# TUTORIAL 9 – OPEN AND CLOSED LOOP LINKS

This tutorial is of interest to any student studying control systems and in particular the EC module D227 – Control System Engineering.

On completion of this tutorial, you should be able to do the following.

- Explain the link between the gain of an open loop and closed loop system.
- Define the Bandwidth of a system.
- Explain and interpret M N Contours.
- Explain and interpret the Nichols Chart.

If you are not familiar with instrumentation used in control engineering, you should complete the tutorials on Instrumentation Systems.

You should complete the previous tutorials before attempting this one.

## 1. RELATIONSHIP BETWEEN OPEN LOOP AND CLOSED LOOP PLOTS

### 1.1 POLAR PLOT

Consider again a closed loop system with unity feedback. The open loop transfer function is G(s) and the closed loop transfer function is $G_{cl} = \dfrac{G(s)}{G(s) + 1}$

A polar plot of the open loop transfer function is much easier to do than a plot of the closed loop system. The plot does, however, yield information of the closed loop gain.
We can determine the vector for G(s) + 1 by simply adding 1 to all the points as shown below to give vector C.

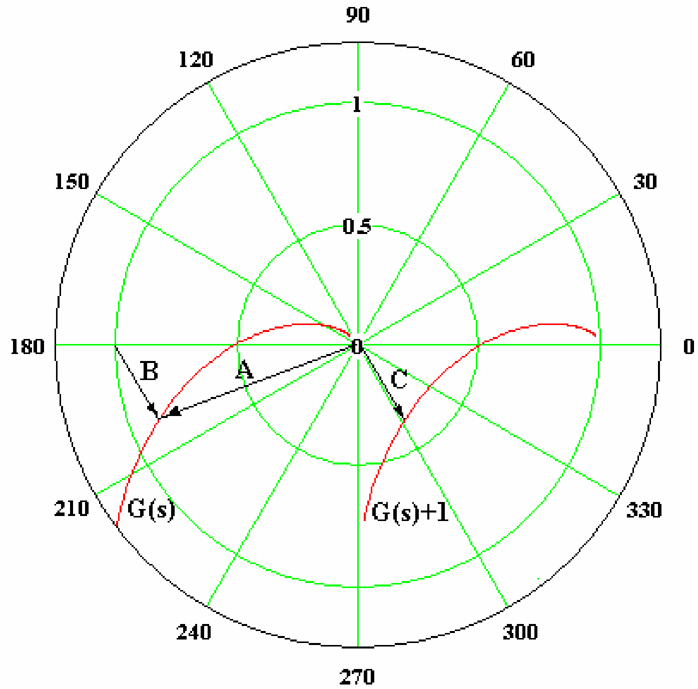


Figure 1

Vector B is the same as vector C. The ratio A/B is the modulus of the closed loop transfer function at any given frequency ω. If this ratio was plotted against frequency it would be seen that it peaks at some critical frequency $\omega_p$.

$$M = A/B = |G_{cl}| = \left|\frac{G(s)}{G(s) + 1}\right|$$

Similarly the phase angle difference between the two plots at any frequency is the phase angle of the closed loop function and this is called an N value.

$$N = \angle G_{cl} = \angle G(s) - \angle\{G(s) + 1\}$$

The modulus of G+1 may be calculated with a bit of trigonometry and is:-

$|G + 1| = \sqrt{\{G \sin\theta\}^2 + G(1 - \sin\theta)^2}$ where θ is 180- ϕ

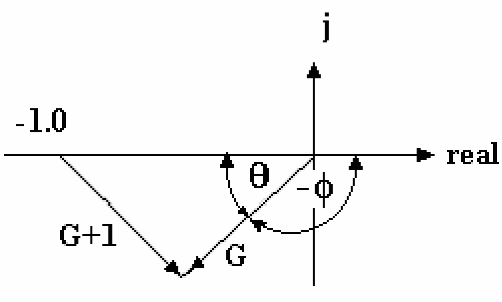


Figure 2

## 1.2 M and N PLOTS

It can be shown that a plot of constant M and N produces circles such that:

For constant M, the centre of the circle is at $\frac{M^2}{1 - M^2}$ and the radius is $\frac{M}{1 - M^2}$

For constant N, the centre of the circle is at -0.5, j0.5/N and the radius is $0.5\sqrt{1 + \frac{1}{N^2}}$ Where N = tanϕ

Below is a polar plot of constant M and N values. The open loop plot may be superimposed on it and information about the closed loop found. The blue line shows a typical closed loop plot. At any point the Modulus and phase angle of the closed loop transfer function may be obtained from the points where it intersects the circles

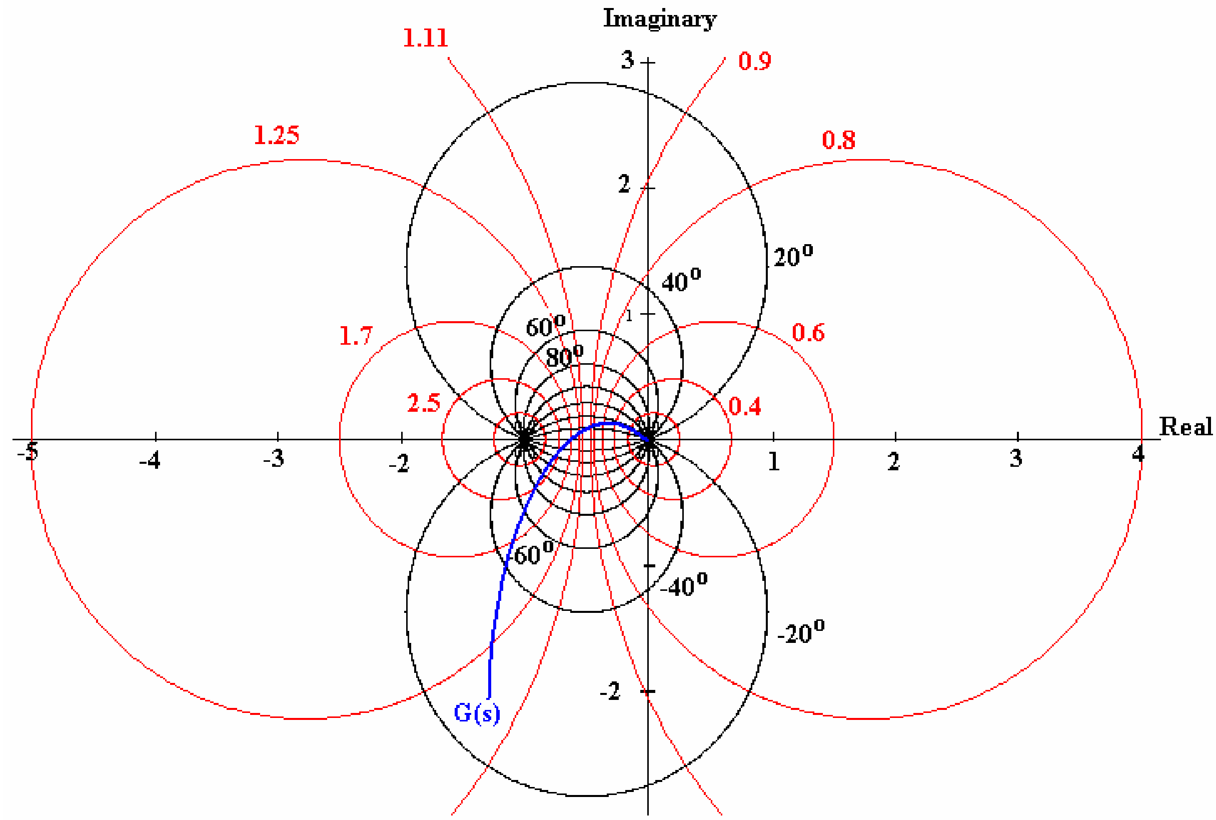


Figure 3

## 1.3 BANDWIDTH

Bandwidth is a way of describing the performance of a system in respect of frequency response. At some frequency the magnification peaks. This might show on a Bode diagram like this.

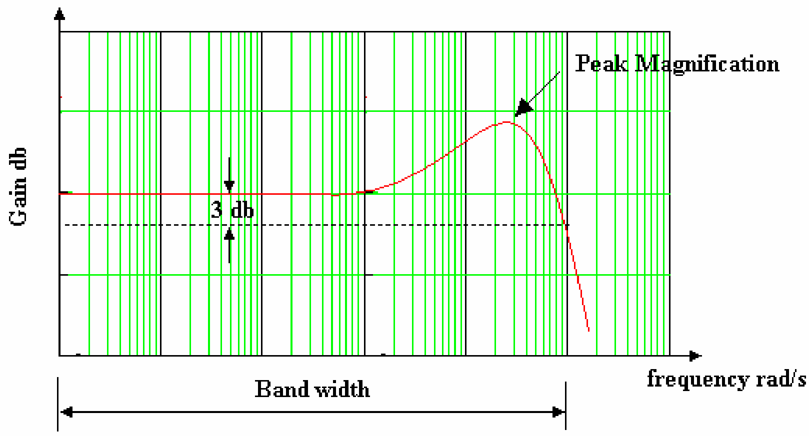


Figure 4

The bandwidth is defined as the frequency at which the magnification (M) is -3db from the peak. This is a factor 0.707 or 1/√2 and comes from electronics terminology governing the frequency band passed by a filter.

## 2. NICHOLS CHART

On the Nichols chart, the open loop gain is plotted vertically in db and the phase angle horizontally in degrees. The M and N contours are superimposed on the chart. When the open loop plot is made, the following information may be obtained:

- Phase Margin
- Gain Margin
- Bandwidth (The frequency at which M = -3 db)
- Peak Magnification and the frequency at which it occurs.

An example open loop plot is shown with these items marked on the plot. Each point on the plot corresponds to a certain frequency.

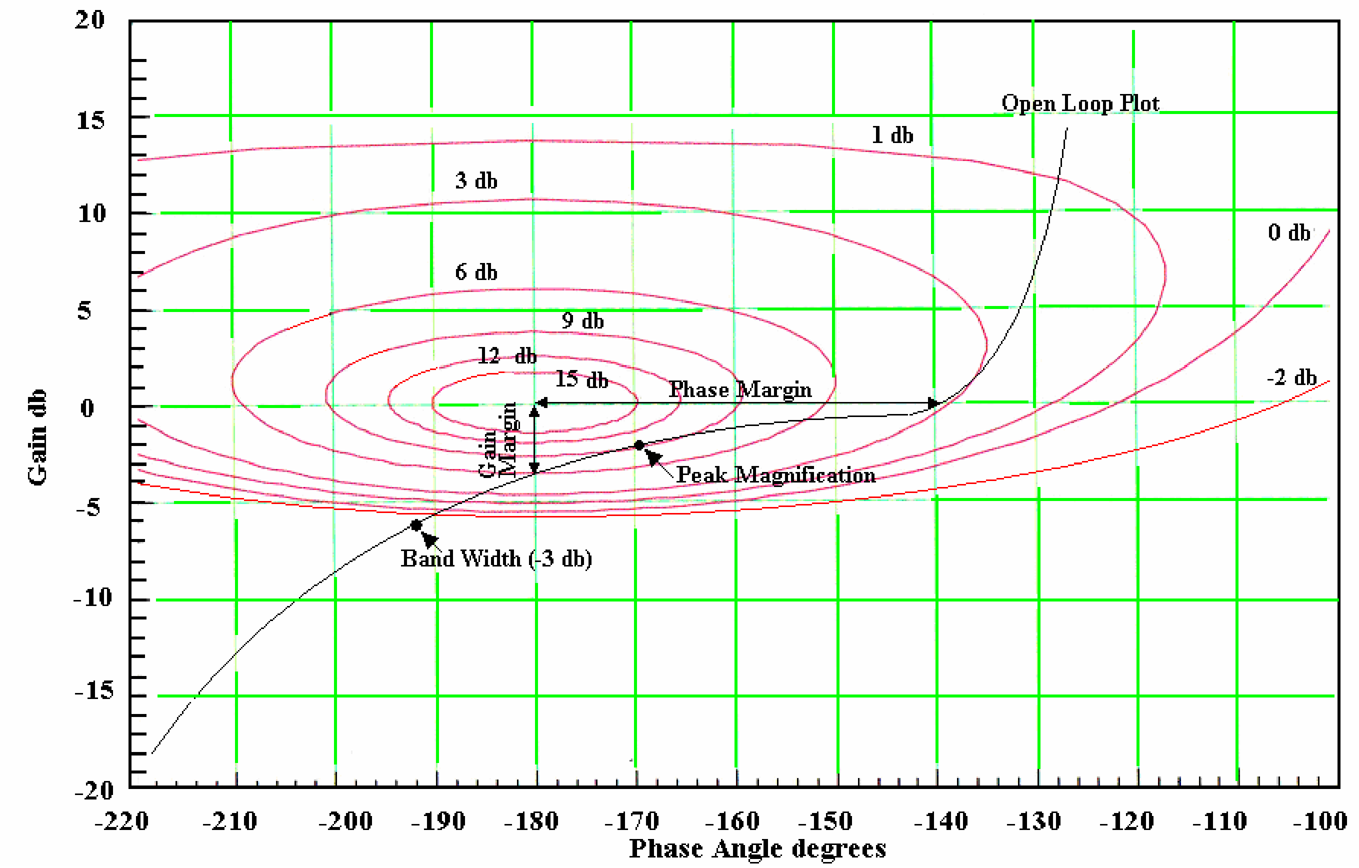


Figure 5

**WORKED EXAMPLE No.1**

Plot the open loop response on the Nichols chart for a system with unity feedback and an open loop transfer function $G(s) = \frac{5}{s(s+2)(s+3)}$

Determine the following.

The phase margin
The gain margin
The peak magnification and the frequency at which it occurs.
The band width

**SOLUTION**

Evaluate the gain and phase angles for $G_1 = \frac{5}{s}$ $G_2 = \frac{1}{(s+2)}$ and $G_3 = \frac{1}{(s+3)}$

A suitable frequency range is 0.4 to 4 rad/s.

In general if we have $\frac{K}{n + Ts}$ the resulting gain and angle are

$$G\,db = 20\log\frac{K}{\sqrt{n^2 + \omega^2 T^2}} \text{ and } \phi - \tan^{-1}\left(\frac{\omega T}{n}\right)$$

(See previous tutorial)

Remember that gains in db are added.

| $\omega$ | $G_1$ | $G_2$ | $G_3$ | $G$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi$ |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 21.938 | -6.191 | -9.619 | 6.128 | -90 | -11.31 | -7.595 | -108.905 |
| 0.6 | 18.416 | -6.395 | -9.713 | 2.309 | -90 | -16.699 | -11.31 | -118.009 |
| 0.8 | 15.918 | -6.665 | -9.841 | -0.588 | -90 | -21.801 | -14.931 | -126.733 |
| 1 | 13.979 | -6.99 | -10 | -3.01 | -90 | -26.565 | -18.435 | -135 |
| 1.2 | 12.396 | -7.356 | -10.187 | -5.147 | -90 | -30.964 | -21.801 | -142.765 |
| 1.4 | 11.057 | -7.752 | -10.398 | -7.094 | -90 | -34.992 | -25.017 | -150.009 |
| 1.6 | 9.897 | -8.169 | -10.63 | -8.902 | -90 | -38.66 | -28.072 | -156.732 |
| 1.8 | 8.874 | -8.597 | -10.878 | -10.601 | -90 | -41.987 | -30.964 | -162.951 |
| 2 | 7.959 | -9.031 | -11.139 | -12.212 | -90 | -45 | -33.69 | -168.69 |
| 2.2 | 7.131 | -9.465 | -11.411 | -13.745 | -90 | -47.726 | -36.254 | -173.98 |
| 2.4 | 6.375 | -9.894 | -11.691 | -15.21 | -90 | -50.194 | -38.66 | -178.854 |
| 2.6 | 5.68 | -10.318 | -11.976 | -16.614 | -90 | -52.431 | -40.914 | -183.346 |
| 2.8 | 5.036 | -10.734 | -12.263 | -17.961 | -90 | -54.462 | -43.025 | -187.487 |
| 3 | 4.437 | -11.139 | -12.553 | -19.255 | -90 | -56.31 | -45 | -191.31 |

Plot the results.

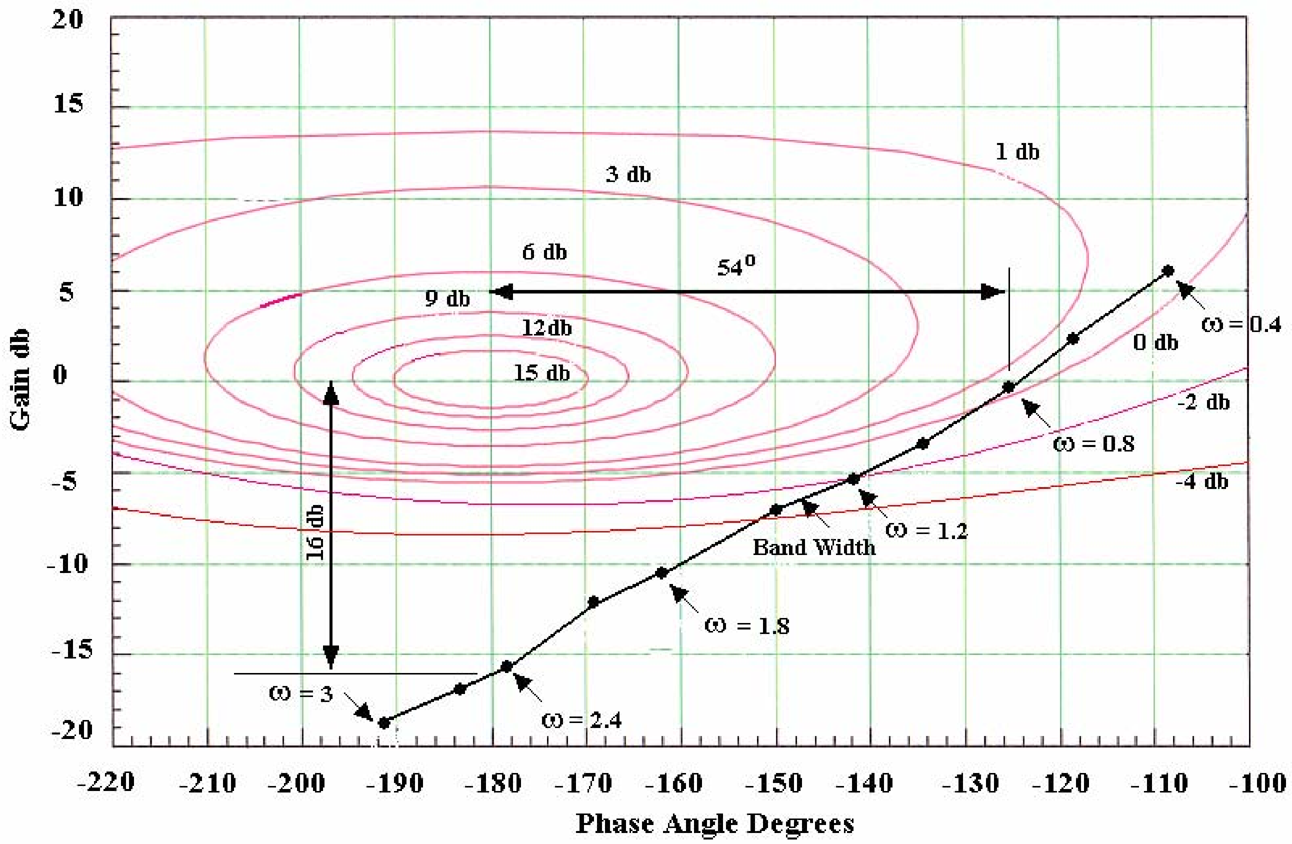


Figure 6

The phase margin is 54° and the gain margin is 16 db. Peak magnification occurs at around $\omega = 0.8$ and has a value of about 0 db. The bandwidth is the frequency where the plot crosses the -3 db contour and is about 1.3 rad/s.

**SELF ASSESSMENT EXERCISE No.1**

1. Plot the open loop response on the Nichols chart for a system with unity feedback and an open loop transfer function $G(s) = \frac{0.5}{(s+0.5)(s+0.2)}$

   Determine the following.
   The phase margin (58°)
   The gain margin (The plot reaches -180° at infinite frequency)
   The peak magnification and the frequency at which it occurs (0 db and 0.6 rad/s)
   The band width (about 1 rad/s)

2. Plot the open loop response on the Nichols chart for a system with unity feedback and an open loop transfer function $G(s) = \frac{5}{(s + 0.5)(s + 1)(s + 1.5)}$

   Determine the following.
   The phase margin (17°)
   The gain margin (-4 db)
   The peak magnification and the frequency at which it occurs (12 db and 1.4 rad/s)
   The band width (about 2 rad/s)