1. What is the variance of the random variable $y \in \mathbb{R}^n$ if $y = Ax$ where $A \in \mathbb{R}^{n \times m}$ is fixed and $x \in \mathbb{R}^m$ has variance $\Sigma_x$?
   (a) $A^\top \Sigma_x A$  (b) $A \Sigma_x A^\top$  (c) $(A \Sigma_x^{-1} A^\top)^{-1}$  (d) $A \Sigma_x^{-1} A^\top$

2. What is the variance of $z = x + y$ if random variables $y, x \in \mathbb{R}^n$ are independent and have variances $\Sigma_x, \Sigma_y$?
   (a) $\Sigma_x + \Sigma_y$  (b) $(\Sigma_x^{-1} + \Sigma_y^{-1})^{-1}$  (c) $\Sigma_x^{1/2} \Sigma_y^{1/2}$  (d) $\Sigma_x^{-1} + \Sigma_y^{-1}$

3. What is the variance of $z = 2x + y$ if random variables $y, x \in \mathbb{R}^n$ are independent and have variances $\Sigma_x, \Sigma_y$?
   (a) $\Sigma_x + \frac{1}{2} \Sigma_y$  (b) $(2 \Sigma_x^{-1} + \Sigma_y^{-1})^{-1}$  (c) $2 \Sigma_x + \Sigma_y$  (d) $4 \Sigma_x + \Sigma_y$

4. Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is not linear or affine?
   (a) $t^2 \dot{y}(t) = u(t)$  (b) $\dot{y}(t)^2 = u(t)$  (c) $\dot{y}(t) = u(t) + \sin(t)$  (d) $\ddot{y}(t) = t^2 u(t)$

5. Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is not time varying?
   (a) $t^2 \dot{y}(t) = u(t)$  (b) $\dot{y}(t)^2 = t^2 u(t)$  (c) $\dot{y}(t) = u(t) + \sin(t)$  (d) $\ddot{y}(t) = u(t)^2$

6. Which of the following dynamic models with inputs $u(t)$ and outputs $y(t)$ is a linear time invariant (LTI) system?
   (a) $\ddot{y}(t) = u(t) + 2$  (b) $\dot{y}(t)^2 = u(t)$  (c) $\dot{y}(t) = u(t) + \sin(t)$  (d) $\dot{y}(t) = 2u(t)$

7. Which of the following models with input $u(k)$ and output $y(k)$ is not linear-in-the-parameters w.r.t. $\theta \in \mathbb{R}^2$?
   (a) $y(k) = \theta_1 y(k-1)^2 + \theta_2 u(k-1)$  (b) $y(k) = \theta_1 (y(k-1) + u(k-1)) + \theta_2 u(k-2)$
   (c) $\tilde{y}(k) = \theta_1 (y(k-1) + \theta_2 u(k-1))$  (d) $y(k) = \theta_1 \sin(y(k-1)) + \theta_2 u(k-1)$

8. Which transfer function $G(s)$ describes the system $\dot{x}(t) = -3x(t) + u(t)$, $y(t) = x(t) + 2u(t)$?
   (a) $\frac{s^2 + 6}{s + 3}$  (b) $\frac{3s + 6}{s + 2}$  (c) $\frac{s^2 - 3}{s + 2}$  (d) $\frac{2s + 7}{s + 3}$

9. Which system is described by the transfer function $G(s) = \frac{s^2 + 2s}{s + 1}$?
   (a) $3\dot{y} + 2\ddot{y} = \ddot{u} + u$  (b) $3\dot{y} + 2y = \ddot{u} + u$  (c) $\dot{y} + y = 3\dot{u} + 2u$  (d) $\dot{y} + y = 3\dot{u} + 2u$

10. What solution $y(t)$ has the system $\dot{y}(t) = \frac{u(t)-y(t)}{\tau}$ with initial value $y(0) = 0$ for constant input $u(t) = 1$?
    (a) $y(t) = 1 - e^{-t/\tau}$  (b) $y(t) = 1 + e^{-t/\tau}$  (c) $y(t) = 1 - e^{-t \tau}$  (d) $y(t) = 1 + e^{-t \tau}$

Please fill in your name above and tick exactly one box for the right answer of each question below.
11. Modelling a warm water boiler: We regard a boiler with constant overall heat capacity \( C_0 \) [J/K] and water temperature \( T(t) \) [K]. Heat losses to the outside result in an energy flow \( Q_0(T(t) - T_0) \) (where \( Q_0 \) [J/s/K] is a constant and \( T_0 \) [K] the outside temperature). We can control the water temperature with an electrical coil, providing heating power \( Q(t) \) [J/s]. Regard \( Q(t) \) as input and \( T(t) \) as output. Which differential equation models this system?

\[
\sum (d) x A \dot{x} - \sum x x_1 x x_2 \log \dot{A} x (b) \theta A^\theta A (b) A \sum \dot{x} (c) (b) (d) A (d) A
\]

12. Regard an oscillator described by the state space equations \( \dot{v}(t) = k[u(t) - p(t)] \) and \( \dot{p}(t) = v(t) \), with state \( x = [p, v]^T \). We want to summarize the equations as \( \dot{x} = Ax + Bu \). What is the matrix \( A \)?

\[
(a) \begin{bmatrix} 0 & k \\ -1 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 \\ 0 & -k \end{bmatrix} \quad (d) \begin{bmatrix} 0 & 1 \\ -k & 0 \end{bmatrix}
\]

13. A carousel is described by the equation \( \dot{\phi}(t) = T(t)/I \), where \( \phi \) is the rotation angle, \( T \) the torque, and \( I \) the constant moment of inertia. Regard \( T \) as input and \( \phi \) as output. In a state space model, what is the state \( x(t) \) of this system?

\[
(a) \begin{bmatrix} \phi \end{bmatrix} \quad (b) \begin{bmatrix} \phi \\ I \end{bmatrix} \quad (c) \begin{bmatrix} \phi, T \end{bmatrix}^T \quad (d) \begin{bmatrix} \phi, \phi \end{bmatrix}^T
\]

14. The temperature \( x(t) \) of a soldering iron is described by \( \dot{x}(t) = K_1 u(t)^2 - K_2 x(t) - K_3 \), where \( u(t) \) is the input voltage and \( K_1, K_2, K_3 \) are constants. If we keep the voltage constant at \( u_{ss} \) for a long time, a constant temperature \( x_{ss} \) is reached. Which?

\[
(a) x_{ss} = \frac{K_1 u_{ss}^2 - K_2}{K_3} \quad (b) x_{ss} = \frac{K_1 u_{ss}^2 - K_2}{K_3} \quad (c) x_{ss} = \frac{K_2 x_{ss} - K_3}{K_1} \quad (d) x_{ss} = \frac{K_1 u_{ss}^2 - K_3}{K_2}
\]

15. We want to linearize the model \( \dot{x}(t) = K_1 u(t)^2 - K_2 x(t) - K_3 \) in an equilibrium point \( (x_{ss}, u_{ss}) \) to obtain the linear dynamics around the steady state, i.e. for \( x(t) = x_{ss} + \Delta x(t) \) and \( u(t) = u_{ss} + \Delta u(t) \) with small \( \Delta x \) and \( \Delta u \). For this aim define the matrices \( A \in \mathbb{R}^{1 \times 1} \) and \( B \in \mathbb{R}^{1 \times 1} \) in \( \Delta \dot{x}(t) = A \Delta x(t) + B \Delta u(t) \).

\[
(a) A = [K_2 x_{ss}], B = [K_2] \quad (b) A = [-K_3], B = [2K_1 u_{ss}]
\]

16. Maximum Likelihood Estimator (MLE): Assume a nominal model \( h_i(\theta) \) and given measurements \( y_i, i = 1, \ldots, N \). The PDF to obtain a measurement value \( y_i \) for a parameter value \( \theta \) is known to be \( \exp\left(-\frac{1}{2\sigma_i^2}(y_i - h_i(\theta))^2\right) \) and measurement noises are uncorrelated. What function of \( \theta \) does the MLE minimize in this case?

\[
(a) \sum_{i=1}^N \frac{1}{\sigma_i^2} (y_i - h_i(\theta))^2 \quad (b) \frac{1}{2} \| h(\theta) - y \|_2^2
\]

17. Maximum Likelihood Estimator (MLE): Assume a nominal model \( h_i(\theta) \) and given measurements \( y_i, i = 1, \ldots, N \). The PDF to obtain a measurement value \( y_i \) for a parameter value \( \theta \) is known to be \( \exp\left(-\|y_i - h_i(\theta)\|\right) \) and measurement noises are uncorrelated. What function of \( \theta \) does the MLE minimize in this case?

\[
(a) \sum_{i=1}^N (y_i - h_i(\theta))^2 \quad (b) \frac{1}{2} \| h(\theta) - y \|_2^2
\]

18. Gauss-Newton: We want to solve the nonlinear least squares problem \( \min_\theta \frac{1}{2} \| h(\theta) - y \|_2^2 \), and have a current parameter guess \( \theta^{[k]} \). What formula delivers the next parameter guess in the Gauss-Newton method?

\[
(a) \theta^{[k+1]} = \theta^{[k]} - \left( \frac{\partial h}{\partial \theta} (\theta^{[k]}) \right)^\top (h(\theta) - y) \quad (b) \theta^{[k+1]} = \theta^{[k]} - \left( \frac{\partial h}{\partial \theta} (\theta^{[k]}) \right)^{-1} (h(\theta) - y)
\]

19. Bayesian estimation: we have a priori information about a parameter in form of a PDF \( g(\theta) \) and know that the PDF to obtain measurements \( y \) given \( \theta \) is given by \( f(y, \theta) \). What function is minimized by a Bayesian estimator in this context?

\[
(a) \log g(\theta) - \log f(y, \theta) \quad (b) - \log g(\theta) - \log f(y, \theta)
\]

\[
(c) g(\theta) + f(y, \theta) \quad (d) - g(\theta) + f(y, \theta)
\]