Linnaeus Center ACCESS
Distinguished Lecture Series

Professor Bart De Moor
Katholieke Universiteit Leuven, Belgium

Friday November 12, 2010, at 13:15

F2, Lindstedtsvägen 26, KTH

ACCESS (Autonomic Complex Communication nEtworks Signals and Systems) is a VR Linnaeus Research Center at KTH, www.access.kth.se. This is the thirty-seven seminar in the ACCESS Distinguished Lecture Series

We have the pleasure of hosting Professor Bart De Moor, Katholieke Universiteit Leuven, Belgium.

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Welcome

Bo Wahlberg

Title: Back to the roots: Solving polynomial systems with numerical linear algebra tools

Speaker: Professor Bart De Moor

Date: Friday November 12, 2010, at 13:15

Location: F2, Lindstedtsvägen 26 , KTH

Abstract: Finding the roots of a set of multivariate polynomials has numerous applications in geometry and optimization, system and control theory, modeling and identification, statistics and bioinformatics, and many other scientific disciplines. It is an old yet fascinating problem, that has intrigued scientists throughout the ages, starting with the Greeks, over Fermat and Descartes, Newton, Leibniz, Bezout and many many others.

It all started with trying to find the roots of a polynomial equation with real coefficients in one unknown. In the beginning of the 19-th century, formulas were known for the roots of polynomials up to degree 4, but it was Galois who showed that no general formulas exist for degree 5 and higher. This implies that roots of polynomials of degree higher than 4 in general can only be found using (iterative) numerical algorithms. Soon thereafter, Sylvester and other contemporains started research on how to find the roots of sets of multivariate polynomials. Sylvester derived an elimination algorithm, in which he eliminates variables one by one, ending up with a 'characteristic equation' in one variable only. If one then has obtained the roots of this last equation, one can then 'back-substitute' root-by-root into the other equations, and hence in principle find all roots. Sylvester's algorithm is the equivalent of Gaussian elimination for linear equations, but more importantly, his results imply that finding the roots of a polynomial system is an eigenvalue problem !

Later on, in the 20th century, there was a booming mathematical development, which gave birth to a discipline that today is called algebraic geometry, with a fabulous rich history, to which famous mathematicians, such as Hilbert, but also many others, contributed. It also led to the machinery of Gröbner bases and the like, which today are ubiquitous in books and symbolic methods in algebraic geometry, with numerous applications in fields like geometric design, combinatorics and integer programming, coding theory, robotics, etc…

We will however not talk about these developments, but return to the very roots and early days of the problem.

In this talk, we will elaborate on a research program, the objective of which is to translate the many – symbolic - algorithms from algebraic geometry, into numerical linear algebra algebra algorithms. Our talk develops ideas on three complementary levels:

- Geometric linear algebra, which deals with column and row vector spaces, dimensions, orthogonality, kernels, eigenvalue problems and the like;
- Numerical linear algebra, where we conceptually deal with tools like Gram-Schmidt orthogonalization, ranks, angles between subspaces, etc… ;
- Numerical algorithms, which implement the linear algebra tools into an efficient and numerically robust method. Here we can exploit matrix structures (e.g. Toeplitz or sparsity), investigate variations of iterative methods (e.g. power methods) or try to speed up convergence (e.g. by FFT).

Starting from a set of multivariate polynomials, we will show how, from the matrix of coefficients, we can construct a 'block-Toeplitz-like' matrix, which we call the 'tableau matrix', the row space of which represents 'the ideal' of the polynomial system, and the kernel its 'variety', which also contains the roots of the polynomial system. Its corank, which is the dimension of the kernel, reveals the number of roots, which can be finite or infinite, and which can have multiplicities larger than 1. We will show how to find them, using insights from system theory (more specifically, we will use realization theory).

This basically leads to 2 approaches to find all roots: a kernel based algorithm, in which one first has to calculate the kernel of the tableau matrix, and then by applying realization theory one can construct an eigenvalue problem from which the roots are obtained; and secondly a 'data-driven' algorithm, in which we directly operate with the tableau matrix, solve a set of linear equations using the QR-decomposition, and then obtain an eigenvalue problem in terms of certain blocks of the triangular factor R.

Our claim is, that in due time, we will have a numerical linear algebra based tool set to efficiently and robustly find all roots of a set of multivariate polynomials. Notions from linear algebra we use are column and row spaces, ranks, kernels and the eigenvalue problem, but also Grassmann's dimension theorem and angles between subspaces. The tool set we use are algorithms such as the QR-, the CS- and the singular value decomposition (SVD).

We will illustrate our results with some motivating examples, such as:

- How to find the minimizing root of polynomial optimization problems, i.e. optimization problems in which the objective function and constraints are multivariate polynomials. In particular, we will show that optimization algorithms in Prediction Error Methods in principle solve an eigenvalue problem, as is also the case in structured total least squares problems.

- Algebraic statistics, in which the maximum likelihood estimation of the parameters of discrete statistical models (such as in Bayesian networks or Hidden Markov Models), leads to finding the roots of a set of multivariate polynomials.
- Multi-way arrays of data or numerical tensors, which arise in statistics, biomedical data and signal processing, system identification, datamining etc… In particular, the problem of approximating a given data tensor in a least squares sense by one of lower multilinear rank, results in a set of multivariate polynomials, and hence is an eigenvalue problem.

Biography: Bart De Moor was born Tuesday July 12, 1960 in Halle, Belgium. He is married and has three children. In 1983, he obtained his Master Degree in Electrical Engineering at the Katholieke Universiteit Leuven, Belgium, and a PhD in Engineering at the same university in 1988. He spent 2 years as a Visiting Research Associate at Stanford University (1988-1990) at the departments of EE (ISL, Prof. Kailath) and CS (Prof.Golub). Currently, he is a full professor at the Department of Electrical Engineering of the Katholieke Universiteit Leuven in the research group SCD, and vice-rector for International Policy of the University of Leuven. His research interests are in numerical linear algebra and optimization, system theory and system identification, quantum information theory, control theory, data-mining, information retrieval and bio-informatics. He is also the coordinator of several research networks and is the promotor of numerous research projects funded by regional, federal and European funding agencies.

Currently, he is leading a research group of 30 PhD students and 8 postdocs and in the recent past, 55 PhDs were obtained under his guidance. He has been teaching at several universities in Europe and the US. He is also a member of several scientific and professional organizations and jury member of several awards (Tech Art Prize, Innovation Award of the Flemish Government, Barco Innovation Award, ICOS Award, Egemin Award). His work has won him several scientific awards (Leybold-Heraeus Prize (1986), Leslie Fox Prize (1989), Guillemin-Cauer best paper Award of the IEEE Transaction on Circuits and Systems (1990), Laureate of the Belgian Royal Academy of Sciences (1992), bi-annual Siemens Award (1994), best paper award of Automatica (IFAC, 1996), IEEE Signal Processing Society Best Paper Award (1999). In November 2010, he receives the 5-annual FWO Excellence Award out of the hands of King Albert II of Belgium. Since 2004 he is a fellow of the IEEE. He is an associate editor of several scientific journals.

From 1991-1999 he was the Head of Cabinet and/or Main Advisor on Science and Technology of several ministers of the Belgian Federal Government (Demeester, Martens) and the Flanders Regional Governments (Demeester, Van den Brande). From December 2005 to July 2007, he was the Head of Cabinet on socio-economic policy of the minister-president of Flanders, Yves Leterme (current Prime Minister), capacity in which he was the coordinator of a new socio-economic business plan for the Flemish region.

He was and/or is in the board of 6 spin-off companies (www.ipcos.be, www.data4s.com, www.tml.be, www.silicos.com, www.dsquare.be, www.cartagenia.be), which he co-founded, of the Flemish Interuniversity Institute for Biotechnology, the Study Center for Nuclear Energy, the Institute for Broad Band Technology. He is also the Chairman of the Industrial Research Fund, Hercules, the Flemish Children Science Center Technopolis, the Alamire Foundation and several other scientific and cultural organizations. As a vice-rector for International Policy, he is a member of the Executive Committee and the Academic Council of the Katholieke Universiteit Leuven and of the Board of Directors of the Association K.U.Leuven.

He made regular television appearances in the Science Show "Hoe?Zo!" on national television in Belgium. Full details on his CV can be found at www.esat.kuleuven.ac.be/~demoor