Katholieke
Universiteit
Leuven

Efficient Numerical Methods for Moving Horizon Estimation

Doctoral presentation – public defense

Niels Haverbeke

Promotor

Prof Dr. Ir. Bart De Moor

Co-promotor

Prof Dr. Moritz Diehl

Chairman

Prof. Dr. Ir. Yves Willems

Jury

Prof. Dr. Ir. Johan Suykens
Prof. Dr. Ir. Jan Willems
Prof. Dr. Ir. Wim Michiels
Prof. Dr. Michel Kinnaert (U.L.B.)
Prof. Dr. Michel Verhaegen (T.U.Delft)
Prof. Dr. Ir. Lieven Vandenberghe (U.C.L.A.)

Overview

Motivation

1. Driven by applications
2. Closed loop control scheme
3. Principle and situation of MHE

Structure exploiting MHE algorithms

4. Optimization problem and KKT system
5. Interior point methods
6. Active-set methods

Convex and nonlinear MHE

7. Huber penalty MHE
8. Joint estimation of piecewise changing inputs
9. Nonlinear MHE and MPC

CONCLUSIONS

MOTIVATION

Driven by applications

Recursive techniques, e.g. Kalman filter

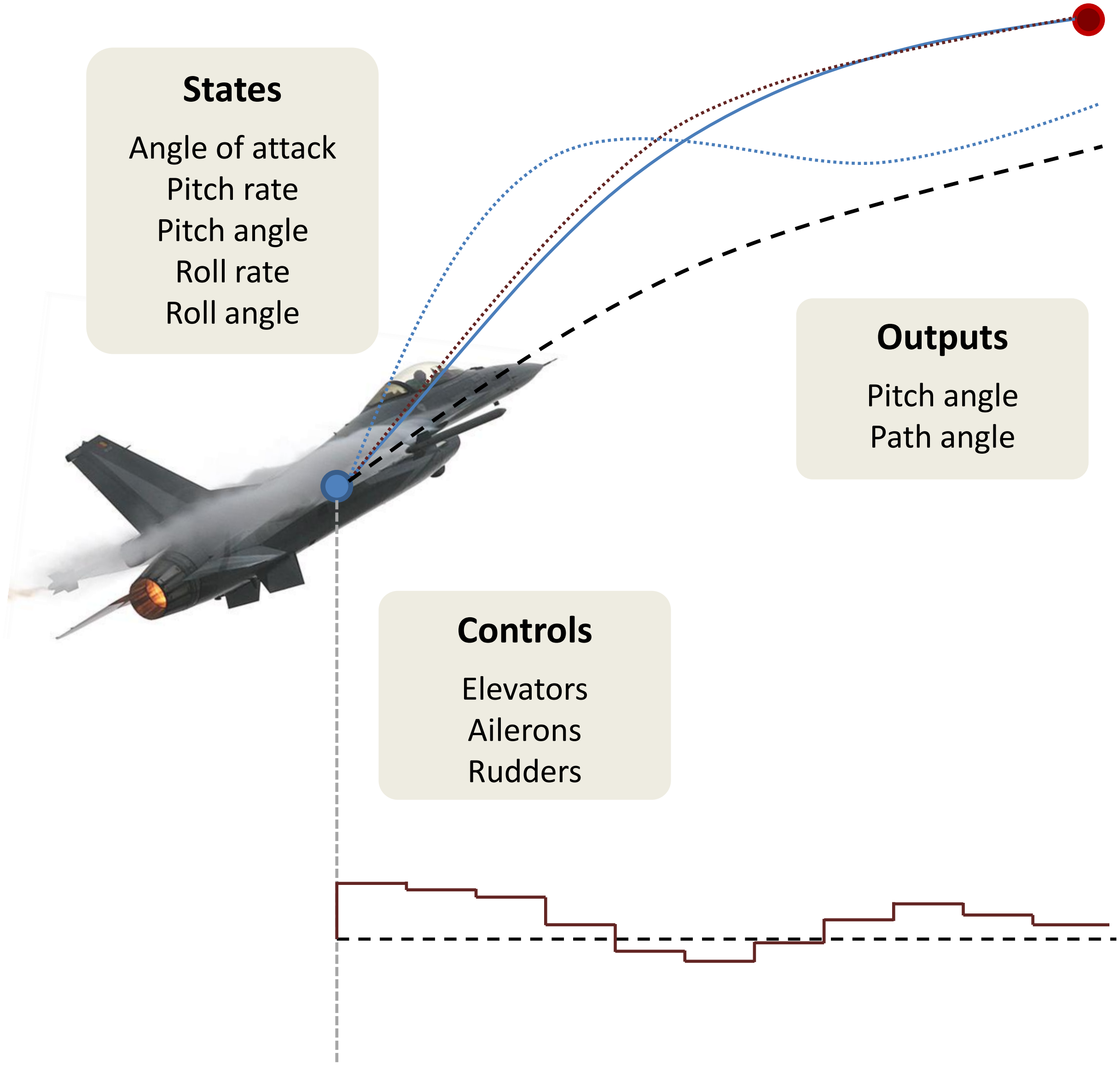
Applied to **fast** systems

Advanced dynamic optimization techniques, e.g. parameter estimation, MPC

Applied to **slow** systems

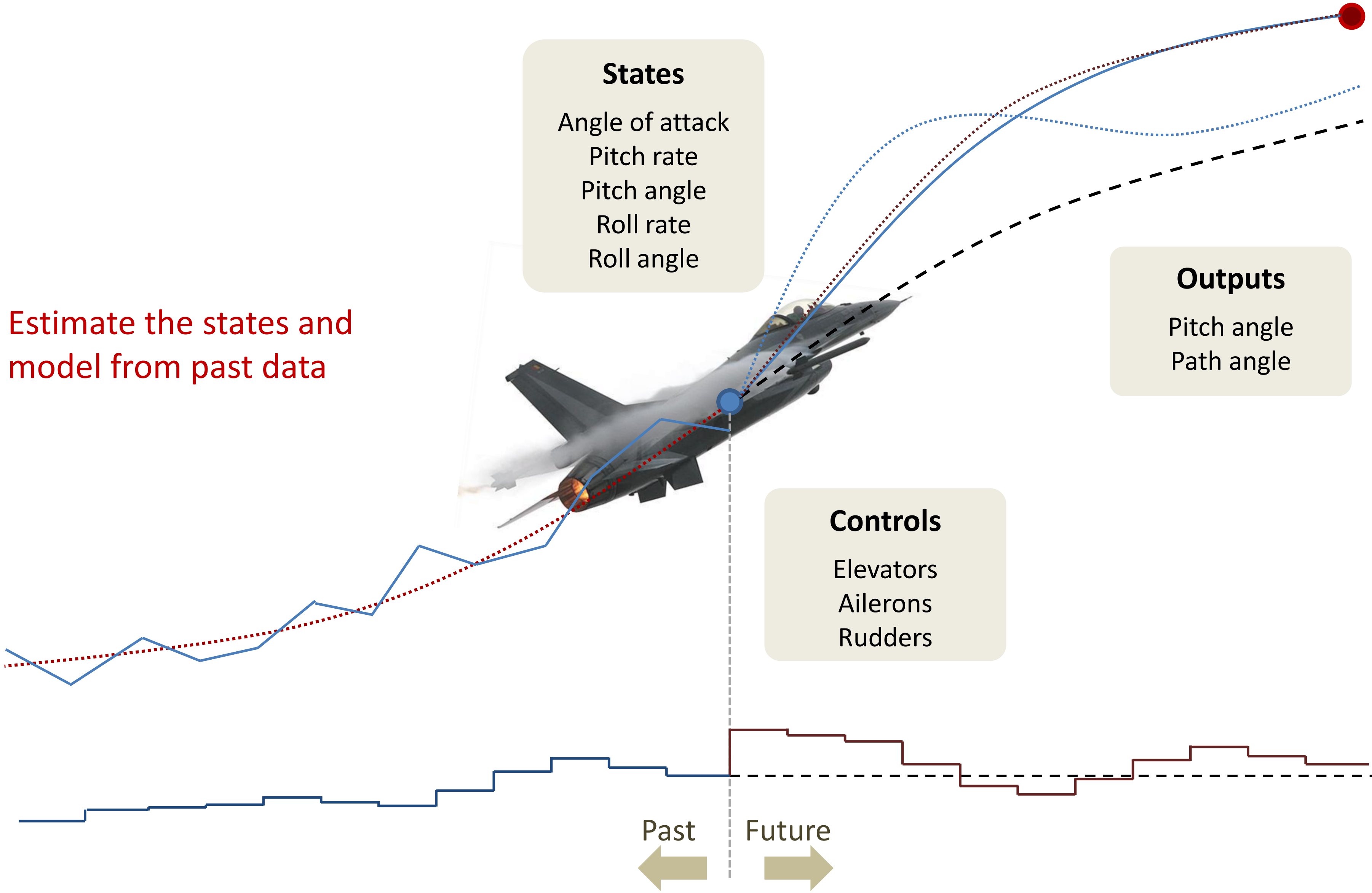
Dynamic optimization for fast real-time systems

Example

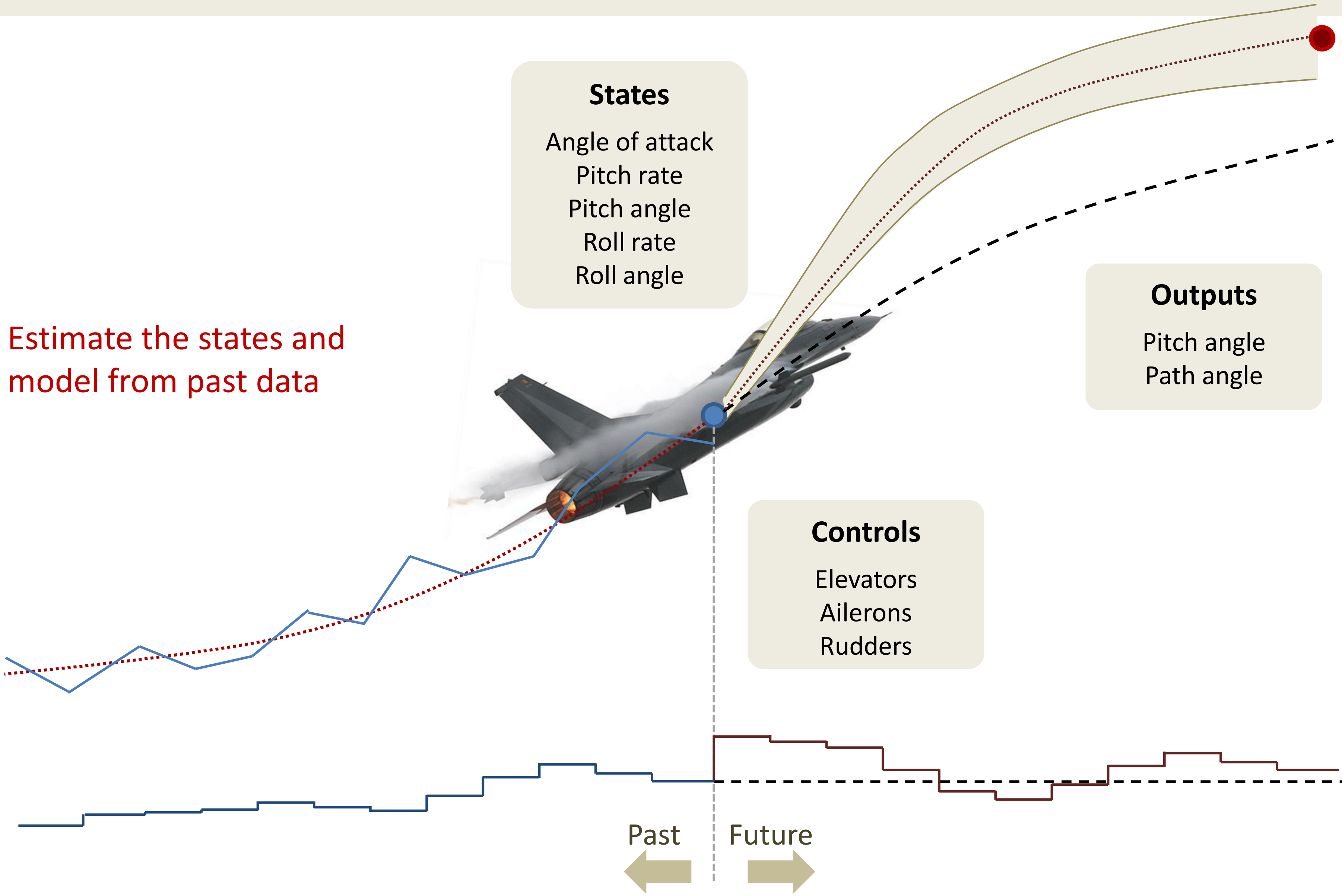

States
Angle of attack
Pitch rate
Pitch angle
Roll rate
Roll angle
Outputs
Pitch angle
Path angle
Controls
Elevators
Ailerons
Rudders

Example

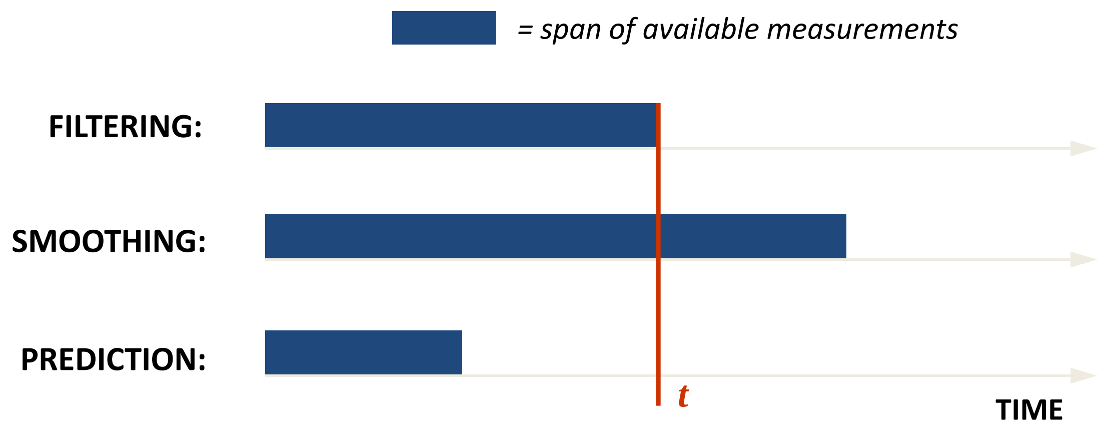
Estimate the states and model from past data



Example


States
Angle of attack
Pitch rate
Pitch angle
Roll rate
Roll angle
Estimate the states and model from past data
Outputs
Pitch angle
Path angle
Controls
Elevators
Ailerons
Rudders
Past
Future

Filtering, smoothing and prediction



Recursive estimation

Window of one time step

Typically online state estimation

Kalman filter and extensions

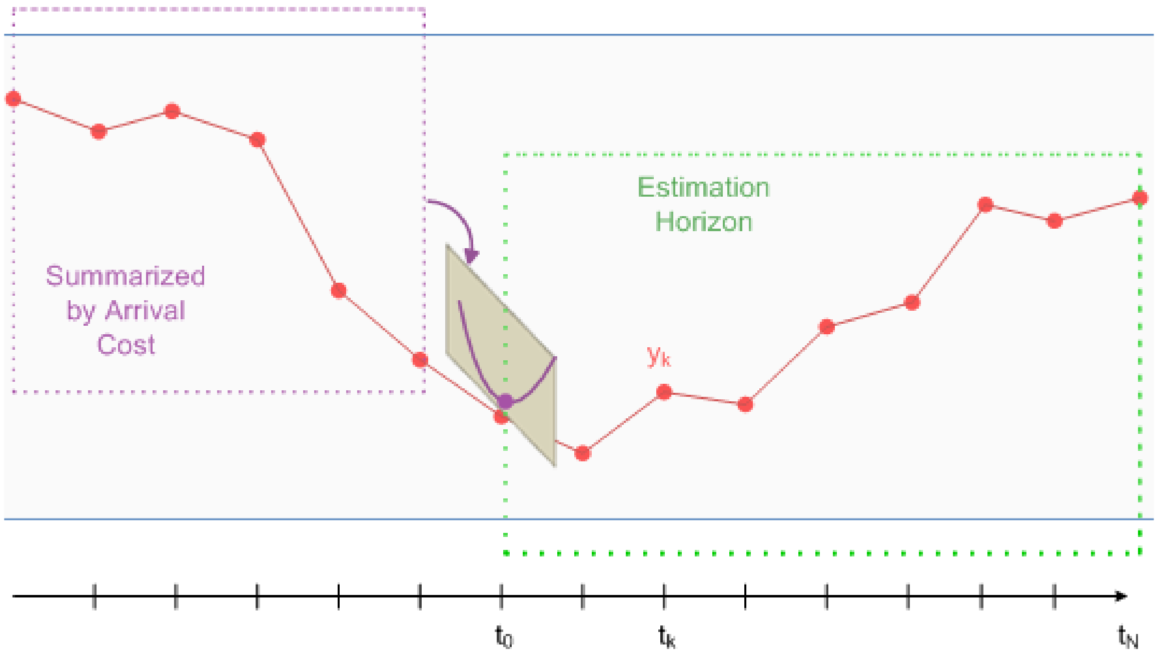
Batch estimation

Large window

Typically offline optimization

Parameter fitting

MHE principle



$$\min_{\mathbf{x},\mathbf{w}} \quad \mathcal{J}_{\text{ic}}(x_0) + \mathcal{J}_{\text{proc}}(N, \mathbf{w}) + \mathcal{J}_{\text{sens}}(N, \mathbf{y}, \mathbf{x})$$

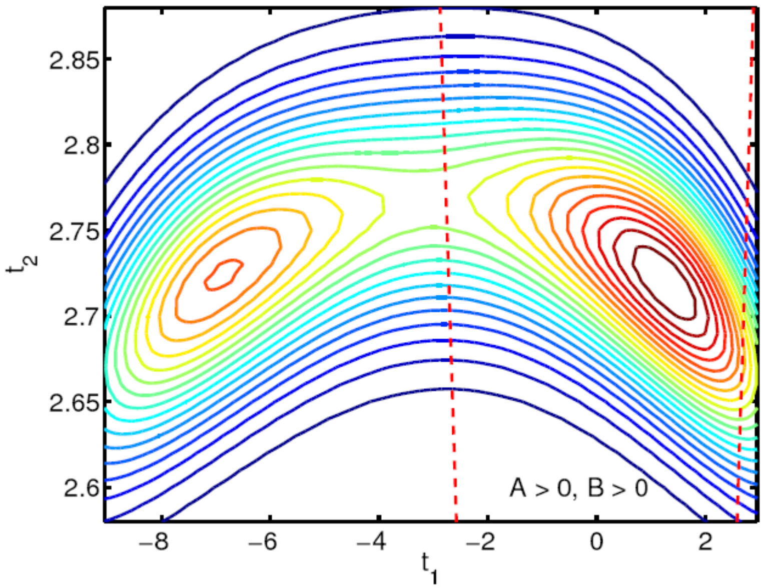
Subject to Dynamic model

Constraints

The role of constraints

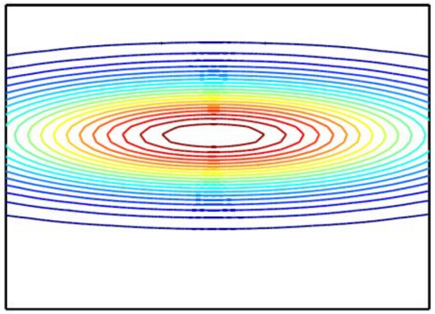
What can go wrong?

→ nonlinear model may give rise to multiple optima

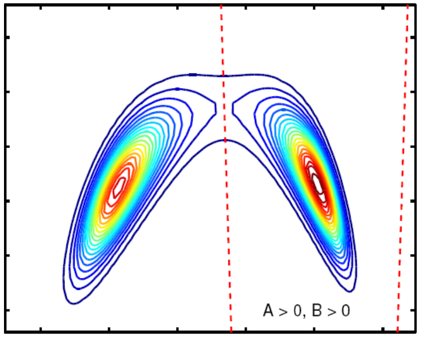


Contours of (rescaled) true conditional probability density $p(x_1|y_0, y_1)$

EKF tries to fit



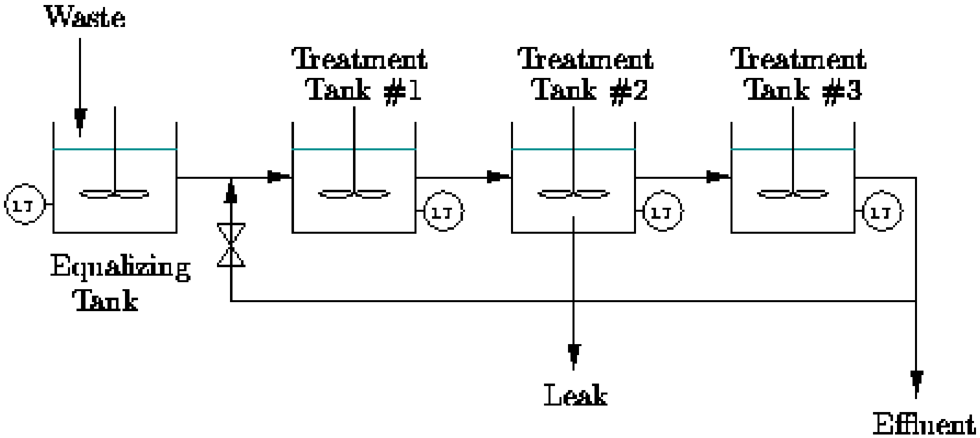
MHE retains dominant characteristics: **multiple optima**

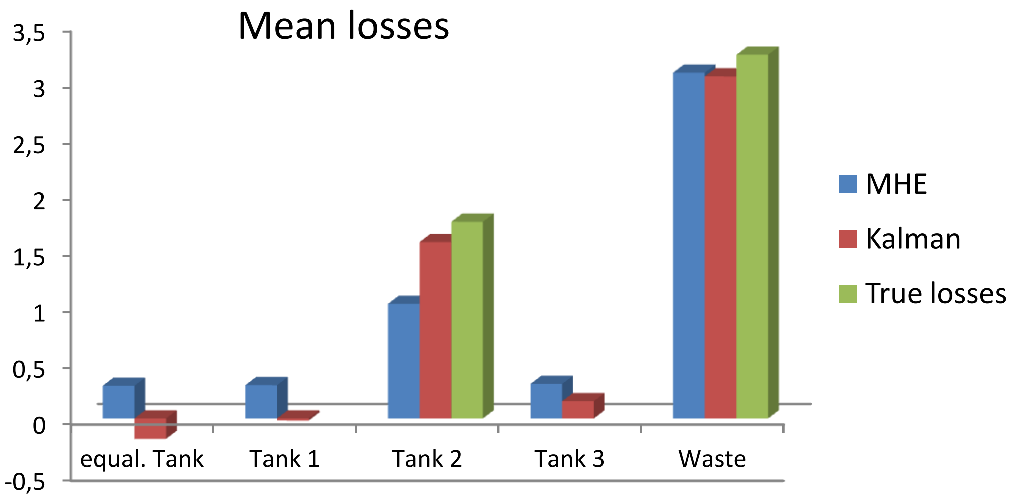


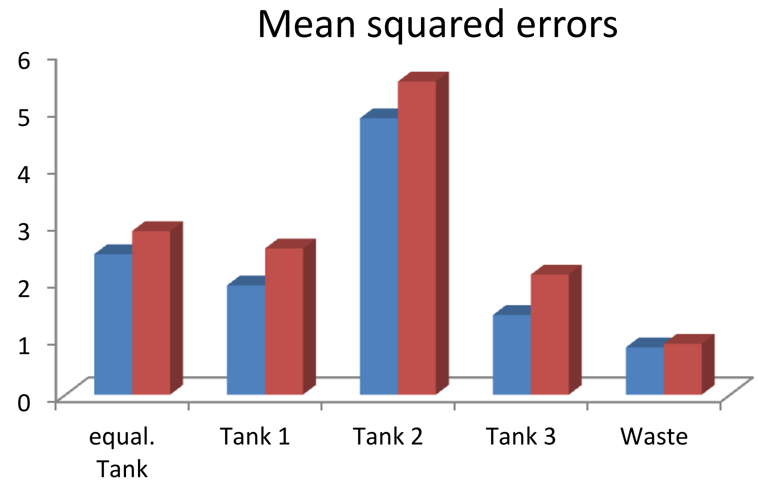
* Source: Haseltine and Rawlings, 2004

Waste water treatment process

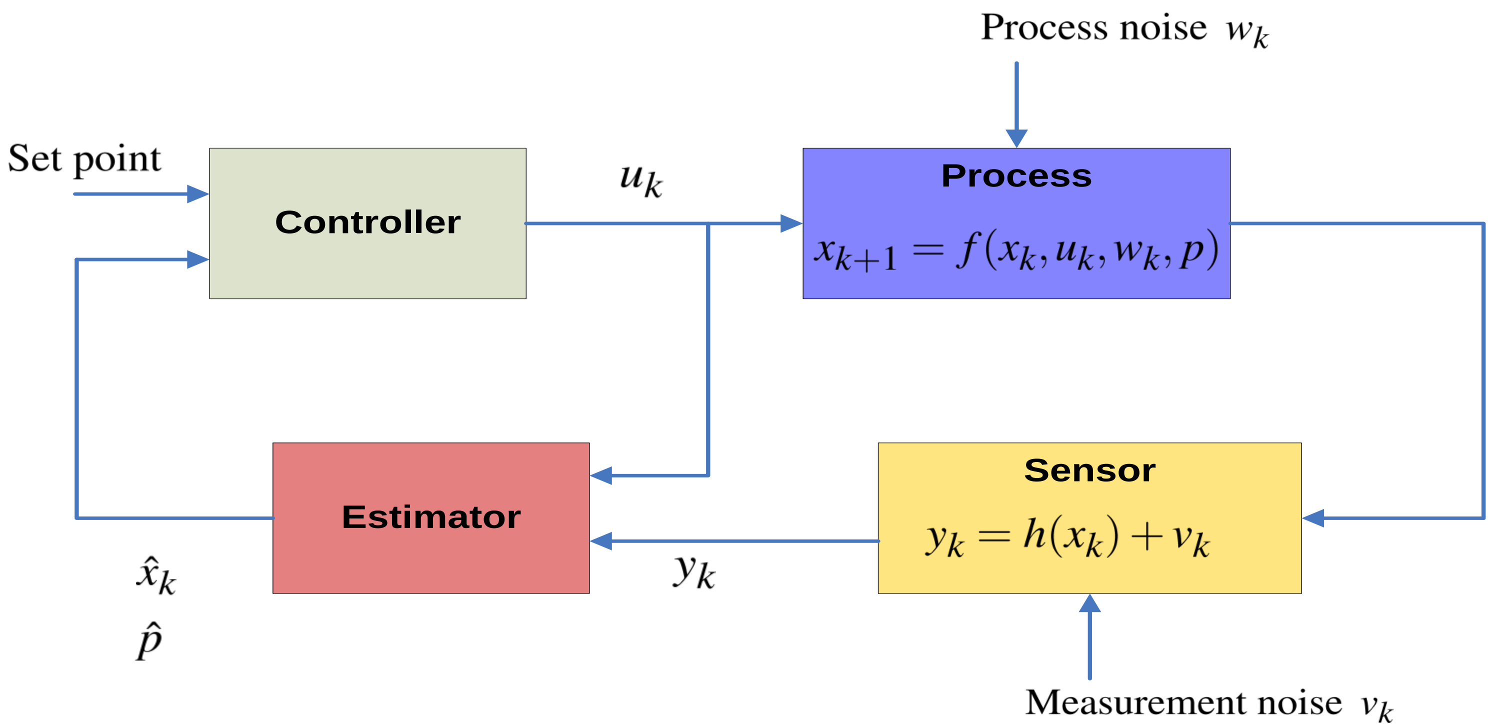
Fifth order system



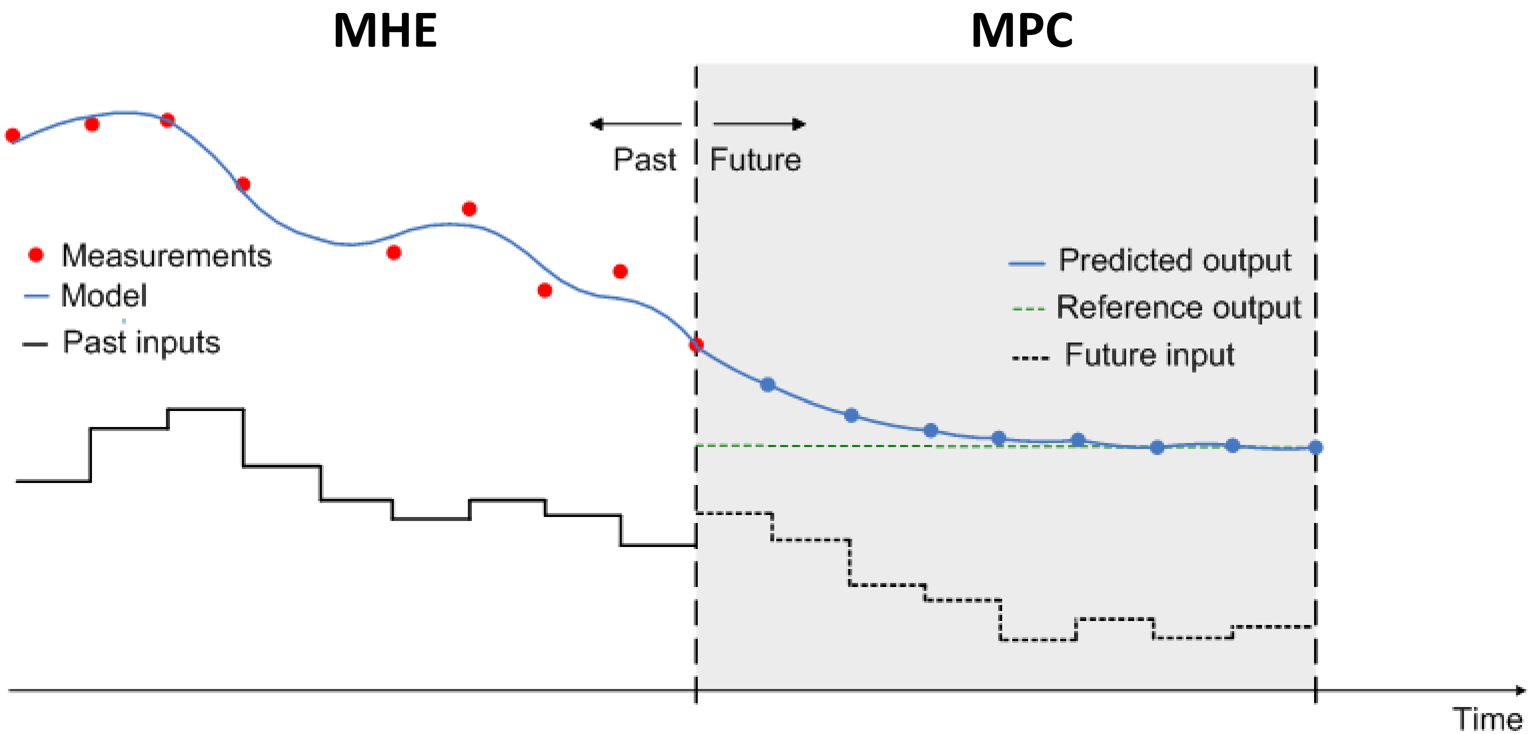




The closed loop control scheme



The closed loop control scheme



Free initial state

Positive semidefinite Hessian

Changing arrival cost

Control dimension ≈ state dimension

Few active constraints

STRUCTURE EXPLOITING MHE ALGORITHMS

The MHE optimization problem

Linear MHE: a quadratic (sub)problem

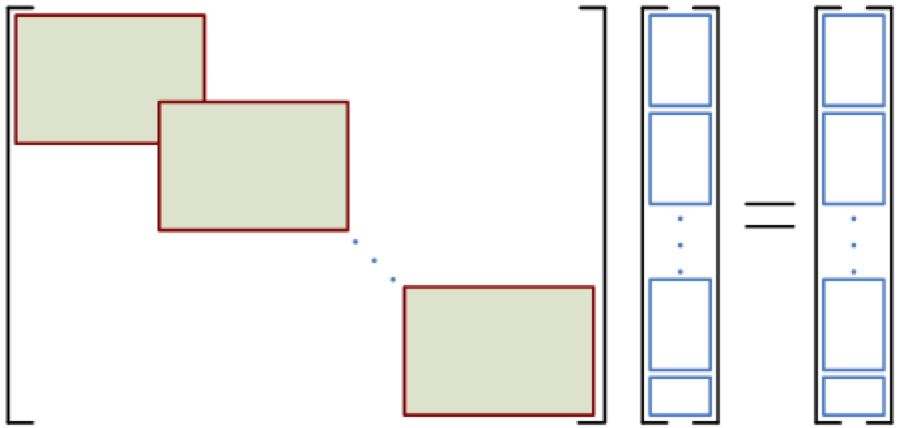
$$\min_{\Delta x, \Delta w} \|S_0^{-T}(\bar{x}_0+\Delta x_0-\hat{x}_0)\|_2^2 + \sum_{k=0}^{N-1} \|W_k^{-T}(\bar{w}_k+\Delta w_k)\|_2^2 + \sum_{k=0}^{N} \|V_k^{-T}(C_k(\bar{x}_k+\Delta x_k)-y_k)\|_2^2$$

$$\begin{aligned} \text{s.t.} \quad & \Delta x_{k+1} = f_k + A_k \Delta x_k + G_k \Delta w_k \quad k = 0, \ldots, N-1 \\ & g_k + D_k \Delta x_k + E_k \Delta w_k \leq 0 \\ & g_N + D_k \Delta x_N \leq 0 \end{aligned}$$

- Writing down the optimality conditions (KKT system), and
- Ordering the block rows,
- … yields a highly structured linear system of equations
- which can be solved with Riccati and vector recursions

A highly structured KKT system

Every time step represents one block in the KKT matrix



Information is translated in three steps

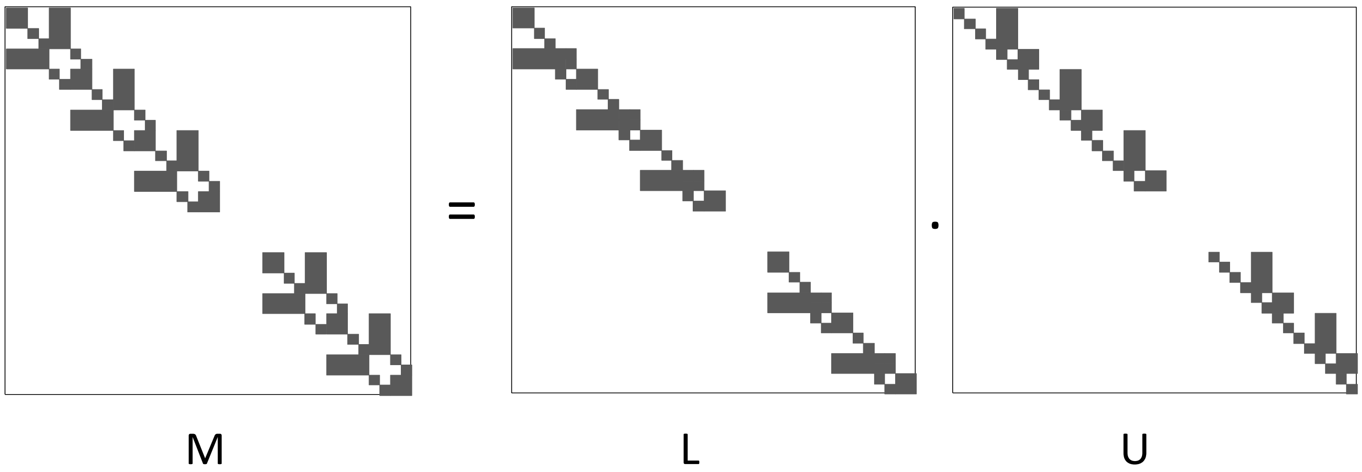
$$\begin{bmatrix} P_0^{-1}+C_0^T C_0 & 0 & A_0^T & 0 \\ 0 & I_{n_w} & G_0^T & 0 \\ A_0 & G_0 & 0 & -I_{n_x} \end{bmatrix}$$

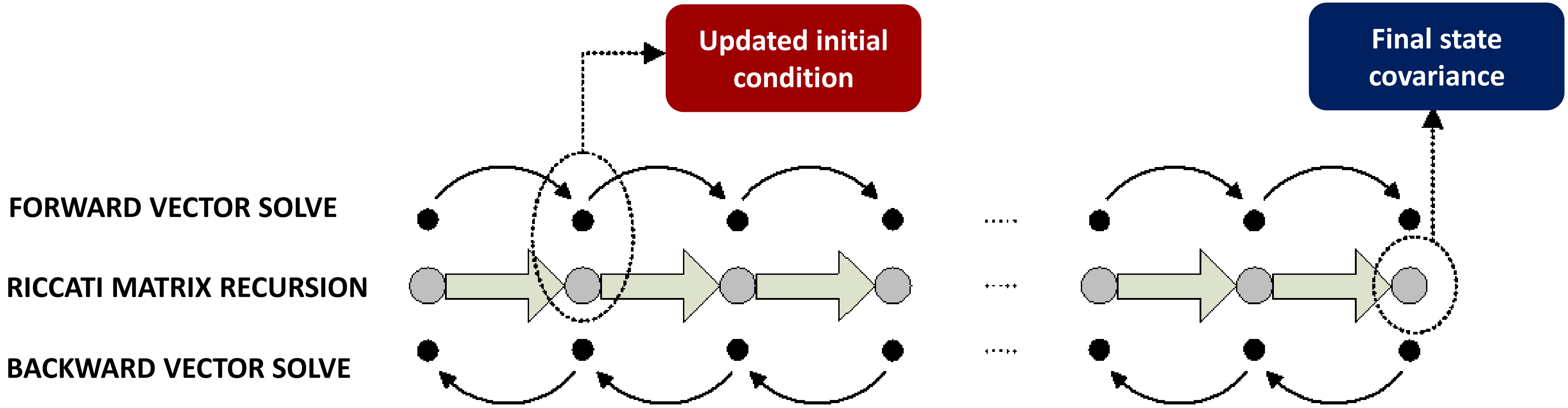
$$\begin{bmatrix} -I_{n_x} & C_1^T C_1 & 0 & A_1^T & 0 \\ 0 & 0 & I_{n_w} & G_1^T & 0 \\ 0 & A_1 & G_1 & 0 & -I_{n_x} \end{bmatrix}$$

1. A priori information
2. Model forwarding
3. Measurement updating

Decomposing the KKT system

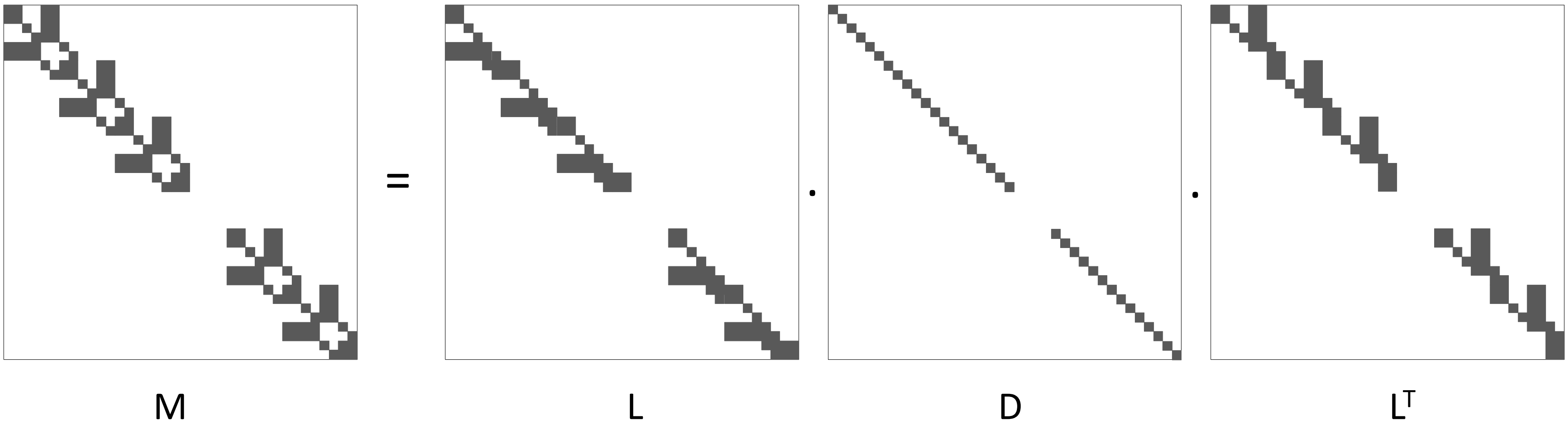
LU decomposition yields the **normal Riccati recursion**





Decomposing the KKT system

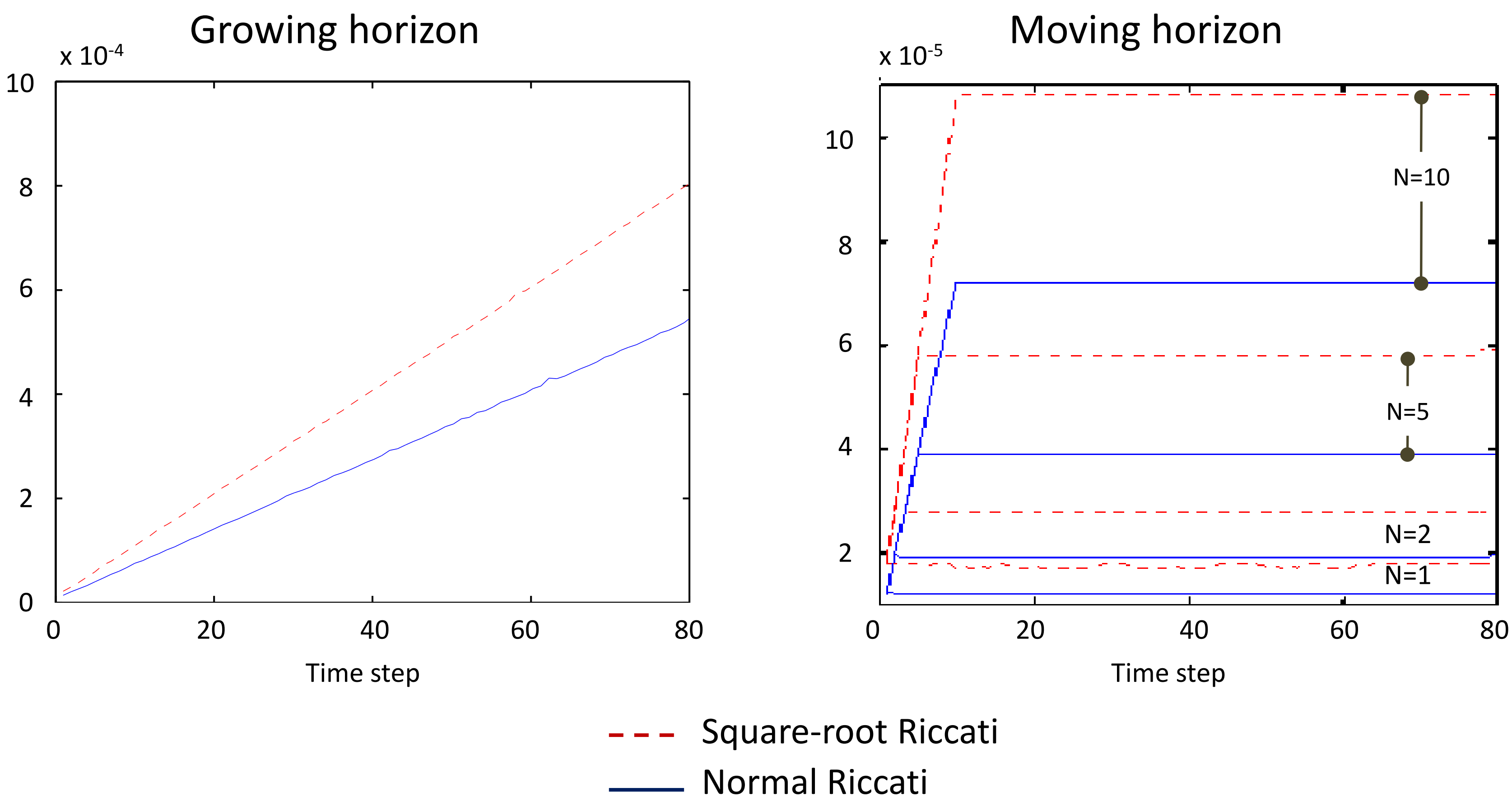
LDL^T decomposition yields the **square-root Riccati recursion**



- Measurement update and time forwarding via *Q-less* QR factorizations
- Fully exploits symmetry
- Yields increased numerical stability

Riccati based MHE

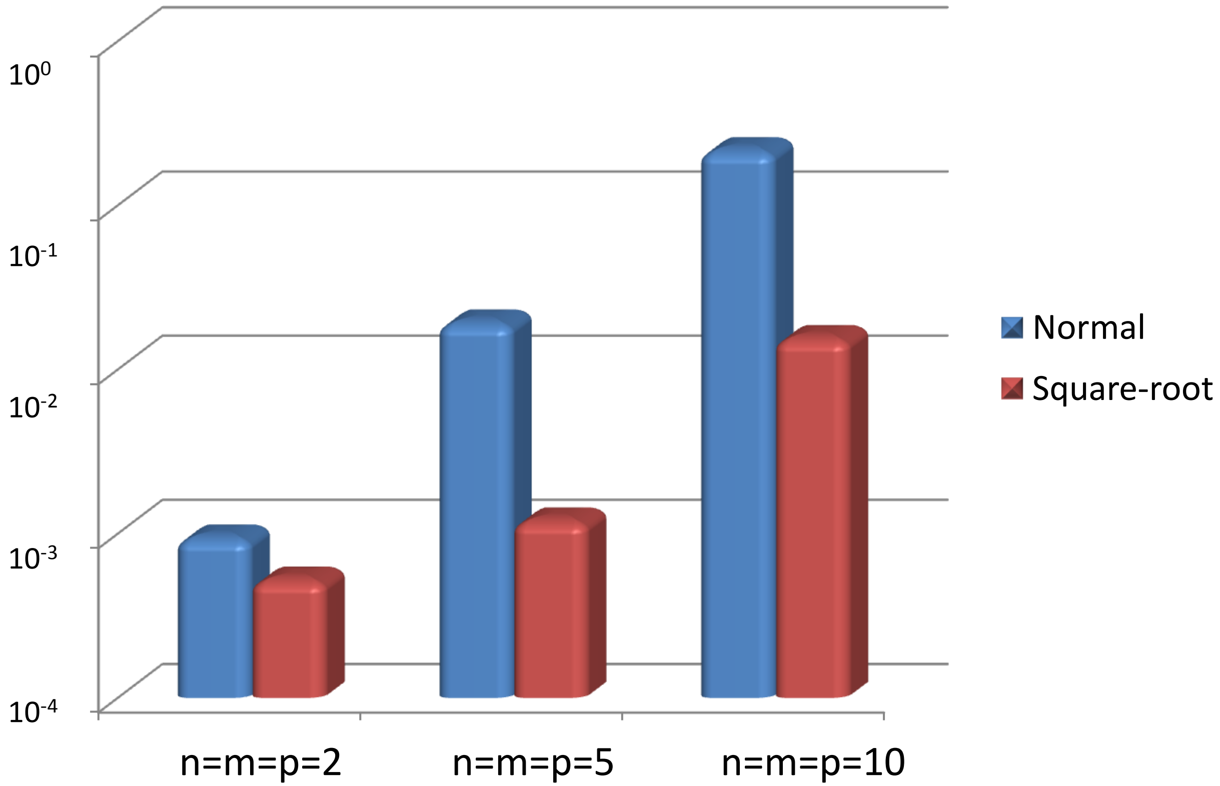
Computation times for 5th order systems



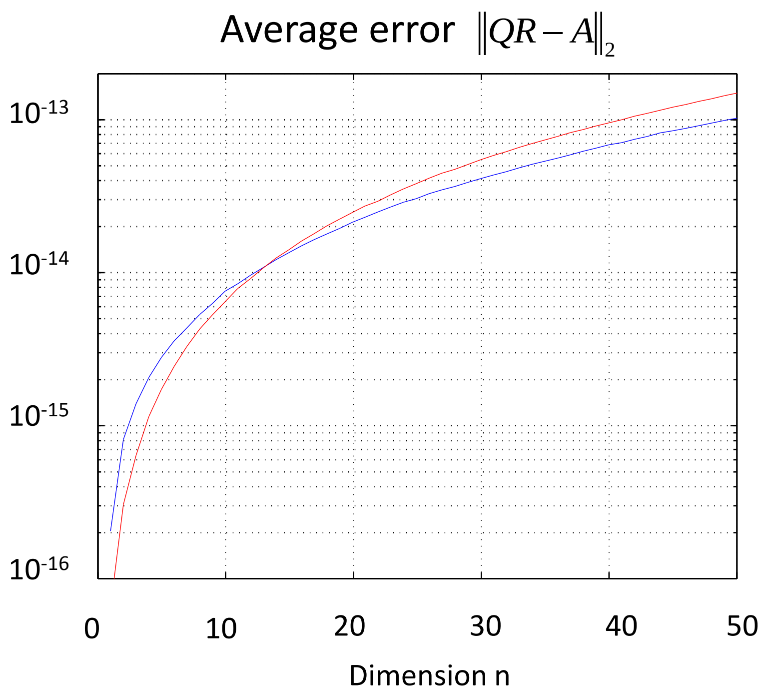
Riccati based MHE

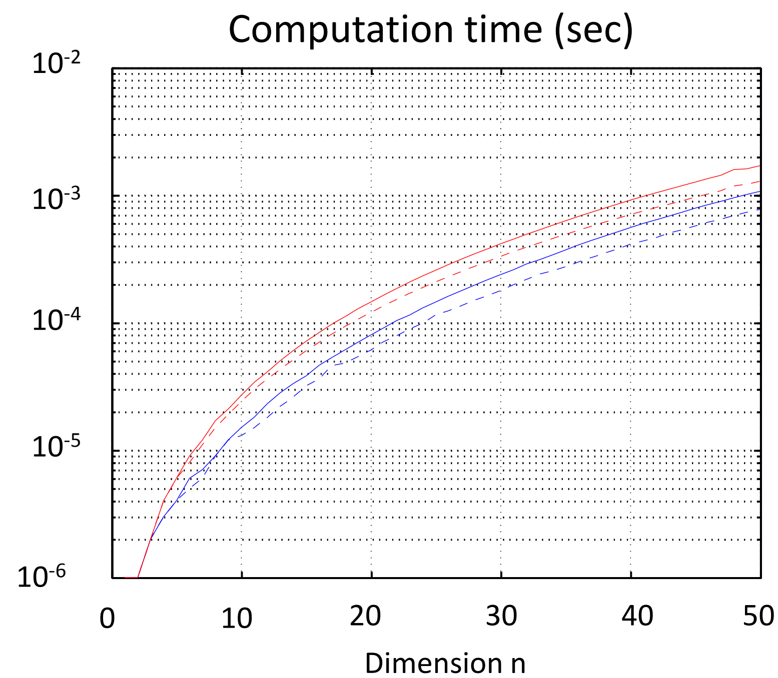
Accuracy

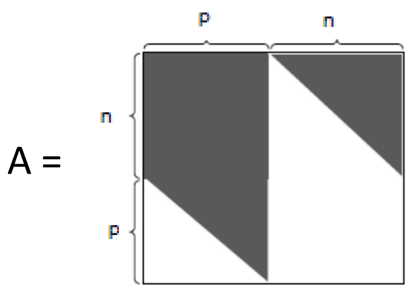
Error $\|P - P_\infty\|_2$



Structured QR factorization







Givens — Structured Givens

Householder — Structured Householder

Interior-point MHE

Primal barrier method

$$
\begin{aligned}
\min_z \quad & \tfrac{1}{2} z^\top H z + g^\top z \\
\text{s.t.} \quad & Cz = d \\
& Pz \le h
\end{aligned}
$$

$$
\begin{aligned}
\min_z \quad & z^T H z + g^T z + \kappa \phi(z) \\
\text{s.t.} \quad & Cz = b
\end{aligned}
$$

$$
\phi(z) = \sum_{i=1}^{p} -\log(h_i - p_i^T z)
$$

Newton method

$$
\begin{bmatrix} H + \kappa P^T \operatorname{diag}(d)^2 P & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta \nu \end{bmatrix} = - \begin{bmatrix} r_d \\ r_p \end{bmatrix}
$$

$$
\kappa \begin{bmatrix} M_0^T M_0 & M_0^T L_0 & & \\ L_0^T M_0 & L_0^T L_0 & & \\ & & \ddots & \\ & & & M_N^T M_N \end{bmatrix}
$$

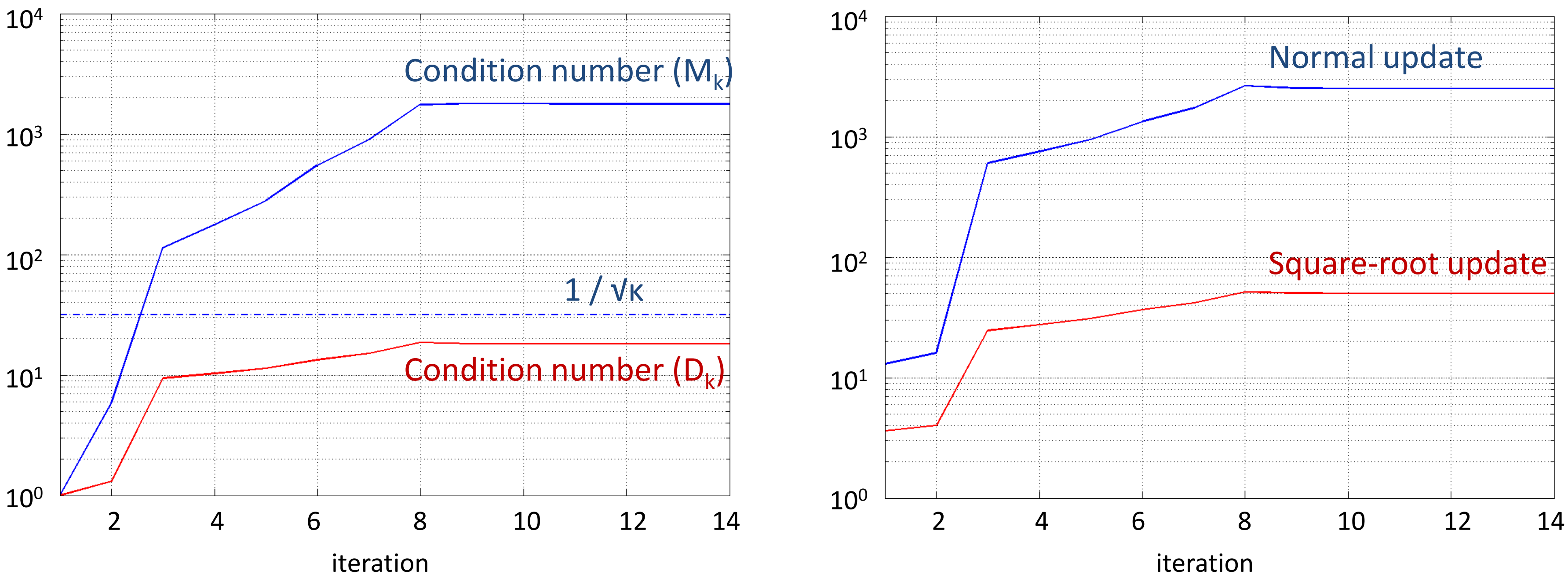
With
$$
\begin{aligned}
r_d &= Hz + g + \kappa P^T d + C^T \nu \\
r_p &= Cz - b
\end{aligned}
$$

Interior-point MHE

Modified Riccati recursion

$$\Sigma_{k+} = \left(\Sigma_k^{-1} + D_k^\top R_k^{-1} D_k\right)^{-1} = \Sigma_k - \Sigma_k D_k^\top \left(\begin{bmatrix} R_k & \\ & I_{ni_k} \end{bmatrix} + D_k \Sigma_k D_k^\top \right)^{-1} D_k \Sigma_k$$

With $\Sigma_k = \begin{bmatrix} P_k & \\ & Q_k \end{bmatrix}$ and $D_k = \begin{bmatrix} C_k & H_k \\ \sqrt{\kappa} M_k & \sqrt{\kappa} L_k \end{bmatrix}$

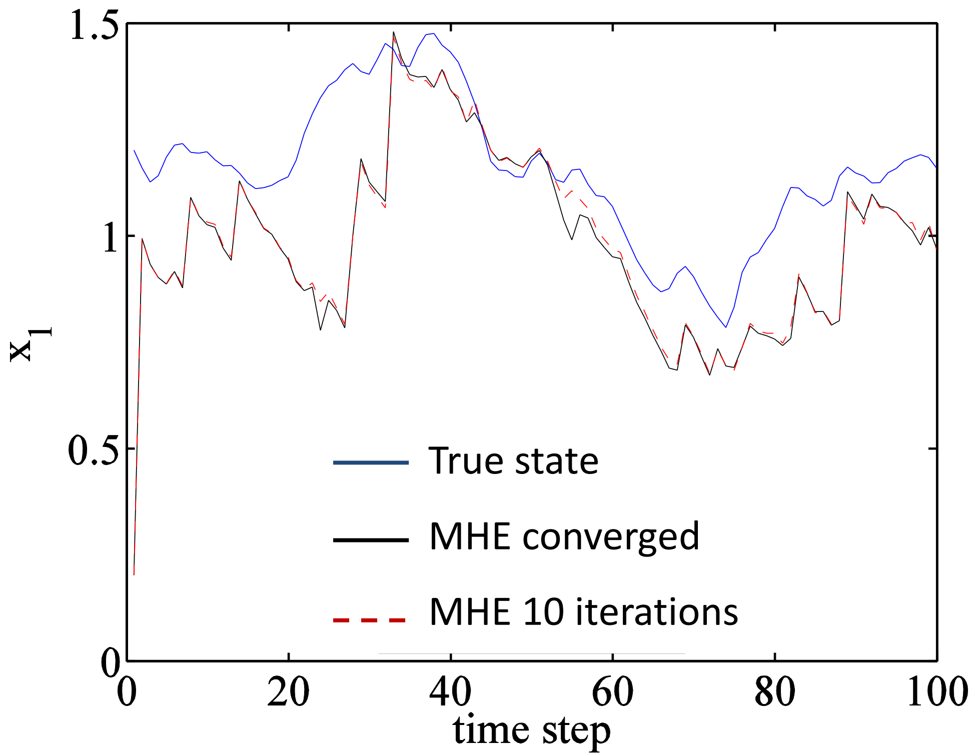


Interior-point MHE

Computation times

Finite number of iterations with decreasing κ

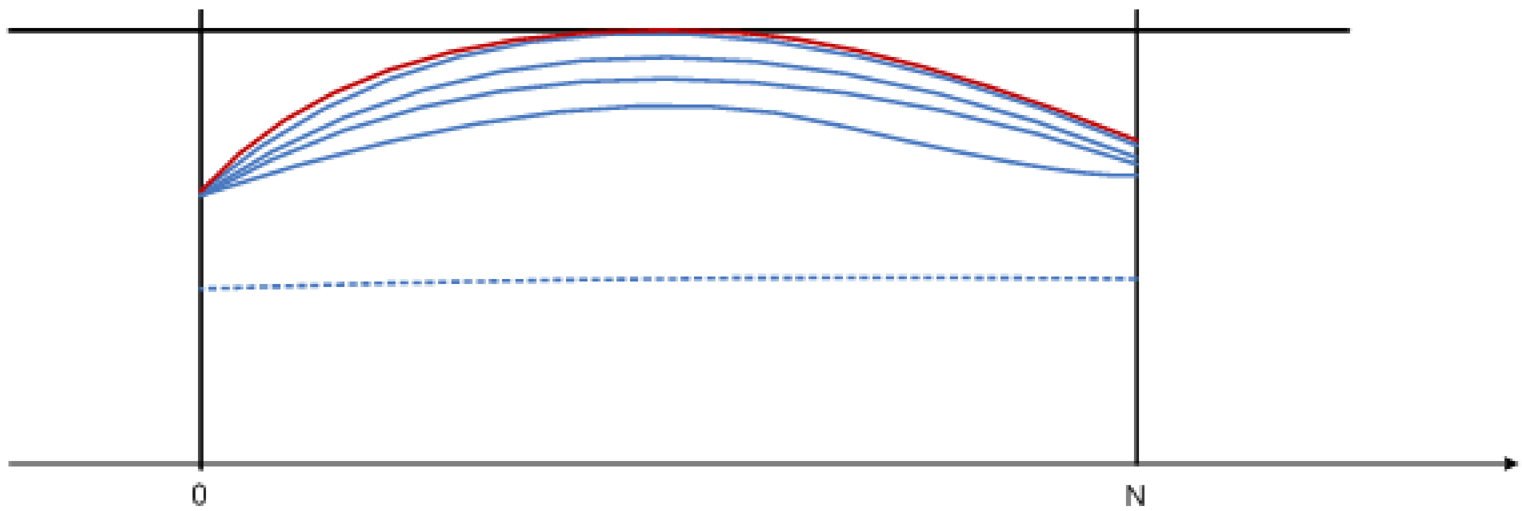
Example - second order system



10 iterations yield satisfying results in this case

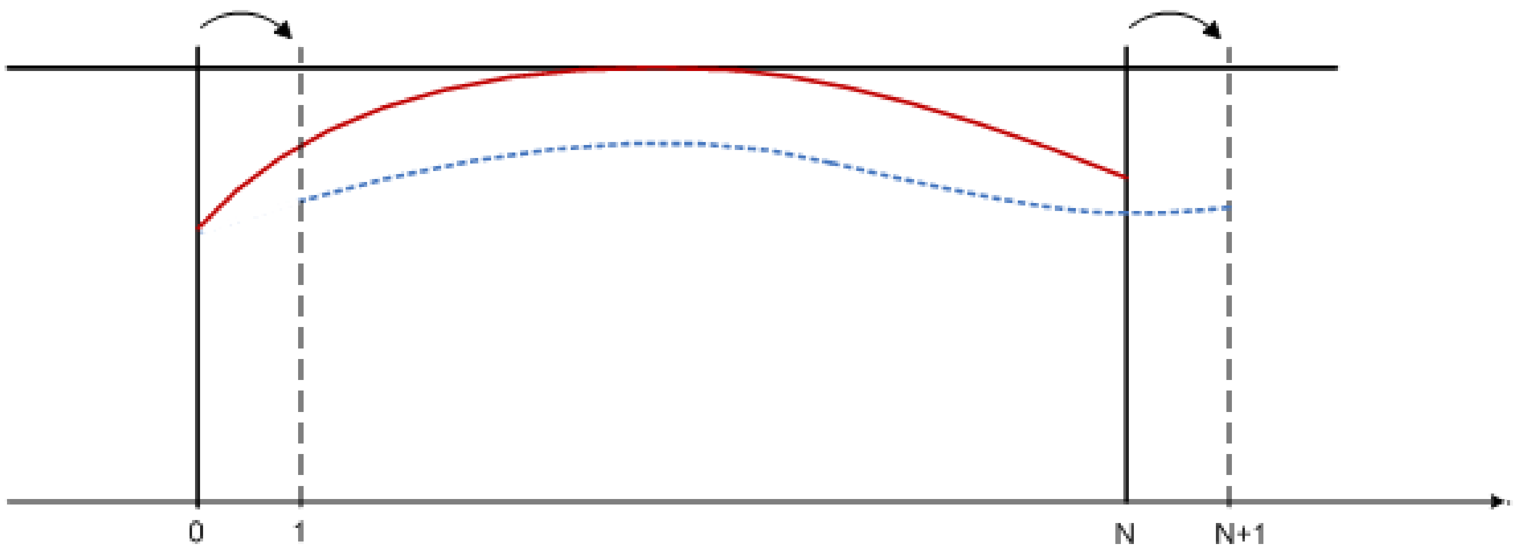
Interior-point MHE

Hot starting



Interior-point MHE

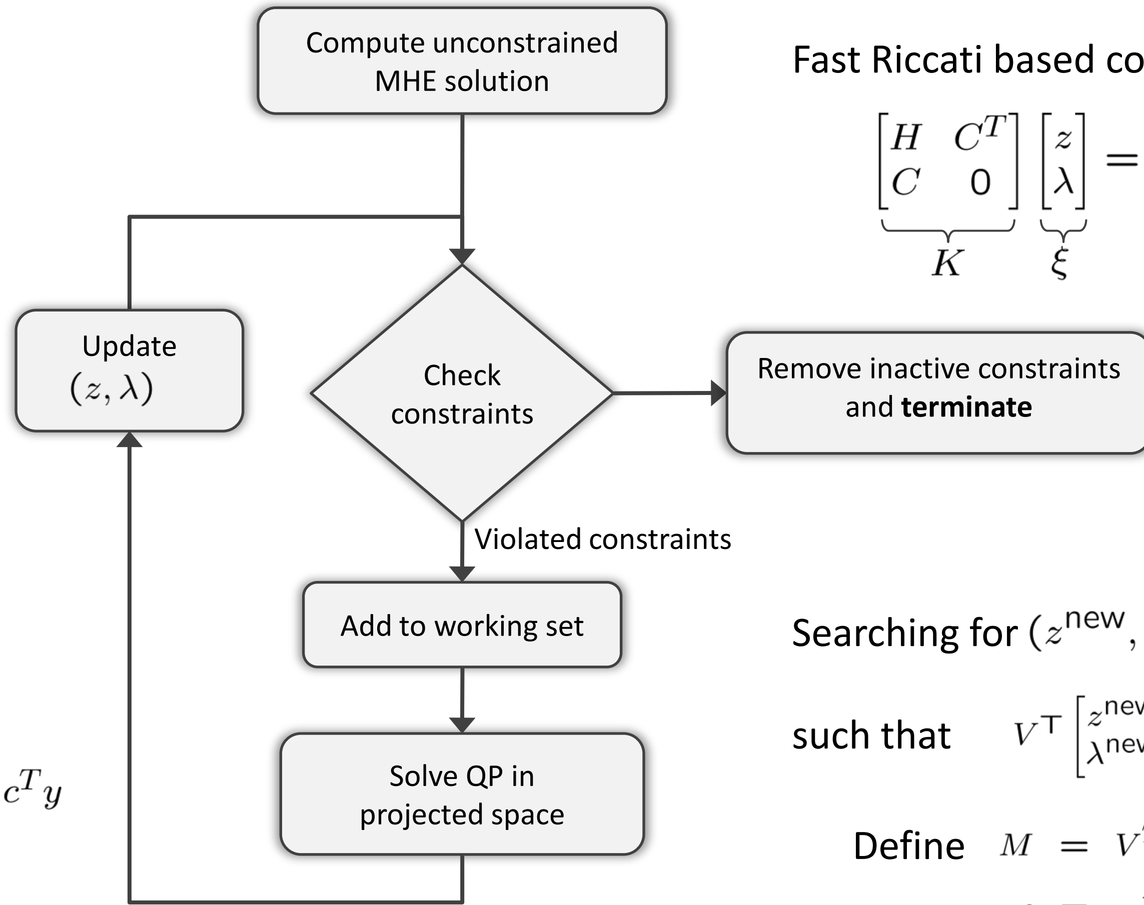
Hot starting



- A good initialization **is** necessary for fast convergence
- Hot starting with the previous solution or the proposed strategy
- Yields convergence improvement for first iterations

A Schur-complement active-set method

Method outline



Fast Riccati based computation

$$\underbrace{\begin{bmatrix} H & C^T \\ C & 0 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} z \\ \lambda \end{bmatrix}}_{\xi} = \underbrace{\begin{bmatrix} -g \\ b \end{bmatrix}}_{r}$$

$$\begin{bmatrix} z^{\text{new}} \\ \lambda^{\text{new}} \end{bmatrix} = \begin{bmatrix} z^0 \\ \lambda^0 \end{bmatrix} - (K^{-1}V)y$$

Searching for $(z^{\text{new}}, \lambda^{\text{new}})$

such that $V^{\mathrm{T}} \begin{bmatrix} z^{\text{new}} \\ \lambda^{\text{new}} \end{bmatrix} \leq h_{\mathcal{I}}$

Define
$$\begin{aligned} M &= V^T(K^{-1}V) \\ c &= -V^T \begin{bmatrix} z^0 \\ \lambda^0 \end{bmatrix} + h_{\mathcal{I}} \end{aligned}$$

Non-negativity constrained QP

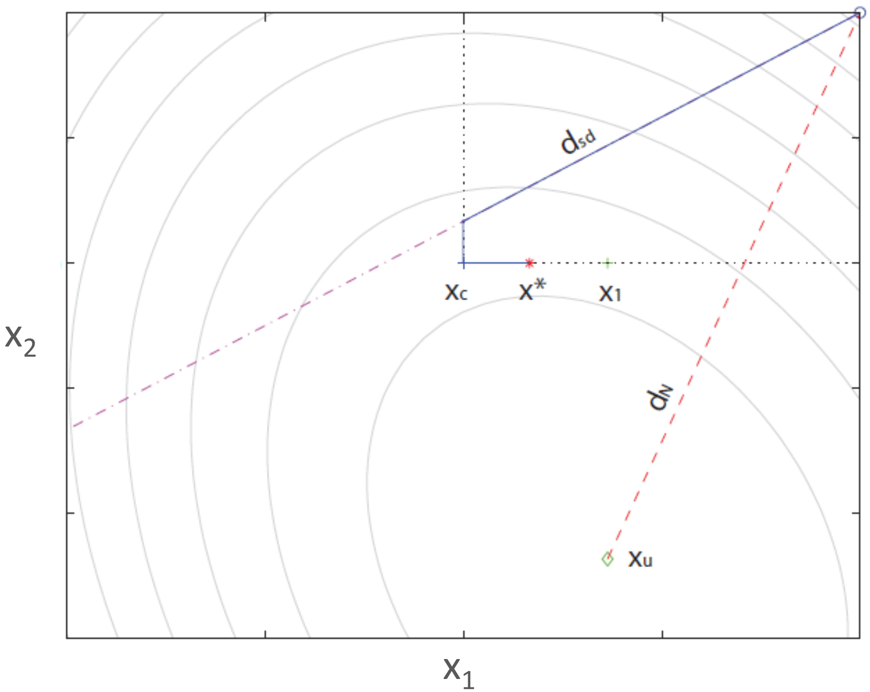
$$\begin{aligned} \min_y \quad & \tfrac{1}{2} y^T M y + c^T y \\ \text{s.t.} \quad & y \geq 0, \end{aligned}$$

A Schur-complement active-set method

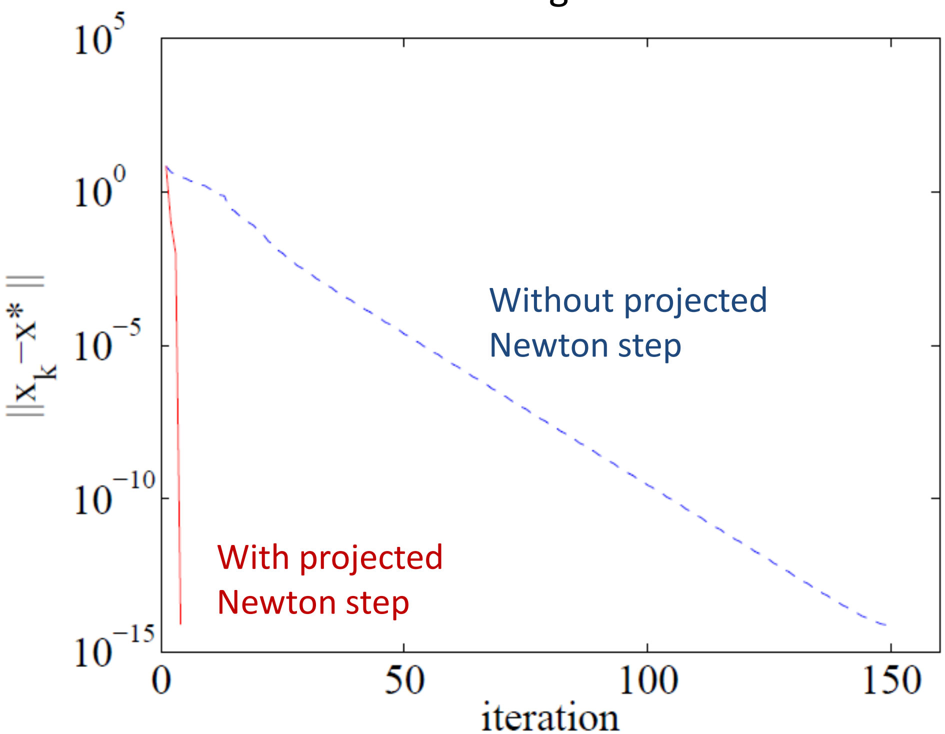
Gradient projection method for non-negativity constrained QP

1. Cauchy calculation step
2. Projected Newton step

Principle



Convergence



A Schur-complement active-set method

Gradient projection method for non-negativity constrained QP

- Projected Newton step

$$\begin{array}{ll} \min_z & \frac{1}{2}x^\top Mx + c^\top x \\ \text{s.t.} & x_i = x_i^c,\ i \in \mathcal{A}(x^c) \\ & x_i \geq 0,\ i \notin \mathcal{A}(x^c) \end{array}$$

$$\Rightarrow$$

$$\begin{array}{ll} \min_z & \frac{1}{2}x^\top Mx + c^\top x \\ \text{s.t.} & x_i = 0,\ i \in \mathcal{W} \end{array}$$

1. Use semidefinite Cholesky factorization of M
2. Set $\mathcal{W} = \mathcal{A}(x^c)$
3. Keep adding non-positive constraints to working set
4. Delete rows and colums of (new) working set constraints
5. Continue until all components non-negative

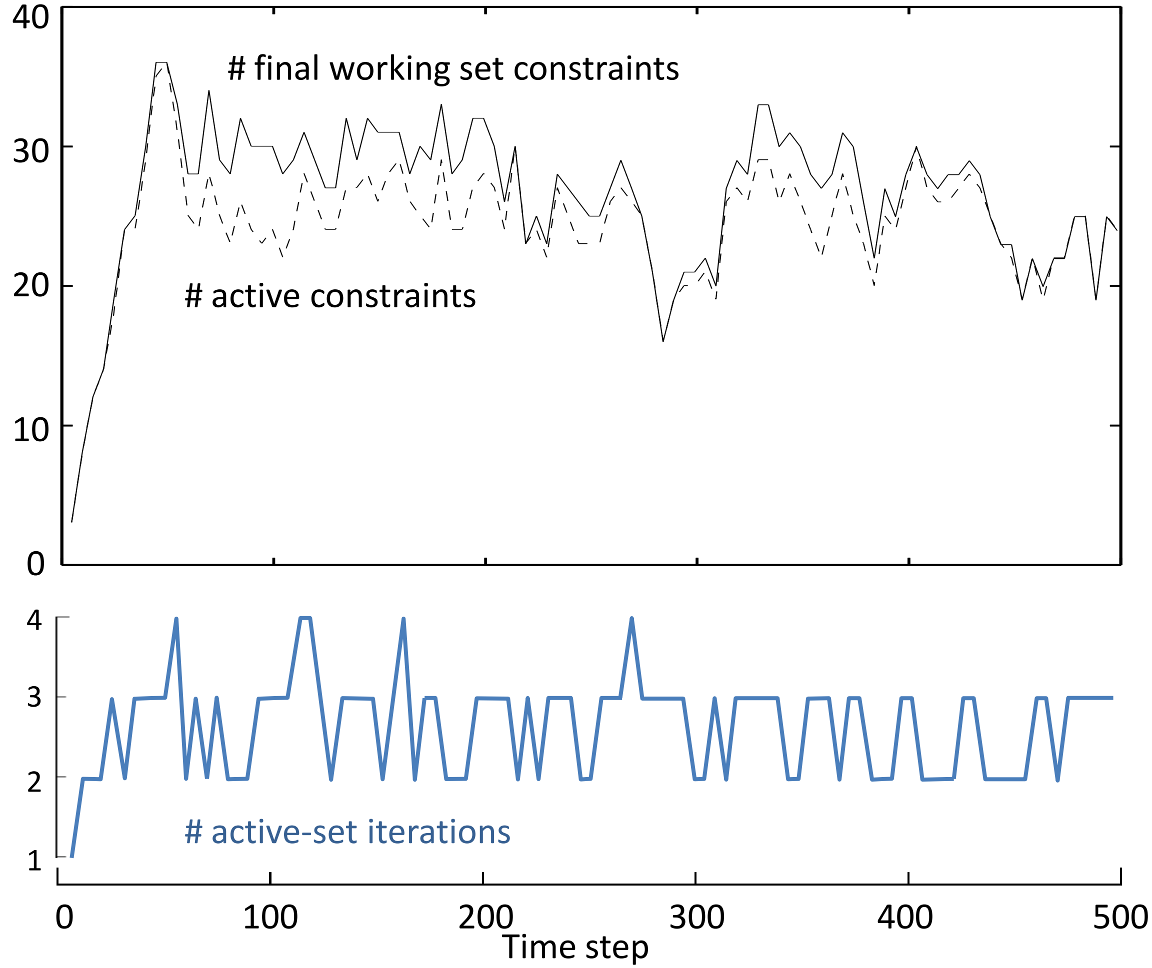
- Between outer active-set iterations: Cholesky block downdating (constraints added)
- Upon termination: Cholesky block updating (constraints removed)

A Schur-complement active-set method

Computational burden

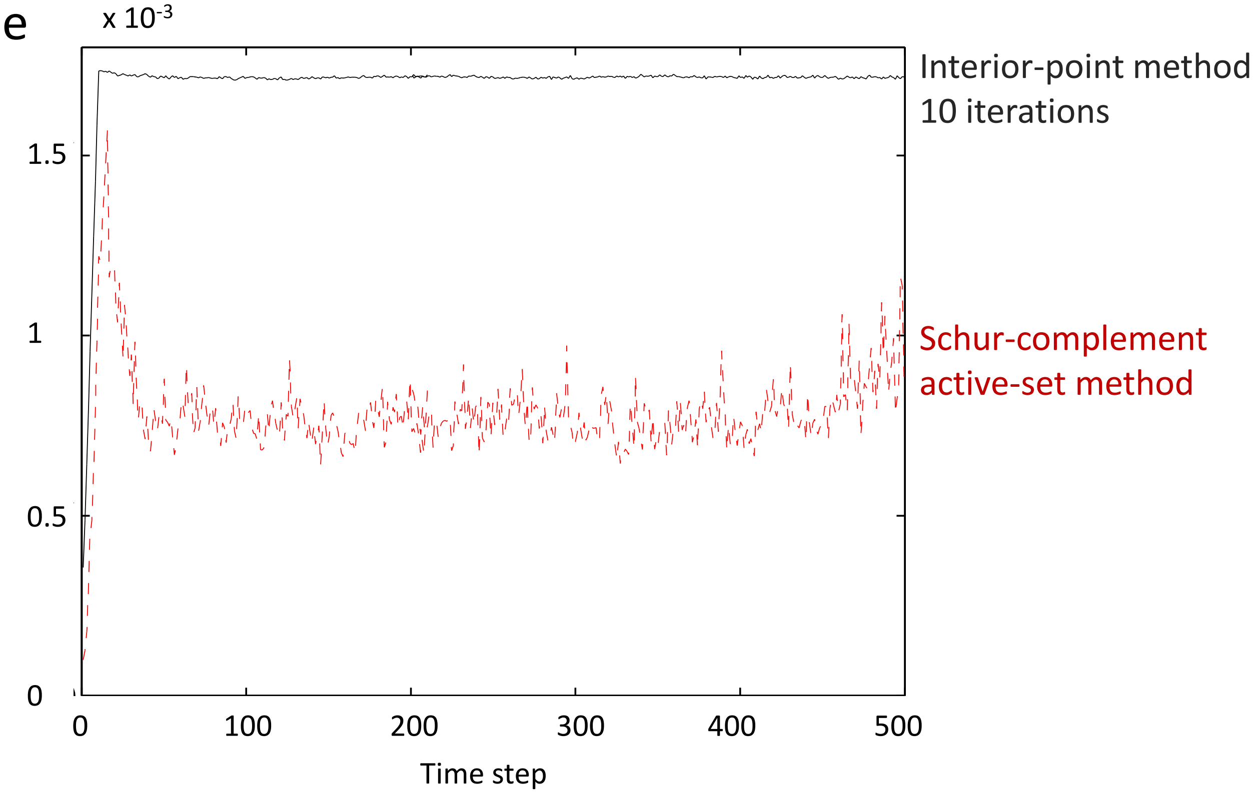
	uMHE	asetMHE	Total
Riccati	1		1
Fsolve	1		1
Partial Fsolve		n_A	n_A
Bsolve	1	n_{it}	$1+n_{it}$
Red. QP		n_{it}	n_{it}

Convergence



A Schur-complement active-set method

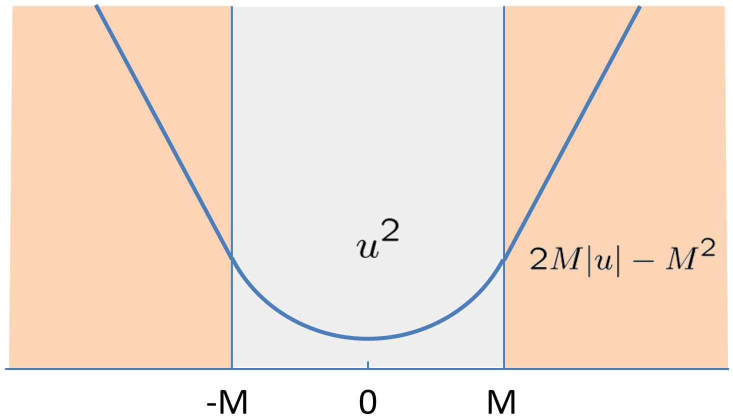
Computation time



CONVEX AND NONLINEAR MHE

Huber penalty MHE

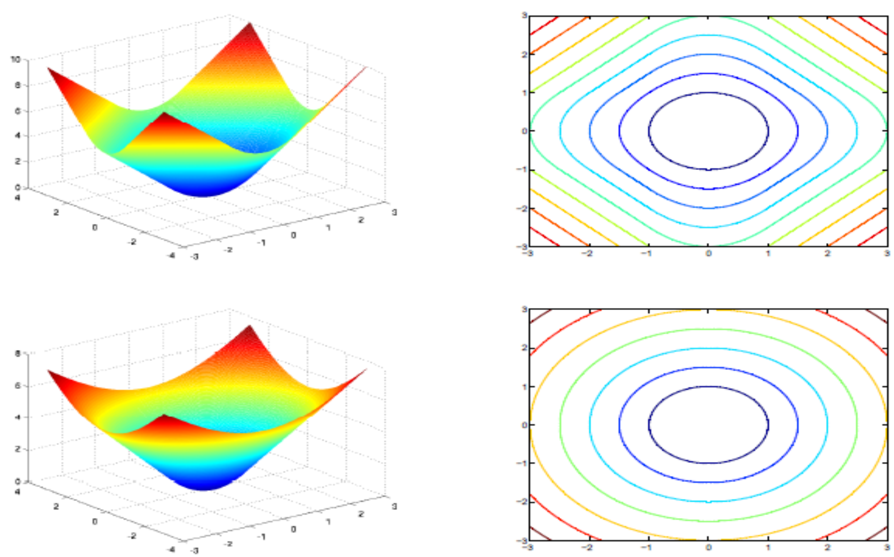
The Huber penalty



- Preserves LS performance
- Increases robustness to outliers

Univariate ➡ QP

$$\begin{aligned} \min_{u,\alpha,\beta} \quad & \alpha^2 + 2M\beta \\ \text{s.t.} \quad & -(\alpha+\beta) \le u \le (\alpha+\beta) \\ & 0 \le \alpha \le M \\ & 0 \le \beta \end{aligned}$$

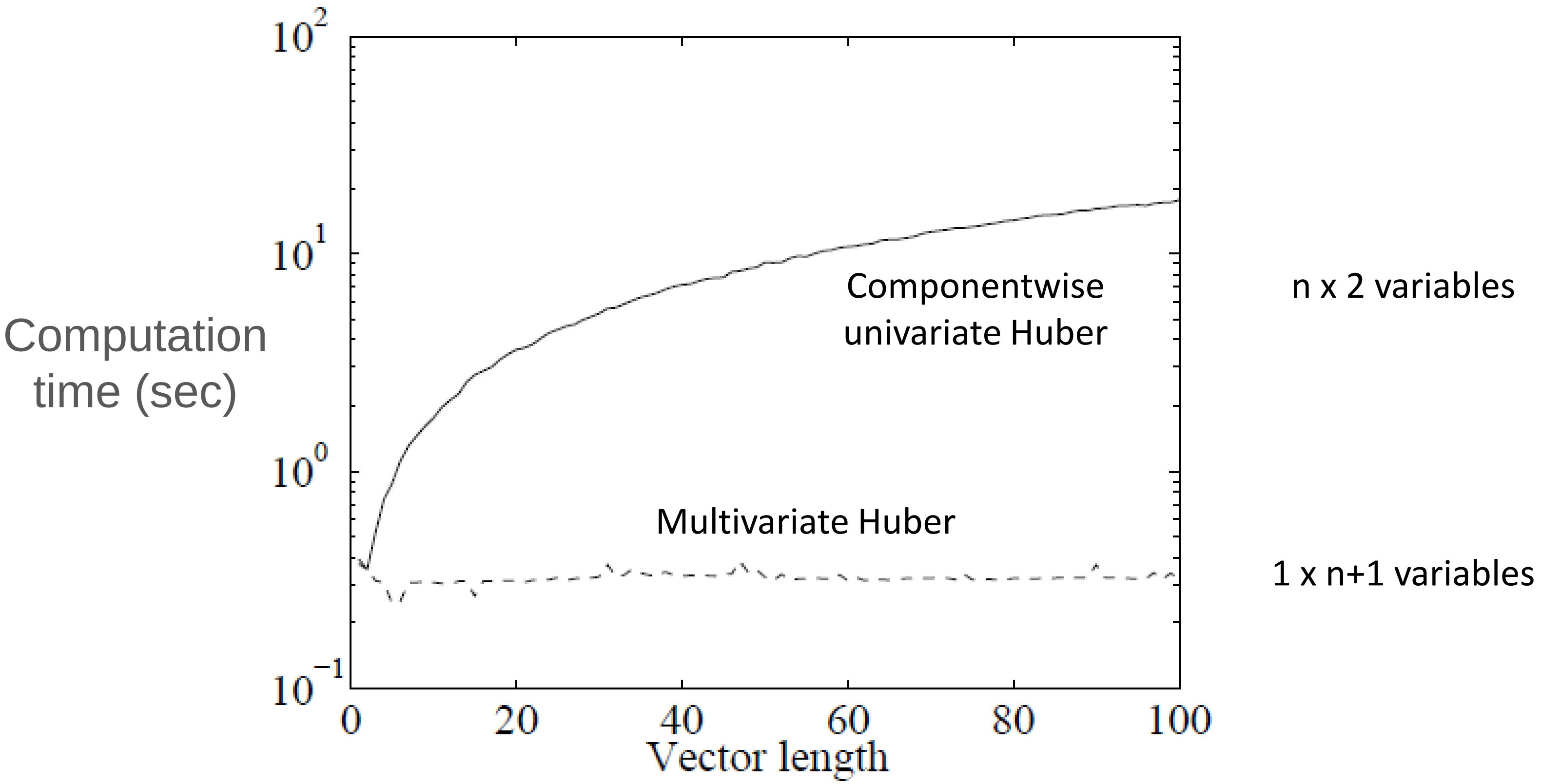


Multivariate ➡ SOCP

$$\begin{aligned} \min_{x,\alpha,\beta} \quad & \|\alpha\|_2^2 + 2M\beta \\ \text{s.t.} \quad & \|x-\alpha\|_2 \le \beta \\ & \|\alpha\|_2 \le M \\ & \alpha \ge 0, \; \beta \ge 0 \end{aligned}$$

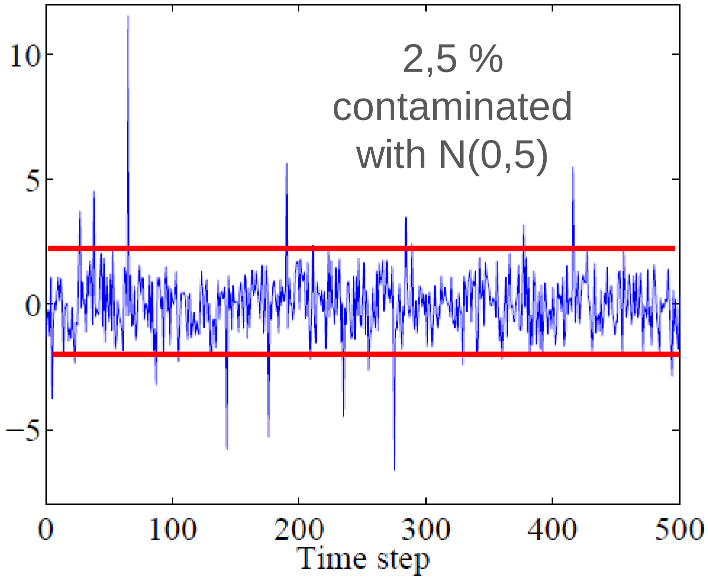
Huber penalty MHE

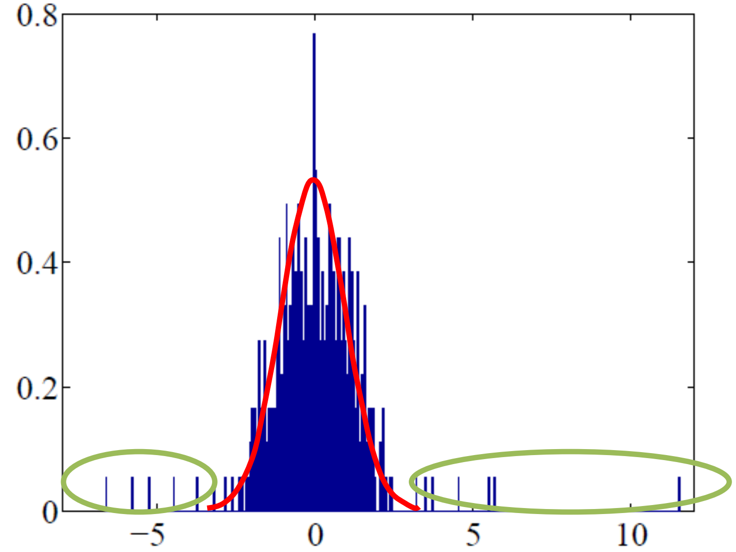
Univariate vs multivariate



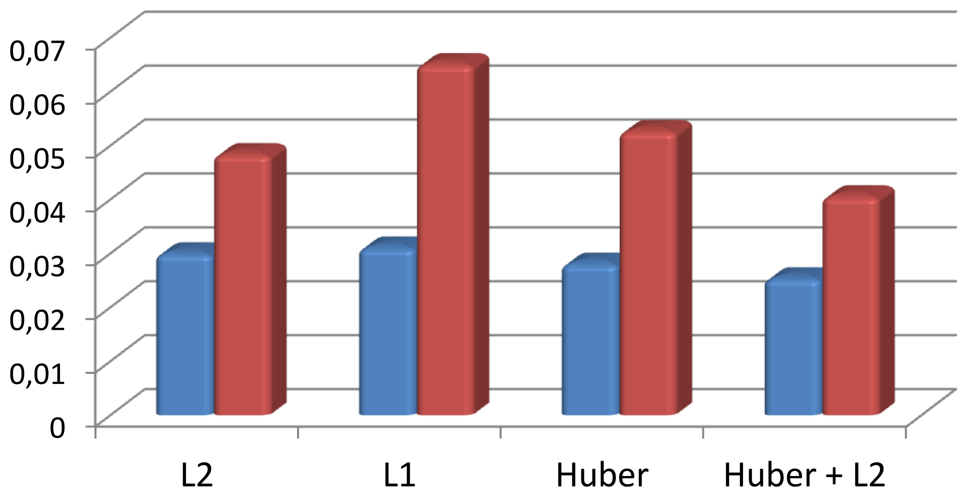
Huber penalty MHE

Output disturbance





Batch estimation
Mean squared error
(50 repetitions)



Simulation 500 time steps

Prediction 50 time steps

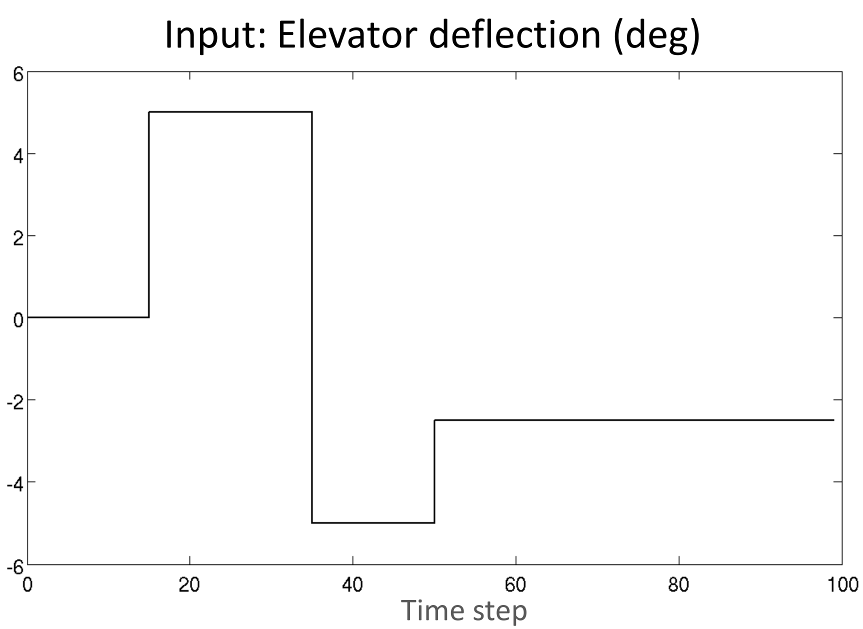
Joint estimation with piecewise inputs

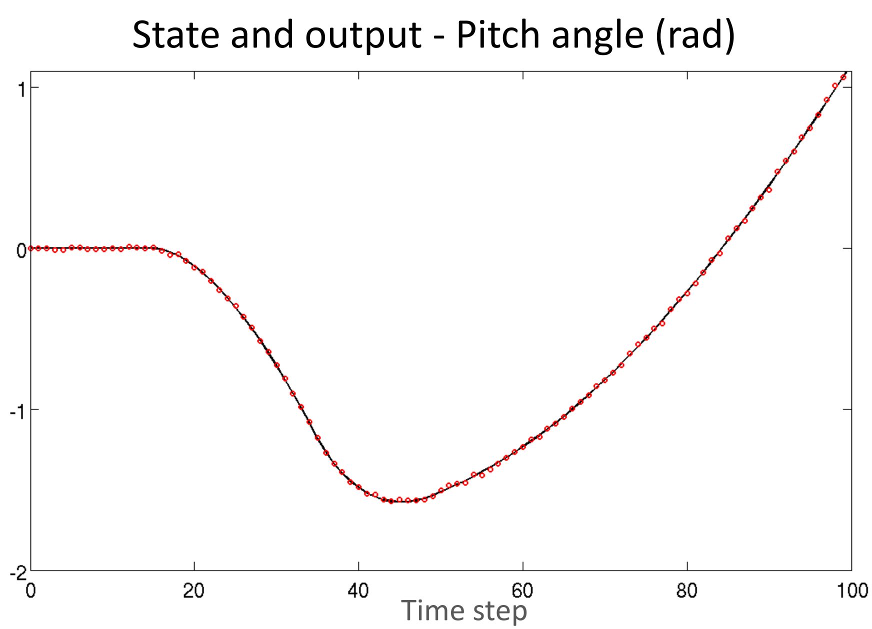
F16 example – linearized longitudinal model

4 states: velocity, angle-of-attack, pitch angle, pitch rate

2 outputs: pitch angle, flight path angle

1 input: elevator deflection

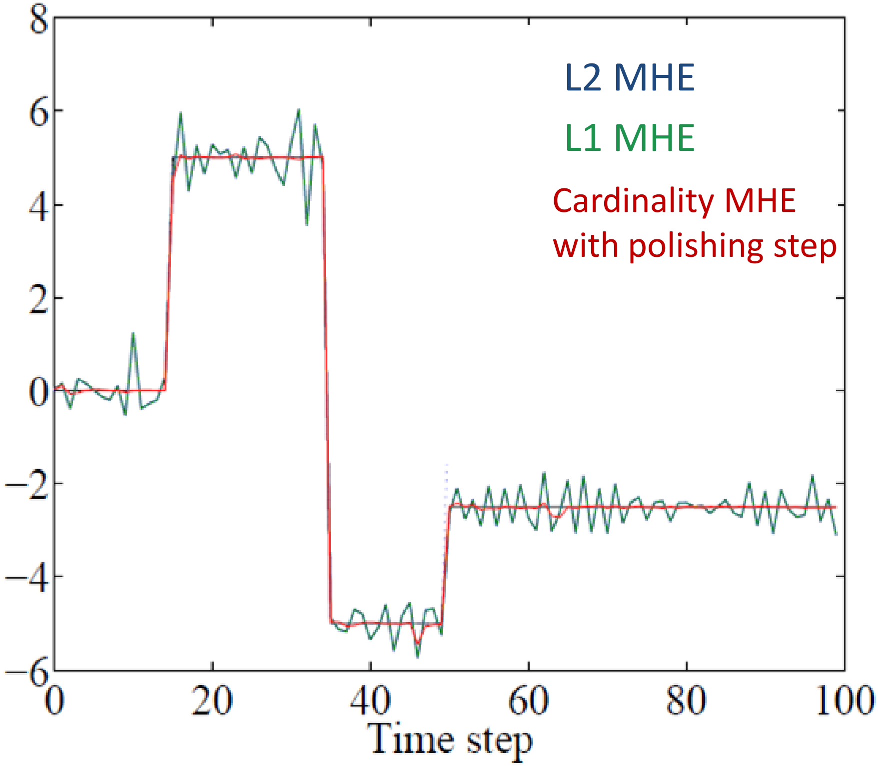




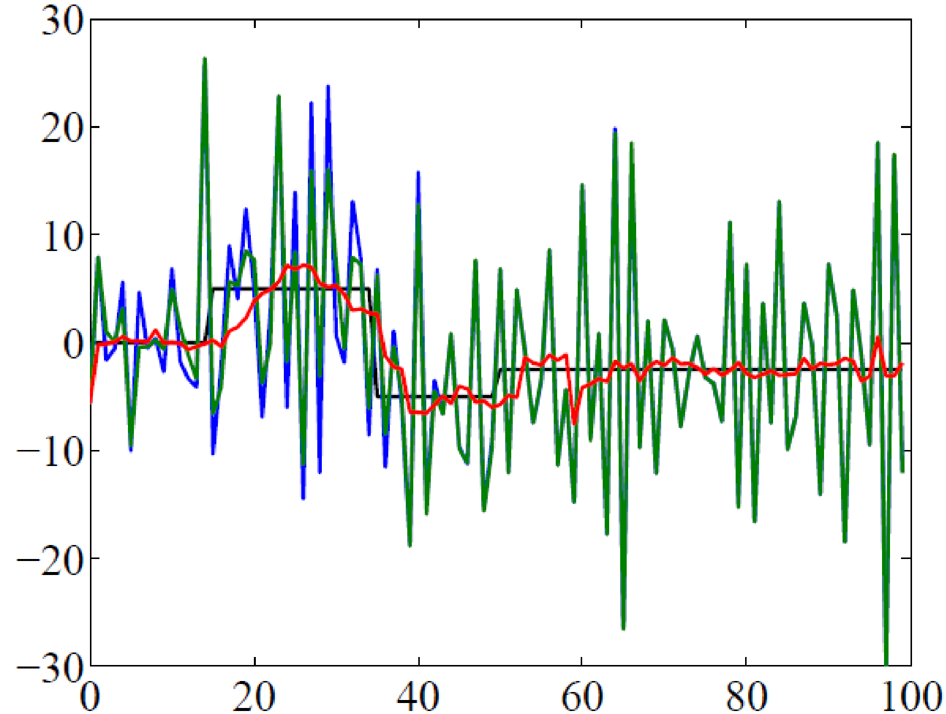
Joint estimation with piecewise inputs

Joint MHE: quality of input estimates

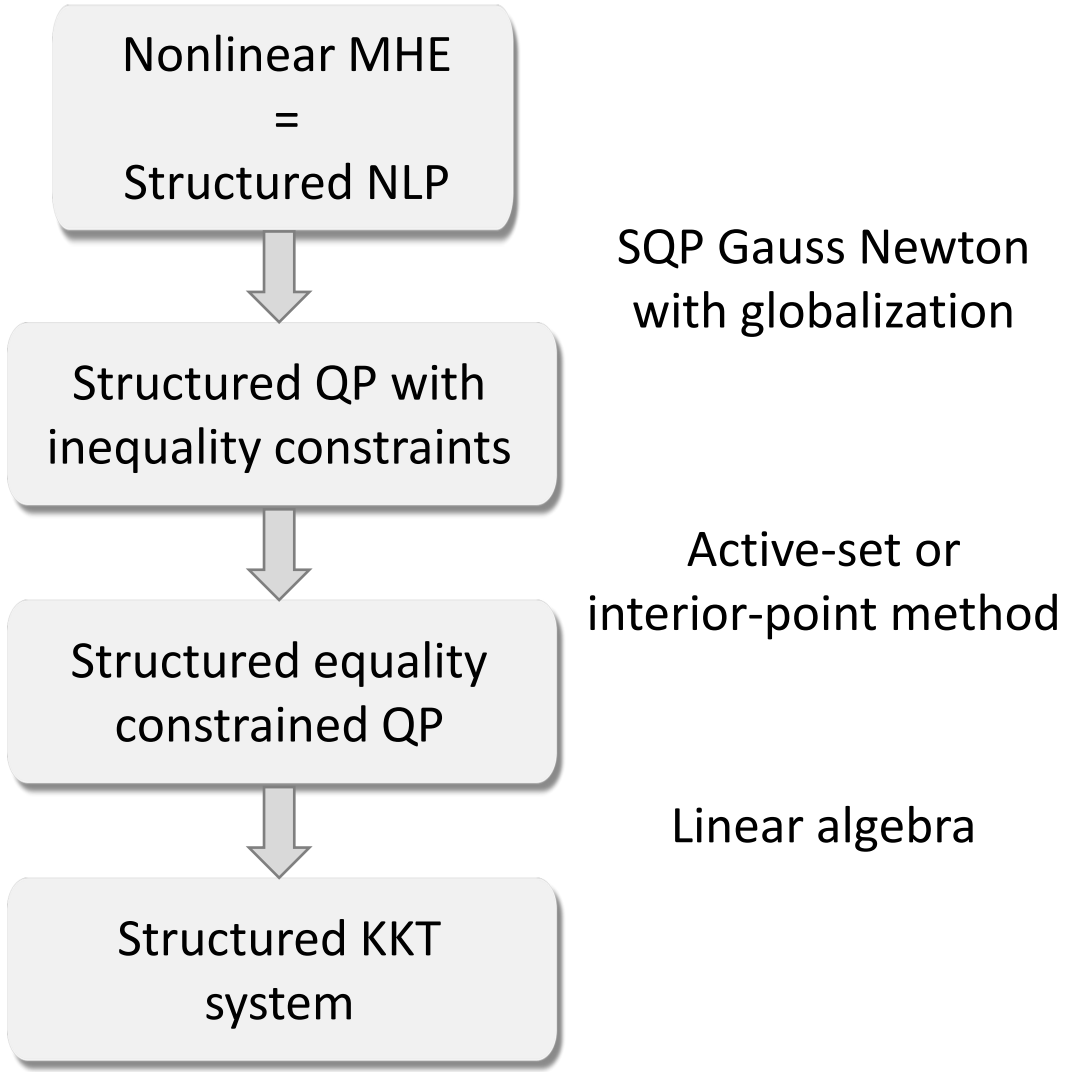
Low output disturbance level R=0.001



High output disturbance level R=0.01

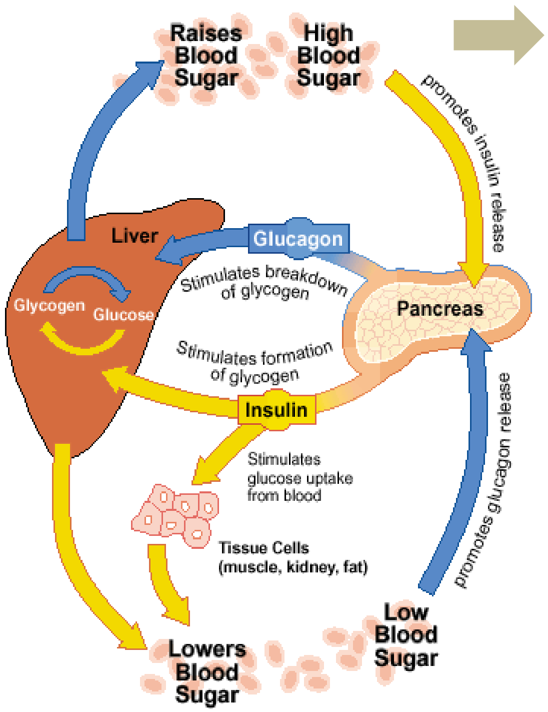


Nonlinear MHE

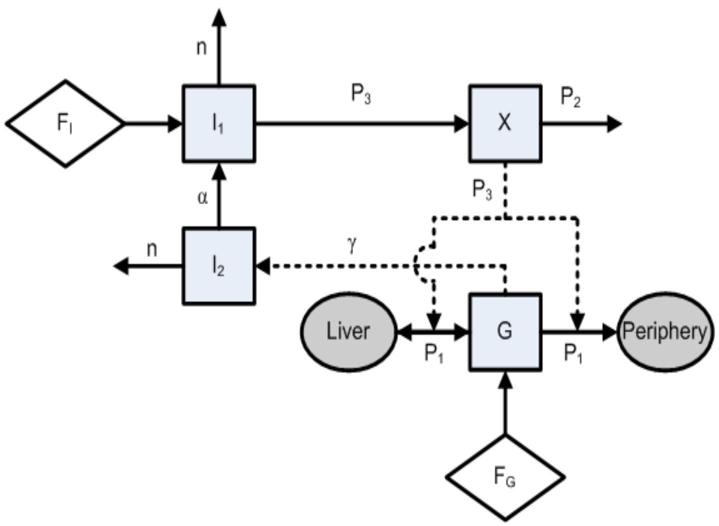


Nonlinear MHE

Estimation and control of glycemia in critically-ill patients



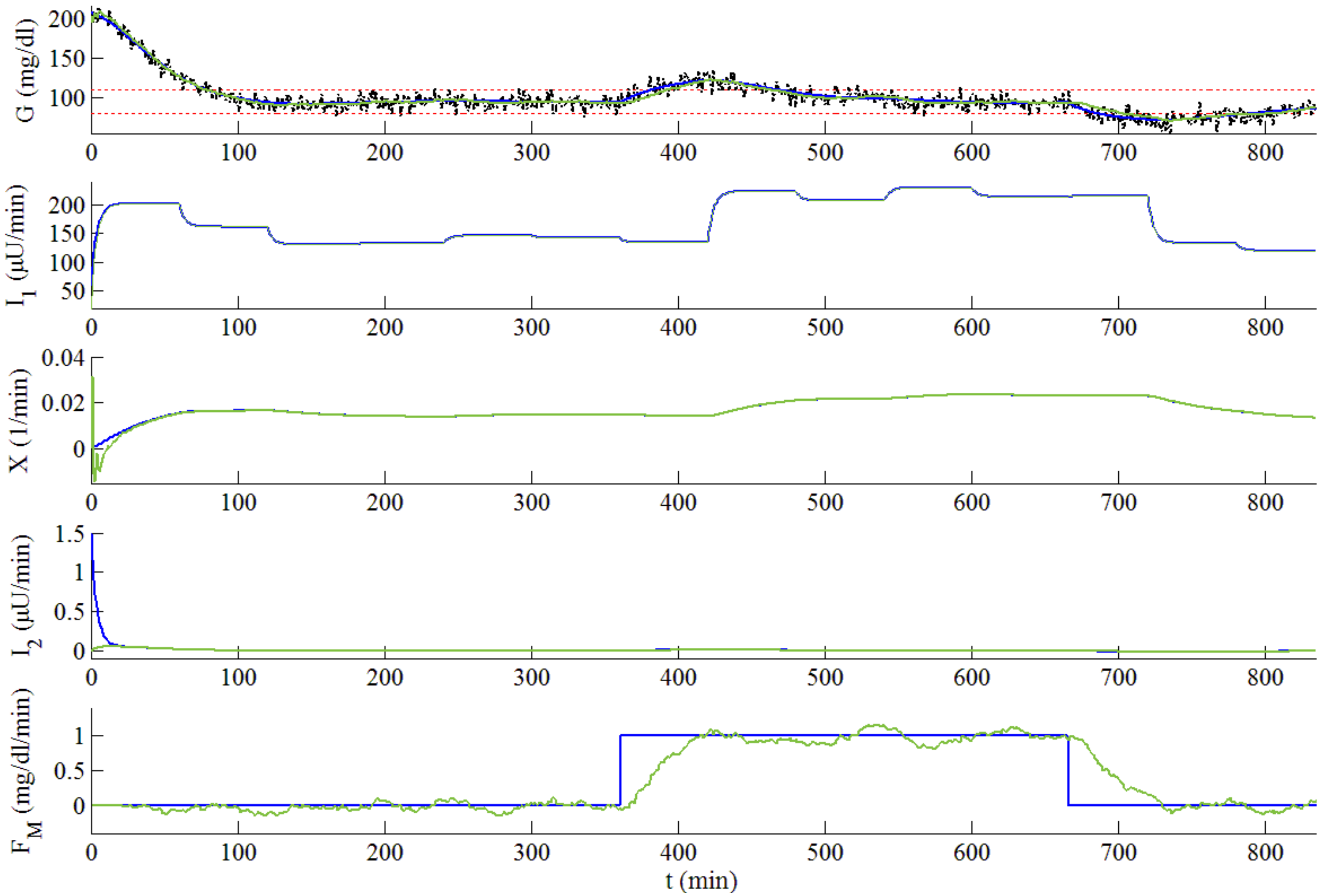
Regulate glycemia to normoglycemic range (80-110 mg/dl)



- Controlled variable: glycemia level (G)
- Known input: carbohydrate calories flow (F_G)
- Unkown input: medication (F_M)
- Manipulated variable: exogenous insulin (F_I)

Nonlinear MHE

Estimation and control of glycemia in critically-ill patients



CONCLUSIONS

Conclusions

- KKT conditions reveal symmetry and structure
  - Decomposition yields Riccati methods
  - Proposed and demonstrated square-root Riccati method using QR factorizations
- Block diagonal structure is preserved in interior-point methods
  - Proposed and demonstrated modified square-root Riccati method
- Block diagonal structure is NOT preserved in active-set methods
  - Proposed and demonstrated a dedicated Schur-complement active-set method

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- Huber penalty increases robustness to outliers
  - Demonstrated Huber penalty MHE
- Joint input estimation with piecewise inputs has finite number of break points
  - Proposed and demonstrated cardinality MHE yielding a sequence of L1-type MHE
- Nonlinear MHE can be solved by SQP Gauss-Newton method
  - Demonstrated NMHE on a biomedical application

Future research

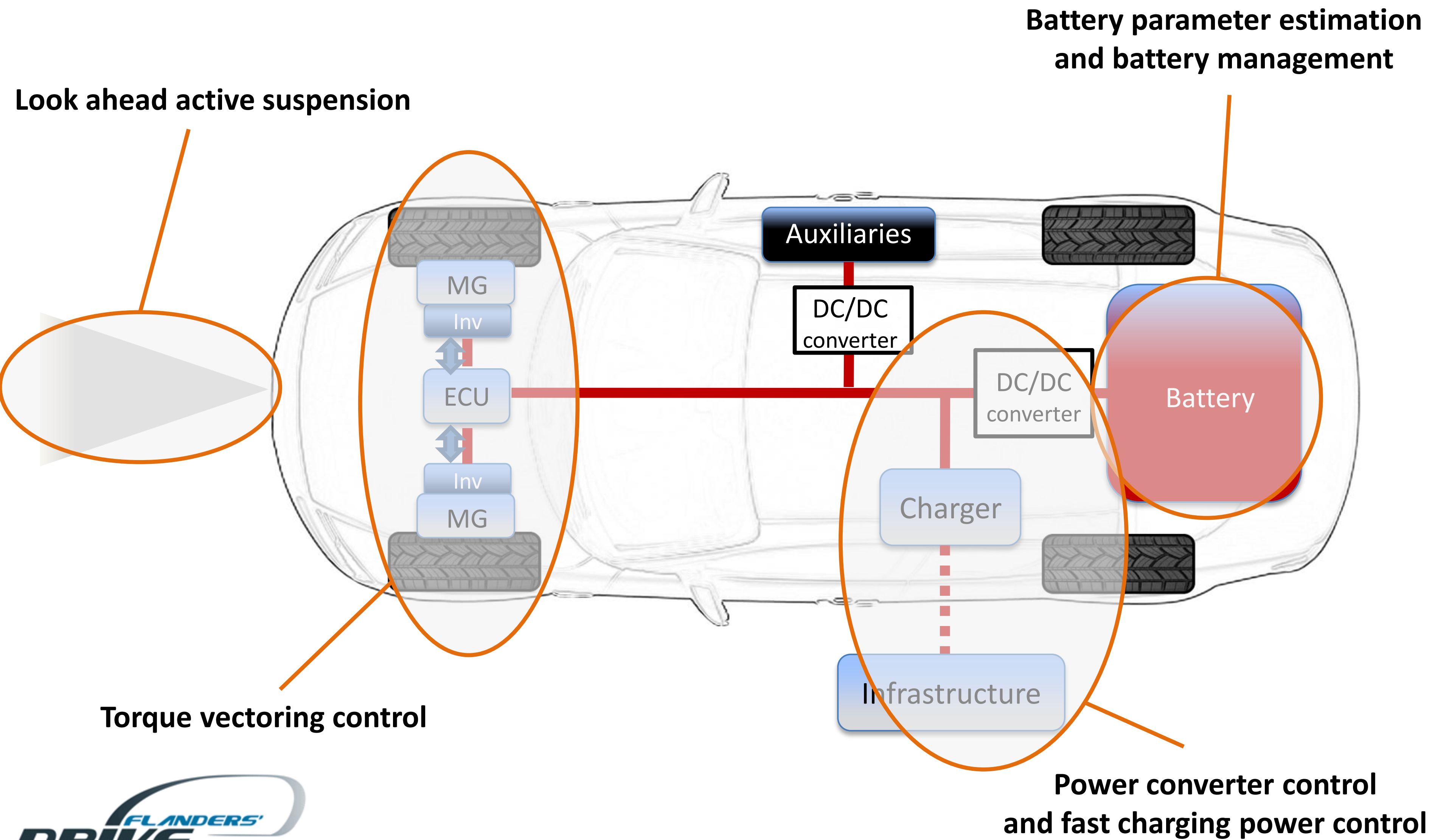
Algorithms

- Ultra-fast nonlinear MHE: fast simulation
- Distributed MHE
- Adaptive control: interaction between MHE and MPC

Applications

- Intensive Care Unit
- Automotive
- Power electronics

Future research



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Co-promotor

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Co-authors

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