

**APPENDIX TO
“SCROLLAR INVARIANTS, SYZYGIES AND REPRESENTATIONS OF THE SYMMETRIC GROUP”**

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Example usage: for a simply branched degree 5 cover $\varphi : C \rightarrow \mathbf{P}^1$ by a curve of genus g with scrollar invariants e_1, e_2, e_3, e_4 and first splitting type $\{b_1, b_2, b_3, b_4, b_5\}$, we find that the 23 scrollar invariants of $\text{res}_{C_5} \varphi$ are given by the multi-set

$$\{e_i + e_j\}_{1 \leq i < j \leq 4} \cup \{e_i + e_j\}_{1 \leq i < j \leq 4} \cup \{g + 4\} \cup \{b_i\}_{1 \leq i \leq 5} \cup \{g + 4 - b_i\}_{1 \leq i \leq 5}.$$

DEGREE 3

List of candidate scrollar invariants:

partition	scrollar invariants	sum	notation
$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array}$	e_1, e_2	$g + 2$	e_i
$\begin{array}{ c } \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array}$	$e_1 + e_2$	$g + 2$	$g + 2$

Scrollar invariants of the resolvents of a simply branched degree 3 cover $\varphi : C \rightarrow \mathbf{P}^1$:

GAP index of subgroup H	structure	generators	remark	index $[S_3 : H]$ $= \text{deg res}_H \varphi$	genus of $\text{res}_H C$	scrollar invariants of $\text{res}_H \varphi$
0	trivial	-	Galois closure	6	$3g + 1$	$e_i (2 \times), g + 2$
1	C_2	(1 2)	given cover	3	g	e_i
2	C_3	(1 2 3)	discriminant cover	2	$g + 1$	$g + 2$

DEGREE 4

List of candidate scrollar invariants:

partition	scrollar invariants	sum	notation
	e_1, e_2, e_3	$g + 3$	e_i
	$e_1 + e_2, e_1 + e_3, e_2 + e_3$	$2g + 6$	e_{ij}
	$e_1 + e_2 + e_3$	$g + 3$	$g + 3$
	$b_1^{(1)}, b_2^{(1)}$	$g + 3$	b_i

Scrollar invariants of the resolvents of a simply branched degree 4 cover $\varphi : C \rightarrow \mathbf{P}^1$:

GAP index of subgroup H	structure	generators	remark	index $[S_4 : H]$ $= \deg \text{res}_H \varphi$	genus of $\text{res}_H C$	scrollar invariants of $\text{res}_H \varphi$
0	trivial	-	Galois closure	24	$12g + 13$	$e_i (3\times), e_{ij} (3\times), g + 3,$ $b_i (2\times)$
1	C_2	$(12)(34)$		12	$6g + 7$	$e_i, e_{ij}, g + 3,$ $b_i (2\times)$
2	C_2	(12)	see Cor. 42	12	$5g + 4$	$e_i (2\times), e_{ij}, b_i$
3	C_3	(123)	see Cor. 32	8	$4g + 5$	$e_i, e_{ij}, g + 3$
4	$C_2 \times C_2$ (normal)	$(12)(34),$ $(13)(24)$		6	$3g + 4$	$g + 3,$ $b_i (2\times)$
5	$C_2 \times C_2$ (non-normal)	$(12), (34)$	see Thm. 4	6	$2g + 1$	e_i, b_i
6	C_4	(1234)		6	$3g + 4$	e_{ij}, b_i

7	S_3	$(12), (123)$	given cover	4	g	e_i
8	D_4	$(12), (1324)$	Lagrange's cubic resolvent	3	$g+1$	b_i
9	A_4	$(123), (234)$	discriminant cover	2	$g+2$	$g+3$

DEGREE 5

List of candidate scollar invariants:

partition	scollar invariants	sum	notation
	e_1, e_2, e_3, e_4	$g + 4$	e_i
	$e_1 + e_2, e_1 + e_3, e_1 + e_4, e_2 + e_3, e_2 + e_4, e_3 + e_4$	$3g + 12$	e_{ij}
	$e_1 + e_2 + e_3, e_1 + e_2 + e_4, e_1 + e_3 + e_4, e_2 + e_3 + e_4$	$3g + 12$	e_{ijk}
	$e_1 + e_2 + e_3 + e_4$	$g + 4$	$g + 4$
	$b_1^{(1)}, b_2^{(1)}, b_3^{(1)}, b_4^{(1)}, b_5^{(1)}$	$2g + 8$	b_i
	$b_1^{(2)}, b_2^{(2)}, b_3^{(2)}, b_4^{(2)}, b_5^{(2)}$	$3g + 12$	$g + 4 - b_i$

Scollar invariants of the resolvents of a simply branched degree 5 cover $\varphi : C \rightarrow \mathbf{P}^1$:

GAP index of subgp H	structure	generators	remark	index $[S_5 : H]$ $= \deg \text{res}_H \varphi$	genus of $\text{res}_H C$	scollar invariants of $\text{res}_H \varphi$
0	trivial	-	Galois closure	120	$60g + 121$	$e_i (4\times), e_{ij} (6\times), e_{ijk} (4\times), g + 4$ $b_i (5\times), g + 4 - b_i (5\times)$
1	C_2	(12)		60	$27g + 49$	$e_i (3\times), e_{ij} (3\times), e_{ijk},$ $b_i (3\times), g + 4 - b_i (2\times)$
2	C_2	(12)(34)		60	$30g + 61$	$e_i (2\times), e_{ij} (2\times), e_{ijk} (2\times), g + 4,$ $b_i (3\times), g + 4 - b_i (3\times)$

3	C_3	(1 2 3)	see Cor. 42	40	$20g + 41$	$e_i (2\times), e_{ij} (2\times), e_{ijk} (2\times), g + 4,$ $b_i, g + 4 - b_i$
4	$C_2 \times C_2$	(1 2)(3 4), (1 3)(2 4)		30	$15g + 31$	$e_i, e_{ijk}, g + 4,$ $b_i (2\times), g + 4 - b_i (2\times)$
5	C_4	(1 2 3 4)		30	$15g + 31$	$e_i, e_{ij}, e_{ijk},$ $b_i, g + 4 - b_i (2\times)$
6	$C_2 \times C_2$	(1 2), (3 4)		30	$12g + 19$	$e_i (2\times), e_{ij},$ $b_i (2\times), g + 4 - b_i$
7	C_5	(1 2 3 4 5)		24	$12g + 25$	$e_{ij} (2\times), g + 4,$ $b_i, g + 4 - b_i$
8	S_3	(1 2), (1 2 3)	see Cor. 42	20	$7g + 9$	$e_i (2\times), e_{ij}, b_i$
9	C_6	(1 2 3), (4 5)	see Cor. 42	20	$9g + 17$	$e_i, e_{ij}, e_{ijk}, b_i$
10	S_3	(1 2 3), (2 3)(4 5)		20	$10g + 21$	$e_i, e_{ijk}, g + 4,$ $b_i, g + 4 - b_i$
11	D_4	(1 2) (1 3 2 4)		15	$6g + 10$	$e_i,$ $b_i, g + 4 - b_i$
12	D_5	(1 2 3 4 5), (3 4)(2 5)		12	$6g + 13$	$g + 4,$ $b_i, g + 4 - b_i$
13	A_4	(1 2 3), (2 3 4)	see Cor. 32	10	$5g + 11$	$e_i, e_{ijk}, g + 4$
14	$C_2 \times S_3$	(1 2), (1 2 3), (4 5)	see Thm. 4	10	$3g + 3$	e_i, b_i
15	$\text{AGL}_1(\mathbf{F}_5)$	(2 3 5 4), (1 2 3 4 5)	Cayley's sextic res.	6	$3g + 7$	$g + 4 - b_i$
16	S_4	(1 2), (1 2 3 4)	given cover	5	g	e_i
17	A_5	(1 2 3), (1 2 3 4 5)	discriminant cover	2	$g + 3$	$g + 4$

1. DEGREE 6

List of candidate scrollar invariants (a_1, a_2, a_3, a_4, a_5 are the exotic invariants discussed in Section 7):

partition	scrollar invariants	sum	notation
	e_1, e_2, e_3, e_4, e_5	$g + 5$	e_i
	$e_1 + e_2, e_1 + e_3, e_1 + e_4, e_1 + e_5, e_2 + e_3,$ $e_2 + e_4, e_2 + e_5, e_3 + e_4, e_3 + e_5, e_4 + e_5$	$4g + 20$	e_{ij}
	$e_1 + e_2 + e_3, e_1 + e_2 + e_4, e_1 + e_2 + e_5, e_1 + e_3 + e_4, e_1 + e_3 + e_5,$ $e_1 + e_4 + e_5, e_2 + e_3 + e_4, e_2 + e_3 + e_5, e_2 + e_4 + e_5, e_3 + e_4 + e_5$	$6g + 30$	e_{ijk}
	$e_1 + e_2 + e_3 + e_4, e_1 + e_2 + e_3 + e_5, e_1 + e_2 + e_4 + e_5,$ $e_1 + e_3 + e_4 + e_5, e_2 + e_3 + e_4 + e_5$	$4g + 20$	e_{ijkl}
	$e_1 + e_2 + e_3 + e_4 + e_5$	$g + 5$	$g + 5$
	$b_1^{(1)}, b_2^{(1)}, b_3^{(1)}, b_4^{(1)}, b_5^{(1)}, b_6^{(1)}, b_7^{(1)}, b_8^{(1)}, b_9^{(1)}$	$3g + 15$	b_i
	$b_1^{(2)}, b_2^{(2)}, b_3^{(2)}, b_4^{(2)}, b_5^{(2)}, b_6^{(2)}, b_7^{(2)}, b_8^{(2)},$ $b_9^{(2)}, b_{10}^{(2)}, b_{11}^{(2)}, b_{12}^{(2)}, b_{13}^{(2)}, b_{14}^{(2)}, b_{15}^{(2)}, b_{16}^{(2)}$	$8g + 40$	b_i^2
	$b_1^{(3)}, b_2^{(3)}, b_3^{(3)}, b_4^{(3)}, b_5^{(3)}, b_6^{(3)}, b_7^{(3)}, b_8^{(3)}, b_9^{(3)}$	$6g + 30$	$g + 5 - b_i$
	$g + 5 - a_1, g + 5 - a_2, g + 5 - a_3, g + 5 - a_4, g + 5 - a_5$	$2g + 10$	$g + 5 - a_i$
	a_1, a_2, a_3, a_4, a_5	$3g + 15$	a_i

Scrollar invariants of the resolvents of a simply branched degree 6 cover $\varphi : C \rightarrow \mathbf{P}^1$:

GAP ind. subgp H	structure	generators	remark	index $[S_6 : H]$ $= \deg \operatorname{res}_H \varphi$	genus of $\operatorname{res}_H C$	scrollar invariants of $\operatorname{res}_H \varphi$
0	trivial	-	Galois closure	720	$360g + 1081$	$e_i (5\times), e_{ij} (10\times), e_{ijk} (10\times),$ $e_{ijkl} (5\times), g + 5,$ $b_i (9\times), b_i^2 (16\times), g + 5 - b_i (9\times),$ $g + 5 - a_i (5\times), a_i (5\times)$
1	C_2	(12)		360	$168g + 481$	$e_i (4\times), e_{ij} (6\times),$ $e_{ijk} (4\times), e_{ijkl},$ $b_i (6\times), b_i^2 (8\times), g + 5 - b_i (3\times),$ $g + 5 - a_i (3\times), a_i (2\times)$
2	C_2	(12)(34)(56)		360	$180g + 541$	$e_i (2\times), e_{ij} (4\times),$ $e_{ijk} (6\times), e_{ijkl} (3\times),$ $b_i (6\times), b_i^2 (8\times), g + 5 - b_i (3\times),$ $g + 5 - a_i, a_i (4\times)$
3	C_2	(12)(34)		360	$180g + 541$	$e_i (3\times), e_{ij} (4\times), e_{ijk} + 4 (4\times),$ $e_{ijkl} (3\times), g + 5,$ $b_i (5\times), b_i^2 (8\times), g + 5 - b_i (5\times),$ $g + 5 - a_i (3\times), a_i (3\times)$
4	C_3	(123)		240	$120g + 361$	$e_i (3\times), e_{ij} (4\times), e_{ijk} (4\times),$ $e_{ijkl} (3\times), g + 5,$ $b_i (3\times), b_i^2 (4\times), g + 5 - b_i (3\times),$ $g + 5 - a_i, a_i$
5	C_3	(123)(456)		240	$120g + 361$	$e_i, e_{ij} (4\times), e_{ijk} (4\times),$ $e_{ijkl}, g + 5,$ $b_i (3\times), b_i^2 (4\times), g + 5 - b_i (3\times),$ $g + 5 - a_i (3\times), a_i (3\times)$
6	$C_2 \times C_2$	(12)(56), (34)(56)	Gassmann equiv. to 7	180	$90g + 271$	$e_i (2\times), e_{ij}, e_{ijk},$ $e_{ijkl} (2\times), g + 5,$

7	$C_2 \times C_2$	(12)(34), (13)(24)	Gassmann equiv. to 6	180	$90g + 271$	$b_i (3\times), b_i^2 (4\times), g + 5 - b_i (3\times),$ $g + 5 - a_i (2\times), a_i (2\times)$ $e_i (2\times), e_{ij}, e_{ijk},$ $e_{ijkl} (2\times), g + 5,$ $b_i (3\times), b_i^2 (4\times), g + 5 - b_i (3\times),$ $g + 5 - a_i (2\times), a_i (2\times)$
8	C_4	(1234)(56)		180	$90g + 271$	$e_i, e_{ij} (2\times), e_{ijk} (2\times),$ $e_{ijkl}, g + 5,$ $b_i (3\times), b_i^2 (4\times), g + 5 - b_i (3\times),$ $g + 5 - a_i, a_i$
9	$C_2 \times C_2$	(12), (34)(56)		180	$84g + 241$	$e_i (2\times), e_{ij} (2\times),$ $e_{ijk} (2\times), e_{ijkl},$ $b_i (4\times), b_i^2 (4\times), g + 5 - b_i,$ $g + 5 - a_i, a_i (2\times)$
10	C_4	(1234)		180	$90g + 271$	$e_i (2\times), e_{ij} (2\times),$ $e_{ijk} (2\times), e_{ijkl},$ $b_i (2\times), b_i^2 (4\times), g + 5 - b_i (3\times),$ $g + 5 - a_i, a_i (2\times)$
11	V_4	(12), (34)		180	$78g + 211$	$e_i (3\times), e_{ij} (3\times), e_{ijk},$ $b_i (4\times), b_i^2 (4\times), g + 5 - b_i,$ $g + 5 - a_i (2\times), a_i$
12	V_4	(12)(34)(56), (13)(24)(56)		180	$90g + 271$	$e_i, e_{ij},$ $e_{ijk} (3\times), e_{ijkl} (2\times),$ $b_i (4\times), b_i^2 (4\times), g + 5 - b_i,$ $a_i (3\times)$
13	C_5	(12345)		144	$72g + 217$	$e_i, e_{ij} (2\times), e_{ijk} (2\times),$ $e_{ijkl}, g + 5,$ $b_i, b_i^2 (4\times), g + 5 - b_i,$ $g + 5 - a_i, a_i$
14	S_3	(12), (123)		120	$48g + 121$	$e_i (3\times), e_{ij} (3\times), e_{ijk},$

15	S_3	$(123)(456),$ $(16)(25)(34)$	120	$60g + 181$	$b_i (3\times), b_i^2 (2\times),$ $g + 5 - a_i$ $e_{ij}, e_{ijk} (3\times), e_{ijkl},$ $b_i (3\times), b_i^2 (2\times),$ $a_i (3\times)$
16	C_6	(123456)	120	$60g + 181$	$e_{ij} (2\times), e_{ijk} (2\times), e_{ijkl},$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i,$ $g + 5 - a_i, a_i (2\times)$
17	S_3	$(123),$ $(23)(45)$	120	$60g + 181$	$e_i (2\times), e_{ij}, e_{ijk},$ $e_{ijkl} (2\times), g + 5,$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i (2\times),$ $g + 5 - a_i, a_i$
18	S_3	$(123)(456),$ $(12)(45)$	120	$60g + 181$	$e_i, e_{ij}, e_{ijk},$ $e_{ijkl}, g + 5,$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i (2\times),$ $g + 5 - a_i (2\times), a_i (2\times)$
19	C_6	$(12), (345)$	120	$56g + 161$	$e_i (2\times), e_{ij} (2\times),$ $e_{ijk} (2\times), e_{ijkl},$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i,$ $g + 1 - a_i$
20	$C_2 \times C_2 \times C_2$	$(12), (34),$ (56)	90	$36g + 91$	$e_i (2\times), e_{ij},$ $b_i (3\times), b_i^2 (2\times),$ $g + 5 - a_i, a_i$
21	$C_2 \times C_2 \times C_2$	$(12)(34),$ $(13)(24), (56)$	90	$42g + 121$	$e_i, e_{ijk}, e_{ijkl},$ $b_i (3\times), b_i^2 (2\times),$ $a_i (2\times)$
22	D_4	$(12)(3456),$ (46)	90	$39g + 106$	$e_i, e_{ij}, e_{ijk},$ $b_i (3\times), b_i^2 (2\times), a_i$
23	$C_2 \times C_4$	$(1234),$ (56)	90	$42g + 121$	$e_i, e_{ij}, e_{ijk}, b_i (2\times),$ $b_i^2 (2\times), g + 5 - b_i, a_i$

24	D_4	(12), (13)(24)		90	$39g + 106$	$e_i (2\times), e_{ij},$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i,$ $g + 5 - a_i, a_i$
25	D_4	(12)(34), (36)(45)		90	$45g + 136$	$e_i, e_{ijkl} + 6, g + 5,$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i (2\times),$ $g + 5 - a_i, a_i$
26	D_4	(12)(34), (3645)		90	$45g + 136$	$e_i, e_{ijk}, e_{ijkl},$ $b_i (2\times), b_i^2 (2\times), g + 5 - b_i,$ $a_i (2\times)$
27	$C_3 \times C_3$	(123), (456)		80	$40g + 121$	$e_i, e_{ij} (2\times), e_{ijk} (2\times),$ $e_{ijkl}, g + 5,$ $b_i, g + 5 - b_i,$ $g + 5 - a_i, a_i$
28	D_5	(12345), (34)(25)		72	$36g + 109$	$e_i, e_{ijkl}, g + 5,$ $b_i, b_i^2 (2\times), g + 5 - b_i,$ $g + 5 - a_i, a_i$
29	A_4	(123), (234)	see Cor. 42	60	$30g + 91$	$e_i (2\times), e_{ij}, e_{ijk},$ $e_{ijkl} (2\times), g + 5,$ $b_i, g + 5 - b_i$
30	A_4	(12)(34), (153)(264)		60	$30g + 91$	$e_{ij}, e_{ijk}, g + 5,$ $b_i, g + 5 - b_i,$ $g + 5 - a_i (2\times), a_i (2\times)$
31	$C_2 \times S_3$	(12), (123), (45)	see Cor. 43	60	$22g + 51$	$e_i (2\times), e_{ij},$ $b_i (2\times), b_i^2 + 4,$ $g + 5 - a_i$
32	$C_2 \times S_3$	(12)(34)(56), (15)(24)		60	$30g + 91$	$e_{ijk}, e_{ijkl},$ $b_i (2\times), b_i^2,$ $a_i (2\times)$
33	$C_2 \times D_4$	(12), (56), (1324)		45	$18g + 46$	$e_i,$ $b_i (2\times), b_i^2, a_i$

34	$\text{Dih}(C_3 \times C_3)$	$(123)(456),$ $(12)(45),$ $(12)(56)$		40	$20g + 61$	$e_i, e_{ijkl}, g + 3$ $b_i, g + 5 - b_i,$ $a_i, g + 5 - a_i$
35	$C_3 \times S_3$	$(123456),$ $(153)(462)$		40	$20g + 61$	$e_{ij}, e_{ijk}, e_{ijkl},$ b_i, a_i
36	$C_3 \times S_3$	$(123),$ $(45), (456)$		40	$16g + 41$	$e_i, e_{ij}, e_{ijk},$ $b_i, g + 5 - a_i$
37	$\text{AGL}_1(\mathbf{F}_5)$	$(12345),$ (1243)		36	$18g + 55$	$e_i, b_i^2,$ $g + 5 - b_i, a_i$
38	$C_2 \times A_4$	$(123456),$ (14)		30	$12g + 31$	$e_{ij}, b_i,$ $a_i, g + 5 - a_i$
39	$C_2 \times A_4$	$(12), (345),$ (456)	see Cor. 42	30	$14g + 41$	$e_i, e_{ijk}, e_{ijkl},$ b_i
40	S_4	$(12)(3456),$ (456)		30	$15g + 46$	$e_i, e_{ijkl}, g + 5,$ $b_i, g + 5 - b_i$
41	S_4	$(12)(3456),$ $(136)(254)$		30	$15g + 46$	$g + 5,$ $b_i, g + 5 - b_i,$ $g + 5 - a_i, a_i$
42	S_4	$(123)(456),$ (1643)		30	$15g + 46$	$e_{ijk},$ $b_i, a_i (2\times)$
43	S_4	(12) (1234)	see Cor. 42	30	$9g + 16$	$e_i (2\times), e_{ij},$ b_i
44	$S_3 \times S_3$	$(12), (123),$ $(45), (456)$		20	$6g + 11$	$e_i, b_i,$ $g + 5 - a_i$
45	$S_3 \times S_3$	$(123456),$ $(24)(35)$		20	$10g + 31$	$e_{ijkl},$ b_i, a_i
46	$(C_3 \times C_3) \rtimes C_4$ = GAP ID [36,9]	$(12)(34),$ $(1364)(25)$		20	$10g + 31$	$g + 5,$ $b_i, g + 5 - b_i$
47	$C_2 \times S_4$	$(123456),$ $(15)(24)(36)$		15	$6g + 16$	b_i, a_i

48	$C_2 \times S_4$	(12), (34) (3456)	see Thm. 4	15	$4g + 6$	e_i, b_i
49	A_5	(123), (12345)	see Cor. 32	12	$6g + 19$	$e_i, e_{ijkl},$ $g + 3$
50	A_5	(123)(456), (12)(34)		12	$6g + 19$	$g + 5,$ $g + 5 - a_i, a_i$
51	$S_3 \wr S_2$	(12), (123), (45), (456), (14)(25)(36)		10	$3g + 6$	b_i
52	S_5	(12), (12345)	given cover	6	g	e_i
53	S_5	(1234), (1562)	exotic resolvent	6	$3g + 10$	a_i
54	A_6	(123), (23456)	discriminant cover	2	$g + 4$	$g + 5$