

The power of low rank MATRIX and TENSOR Approximations in *Smart Diagnostics*



HCM workshop: Low-rank Optimization and Applications
Bonn, Germany
June 8-12, 2015

Prof. Sabine Van Huffel



Contents Overview

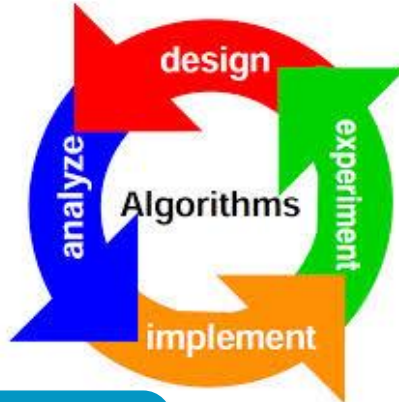
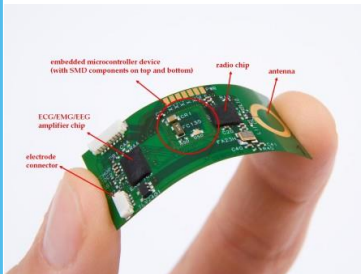
- Introduction

- Smart Patient Monitoring
- Blind Source Separation
- Tensor Decompositions

- Examples in Neonatal Brain Monitoring

- Examples in MR-based Brain Tumor Diagnosis

- Conclusions and new directions



Brain monitoring for neurological disease



Vital signs monitoring: sleep, stress, cardio risk stratification

Sensors
(Carriers)

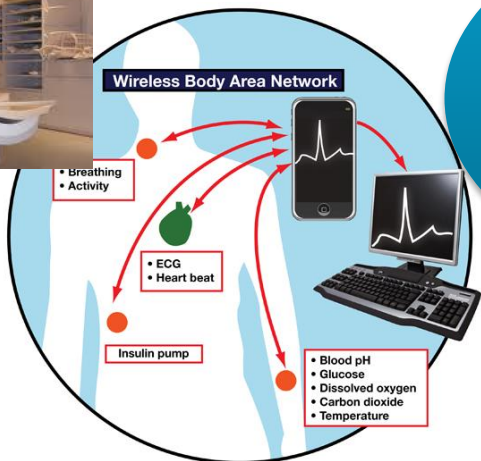
Algorithms
(Technology)

Pathologies
(Applications)

Smart
Diagnostics



Oncology: cancer diagnosis and prognosis



Chronic disease management & telemonitoring application

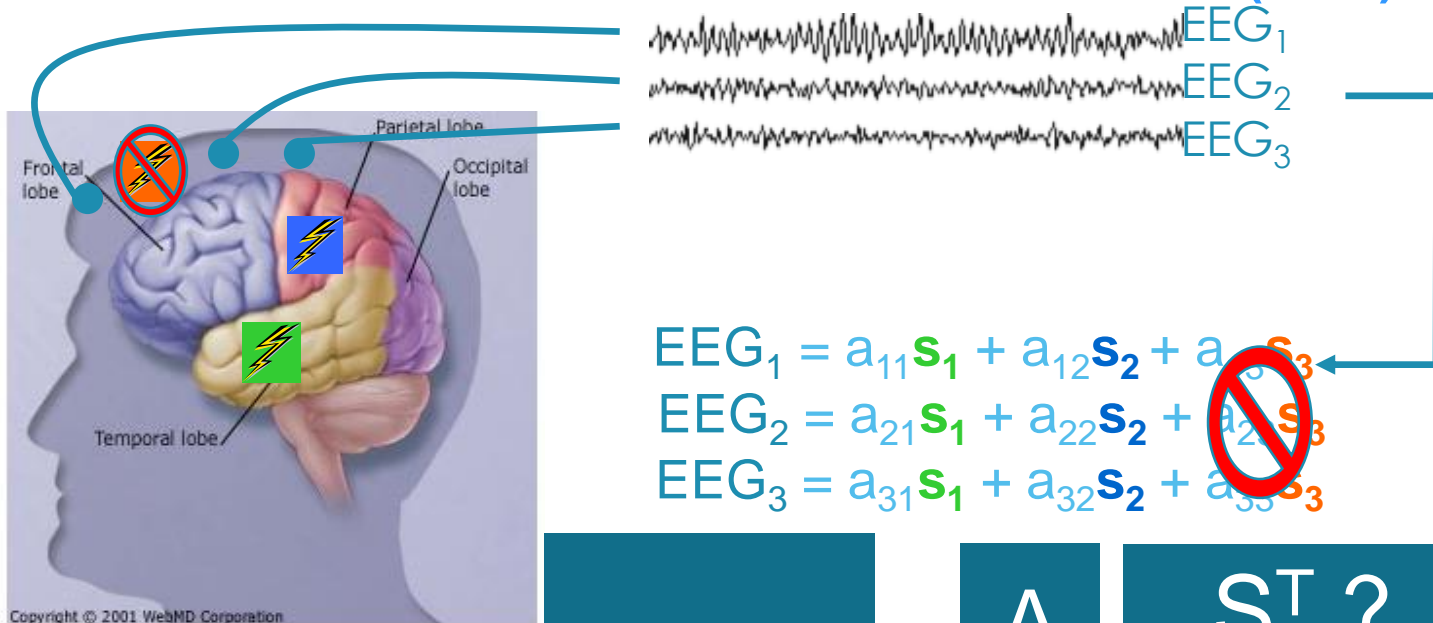
Blind source separation

Signal analysis difficult because of artefacts → REMOVE

Matrix based Blind Source Separation (BSS)

Non-unique → Constraints are needed!

- sources orthogonal (PCA),
- sources statistically independent (ICA)
- **sources uncorrelated and of different autocorrelation (CCA)**



EEG

=

A
?

S^T ?

KU LEUVEN

From Matrix to Tensor rank

At its core, a matrix decomposition is

$$X = \begin{matrix} \square \\ | \\ | \\ | \end{matrix} \begin{matrix} \square \\ \square \\ \square \end{matrix} + \dots + \begin{matrix} \square \\ | \\ | \\ | \end{matrix} \begin{matrix} \square \\ \square \\ \square \end{matrix}$$

or

$$X = \begin{matrix} \square \\ \square \\ \square \\ \square \end{matrix} \begin{matrix} \square \\ \square \\ \square \end{matrix}$$

with some constraints

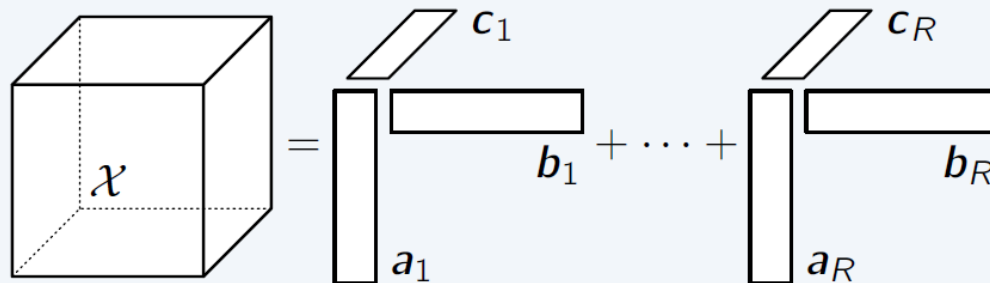
Two equivalent ways to define matrix rank

- ▶ Minimal number of rank-one matrices that sum to X
- ▶ Dimension of column (or row) space of X

For tensors, these are two different concepts!

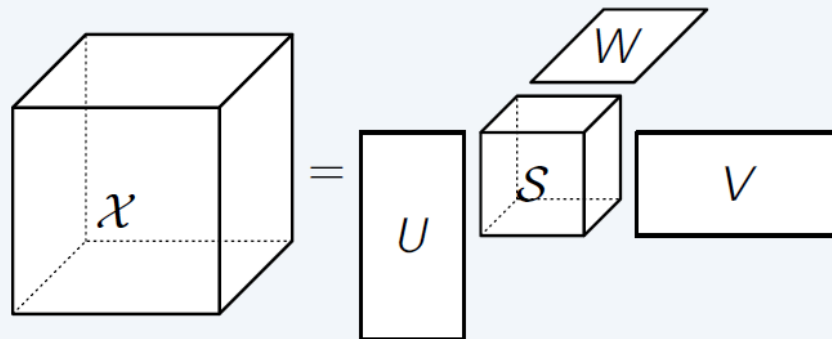
Tensor Decompositions

The **canonical polyadic decomposition** (CPD) decomposes a tensor into a minimal number of rank-one tensors R



The tensor's **rank** is defined as R

A **low multilinear rank approximation** (LMLRA) decomposes a tensor into a core tensor \mathcal{S} and matrices U , V and W



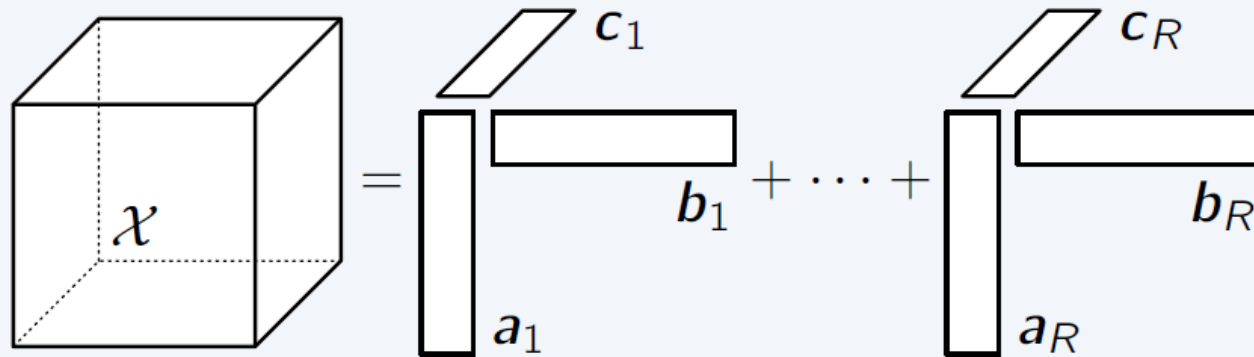
The tensor's **multilinear rank** is defined as the triplet $(\text{rank}(U), \text{rank}(V), \text{rank}(W))$

Uniqueness means Interpretable

Without constraints, matrix decompositions are **not unique**

$$X = A \cdot B = (A \cdot M) \cdot (M^{-1} \cdot B) = \hat{A} \cdot \hat{B}$$

Tensor decompositions can be **unique under mild conditions!**
For example, the vectors \mathbf{a}_r , \mathbf{b}_r and \mathbf{c}_r in the CPD



are generically unique when $k_A + k_B + k_C \geq 2 \cdot R + 2$

$$A = [\mathbf{a}_1, \dots, \mathbf{a}_R]$$

$$B = [\mathbf{b}_1, \dots, \mathbf{b}_R]$$

$$C = [\mathbf{c}_1, \dots, \mathbf{c}_R]$$

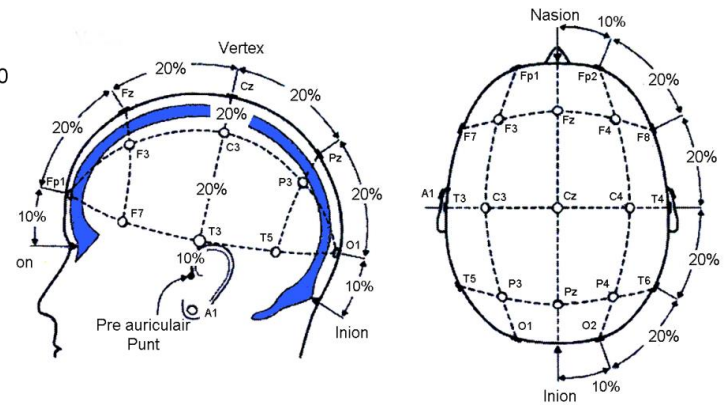
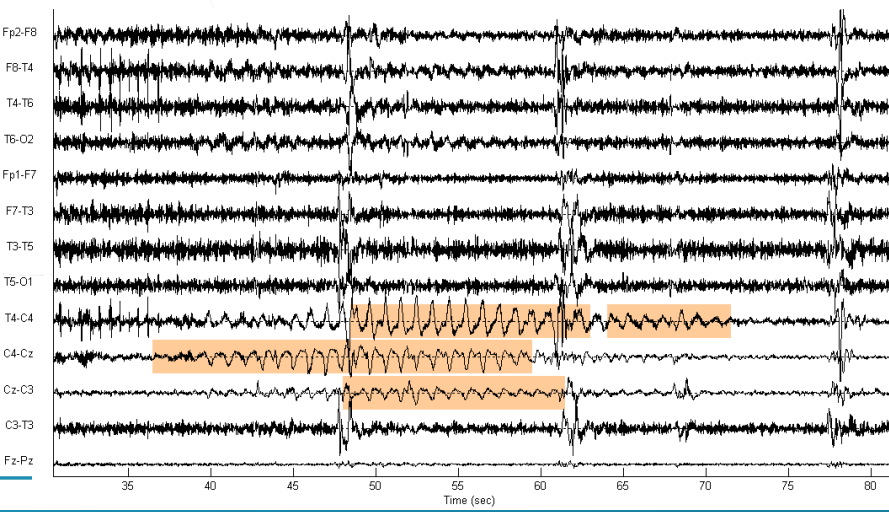
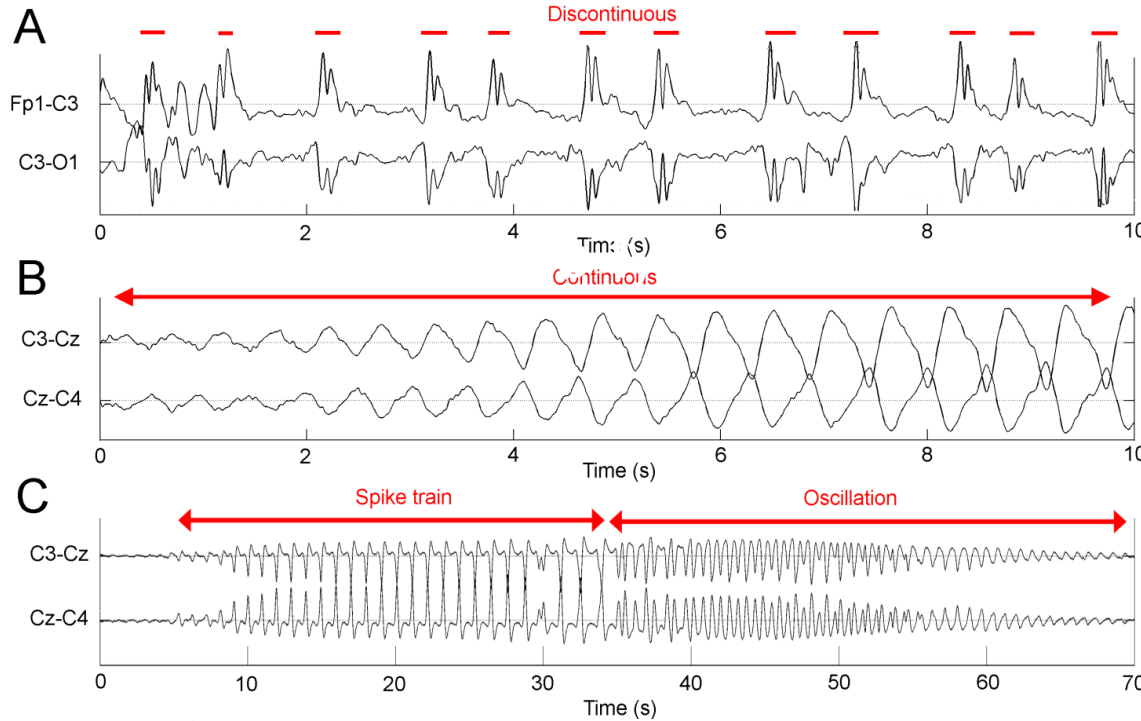
Contributors (nonexhaustive list):

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Contents Overview

- Introduction
- Examples in Neonatal Brain Monitoring
 - Seizure detection
 - Seizure onset localization
 - Background EEG grading
- Examples in MR-based Brain Tumor Diagnosis
- Conclusions and new directions

Neonatal Brain Monitoring: Seizure detection



*Deburchgraeve, PhD thesis, 2011; P.J. Cherian 2011; W. Deburchgraeve et al, Clin. Neurophys. 2008 & 2009
Alternatives using classification: See pubs of Temko, G. Boylan, etc.*

Artefact removal: ECG, respiration, pulsation

BSS based algorithm removes these 3 artifacts **before** seizure detection starts

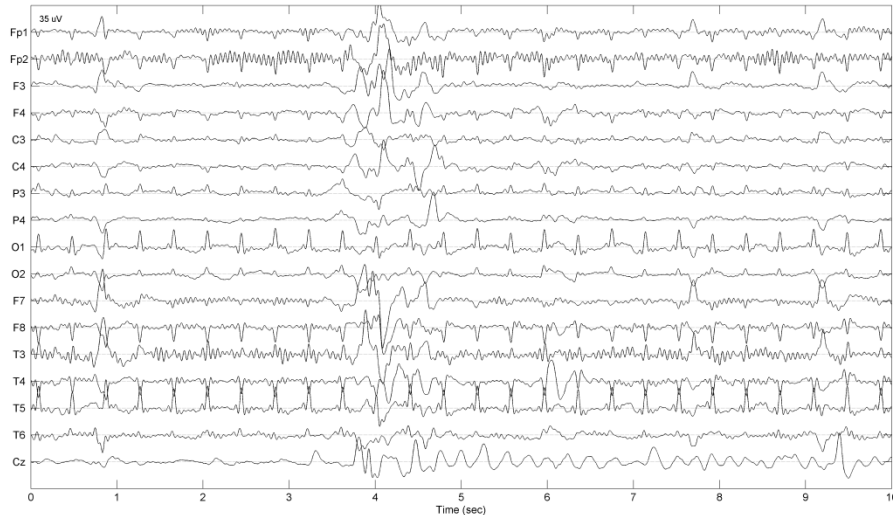
ECG artefact removal:

Respiration & Pulsation removal:

Number of sources:

IC source recognition:

ECG spike artefact



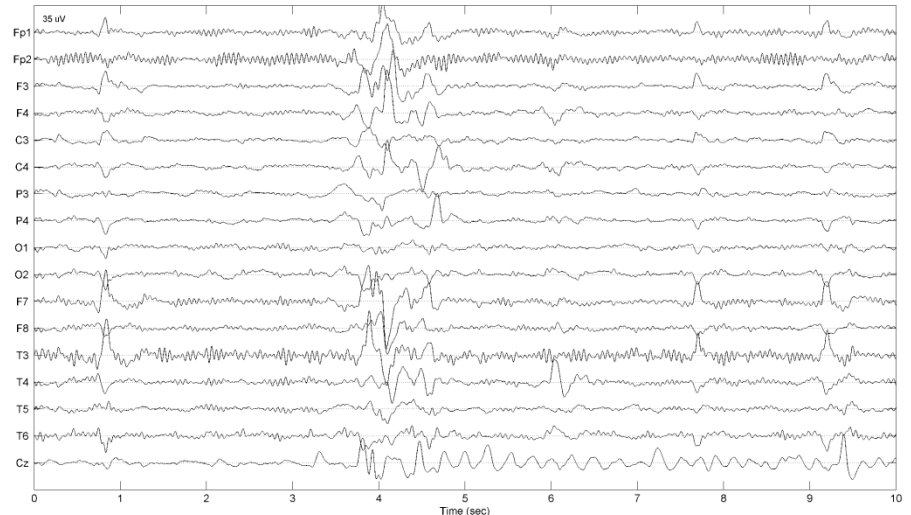
with RobustICA (spiky)

with SOBI (oscillatory, autocorrelated)

estimated with PCA and variance threshold

correlation with reference after transformation

Cleaned EEG

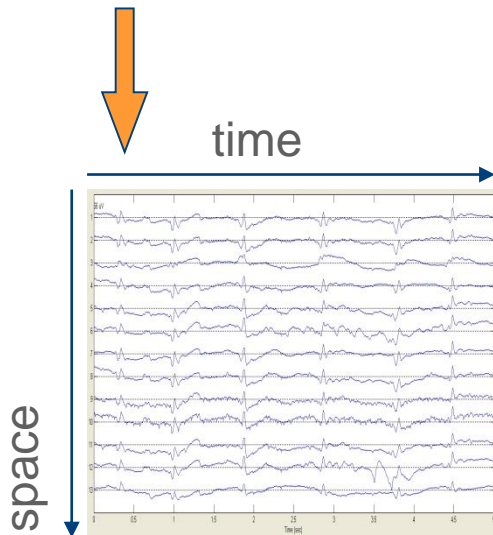
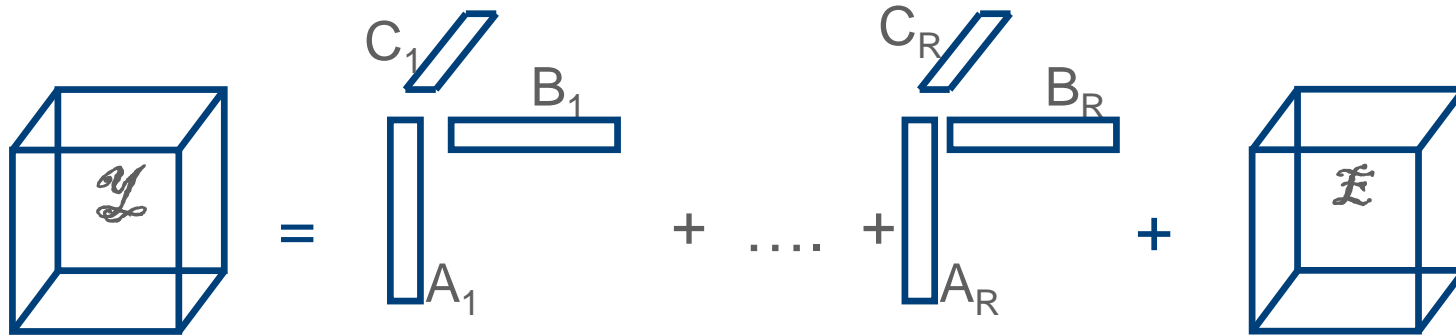


VALIDATION study artefact-removal: *M. De Vos et al., Clinical Neurophys., Dec. 2011*

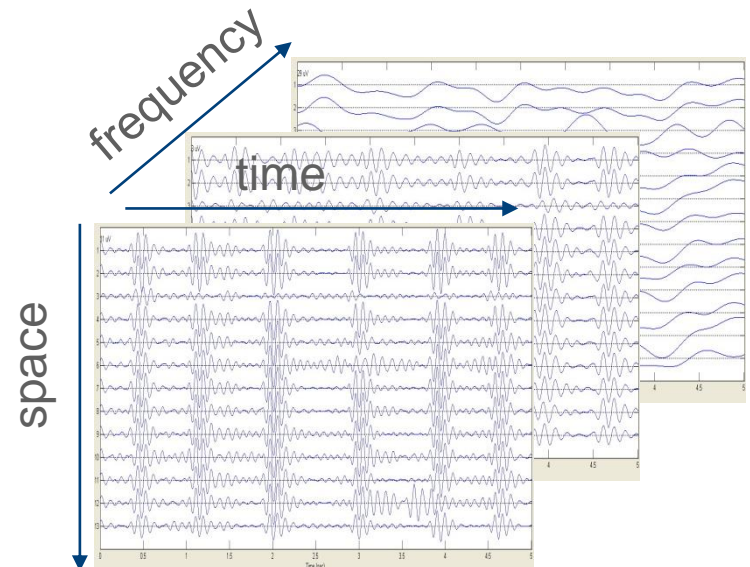
•13 patients: asphyxia (9 with artefacts, 4 artefact-free), 8h EEG per patient

	<u>Fp/h</u>	<u>Seizure detection Rate (%)</u>
No-AR:	0.38 (0-5)	87.1 (20-100)
With-AR:	0.00 (0-0.875)	100 (20-100)

Seizure onset localization: CPD



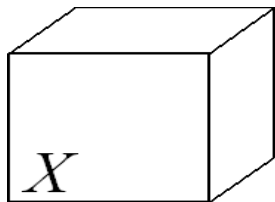
Split EEG in different frequencies using wavelets.



=> Analysis in 3 dimensions instead of just 2

Interpretation of a trilinear component

CPD: Example extracting 1 component

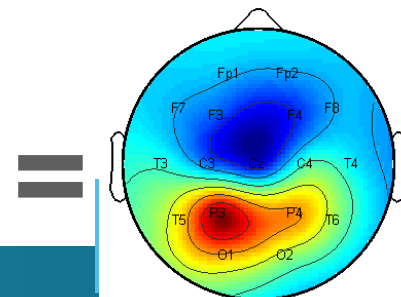
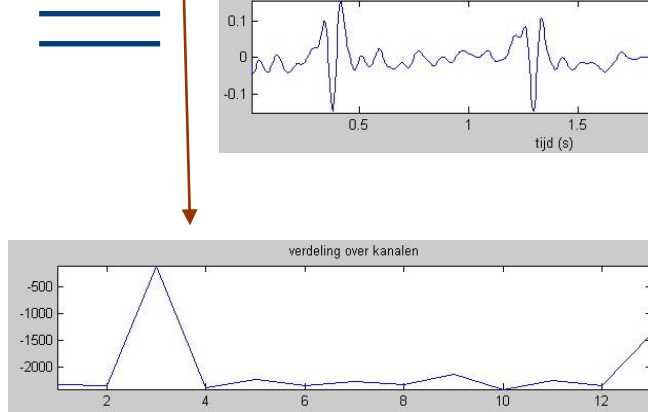
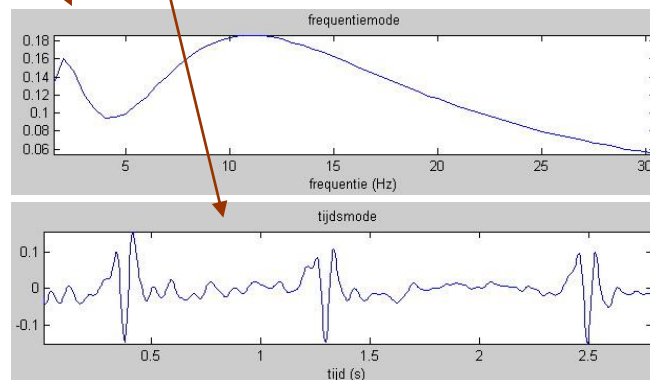
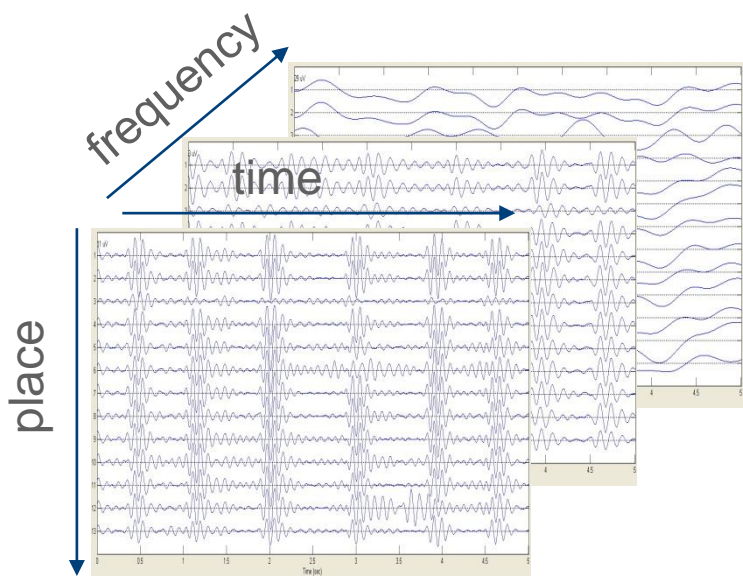


$$X = C_1 \cdot A_1 \cdot B_1$$

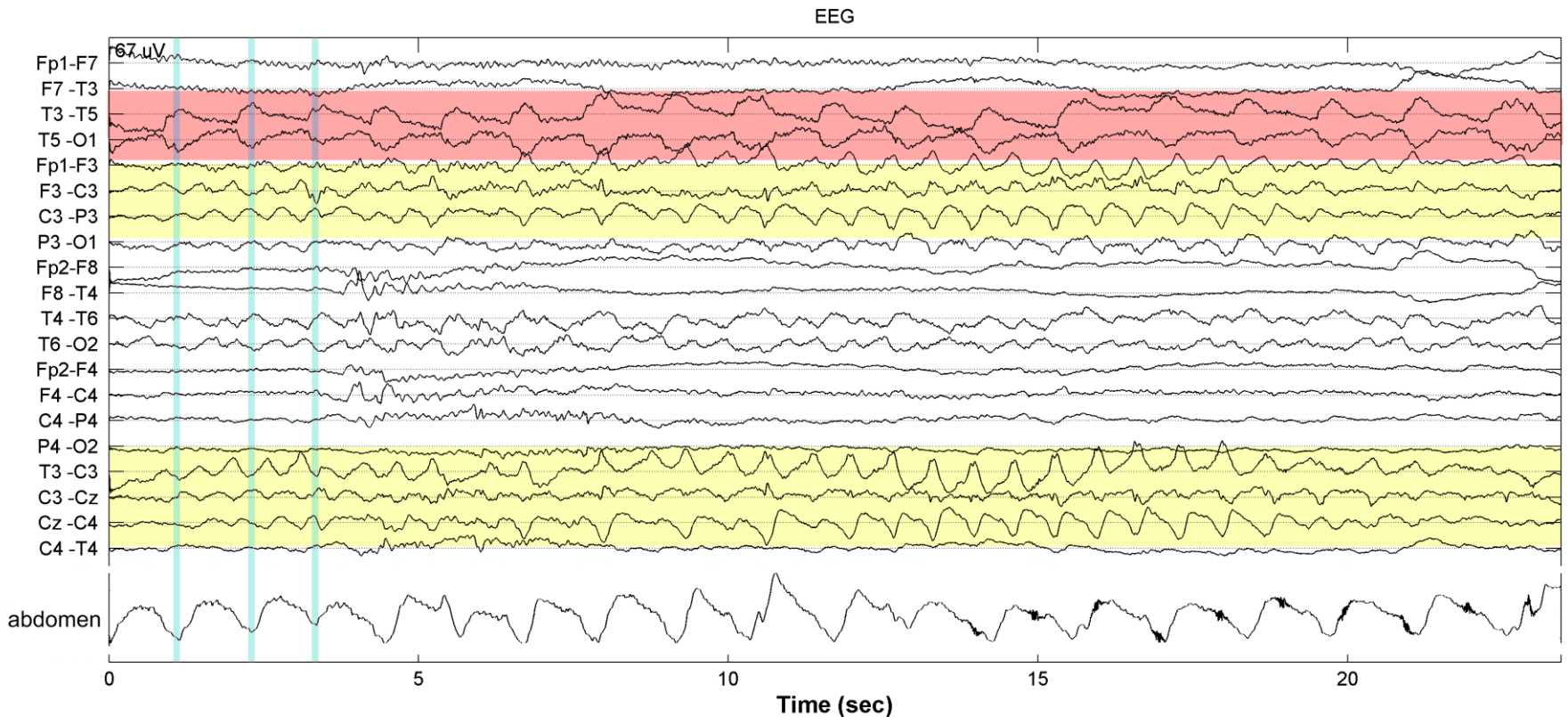
B_1 : time course

A_1 : distribution over channels

C_1 : frequency content (distribution across scales).

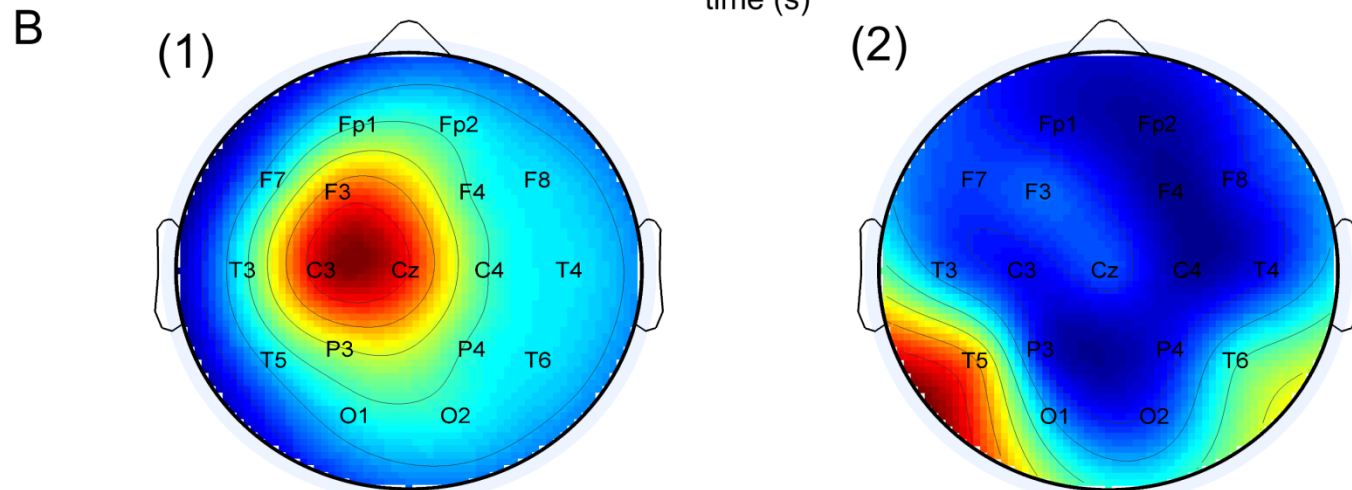
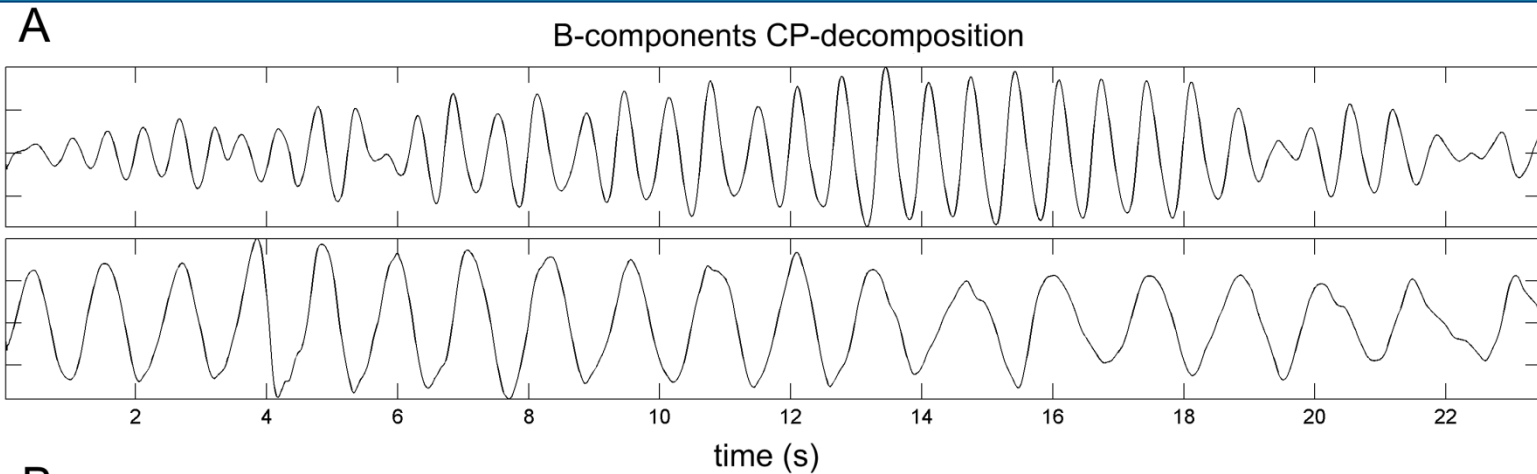


separation seizure from background activity



⇒ Extract spatial distribution of the seizure without distortion of the artifact

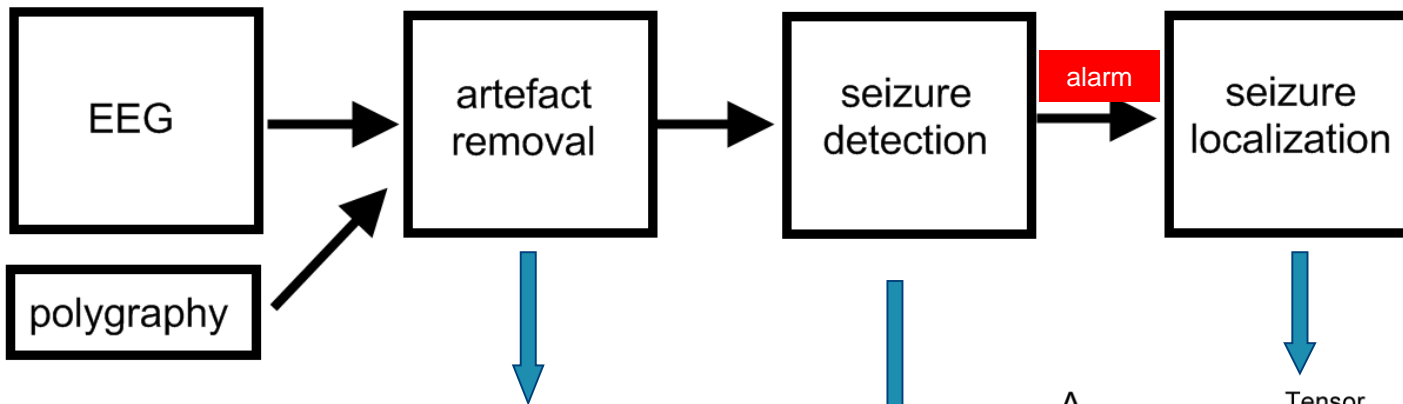
CPD: spatial source components



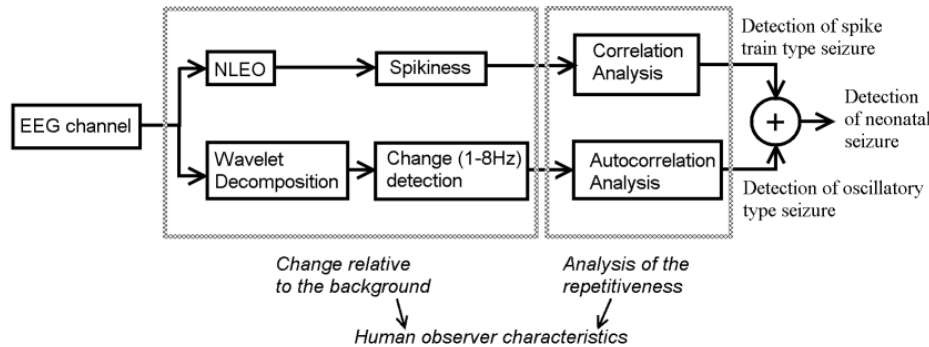
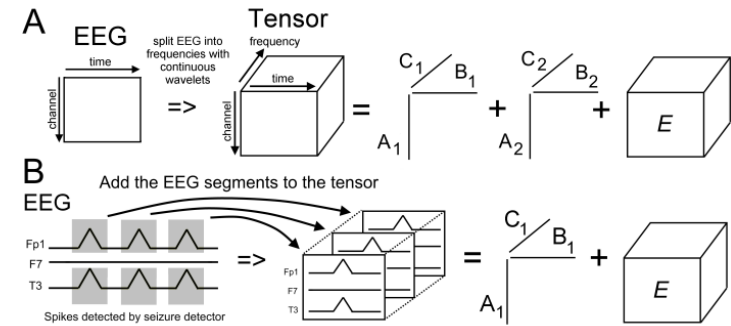
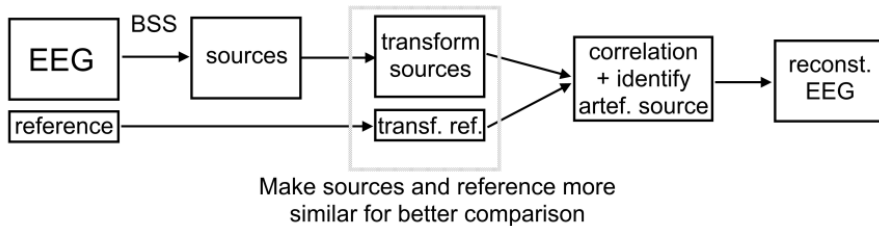
VALIDATION Study: [W. Deburchgraeve et al., Clinical Neurophys., 2009](#)

good qualitative correspondence between visual analysis by expert and algorithm.
Together with seizure detector useful for bedside brain monitoring

Automated seizure monitoring



Artefact removal: general strategy



NeoGuard : decision support

Brain injury estimate

- Detection of neonatal epileptic seizures
- **Seizure onset localization**
- Inter-burst intervals

Clinician's expertise

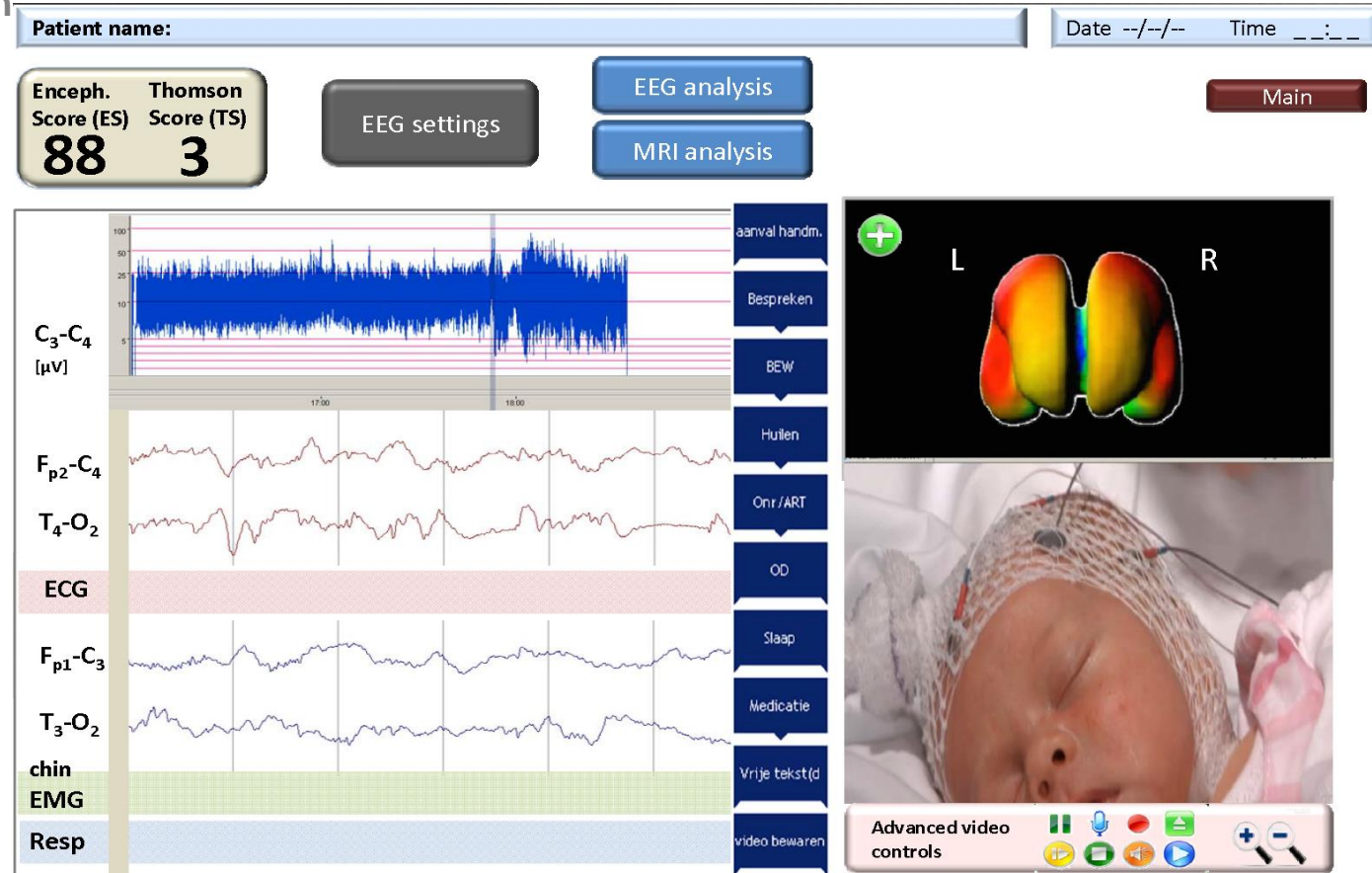
- Neurophysiological knowledge included in algorithms

Brain Monitoring

- **Recovery after damage**
- Maturation in preterms

Outcome prediction

- Good
- Poor

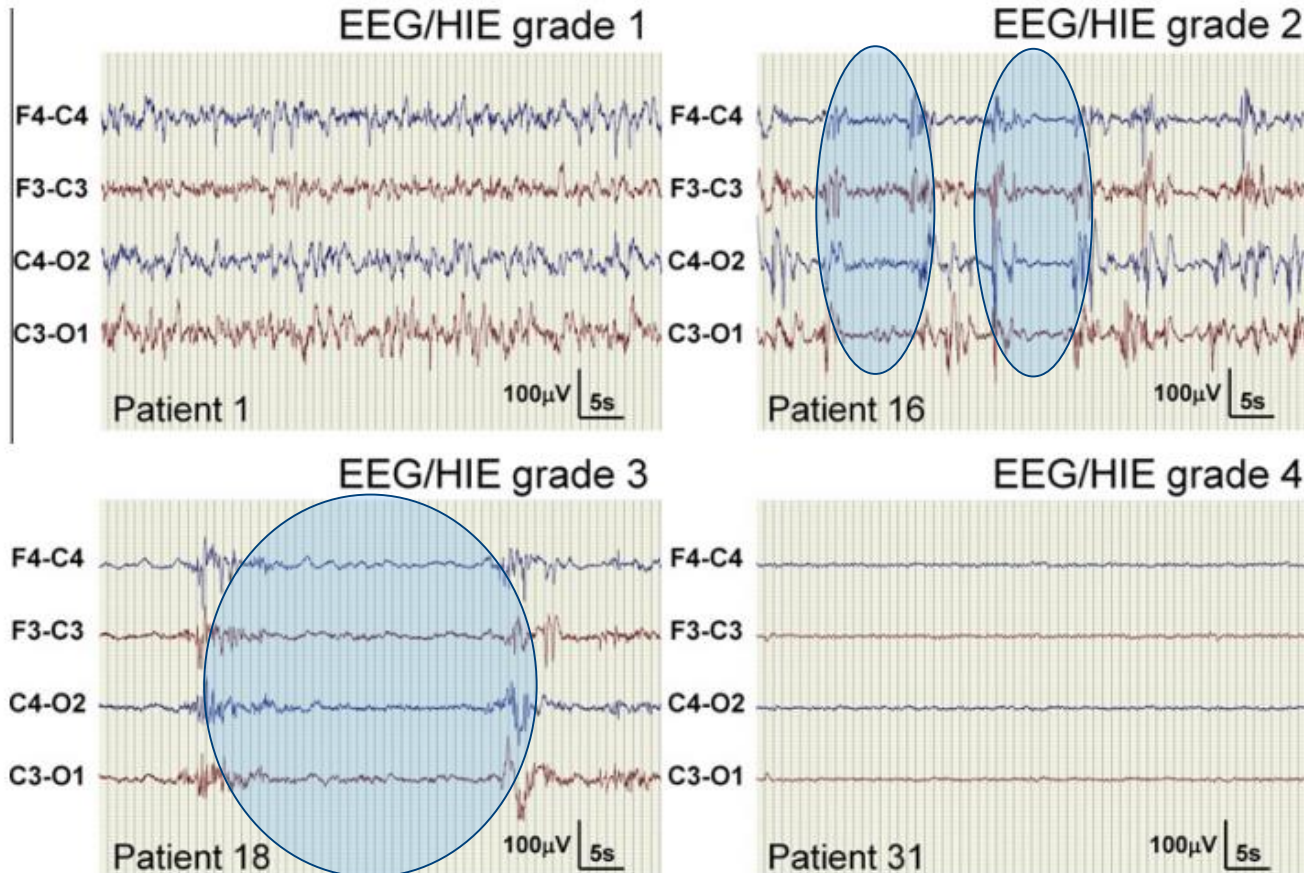


How Assessing Brain Recovery?

→ monitor abnormality of Background EEG

mildly abnormal

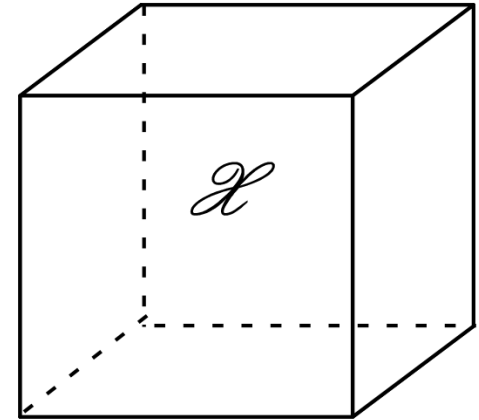
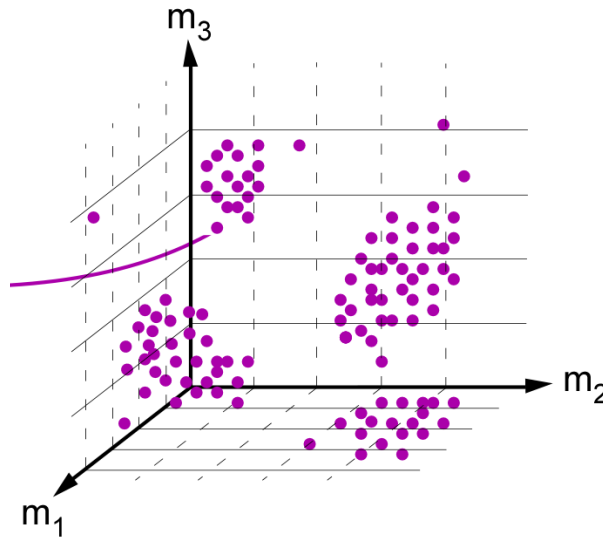
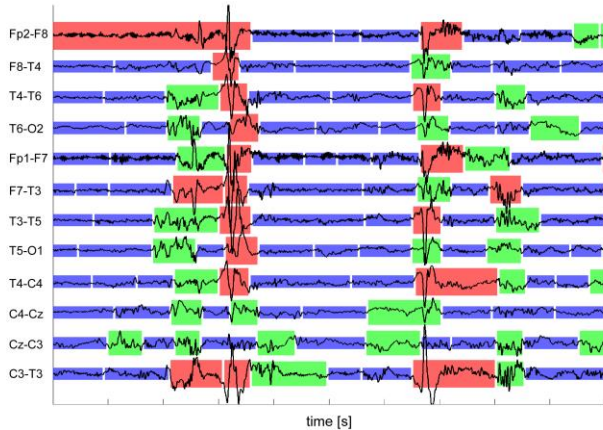
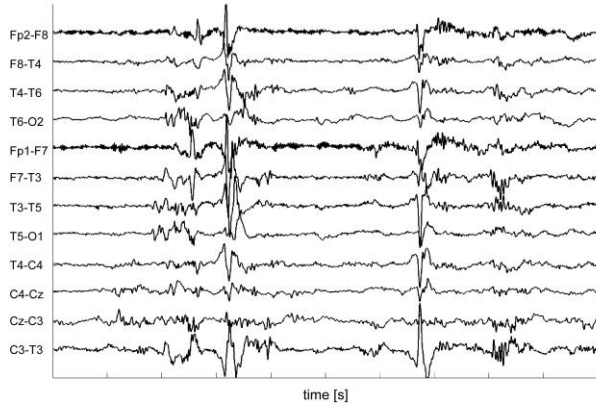
moderately abnormal



severely abnormal

Ideal examples, taken from [Korotchikova et al., 2011]

Monitoring neonatal background EEG: The power of structuring data

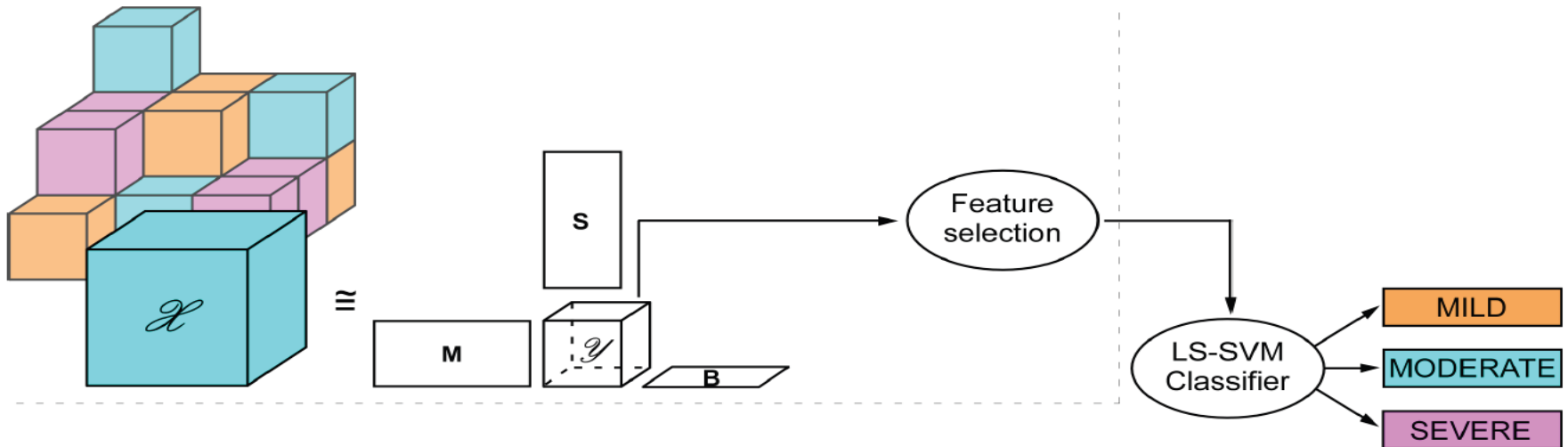


V. Matic et al., J. Neural Engineering, Oct. 2014

Higher Order Discriminant Analysis

- > compute simultaneous LMLRA
- > factors M , S , B common and orthogonal
- > maximizing the Fisher ratio between core tensors

TRAINING

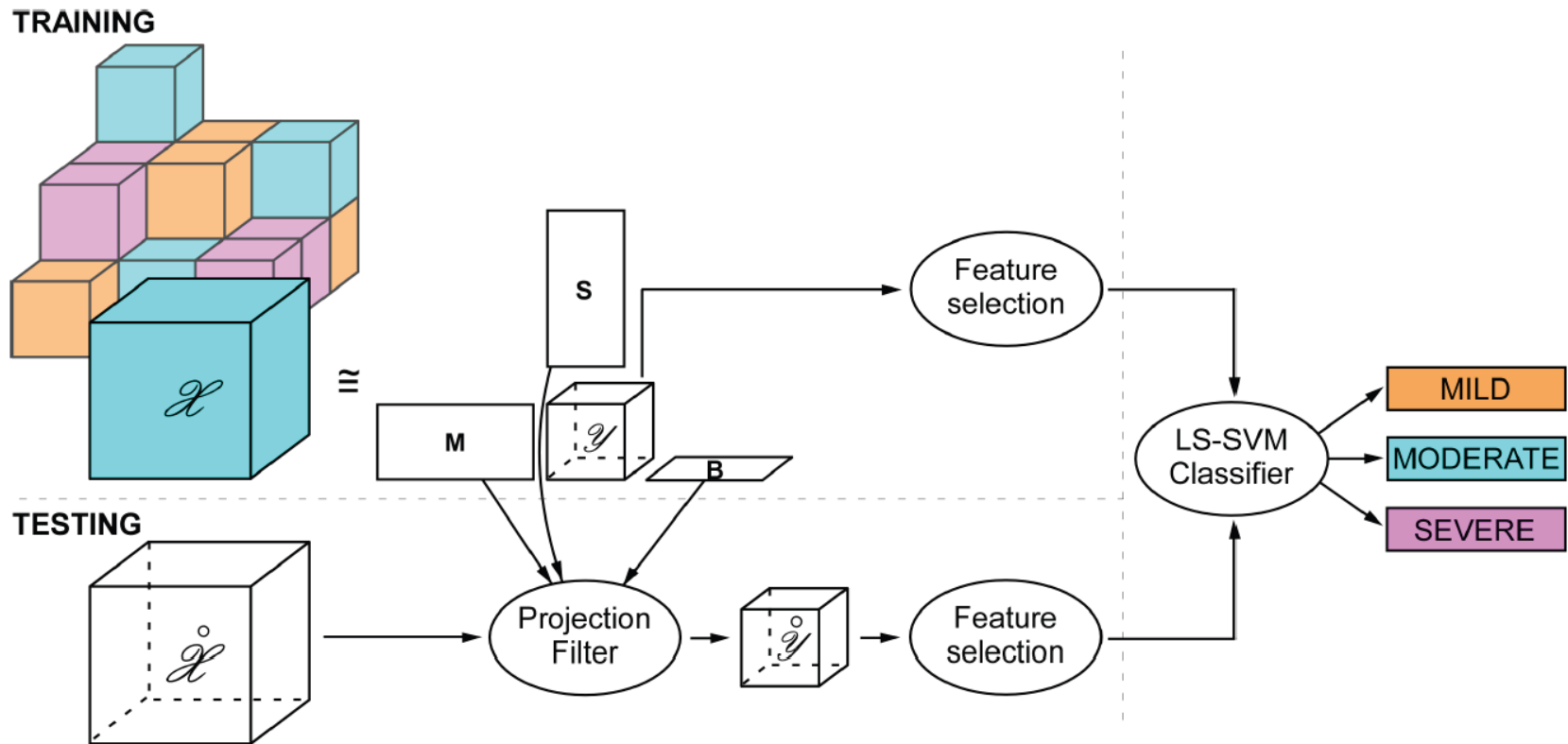


Phan A and Cichocki A, Nonlinear Theory Appl., IEICE, 2010

Phan A, 2011, Matlab Software Toolbox

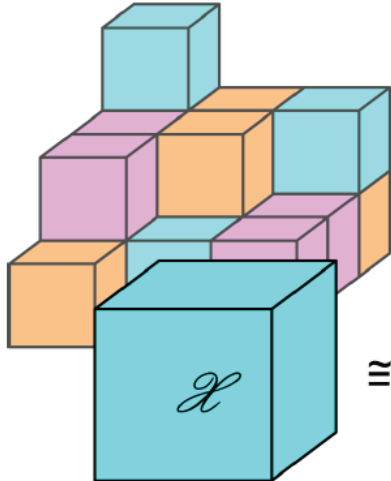
(www.bsp.brain.riken.jp/~phan/nfea/nfea.html)

Higher Order Discriminant Analysis

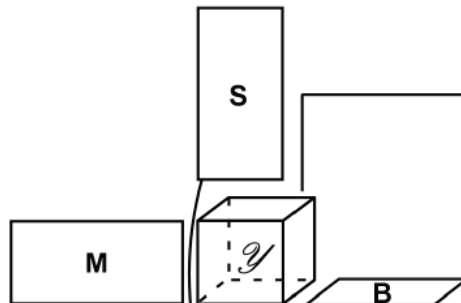


Higher Order Discriminant Analysis

TRAINING



IR

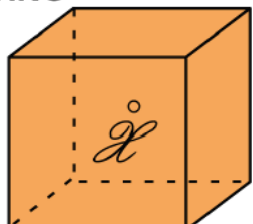


Feature selection

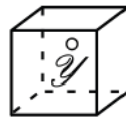
LS-SVM Classifier

- MILD
- MODERATE
- SEVERE

TESTING



Projection Filter



Feature selection

classification accuracy = 89%

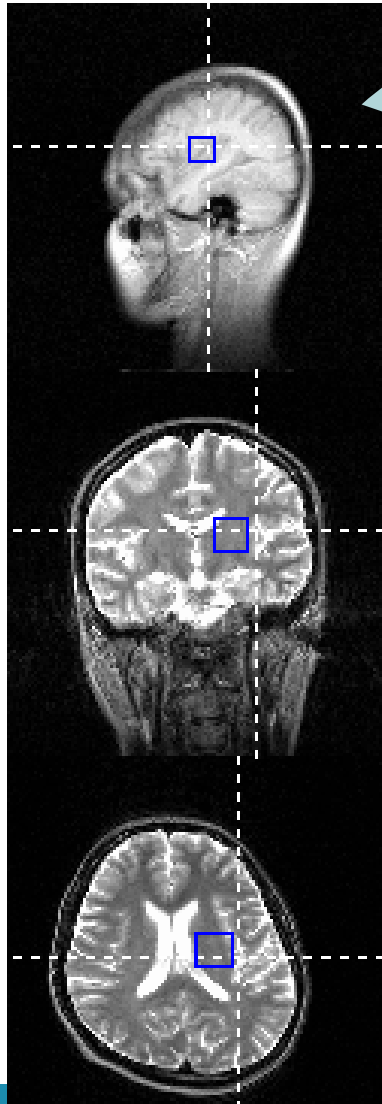
<i>Automated \ Expert EEG</i> reader	MILD	MODERATE	SEVERE
MILD	73 (91%)	6	1
MODERATE	7	44 (76%)	7
SEVERE	0	8	126 (94%)
Achieved accuracy	91%	76%	(94%)

(V. Matic et al,
J. Neural Eng. 11, 2014)

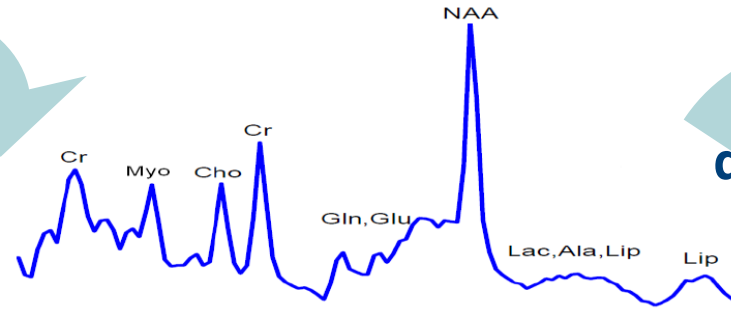
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- Examples in MR-based Brain tumor diagnosis
 - NMF
 - Hierarchical(h) NMF
 - Multimodal hNMF
 - Non-negative Tensor Factorization
- Conclusions and new directions

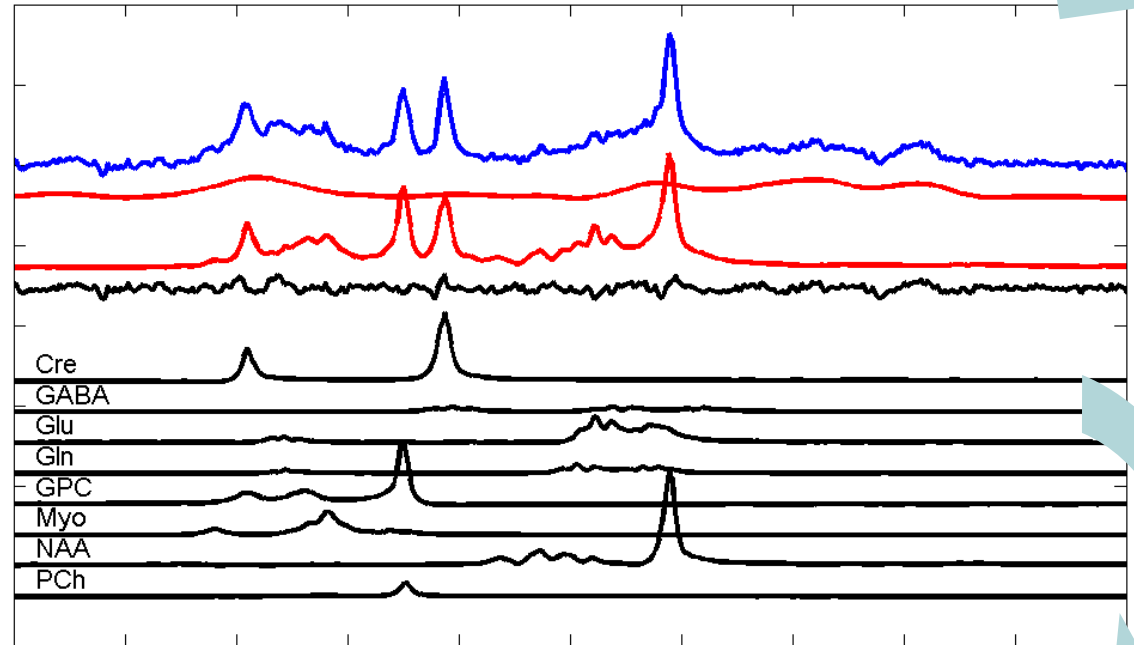
Metabolite quantification for MR Spectroscopy (MRS)



Single-voxel
MRS



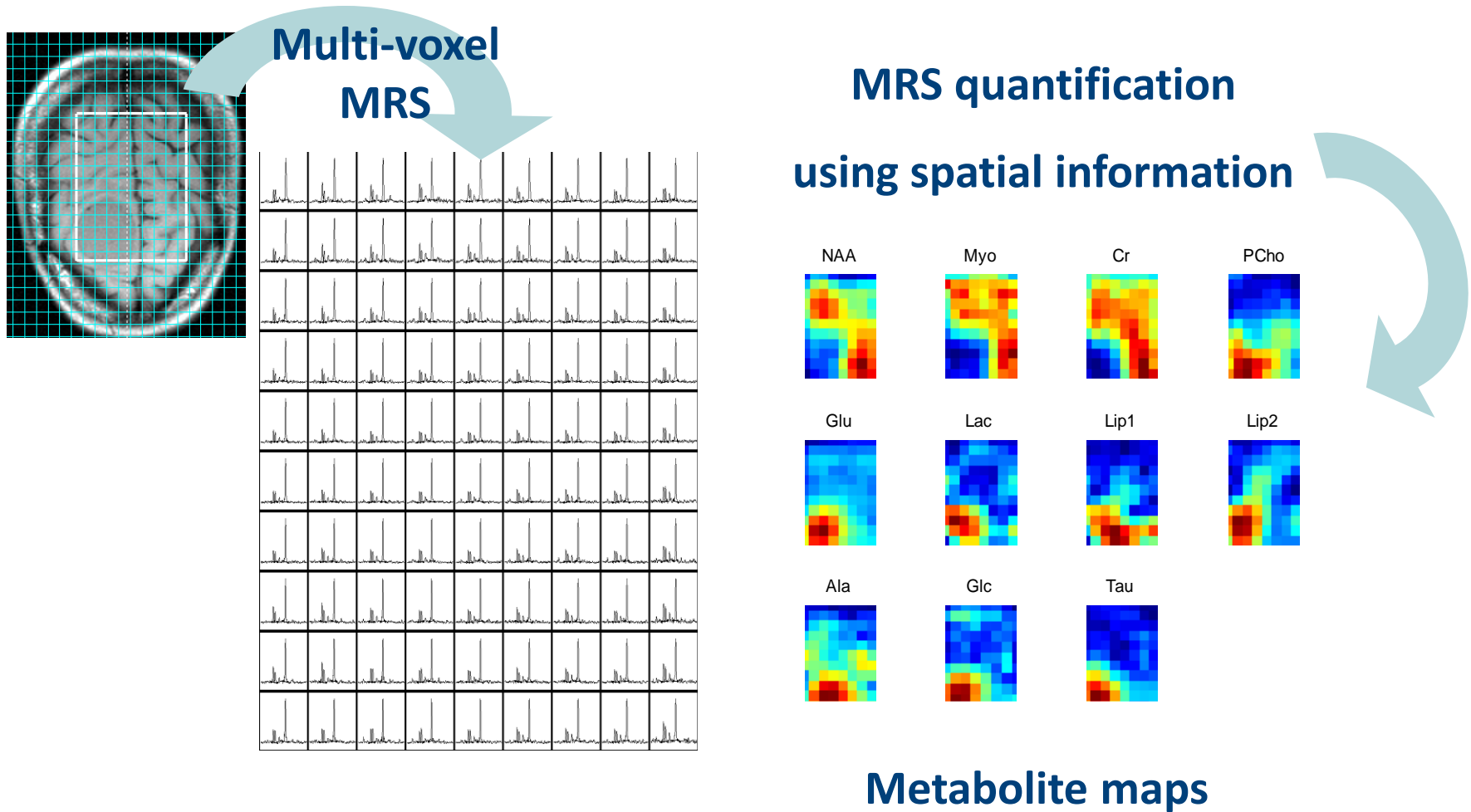
MRS
quantification



Metabolite conce

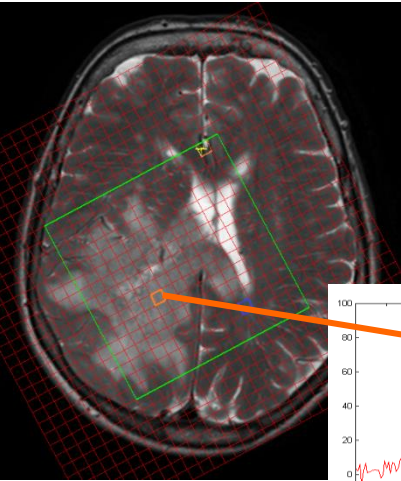
KU LEUVEN

Metabolite quantification for MRS Imaging (MRSI)



Metabolite concentrations = biomarkers of disease

Unsupervised Brain Tumor Diagnosis using NMF

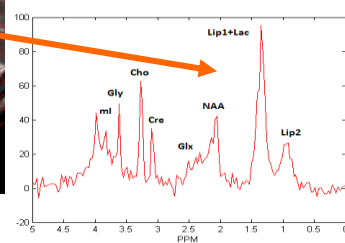


MRSI

$Y =$ matrix of spectra, $Y \approx WH$

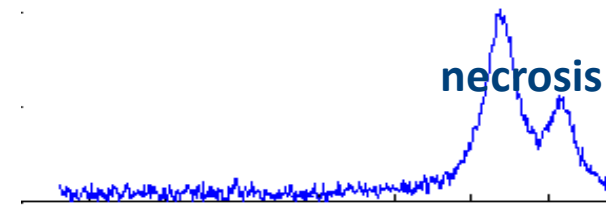
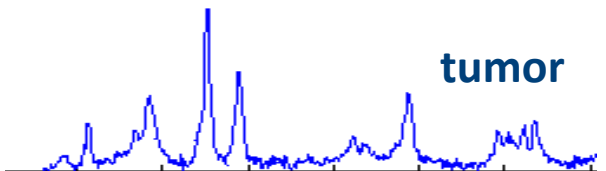
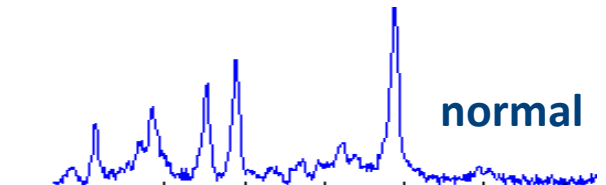
$\min ||Y - WH||$
such that $W \geq 0, H \geq 0$

non-negative matrix factorization



$W =$ tissue-specific spectral patterns:

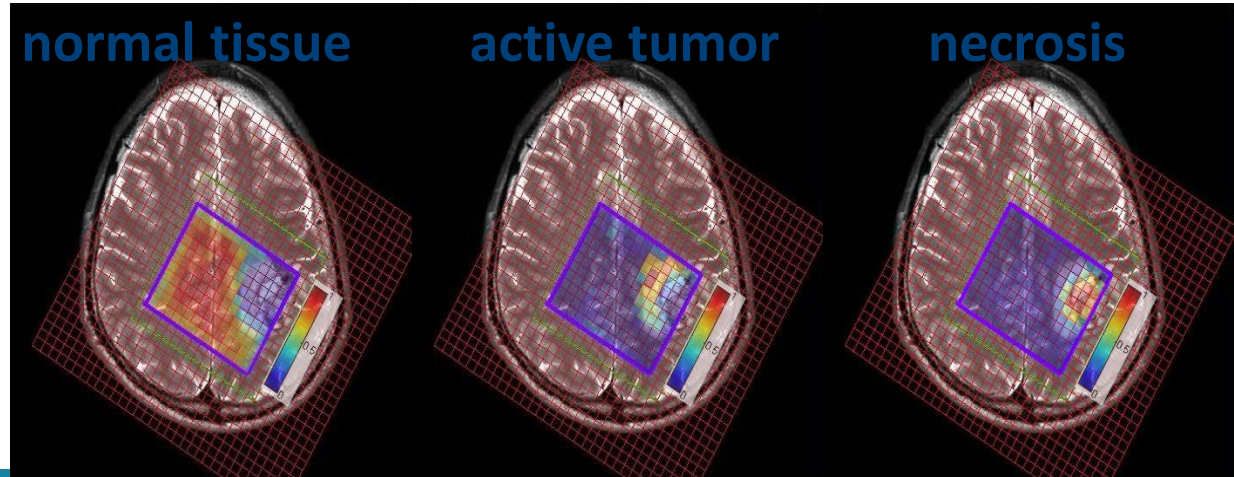
$H =$ spatial distribution of tissue types:



normal tissue

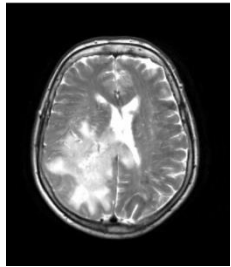
active tumor

necrosis

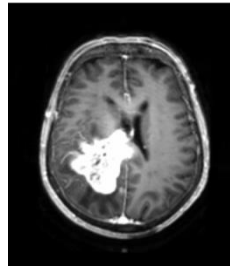


glioblastoma multiforme patient

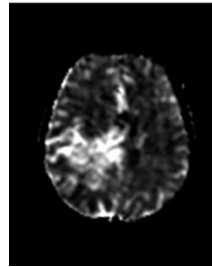
Multi-Parametric (MP) NMF



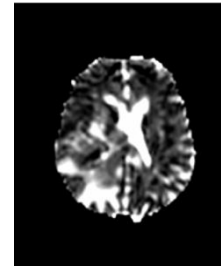
T2



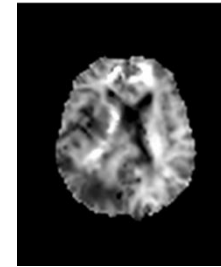
T1



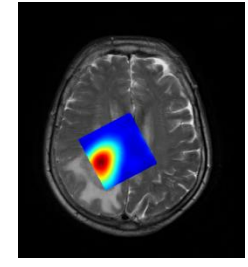
CBV
Cerebral Blood
Volume



MD
Mean Diffusion



MK
Mean Kurtosis



NAA, Cre,
tCho, Lip1,...



Anatomical MRI



Diffusion MRI

Spectroscopic
MRI

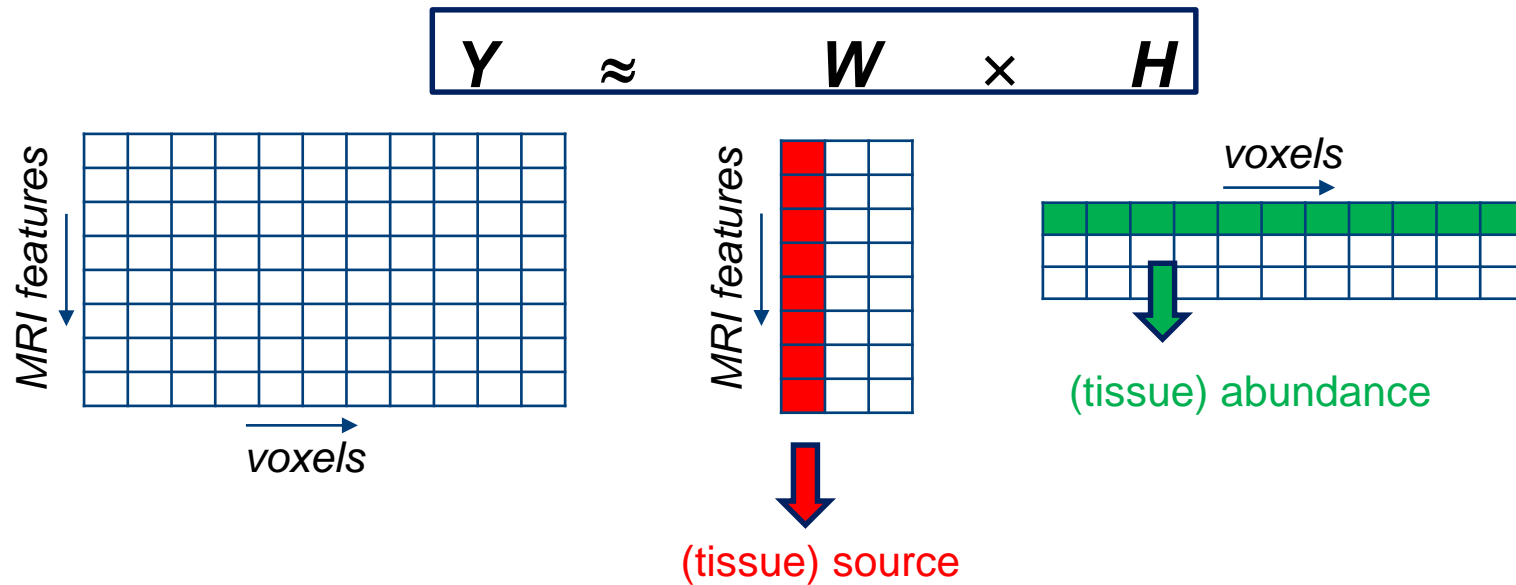
Research study:

- 14 high-grade glioma (HGG) patients with complete MP-MRI dataset (3 grade III, 11 grade IV, UZ Leuven) and 10 low-grade gliomas

Research questions:

- *Do we get valid tissue differentiation with NMF on MP-MRI?*
- *What is added value of individual MRI modalities?*
- *How to improve NMF → hierarchical NMF ? Tensor Factorisation?*

Non-negative matrix factorization (NMF)

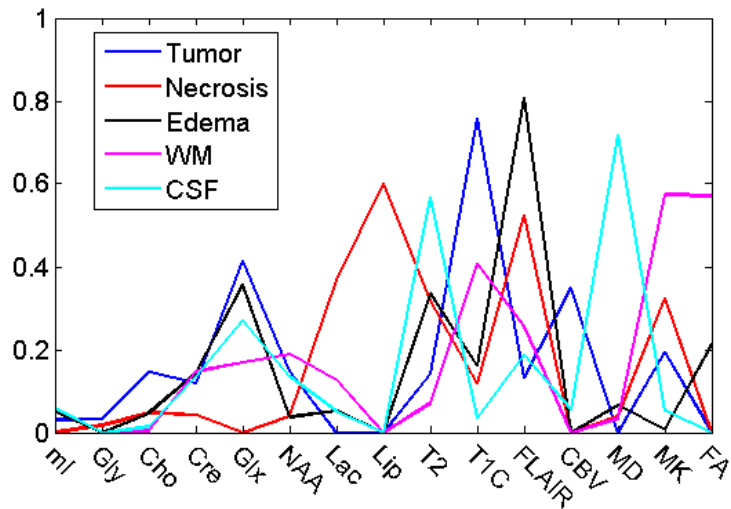


- Non-negativity constraint: $Y_{i,j}, W_{i,j}, H_{i,j} \geq 0, \forall i,j$
- Unsupervised: \rightarrow applicable patient-by-patient,
 \rightarrow tissue classes not a priori known
- MP-NMF: 1. integrate ALL features of each modality into one vector
2. use NMF

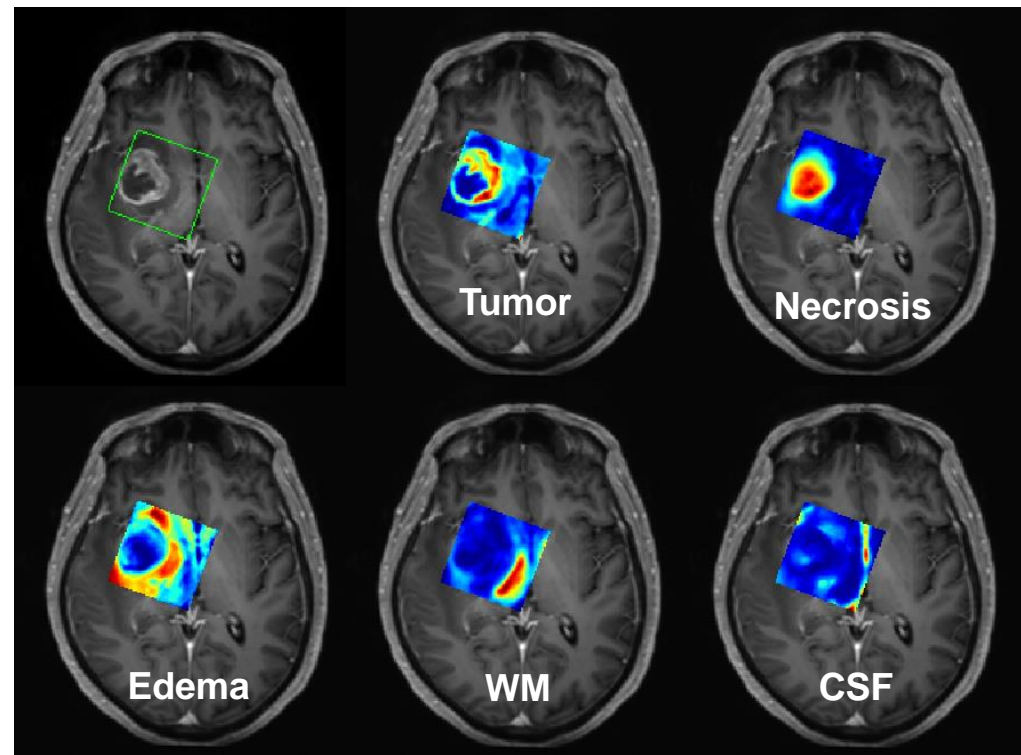
Case study: single stage NMF

5 tissue types within region of interest: active tumor, necrosis, edema, white matter, CSF

Tissue sources (W)



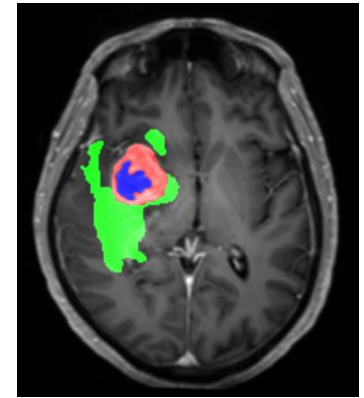
Tissue abundances (H)



Is this good/bad result?

Validation

Based on manual segmentation by radiologist
(only pathological tissue types)



1) Dice-scores (based on H)

$$Dice = 2x \frac{area(A \cap B)}{area(A) \cup area(B)}$$



2) Correlation coefficients (based on W)

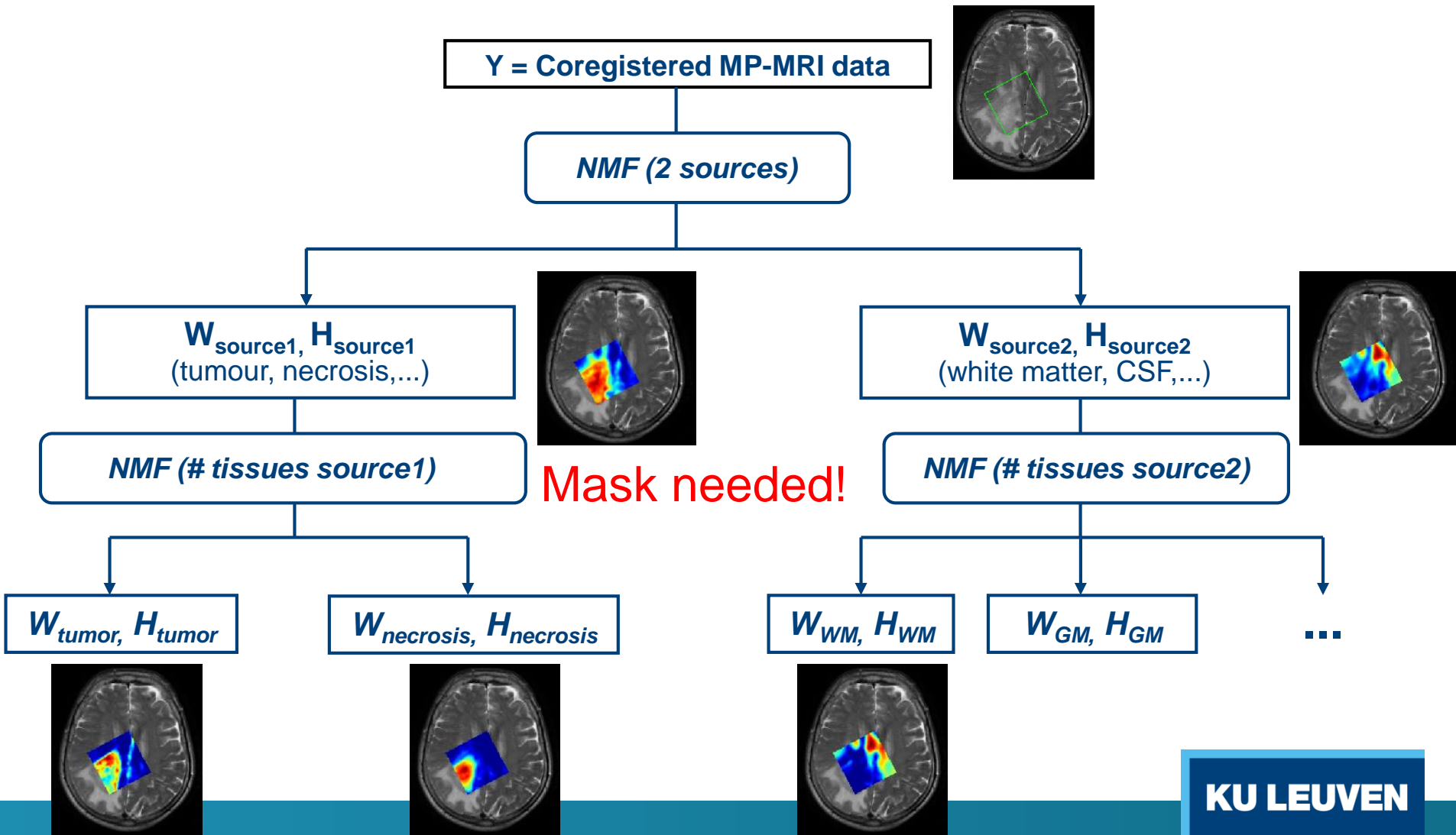
$$Corr = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

\vec{a} : tissue source vector

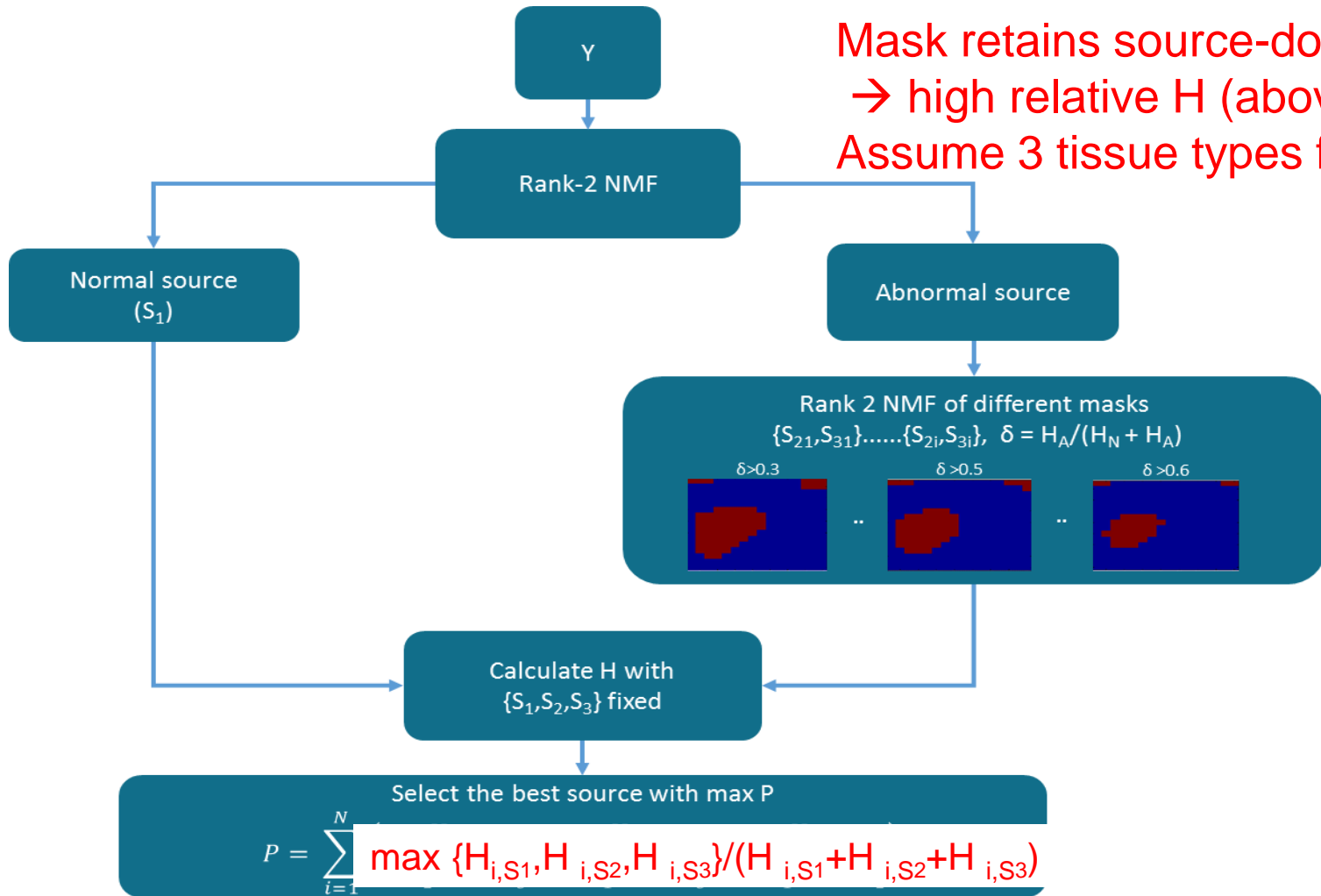
\vec{b} : average feature vector over corresponding tissue region

Hierarchical NMF (hNMF)

- Improved results on MRSI data only (Li et al., NMR in BioMed. 2013)



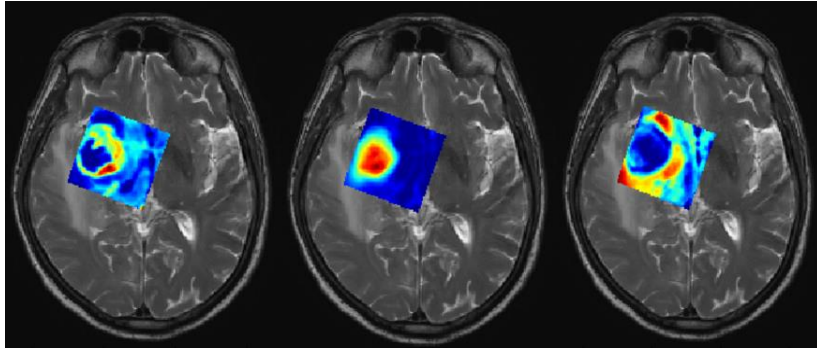
Hierarchical NMF: Select best Mask



Sauwen N, Sima D, Van Cauter S, Veraart J, Leemans A, Maes F, Himmelreich U, Van Huffel S. Hierarchical non-negative matrix factorization to characterize brain tumor heterogeneity using multi-parametric MRI. *NMR in BioMedicine*, 2015, paper in review

Case study: single stage NMF vs hNMF

Single stage NMF



$$\text{Dice}_{\text{tumor}} = 71\%$$

$$\text{Dice}_{\text{tumor+necrosis}} = 83\%$$

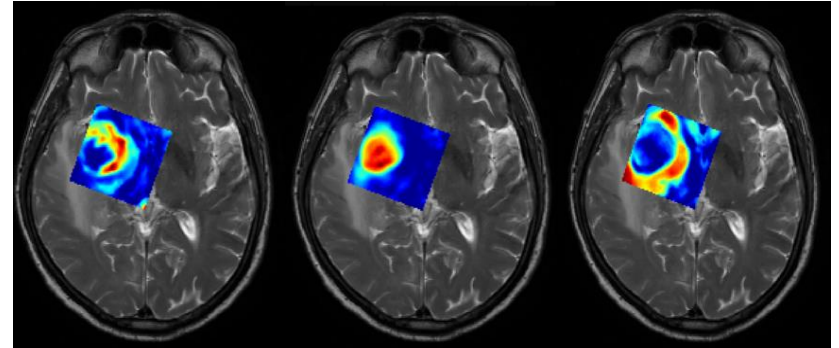
$$\text{Dice}_{\text{complete tumor}} = 75\%$$

$$\text{Corr}_{\text{tumor}} = 0.60$$

$$\text{Corr}_{\text{necrosis}} = 0.97$$

$$\text{Corr}_{\text{edema}} = 0.93$$

hNMF



$$\text{Dice}_{\text{tumor}} = 81\%$$

$$\text{Dice}_{\text{tumor+necrosis}} = 92\%$$

$$\text{Dice}_{\text{complete tumor}} = 83\%$$

$$\text{Corr}_{\text{tumor}} = 0.78$$

$$\text{Corr}_{\text{necrosis}} = 0.98$$

$$\text{Corr}_{\text{edema}} = 0.97$$

Full study results: Dice-scores

HGG	Dice-score active tumor [%]						Dice-score tumor core [%] (active tumor + necrosis)						Dice-score complete tumor [%] (active tumor + necrosis + edema)					
	Full MP-MRI	no-cMRI	no-PWI	no-MRSI	no-DKI	cMRI only	Full MP-MRI	no-cMRI	no-PWI	no-MRSI	no-DKI	cMRI only	Full MP-MRI	no-cMRI	no-PWI	no-MRSI	no-DKI	cMRI only
Average	78	65	76	71	76	69	85	77	81	79	79	71	83	75	79	76	77	68
Std dev	10	14	19	16	12	18	11	14	18	17	14	17	14	15	17	18	14	18
p-value	-	0.001*	0.54	0.03*	0.18	0.004*	-	0.003*	0.28	0.01*	0.002*	0.001*	-	0.002*	0.03*	0.001*	0.003*	0.001*

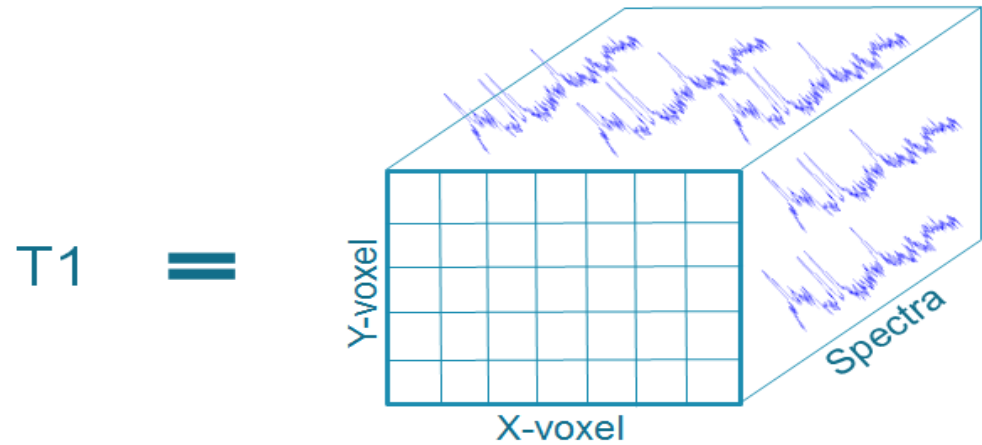
* Statistical significance of lower Dice-scores compared to full MP-MRI based on one-tailed Wilcoxon signed rank test, $p < 0.05$

Combining 4 MRI modalities
improves brain tissue differentiation

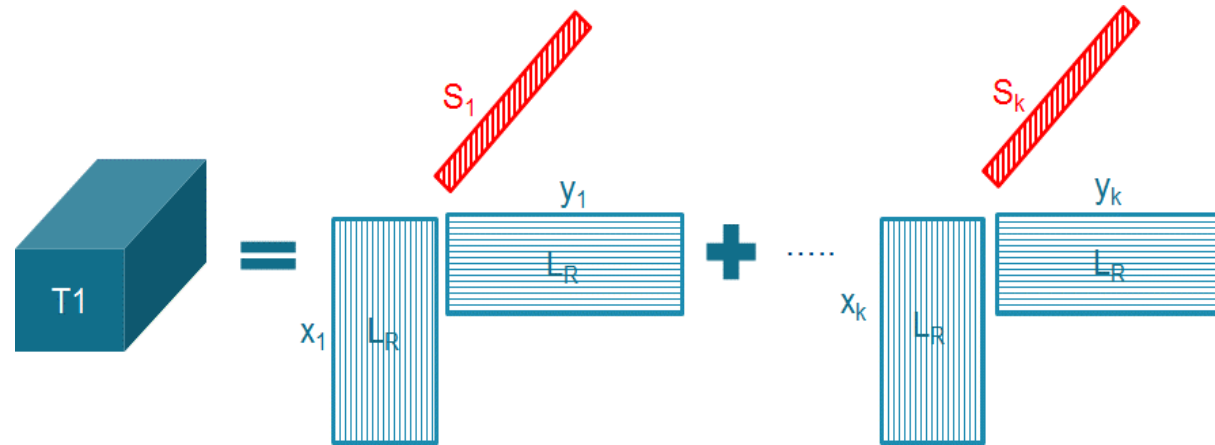
Similar conclusions when comparing correlation coefficients

Spatial Tensor Representation (MRSI only)

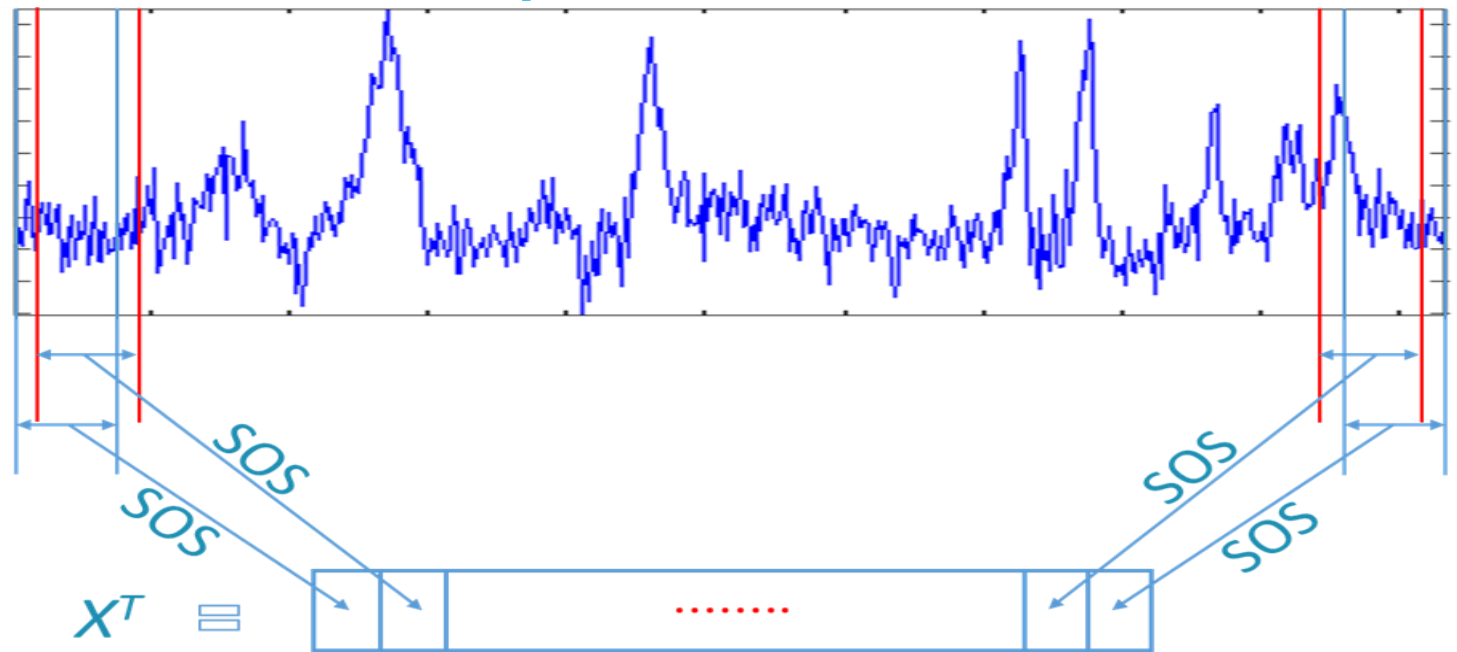
- Frontal slices (XY^T) representing the spatial distribution of a tissue type does not have low rank structure.



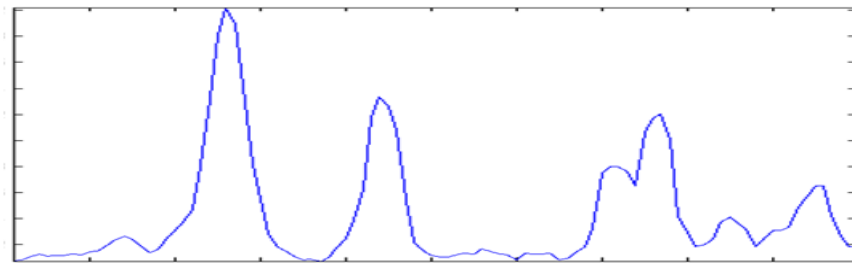
- Difficult to find the rank L_R for a particular tissue type distribution.



XX^T based Tensor Representation (MRSI only)

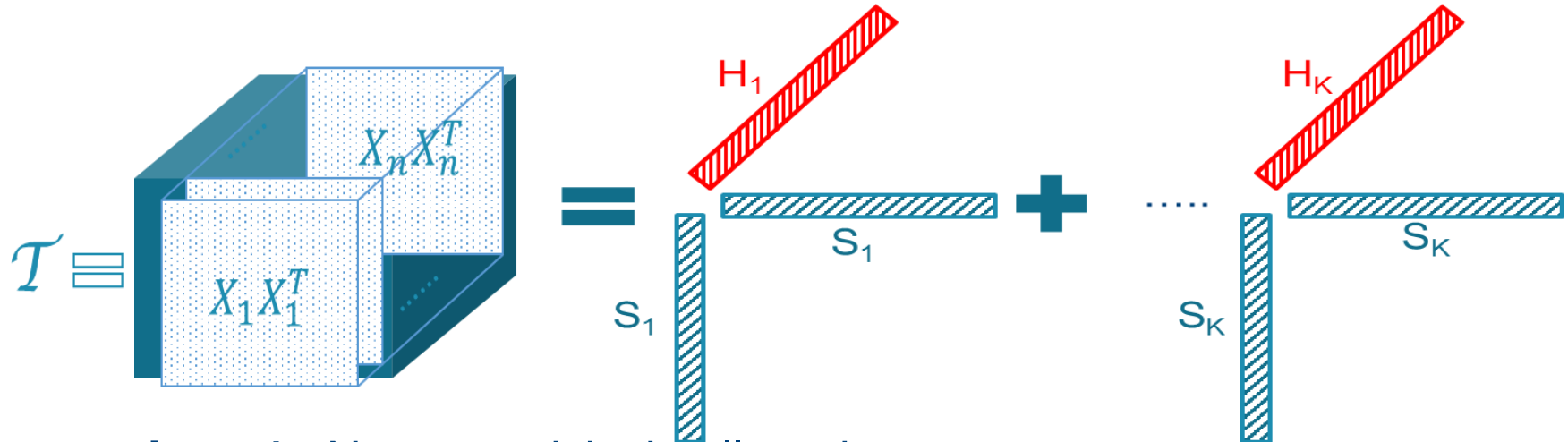


$$X(i) = \sum_{j=1}^L s_{ij}^2$$



- Spectra reduced in length and denoised without losing vital information,
- Peaks get higher weights,
- Peaks coupled because of XX^T in the frontal slices³⁵

Non-negative CPD for Tumor Differentiation



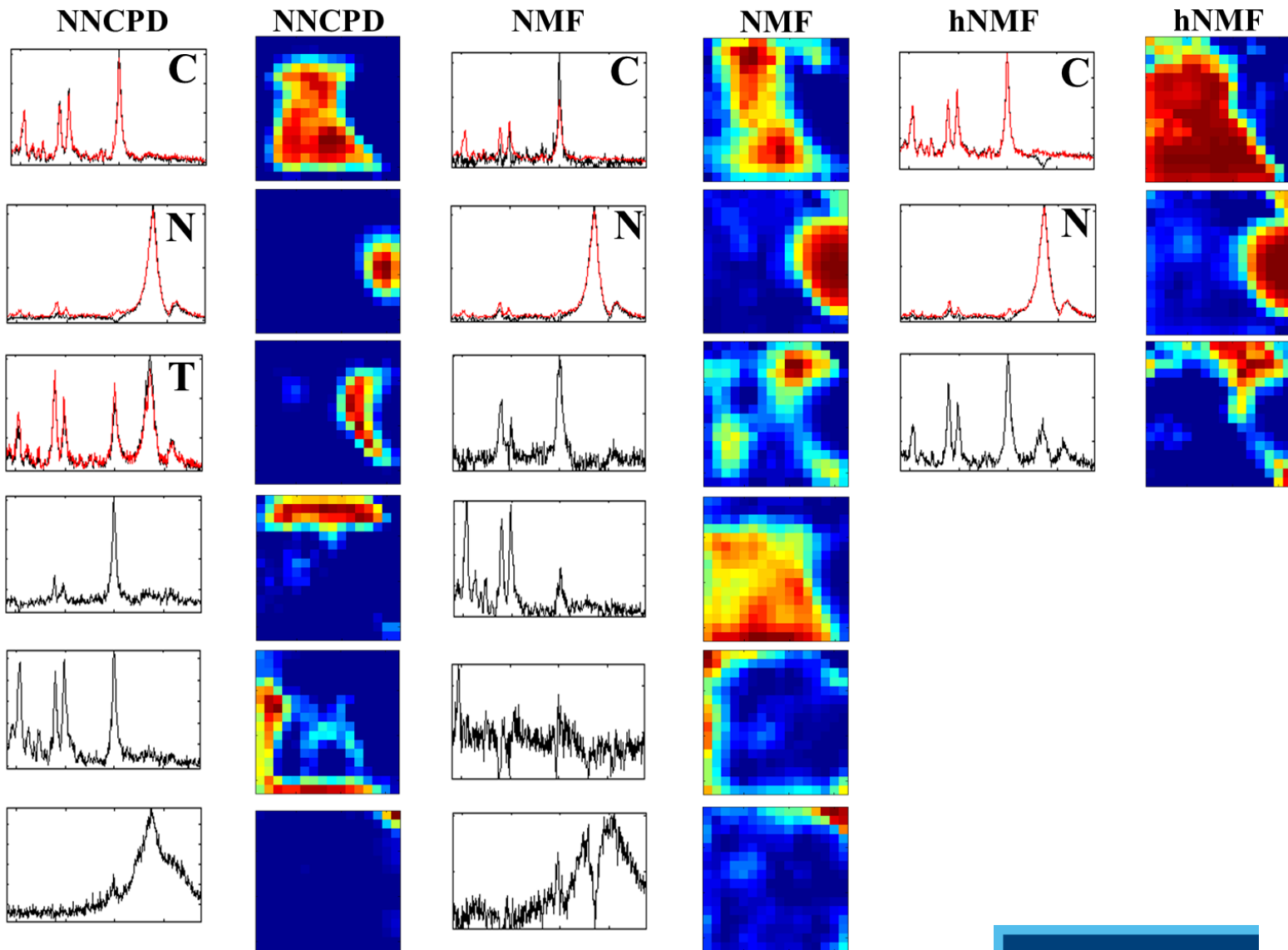
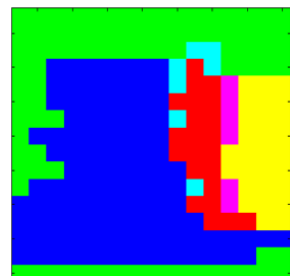
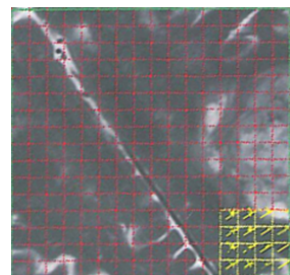
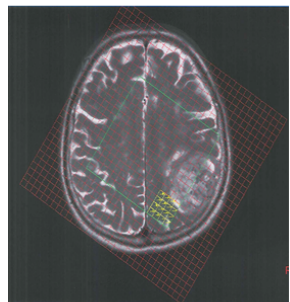
- **Constraints:**
 1. Non-negativity in all modes
 2. Symmetry in frontal slices
 3. Sparsity in H-factor

→ apply Non-negative CPD with L_1 regularization using Tensorlab¹

$$[S^*, H^*] = \min_{S \geq 0, H \geq 0} \left\| T - \sum_{r=1}^K S(:, r) \circ S(:, r) \circ H(:, r) \right\|_2^2 + \lambda |Vec(H)|_1$$

- Using H^* , recover tissue-specific spectra W from Y via LS
- Using W , recover tissue-type spatial distributions H from Y via NN-LS

Result: Patient-2



Source Correlation: Algorithm vs Expert labeling (MRSI only)

W		NCPD	Single stage NMF	hNMF	Grade
PATIENT-2	T	0.99	X	X	High
PATIENT-2	N	0.9975	0.9971	0.9972	High
Median/MAD	T	0.98 /0.0112	0.67 /0.0514	0.87 /0.0093	High
Median/MAD	N	0.9969 /0.0013	0.9945/0.0028	0.9967/0.0005	

Abundance map Correlation: Algorithm vs Expert labeling

H		NCPD	Single stage NMF	hNMF	Grade
PATIENT-2	T	0.80	X	X	High
PATIENT-2	N	0.9552	0.8949	0.8967	High
Median/MAD	T	0.79 /0.0380	0.69 /0.0849	0.69 /0.0731	High
Median/MAD	N	0.8742 /0.0276	0.7876/0.0662	0.8009/0.0716	

Contents Overview



European Research Council
Established by the European Commission

- Introduction
- Examples in Neonatal brain monitoring
- Examples in MR-based Brain tumor diagnosis
- **Conclusions and new directions**

Conclusions and new directions

- Many BSS problems in Smart Diagnostics are low rank
→ solve via matrix or tensor factorization plus constraints
- Successful examples shown, e.g., in neonatal brain monitoring, brain tissue typing
- Extensions to biomedical data fusion emerge, e.g. EEG-fMRI
→ solve via coupled matrix /tensor decompositions
- Other BSS applications: *bioinformatics (O. Alter, E. Acar), BCI (Cichocki, Mørup, Martinez-Montes), mobile EEG, multichannel ECG*

New directions?

- *Adaptive tensor decompositions, rank & structure estimation*
- *Applications increasing in BCI, (single-trial) ERP, ECG, MRSI*
- *exploit full potential of Tensor toolbox*



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Thank you!



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**Workshop on Tensor Decompositions and Applications
January 18 - 22, 2016, Leuven, Belgium**

Local Organisers: Sabine Van Huffel and Lieven De Lathauwer

Confirmed Speakers

Orly Alter
Pierre Comon
Eva Ceulemans
Harm Derksen

Nicolas Gillis
Daniel Kressner
Lek-Heng Lim
Ivan Markovsky

Morten Mørup
Nikos Sidiropoulos
Bart Vandereycken
Frank Verstraete

