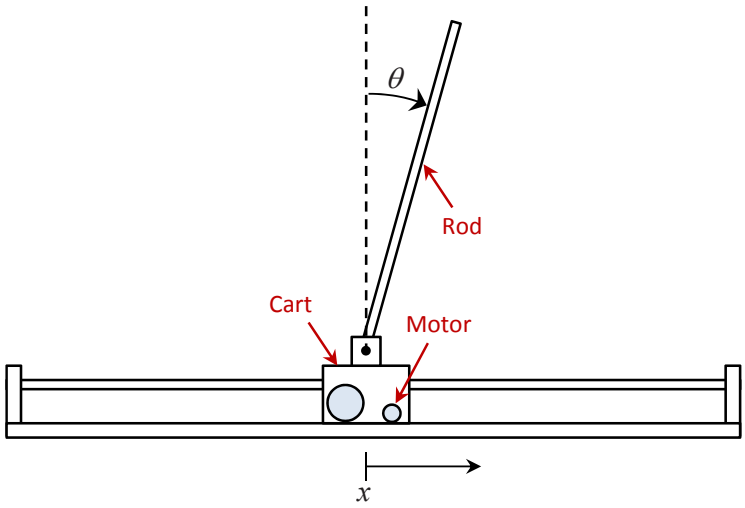

Figure 1: Inverted pendulum setup.

1 Brief description

This setup consists of a rod (the pendulum), mounted on a cart which slides on a shaft (see Figure 1). The cart is equipped with a DC motor, which allows the cart to move horizontally. The **control goal** is to move the cart from one desired position to another while keeping the rod upright.

2 Linear model

The linear state-space model of the system is given by $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$,

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4.5231 & -16.8835 & 0 \\ 0 & 46.9609 & 55.3557 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3.7778 \\ -12.3862 \end{bmatrix} u$$

where $\mathbf{x} = [x, \theta, \dot{x}, \dot{\theta}]^T$ is the state vector, x is position of the cart in [m], θ is the angle of the rod in [rad], $\dot{x}$ is the velocity of the cart in [m/s], $\dot{\theta}$ is the angular velocity of the rod in [rad/s] and u is the voltage in [v] applied to DC motor attached to the cart. This model was obtained after linearizing the nonlinear differential equations of the system around $\theta = 0$.

2.1 Physical Constraints

$$
\begin{aligned}
-0.45\,\mathrm{m} \leq x \leq 0.45\,\mathrm{m} \\
-\frac{\pi}{2}\,\mathrm{rad} \leq \theta \leq \frac{\pi}{2}\,\mathrm{rad} \\
-5\,\mathrm{v} \leq u \leq 5\,\mathrm{v}
\end{aligned}
$$

3 Control Law

The control law currently implemented in the Lab is the following one:

$$u = -\mathbf{K}\left(\mathbf{x} - \mathbf{x}_{\mathrm{d}}\right)$$

where $\mathbf{x}_{\mathrm{d}} = [x_{\mathrm{d}}, 0, 0, 0]^T$ is the desired state vector, x_{d} is the desired cart position in [m], and $\mathbf{K} = [k_1, k_2, k_3, k_4]$ is the controller gain.

4 Additional info

For more info about the virtual control labs you can contact Dr. Oscar Mauricio Agudelo (mauricio.agudelo@esat.kuleuven.be) or Prof. Bart De Moor (bart.demoor@esat.kuleuven.be).