DSP-CIS

Part-I : Introduction

Chapter-3: Acoustic Modem Project

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Chapter-3: Acoustic Modem Project

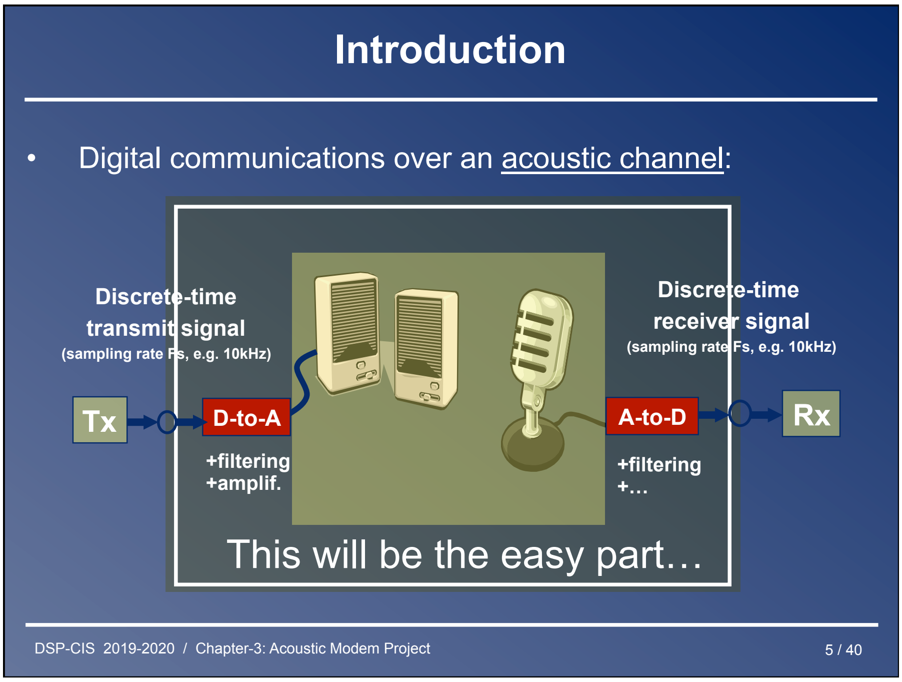
- **Introduction**
  Overview & Target
- **Work Plan**
  Week-1
  Week-2: Channel modeling & evaluation
  Week 3-4: OFDM modulation
  Week 5-6
  Week 7-8

Introduction/Overview

– Digital communication over an acoustic channel (from loudspeaker to microphone)
– FFT/IFFT-based modulation format : OFDM (as in ADSL/VDSL, WiFi, DAB, DVB…)
– Channel estimation, equalization, etc…

Introduction

- Digital communications over an acoustic channel:


Introduction
Digital communications over an acoustic channel:
Discrete-time transmit signal
(sampling rate Fs, e.g. 10kHz)
Tx
D-to-A
+filtering
+amplif.
Discrete-time receiver signal
(sampling rate Fs, e.g. 10kHz)
A-to-D
+filtering
+…
Rx
This will be the easy part…
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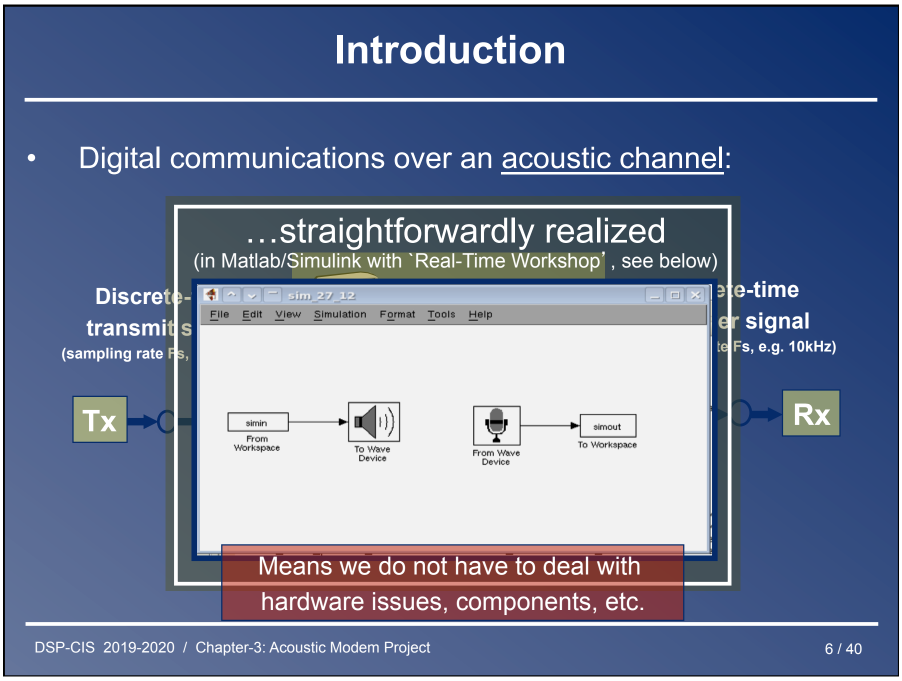
Introduction
Digital communications over an acoustic channel:
…straightforwardly realized
(in Matlab/Simulink with `Real-Time Workshop', see below)
sim_27_12
File Edit View Simulation Format Tools Help
simin
From Workspace
To Wave Device
From Wave Device
simout
To Workspace
Tx
Rx
Means we do not have to deal with hardware issues, components, etc.
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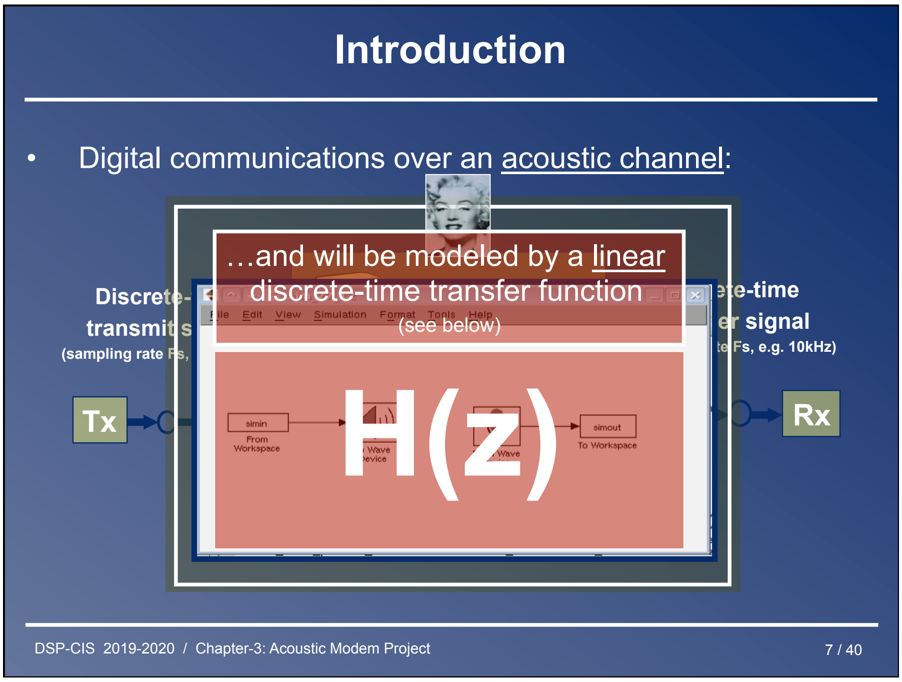
Introduction
Digital communications over an acoustic channel:
…and will be modeled by a linear discrete-time transfer function
(see below)
H(z)
Tx
Rx
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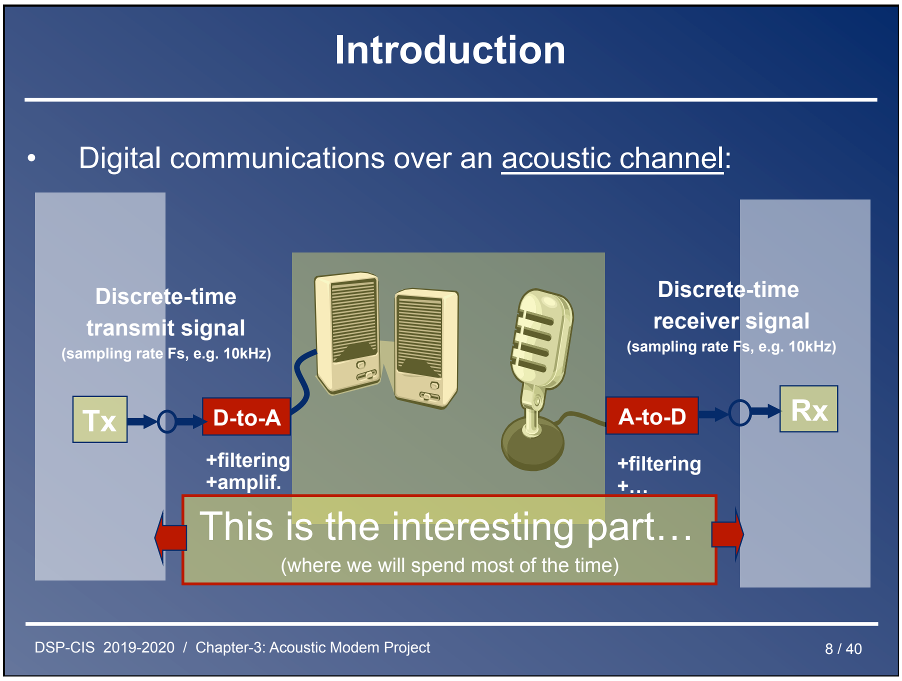
Introduction
Digital communications over an acoustic channel:
Discrete-time transmit signal
(sampling rate Fs, e.g. 10kHz)
Discrete-time receiver signal
(sampling rate Fs, e.g. 10kHz)
Tx
D-to-A
+filtering
+amplif.
A-to-D
+filtering
+…
Rx
This is the interesting part…
(where we will spend most of the time)
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Introduction

- Will use OFDM as a modulation format

Orthogonal frequency-division multiplexing
From Wikipedia, the free encyclopedia

Orthogonal frequency-division multiplexing (**OFDM**), essentially identical to (…) **discrete multi-tone modulation** (**DMT**), is a frequency-division multiplexing (FDM) scheme used as a digital multi-carrier modulation method. A large number of closely-spaced orthogonal sub-carriers are used to carry data. The data is divided into several parallel data streams or channels, one for each sub-carrier. Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation or phase-shift keying) at a low symbol rate, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth. OFDM has developed into a popular scheme for wideband digital communication, whether wireless or over copper wires, used in applications such as digital television and audio broadcasting, wireless networking and broadband internet access.

OFDM/DMT is used in ADSL/VDSL, WiFi, DAB, DVB …
OFDM heavily relies on DSP functionalities (FFT/IFFT, …)

Introduction

Target:
Design efficient OFDM based modem (Tx/Rx)
for transmission over acoustic channel

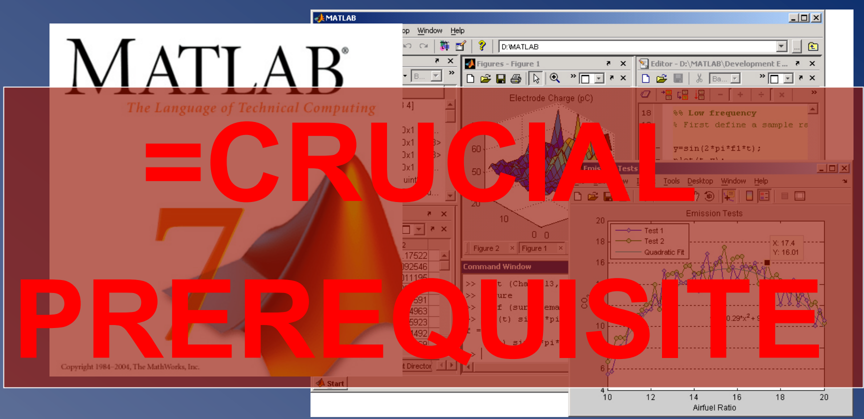
Specifications:
Data rate (e.g. 1kbits/sec), bit error rate (e.g. 0.5%), channel tracking speed, synchronisation, …

Work Plan

8 Weeks:

- Week 0: Introduction Matlab/Simulink
- Week 1: Audio playback, recording and analysis
- Week 2: Acoustic channel measurement & modeling
  deliverable
- Week 3-4: OFDM transmitter/receiver design
  deliverable
- Week 5-6: OFDM over acoustic channel
  deliverable
- Week 7-8: OFDM with adaptive equalization
  deliverable

Week 0 / Introduction to Matlab & Simulink

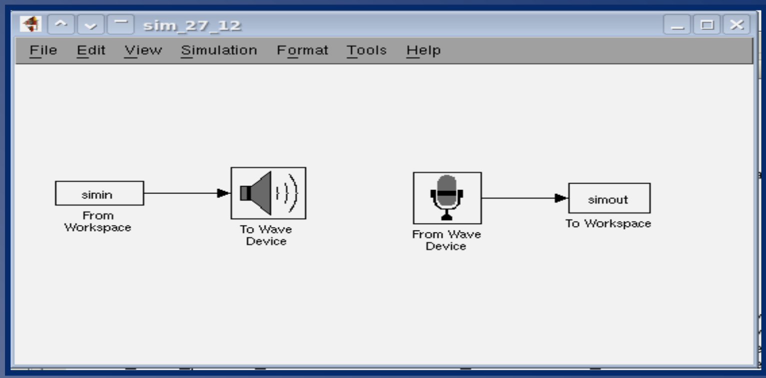


Matlab tutorial provided..

Self-test = exercise 6 (IF 'failure',THEN 'brush up your Matlab skills!')

Week 1 / Audio playback, recording and analysis

Will provide basic Simulink scheme… (`Real-Time Workshop’)



✪Time-frequency analysis of recorded signals

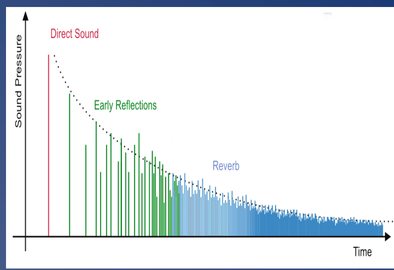
Week 2 / Channel Modeling & Evaluation

Transmission channel consist of

- Tx `front end’ : Digital-to-Analog conv./filtering/amplification
- Loudspeaker (ps: cheap loudspeakers mostly have a non-linear characteristic ☹)
- <u>Acoustic channel</u>
- Microphone
- Rx `front end’ : filtering/Analog-to-Digital conv.

Week 2 / Channel Modeling & Evaluation

Acoustic channel (`room acoustics'):
Acoustic path between loudspeaker and microphone is represented by the **acoustic impulse response** (which can be recorded/measured)



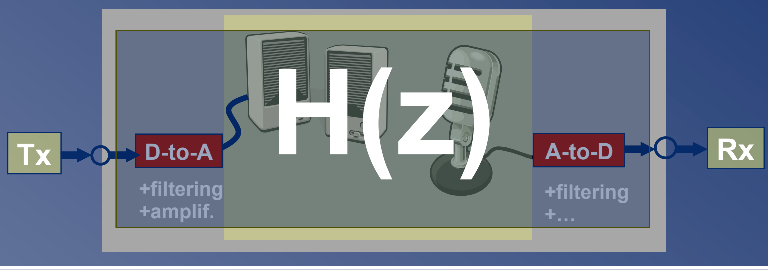
- First there is a dead time
- Then come the direct path impulse and some early reflections, which depend on the geometry of the room
- Finally there is an exponentially decaying tail called reverberation, corresponding to multiple reflections on walls, objects,...

Week 2 / Channel Modeling & Evaluation

Complete transmission channel will be modeled by a discrete-time (FIR `finite impulse response') transfer function

$$H(z) = h_0 + h_1.z^{-1} + h_2.z^{-2} + ... + h_L.z^{-L}$$

- Pragmatic & good-enough approximation
- Model order L depends on sampling rate (e.g. L=100…1000…)



PS: will use shorthand notation here, i.e. h_k, x_k, y_k , instead of h[k], x[k], y[k]

Week 2 / Channel Modeling & Evaluation

When a discrete-time (Tx) signal x_k is sent over a channel…

$$H(z) = h_0 + h_1.z^{-1} + h_2.z^{-2} + \ldots + h_L.z^{-L}$$

..then channel output signal (=Rx input signal) y_k is

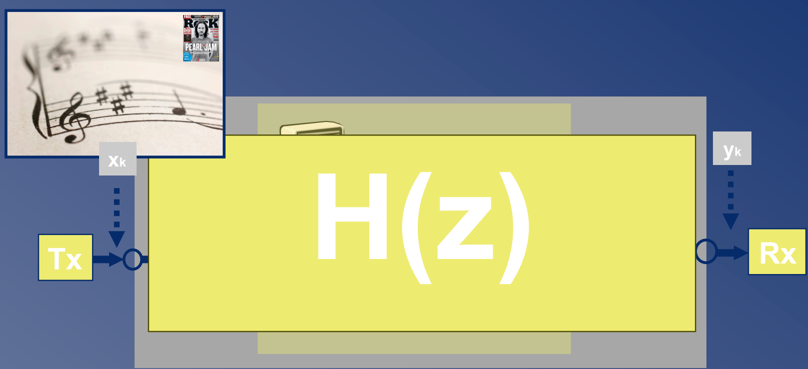
$$\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+K} \end{bmatrix} = \begin{bmatrix} x_k & x_{k-1} & x_{k-2} & \cdots & x_{k-L} \\ x_{k+1} & x_k & x_{k-1} & \cdots & x_{k+1-L} \\ x_{k+2} & x_{k+1} & x_k & \cdots & x_{k+2-L} \\ x_{k+3} & x_{k+2} & x_{k+1} & \cdots & x_{k+3-L} \\ x_{k+4} & x_{k+3} & x_{k+2} & \cdots & x_{k+4-L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{k+K} & x_{k+K-1} & x_{k+K-2} & \cdots & x_{k+K-L} \end{bmatrix} \cdot \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix}$$

=`convolution'

Week 2 / Channel Modeling & Evaluation

Can now run parameter estimation experiment:

1. Transmit your favorite signal x_k
2. Record corresponding signal y_k

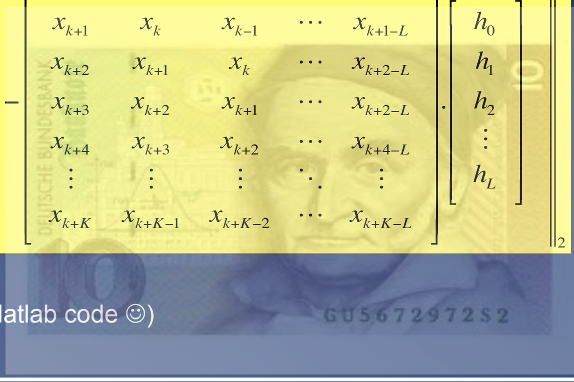


Week 2 / Channel Modeling & Evaluation

3. Least squares estimation

$$\min_{h_0,h_1,h_2,\ldots,h_L} \left\| \begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+K} \end{bmatrix} - \begin{bmatrix} x_k & x_{k-1} & x_{k-2} & \cdots & x_{k-L} \\ x_{k+1} & x_k & x_{k-1} & \cdots & x_{k+1-L} \\ x_{k+2} & x_{k+1} & x_k & \cdots & x_{k+2-L} \\ x_{k+3} & x_{k+2} & x_{k+1} & \cdots & x_{k+2-L} \\ x_{k+4} & x_{k+3} & x_{k+2} & \cdots & x_{k+4-L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{k+K} & x_{k+K-1} & x_{k+K-2} & \cdots & x_{k+K-L} \end{bmatrix} \cdot \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} \right\|_2^2$$

(i.e. one line of Matlab code ☺)



Carl Friedrich Gauss (1777 – 1855)

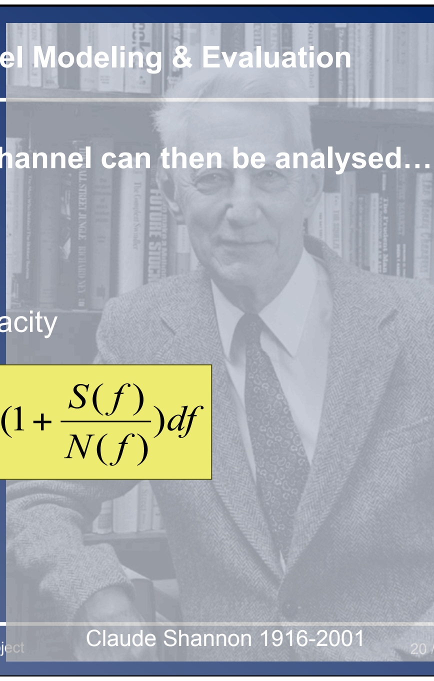
Week 2 / Channel Modeling & Evaluation

Estimated transmission channel can then be analysed…

- Frequency response
- Information theoretic capacity

$$C(\text{bits/sec}) = \int_{f_{\min}}^{f_{\max}} \log_2\left(1+\frac{S(f)}{N(f)}\right) df$$

ps: noise spectrum?



Claude Shannon 1916-2001

Week 3-4 / OFDM modulation

OFDM – Orthogonal Frequency Division Multiplexing
a.k.a.
DMT – Discrete Multitone Modulation

Basic idea is to (QAM-)modulate (many) different carriers with low-rate bit streams. The modulated carriers are summed and then transmitted.
A high-rate bit stream is thus carried by dividing it into hundreds of low-rate streams.
Modulation/demodulation is performed by FFT/IFFT (see below)

Now 14 pages of (simple) maths/theory…

OFDM Modulation

Consider the modulation of
a complex exponential carrier (with period N)

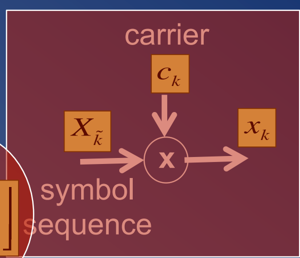
$$c_k = (e^{j2\pi/N})^k \quad \text{for} \quad k = 0,1,\ldots$$

by a `symbol sequence' (see p.27)

$$X_{\tilde{k}} \quad \text{for} \quad \tilde{k} = 0,1,\ldots$$

defined as

$$x_k = c_k . X_{\tilde{k}} \quad \text{for} \quad k = 0,1,\ldots \quad \text{and} \quad \tilde{k} = \left\lfloor \frac{k}{N} \right\rfloor$$



(i.e. "1 symbol per N samples of the carrier")

- PS: remember that modulation of sines and cosines is similar/related to modulation of complex exponentials (see also p.26, 2nd 'PS')

OFDM Modulation

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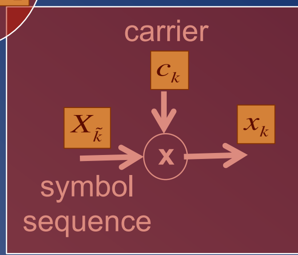
$$x_k = c_k . X_{\tilde{k}} \quad \text{for } k = 0,1,\ldots \quad \text{and} \quad \tilde{k} = \left\lfloor \frac{k}{N} \right\rfloor$$

This corresponds to (for k=0,N,2N,..):

$$\underbrace{\begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ x_{k+4} \\ \vdots \\ x_{k+N-1} \end{bmatrix}}_{\text{time-domain signal segment}} = \begin{bmatrix} (e^{j2\pi/N})^0 \\ (e^{j2\pi/N})^1 \\ (e^{j2\pi/N})^2 \\ (e^{j2\pi/N})^3 \\ (e^{j2\pi/N})^4 \\ \vdots \\ (e^{j2\pi/N})^{N-1} \end{bmatrix} . X_{\tilde{k}}$$



OFDM Modulation

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Now consider the modulation of
N such complex exponential carriers

$$c_k^{(n)} = (e^{j2\pi n/N})^k \quad \text{for } k = 0,1,\ldots \quad \text{and} \quad n = 0,1,..,N-1$$

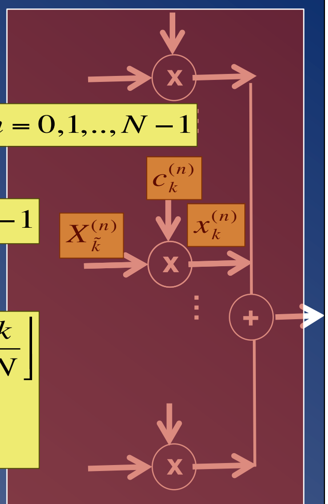
by `symbol sequences'

$$X_{\tilde{k}}^{(n)} \quad \text{for } \tilde{k} = 0,1,\ldots \quad \text{and} \quad n = 0,1,..,N-1$$

defined as

$$x_k^{(n)} = c_k^{(n)} . X_{\tilde{k}}^{(n)} \quad \text{for } k = 0,1,\ldots \quad \text{and} \quad \tilde{k} = \left\lfloor \frac{k}{N} \right\rfloor$$

$$x_k = \sum_{n=0}^{N-1} x_k^{(n)}$$



OFDM Modulation

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This corresponds to (for k=0,N,2N,..):

$$\underbrace{\begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ x_{k+4} \\ \vdots \\ x_{k+N-1} \end{bmatrix}}_{\text{time-domain signal segment}} = \underbrace{\begin{bmatrix} (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^0 & \ldots & (e^{j\frac{2\pi}{N}})^0 \\ (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^1 & (e^{j\frac{2\pi}{N}})^2 & (e^{j\frac{2\pi}{N}})^3 & (e^{j\frac{2\pi}{N}})^4 & \ldots & (e^{j\frac{2\pi}{N}})^{N-1} \\ (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^2 & (e^{j\frac{2\pi}{N}})^4 & (e^{j\frac{2\pi}{N}})^6 & (e^{j\frac{2\pi}{N}})^8 & \ldots & (e^{j\frac{2\pi}{N}})^{2(N-1)} \\ (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^3 & (e^{j\frac{2\pi}{N}})^6 & (e^{j\frac{2\pi}{N}})^9 & (e^{j\frac{2\pi}{N}})^{12} & \ldots & (e^{j\frac{2\pi}{N}})^{3(N-1)} \\ (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^4 & (e^{j\frac{2\pi}{N}})^8 & (e^{j\frac{2\pi}{N}})^{12} & (e^{j\frac{2\pi}{N}})^{16} & \ldots & (e^{j\frac{2\pi}{N}})^{4(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ (e^{j\frac{2\pi}{N}})^0 & (e^{j\frac{2\pi}{N}})^{N-1} & (e^{j\frac{2\pi}{N}})^{2(N-1)} & (e^{j\frac{2\pi}{N}})^{3(N-1)} & (e^{j\frac{2\pi}{N}})^{4(N-1)} & \ldots & (e^{j\frac{2\pi}{N}})^{..} \end{bmatrix}}_{=\ (N)\ *\ IDFT\ \text{-matrix}} \cdot \begin{bmatrix} X_{\tilde{k}}^{(0)} \\ X_{\tilde{k}}^{(1)} \\ X_{\tilde{k}}^{(2)} \\ X_{\tilde{k}}^{(3)} \\ X_{\tilde{k}}^{(4)} \\ \vdots \\ X_{\tilde{k}}^{(N-1)} \end{bmatrix}$$

..and so can be realized by means of an N-point
`Inverse Discrete Fourier Transform' (IDFT) !!!

OFDM Modulation

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- PS: Note that $X_{\tilde{k}}^{(0)}$ modulates a DC signal (hence often set to zero)
- PS: To ensure time-domain signal is real-valued, have to choose

$$X_{\tilde{k}}^{(N-1)} = (X_{\tilde{k}}^{(1)})^* \ ,\ X_{\tilde{k}}^{(N-2)} = (X_{\tilde{k}}^{(2)})^* \ ,\ \ldots$$

- **PS: The <u>IDFT matrix</u> is a cool matrix:**
  - For any chosen dimension N, an IDFT matrix can be constructed as given on the previous slide.
  - Its inverse is the DFT matrix (symbol `**F**').
    DFT and IDFT matrices are <u>unitary</u> (up to a scalar), i.e.

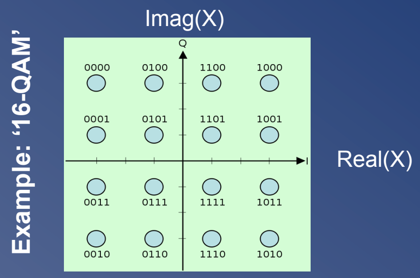
$$\mathrm{F} = (\mathrm{IDFT}-\mathrm{matrix})^{-1} = N.(\mathrm{IDFT}-\mathrm{matrix})^H$$

  - The structure of the IDFT matrix allows for a cheap (complexity N.logN instead of N.N) algorithm to compute the matrix-vector product on the previous slide (=**<u>IFFT</u>** =inverse fast Fourier transform)

OFDM Modulation

So this will be the basic modulation operation at the Tx :

- The X's are (QAM-symbols) defined by the input bit stream



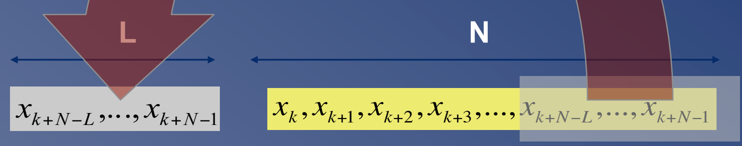
- The time-domain signal segments $x_k, x_{k+1}, x_{k+2}, \ldots, x_{k+N-1}$ are obtained by IDFT/<u>IFFT</u> and then transmitted over the channel, one after the other. At the Rx, demodulation is done with an inverse operation (i.e. DFT/<u>FFT</u>=fast Fourier transform, see also p.33).

OFDM Modulation

Sounds simple, but forgot one thing: <u>channel</u> H(z) !!

OFDM has an ingenious way of dealing with the channel effect, namely through the insertion of a so-called `**cyclic prefix**' at the Tx :

If the channel is FIR with order L (see p.16), then per segment, instead of transmitting N samples, **<u>N+L</u>** sampes are transmitted (assuming L<<N), where the last L samples are copied and put up front…



OFDM Modulation

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At the Rx, throw away L samples corresponding to cyclic prefix, keep the other N samples, which correspond to

prefix

N

N+L

$$\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+N-1} \end{bmatrix} = \begin{bmatrix} h_L & h_{L-1} & \dots & h_1 & h_0 & 0 & 0 & 0 & \dots & 0 \\ 0 & h_L & \dots & h_2 & h_1 & h_0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & h_3 & h_2 & h_1 & h_0 & 0 & \dots & 0 \\ 0 & 0 & \dots & h_4 & h_3 & h_2 & h_1 & h_0 & \dots & 0 \\ 0 & 0 & \dots & h_5 & h_4 & h_3 & h_2 & h_1 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & & \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & h_0 \end{bmatrix} \cdot \begin{bmatrix} x_{k+N-L} \\ x_{k+N-L+1} \\ \vdots \\ x_{k+N-1} \\ x_k \\ x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ \vdots \\ x_{k+N-1} \end{bmatrix}$$

This is equivalent to …

OFDM Modulation

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N

N

$$\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+N-1} \end{bmatrix} = \underbrace{\begin{bmatrix} h_L & h_{L-1} & \dots & h_1 & h_0 & 0 & 0 & 0 & \dots & 0 \\ 0 & h_L & \dots & h_2 & h_1 & h_0 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & h_3 & h_2 & h_1 & h_0 & 0 & \dots & 0 \\ 0 & 0 & \dots & h_4 & h_3 & h_2 & h_1 & h_0 & \dots & 0 \\ 0 & 0 & \dots & h_5 & h_4 & h_3 & h_2 & h_1 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & & \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & h_0 \end{bmatrix} \cdot \begin{bmatrix} 0 \mid I_{LxL} \\ I_{NxN} \end{bmatrix}}_{\begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & \dots & h_3 & h_2 & h_1 \\ h_1 & h_0 & 0 & 0 & 0 & \dots & h_4 & h_3 & h_2 \\ h_2 & h_1 & h_0 & 0 & 0 & \dots & & h_4 & h_3 \\ h_3 & h_2 & h_1 & h_0 & 0 & \dots & & & h_4 \\ h_4 & h_3 & h_2 & h_1 & h_0 & \dots & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & h_2 & h_1 & h_0 \end{bmatrix}} \cdot \begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ x_{k+4} \\ \vdots \\ x_{k+N-1} \end{bmatrix} \quad (*)$$

The resulting matrix (call it `**H**' ) is an NxN `circulant matrix' =every row is the previous row up to a 'cyclic shift'

OFDM Modulation

- PS: Cyclic prefix converts a (linear) convolution (see p.29) into a so-called 'circular convolution' (see p.30)
- **Circulant matrices** are cool matrices…

  A weird property (proof by Matlab!) is that when a circulant matrix H is pre-/post-multiplied by the DFT/IDFT matrix, a diagonal matrix is always obtained: $\mathbf{F}\mathbf{H}.\mathbf{F}^{-1} = [\text{diagonal matrix}]$

  Hence, a circulant matrix can **always** be written as (=eigenvalue decomposition!)

$$\mathbf{H} = \mathbf{F}^{-1}.\begin{bmatrix} H_0 & 0 & \dots & 0 \\ 0 & H_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & H_{N-1} \end{bmatrix}.\mathbf{F}$$

OFDM Modulation

Combine previous formulas, to obtain…

$$\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+N-1} \end{bmatrix} = \mathbf{F}^{-1}.\begin{bmatrix} H_0 & 0 & \dots & 0 \\ 0 & H_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & H_{N-1} \end{bmatrix}.\mathbf{F}.\begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ x_{k+4} \\ \vdots \\ x_{k+N-1} \end{bmatrix}$$

$$\mathbf{F}.\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+N-1} \end{bmatrix} = \begin{bmatrix} H_0 & 0 & \dots & 0 \\ 0 & H_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & H_{N-1} \end{bmatrix}.\mathbf{F}.\begin{bmatrix} x_k \\ x_{k+1} \\ x_{k+2} \\ x_{k+3} \\ x_{k+4} \\ \vdots \\ x_{k+N-1} \end{bmatrix}$$

OFDM Modulation

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In other words…

$$\begin{bmatrix} Y_{\tilde{k}}^{(0)} \\ Y_{\tilde{k}}^{(1)} \\ Y_{\tilde{k}}^{(2)} \\ Y_{\tilde{k}}^{(3)} \\ Y_{\tilde{k}}^{(4)} \\ \vdots \\ Y_{\tilde{k}}^{(N-1)} \end{bmatrix} \triangleq \mathbf{F}.\begin{bmatrix} y_k \\ y_{k+1} \\ y_{k+2} \\ y_{k+3} \\ y_{k+4} \\ \vdots \\ y_{k+N-1} \end{bmatrix} = \begin{bmatrix} H_0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & H_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & H_2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & H_3 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & H_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & H_{N-1} \end{bmatrix} . \begin{bmatrix} X_{\tilde{k}}^{(0)} \\ X_{\tilde{k}}^{(1)} \\ X_{\tilde{k}}^{(2)} \\ X_{\tilde{k}}^{(3)} \\ X_{\tilde{k}}^{(4)} \\ \vdots \\ X_{\tilde{k}}^{(N-1)} \end{bmatrix}$$

This means that after removing the prefix part and performing a DFT in the Rx, the obtained Y's are equal to the transmitted X's, up to (scalar) channel attenuations Hn (!!)

$$\Rightarrow \quad Y_{\tilde{k}}^{(n)} = H_n . X_{\tilde{k}}^{(n)}$$

OFDM Modulation

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- PS: It can be shown (check first column of $\mathbf{F}.\mathbf{H} = [\text{diagonal matrix}].\mathbf{F}$) that Hn is the channel frequency response evaluated at the n-th carrier !

$$\begin{bmatrix} H_0 \\ H_1 \\ H_2 \\ H_3 \\ \vdots \\ H_{N-1} \end{bmatrix} = \mathbf{F}.\ (1^{st}\text{column of } \mathbf{H}) = \mathbf{F}. \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_L \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \Rightarrow \quad H_n = H(z)\big|_{z=e^{j2\pi n/N}}$$

(p.33 then represents 'frequency domain version' of circular convolution, i.e. 'component-wise multiplication in the frequency domain')

OFDM Modulation

- PS: It can be shown (check first column of $\mathbf{F.H} = [\text{diagonal matrix}].\mathbf{F}$) that H_n is the channel frequency response evaluated at the n-th carrier !

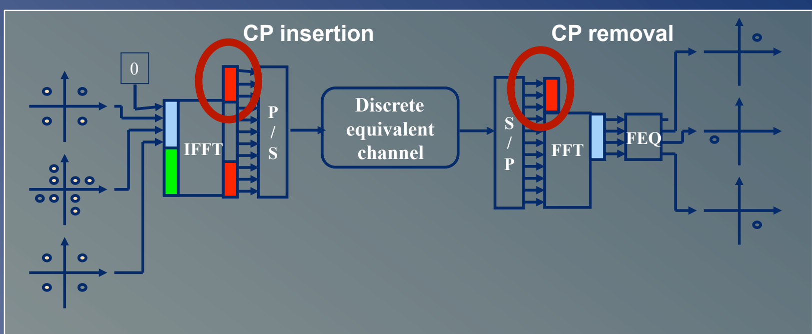
$$\begin{bmatrix} H_0 \\ H_1 \\ H_2 \\ H_3 \\ \vdots \\ H_{N-1} \end{bmatrix} = \mathbf{F}.\,(1^{st}\text{column of } \mathbf{H}) = \mathbf{F}.\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_L \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \Rightarrow \quad H_n = H(z)|_{z=e^{j2\pi n/N}}$$

**`Channel equalization'** may then be performed after the DFT (=in the frequency domain), by component-wise division (divide by $H_n$ for carrier-n). This is referred to as `1-tap FEQ' (Freq.-domain EQualization)

$$Y_{\tilde{k}}^{(n)} = H_n .\, X_{\tilde{k}}^{(n)} \quad \Rightarrow \quad \text{estimate}\{X_{\tilde{k}}^{(n)}\} = (H_n)^{-1}.Y_{\tilde{k}}^{(n)}$$

OFDM Modulation

- Conclusion: DMT-modulation with cyclic prefix leads to a simple (trivial) channel equalization problem (!!)



Week 3-4

Target Week 3-4:

– Study/understand OFDM scheme

Surf around, use IEEE Xplore, Wikipedia, etc.

– Simulate basic OFDM Transceiver in Matlab

First without channel dispersion & without noise, then with noise, then with channel (model from Week-2)

– Optional : Extend OFDM Tx/Rx with `bit-loading ‘

=Carriers with a high SNR transmit more bits/sec

Week 5-6

Target Week 5-6:

– OFDM over acoustic channel, with basic Simulink (Real-time Workshop) scheme (Week-1)

– Extend OFDM Tx/Rx with mechanism for channel estimation and/or equalizer (FEQ) initialization/updating based on transmitted training symbols

Week 7-8

Target Week 7-8:

- OFDM over acoustic channel, with decision-directed <u>adaptive equalization</u> (see Part III)

Important !

PS: groups of 2

- Runs over 8 weeks
- Each week
  - 1 PC/Matlab session (supervised, 2.5hrs)
  - 2 'Homework' sesions (unsupervised, 2*2.5hrs)

PS: Time budget = 8*(2.5hrs+5hrs) = 6[illegible]rs

- 'Deliverables' after week 2, 4, 6, 8
- Grading: based on deliverables, evaluated during sessions
  main part=80%, optional part=20%
- <u>TAs</u>: amin.hassani@kuleuven.be (English+Persian)
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BE THERE !!