DSP-CIS

Part-I: Introduction

Chapter-2 : Signals & Systems Review

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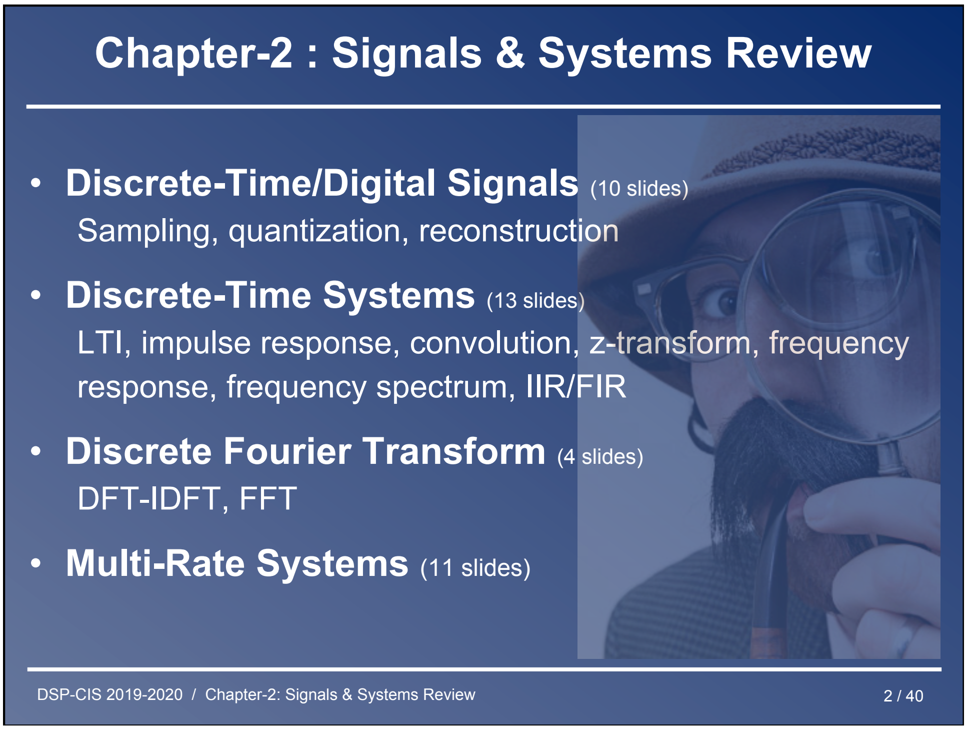
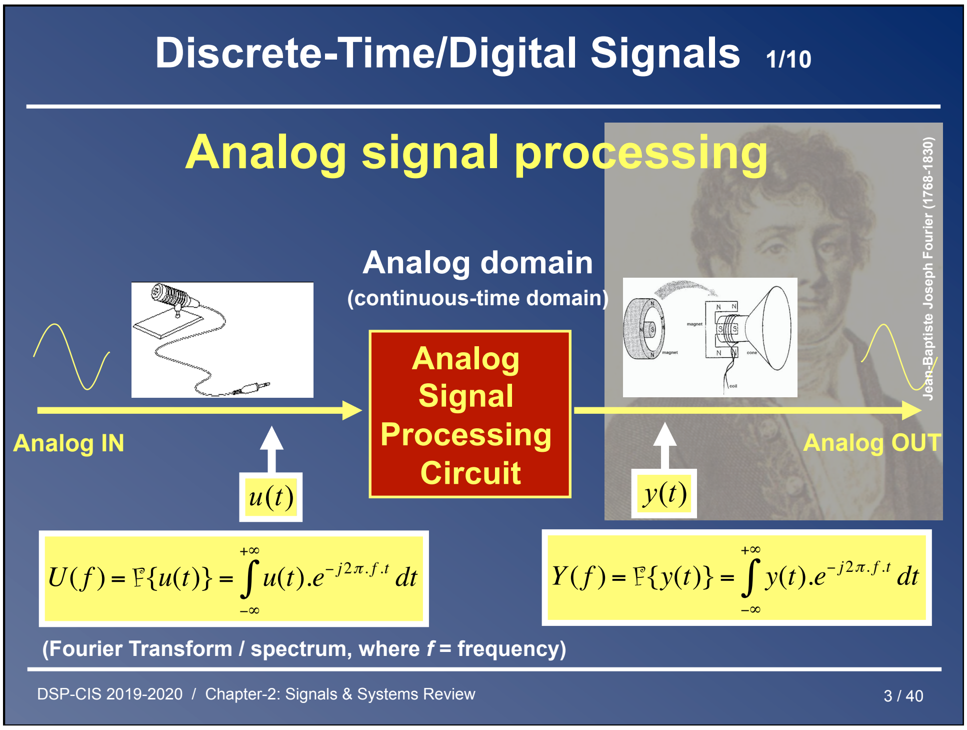


Chapter-2 : Signals & Systems Review

- **Discrete-Time/Digital Signals** (10 slides)
  Sampling, quantization, reconstruction
- **Discrete-Time Systems** (13 slides)
  LTI, impulse response, convolution, z-transform, frequency response, frequency spectrum, IIR/FIR
- **Discrete Fourier Transform** (4 slides)
  DFT-IDFT, FFT
- **Multi-Rate Systems** (11 slides)



Discrete-Time/Digital Signals 1/10

Analog signal processing

Analog domain

(continuous-time domain)

Analog IN

Analog Signal Processing Circuit

Analog OUT

$u(t)$

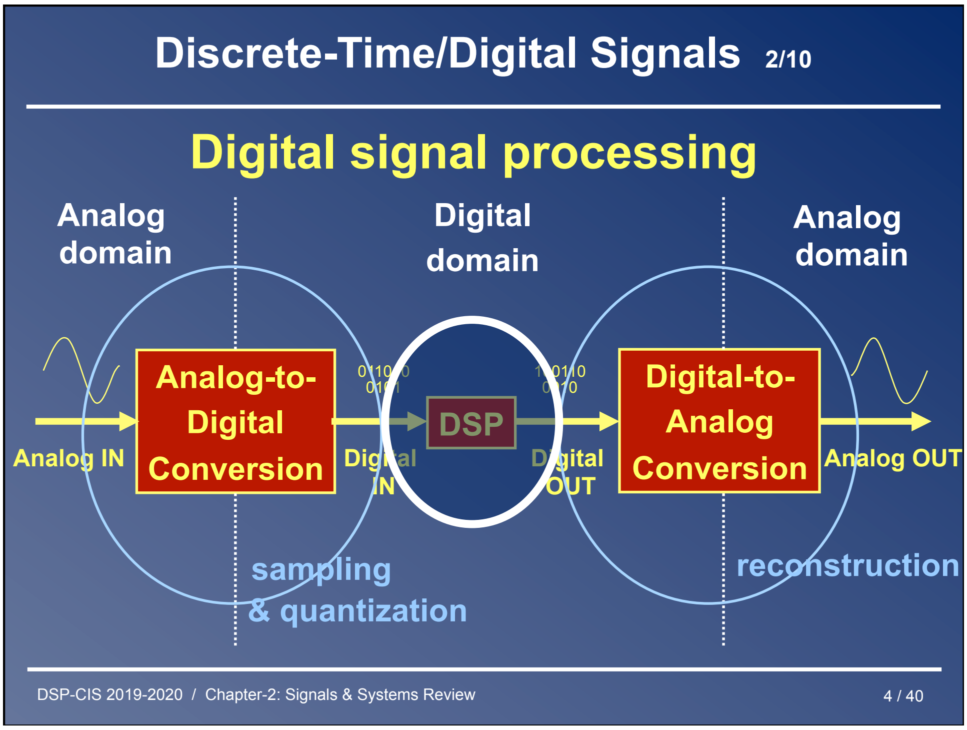
$y(t)$

Jean-Baptiste Joseph Fourier (1768-1830)

$$U(f)=\mathcal{F}\{u(t)\}=\int_{-\infty}^{+\infty} u(t).e^{-j2\pi.f.t}\,dt$$

$$Y(f)=\mathcal{F}\{y(t)\}=\int_{-\infty}^{+\infty} y(t).e^{-j2\pi.f.t}\,dt$$

(Fourier Transform / spectrum, where *f* = frequency)



Discrete-Time/Digital Signals 2/10

Digital signal processing

Analog domain

Digital domain

Analog domain

Analog IN

Analog-to-Digital Conversion

Digital IN

DSP

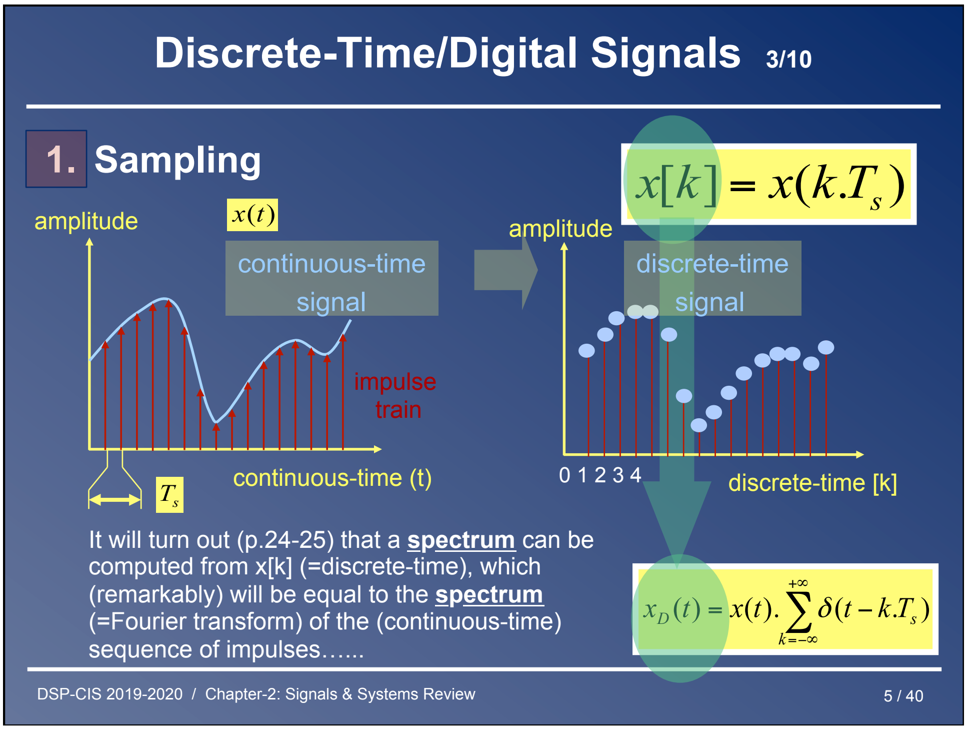
Digital OUT

Digital-to-Analog Conversion

Analog OUT

sampling & quantization

reconstruction



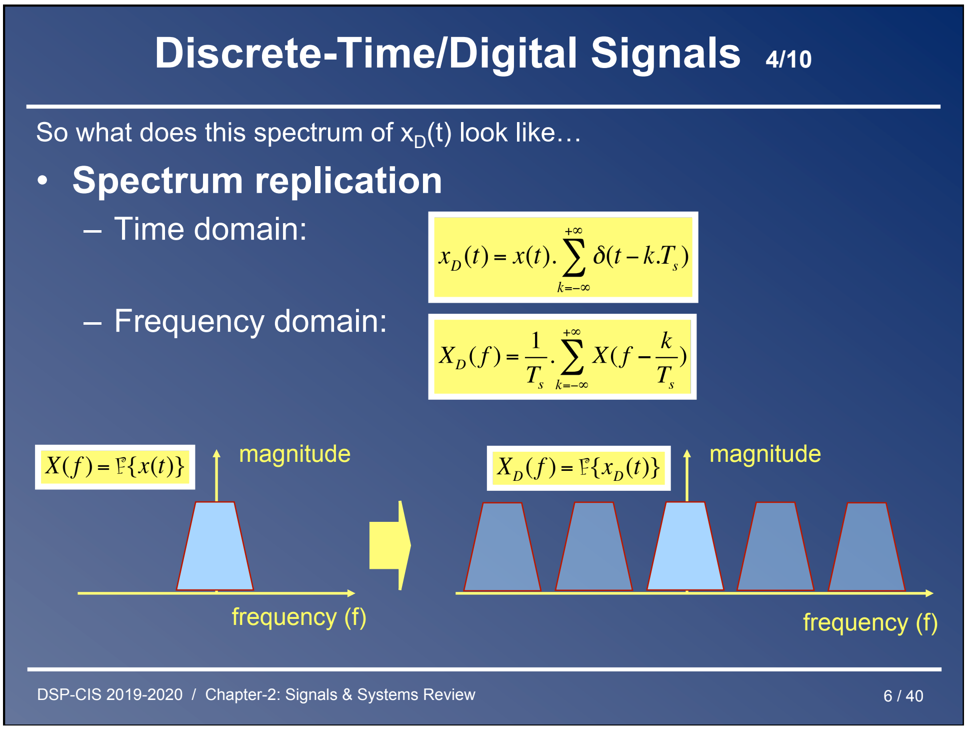
Discrete-Time/Digital Signals 3/10

1. Sampling

$$x[k] = x(k.T_s)$$

It will turn out (p.24-25) that a **spectrum** can be computed from x[k] (=discrete-time), which (remarkably) will be equal to the **spectrum** (=Fourier transform) of the (continuous-time) sequence of impulses……

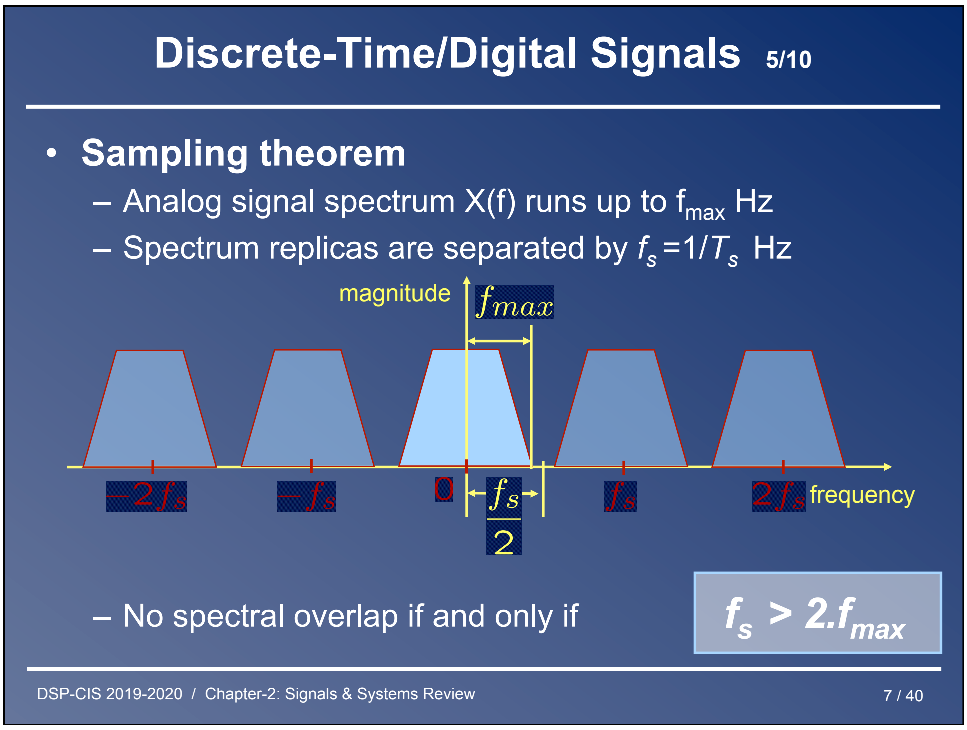
$$x_D(t) = x(t).\sum_{k=-\infty}^{+\infty}\delta(t-k.T_s)$$



Discrete-Time/Digital Signals 4/10

So what does this spectrum of $x_D(t)$ look like…

- **Spectrum replication**
  - Time domain: $x_D(t) = x(t).\sum_{k=-\infty}^{+\infty}\delta(t-k.T_s)$
  - Frequency domain: $X_D(f) = \frac{1}{T_s}.\sum_{k=-\infty}^{+\infty} X(f-\frac{k}{T_s})$

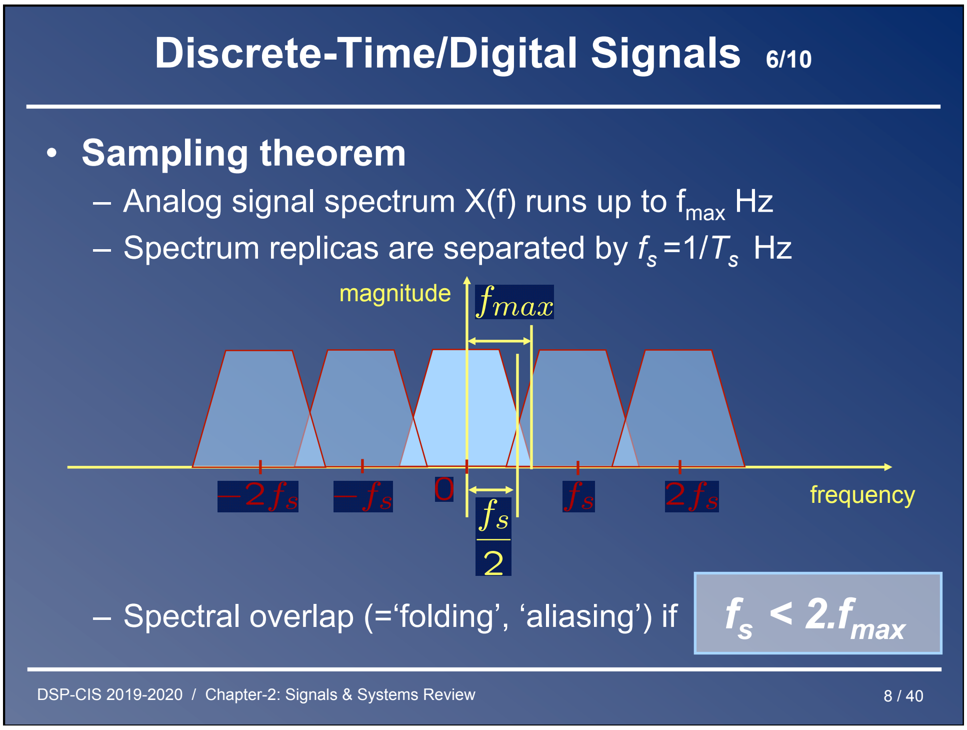


Discrete-Time/Digital Signals 5/10

- **Sampling theorem**
  - Analog signal spectrum X(f) runs up to f_{max} Hz
  - Spectrum replicas are separated by $f_s = 1/T_s$ Hz

  - No spectral overlap if and only if

$f_s > 2.f_{max}$

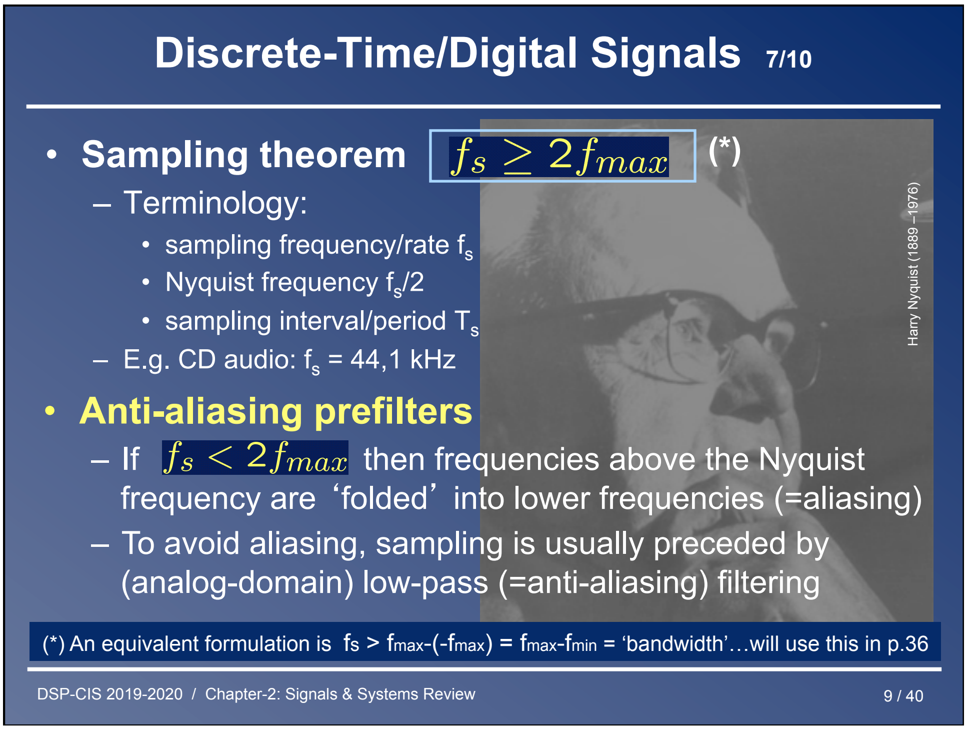


Discrete-Time/Digital Signals 6/10

- **Sampling theorem**
  - Analog signal spectrum X(f) runs up to f_{max} Hz
  - Spectrum replicas are separated by $f_s = 1/T_s$ Hz

  - Spectral overlap (='folding', 'aliasing') if

$f_s < 2.f_{max}$

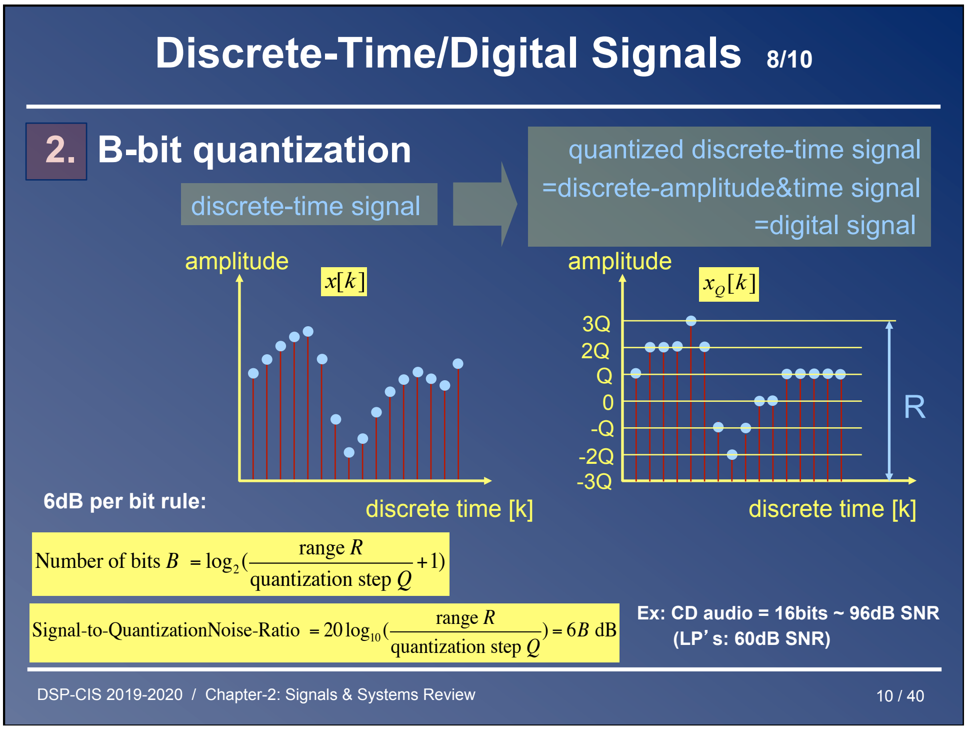


Discrete-Time/Digital Signals 7/10

- **Sampling theorem** $f_s \geq 2f_{max}$ (*)
  - Terminology:
    - sampling frequency/rate f_s
    - Nyquist frequency $f_s/2$
    - sampling interval/period T_s
  - E.g. CD audio: f_s = 44,1 kHz
- **Anti-aliasing prefilters**
  - If $f_s < 2f_{max}$ then frequencies above the Nyquist frequency are 'folded' into lower frequencies (=aliasing)
  - To avoid aliasing, sampling is usually preceded by (analog-domain) low-pass (=anti-aliasing) filtering

Harry Nyquist (1889 -1976)

(*) An equivalent formulation is $f_s > f_{max}-(-f_{max}) = f_{max}-f_{min}$ = 'bandwidth'…will use this in p.36



Discrete-Time/Digital Signals 8/10

2. B-bit quantization

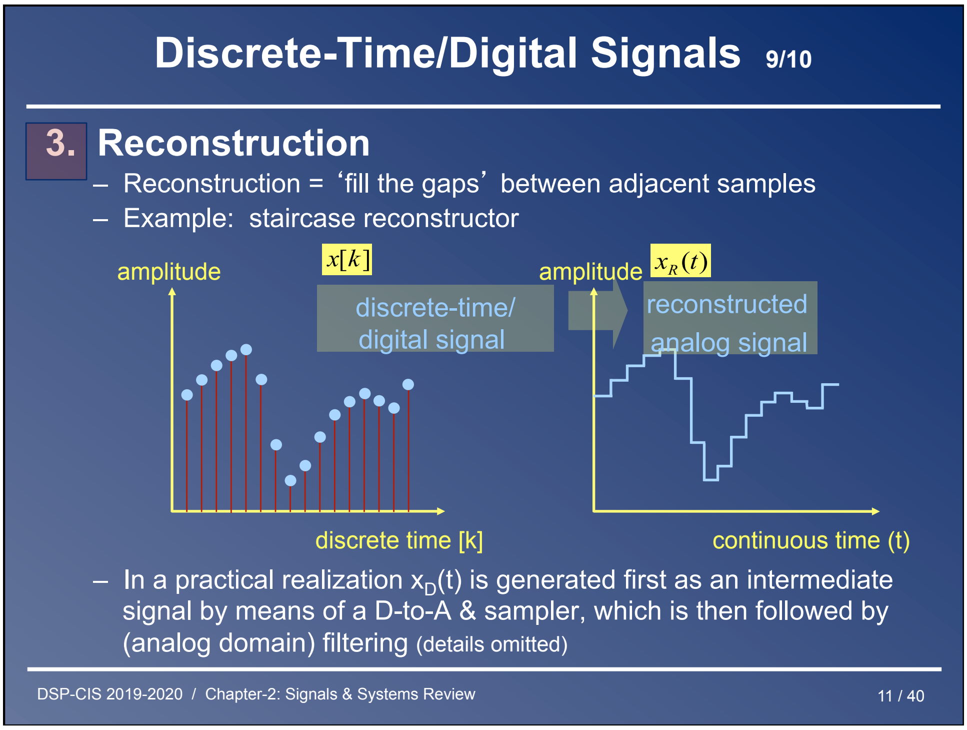
discrete-time signal → quantized discrete-time signal =discrete-amplitude&time signal =digital signal

6dB per bit rule:

$$\text{Number of bits } B = \log_2\left(\frac{\text{range } R}{\text{quantization step } Q}+1\right)$$

$$\text{Signal-to-QuantizationNoise-Ratio} = 20\log_{10}\left(\frac{\text{range } R}{\text{quantization step } Q}\right) = 6B \text{ dB}$$

Ex: CD audio = 16bits ~ 96dB SNR (LP's: 60dB SNR)



Discrete-Time/Digital Signals 9/10

3. Reconstruction

- Reconstruction = ‘fill the gaps’ between adjacent samples
- Example: staircase reconstructor

- In a practical realization $x_D(t)$ is generated first as an intermediate signal by means of a D-to-A & sampler, which is then followed by (analog domain) filtering (details omitted)

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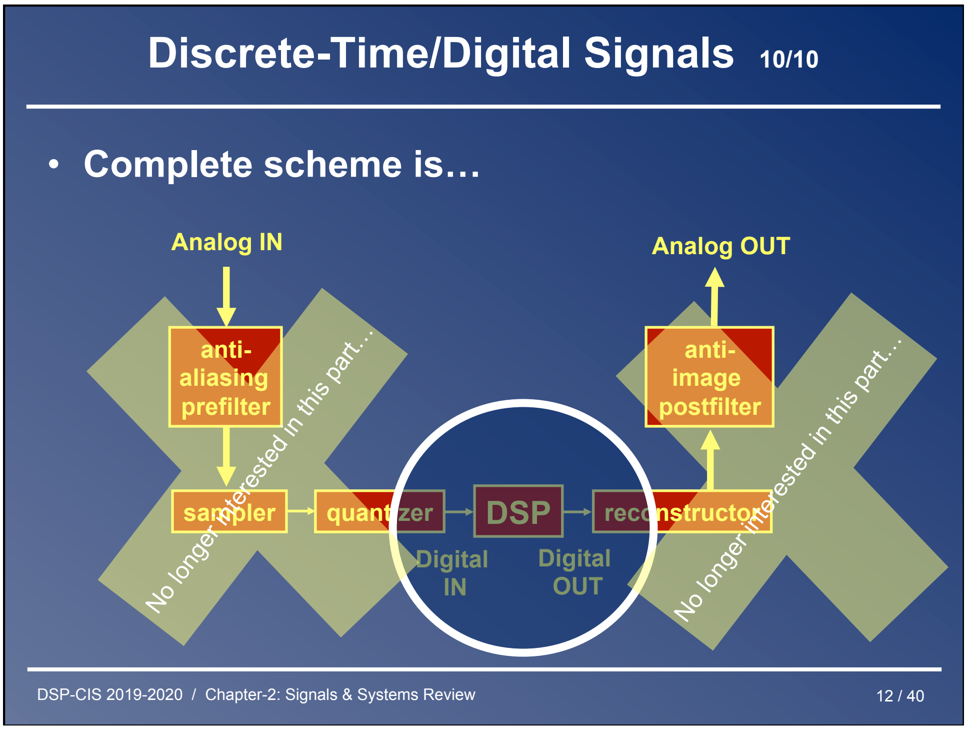

Discrete-Time/Digital Signals 10/10

- Complete scheme is…

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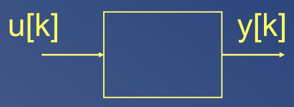
Discrete-Time Systems 1/13

Discrete-time system is `sampled data' system



Input signal u[k] is a sequence of samples (=numbers)

..,u[-2],u[-1],u[0], u[1],u[2],…

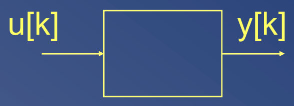
System then produces a sequence of output samples y[k]

..,y[-2],y[-1],y[0], y[1],y[2],…

Example: `DSP' block in previous slide

Discrete-Time Systems 2/13

Will consider linear time-invariant (LTI) systems



Linear :

input u1[k] -> output y1[k]

input u2[k] -> output y2[k]

hence a.u1[k]+b.u2[k]-> a.y1[k]+b.y2[k]

Time-invariant (shift-invariant)

input u[k] -> output y[k]

hence input u[k-T] -> output y[k-T]

Discrete-Time Systems 3/13

Will consider causal systems

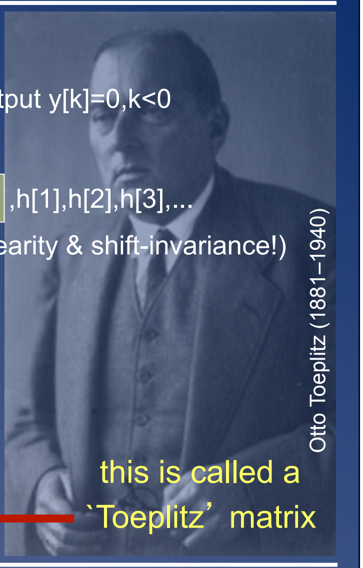
iff for all input signals with u[k]=0,k<0 -> output y[k]=0,k<0

Impulse response

input …,0,0, $\overset{K=0}{1}$,0,0,0,...-> output …,0,0, $\overset{K=0}{h[0]}$,h[1],h[2],h[3],...

General input u[0],u[1],u[2],u[3] (cfr. linearity & shift-invariance!)

$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \\ y[5] \end{bmatrix} = \begin{bmatrix} h[0] & 0 & 0 & 0 \\ h[1] & h[0] & 0 & 0 \\ h[2] & h[1] & h[0] & 0 \\ 0 & h[2] & h[1] & h[0] \\ 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & h[2] \end{bmatrix} . \begin{bmatrix} u[0] \\ u[1] \\ u[2] \\ u[3] \end{bmatrix}$$

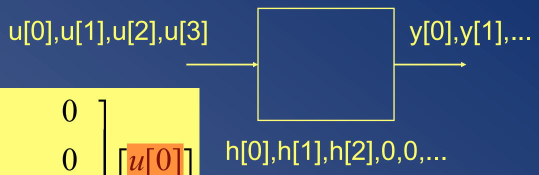


Otto Toeplitz (1881–1940)

this is called a `Toeplitz’ matrix

Discrete-Time Systems 4/13

Convolution

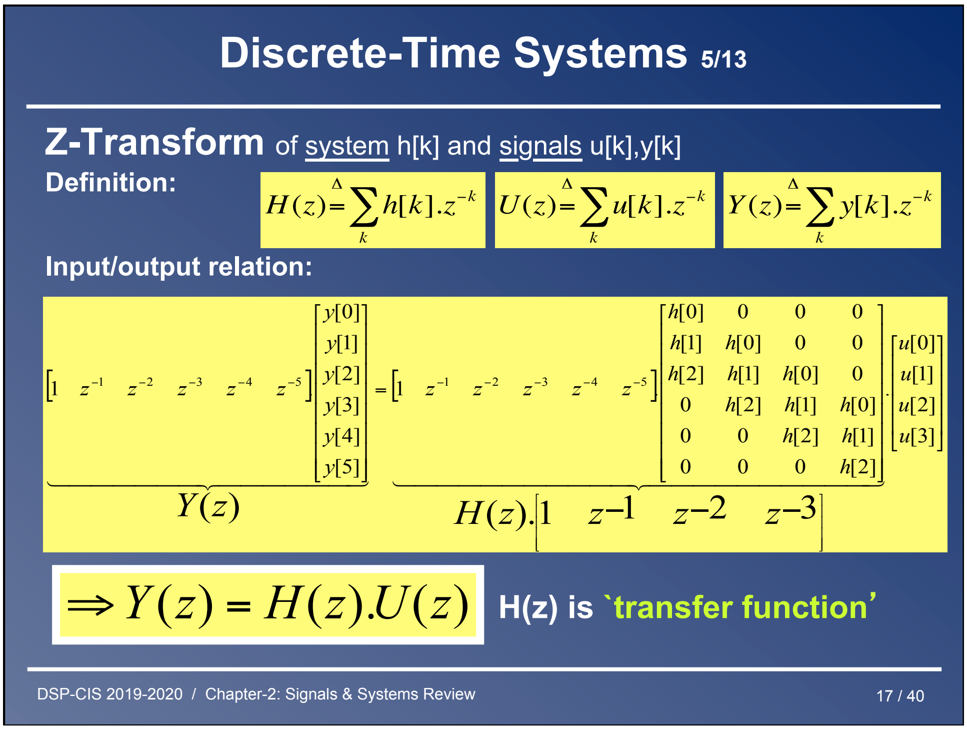


$$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \\ y[5] \end{bmatrix} = \begin{bmatrix} h[0] & 0 & 0 & 0 \\ h[1] & h[0] & 0 & 0 \\ h[2] & h[1] & h[0] & 0 \\ 0 & h[2] & h[1] & h[0] \\ 0 & 0 & h[2] & h[1] \\ 0 & 0 & 0 & h[2] \end{bmatrix} . \begin{bmatrix} u[0] \\ u[1] \\ u[2] \\ u[3] \end{bmatrix}$$

$$y[k] = \sum_{\bar{k}} h[k-\bar{k}].u[\bar{k}] \overset{\Delta}{=} h[k] * u[k]$$

= `convolution sum ‘

(=more convenient than Toeplitz matrix notation when considering (infinitely) long input and impulse response sequences)



Discrete-Time Systems 5/13

Z-Transform of system h[k] and signals u[k],y[k]

Definition:

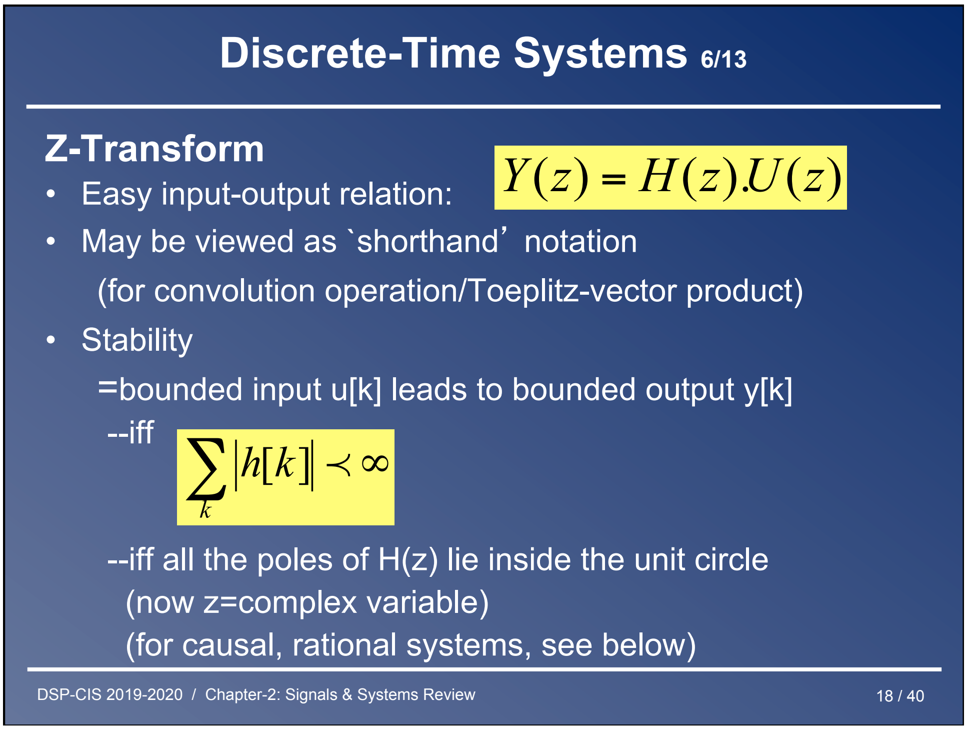
$$H(z)\overset{\Delta}{=}\sum_k h[k].z^{-k} \qquad U(z)\overset{\Delta}{=}\sum_k u[k].z^{-k} \qquad Y(z)\overset{\Delta}{=}\sum_k y[k].z^{-k}$$

Input/output relation:

$$\underbrace{\begin{bmatrix}1 & z^{-1} & z^{-2} & z^{-3} & z^{-4} & z^{-5}\end{bmatrix}\begin{bmatrix}y[0]\\y[1]\\y[2]\\y[3]\\y[4]\\y[5]\end{bmatrix}}_{Y(z)} = \underbrace{\begin{bmatrix}1 & z^{-1} & z^{-2} & z^{-3} & z^{-4} & z^{-5}\end{bmatrix}\begin{bmatrix}h[0] & 0 & 0 & 0\\h[1] & h[0] & 0 & 0\\h[2] & h[1] & h[0] & 0\\0 & h[2] & h[1] & h[0]\\0 & 0 & h[2] & h[1]\\0 & 0 & 0 & h[2]\end{bmatrix}}_{H(z).\begin{bmatrix}1 & z-1 & z-2 & z-3\end{bmatrix}}.\begin{bmatrix}u[0]\\u[1]\\u[2]\\u[3]\end{bmatrix}$$

$$\boxed{\Rightarrow Y(z)=H(z).U(z)}$$

H(z) is `transfer function'



Discrete-Time Systems 6/13

Z-Transform

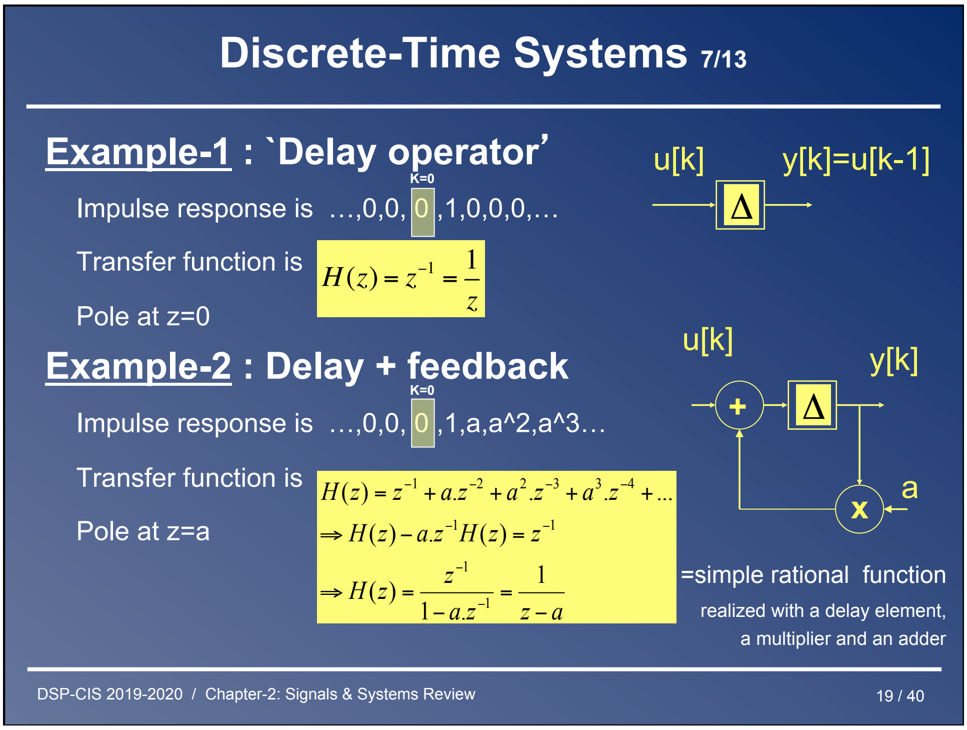
- Easy input-output relation: $Y(z)=H(z).U(z)$
- May be viewed as `shorthand' notation

  (for convolution operation/Toeplitz-vector product)
- Stability

  =bounded input u[k] leads to bounded output y[k]

  --iff $\sum_k |h[k]| \prec \infty$

  --iff all the poles of H(z) lie inside the unit circle
  (now z=complex variable)
  (for causal, rational systems, see below)



Discrete-Time Systems 7/13

Example-1 : `Delay operator'

Impulse response is …,0,0, $\overset{K=0}{\boxed{0}}$,1,0,0,0,…

Transfer function is $H(z)=z^{-1}=\frac{1}{z}$

Pole at z=0

Example-2 : Delay + feedback

Impulse response is …,0,0, $\overset{K=0}{\boxed{0}}$,1,a,a^2,a^3…

Transfer function is

$$H(z)=z^{-1}+a.z^{-2}+a^{2}.z^{-3}+a^{3}.z^{-4}+...$$

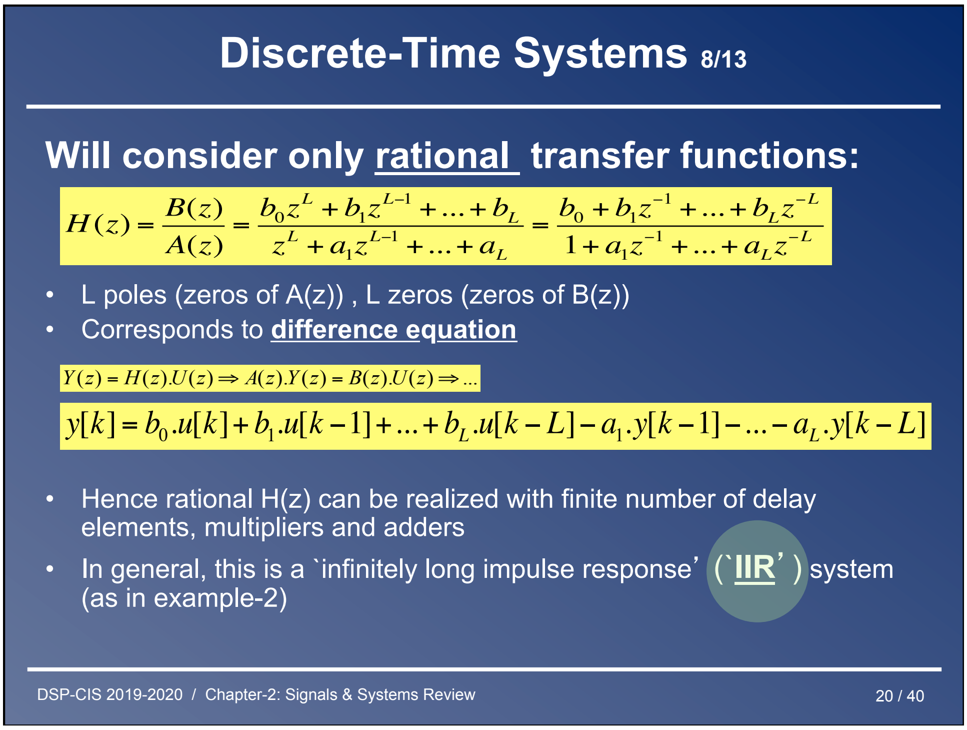
$$\Rightarrow H(z)-a.z^{-1}H(z)=z^{-1}$$

$$\Rightarrow H(z)=\frac{z^{-1}}{1-a.z^{-1}}=\frac{1}{z-a}$$

Pole at z=a

=simple rational function

realized with a delay element,
a multiplier and an adder



Discrete-Time Systems 8/13

Will consider only rational transfer functions:

$$H(z)=\frac{B(z)}{A(z)}=\frac{b_0z^{L}+b_1z^{L-1}+...+b_L}{z^{L}+a_1z^{L-1}+...+a_L}=\frac{b_0+b_1z^{-1}+...+b_Lz^{-L}}{1+a_1z^{-1}+...+a_Lz^{-L}}$$

- L poles (zeros of A(z)) , L zeros (zeros of B(z))
- Corresponds to **difference equation**

$$Y(z)=H(z).U(z)\Rightarrow A(z).Y(z)=B(z).U(z)\Rightarrow ...$$

$$y[k]=b_0.u[k]+b_1.u[k-1]+...+b_L.u[k-L]-a_1.y[k-1]-...-a_L.y[k-L]$$

- Hence rational H(z) can be realized with finite number of delay elements, multipliers and adders
- In general, this is a `infinitely long impulse response' (`**IIR**') system (as in example-2)

Discrete-Time Systems 9/13

Special case is

$$H(z)=\frac{B(z)}{z^L}=b_0+b_1z^{-1}+\ldots+b_Lz^{-L}$$

- L poles at the origin z=0 (hence guaranteed stability)
- L zeros (zeros of B(z)) = `all zero' filter
- Corresponds to **difference equation**

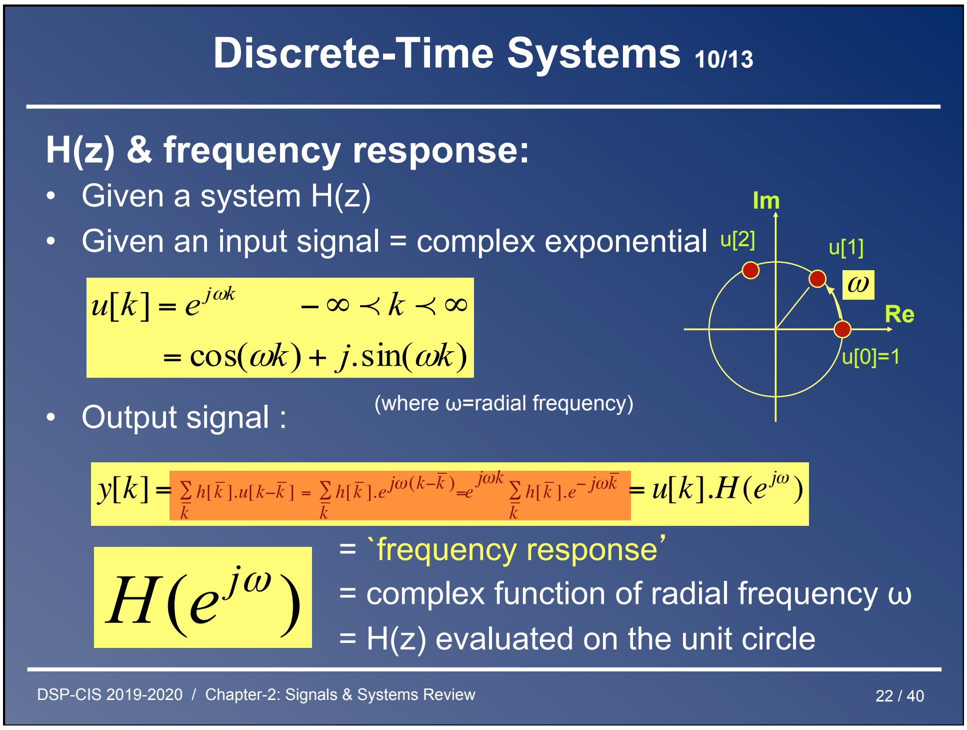
$$Y(z)=H(z).U(z)\Rightarrow y[k]=b_0.u[k]+b_1.u[k-1]+\ldots+b_L.u[k-L]$$

=`moving average' (MA) filter

- Impulse response h[k] is

$$0,0,0,b_0,b_1,\ldots,b_{L-1},b_L,0,0,0,\ldots$$

= `finite impulse response' (`**FIR**') filter



Discrete-Time Systems 10/13

H(z) & frequency response:

- Given a system H(z)
- Given an input signal = complex exponential

$$u[k]=e^{j\omega k} \qquad -\infty \prec k \prec \infty$$
$$=\cos(\omega k)+j.\sin(\omega k)$$

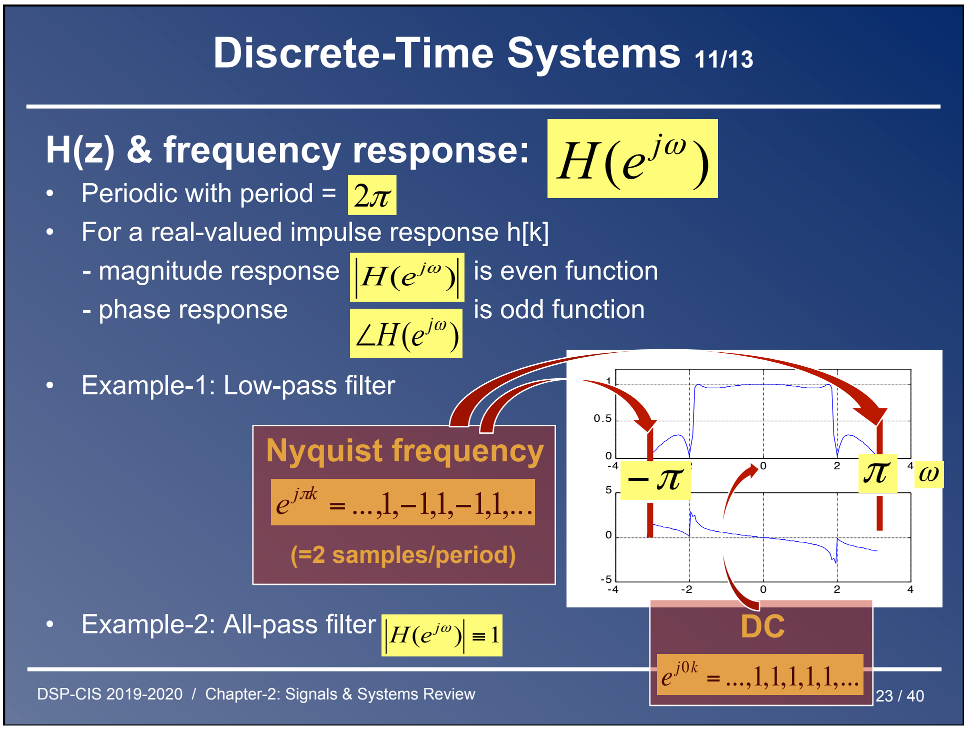
(where ω=radial frequency)

- Output signal :

$$y[k]=\sum_{\bar{k}} h[\bar{k}].u[k-\bar{k}]=\sum_{\bar{k}} h[\bar{k}].e^{j\omega(k-\bar{k})}=e^{j\omega k}\sum_{\bar{k}} h[\bar{k}].e^{-j\omega\bar{k}}=u[k].H(e^{j\omega})$$

$$H(e^{j\omega})$$

= `frequency response'
= complex function of radial frequency ω
= H(z) evaluated on the unit circle



Discrete-Time Systems 11/13

H(z) & frequency response: $H(e^{j\omega})$

- Periodic with period = 2π
- For a real-valued impulse response h[k]
  - magnitude response $|H(e^{j\omega})|$ is even function
  - phase response $\angle H(e^{j\omega})$ is odd function
- Example-1: Low-pass filter

Nyquist frequency

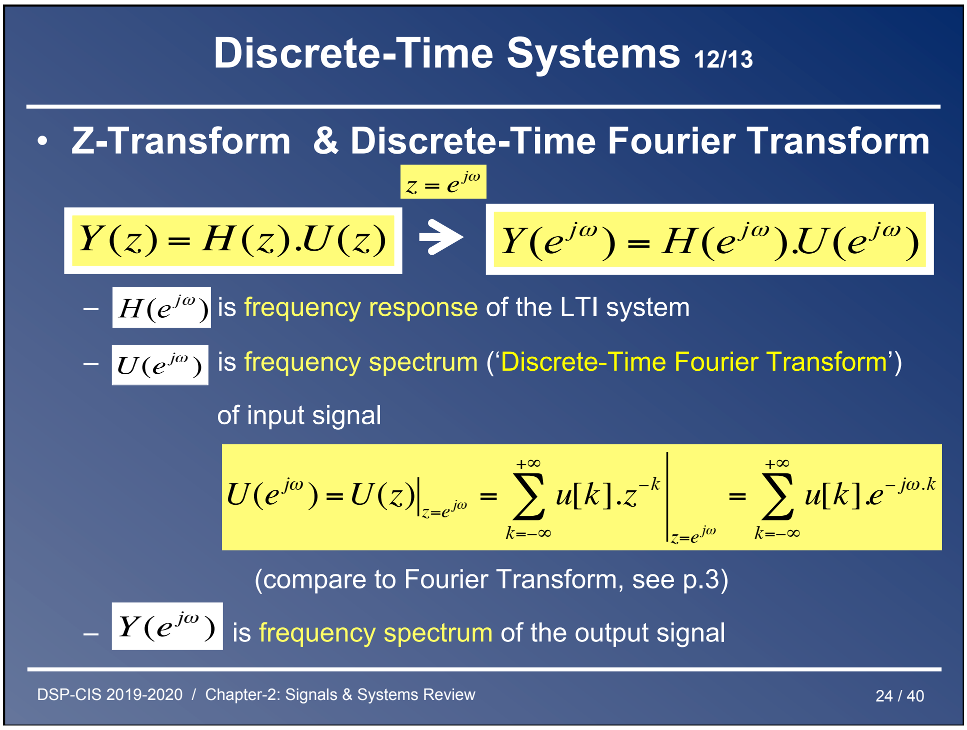
$e^{j\pi k} = \ldots,1,-1,1,-1,1,\ldots$

(=2 samples/period)

DC

$e^{j0k} = \ldots,1,1,1,1,1,\ldots$

- Example-2: All-pass filter $|H(e^{j\omega})| \equiv 1$



Discrete-Time Systems 12/13

- **Z-Transform & Discrete-Time Fourier Transform**

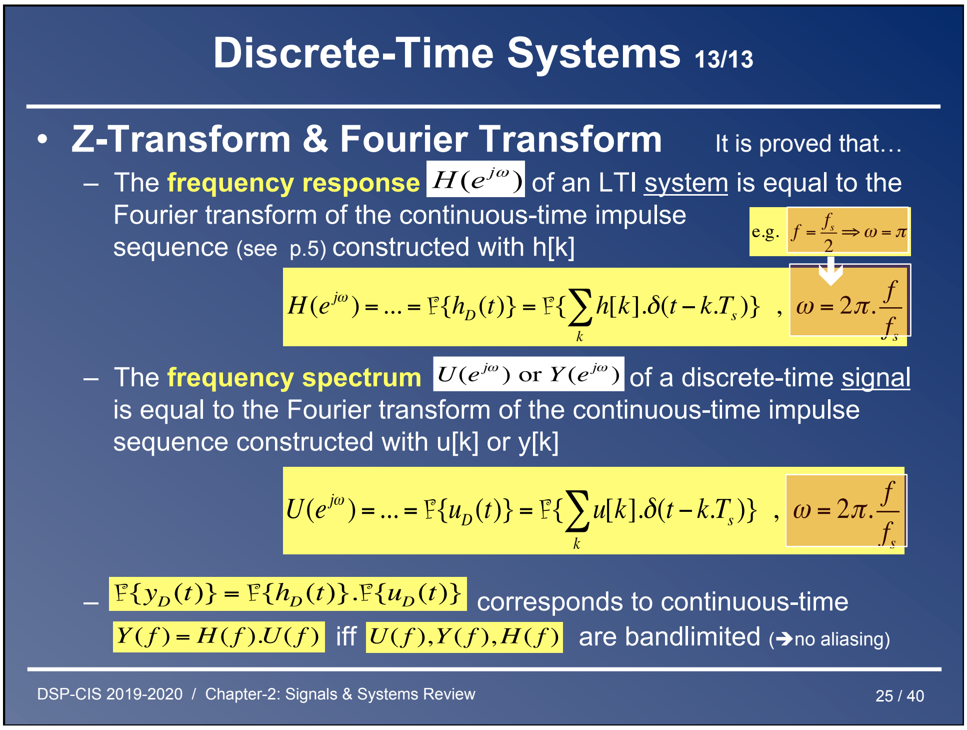
$$Y(z) = H(z).U(z) \xrightarrow{z=e^{j\omega}} Y(e^{j\omega}) = H(e^{j\omega}).U(e^{j\omega})$$

- $H(e^{j\omega})$ is frequency response of the LTI system
- $U(e^{j\omega})$ is frequency spectrum ('Discrete-Time Fourier Transform') of input signal

$$U(e^{j\omega}) = U(z)\big|_{z=e^{j\omega}} = \sum_{k=-\infty}^{+\infty} u[k].z^{-k}\Big|_{z=e^{j\omega}} = \sum_{k=-\infty}^{+\infty} u[k].e^{-j\omega.k}$$

(compare to Fourier Transform, see p.3)

- $Y(e^{j\omega})$ is frequency spectrum of the output signal



Discrete-Time Systems 13/13

- **Z-Transform & Fourier Transform** It is proved that…
  - The **frequency response** $H(e^{j\omega})$ of an LTI <u>system</u> is equal to the Fourier transform of the continuous-time impulse sequence (see p.5) constructed with h[k]

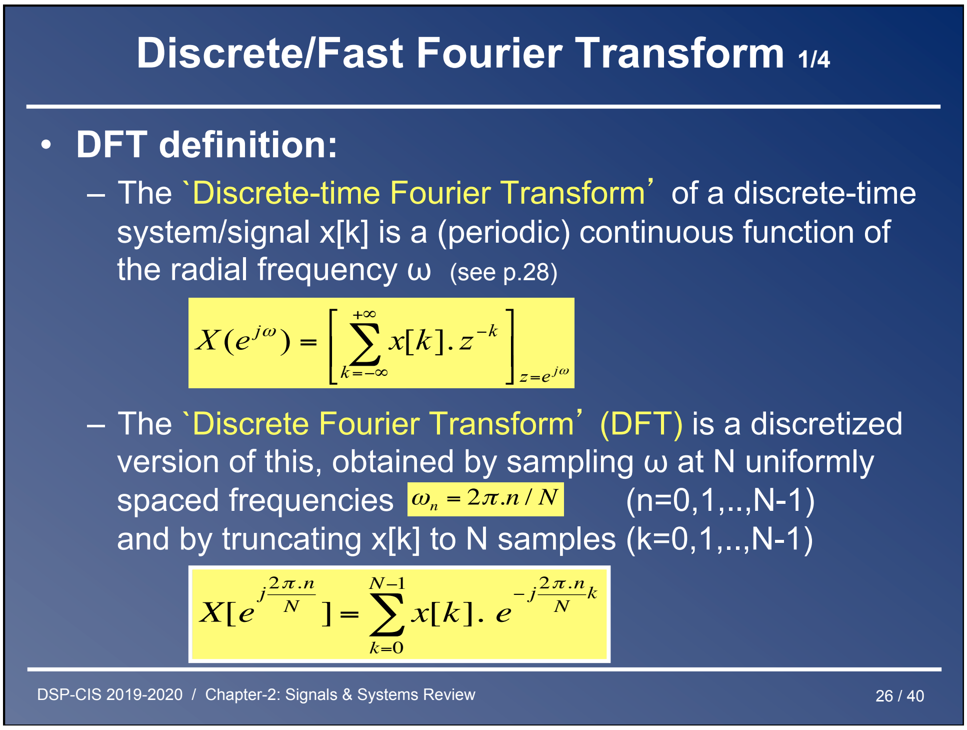
    e.g. $f=\frac{f_s}{2} \Rightarrow \omega=\pi$

    $$H(e^{j\omega})=...=\mathbb{F}\{h_D(t)\}=\mathbb{F}\{\sum_k h[k].\delta(t-k.T_s)\} \quad , \quad \omega=2\pi.\frac{f}{f_s}$$

  - The **frequency spectrum** $U(e^{j\omega})$ or $Y(e^{j\omega})$ of a discrete-time <u>signal</u> is equal to the Fourier transform of the continuous-time impulse sequence constructed with u[k] or y[k]

    $$U(e^{j\omega})=...=\mathbb{F}\{u_D(t)\}=\mathbb{F}\{\sum_k u[k].\delta(t-k.T_s)\} \quad , \quad \omega=2\pi.\frac{f}{f_s}$$

  - $\mathbb{F}\{y_D(t)\}=\mathbb{F}\{h_D(t)\}.\mathbb{F}\{u_D(t)\}$ corresponds to continuous-time $Y(f)=H(f).U(f)$ iff $U(f),Y(f),H(f)$ are bandlimited (➔no aliasing)



Discrete/Fast Fourier Transform 1/4

- **DFT definition:**
  - The \`Discrete-time Fourier Transform' of a discrete-time system/signal x[k] is a (periodic) continuous function of the radial frequency ω (see p.28)

    $$X(e^{j\omega})=\left[\sum_{k=-\infty}^{+\infty} x[k].z^{-k}\right]_{z=e^{j\omega}}$$

  - The \`Discrete Fourier Transform' (DFT) is a discretized version of this, obtained by sampling ω at N uniformly spaced frequencies $\omega_n=2\pi.n/N$ (n=0,1,..,N-1) and by truncating x[k] to N samples (k=0,1,..,N-1)

    $$X[e^{j\frac{2\pi.n}{N}}]=\sum_{k=0}^{N-1} x[k].\ e^{-j\frac{2\pi.n}{N}k}$$

Discrete/Fast Fourier Transform 2/4

- **DFT & Inverse DFT (IDFT):**
  - An *N*-point DFT sequence can be calculated from an *N*-point time sequence:

$$X[e^{j\frac{2\pi.n}{N}}]=\sum_{k=0}^{N-1}x[k].\,e^{-j\frac{2\pi.n}{N}k}$$

= DFT

  - Conversely, an *N*-point time sequence can be calculated from an *N*-point DFT sequence:

$$x[k]=\frac{1}{N}\sum_{n=0}^{N-1}X[e^{j\frac{2\pi.n}{N}}].\,e^{j\frac{2\pi.n}{N}k}$$

= IDFT

Discrete/Fast Fourier Transform 3/4

- **DFT/IDFT in matrix form**
  - Using shorthand notation..
  - ..the **DFT** can be rewritten as

$$\left\{\begin{array}{rcl} X[n] & = & X\left[e^{j\frac{2\pi}{N}n}\right] \\ W_N & = & e^{-j\frac{2\pi}{N}} \end{array}\right.$$

$$\begin{bmatrix} X[0] \\ X[1] \\ \dots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^1 & \dots & W_N^{(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^0 & W_N^{(N-1)} & \dots & W_N^{(N-1)^2} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

$$\mathbf{X} = \mathbf{F}.\mathbf{x}$$

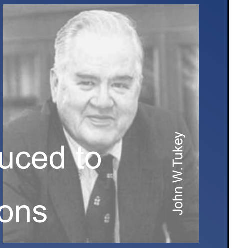
  - ..the **IDFT** can be rewritten as

$$\begin{bmatrix} x[0] \\ x[1] \\ \dots \\ x[N-1] \end{bmatrix} = \frac{1}{N} \begin{bmatrix} W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^{-1} & \dots & W_N^{-(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W_N^0 & W_N^{-(N-1)} & \dots & W_N^{-(N-1)^2} \end{bmatrix} \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}$$

$$\mathbf{x} = \mathbf{F}^{-1}.\mathbf{X} = \frac{1}{N}\mathbf{F}^*.\mathbf{X}$$

Discrete/Fast Fourier Transform 4/4

- **Fast Fourier Transform (FFT)** (1805/1965)
  - Divide-and-conquer approach:
    - Split up N-point DFT in two N/2-point DFT's
    - Split up two N/2-point DFT's in four N/4-point DFT's
    - …
    - Split up N/2 2-point DFT's in N 1-point DFT's
    - **Calculate N 1-point DFT's**
    - Rebuild N/2 2-point DFT's from N 1-point DFT's
    - …
    - Rebuild two N/2-point DFT's from four N/4-point DFT's
    - Rebuild N-point DFT from two N/2-point DFT's
  - **DFT complexity** of N^2 multiplications is reduced to **FFT complexity** of $O(N.\log_2(N))$ multiplications
  - Similar IFFT



Multi-Rate Systems 1/11

- **Decimation : decimator (=downsampler)**

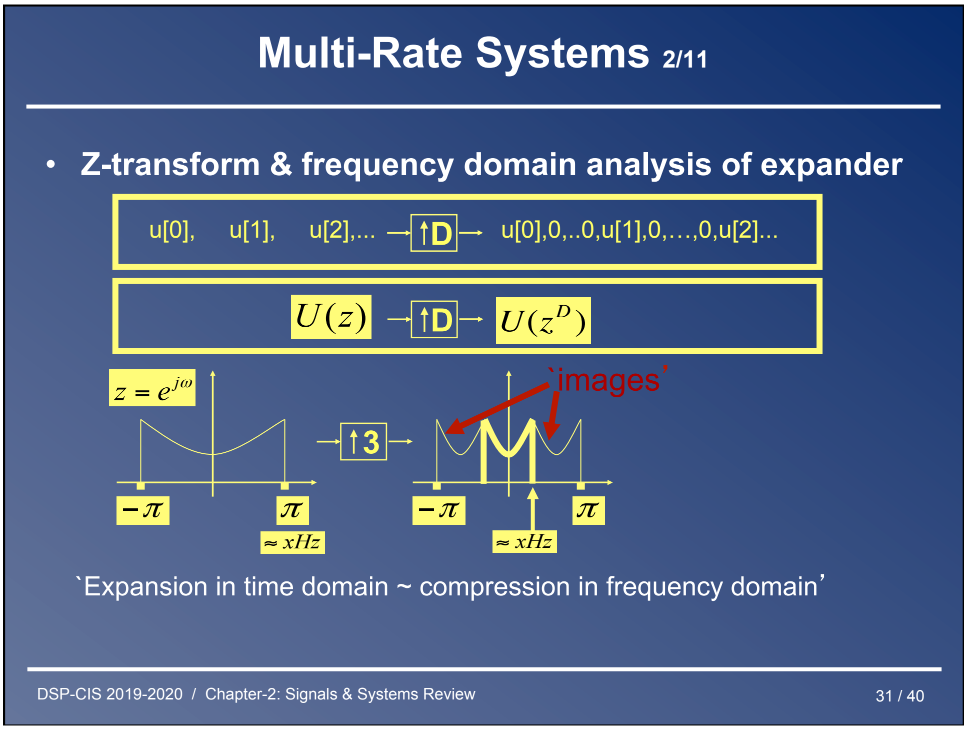
$$u[0],u[1],u[2]... \rightarrow \boxed{\downarrow D} \rightarrow u[0],\quad u[D],\quad u[2D]...$$

Example : u[k]: 1,2,3,4,5,6,7,8,9,…
2-fold downsampling: 1,3,5,7,9,...

- **Interpolation : expander (=upsampler)**

$$u[0],\quad u[1],\quad u[2],... \rightarrow \boxed{\uparrow D} \rightarrow u[0],0,..0,u[1],0,\dots,0,u[2]...$$

Example : u[k]: 1,2,3,4,5,6,7,8,9,…
2-fold upsampling: 1,0,2,0,3,0,4,0,5,0...



Multi-Rate Systems 2/11

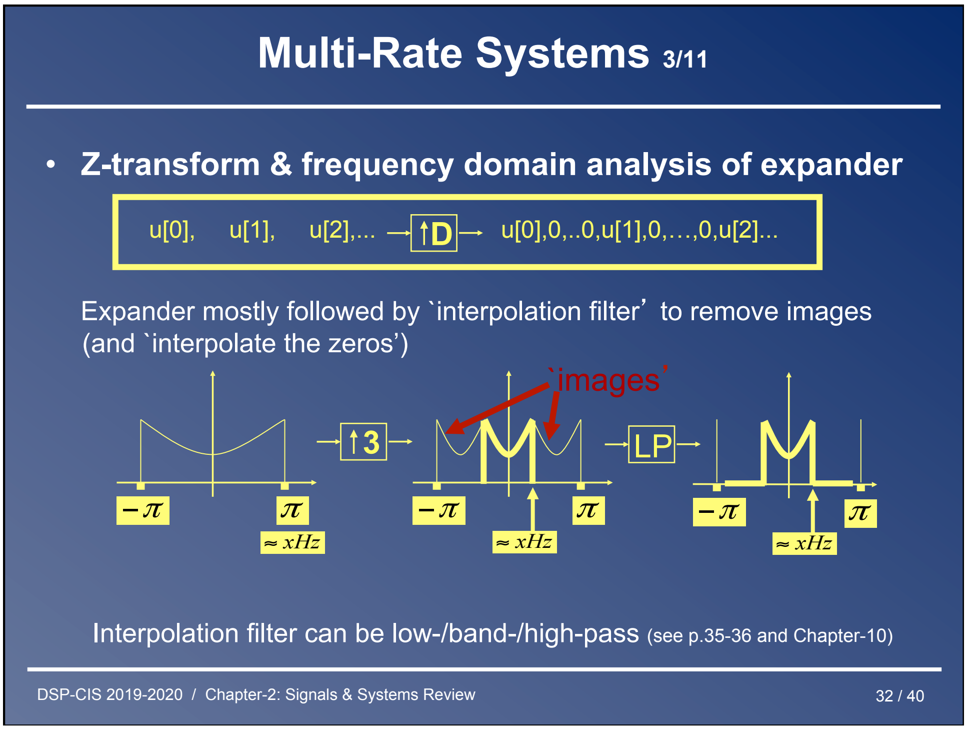
- **Z-transform & frequency domain analysis of expander**

u[0], u[1], u[2],... $\rightarrow \boxed{\uparrow D} \rightarrow$ u[0],0,..0,u[1],0,…,0,u[2]...

$U(z) \rightarrow \boxed{\uparrow D} \rightarrow U(z^D)$

$z = e^{j\omega}$

`Expansion in time domain ~ compression in frequency domain'



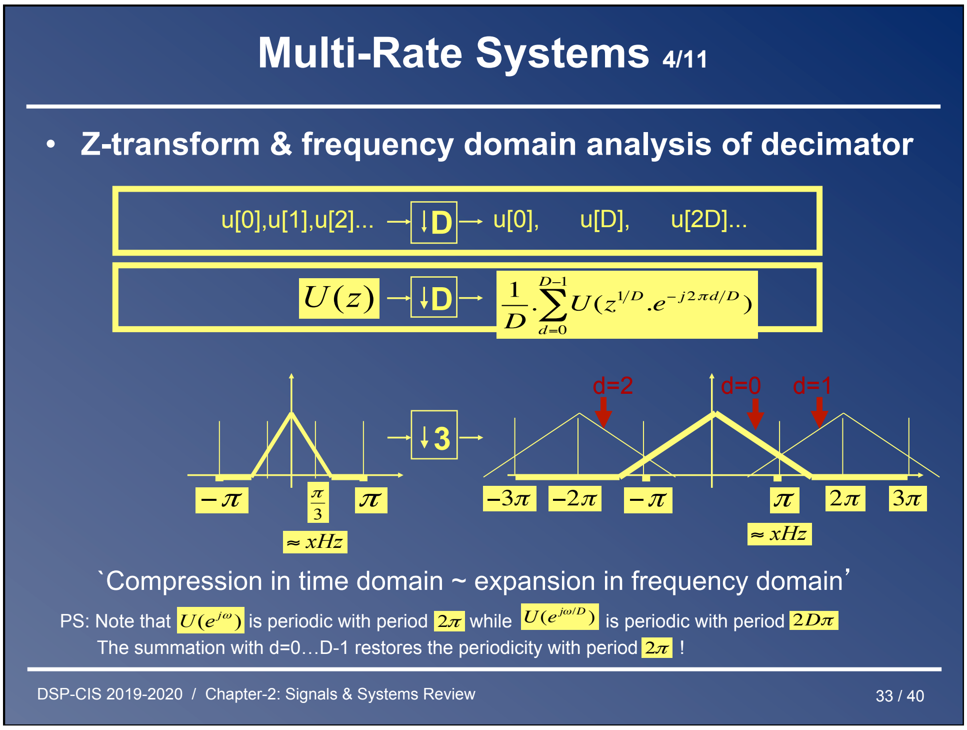
Multi-Rate Systems 3/11

- **Z-transform & frequency domain analysis of expander**

u[0], u[1], u[2],... $\rightarrow \boxed{\uparrow D} \rightarrow$ u[0],0,..0,u[1],0,…,0,u[2]...

Expander mostly followed by `interpolation filter' to remove images (and `interpolate the zeros')

Interpolation filter can be low-/band-/high-pass (see p.35-36 and Chapter-10)



Multi-Rate Systems 4/11

- **Z-transform & frequency domain analysis of decimator**

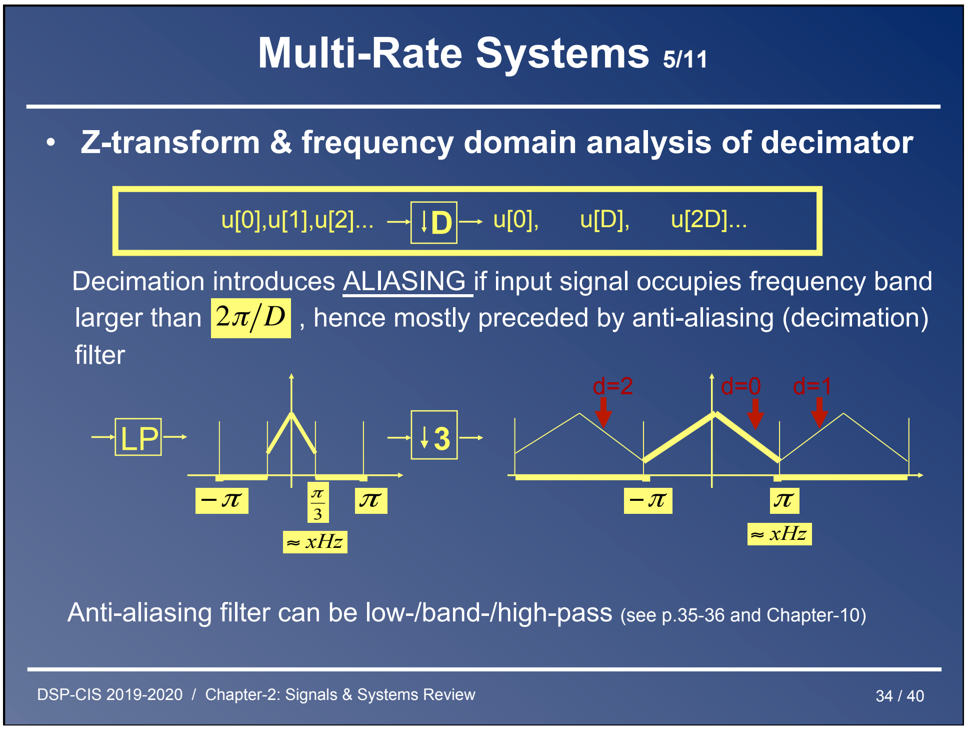
u[0],u[1],u[2]... $\rightarrow \boxed{\downarrow D} \rightarrow$ u[0], u[D], u[2D]...

$$U(z) \rightarrow \boxed{\downarrow D} \rightarrow \frac{1}{D}.\sum_{d=0}^{D-1} U(z^{1/D}.e^{-j2\pi d/D})$$

`Compression in time domain ~ expansion in frequency domain'

PS: Note that $U(e^{j\omega})$ is periodic with period 2π while $U(e^{j\omega/D})$ is periodic with period $2D\pi$

The summation with d=0…D-1 restores the periodicity with period 2π !



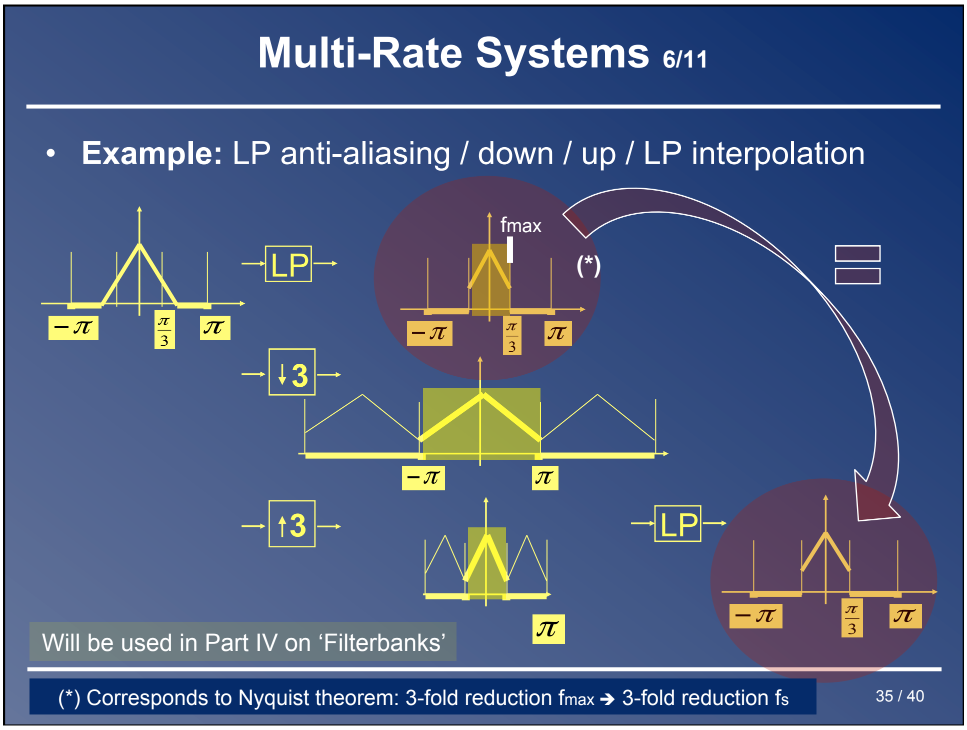
Multi-Rate Systems 5/11

- **Z-transform & frequency domain analysis of decimator**

u[0],u[1],u[2]... $\rightarrow \boxed{\downarrow D} \rightarrow$ u[0], u[D], u[2D]...

Decimation introduces ALIASING if input signal occupies frequency band larger than $2\pi/D$, hence mostly preceded by anti-aliasing (decimation) filter

Anti-aliasing filter can be low-/band-/high-pass (see p.35-36 and Chapter-10)



Multi-Rate Systems 6/11

- **Example:** LP anti-aliasing / down / up / LP interpolation

Will be used in Part IV on 'Filterbanks'

(*) Corresponds to Nyquist theorem: 3-fold reduction f_{max} ➔ 3-fold reduction f_s

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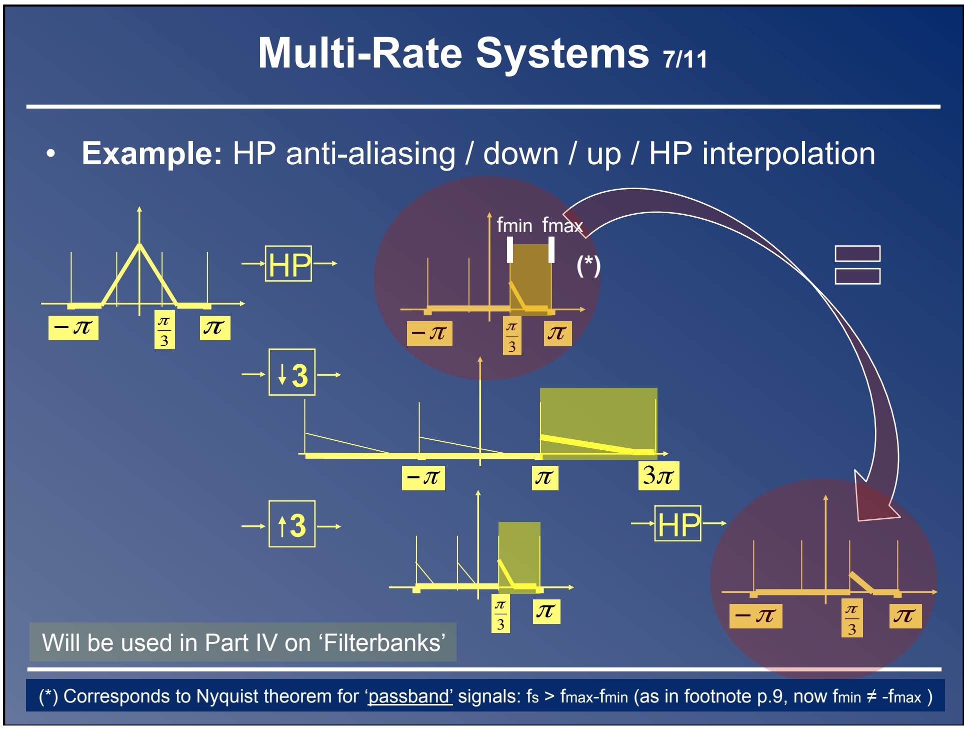
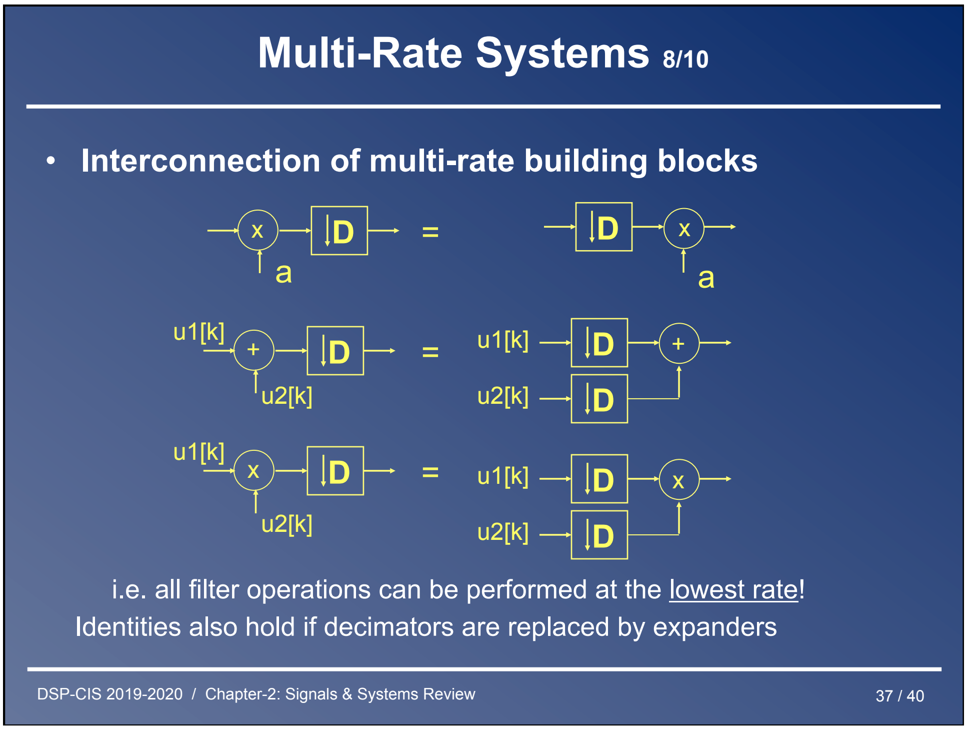


Multi-Rate Systems 7/11

- **Example:** HP anti-aliasing / down / up / HP interpolation

Will be used in Part IV on 'Filterbanks'

(*) Corresponds to Nyquist theorem for '<u>passband</u>' signals: $f_s > f_{max}-f_{min}$ (as in footnote p.9, now $f_{min} \neq -f_{max}$)

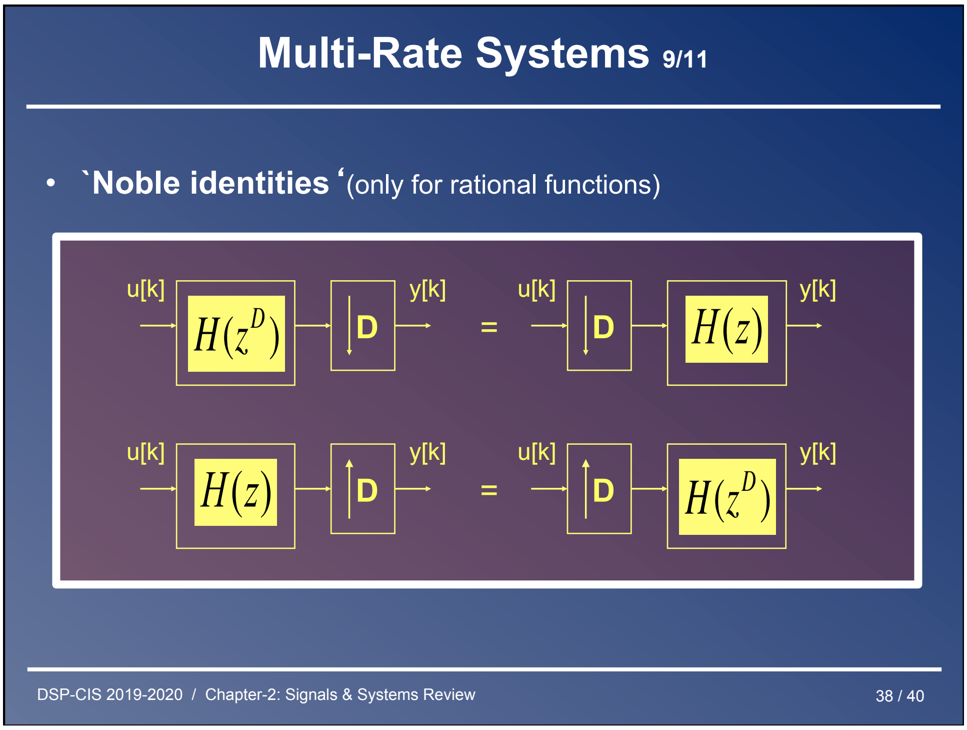


Multi-Rate Systems 8/10

- **Interconnection of multi-rate building blocks**

i.e. all filter operations can be performed at the <u>lowest rate</u>!

Identities also hold if decimators are replaced by expanders



Multi-Rate Systems 9/11

- **`Noble identities ‘**(only for rational functions)

Multi-Rate Systems 10/10

Application of `noble identities : efficient multi-rate realizations of FIR filters through…

- **Polyphase decomposition:**

Example : (2-fold decomposition)

$$\begin{aligned} H(z) &= h[0]+h[1].z^{-1}+h[2].z^{-2}+h[3].z^{-3}+h[4].z^{-4}+h[5].z^{-5}+h[6].z^{-6} \\ &= \underbrace{(h[0]+h[2].z^{-2}+h[4].z^{-4}+h[6].z^{-6})}_{E_0(z^2)}+z^{-1}.\underbrace{(h[1]+h[3].z^{-2}+h[5].z^{-4})}_{E_1(z^2)} \end{aligned}$$

Example : (3-fold decomposition)

$$\begin{aligned} H(z) &= h[0]+h[1].z^{-1}+h[2].z^{-2}+h[3].z^{-3}+h[4].z^{-4}+h[5].z^{-5}+h[6].z^{-6} \\ &= \underbrace{(h[0]+h[3].z^{-3}+h[6].z^{-6})}_{E_0(z^3)}+z^{-1}.\underbrace{(h[1]+h[4].z^{-3})}_{E_1(z^3)}+z^{-2}.\underbrace{(h[2]+h[5].z^{-3})}_{E_2(z^3)} \end{aligned}$$

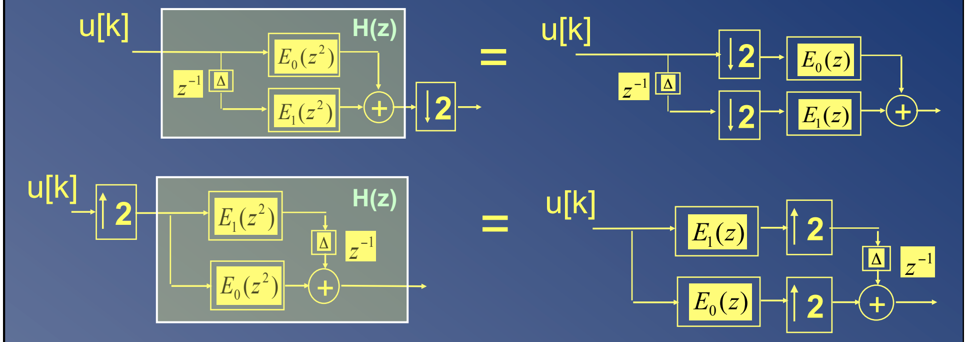
General: (D-fold decomposition)

$$H(z)=\sum_{k=-\infty}^{\infty} h[k].z^{-k}=\sum_{d=0}^{D-1} z^{-d}.E_d(z^D) \qquad , \qquad E_d(z)=\sum_{k=-\infty}^{\infty} h[D.k+d].z^{-k}$$

Multi-Rate Systems 11/11

- **Polyphase decomposition:**

Example : efficient realization of FIR decimation/interpolation filter



i.e. all filter operations can be performed at the lowest rate!