Exercise 7

The aim of this exercise is to identify the dynamics of the heating system inside the building depicted below:

![Diagram of the heating system with rooms, heater, and flow rates]

The building is composed by two rooms and a heater is installed only in Room 1 ($q_h$ represents the power released by the heater to Room 1). In Room 2 there is an unknown flow $q_d$ exchanging heat with the external ambient. We are interested in characterizing the temperature in Room 2, which we denote as $T_2$.

We make the following assumptions:

- the external ambient temperature is equal to zero during the experiments ($T_a = 0$);
- $q_d$ is modeled as a i.i.d. white Gaussian noise with zero mean.

The room temperature dynamics are characterized by the following equations

\[
\begin{align*}
C_1 \frac{dT_1(t)}{dt} &= q_h(t) - q_{12}(t) - q_{1a}(t) \\
C_2 \frac{dT_2(t)}{dt} &= q_d(t) + q_{12}(t) - q_{2a}(t)
\end{align*}
\]

\( q_{2a}(t) = \frac{T_2(t) - T_a(t)}{R_{2a}} \quad q_{1a}(t) = \frac{T_1(t) - T_a(t)}{R_{1a}} \)

\( q_{12}(t) = \frac{T_1(t) - T_2(t)}{R_{12}} \)

where

- $q_h(t)$ is the heat flow from the heater to Room 1;
- $q_{12}(t)$ is the heat flow from Room 1 to Room 2;
- $q_{1a}(t)$ is the heat flow from Room 1 to the ambient;
- $q_d(t)$ is the disturbance heat flow from Room 2 to the ambient;
- $q_{2a}(t)$ is the heat flow from Room 2 to the ambient through the wall;
- $C_1$ is the thermal capacity of Room 1;
- $C_2$ is the thermal capacity of Room 2;
- $R_{12}$ is the thermal resistance between Room 1 and Room 2;
- $R_{1a}$ is the thermal resistance between Room 1 and the ambient;
- $R_{2a}$ is the thermal resistance between Room 2 and the ambient.
Model the system

1. Write the system dynamics in the state-space form

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &=Cx(t) + Du(t)
\end{align*}
\]

where

\[
x(t) = [T_1(t)\ T_2(t)]^T, \quad u(t) = [q_h(t)\ q_d(t)]^T, \quad \text{and} \quad y(t) = T_2(t)
\]

(here we treat \(q_d(t)\) as an input).

Hint: Use the equations (2) to substitute in (1) the heat flows which are not considered as inputs.

2. Construct the matrices \(A, B, C\) and \(D\) in Matlab using the following values for the constants

- \(R_{1a} = 5 \times 10^{-3} \text{[K/W]}\);
- \(R_{2a} = 10 \times 10^{-3} \text{[K/W]}\);
- \(R_{12} = 10 \times 10^{-3} \text{[K/W]}\);
- \(C_1 = 1 \times 10^6 \text{[J/K]}\);
- \(C_2 = 7 \times 10^5 \text{[J/K]}\).

With the matrices created define the system \(\text{sys}\) using the command \(\text{ss}\).

Simulate the system

3. Generate the input signals to simulate an estimation experiment. Set the sampling time \(t_s\) to 60 seconds. The experiment should last 6 days (therefore there should be 8640 samples).

- \(q_h(t)\) should be periodic with a period \(N=2880\) samples (which corresponds to 2 days). The signal should be a multisine with constant amplitude and random phase. Scale the signal such that its value is contained in the interval \([0, 6000]\). To define the periodic part of \(q_h(t)\) you can use the following code:

```matlab
% the DFT of q_h is initialized such that it contains N zeros
Q_h = zeros(N, 1);
% the amplitude of the frequency components are set to 1 and the phases
% are set to a random value
Q_h(2:N/2) = exp(i*2*pi*rand(N/2-1,1));
% Q_h is made conjugate symmetric
Q_h(end:-1:N/2+2) = conj(Q_h(2:N/2));
% q_h is defined using the inverse FFT
q_h = ifft(Q_h);
% q_h is rescaled in the interval [0, 6000]
q_h = 3000*(1+q_h/max(abs(q_h)));
% Q_h is updated
Q_h = fft(q_h);
```

- \(q_d(t)\) should be white Gaussian noise (use the function \(\text{wgn}\) and set the power to 25).

Simulate the system using the command \(\text{lsim}\).

4. Verify that in the last two days of the experiment the effects of the transient response are negligible. To do this you can consider the following signal

\[
y_{\text{diff}} = \text{abs}(y(1:(n\_periods-1)*N)-y(N+1:end))
\]

where \(y\) is the output signal resulting from the simulation and \(n\_periods=3\) is the number of repetitions of the input signal. If there was no disturbance, after the transient effects have disappeared, the output signal should be periodic of period \(N\). Plot \(y_{\text{diff}}\) in logarithmic scale and verify that after \(N\) samples the signal is small enough.
Identify the transfer function

5. Calculate the Estimated Transfer Function using the output data corresponding to the last two days. In this task you should consider only the input \( q_h(t) \). To calculate the ETF compute the DFT of the last part of the signal

\[
Y = \text{fft}(y((n\_periods-1)*N+1:end));
\]

and use only the significant part of \( Y \) (which is \( Y(2:N/2-1) \)).

6. Compare the Bode plots obtained from the ETF from the system used for simulating the data (using \( \text{sys}(1) \) instead of \( \text{sys} \) will neglect the input used to simulate the disturbance \( q_d(t) \)).

Hint: follow these steps

- define a vector \( \omega \) containing the frequencies (in [rad/s]) corresponding to the ETF points;
- use the command \([\text{mag}, \text{phase}] = \text{bode}(\text{sys}(1), \omega)\); to generate the transfer function data for \( \text{sys}(1) \);
- using the commands \text{semilogx} and \text{subplot} generate the Bode plot corresponding to the ETF and \( \text{sys}(1) \) on the same figure.

7. Repeat the previous four tasks but using a multisine where the amplitudes in the frequency interval \([800 \cdot t_s \cdot N, 1200 \cdot t_s \cdot N]\) are 20 times bigger than the amplitudes outside this interval. What can you observe from the plots?