Session 3: Reference introduction

- LQR and Kalman again, but with
  - Reference introduction (chapter 8)
  - Robustness? ⇒ Integral control
  - Black box system identification

    (vs. linearization of physical model equations)
- Wanted ss vals for u and x are u_{ss} and x_{ss}:

  $$u = u_{ss} - \mathbf{K}(x - x_{ss})$$
- Values for which $y_{ss} = r_{ss}$ for any r_{ss}?

- Assume step reference

$$x_{ss} = Ax_{ss} + Bu_{ss}$$

$$r_{ss} = Cx_{ss} + Du_{ss}$$

Let $x_{ss} = N_x r_{ss}$ and $u_{ss} = N_u r_{ss}$, then

$$\begin{bmatrix} A-I & B \\ C & D \end{bmatrix} \begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

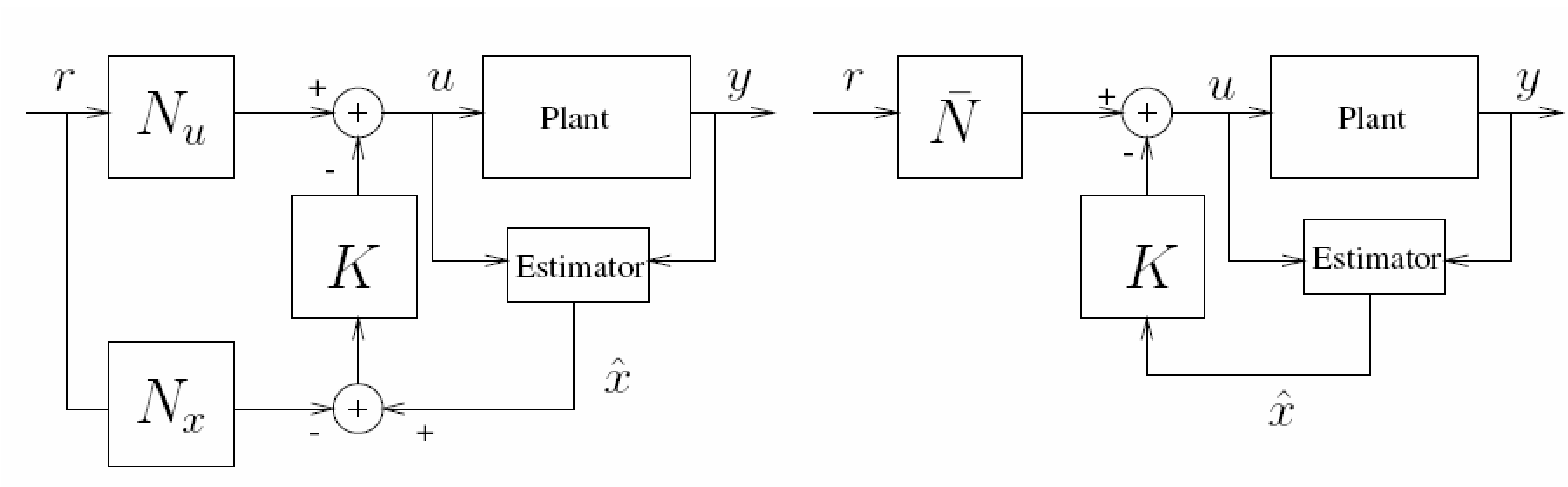
$$\Rightarrow$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A-I & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$u_k = N_u r_k - K(\hat{x}_k - N_x r_k) = -K\hat{x}_k + \underbrace{(N_u + KN_x)}_{\bar{N}} r_k$$

$$\text{Type I:} \quad u_k = N_u r_k - K(\hat{x}_k - N_x r_k)$$

$$\text{Type II:} \quad u_k = -K\hat{x}_k + \underbrace{(N_u + KN_x)}_{\bar{N}} r_k$$



More robust to parameter errors!

$$\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} B\bar{N} \\ B\bar{N} \end{bmatrix} r_k$$

$$y_k = \begin{bmatrix} C & -DK \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + D\bar{N} r_k.$$

Extra w.r.t page 194

If stable ⇨

$$x_{ss} = Ax_{ss} - BK\hat{x}_{ss} + B\bar{N}r_{ss}$$

$$\hat{x}_{ss} = x_{ss}$$

$$y_{ss} = (C - DK)x_{ss} + D\bar{N}r_{ss}$$

$$u_{ss} = -Kx_{ss} + \bar{N}r_{ss}$$

$$\Downarrow$$

$$y_{ss} = Cx_{ss} + D(-Kx_{ss} + \bar{N}r_{ss}) = Cx_{ss} + Du_{ss} = r_{ss}$$

$$\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A-BK-LC \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} B\bar{N} \\ M \end{bmatrix} r_k,$$

$$y_k = \begin{bmatrix} C & -DK \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + D\bar{N} r_k.$$

Introduce freedom

If $M = B\bar{N}$

$\Rightarrow$ $\tilde{x}_{k+1} = (A - LC)\tilde{x} + B\overline{N} r_k - M r_k = (A - LC)\tilde{x}$

error dynamics independent of reference input!

Introducing references does not change poles,
BUT it does change zeros! (p 225)

- $M = B\bar{N}$
  - T Zeros from r to u cancel out poles of state estimator (p 227)
  - Uncontrollability of estimator modes because of this pole zero cancellation
- $\bar{N} = 0,\ M = -L$
  - Tracking error estimator (p 228)
  - No way to choose zeros (fixed if K, L fixed)
- Zero assignment estimator (p 230)
  - Place zeros cf. pole placement

So far, based reference introduction on wish to make steady state error zero..

BUT the result is not robust: any change in parameters results in steady state error!

→ **Error space approach**, in particular **Integral Control** with additional integrator states! However, the augmented system is not always stabilizable (p 237)