Chapter 13: The Kalman Filter

So far, \( \Theta \) did not change \([\Theta_{n+1} = \Theta_n]\).

Now, we are interested in state estimation for LTI systems \( x(n) \) with known dynamics:

\[
\begin{align*}
x(n+1) &= A \cdot x(n) + B \cdot u(n) + v(n) \\
y(n) &= C \cdot x(n) + n(n)
\end{align*}
\]

Given \( y(1), \ldots, y(n) \), what is best estimate for \( x(1) \)?
ASSUMPTIONS:

1) prior for $X(t) \sim \mathcal{N}(X(0), P(0))$

$$\text{PDF} \approx \exp \left( - (x - X(0))^T P(0)^{-1} (x - X(0)) \right)$$

2) $V(u)$ i.i.d. $V(u) \sim \mathcal{N}(0, R_v)$

3) $W(u)$ i.i.d. $W(u) \sim \mathcal{N}(0, R_w)$

$[P(0), R_v, R_w]$ positive definite matrices

Also $A, B, C$ are assumed known.
RELATION TO PREVIOUS CHAPTER

\[ \Theta(n) \equiv x(n) \]  \quad A = I, \quad B = \text{empty} \\
\text{DYNAMICS ARE TRIVIAL} \\
\[ \Theta(n+1) = \Theta(n) \]  \quad x(n+1) = A \cdot x(n) \\
\[ y(n) = C \cdot \Theta(n) + u(n) \]  \quad y(n) = C \cdot x(n) \\
So \text{ previous ch. was special case!}
IDEA: APPLY RECURSIVE BAYESIAN ESTIMATION PHILOSOPHY

\[ x_{(k)} \sim N(x_{(k+1)}, P_{(k+1)}) \]

\[ x_{(k+1)} \sim N(x_{-(k+1)}, Q_{(k+1)}) \]

BEFORE MEASUREMENT

Step 1

STEP 2

AFTER MEASUREMENT
Step 1: \[ X_{-(h+1)} = 1E (A \cdot x(u) + B \cdot u(u) + v(u)) \]
\[ = A \cdot \bar{X}(u) + B \cdot u(u) + \bar{v} \]
\[ Q(u+1) = A \cdot P(u) A^T + R_v \]

Step 2: Given \( X_{-(h+1)} \), \( Q(u+1) \)
And measurement \( y(u+1) \) with covariance \( R_n \)
\[ P_{u+1}^{-1} = Q_{u+1}^{-1} + C^T \cdot R_n^{-1} \cdot C \]
\[ X(u+1) = X_{-(h+1)} + P(u+1) \cdot (y(u+1) - \cdot C \cdot X_{u+1}) \]
\begin{align*}
\text{Step 1:} & \quad Q(h+1) = A P(h) A^T + R_v \\
& \quad K(h+1) = Q(h+1) C^T (C Q(h+1) C^T + R_v)^{-1} \\
\text{Step 2:} & \quad P(h+1) = (I - K(h+1) C) Q(h+1) \\
& \quad X(h+1) = AX(h) + Bu(h) \\
& \quad \underbrace{+ K(h+1)(Y(h+1) - C[X(h) + Bu(h)])}
\end{align*}