Back to the Roots:
Bart's legacy in the Quantum World

Frank Verstraete

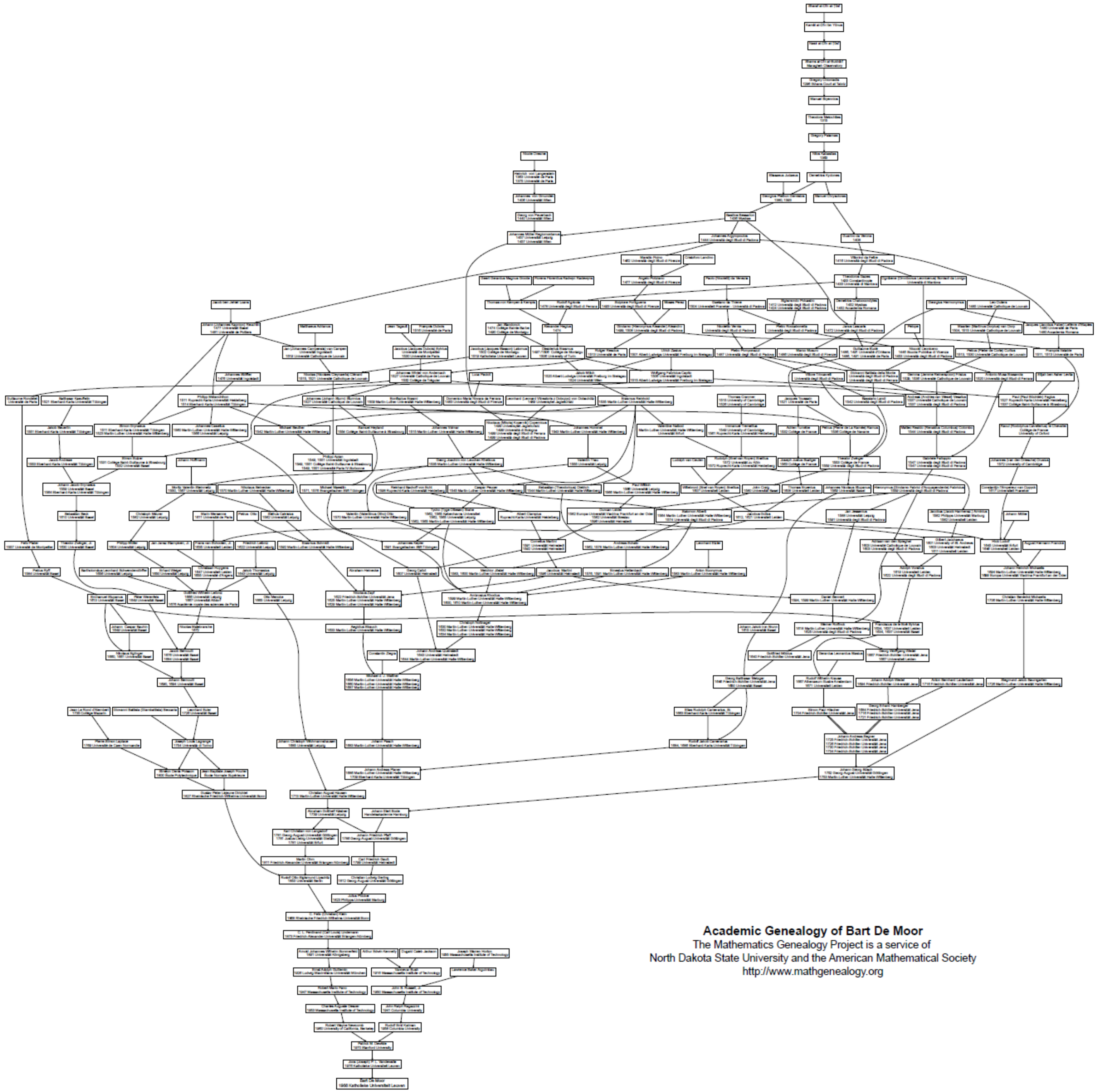
Ghent University

Leuven 08/07/22

It's the linear algebra, stupid!

(Carville/Clinton/De Moor)

- I was raised in the group of Bart with the mantra "It's the linear algebra, stupid"
- Took this advice to heart, and applied it to the then (1999) new field of quantum entanglement / quantum information / quantum computation
- I was - among many many others - extremely fortunate to have an advisor like Bart who believed in me – and gave me complete freedom in exploring my own interests
- The passion, enthusiasm and interests of Bart are unsurpassed – he is the ultimate post-Renaissance-man and role model - and therefore he had to understand quantum physics, the most successful theory of all matter



Academic Genealogy of Bart De Moor

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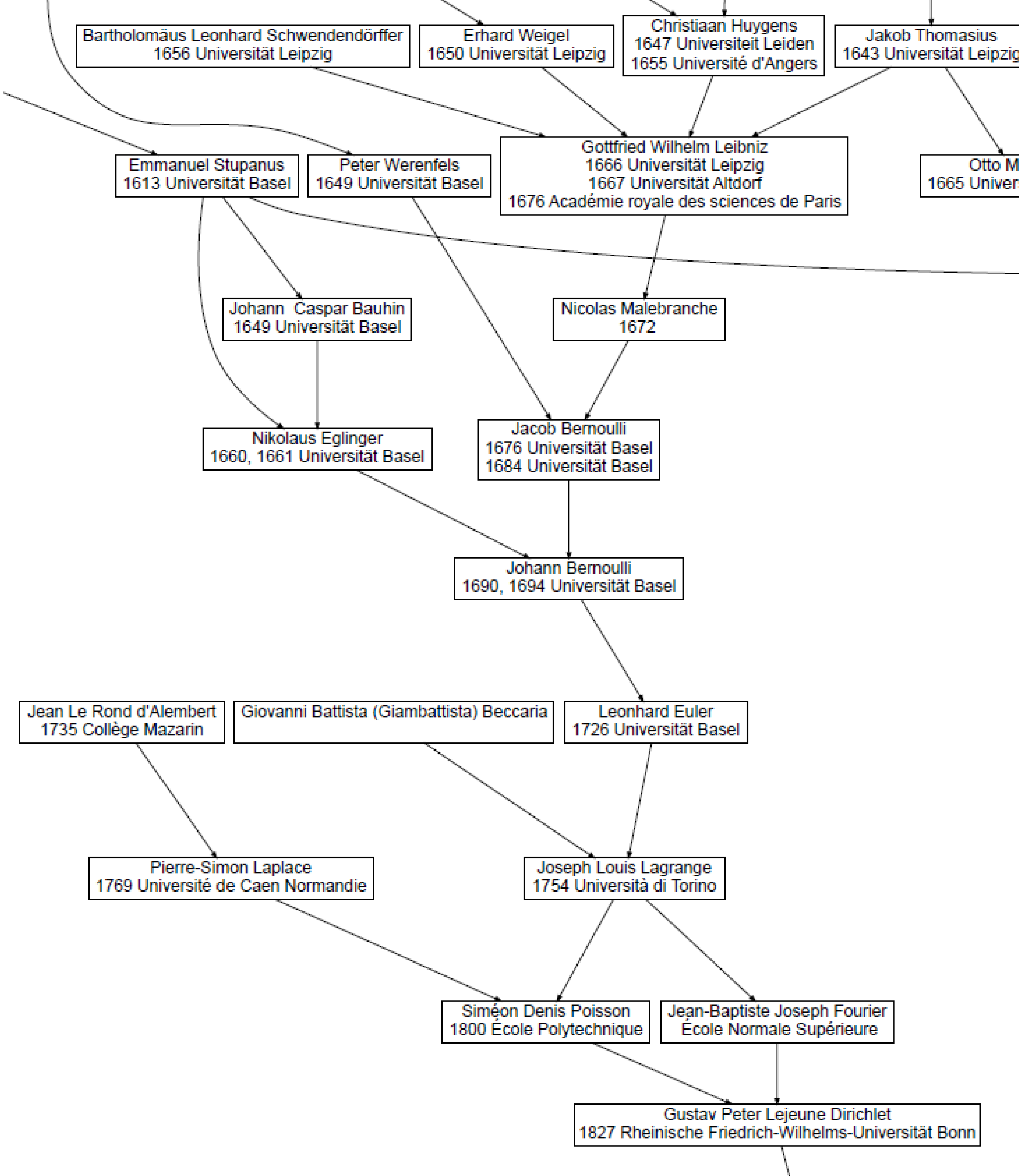
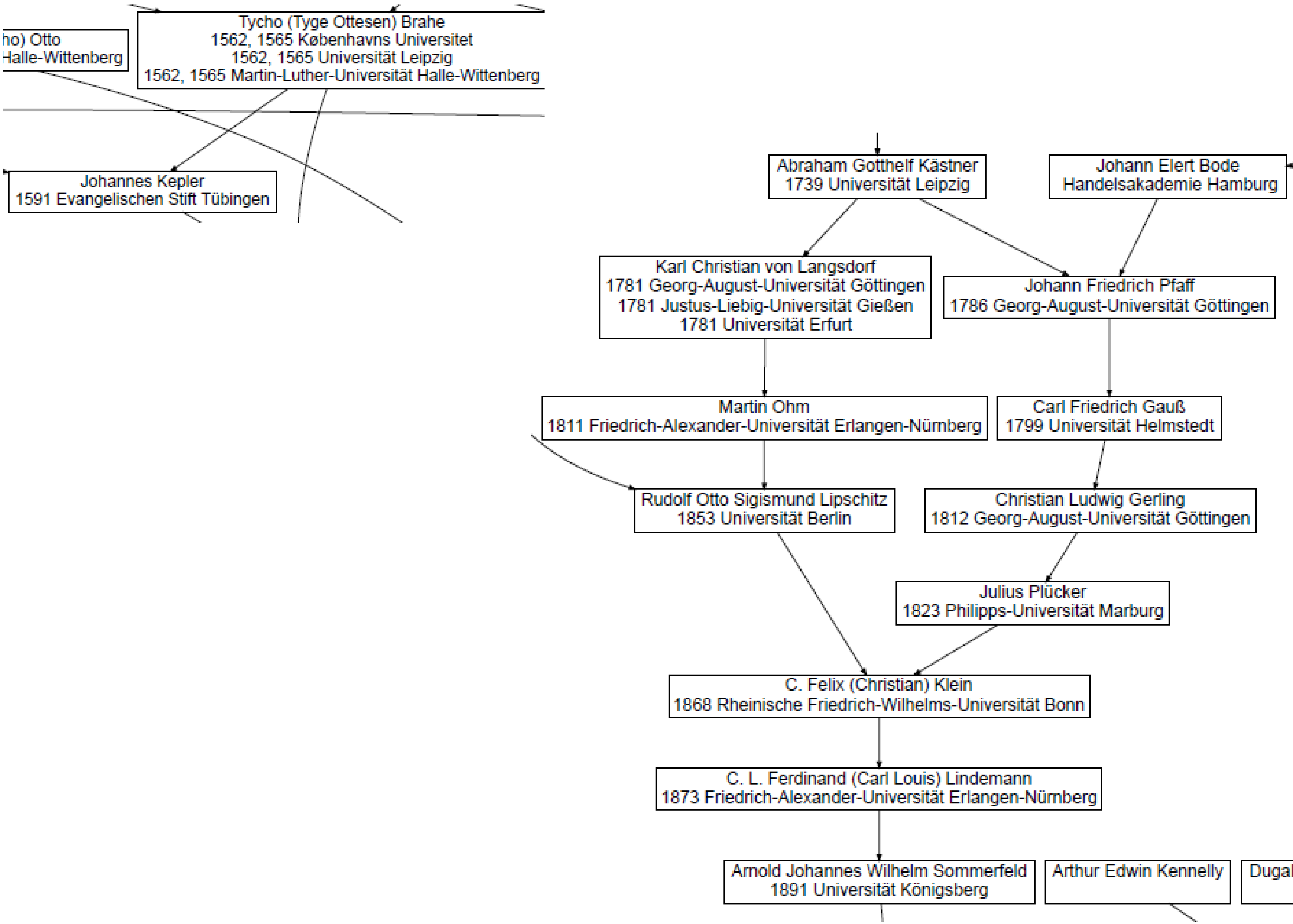
North Dakota State University and the American Mathematical Society
http://www.mathgenealogy.org



Bartholomäus Leonhard Schwendendörffer
1656 Universität Leipzig
Erhard Weigel
1650 Universität Leipzig
Christiaan Huygens
1647 Universiteit Leiden
1655 Université d'Angers
Jakob Thomasius
1643 Universität Leipzig
Emmanuel Stupanus
1613 Universität Basel
Peter Werenfels
1649 Universität Basel
Gottfried Wilhelm Leibniz
1666 Universität Leipzig
1667 Universität Altdorf
1676 Académie royale des sciences de Paris
Otto M
1665 Univer
Johann Caspar Bauhin
1649 Universität Basel
Nicolas Malebranche
1672
Nikolaus Eglinger
1660, 1661 Universität Basel
Jacob Bernoulli
1676 Universität Basel
1684 Universität Basel
Johann Bernoulli
1690, 1694 Universität Basel
Jean Le Rond d'Alembert
1735 Collège Mazarin
Giovanni Battista (Giambattista) Beccaria
Leonhard Euler
1726 Universität Basel
Pierre-Simon Laplace
1769 Université de Caen Normandie
Joseph Louis Lagrange
1754 Università di Torino
Siméon Denis Poisson
1800 École Polytechnique
Jean-Baptiste Joseph Fourier
École Normale Supérieure
Gustav Peter Lejeune Dirichlet
1827 Rheinische Friedrich-Wilhelms-Universität Bonn


Tycho (Tyge Ottesen) Brahe
1562, 1565 Københavns Universitet
1562, 1565 Universität Leipzig
1562, 1565 Martin-Luther-Universität Halle-Wittenberg
ho) Otto
Halle-Wittenberg
Johannes Kepler
1591 Evangelischen Stift Tübingen
Abraham Gotthelf Kästner
1739 Universität Leipzig
Johann Elert Bode
Handelsakademie Hamburg
Karl Christian von Langsdorf
1781 Georg-August-Universität Göttingen
1781 Justus-Liebig-Universität Gießen
1781 Universität Erfurt
Johann Friedrich Pfaff
1786 Georg-August-Universität Göttingen
Martin Ohm
1811 Friedrich-Alexander-Universität Erlangen-Nürnberg
Carl Friedrich Gauß
1799 Universität Helmstedt
Rudolf Otto Sigismund Lipschitz
1853 Universität Berlin
Christian Ludwig Gerling
1812 Georg-August-Universität Göttingen
Julius Plücker
1823 Philipps-Universität Marburg
C. Felix (Christian) Klein
1868 Rheinische Friedrich-Wilhelms-Universität Bonn
C. L. Ferdinand (Carl Louis) Lindemann
1873 Friedrich-Alexander-Universität Erlangen-Nürnberg
Arnold Johannes Wilhelm Sommerfeld
1891 Universität Königsberg
Arthur Edwin Kennelly
Dugal

Entanglement: what is it?

- Quantum states are described as vectors in a Hilbert space (superposition principle):

$$|\psi_A\rangle = \sum_i \psi_i |i\rangle$$

- Most relevant feature of Hilbert space for our purposes: the tensor product structure for many body systems

$$|\psi_{AB}\rangle = \sum_{ij} \psi_{ij} |i\rangle|j\rangle \qquad |\psi\rangle = \sum_{ijk\ldots} X_{ijk\ldots} |i\rangle|j\rangle\ldots$$

  - Any state which does not factorize with respect to the preferred basis ($\psi_{ij} \neq \chi_i \, . \, \xi_j$) is called entangled

Four qubits can be entangled in nine different ways
F Verstraete, J Dehaene, B De Moor, H Verschelde - Physical Review A, 2002 - APS
We consider a single copy of a pure four-partite state of qubits and investigate its behavior under the action of stochastic local quantum operations assisted by classical communication (...
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A comparison of the entanglement measures negativity and concurrence
F Verstraete, K Audenaert, J Dehaene... - Journal of Physics A ..., 2001 - iopscience.iop.org
In this paper we investigate two different entanglement measures in the case of mixed states of two qubits. We prove that the negativity of a state can never exceed its concurrence and ...
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Variational characterizations of separability and entanglement of formation
K Audenaert, F Verstraete, B De Moor - Physical Review A, 2001 - APS
In this paper we develop a mathematical framework for the characterization of separability and entanglement of formation (EOF) of general bipartite states. These characterizations are ...
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Normal forms and entanglement measures for multipartite quantum states
F Verstraete, J Dehaene, B De Moor - Physical Review A, 2003 - APS
A general mathematical framework is presented to describe local equivalence classes of multipartite quantum states under the action of local unitary and local filtering operations. This ...
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Maximally entangled mixed states of two qubits
F Verstraete, K Audenaert, B De Moor - Physical Review A, 2001 - APS
We consider mixed states of two qubits and show under which global unitary operations their entanglement is maximized. This leads to a class of states that is a generalization of the ...
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Lorentz singular-value decomposition and its applications to pure states of three qubits
F Verstraete, J Dehaene, B De Moor - Physical Review A, 2002 - APS
All mixed states of two qubits can be brought into normal form by the action of local operations and classical communication operations of the kind $\rho'=(A\otimes B)\,\rho\,(A\otimes B)^\dagger$. These normal ...
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Local permutations of products of Bell states and entanglement distillation
J Dehaene, M Van den Nest, B De Moor, F Verstraete - Physical Review A, 2003 - APS
We present different algorithms for mixed-state multicopy entanglement distillation for pairs of qubits. Our algorithms perform significantly better than the best-known algorithms. Better ...
Save Cite Cited by 81 Related articles All 10 versions Web of Science: 47

On the geometry of entangled states
F Verstraete, J Dehaene, B De Moor - journal of modern optics, 2002 - Taylor & Francis
The basic question that is addressed in this paper is finding the closest separable state for a given entangled state, measured with the Hilbert-Schmidt distance. While this problem is in ...
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Local filtering operations on two qubits
F Verstraete, J Dehaene, B DeMoor - Physical Review A, 2001 - APS
We consider one single copy of a mixed state of two qubits and investigate how its entanglement changes under local quantum operations and classical communications (LQCC) of the ...
Save Cite Cited by 220 Related articles All 7 versions Web of Science: 139

Four qubits can be entangled in nine different ways

F. Verstraete, J. Dehaene, B. De Moor, and H. Verschelde
Phys. Rev. A **65**, 052112 – Published 25 April 2002

ABSTRACT

We consider a single copy of a pure four-partite state of qubits and investigate its behavior under the action of stochastic local quantum operations assisted by classical communication (SLOCC). This leads to a complete classification of all different classes of pure states of four qubits. It is shown that there exist nine families of states corresponding to nine different ways of entangling four qubits. The states in the generic family give rise to Greenberger-Horne-Zeilinger-like entanglement. The other ones contain essentially two-or three-qubit entanglement distributed among the four parties. The concept of concurrence and 3-tangle is generalized to the case of mixed states of four qubits, giving rise to a seven-parameter family of entanglement monotones. Finally, the SLOCC operations maximizing all these entanglement monotones are derived, yielding the optimal single-copy distillation protocol.

Received 29 November 2001

Workshop: Recognizing wildness in all its guises

- Workshop in Santa Fe, 2015

The workshop will focus on wildness in physics, math, and computer science. A classification problem is wild if it contains the problem of classifying the representations of any finite-dimensional algebra. In physics, the perhaps more interesting example of wildness is the classification of entangled states up to local unitaries or local operations with classical communication (as much as possible - a complete classification in general is incredibly unlikely). The geometry of the local unitary orbits are current frontier examples to try to understand in the Geometric Complexity Theory program towards P vs NP. In CS, this same problem is furthermore closely related to the complexity of Graph Isomorphism, which is neither known to be in P nor known to be NP-complete (a rare gem). These questions are also essentially the same as those arising at the intersection of representation theory and algebraic geometry that mathematicians have been interested in for decades.

Despite these connections, these three disciplines - physics, CS, and math - have largely been studying these problems from different angles and with different tools. The workshop will bring them together, with special attention paid to bridging the language barriers between these fields, in the hope that combining ideas and methods from different fields can lead to a better understanding in all of them.

$$|\psi'\rangle = A_1 \otimes A_2 \otimes A_3 \otimes A_4 |\psi\rangle$$

Theorem 1. Given a complex $n \times n$ matrix R, then there always exist complex square orthogonal matrices O_1 and O_2 such that $R' = O_1 R O_2$ is a unique direct sum of blocks of the form

(1) $m \times m$ blocks of the form $(\lambda_j I_m + S_m)$ being symmetric Jordan blocks (see, for example, [14] 4.4.9), and λ_j is a complex parameter (note that the case $m = 1$ corresponds to the scalar case);

(2) $m \times m$ blocks consisting of an upper left $(m_1 + 1) \times m_1$ part being the matrix obtained by taking the odd rows and even columns of an $(2m_1 + 1) \times (2m_1 + 1)$ symmetric Jordan block, and a lower right $(m - m_1 - 1) \times (m - m_1)$ part being the transpose of the matrix obtained by taking the odd rows and even columns of a $[2(m - m_1) - 1] \times [2(m - m_1) - 1]$ symmetric Jordan block.

$$G_{abcd} = \frac{a+d}{2}(|0000\rangle + |1111\rangle) + \frac{a-d}{2}(|0011\rangle + |1100\rangle) + \frac{b+c}{2}(|0101\rangle + |1010\rangle) + \frac{b-c}{2}(|0110\rangle + |1001\rangle),$$

$$L_{abc_2} = \frac{a+b}{2}(|0000\rangle + |1111\rangle) + \frac{a-b}{2}(|0011\rangle + |1100\rangle) + c(|0101\rangle + |1010\rangle) + |0110\rangle,$$

$$L_{a_2b_2} = a(|0000\rangle + |1111\rangle) + b(|0101\rangle + |1010\rangle) + |0110\rangle + |0011\rangle,$$

$$L_{ab_3} = a(|0000\rangle + |1111\rangle) + \frac{a+b}{2}(|0101\rangle + |1010\rangle) + \frac{a-b}{2}(|0110\rangle + |1001\rangle) + \frac{i}{\sqrt{2}}(|0001\rangle + |0010\rangle + |0111\rangle + |1011\rangle),$$

$$L_{a_4} = a(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle) + (i|0001\rangle + |0110\rangle - i|1011\rangle),$$

$$L_{a_2 0_{3\oplus 1}} = a(|0000\rangle + |1111\rangle) + (|0011\rangle + |0101\rangle + |0110\rangle),$$

$$L_{0_{5\oplus 3}} = |0000\rangle + |0101\rangle + |1000\rangle + |1110\rangle,$$

$$L_{0_{7\oplus 1}} = |0000\rangle + |1011\rangle + |1101\rangle + |1110\rangle,$$

$$L_{0_{3\oplus 1}\bar{0}_{3\oplus 1}} = |0000\rangle + |0111\rangle.$$

A complete set of covariants of the four qubit system

Emmanuel Briand, Jean-Gabriel Luque and Jean-Yves Thibon
Institut Gaspard Monge, Université de Marne-la-Vallée
77454 Marne-la-Vallée cedex, France

We shall give here the first complete solution to the problem of describing the polynomial covariants, an investigation started by Le Paige as early as 1881 [7]. The theory was further advanced by C Segre in 1922 [16], using only geometric methods which led him close to a complete classification of the orbits. Such a complete classification was obtained only recently by Verstratete et al. [21], by exploiting the local isomorphism between SO_4 and $SL_2 \times SL_2$, which permits a reduction of the problem to the classification of complex symmetric matrices up to orthogonal transformations.

Constantin Le Paige

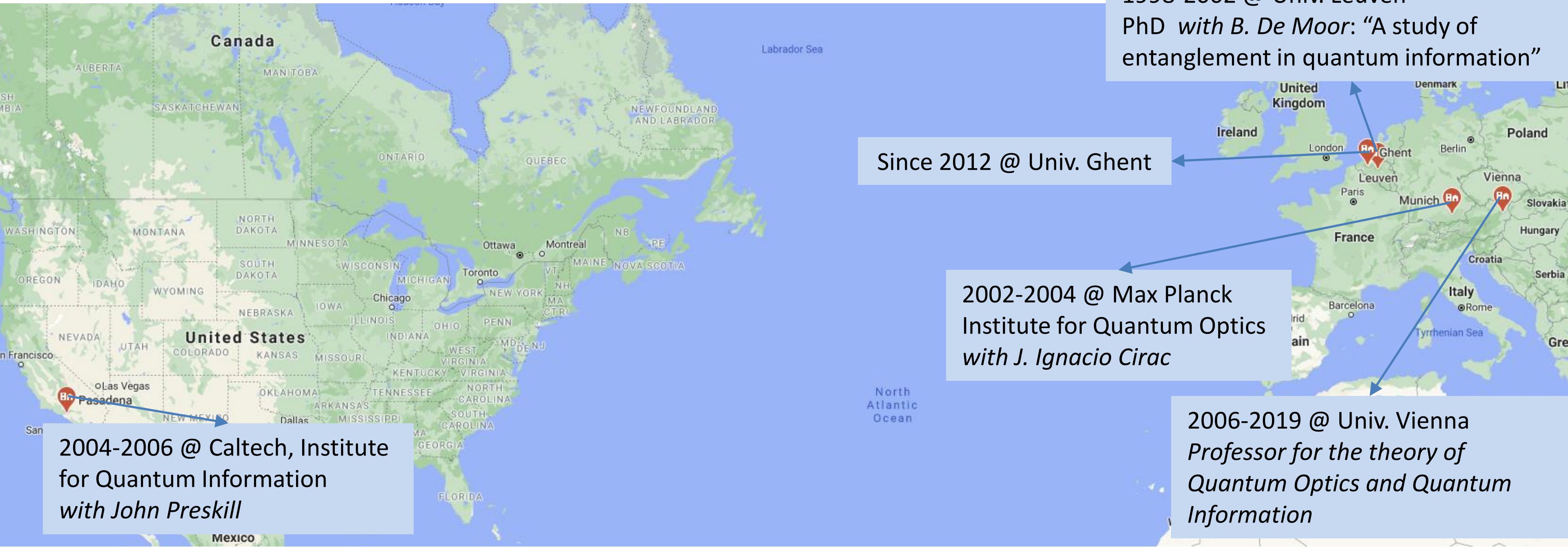
From Wikipedia, the free encyclopedia

Constantin Marie Le Paige (9 March 1852 – 26 January 1929) was a Belgian mathematician.

Born in Liège, Belgium, Le Paige began studying mathematics in 1869 at the University of Liège. After studying analysis under Professor Eugène Charles Catalan, Le Paige became a professor at the Université de Liège in 1882.

While interested in astronomy and the history of mathematics, Le Paige mainly worked on the theory of algebraic form, especially algebraic curves and surfaces and more particularly for his work on the construction of cubic surfaces. Le Paige remained at the university until his retirement in 1922.

My Trajectory



1998-2002 @ Univ. Leuven
PhD *with B. De Moor*: “A study of entanglement in quantum information”

Since 2012 @ Univ. Ghent

2002-2004 @ Max Planck Institute for Quantum Optics
with J. Ignacio Cirac

2006-2019 @ Univ. Vienna
Professor for the theory of Quantum Optics and Quantum Information

2004-2006 @ Caltech, Institute for Quantum Information
with John Preskill

- Quantum Entanglement
- Quantum Many-Body Physics
- Quantum Computing
- Strongly Correlated Systems

} Tensor Networks

- Key contribution: realization that entanglement gives a new perspective on fundamental problems plaguing traditional approaches to the quantum many-body problem, and at the same time provides a completely new language for tackling it
  - Vocabulary consists of entangled qubits, syntax of tensor networks
  - Grand goal: reveal semantics of entanglement patterns

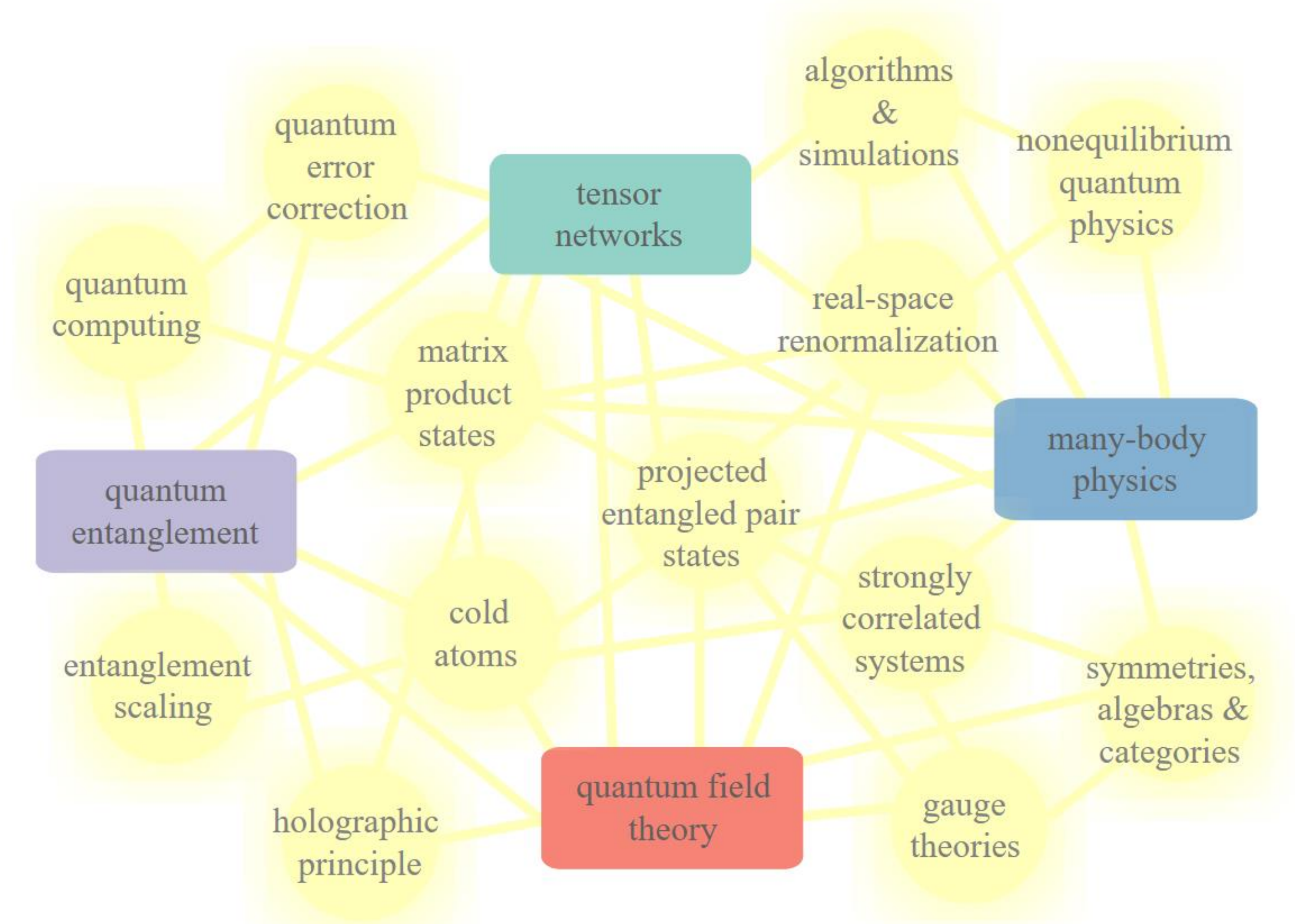
quantumghent.github.io/research/

QuantumGroup@UGent

Research Team BEC Lab Publications Software Links Student Info

Research

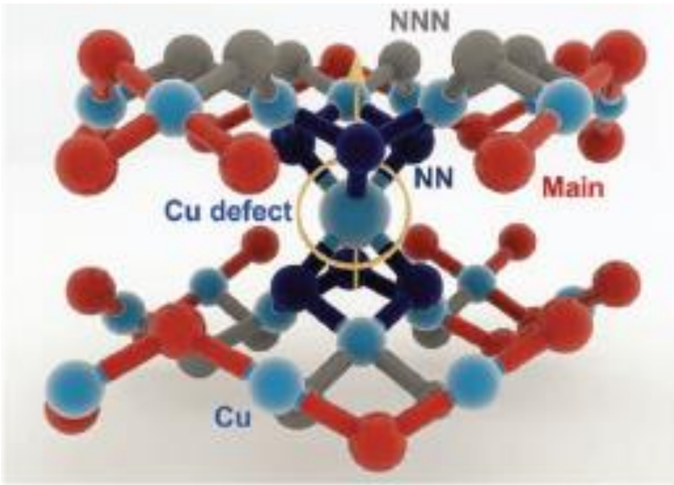
The QuantumGroup@UGent pursues a variety of research goals in the realm of theoretical physics; click on the items to find out more:



The Quantum Many-Body Problem

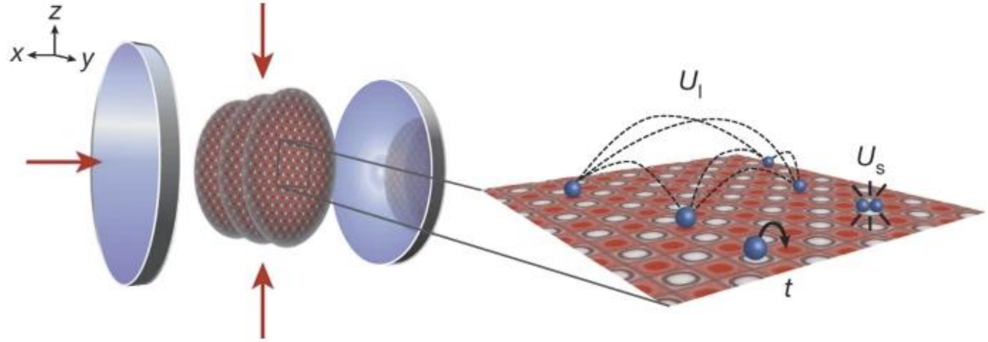
- Problem: describe a large collection of interacting quantum particles
  - Is the central problem in quantum mechanics / physics since 1930’s
    - Quantum chemistry, condensed matter, quantum field theory, … : the whole is more than the sum of its parts
  - We know the (Schrödinger) equations, but cannot solve them: complexity increases exponentially in # particles
    - This scaling is both a curse (physics) and a blessing (computing)
  - Biggest breakthroughs in 20^{th} century theoretical physics almost always consisted of the discovery of approximate methods for solving those equations
    - Hartree-Fock, Monte Carlo, Feynman’s diagrammatic methods, Density Functional Theory, Renormalization Group, …
    - All of those methods rely on the existence of a good fiducial noninteracting state, and those do not exist for e.g. the Hubbard model (high Tc superconductivity)
  - Strongly correlated systems resist solutions using those traditional methods

=> New methods needed!

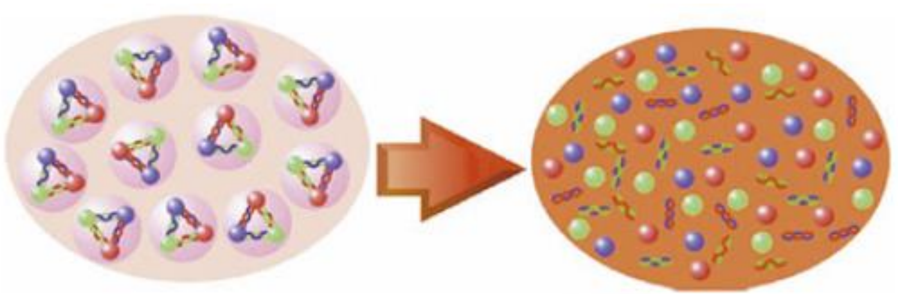


$ZnCu_3(OH)_6Cl_2$.

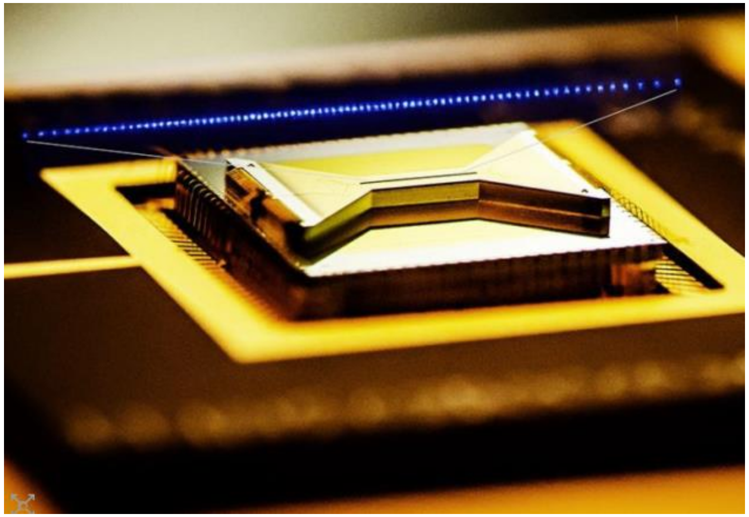
“Evidence for a gapped spin-liquid ground state in a kagome Heisenberg antiferromagnet”
M. Fu et al., Science 350, 355 (2015)



“Quantum phases from competing short- and long-range interactions in an optical lattice”
Landig et al, Nature 532, 476 (2016)



Quark-Gluon Plasma (taken from Riken)



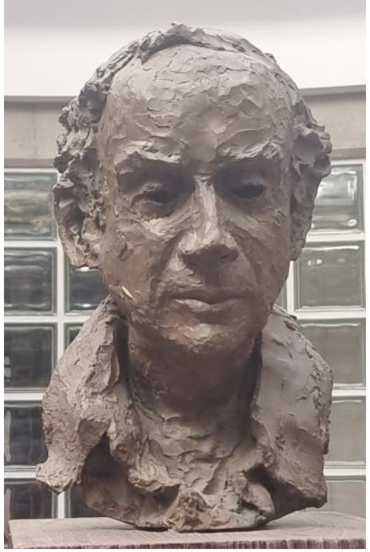
Ion trap quantum computer
(taken from Rainer Blatt)

- Quantum physics of the 1920's:

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

  - Hilbert space is endowed with a tensor product structure

$$|\psi\rangle = \sum_{i_1 i_2 \cdots i_n} \psi_{i_1 i_2 i_3 \cdots} |i_1\rangle |i_2\rangle$$

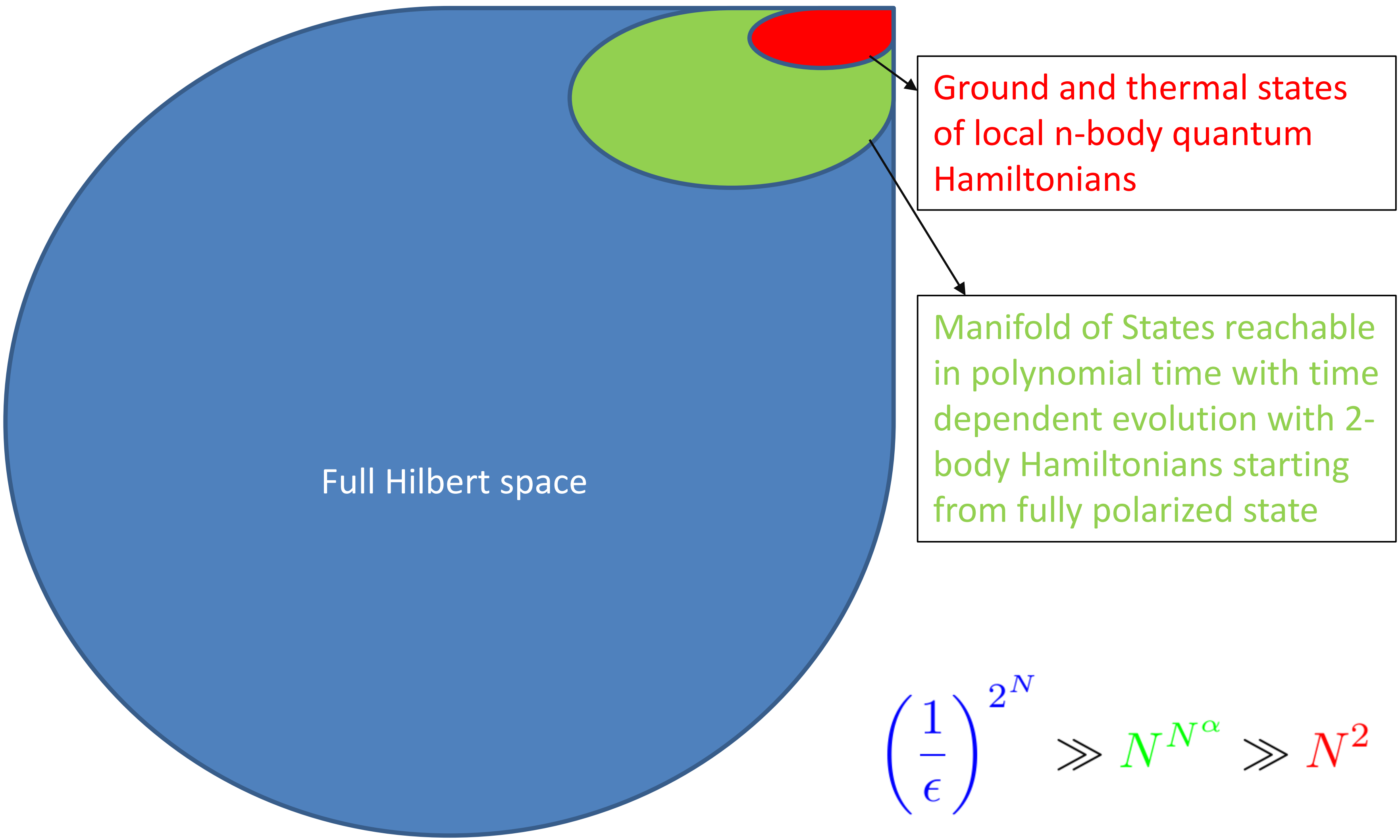


"The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems without too much computation." [Dirac '29]

PRL 106, 170501 (2011) PHYSICAL REVIEW LETTERS week ending 29 APRIL 2011

Quantum Simulation of Time-Dependent Hamiltonians and the Convenient Illusion of Hilbert Space

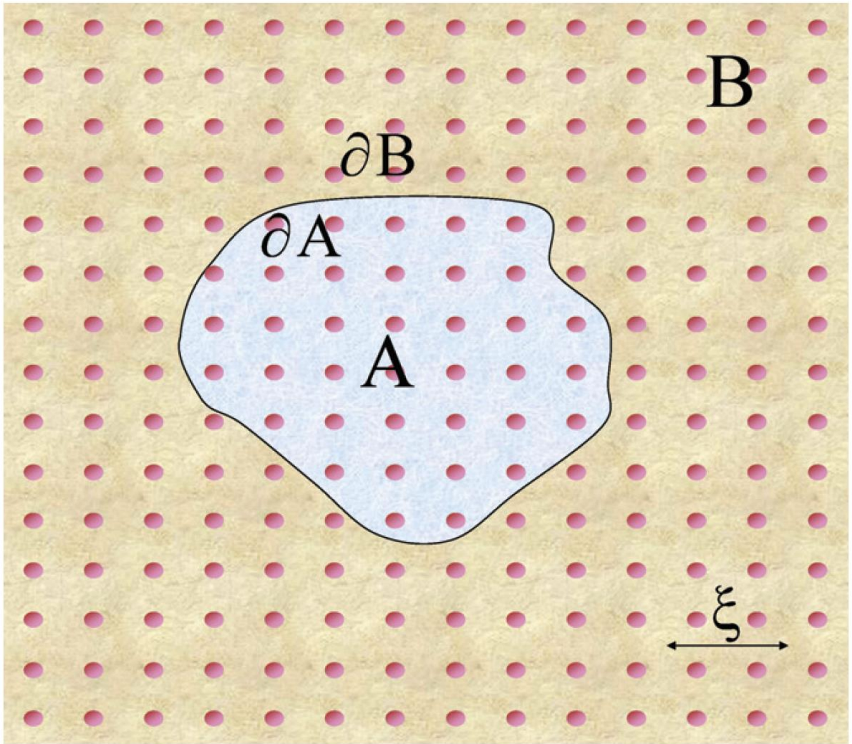
David Poulin,[1] Angie Qarry,[2,3] Rolando Somma,[4] and Frank Verstraete[2]



$$\left(\frac{1}{\epsilon}\right)^{2^N} \gg N^{N^\alpha} \gg N^2$$

Area Laws for the entanglement entropy

- Ground and Gibbs states of interacting quantum many body Hamiltonians with ***local interactions*** have very peculiar properties
  - Area law for the entanglement entropy (ground states) or for mutual information (Gibbs states)
  - "explains" why physics is possible at all



Ground states:

$$S(\rho_A) = c.\partial A$$

Srednicki '93; Hastings '07; …

$$S(\rho_A) = \frac{c}{6}.\log(A/\epsilon)$$

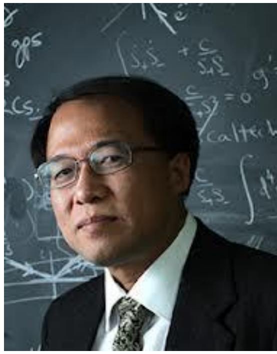
Holzhey, Larsen, Wilczek '94; …

Gibbs states:

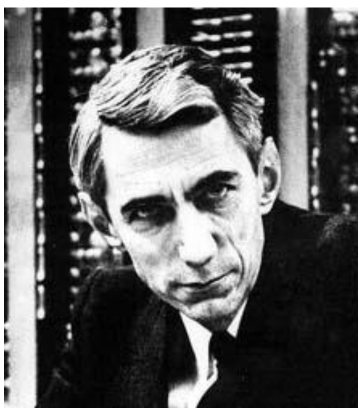
$$\begin{aligned} I(A,B) &= S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \\ &= c.\partial A \end{aligned}$$

Wolf, Hastings, Cirac, FV '08

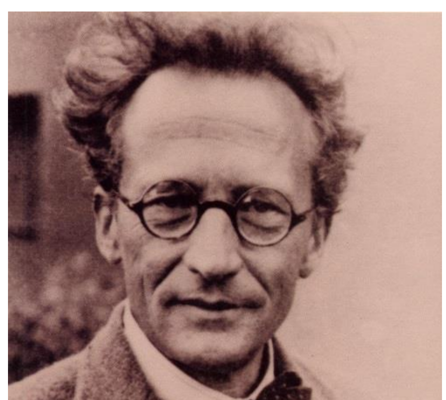
Quantum Physics in the 21st century







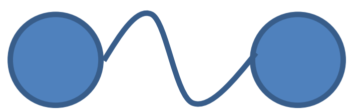
Entanglement

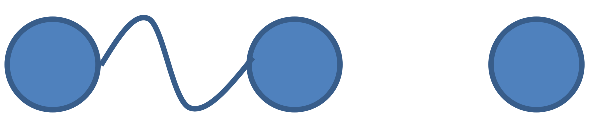


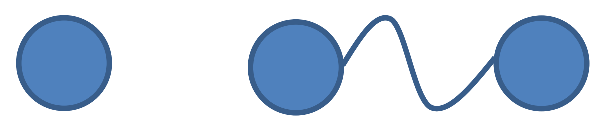
Entanglement in an antiferromagnet

- Heisenberg antiferromagnet (spin ½):

$$\mathcal{H} = \sum_i \vec{S}_i \vec{S}_{i+1}$$

- What is the ground state?
  - 2 spins:  $(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$
  - 3 spins:



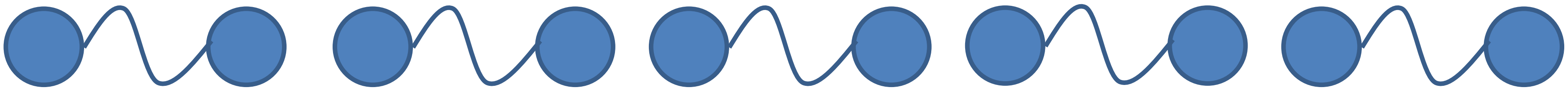


$(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)\,|\uparrow\rangle$ - $|\uparrow\rangle\,(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$

- Monogamy of entanglement: impossibility of sharing a singlet with two spin ½'s. Emerging phenomena arise because of this competition.

Tensor Calculus for Quantum Spin Chains

- Systematic way of creating states which have extremal local marginals but keep translational invariance: ***matrix product states*** ***(MPS)***



Tensor Calculus for Quantum Spin Chains

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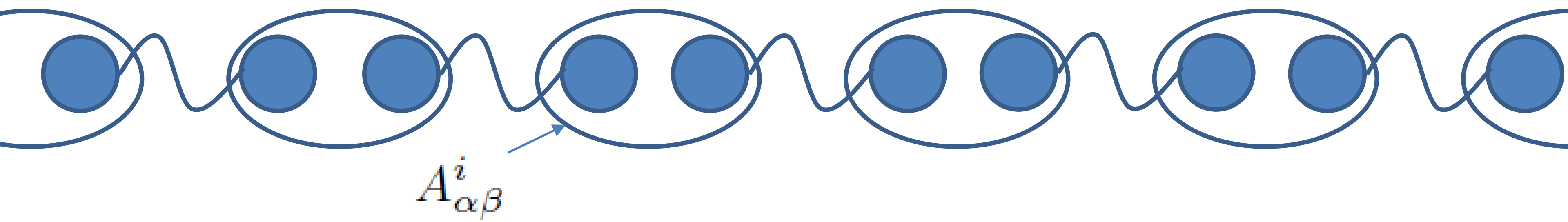
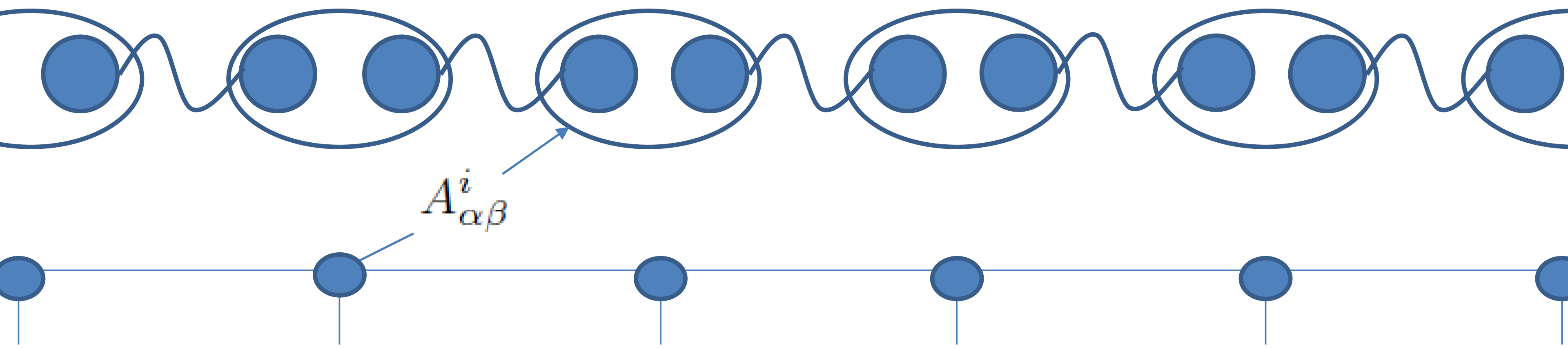


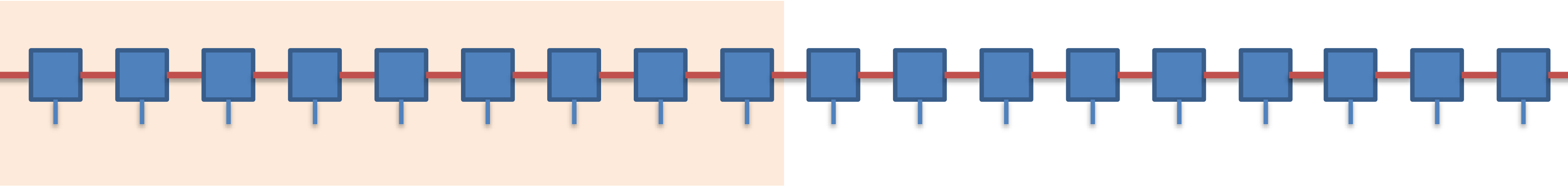
Tensors for Quantum Spin Chains

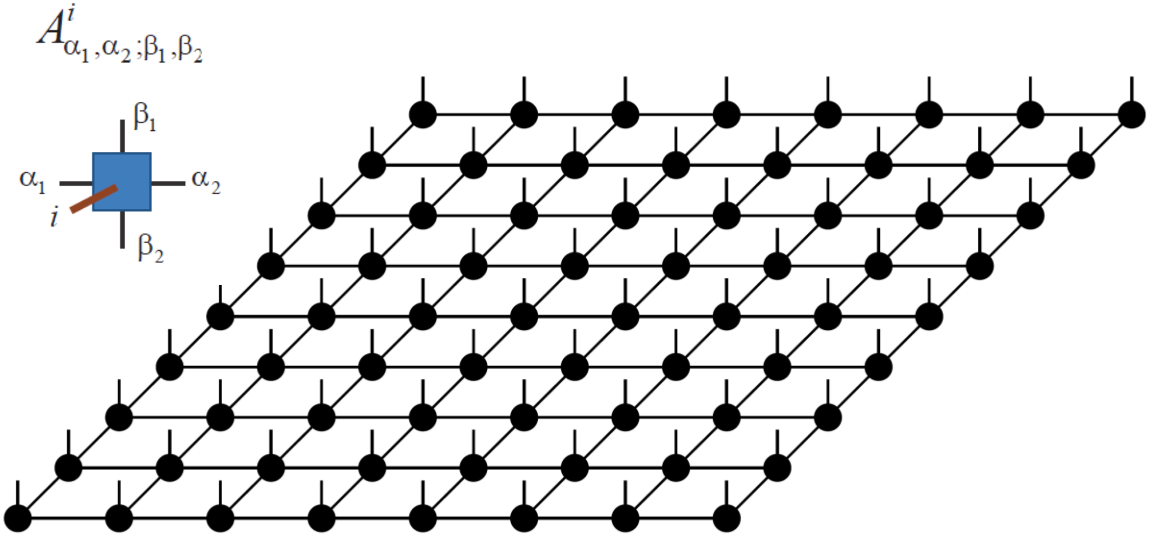
- Systematic way of creating states which have extremal local marginals but keep translational invariance: ***matrix product states*** ***(MPS)***

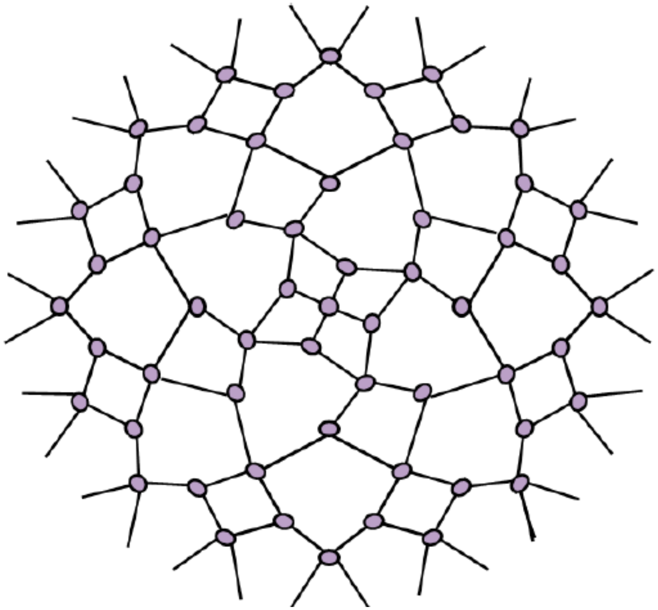


$$|\psi\rangle = \sum_{i_1 i_2 i_3 \cdots} \mathrm{Tr}\left(A^{i_1} A^{i_2} A^{i_3} \cdots\right) |i_1\rangle |i_2\rangle |i_3\rangle \cdots$$

Entanglement as building block of matter: Quantum Tensor Networks



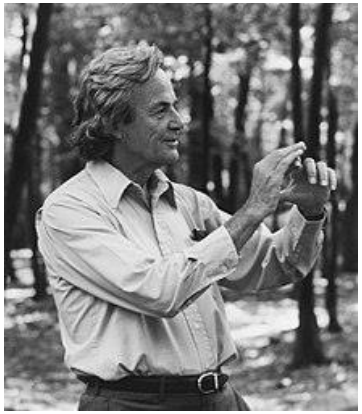




- ***tensor networks : crucial concepts***
  - Tensors model the entanglement structure: modelling correlations makes much more sense than modelling wavefunction directly
    - Tensors dictate the ***entanglement patterns***
  - Tensor networks can be efficiently contracted due to ***holographic property***: map quantum 3D -> 2D -> 1D -> 0D problems, and this can be done efficiently due to area laws
  - States are defined in thermodynamic limit; *finite size scaling* is replaced by ***finite entanglement scaling***
  - *Local tensor contains all global information* about quantum many body state
    - different phases of matter can be distinguished by ***symmetries of those local tensors***, including topological phases
    - Tensor networks provide a natural way of dealing with gauge theories: enforcing symmetries

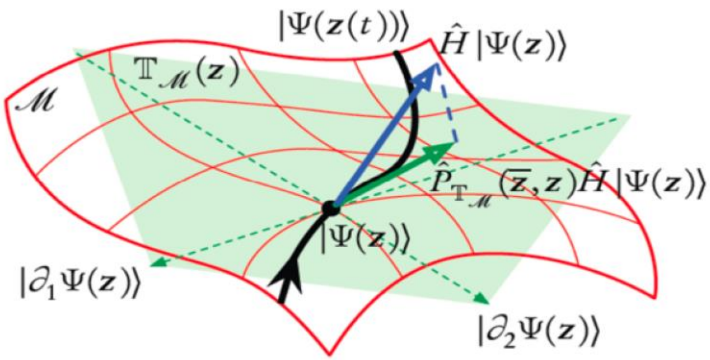
Feynman's dream

"Now, in field theory, what's going on over here and what's going on over there and all over space is more or less the same. Why do we have to keep track in our functional of all things going on over there while we are looking at the things that are going on over here? …. It's really quite insane actually: we are trying to find the energy by taking the expectation of an operator which is located here and we present ourselves with a functional which is dependent on everything all over the map. That's something wrong. Maybe there is some way to surround the object, or the region where we want to calculate things, by a surface and describe what things are coming in across the surface. It tells us everything that's going on outside.
I'm talking about a new kind of idea but that's the kind of stuff we shouldn't talk about at a talk, that's the kind of stuff you should actually do!"



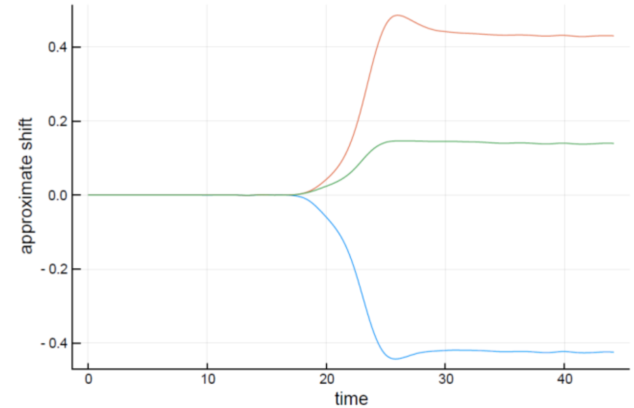
"Difficulties in Applying the Variational Principle to Quantum Field Theories", Wangerooge 1987, Proceedings,
Variational calculations in quantum field theory

MPS: computational aspects

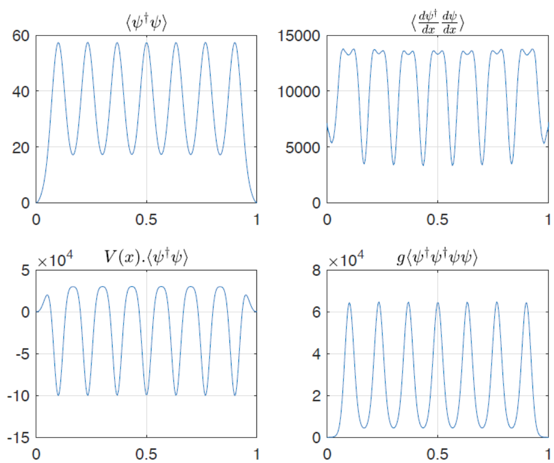


$$(i)\frac{d}{dt}|\psi\rangle = -\mathcal{P}_{T(A)}\hat{H}|\psi\rangle$$

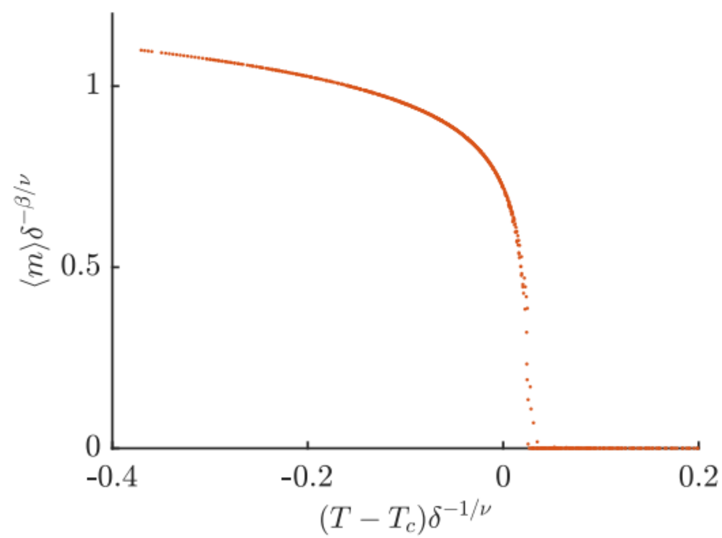
Time-Dependent Variational Principle for MPS
PRL 107, 70601 (2011)



Scattering of magnons in spin 1 Heisenberg model
PRR 3, 013078 (2021)

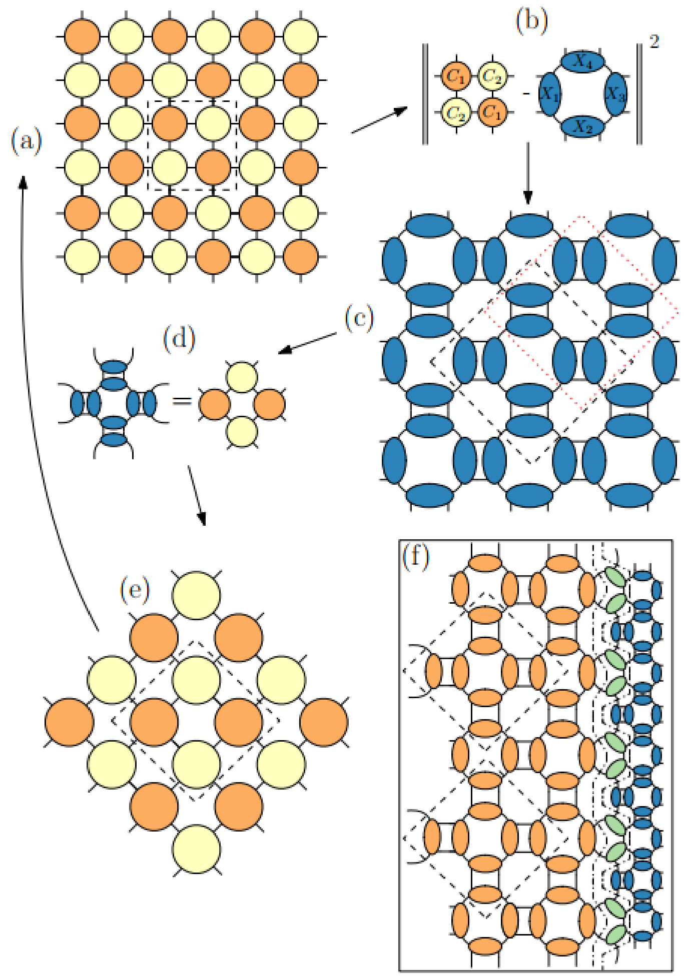


Interacting bosons with OBC:
cMPS and quantum Gross-Pitaevskii
PRL 128, 020501 (2022)

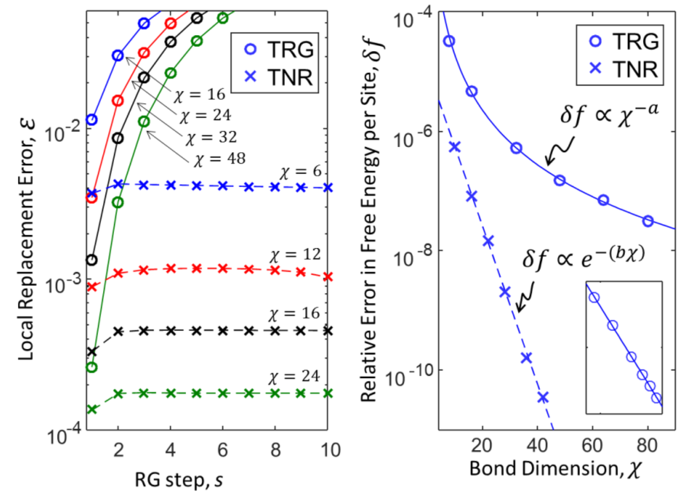


Finite Entanglement Scaling for critical Potts model
PRL 123, 250604 (2019)

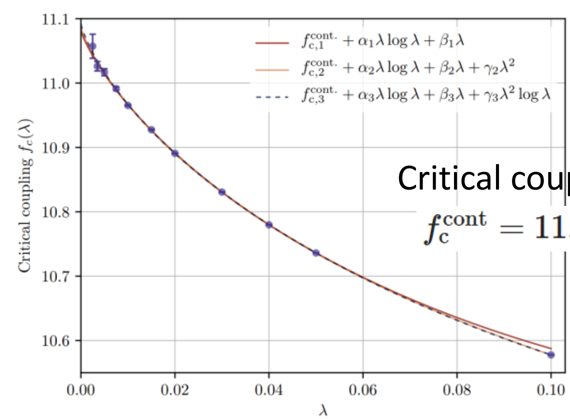
Multiscale Entanglement Renormalization Ansatz (MERA), Tensor Renormalization Group (TRG), Tensor Network Renormalization (TNR)



Renormalization group flows of Hamiltonians using tensor networks
PRL 118, 250602 (2017)



Tensor Network Renormalization
Evenbly, Vidal PRL 115, 180405 (2015)



Critical coupling cst:

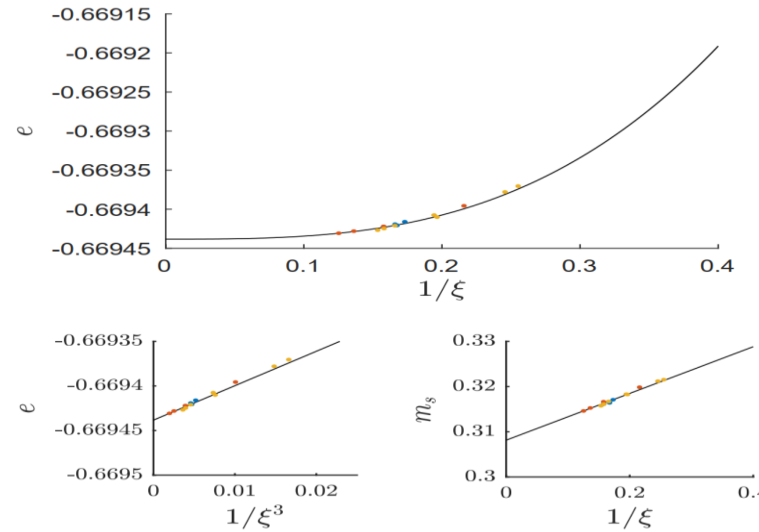
$$f_c^{\text{cont}} = 11.0861\,(90)$$

Computing the renormalization group flow of two-dimensional Φ^4 theory with tensor networks
Delcamp, Tilloy PRR 2, 033278 (2020)

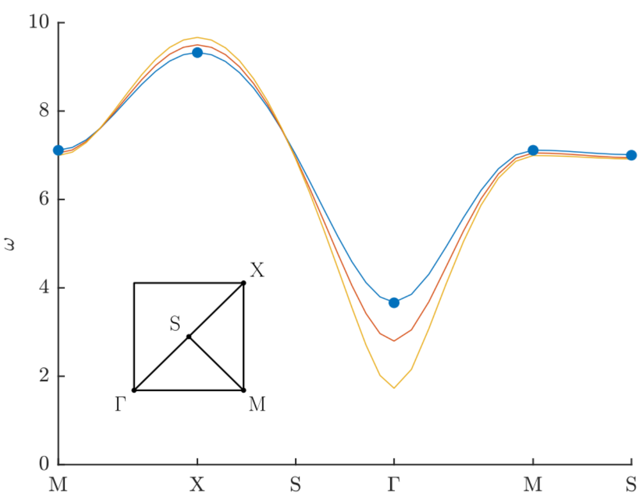
Projected Entangled Pair States (PEPS)

Pauling[13]	mean field	1.5
Nagle[15]	series expansion	1.50685(15)
Berg et.al.[19]	multicanonical	1.507117(35)
Herrero et.al.[16]	num. integration	1.50786(12)
Kolafa[17]	num. integration	1.5074660(36)
PEPS	$D = 2$	1.50735
	$D = 3$	1.507451
	$D = 4$	1.507456

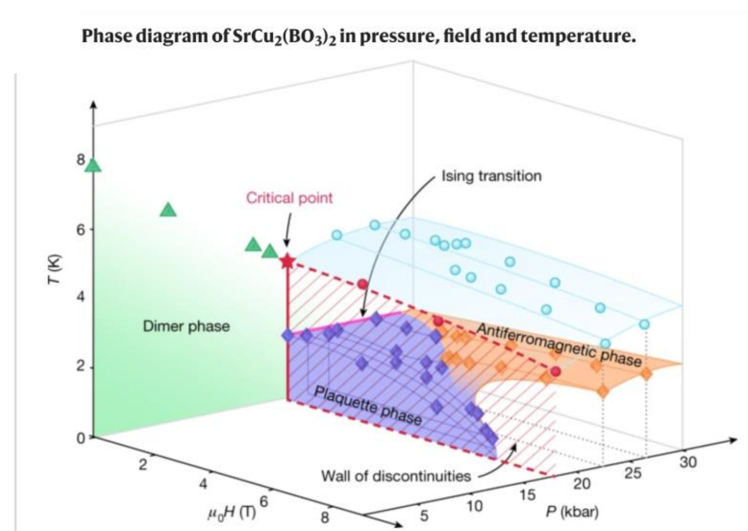
Spin Ice: residual entropy
PRE 98, 042145 (2018)



Entanglement scaling for 2D Heisenberg model
arXiv:2102.03143



Dispersion relation for 2D Ising with transverse field
PRB 99, 165121 (2019)



Physical realization of Shastry-Sutherland model
Mila et al., Nature 592, 370 (2021)

It’s all about symmetries

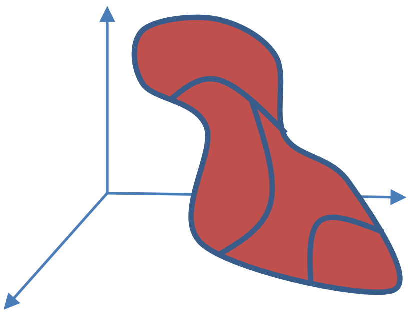
- Anderson ‘72: *“physics is an applied form of group theory”*

- Essential paradigm: detect the ***global*** features of a system through its entanglement degrees of freedom / ***local*** tensor network description

  - The symmetries of the local tensor in the tensor network reveal the emerging properties of the system

  - Even for the case of topological order: *local* order parameters arise in the form of different representations of groups / fusion algebras

- This has led to the program of classifying phases of matter

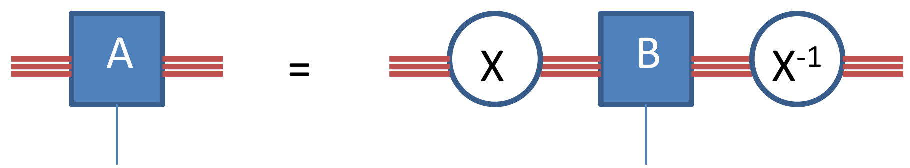


- QIT tools enable the correct definition of phases of matter: in terms of low-depth quantum circuits
  Bravyi, Hastings, FV PRL 97, 050401 (2006)

Tensor Networks: theoretical aspects

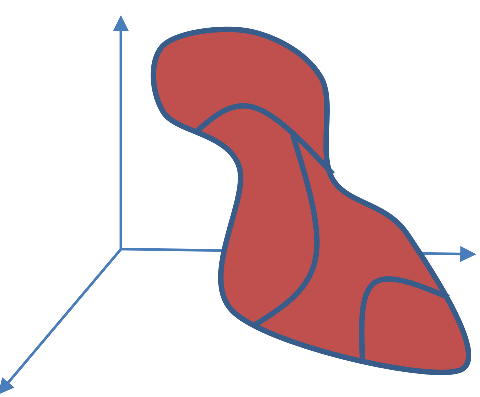
- MPS: fundamental theorem

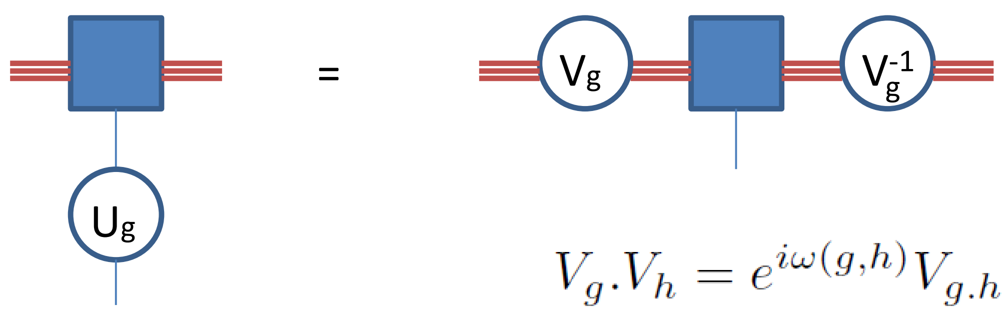
$$|\psi(A^i)\rangle = |\psi(B^i)\rangle \Leftrightarrow \exists X : A^i = XA^iX^{-1}$$



Cirac, Perez-Garcia, Schuch, FV ‘16

- Classification of 1D SPT phases



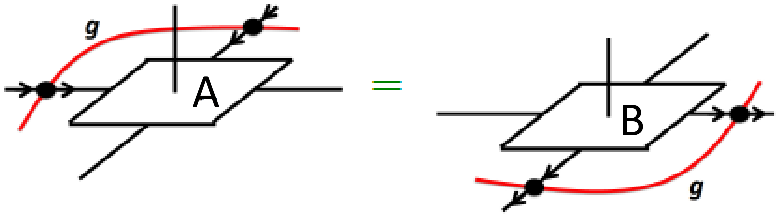


$$V_g.V_h = e^{i\omega(g,h)}V_{g.h}$$

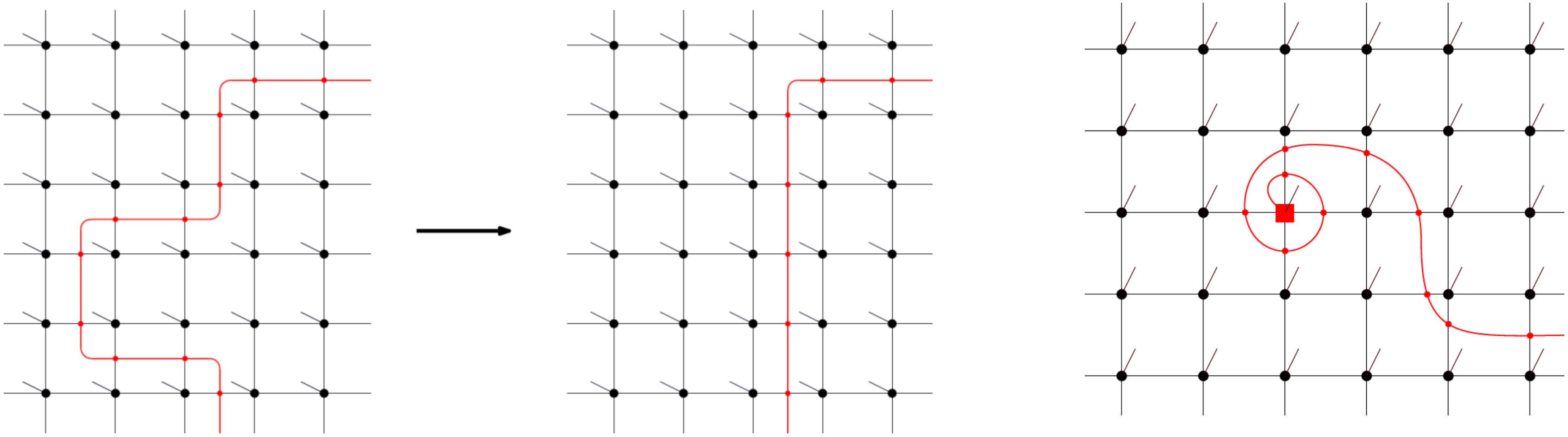
Perez-Garcia, Wolf, Sanz, FV, Cirac ‘08
Pollmann, Turner, Berg, Oshikawa ’10; Chen, Gu, Wen ‘11; Schuch, Perez-Garcia, Cirac ’11

- PEPS: fundamental theorem

$$|\psi(A^i)\rangle = |\psi(B^i)\rangle \Leftarrow$$



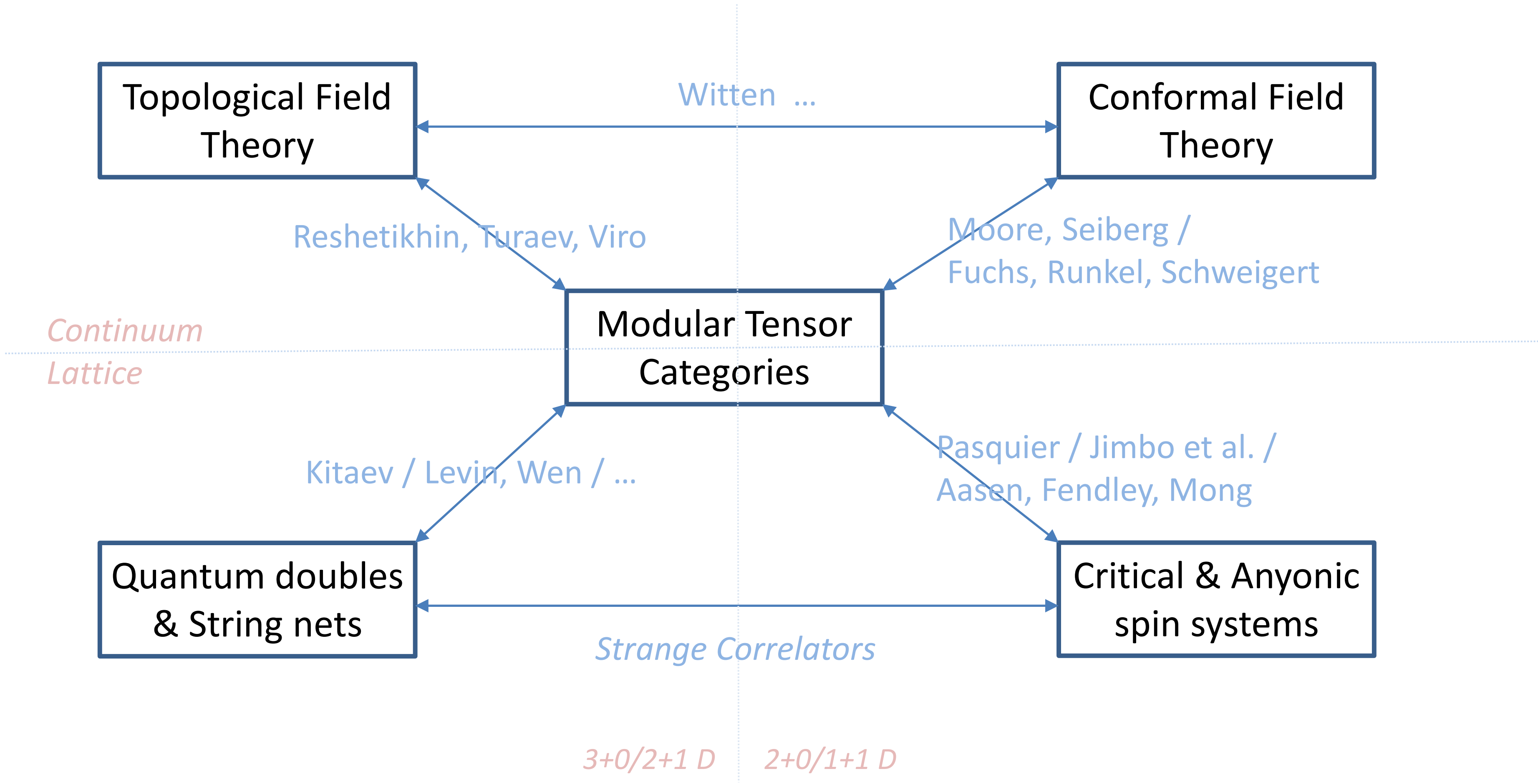
- Topological order: Levin-Wen string nets / Turaev-Viro state sums (TFT)



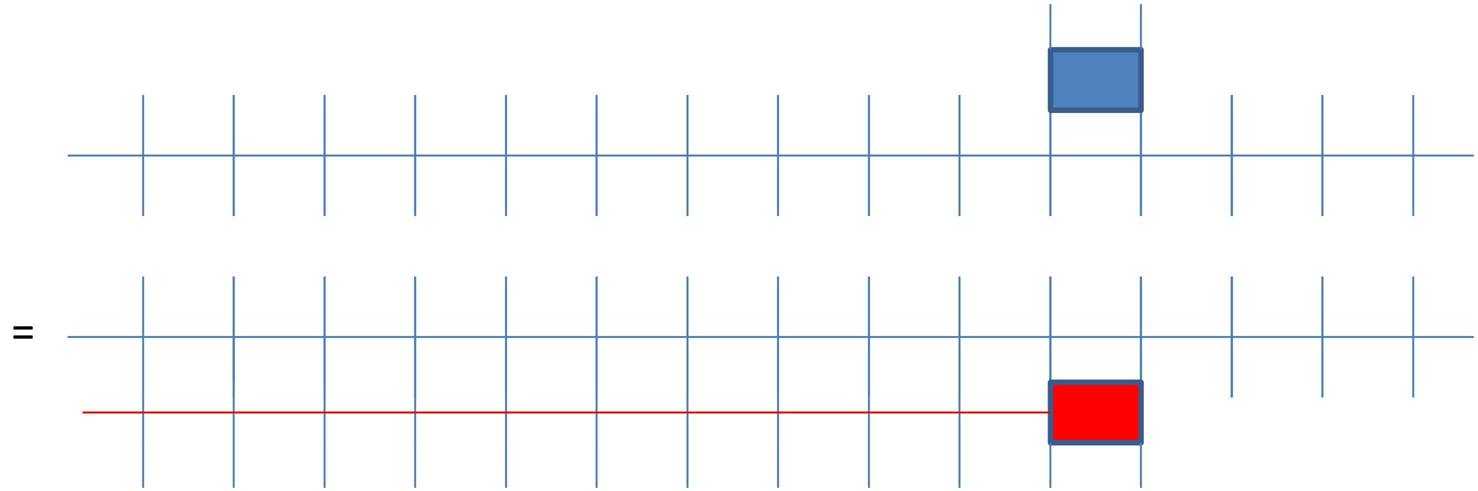
$$O_a . O_b = \sum_c N_{ab}^c O_c$$

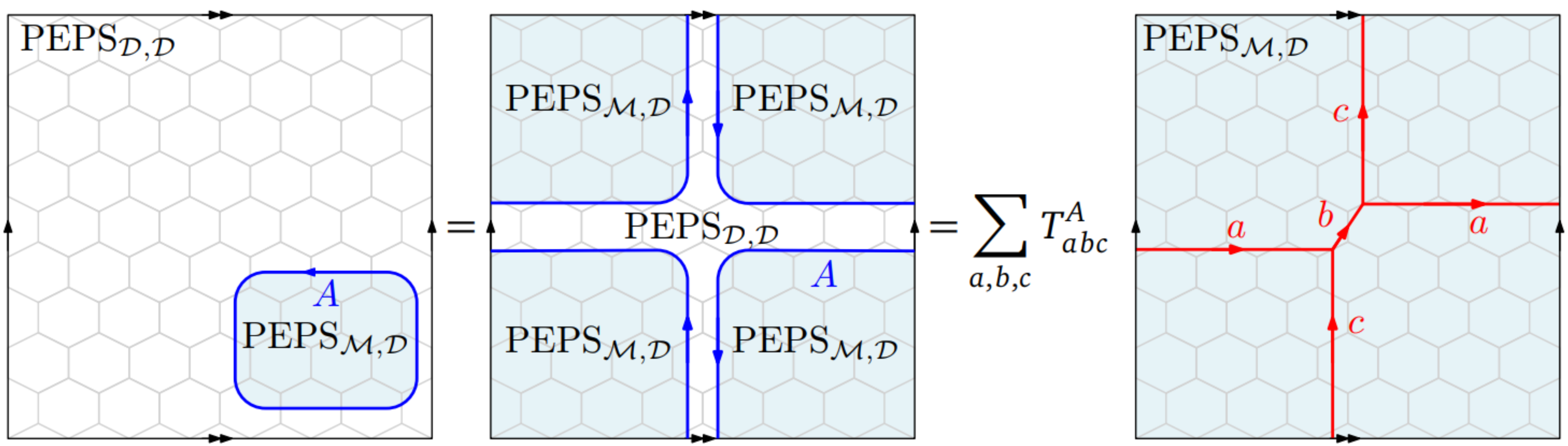
Bultinck, Marien, Sahinoglu, Williamson, Haegeman, FV '17
Lootens, Fuchs, Haegeman, Schweigert, FV '21

Topological / Categorical symmetries: TFT & CFT



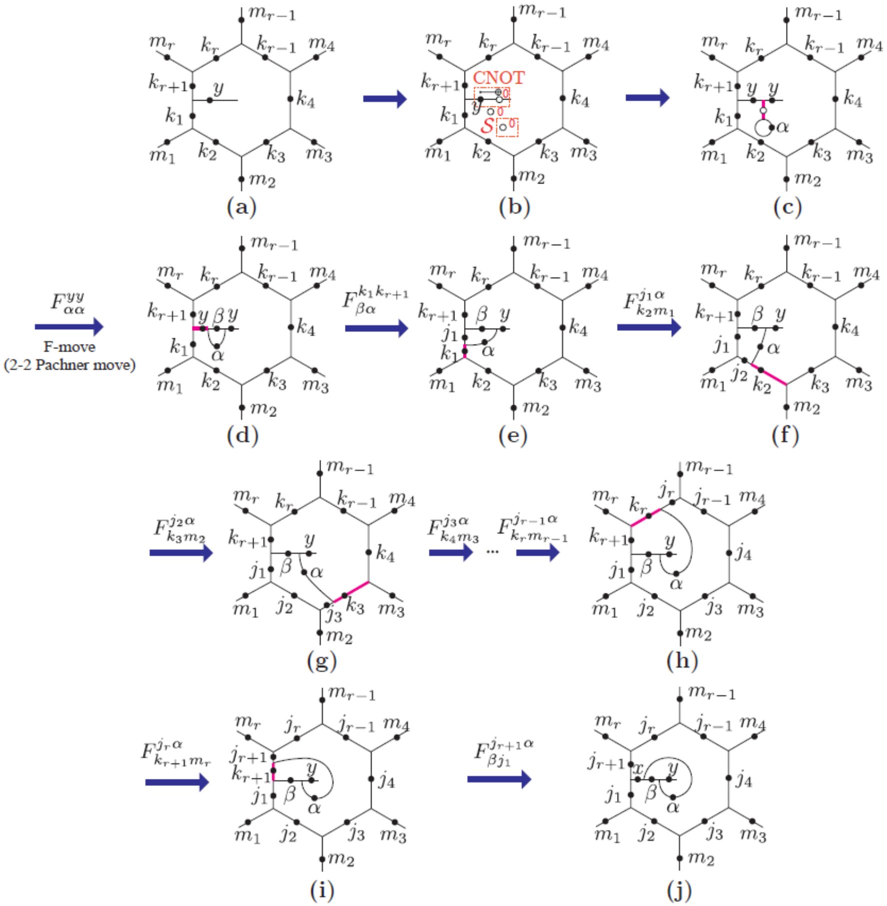
Dualities and MPO intertwiners: bimodule categories

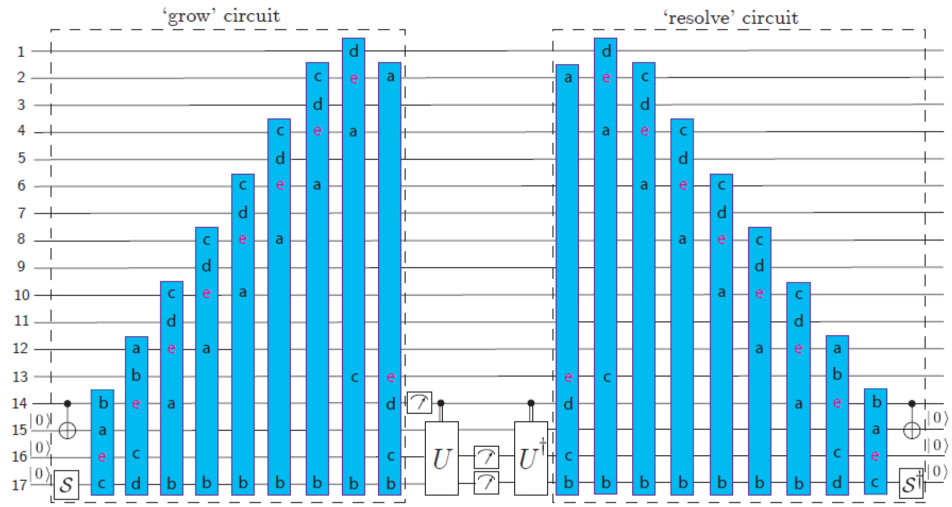


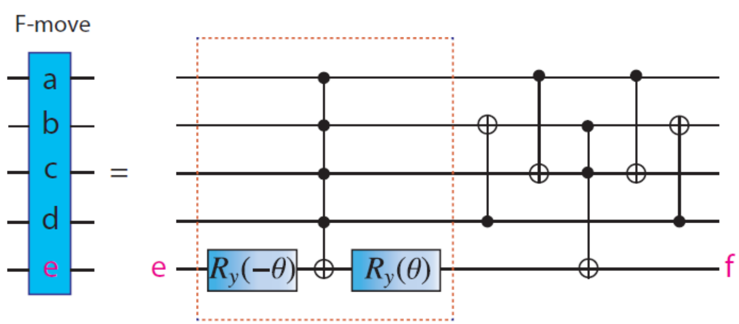


Category-theoretic recipe for dualities in one-dimensional quantum lattice models
Lootens, Delcamp, Ortiz, FV arXiv:2112.09091

Application: Fibonacci Turaev-Viro error correcting code



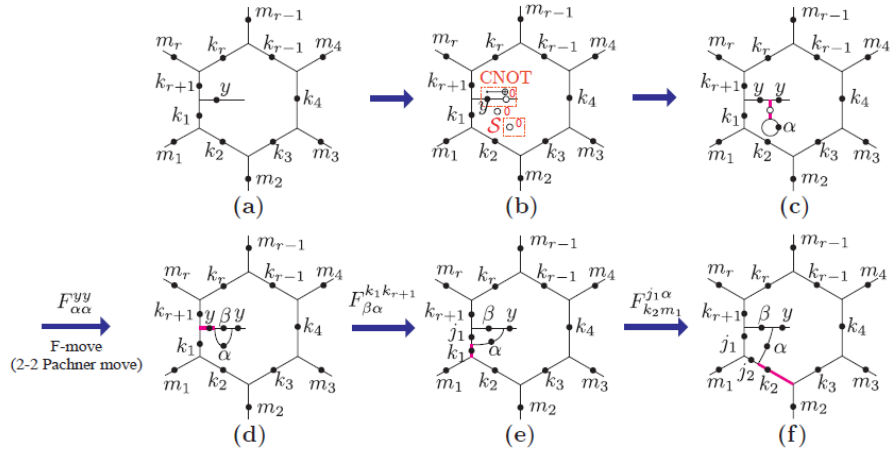


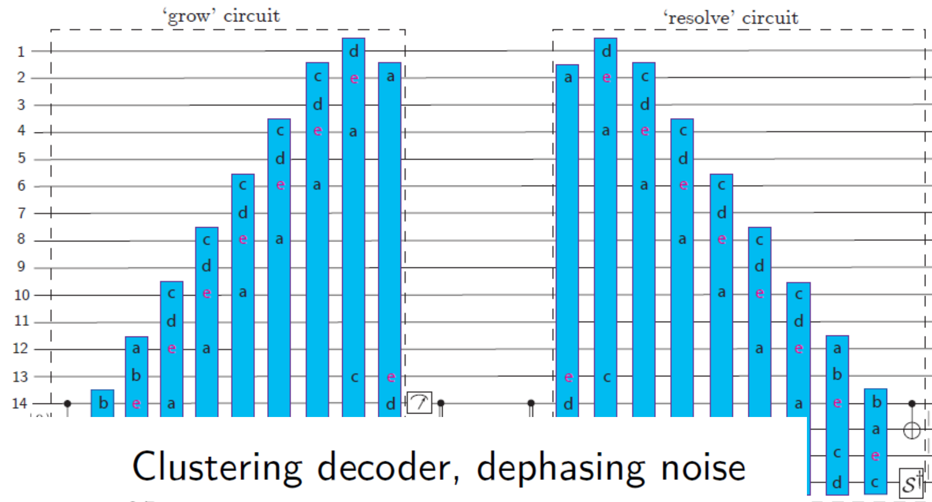


Measuring anyon charges

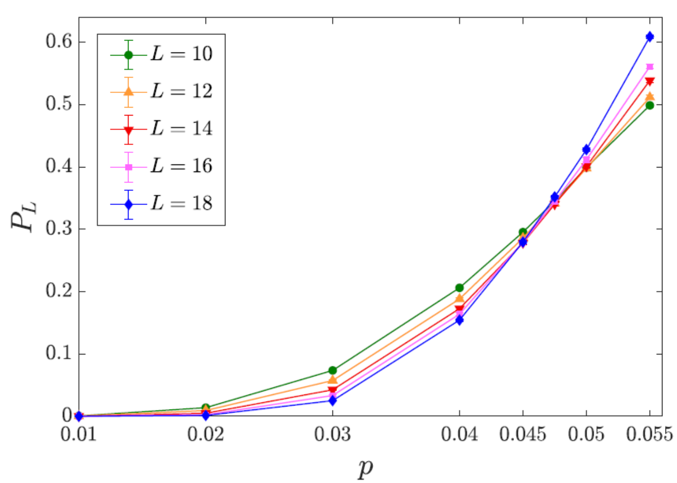
Schotte, Zhu, Burgelman, FV PRX 12, 021012 (2022)

Application: Fibonacci Turaev-Viro error correcting code

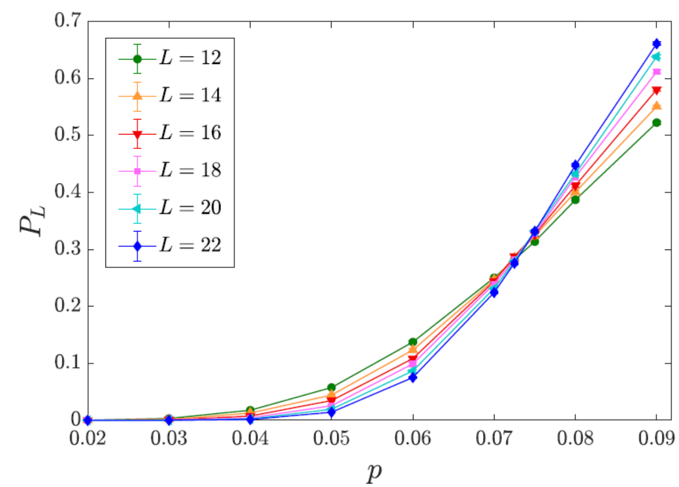




Clustering decoder, depolarizing noise



Clustering decoder, dephasing noise



$p_c \approx 0.0475$

$p_c \approx 0.0725$

Schotte, Zhu, Burgelman, FV PRX 12, 021012 (2022)

“the unreasonable effectiveness of ~~tensor networks~~ in physics”

Linear Algebra

Real-Space Renormalization Group

Symmetries and dualities

Tensor Networks

Quantum Field Theory
Statistical Mechanics

- High-Energy Physics
- Condensed Matter
- Cold Atoms