Signal Processing based on Coupled Tensor Decompositions

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Biotensors Kickoff Meeting - ESAT, Leuven, Belgium
May 13, 2014

European Research Council
Established by the European Commission
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Standard Tensor based Signal Processing (SP)

Tensors are now common in signal processing, such as

- Sensor Array processing
- Wireless Communication
- Radar Processing
- Independent Component Analysis

Typically formulated as a CPD problem. Roughly speaking,

Standard Tensor based SP $=$ CPD modeling

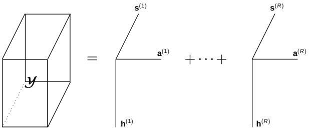
The CPD paradigm either *limits* or *ignores*

- Signal Structure and Properties.
- Transmit/Receive Array Structure and Properties.
- Channel/Propagation Models.

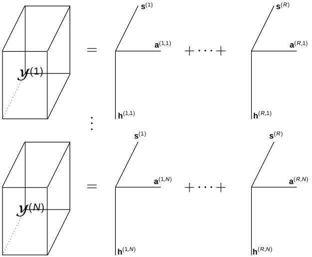
What lies beyond the CPD modeling paradigm ?

Tensor based SP: Beyond the CPD Paradigm

Standard CPD solution for tensor based SP:



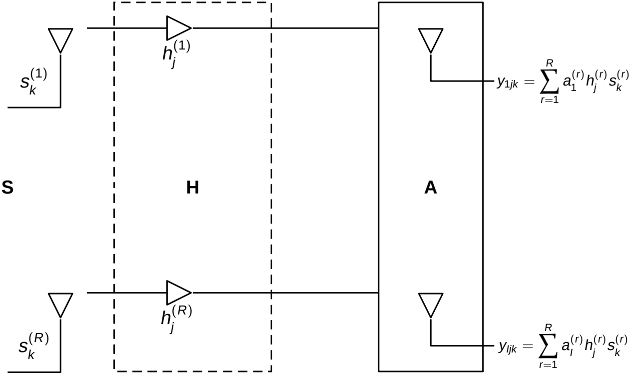
We propose the coupled tensor decomposition modeling solution, e.g.:



Is coupled tensor modeling a good idea?

but let us first review “CPD” based signal processing.

Case study: Blind Separation of Oversampled Signals (BSOS)



R = number of users

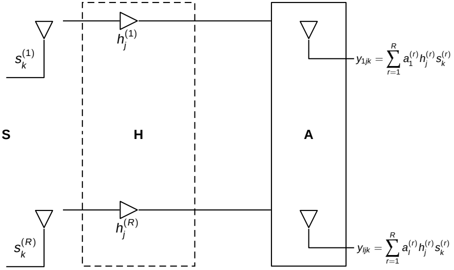
I = number of receivers

J = oversampling factor

K = symbol periods

(A biotensor CPD based example is EEG: Time × Frequency × Channel)

BSOS: A Trilinear Wireless Communication System



Input-Output Relation [Sidiropoulos, Giannakis and Bro, 2000]

$$y_{ijk} = \sum_{r=1}^{R} a_i^{(r)} h_j^{(r)} s_k^{(r)}$$

antenna number

symbol period

oversampling period

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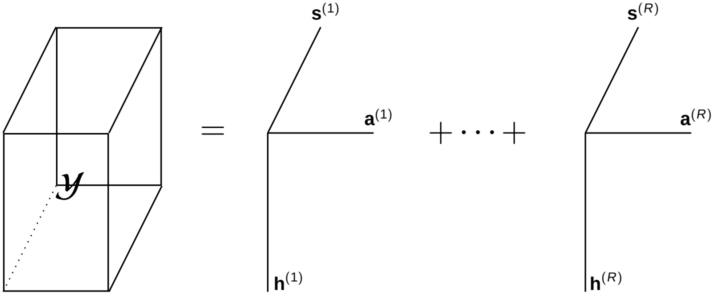
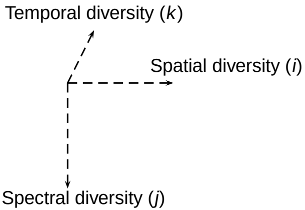
BSOS Leads Naturally to a CPD Problem





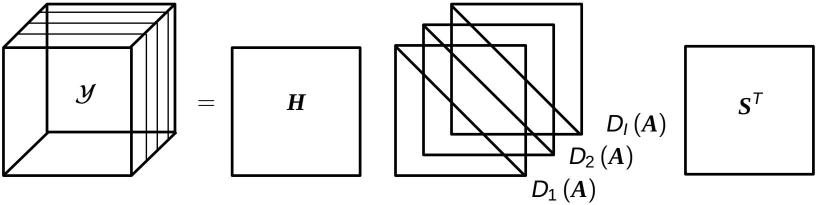
CPD

$$\mathcal{Y} = \sum_{r=1}^{R} \mathbf{a}^{(r)} \circ \mathbf{h}^{(r)} \circ \mathbf{s}^{(r)}$$

Vectors

$$\mathbf{a}^{(r)} = \left[a_1^{(r)}, \quad \cdots \quad , a_I^{(r)} \right]^T$$
$$\mathbf{h}^{(r)} = \left[h_1^{(r)}, \quad \cdots \quad , h_J^{(r)} \right]^T$$
$$\mathbf{s}^{(r)} = \left[s_1^{(r)}, \quad \cdots \quad , s_K^{(r)} \right]^T$$

Recall: Matrization of CPD of Tensor



Matrix representation of CPD of $\mathcal{Y}$

$$D_i(\mathbf{A}) = \operatorname{diag}(\mathbf{A}(i,:))$$

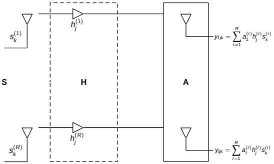
$$\mathbf{Y}^{(i\cdot\cdot)} = \mathbf{H}D_i(\mathbf{A})\mathbf{S}^T$$

$$\mathbf{Y}^{(1)} = \begin{bmatrix} \mathbf{Y}^{(1\cdot\cdot)} \\ \vdots \\ \mathbf{Y}^{(I\cdot\cdot)} \end{bmatrix} = \begin{bmatrix} \mathbf{H}D_1(\mathbf{A}) \\ \vdots \\ \mathbf{H}D_I(\mathbf{A}) \end{bmatrix} \mathbf{S}^T = (\mathbf{A} \odot \mathbf{H})\mathbf{S}^T$$

Factor Matrices

$$\mathbf{A} = \left[\mathbf{a}^{(1)}, \ldots, \mathbf{a}^{(R)}\right], \quad \mathbf{H} = \left[\mathbf{h}^{(1)}, \ldots, \mathbf{h}^{(R)}\right], \quad \mathbf{S} = \left[\mathbf{s}^{(1)}, \ldots, \mathbf{s}^{(R)}\right].$$

Unique CPD ⇒ Recovery of signal matrix **S**



Necessary signal recovery conditions

- **A** and **H** do not contain collinear columns, $\min(k(\mathbf{H}), k(\mathbf{A})) \geq 2$.
- $\mathbf{A} \odot \mathbf{H}$ has full column rank, $r(\mathbf{A} \odot \mathbf{H}) = R$.
- R is not too large.

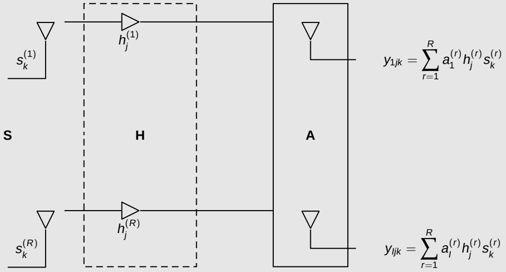
Uniqueness [Kruskal, 1977], [Jiang and Sidiropoulos, 2004], [Domanov and De Lathauwer, 2013]

If the necessary signal recovery conditions are satisfied, then under mild conditions we can recover **S** based on only $\mathbf{Y}^{(1)} = (\mathbf{A} \odot \mathbf{H})\mathbf{S}^T$.
⇒ Recovery of **S** up to column scaling and permutation.

Note: CPD uniqueness does not in general imply identifiability!

"CPD" based signal processing has limitations

Classical tensor-based signal processing (CPD):



Standard CPD methods have practical limitations, such as

- Only valid for very basic propagation channels.
- Few allowed transmit/receive antenna array configurations.
- Ignores transmit/receive antenna array structure.
- Ignores signal structure and statistical properties.

Coupled tensor decompositions:

A framework for tensor-based signal processing

Some suggestions to overcome the restrictions of standard CPD:

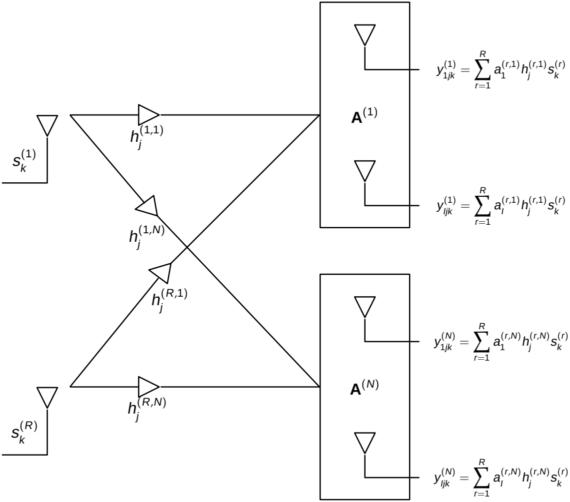
1. Increase the spatial diversities of the system (“MIMO” techniques).
2. Increase the temporal, transmitter or receiver diversities of the system (higher-order tensors).
3. Incorporate system structures (constrained decompositions).

We briefly demonstrate in this talk

Coupled tensor decompositions provide a framework that can

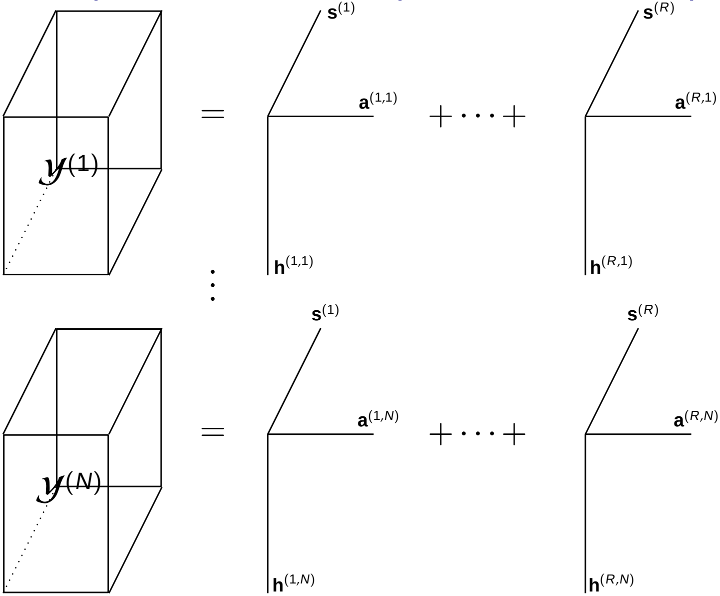
- deal with more elaborate transmit/receive antenna array configurations (e.g. multiple antenna arrays).
- handle more challenging propagation channels (e.g. large angle spread).
- integrate multiple diversities (e.g. polarization).
- incorporate signal and transmit/receive antenna array structures (e.g. uniform linear arrays).

Increase spatial diversity: Widely Separated Antenna Arrays



(A biotensor example is multimodal data fusion: EEG $\times$ fMRI$\times$ MEG $\times \cdots$)

Signal Separation from a Coupled Tensorial Perspective



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Coupled Decompositions ⇒ Improved Identifiability

Coupled CPD [Sørensen and De Lathauwer, 2013]

$$\mathcal{Y}^{(n)} = \sum_{r=1}^{R} \mathbf{a}^{(r,n)} \circ \mathbf{h}^{(r,n)} \circ \mathbf{s}^{(r)}, \quad n \in \{1, \ldots, N\}.$$

Matrix representation for coupled CPD

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{(1)} \\ \vdots \\ \mathbf{Y}^{(N)} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{(1)} \odot \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{A}^{(N)} \odot \mathbf{H}^{(N)} \end{bmatrix} \mathbf{S}^T = \mathbf{F}\mathbf{S}^T, \quad \mathbf{F} = \begin{bmatrix} \mathbf{A}^{(1)} \odot \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{A}^{(N)} \odot \mathbf{H}^{(N)} \end{bmatrix}.$$

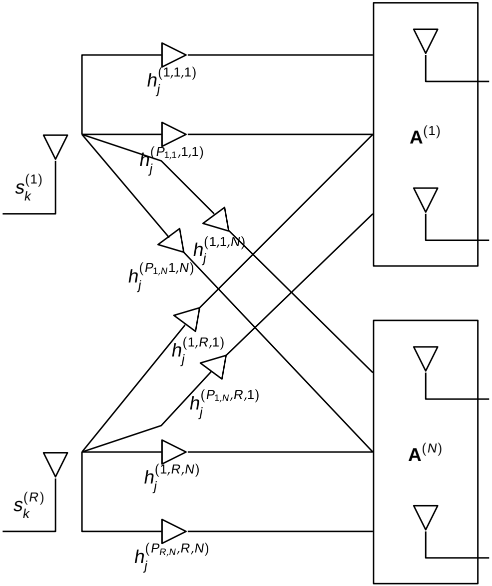
Full column rank of **F** is necessary signal recovery condition:

- Does not prevent $r\left(\mathbf{A}^{(n)} \odot \mathbf{H}^{(n)}\right) < R$ (e.g. deep channel fadings).
- Does not prevent $k\left(\mathbf{A}^{(n)}\right) = 1$ (e.g. collinear steering vectors).
- More relaxed bound on R (e.g. more signals can be separated).

Uniqueness [Sørensen and De Lathauwer, 2013]

Under mild conditions we can recover **S** based on only $\mathbf{Y} = \mathbf{F}\mathbf{S}^T$ ⇒
Recovery of **S** up to column scaling and permutation.

Extension to Incoherent Multipath with Small Delay Spread



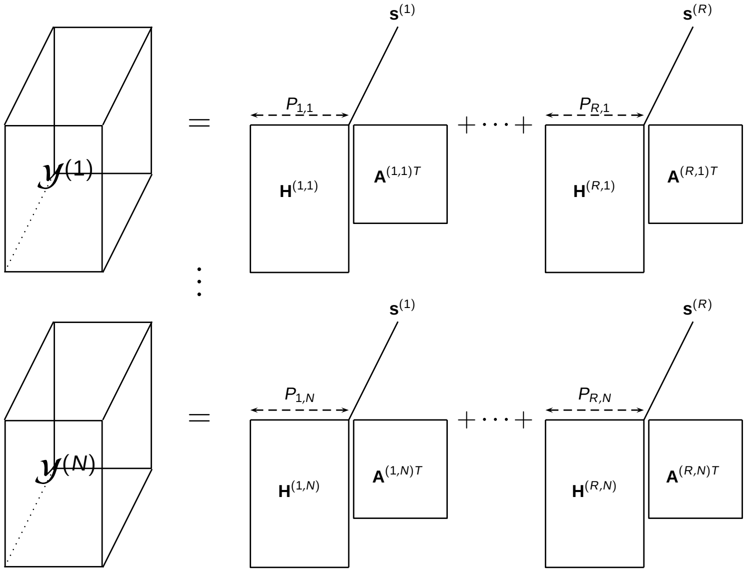
$$y_{1jk}^{(1)} = \sum_{r=1}^{R} \sum_{p=1}^{P_{r,1}} a_1^{(p,r,1)} h_j^{(p,r,1)} s_k^{(r)}$$

$$y_{Ijk}^{(1)} = \sum_{r=1}^{R} \sum_{p=1}^{P_{r,1}} a_I^{(p,r,1)} h_j^{(p,r,1)} s_k^{(r)}$$

$$y_{1jk}^{(N)} = \sum_{r=1}^{R} \sum_{p=1}^{P_{r,N}} a_1^{(p,r,N)} h_j^{(p,r,N)} s_k^{(r)}$$

$$y_{Ijk}^{(N)} = \sum_{r=1}^{R} \sum_{p=1}^{P_{r,N}} a_I^{(p,r,N)} h_j^{(p,r,N)} s_k^{(r)}$$

Signal Separation from a Coupled Tensorial Perspective



Coupled Decompositions ⇒ Improved Performance

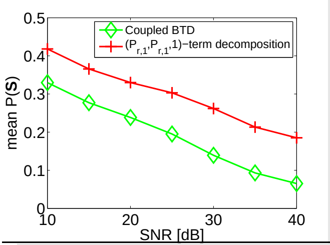
Coupled BTD [Sørensen and De Lathauwer, 2013]

$$\mathcal{Y}^{(n)} = \sum_{r=1}^{R} \sum_{p=1}^{P_{r,n}} \mathbf{a}^{(p,r,n)} \circ \mathbf{h}^{(p,r,n)} \circ \mathbf{s}^{(r)}$$

$$= \sum_{r=1}^{R} \left(\mathbf{A}^{(r,n)} \mathbf{H}^{(r,n)T} \right) \circ \mathbf{s}^{(r)}, \quad n \in \{1, \ldots, N\}. \quad (1)$$

Remark: If $N = 1$, then (1) reduces to the multilinear rank-$(P_r, P_r, 1)$ decomposition [De Lathauwer, 2008].

Computation via Simultaneous Diagonalization [De Lathauwer, 2006], [Sørensen, Domanov, Nion and De Lathauwer, 2013]



Variable	Value	Description
R	2	# users
$P_{r,n}$	2	# paths
N	2	# arrays
I_n	3	# antennas
J_n	5	oversampling
K	50	# symbols

Higher-Order/Constrained Decompositions ⇒ Improved Identifiability

Multiple Diversities [Sørensen and De Lathauwer, 2013]

$$\mathcal{Y}^{(n)} = \sum_{r=1}^{R} \mathbf{a}^{(r,n)} \circ \mathbf{b}^{(r,n)} \circ \mathbf{h}^{(r,n)} \circ \mathbf{s}^{(r)}, \quad n \in \{1, \ldots, N\},$$

where $\mathbf{b}^{(r,n)}$ is the added diversity vector due to for instance *"Sensor Fusion"* (e.g. polarization sensitivity [Guo, Miron, Brie, Zhu and Liao, 2011]).

Constrained Decompositions [Sørensen and De Lathauwer, 2013]

In practice, the factors are often constrained, e.g., ULAs imply that

$$\mathcal{Y}^{(n)} = \sum_{r=1}^{R} \mathbf{a}^{(r,n)} \circ \mathbf{h}^{(r,n)} \circ \mathbf{s}^{(r)}, \quad n \in \{1, \ldots, N\},$$

with

$$\mathbf{A}^{(r,n)} = \begin{bmatrix} a_1^{(r,n)} & \cdots & a_1^{(r,n)} \\ a_2^{(r,n)} & \cdots & a_2^{(r,n)} \\ \vdots & \ddots & \vdots \\ a_{I_n}^{(r,n)} & \cdots & a_{I_n}^{(r,n)} \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 \\ z_{r,n} & \cdots & z_{r,n} \\ \vdots & \ddots & \vdots \\ z_{r,n}^{I_n-1} & \cdots & z_{r,n}^{I_n-1} \end{bmatrix}, \quad z_{r,n} \in \mathbb{C}.$$

(A biotensor example could be exponential modeling of EEG.)

Summary

Motivation of work

Develop coupled tensor decomposition models (coupled CPD, coupled BTD, . . .) to deal with a large class of array processing and wireless communication problems (signal separation, DOA/DOD, system identification, . . .).

Benefits of the coupled tensor decomposition framework include

- Improved identifiability.
- Improved performance.
- Can handle more general propagation channels.
- Valid for a broader range of antenna array configurations.

Coupled tensor decompositions provides a unified framework for

- Coupled decompositions.
- Higher-order tensors.
- Constrained decompositions.

Initial work on coupled decompositions:

- *Coupled Canonical Polyadic Decompositions and (Coupled) Decompositions in Multilinear rank-$(L_{r,n}, L_{r,n}, 1)$ terms — Part I: Uniqueness*, M. Sørensen and L. De Lathauwer, Technical Report 13-143, ESAT-STADIUS, KU Leuven, Belgium.
- *Coupled Canonical Polyadic Decompositions and (Coupled) Decompositions in Multilinear rank-$(L_{r,n}, L_{r,n}, 1)$ terms — Part II: Algorithms*, M. Sørensen, I. Domanov, D. Nion and L. De Lathauwer, Technical Report 13-144, ESAT-STADIUS, KU Leuven, Belgium.
- *Multidimensional harmonic retrieval via coupled canonical polyadic decompositions*, M. Sørensen and L. De Lathauwer, Technical Report 13-240, ESAT-STADIUS, KU Leuven, Belgium.

(More reports are in the pipeline . . .)